Lawrence Berkeley National Laboratory

Recent Work

Title
CHAOTIC BEHAVIOR OF A DRIVEN P-N JUNCTION

Permalink
https://escholarship.org/uc/item/82k4t5pm

Author
Perez, J.M.

Publication Date
1983-11-01
CHAOTIC BEHAVIOR OF A DRIVEN P–N JUNCTION

J.M. Perez
(Ph.D. Thesis)

November 1983

TWO-WEEK LOAN COPY
This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 6782.
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
CHAOTIC BEHAVIOR OF A DRIVEN P-N JUNCTION

Jose Maria Perez

Materials and Molecular Research Division
Lawrence Berkeley Laboratory

and

Department of Physics
University of California
Berkeley, California 94720

ABSTRACT

The chaotic behavior of a driven p-n junction is experimentally examined. Bifurcation diagrams for the system are measured, showing period doubling bifurcations up to $f/32$, onset of chaos, reverse bifurcations of chaotic bands, and periodic windows. Some of the measured bifurcation diagrams are similar to the bifurcation diagram of the logistic map $x_{n+1} = \lambda x_n (1 - x_n)$. A return map is also measured showing approximately a one-dimensional map with a single extremum at low driving voltages.

The intermittency route to chaos is experimentally observed to occur near a tangent bifurcation as the system approaches a period 5 window at $\lambda = \lambda_5$. Data are presented for the dependence of the average laminar length $\langle \lambda \rangle$ on $\epsilon = \lambda_5 - \lambda$, and for the probability distribution $P(\lambda)$ vs. $\lambda$.

The effects of additive stochastic noise on period doubling, chaos, windows, and intermittency are examined and are found to agree with the logistic
model and universal predictions. Three examples of crisis of the attractor are observed. The crises occur when an unstable orbit intersects the chaotic attractor.

A period adding sequence is reported in which wide periodic windows of period 2, 3, 4, ... are observed for increasing driving voltage. The initial period doubling cascade and the period adding sequence are compared to two theoretical models, with reasonable success.
This thesis is dedicated

to my parents.
ACKNOWLEDGMENTS

I thank Professor Carson Jeffries for suggesting the topic of this thesis and for his continued advice and encouragement. I also thank members of our group for assistance and useful discussions; specifically, Glenn Held for his work on the SPICE program and suggestions regarding the period adding sequence and James Testa for his work on the period doubling route. Paul Bryant, James Crutchfield, James Culbertson, Elga Pakulis, Robert Van Buskirk, and other members of our group contributed to discussions of the theory and experimental results. James Crutchfield critically read the text. Sandy Ewing accurately typed the manuscript correcting numerous errors.
CHAOTIC BEHAVIOR OF A DRIVEN P-N JUNCTION

Table of Contents

CHAPTER I INTRODUCTION .................................................. 1
  1.1 The Physical System .................................................. 2
  1.2 Analog Computers .................................................. 5
  1.3 Qualitative Behavior ................................................ 5

CHAPTER II PERIOD DOUBLING ROUTE TO CHAOS IN THE DRIVEN
  P-N JUNCTION ............................................................ 7
  2.1 Introduction ......................................................... 7
  2.2 Experimental Bifurcation Diagrams and Return Maps ............ 8
  2.3 Window Sequences and Patterns .................................... 11
  2.4 Power Spectral Density ............................................ 11
  2.5 Conclusions .......................................................... 12

CHAPTER III INTERMITTENCY ROUTE TO CHAOS IN THE DRIVEN
  P-N JUNCTION ............................................................ 13
  3.1 Introduction .......................................................... 13
  3.2 Experimental Observation of Intermittency and Tangent Bifurcation ........................................... 15
  3.3 Quantitative Analysis of $\lambda$ and $P(\lambda)$ ............... 17
  3.4 Conclusions .......................................................... 20

CHAPTER IV EFFECTS OF ADDITIVE NOISE ............................. 21
  4.1 Introduction .......................................................... 21
  4.2 Experimental Effects of Additive Noise .......................... 23
  4.3 Effects of Adding a Sinusoidal Voltage ......................... 25
  4.4 Conclusions .......................................................... 25
<table>
<thead>
<tr>
<th>CHAPTER V</th>
<th>CRISIS OF THE ATTRACTOR IN THE DRIVEN P-N JUNCTION</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Introduction</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>5.2 Crisis Following the Period 3 Window</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>5.3 Hysteresis Crisis of the Period 3 Window</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>5.4 Crisis at the $4 \rightarrow 2$ Band Merging</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>CHAPTER VI</td>
<td>PERIOD ADDING SEQUENCE IN THE DRIVEN P-N JUNCTION</td>
<td>29</td>
</tr>
<tr>
<td>6.1 Experimental Observations</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>CHAPTER VII</td>
<td>MODELS OF THE DRIVEN P-N JUNCTION</td>
<td>31</td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>7.2 One-Dimensional Iterative Map</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>7.3 Three Coupled Ordinary Differential Equations</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>CHAPTER VIII</td>
<td>SUMMARY AND CONCLUSIONS</td>
<td>34</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>36</td>
</tr>
<tr>
<td>FIGURE CAPTIONS</td>
<td></td>
<td>43</td>
</tr>
<tr>
<td>FIGURES</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td></td>
<td>106</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td></td>
<td>108</td>
</tr>
<tr>
<td>APPENDIX C</td>
<td></td>
<td>111</td>
</tr>
<tr>
<td>APPENDIX D</td>
<td></td>
<td>114</td>
</tr>
</tbody>
</table>
CHAPTER I. INTRODUCTION

Recently, progress has been made in understanding routes to chaos by studying simple models such as nonlinear ordinary differential equations\(^1\) or one-dimensional iterative maps\(^2-4\) which display surprisingly varied periodic and chaotic behavior. For models which follow the period doubling route to chaos, quantitative universal features have been discovered\(^5,6\) which are believed to be displayed by real physical systems. Other routes to chaos have been found to proceed in a characteristic way\(^7\) which may also be observed experimentally. The objective of this thesis is to test some of the theoretical results by examining in detail the transition to chaos in a physical system, a semiconducting p-n junction.

In the next section we describe the system and the physical processes which determine its behavior. We show what happens as the system is driven harder: a cascade of period doubling bifurcations occurs followed by chaos and periodic windows in agreement with the logistic map; at higher driving values, a period adding sequence of wide windows is observed. In Chapter II, we examine the initial period doubling and the periodic windows. Experimentally determined bifurcation diagrams, return maps, probability distributions, and frequency spectra are compared with theoretical predictions. In Chapter III, the intermittency route to chaos\(^8,9\) is examined. In Chapter IV, the effects of additive noise on the period doubling cascade, on windows, and on intermittency are studied. Chapter V presents some experimental examples of crises of the attractor as conjectured by Grebogi et al.\(^10\) In
Chapter VI, the period adding sequence is discussed. Finally, in Chapter VII, two different models are presented: a set of three coupled ordinary differential equations, and a one-dimensional iterative map.

1.1 The Physical System

The physical system, Fig. 1.1, is a nonlinear oscillator consisting of a series connected resistor, inductor, and p-n junction which is driven sinusoidally by a voltage \( V_0 \sin \omega t \) where \( V_0 \approx 0-10 \) volts and \( \omega = 100 \) kHz. Typical components used are a 100\( \Omega \) resistor, a 10 mH inductor, and a type 1N4004 Si rectifier diode, or a type 1N953 Si varactor diode. Varactor diodes have a greater junction capacitance during reverse bias than rectifier diodes as explained below. The dynamical variables chosen to describe the system are the series current \( I \), the voltage across the p-n junction \( V \), and the phase of the sinusoidal driver, \( \Omega = \omega t \). The behavior of the circuit is very sensitive to stray capacitance and resistive loading so buffers are used to measure the dynamical variables as shown in Fig. 1.1. A detailed schematic appears in Appendix A.

When the circuit is in operation, the resistor and inductor act as linear elements. The only nonlinear component is the p-n junction, which has three characteristic nonlinearities:

1) When the junction is reverse-biased, the depletion of charge near the junction barrier results in a capacitance known as the junction capacitance

\[
C_j = C_1/(1 - V/\phi)^n
\]  

(1.1)
where \( V \) is the voltage across the p-n junction. The value of \( n \) depends on the doping profile near the junction barrier.\(^{11}\) For diffused type junctions with a linearly graded profile, \( n = 1/3 \). For an abrupt junction, \( n = 1/2 \), and for a hyperabrupt junction, \( n = 2 \). Rectifier diodes can be of the diffused type. Varactor diodes, which are specially made to be used as voltage-controlled capacitors, have either abrupt or hyperabrupt doping profiles. For varactor diodes such as the \( \text{1N953} \), \( C_1 \approx 300 \text{ pf} \) and \( \phi \approx 0.6 \text{ volts} \). For rectifier diodes such as the \( \text{1N4004} \), \( C_1 \approx 40 \text{ pf} \) and \( \phi \approx 0.6 \text{ volts} \).

2) When the junction is forward biased, minority charge injection results in a charge storage capacitance\(^{11}\) which may be approximately calculated as follows. The minority charge distribution per unit volume is assumed to have the form\(^{11}\)

\[
p(x,t) = p_0(e^{qV/kT} - 1)e^{-x/\lambda}
\]

(1.2)

where \( V \) is the voltage across the p-n junction, \( \lambda \) is the diffusion length, and \( x \) is the distance measured from the junction barrier. In principle, \( p(x,t) \) is the solution of the time dependent diffusion equation

\[
\frac{\partial p(x,t)}{\partial t} = D \frac{\partial^2 p(x,t)}{\partial x^2} - \frac{p(x,t)}{\tau}
\]

(1.3)

where \( \tau \) is the carrier lifetime and \( D \) is the diffusion constant. The total minority charge can be written

\[
Q_s = qA \int_0^\infty p(x,t)dx
\]

(1.4)

where \( A \) is the cross-sectional area. The effective capacitance is then
where \( C_0 \) is a constant.

The magnitude of the charge storage capacitance can be estimated from the switching transients in the circuit shown in Fig. 1.2. An ideal diode rectifies instantaneously when the driving voltage swings negatively as shown in Fig. 1.3(b). A diode which stores minority charge during forward bias must discharge by conducting current in the reverse direction before it can rectify. This behavior is shown in Fig. 1.3(c). The time during which the diode conducts in the reverse direction is called the reverse recovery time, \( \tau_R \). Varactor diodes usually have \( \tau_R \sim 1 \mu s \). Ordinary rectifier diodes have \( \tau_R \sim 1-2 \mu s \), and fast signal diodes usually have \( \tau_R \sim 1 \text{ ns} \).

The switching transients introduced by the charge storage capacitance are necessary for chaotic behavior to arise in the driven p-n junction. For example, fast signal diodes with small reverse recovery times do not bifurcate. Two diodes in series back to back do not bifurcate, and diodes always reverse biased do not bifurcate. Diodes which bifurcate when driven at 100 kHz do not bifurcate when driven at 10 kHz. At 10 kHz the ratio of the reverse recovery time to the driving period is ten times smaller than the ratio at 100 kHz. Therefore the nonlinearity introduced by the switching transients appears over a smaller fraction of the period when the system is driven at 10 kHz.

3) Another nonlinearity present in the p-n junction is the built-in potential barrier which results in the Shockley equation

\[
I_d(V) = I_0(e^{qV/kT} - 1)
\]  

(1.6)
In Chapter VII we derive a set of three coupled differential equations based on the nonlinearities given by Eqs. (1.1), (1.5), and (1.6).

1.2 Analog Computers

In this section we explain the distinction between the driven p-n junction and an analog computer. Analog computers have been used to study nonlinear differential equations such as the van der Pol equations and the Lorenz equations. They typically consist of building blocks such as integrators, multipliers, and adders. The driven p-n junction is not an analog computer, although at first glance it may appear to be. It is a physical system whose behavior is approximately given by Eqs. (1.1), (1.5), and (1.6); differential equations derived from these equations are approximate models for the behavior of the system. Thus analog computers are electrical models of differential equations, whereas our nonlinear oscillator is a physical system whose behavior is approximately modelled by differential equations. One may argue that since the oscillator is described by a set of differential equations it is an analog computer for that set of equations. This argument is incorrect for the following reasons: 1) It would imply that all physical systems are analog computers; 2) It is impossible to examine the set of equations in all respects to see if it is in agreement in every way with the physical system.

1.3 Qualitative Behavior

In this section we describe the qualitative behavior of the current signal $I(t)$ as the amplitude of the driving voltage is increased.
The behavior is analyzed in quantitative detail in later chapters. Figure 1.4(b) shows the current $I(t)$ in the oscillator circuit with diode 1N953 at $V_0 = 1.5$ volts. The current is periodic with period $T = 2\pi/\omega$ equal to that of the driving signal. As the amplitude of the driving voltage is increased to $V_o = 2.5$ volts, the period of the current signal doubles as shown in Fig. 1.4(c). As the driving voltage is further increased, the period is observed to double a total of five times before the signal appears chaotic at $V_0 = 3.5$ volts as shown in Fig. 1.4(e). At higher driving voltages periodic windows appear such as the period 3 window at $V_0 = 4.1$ volts shown in Fig. 1.4(f). Further increases in driving voltage result in a sequence of wide periodic windows of period 2, 3, 4, 5..., i.e., a period adding sequence. For example, in diode 1N4004, the period 2 window appears at $V_o = 1.2$ volts, the period 3 at $V_o = 3.4$ volts, the period 4 at $V_o = 5.8$ volts, and the period 5 at $V_o = 8.1$ volts. Figure 1.5 shows the current signal at the period 2, 3, and 4 windows. The current signal and voltage across the p-n junction at the period 5 window are shown in Fig. 6.2.
CHAPTER II. PERIOD DOUBLING ROUTE TO
CHAOS IN THE DRIVEN P-N JUNCTION

2.1 Introduction

One-dimensional iterative maps have been used to describe a wide variety of phenomena: nonoverlapping insect populations, the propagation of rumors, and the wearing of drill bits are three examples.\textsuperscript{1,2} Their interest in physics is their use as models for routes to chaos. For example, the behavior of the logistic equation

\[ x_{n+1} = \lambda x_n (1 - x_n) \]  

(2.1)

is summarized in Fig. 2.1 by a bifurcation diagram which is a plot of the iterates of Eq. (2.1), \( \{x_n\} \), vs. the control parameter \( \lambda \). As \( \lambda \) increases, \( \{x_n\} \) undergoes a series of period doubling pitchfork bifurcations.\textsuperscript{1} The value of \( \lambda \) at which the \( n^{th} \) bifurcation occurs, \( \lambda_n \), converges geometrically as \( \lambda_c - \lambda_n = \delta^{-n} \) to the onset of chaos as \( \lambda_c \) where \( \{x_n\} \) becomes chaotic.\textsuperscript{3} In the chaotic regime, \( \lambda > \lambda_c \), chaotic bands merge in a series of reverse bifurcations\textsuperscript{4} which mirror the period doubling bifurcations for \( \lambda < \lambda_c \), and there exist narrow periodic windows with a specific pattern.\textsuperscript{5} The logistic model is quantified by universal numbers as \( n \to \infty \) which apply to all one-dimensional maps with a quadratic extremum: the convergence rate

\[ \delta = \lim_{n \to \infty} \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+2} - \lambda_{n+1}} = 4.669 \ldots, \]

and the pitchfork scaling parameter \( \alpha = 2.502 \ldots \), first computed by Feigenbaum.\textsuperscript{3} The scaling parameter \( \alpha \) is the asymptotic ratio as \( n \to \infty \).
of the two distances shown in Fig. 2.1. The distances are measured at the superstable cycles which are cycles where the critical point 1/2 is one of the iterates. Other universal numbers characterize the spectral power density and effects of additive noise. The pattern of periodic cycles is a universal feature of maps with a single extremum. The pattern of a superstable cycle is defined in Ref. 5 as follows. Starting at the critical point 1/2, iterates less than 1/2 are denoted by L and those greater than 1/2 are denoted by R. For example, the period 5 cycle

has the pattern RLLR. Patterns for periodic cycles up to period 11 are given in Ref. 5. These patterns depend only on the map having a single extremum.

2.2 Experimental Bifurcation Diagrams and Return Maps

We make the following correspondences between the logistic map and experimental quantities:

\[ I_p(t) \leftrightarrow x_n \]
\[ I_p(t + mT) \leftrightarrow x_{n+m} \]
\[ V_o \leftrightarrow \lambda \]

where \( I_p(t) \) is a peak in the current signal \( I(t) \), and \( T \) is the period of the driving voltage. A bifurcation diagram is made on an oscilloscope by strobing the current signal at each peak and using the rms amplitude of the driving voltage as the horizontal deflection voltage.
As the amplitude of the driving voltage is increased, a bifurcation diagram is traced. A return map, $x_{n+1}$ vs. $x_n$, is made by holding the value of the current peak for one period on the vertical input while keeping the present value of the current on the horizontal input and strobing the current at every other peak, as shown in Fig. 2.2(a). Every other peak was strobed to avoid using additional circuit components such as switches which can add glitches to the signal. Strobing each peak has the advantage that every value is plotted, but both methods give the same return map when the system is at a one-band attractor. Both of these experimental methods are outlined in Fig. 2.2(b). A detailed schematic appears in Appendix B. The $n^{th}$ iterate of the return map is made the same way as the return map except that the current peak is held for $n$ periods, and the current is strobed every $2n^{th}$ peak. These methods rely on the current signal staying in phase with the driving voltage from where the strobe pulse is derived. The voltage across the p-n junction, $V$, is phase shifted by an amount depending on the magnitude of $V$ so it is not possible to make a bifurcation diagram of the voltage signal using the method described for the current signal. A bifurcation diagram of the voltage signal can be made indirectly by scanning $V$ with a window comparator and strobing whenever a peak is detected. The scanning frequency can be made much smaller than the oscillator driving frequency which is sometimes advantageous to do because it has the effect of bringing out faint details.
A bifurcation diagram made from the current signal at small driving voltages by the method described earlier is shown in Fig. 2.3 for diode 1N4004. It is similar to the diagram computed from the logistic map, Fig. 2.1, including bifurcation thresholds, onset of chaos, band mergings, noise-free windows, and the veiled structure corresponding to regions of high probability. Figure 2.4(a) shows a bifurcation diagram of the current signal for varactor diode 1N953. From the first four threshold voltages, $V_n$, the convergence rate is calculated in Ref. 10 to be

$$\delta_1 = \frac{V_2 - V_1}{V_3 - V_2} = 4.3 \pm 0.1$$

$$\delta_2 = \frac{V_3 - V_2}{V_4 - V_3} = 4.3 \pm 0.1$$

The predicted values from the logistic map are $\delta_1 = 4.751$, and $\delta_2 = 4.656$. The discrepancy between measured and predicted values is due to small $n$. Figure 2.4(b) shows an expanded view of the period 3 window in Fig. 2.4(a). In Ref. 10 the pitchfork scaling ratio is measured to be $\alpha = 2.41 \pm 0.1$, to be compared with the predicted value as $n \to \infty \alpha = 2.502$.

Figure 2.5(a) shows the return map made at small driving voltages for diode 1N4004 corresponding to the bifurcation diagram of Fig. 2.3. It has a single extremum and folds over at one end as observed in the Henon map at high dissipation. Figure 2.5(b) shows the return map for diode 1N953 corresponding to the bifurcation diagram of Fig. 2.4(a).

Figure 2.6 shows a bifurcation diagram of the voltage for diode 1N953 made with a scanning window comparator. Only the upper half of
the bifurcation diagram appears for the voltage because during the conducting half cycle of the diode the voltage is small and is compressed toward the zero line in Fig. 2.6.

2.3 Window Sequences and Patterns

The bifurcation diagram of Fig. 2.3 shows three noise-free windows of periods 6, 5, and 3. Windows of higher periodicity are narrower and more difficult to observe. Figure 2.7 shows the voltage \( V \) across the p-n junction and the pattern of visitation of the period 6 and 5 windows, in agreement with the theory of Metropolis, Stein and Stein\(^5\) (MSS). We make the following correspondences between experimental data and the notation of MSS: \( L \leftrightarrow \) the diode is forward biased (the voltage peaks are positive); \( R \leftrightarrow \) the diode is reverse biased (the voltage peaks are negative). The data for the period 12, 7, 3, and 9 windows also agree\(^\text{10} \) with the theory of MSS.

2.4 Power Spectral Density

Figure 2.8 shows a frequency spectrum from Ref. 10 of the current when the system has bifurcated to \( f/32 \). The spectrum analyzer used was a Hewlett-Packard 3580A which has a dynamic range of 85 db, a sensitivity of 300 nV, and a frequency range from 0 to 50 kHz, thus allowing observation of spectral components 95 db below \( V_0 \). It is predicted that the average heights of the peaks for a period is 13.21 db below the previous period\(^\text{12} \). The data are consistent with this; however, the region between \( f/2 \) and \( f \) is out of the range of the spectrum analyzer. The predicted\(^\text{13} \) values of the spectral components are shown in Fig. 2.8. The experimental values lie between 11 db and 15 db.\(^\text{10} \)
2.5 Conclusions

The similarity between the experimental bifurcation diagrams and return maps and the logistic model supports the use of one-dimensional quadratic maps as models of chaotic behavior. It is also clear, however, from the experimental return map, Figs. 2.5(a) and (b), that a higher dimensional map is necessary for a better understanding.
CHAPTER III. INTERMITTENCY ROUTE TO CHAOS
IN THE DRIVEN P-N JUNCTION

3.1 Introduction

Intermittency is characterized by periodic behavior interrupted by aperiodic behavior. As the periodic regions become smaller, the system becomes more chaotic. According to Manneville and Pomeau, intermittency arises when a tangent bifurcation occurs.\(^1,2\) This can be illustrated by considering the logistic equation

\[ f(x,\lambda) = \lambda x (1 - x) . \]  

Figure 3.1 is a plot of the fifth iterate \( f^5(x) \) vs. \( x \) at \( \lambda_5 = 3.73775 \) where \( f^5(x) \) just becomes tangent to the 45° line, giving rise to five fixed points and a period 5 window. For a small positive value of \( \varepsilon = \lambda_5 - \lambda \) the five points which were tangent at \( \lambda = \lambda_c \) are slightly above or below the 45° line. Figure 3.2 is an expansion of \( f^5(x,\lambda) \) when \( \varepsilon > 0 \) near one of the tangent points, showing a channel formed by the 45° line and one side of \( f^5(x) \) through which a trajectory traverses from A to B. The dots and arrows in Figure 3.2 represent a sequence of five-fold iterations through the channel during which the iterates are close to one another and the behavior is periodic. After the iterates exit at B, they move erratically producing aperiodic behavior, until they reenter one of the five channels at A. Hirsh, Huberman, and Scalapino\(^3\) (HHS) find that the average periodic length \( \langle \lambda \rangle \) decreases with \( \varepsilon \) with scaling behavior

\[ \langle \lambda \rangle \propto 1/\varepsilon^{1/2} \]  

(3.2)
for the logistic map, or more generally as $\langle \lambda \rangle \sim 1/\epsilon^{1-1/Z}$ for a map which expands about the point of tangency as $x + a|x|^Z$. They find that this scaling is universal, i.e., it is the same for all maps which have an expansion whose leading nonlinear term is of order $Z$.

Equation (3.1) is derived in Ref. 3 as follows. For $\epsilon = \lambda_5 - \lambda \approx 0$ one expands the fifth iterate about the tangency point $x_i$:

$$f^5(x,\lambda) = f^5(x_i,\lambda_5) + \left. \frac{df^5(x,\lambda_5)}{dx} \right|_{x=x_i} (x-x_i) + a(x-x_i)^2 + b(\lambda - \lambda_5)$$

(3.3)

where $a$ and $b$ are constants. Since the $45^\circ$ line is tangent to $f^5(x,\lambda_5)$ at $x_i$, we have that

$$f^5(x_i,\lambda_5) = x_i$$

(3.4)

$$\left. \frac{df^5(x,\lambda_5)}{dx} \right|_{x=x_i} = 1$$

(3.5)

thus we can write

$$y_{n+1} = y_n + cy_n^2 + \epsilon$$

(3.6)

where $y_n = (x_n - x_i)/b$ and $c$ is a constant. Since the step size near $x_i$ is small, Eq. (3.6) may be approximated by the differential equation

$$\frac{dy}{d\lambda} = cy^2 + \epsilon$$

(3.7)

Integrating (3.7) gives for the number of steps between $y_{in}$ and $y_{out}$
\[ \ell = \frac{1}{\sqrt{c\varepsilon}} \left( \tan^{-1}\left( \frac{y_{\text{out}}}{\sqrt{c\varepsilon}} \right) - \tan^{-1}\left( \frac{y_{\text{in}}}{\sqrt{c\varepsilon}} \right) \right) \]  

(3.8)

The average length of a periodic region \( \langle \ell \rangle \) is found by averaging \( y_i \) over a probability distribution uniform over some acceptance region \((-G,G)\) and by setting \( y_{\text{out}} = G \),

\[ \langle \ell \rangle = \frac{1}{\sqrt{c\varepsilon}} \tan^{-1}\left( \frac{G}{\sqrt{c\varepsilon}} \right) \]  

(3.9)

For \( \sqrt{c\varepsilon} < G \) we have

\[ \langle \ell \rangle = \frac{\pi}{2} \frac{1}{\sqrt{c\varepsilon}} \]  

(3.10)

HHS also calculate an expression for the probability of observing a laminar region of length \( \ell \), \( P(\ell) \), and plot the correlation function

\[ c(\ell) = \frac{1}{N} \sum_{n=1}^{N} \langle x_{n+\ell} x_n \rangle \text{ for two values of } \varepsilon. \]

Intermittency has been observed in thermal convection experiments,\(^4\),\(^5\) chemical reactions,\(^6\) and other systems, but evidence linking intermittency with a tangent bifurcation and data supporting the scaling of \( \langle \ell \rangle \) with \( \varepsilon \) and the behavior of \( P(\ell) \) vs. \( \ell \) has been lacking.

3.2 Experimental Observation of Intermittency and Tangent Bifurcation

The period 3 window displays hysteresis shown in Fig. 6.5 which prevents intermittency from being observed. The period 5 window does not display hysteresis and intermittency can be observed.\(^7\),\(^8\) The fifth iterate of the current is measured as described in Section 2.2.
A detailed schematic of the circuit appears in Appendix B. Figures 3.3(a)-(d) show a series of oscilloscope photographs taken at four different values of $V_0$ near the value $V_{oc}$ for the period 5 window. The diagonal line $I(t + 5T) = I(t)$ which appears is hand drawn on the photograph. In Fig. 3.3(a) the system is fully chaotic before the period 5 window, $V_0 < V_{oc}$. The entire map is visited fairly uniformly by the iterates, so the map appears on the photograph with uniform intensity. The photo trace appears to touch at tangent points the $I(t + 5T) = I(t)$ line which would contradict $V_0 < V_{oc}$, but this is because of the finite line width of the trace. Careful measurement with a window comparator shows that these points lie off the $I(t + 5T) = I(t)$ line by an amount too small to be resolved in the photograph. The discontinuities and doublevaluedness are due to the higher dimensional phase space. Otherwise, the photograph has a good correspondence with the fifth iterate of the logistic map, Fig. 3.1. In Fig. 3.3(b) the system is intermittent, $V_0 \leq V_{oc}$. The channels are visited more slowly than the rest of the map which makes the channels appear brighter on the oscilloscope. This photo offers direct evidence for Manneville and Pomeau's interpretation of intermittency, Fig. 3.2. In Fig. 3.3(c) the system is periodic near the start of the period 5 window $V_0 = V_{oc}$. Only one stable fixed point of the fifth iterate is visited whenever the system is periodic. All five fixed points were photographed by reducing $V_0$ slightly to make the system sample all five and taking a multiple exposure. In Fig. 3.3(d) the system is chaotic after the period 5 window has bifurcated into chaos, $V_0 > V_{oc}$. The map has grown across the $I(t + 5T) = I(t)$ line as expected. Figures 3.4(a)-(c)
show the intermittency as it appears in the current signal. Figure 3.4(a) is an oscilloscope photograph of \( I(t) \) at the period 5 window taken at 15 milliseconds per sweep. The relatively slow sweep makes the signal appear smooth or laminar with dark horizontal lines corresponding to the peaks and valleys of the current signal. Figures 3.4(b) and (c) show the intermittency as it appears in the current signal.

3.3 Quantitative Analysis of \( \langle \lambda \rangle \) and \( P(\lambda) \)

The voltage \( V \) across the varactor is used in the quantitative analysis rather than the series current \( I \) because the voltage's sharp pulses are more easily analyzed with existing equipment. To determine \( \langle \lambda \rangle \), it is necessary to know the total length and the total number of periodic laminar regions in an interval of time. Since we are also measuring \( P(\lambda) \), we find it convenient to know the length and the start and end of each laminar region. Figure 3.5 shows the voltage during intermittency. The largest voltage pulses are attained only during a laminar phase and are used to trigger an event pulse \( P \) which can be seen directly below the voltage signal in Fig. 3.5. The event pulses \( P \) are triggered by a discriminator set to a value 1% below the largest voltage pulse. This was done to facilitate comparison with the work of Hirsch, Huberman, and Scalapino\(^3\) which uses a 1% acceptance gate \((-G,G)\) [see Eq. (3.9)].

Additional logic circuitry detects the beginning of a periodic train and outputs a pulse \( B \), and detects the end of the periodic train and outputs a a pulse \( E \). The average length \( \langle \lambda \rangle \) is measured as a
function of \( \varepsilon \), which we define experimentally as

\[
\varepsilon \equiv \lambda_5 - \lambda_0 = (V_{o5} - V_o) \left[ \frac{\Delta \lambda}{\Delta V_0} \right]
\]

to facilitate comparison with Hirsch, Huberman, and Scalapino. \(^3\)

Here \( V_{o5} \) is the amplitude of the driver at the period 5 window threshold, and \( V_o \) is the amplitude voltage below threshold. Both are measured with a Fluke 8520A six-digit programmable voltmeter. The scaling factor, \( \Delta \lambda/\Delta V_0 \), is used to establish a correspondence between the experimental quantity \( (V_{o5} - V_o) \) and the quantity \( \lambda_5 - \lambda \) computed from the logistic map. \( \Delta \lambda = \lambda_{10} - \lambda_5 \) is computed from the logistic map where \( \lambda_{10} \) is the threshold for bifurcation to period 10 and \( \Delta V_0 \) is the measured voltage increment between the same thresholds.

For our system \( \Delta \lambda/\Delta V_0 \approx 0.103 V^{-1} \).

\( \varepsilon \) was varied by a three-stage helipot attenuator driven by a digitally controlled stepper motor with a resolution of \( 10^{-5} \) in \( \varepsilon \). \( \langle \lambda \rangle \) vs. \( \varepsilon \) was measured as follows. The \( P \) pulses were inputted into a 512 channel type NS-900 multichannel analyzer made by Northern Scientific which was advanced by one channel for every 2048 \( B \) pulses; the stepper motor then advanced to \( \varepsilon + \Delta \varepsilon \), etc. A plot of the probability distribution \( P(\lambda) \) was measured in this way: pulse \( B \) triggered a linear ramp \( V_R(t) = K(t - t_B) \) at time \( t_B \), and pulse \( E \) sampled the ramp voltage \( V_R(t_E) = K(t_E - t_B) \) at time \( t_E \) and generated a pulse with magnitude equal to \( V_R(t_E) \). Pulse \( E \) also reset \( V_R \) to zero, ready to be triggered by the next \( B \) pulse, etc. The pulse which is equal to \( V_R(t_E) \) is input into a pulse height analyzer which displays \( P(\lambda) \) vs. \( \lambda \).
A block diagram which describes how $\langle \lambda \rangle$ and $P(\lambda)$ are measured is shown in Fig. 3.6. A detailed schematic of the circuit appears in Appendix C.

Figure 3.7 is a plot of $\log_{10}(\lambda)$ vs. $\log_{10}c$. After an initial steep slope, the data are fit by $\langle \lambda \rangle \approx 1/c^\beta$, where $\beta = 0.43$ is the slope of the drawn line. From similar runs an average value is found to be $\bar{\beta} = 0.45 \pm 0.05$. We have no explanation for the initial steep slope and the deviation from the predicted value $\beta = 0.5$. Figure 3.8 shows $P(\lambda)$ vs. $\lambda$ in units of $5T = 62.5$ $\mu$s for $c = 2.5 \times 10^{-4}$. After an initial steep decay there is a slight hump at $\lambda \approx 9$ followed by a fast roll-off to very small values of $P(\lambda)$ for large $\lambda$. $\lambda$ as large as 5000 occurred, but the probability is too small to appear in Fig. 3.8. This figure is to be compared with the theoretical prediction, Fig. 7 of Ref. 3, which shows $P(\lambda)$ large at small and large values of $\lambda$ with a dip at $\lambda \approx \langle \lambda \rangle \approx 10$. However, if a small amount of random noise is added, then the predicted $P(\lambda)$ has a hump at $\lambda \approx 10$ and then rolls off at large $\lambda$. This is qualitatively similar to our data of Fig. 3.8, which therefore may be explained by the presence of noise in the nonlinear circuit. Data taken at $c = 8 \times 10^{-5}$ show $P(\lambda)$ vs. $\lambda$ extending to $\lambda = 5000$, but with a modulation at 60 Hz. This indicates that at small values of $c$ the intermittency is very sensitive to the amplitude modulation of the driver voltage at the power line frequency. The amplitude modulation is small but unavoidable.
3.4 Conclusions

To summarize, in this chapter we have presented evidence for a tangent bifurcation in a driven nonlinear oscillator and have shown that it gives rise to intermittency according to the geometrical picture of Manneville and Pomeau. We find the reasonable correspondence between the data

\[ \langle \ell \rangle \propto \varepsilon^{-0.45 \pm 0.05} \]

and the predicted expression \( \langle \ell \rangle \propto \varepsilon^{-1/2} \). Furthermore, the observed probability distribution \( P(\ell) \) vs. \( \ell \) is qualitatively similar to the theoretical prediction. We have no explanation for the initial steep decay of \( \langle \ell \rangle \) vs. \( \varepsilon \) (Fig. 3.7), for the small deviation of the observed scaling of \( \langle \ell \rangle \) vs. \( \varepsilon \) with the prediction \( \langle \ell \rangle \propto \varepsilon^{-1/2} \), and for the absence of a second peak at large \( \ell \) in the probability distribution \( P(\ell) \). These discrepancies are not due to the small amount of thermal and electrical noise present in the circuit. \(^9\)
CHAPTER IV. EFFECTS OF ADDITIVE NOISE

4.1 Introduction

At \( \lambda > \lambda_c \) the iterates of the logistic equation, \( \{x_n\} \), display sensitivity to initial conditions. Two points which are approximately equal will differ by a large amount after a few iterations. Thus the sequence \( \{x_n\} \) appears random.\(^1\) Since \( \{x_n\} \) is computed from Eq. (2.1), the randomness or chaos is sometimes called deterministic. Physical systems which display turbulence or chaos usually also contain fluctuations of a stochastic nature such as thermal and electrical noise. The effects of added noise on the logistic model have been investigated by several authors\(^2-7\) who report its effect on the stability and observability of period doubling, on windows, on single-band chaos, on the Lyapunov exponent, and on intermittency. In this chapter we report the effects of added noise on the driven p-n junction.\(^8\) We find close agreement between experiment and the logistic model and universal predictions.

Theoretical models\(^2-7\) add a term \( p_n \) to Eq. (2.1):

\[
x_{n+1} = \lambda x_n (1-x_n) + p_n
\]

where \( p_n \) is either a Gaussian or a uniform random distribution with standard deviation \( \sigma \) and mean value zero. In our experiments, a random noise voltage is added to the driving voltage, so the equation

\[
x_{n+1} = (\lambda + q_n) x_n (1-x_n)
\]

is more appropriate. Here \( q_n \) is a random distribution with standard deviation \( \sigma_q \) and mean value zero. Crutchfield, Farmer, and Huberman\(^5\)
show that the effects of $p_n$ and $q_n$ are approximately the same and that

$$\sigma_q \approx 8\sigma.$$  

Theoretical computations are displayed as: 1) Plots of

spectral power density $S(f)^{2,6}$; 2) plots of probability density

$P(x_n)$ at fixed $\lambda^{5,6}$; and 3) noisy bifurcation diagrams.\(^5\) We review

the predicted behavior of $P(x_n)$ for the three cases which we investigate.

(i) For $\sigma = 0$ and $\lambda_c > \lambda = \text{value midway into period } 2^n$, $P(x_n)$

consists of $2^n$ singularities. As the noise is increased to value $\sigma$,

the singularities broaden and merge to $2^{n-1}$ peaks, with $P(x_n) = 0$

between the peaks. The system has become semiperiodic.\(^6,9\) If the

noise is further increased to $K\sigma$, the $2^{n-1}$ peaks merge to $2^{n-2}$ peaks

with $P(x_n) = 0$ between the peaks, where $K = 6.619...$ is a universal

number, first computed by Crutchfield, Nauenberg, and Rudnick.\(^3\)

This process continues as $\sigma$ is increased until eventually all peaks

merge to one band and the system becomes aperiodic.

(ii) For $\sigma = 0$ and $\lambda_c < \lambda = \text{value for a window of period } k$, $P(x_n)$

consists of $k$ singularities. As $\sigma$ is increased, the singularities

broaden only slightly before $P(x_n)$ becomes non-zero throughout its

domain, and the system is aperiodic.

(iii) For $\sigma = 0$ and $\lambda_c < \lambda = \text{value for a one-band attractor},$ $P(x)$

consists of a high base line with structures and singularities

corresponding to mappings of the critical point $x_c = 1/2$. As $\sigma$ is

increased, the singularities disappear but the gross features remain.

Hirsch, Huberman, and Scalapino\(^7\) find that for $\varepsilon = 0$, intermittency

is induced by adding a noise term $g\xi(t)$ to the right side of the

logistic equation, where $\xi(t)$ is a white noise source and $g$ is the
standard deviation. The predicted scaling behavior for the average laminar length is

$$\langle \ell \rangle \propto q^{-2/3}$$

4.2 Experimental Effects of Additive Noise

The noise voltage is generated by a reverse biased Zener diode and added to the oscillator circuit by a summing amplifier. A circuit diagram appears in Appendix D. The varactor diode voltage $V$ is observed by a spectrum analyzer which gives $S(f)$; by a pulse height analyzer which gives the probability $P(V_p)$ corresponding to $P(x_n)$ where $V_p$ is a voltage peak; and by the set-up described in Section 2.2 which plots the bifurcation diagram $\{V_p\}$ vs. $V_0$. Data are taken for various values of noise voltage $V_n$, and recorded as $\sigma_m = \sigma_q/8 = V_n/(8V_{cm})$ to facilitate comparison with theoretical computations. Here $V_{cm}$ is a scaling factor equal to the maximum voltage $V$ at maximum driving voltage $V_0$ which corresponds in the logistic model to a range of order unity for $\{x_n\}$.

Figure 4.1 shows $\log_{10} P(V_p)$ observed at $f/16$. As noise is added, the behavior is similar to that predicted. In Fig. 4.1(a), $\sigma_m = 0$; the system is periodic with sharp peaks as expected. The peaks are not singularities because of the small amount of thermal and electrical noise in the circuit. In Fig. 4.1(b), $\sigma_m = 1.4 \times 10^{-4}$; the system has semiperiodicity 8. In Fig. 4.1(c), $\sigma_m = 8.7 \times 10^{-4}$: increasing the noise voltage by the factor 6.3 reduces the semiperiodicity to 4. Figure 4.1(d) shows that another increase in $\sigma_m$ by a factor of 6.3 reduces the semiperiodicity to 2. These features are similar to
Fig. 20 of Ref. 5. Figures 4.2(a)-(d) show the observed power spectral density $S(f)$ measured simultaneously under the same conditions as Figs. 4.1(a)-(d). Successive factors of 6.3 in added noise voltage eliminate the sharp spectral components, reducing the period from $8 \rightarrow 4 \rightarrow 2$. A series of 15 measurements gives the average value $\kappa = 6.4 \pm 0.2$. Figure 4.3(a) shows the observed bifurcation diagram with noise voltage added to set the semiperiodicity at 8. In Fig. 4.3(b) the noise voltage is increased by a factor 6.3, reducing the semiperiodicity to 4. These measurements are similar to the computed results, Fig. 7 of Ref. 5.

Figure 4.4(a) shows $\log_{10} P(V_p)$ at $\sigma_m = 0$ and $V_o$ set just after the beginning of the period 5 window. In Fig. 4.4(b) the addition of a small amount of noise $\sigma_m = 1.1 \times 10^{-4}$ raises $P(V_p) \neq 0$ for all $V_p$. This confirms prediction (ii). To confirm prediction (iii), Fig. 4.4(c) shows $\log_{10} P(V_p)$ observed at a one-band attractor with $\sigma_m = 0$. Addition of noise $\sigma_m = 1 \times 10^{-3}$ in Fig. 4.4(d), washes out the peaks but does not change the gross features. These results are similar to Figs. 11 and 12 of Ref. 6.

Figure 4.5 shows a plot of $\log_{10} \langle \ell \rangle$ vs. $\log_{10} g$ with $\epsilon = 0$, where $\ell$ is proportional to the additive noise voltage $V_n$. The data are fit by $\langle \ell \rangle \approx 1/g^\gamma$, where $\gamma = 0.65$ is the slope of the drawn line. Similar runs give an average value $\overline{\gamma} = 0.65 \pm 0.05$. Figure 4.6 shows the probability distribution $P(V_p)$ measured for $\epsilon = 0$ and $g = 10^{-4}$. There is a small hump at $\ell \approx 10$ which is not observed at $g = 3.5 \times 10^{-4}$. 
4.3 Effects of Adding a Sinusoidal Voltage

We have examined the effects of adding a sinusoidal rather than a random noise voltage. If the system is at the period 3 window, it follows the period doubling route to chaos as an additive voltage at $f/2$ is increased. For the system chaotic at a one-band attractor after the period 3 window, an additive voltage at $f/3$ induces periodic behavior at $f/12$, $f/6$, and $f/3$ as the additive voltage is increased.

4.4 Conclusions

In conclusion, we observe that increasing the noise voltage by a factor $\kappa = 6.4 \pm 0.2$ produces a transition to half the semiperiodicity. As predicted, additive noise produces sudden aperiodicity at windows, but has a small effect at a one-band chaotic attractor. Additive noise at $\varepsilon = 0$ induces intermittency with scaling law $\langle \ell \rangle = 1/g^\gamma$ where $\gamma = 0.65 \pm 0.05$ in reasonable agreement with the predicted value $\gamma = 0.666...$. The observed distribution $P(V_p)$ is qualitatively similar to theoretical predictions. Finally, if the system is chaotic, it is not stable against a sinusoidal perturbation at a subharmonic frequency, which induces periodic behavior.
CHAPTER V. CRISIS OF THE ATTRACTOR IN THE DRIVEN P-N JUNCTION

5.1 Introduction

Sudden qualitative changes, called crises, in the size and shape of the chaotic attractor of the logistic map and of the two-dimensional Henon map have been investigated by Grebogi, Ott, and Yorke.\(^1\),\(^2\) They conclude that crises can occur whenever the chaotic attractor is intersected by an unstable periodic orbit. In this chapter we examine three cases of crisis of the attractor for the driven p-n junction\(^3\):

1) crisis following the period 3 window which is predicted by the logistic and Henon maps\(^1\),\(^2\); 2) hysteresis crisis of the period 3 window which is predicted by the Henon map\(^2\); and, 3) crisis at the 4→2 band merging which is not seen in the logistic or Henon maps.

5.2 Crisis Following the Period 3 Window

Grebogi et al.\(^1\) show that a crisis occurs in the attractor of the logistic map when the period 3 unstable orbits intersect the three-band chaotic attractor. This occurs at points C in Fig. 5.1. Figure 5.2 shows the upper part of a bifurcation diagram of the voltage for varactor diode 1N953 as the driving voltage is increased past the value for the onset of the window at point A, to the value at point B where the crisis occurs. \(\{V_p\}\) suddenly takes on a set of values between the semiperiodic bands. The Poincaré section of the flow in phase space is made by strobing on an oscilloscope the current \(I(t)\) vs.
voltage $V(t)$. This method is illustrated in Fig. 5.3. Figure 5.4(a) is a Poincaré section for the system at a three-band chaotic attractor following the period 3 window. In Fig. 5.4(b) the crisis occurs: the attractor suddenly becomes a one-band attractor.

5.3 **Hysteresis Crisis of the Period 3 Window**

Figure 5.5 shows a high resolution picture of the hysteresis. In Fig. 5.5(a) $V_o$ is decreasing and in Fig. 5.5(b) $V_o$ is increasing. Figure 5.5(c) is a composite drawing showing the parabolic shape of the stable periodic attractor and the associated unstable orbit (dashed line). The hysteresis crisis is explained by the intersection of the unstable period 3 orbit with the chaotic attractor. Figure 5.6 is a Poincaré section under the conditions of Fig. 5.5. The three dots correspond to the period 3 window. As $V_o$ is reduced below the threshold value, the attractor jumps discontinuously at point A to the solid lines. The solid lines are similar to the Henon map at high dissipation.

5.4 **Crisis at the 4→2 Band Merging**

Figure 5.1 shows unstable period 2 orbits intersecting the chaotic attractor at points A and B where there is a 4→2 band merging. Figure 5.7 shows the Poincaré section as the voltage is increased to $V_{om}$, the value at band merging. Figure 5.7(a) shows a four-band chaotic attractor for $V_o < V_{om}$. In Figures 5.7(b) and (c) the bands come closer to the merge point. In Fig. 5.7(d) the four bands merge to two bands, and there is what appears to be a crisis: observation
of the Poincaré section shows a fountainlike expansion where the bands merge. Bifurcation diagrams of the logistic map and Henon map do not show this type of behavior.
CHAPTER VI. PERIOD ADDING SEQUENCE
IN THE DRIVEN P-N JUNCTION

6.1 Experimental Observations

Figure 6.1 shows a bifurcation diagram for diode 1N4004 made at driving voltage amplitudes in the range 0 to 10 volts and at a driving frequency $\omega = 150$ kHz. The circuit is that of Fig. 1.1 with $L = 10$ mH and $R = 100\Omega$. The initial period doubling cascade examined in Chapter II is indicated for comparison in Fig. 6.1. The most prominent feature is the sequence of wide windows whose periodicity increases by one period. Each wide window appears at a tangent bifurcation and period doubles as the driving voltage is increased. The current signal at a period $n$ window consists of staircase waveforms with $n$ ascending steps (an "$n$ staircase") as shown in Fig. 6.2(a). The voltage across the p-n junction is approximately constant near zero volts during the first $n-1$ peaks of the staircase and then rises in a sharp pulse at the highest peak as shown in Fig. 6.2(b).

Between all observed windows of period $n$ and period $n+1$ there occurred smaller windows of period $2n-1$ and $2n+1$. The period $2n+1$ window consists of an $n$ staircase followed by an $n+1$ staircase as shown in Fig. 6.3. The period $2n-1$ window consists of an $n-1$ staircase followed by an $n$ staircase. The chaos between windows of period $n$ and $n+1$ was observed to consist of $n-1$, $n$, and $n+1$ staircases of varying height as shown in Fig. 6.4.

The return map, $I_p(t)$ vs. $I_p(t+T)$, is shown in Fig. 6.5 just before the period 5 window. It consists, to a first approximation,
of four parabolas with a common maximum joined to a straight line. In
general, the map is observed to have \( n-1 \) parabolas just before the
period \( n \) window. Both a maximum and minimum are observed, unlike the
logistic map which has a single maximum. At a period \( n \) window the
return map has \( n-1 \) iterates on the straight line part of the map and
one iterate on the quadratic maximum as shown in Fig. 6.6.

All of these features are explained in the next chapter by a one­
dimensional map similar to the experimental return map. Period adding
sequences have been observed in the Belousov-Zhabotinsky (B-Z)
reaction\(^2,3\) and in phase-locking iterative maps.\(^4,5\) The B-Z reaction
has many of the same features we have observed; however, the bifurcation
diagram of a fitted return map is very different from Fig. 6.1. Phase­
locking iterative maps also have period adding bifurcation diagrams with
a structure rather different from what we observe.\(^6\)
7.1 Introduction

In this chapter we present two models which predict the period adding sequence of windows: 1) a one-dimensional map consisting of a parabola and straight line, and 2) a set of three coupled ordinary differential equations which model the charge storage properties of p-n junctions. Two models have appeared in the literature which deal with driven nonlinear circuits: 1) Rollins and Hunt\(^1\) model the p-n junction in our circuit by a constant capacitance under reverse bias and by a perfect conductor under forward bias. There is a specified threshold voltage for conduction and an assumed expression for the reverse recovery time. Their model predicts period doubling, chaos, and periodic windows in agreement with the logistic map, but not the period adding sequence which we observe. 2) N. Robinson\(^2\) examines a circuit similar to ours except that the diode is replaced by a piecewise constant capacitance. Period doubling and period adding are observed; however, their circuit is physically different from ours. One model for the B-Z reaction\(^3\) predicts period adding and a return map similar to our experimental map.

7.2 One-Dimensional Iterative Map

A one-dimensional map similar to the experimental map is shown in Fig. 7.1(a). The maximum of the parabola and the point where the parabola joins the straight line are held at fixed $x$. The control
parameter steepens the parabola and increases the height. The increased height allows the iterates to visit further up the straight line and produce successively higher periodic windows as illustrated in Figs. 7.1(b) and (c). The equation for the map is

\[ x_{n+1} = A x_n + F \quad \text{for } x_n \geq K \]

\[ x_{n+1} = A K + F + L \left( (x_n - B)^2 - (K - B)^2 \right) \quad \text{for } x_n < K \quad (7.1) \]

where L is the control parameter. Figure 7.2 shows the computed bifurcation diagram for parameter values \( A = 0.85, B = 8, F = 2.7, Q = 2, K = 7 \). It is similar to the experimental diagram. There is a sequence of windows whose periodicity increases by one and narrow windows of periodicity \( 2n+1 \) between wide windows of period \( n \) and \( n+1 \). The period \( n \) window consists of \( n \) staircases, as defined in Chapter VI, and the period \( 2n+1 \) window consists of an \( n \) staircase followed by an \( n+1 \) staircase as shown in Fig. 7.3. The chaos is also similar to the experimental data described in Chapter VI as shown in Fig. 7.4.

### 7.3 Three Coupled Ordinary Differential Equations

Assuming the only nonlinearities of the p-n junction are given by Eqs. (1.1), (1.5), and (1.6), the diode in Fig. 1.1 may be replaced by the parallel combination of current source \( I_d \) and capacitance \( C \) as shown in Fig. 7.5. \( I_d \) is determined by the d-c characteristics of the diode, Eq. (1.6), and \( C = C_j + C_s \) where \( C_j \) and \( C_s \) are given by Eqs. (1.1) and (1.5), respectively. The differential equations governing this system are:
\[ \dot{i} = \frac{V_0 \sin \Omega - V - IR}{L} \]  
\[ \dot{V} = \frac{I - I_0 (e^{qV/kT} - 1)}{C_0 e^{qV/kT} + C_1/(1 - V/\phi)} \]  
\[ \dot{\Omega} = \omega \]  
(7.2)  
(7.3)  
(7.4)

I, V, and \( \Omega \) are the current, voltage across the p-n junction, and the phase of the sinusoidal driver, respectively; \( V_0 \) and \( \omega \) are the amplitude and frequency of the driver, respectively. These equations were solved numerically using a circuit analysis program called SPICE4 and were found to display a period adding sequence as the amplitude of the driver was increased. Figure 7.6 shows the experimental and computed current waveforms at the period 2, 3, 4, and 5 windows and the voltage across the p-n junction at the period 5 window. The computed current and voltage waveforms are similar to the experimental data. The chaos between windows of period \( n \) and \( n+1 \) consists mostly of \( n-1, n, \) and \( n+1 \) staircases. Other features such as bifurcation diagram and return map require greater accuracy and longer computer runs and were not computed.
CHAPTER VIII. SUMMARY AND CONCLUSIONS

To summarize, experimentally determined bifurcation diagrams and return maps for the driven p-n junction are similar to those of the logistic map. This fact supports the use of one-dimensional quadratic maps as models of chaotic behavior, but it is also clear from the data that the system is more accurately described by a higher dimensional map.

Intermittency was observed to occur near the onset of a period 5 window. Direct observation of the tangent bifurcation was made possible by the high data rate and good signal-to-noise ratio of the system. We found a reasonable correspondence between the data and the predicted expression $\langle x \rangle \propto \epsilon^{-1/2}$. The observed probability distribution $P(x)$ vs. $x$ was found similar to the theoretical prediction.

The effects of additive noise on period doubling, chaos, windows, and intermittency were all in agreement with calculations based on the logistic model and with universal predictions. The universal number $\kappa = 6.619\ldots$ was measured for the experimental system to be $\kappa = 6.4 \pm 0.2$. Additive noise at $\epsilon = 0$ induced intermittency with scaling law $\langle x \rangle \propto g^{-\gamma}$ where experimentally $\gamma = 0.65 \pm 0.05$, in reasonable agreement with the predicted value $\gamma = 0.666$.

Three examples of crisis of the attractor were observed for the system. In the period 3 window, an interior crisis leading to a one-band attractor was observed as in the logistic model. Hysteresis crisis was observed at the onset of the period 3 window as in the Henon map. Crisis was also observed at the 4 to 2 band merging.
A period adding sequence of windows was observed at high driving voltages. The initial period doubling and subsequent period adding were demonstrated in two theoretical models: a one-dimensional map and a system of three coupled differential equations. These models were based on the data and nonlinear properties of p-n junctions.

Interesting areas which were not experimentally examined include band hopping, reverse bifurcations, the Lyapunov exponent, and the dimension of the attractor.
CHAPTER I REFERENCES

CHAPTER II REFERENCES

CHAPTER III REFERENCES

CHAPTER IV REFERENCES

CHAPTER V REFERENCES

2. C. Grebogi, E. Ott, and J. A. Yorke, Physics publication number 83-037, University of Maryland (1982).
CHAPTER VI REFERENCES

REFERENCES

4. SPICE, Dept. of Electrical Engineering and Computer Sciences, University of California, Berkeley.
CHAPTER I FIGURE CAPTIONS

Fig. 1.1 Nonlinear oscillator circuit consisting of a sinusoidal driver, resistor, inductor, and p-n junction connected in series. Buffers are used to measure the current and voltage across the p-n junction. Typically, \( R = 100\, \Omega \), \( L = 10\, \text{mH} \), \( \omega = 100\, \text{kHz} \), \( V_0 = 0-10 \) volts, and the p-n junction is a Type 1N953 Si varactor diode or a Type 1N4004 Si rectifier diode.

Fig. 1.2 Circuit used to measure switching transients. The current \( I \) is measured as the voltage drop across the resistor \( R \). The frequency of the generator is typically 100 kHz.

Fig. 1.3 (a) The square wave output of the signal generator in Fig. 1.2. (b) The voltage across the resistor \( R \) with an ideal diode in the circuit of Fig. 1.2. (c) The voltage across the resistor \( R \) with a real diode showing switching transients with reverse recovery time \( \tau_R \). Typically, diodes with \( \tau_R \sim 1-2\, \mu\text{s} \) were used.

Fig. 1.4 (a) The sinusoidal driver voltage, Fig. 1.1. (b) The current \( I \) before it has period doubled at \( V_0 \approx 1.5 \) volts. (c) The current after it has period doubled once at \( V_0 \approx 2.5 \) volts. (d) The current after it has period doubled twice (period 4) at \( V_0 \approx 2.8 \) volts. (e) The current when the system is chaotic at \( V_0 \approx 3.1 \) volts. The voltage peaks appear at varying heights. (f) The current at the period 3 window at \( V_0 \approx 7.8 \) volts.
Fig. 1.5 Current signal for diode IN4004 at wide windows which appear in a period adding sequence. (a) Period 2 current at \( V_o \approx 1.2 \text{ volts} \). (b) Period 3 current at \( V_o \approx 3.4 \text{ volts} \). (c) Period 4 current at \( V_o \approx 5.8 \text{ volts} \).
CHAPTER II  FIGURE CAPTIONS

Fig. 2.1  Computed bifurcation diagram \( \{x_n\} \) vs. \( \lambda \) for Eq. (2.1), showing successive period doubling bifurcations, chaos at \( \lambda_c \), and periodic windows of period 6, 5, and 3.

Fig. 2.2 (a) (i) Current signal at the period 5 window. (ii) Output of the sample and hold circuit with waveform in (i) as the input. The sample and hold circuit samples the current at a peak and holds the peak value for a little over one driving period. Waveform (i) is fed to the vertical input of the scope and waveform (ii) to the horizontal input. The scope is strobed at points indicated in the figure by dots and dashed lines. The result is a plot on the scope of \( I_{n+1} \) vs. \( I_n \). A similar method is used to obtain \( I_{n+m} \) vs. \( I_n \).
(b) Block diagram used to obtain bifurcation diagrams and return maps directly on an oscilloscope.

Fig. 2.3  Bifurcation diagram made from the current signal at small driving voltages for diode 1N4004. \( I_n \) are the values of the current peaks obtained by strobing the oscilloscope. Period doubling, chaos, band mergings, and periodic windows of period 6, 5, and 3 are visible. \( R = 100\Omega \), \( L = 10 \text{ mH} \), \( \omega = 150 \text{ kHz} \), and \( V_o = 0-1 \text{ volt} \).

Fig. 2.4 (a) Bifurcation diagram made from the current signal for varactor diode 1N953. Period doubling, chaos, band mergings, and periodic windows of period 6, 5, and 3 are visible. \( R = 100\Omega \), \( L = 10 \text{ mH} \), \( \omega = 100 \text{ kHz} \), and \( V_o \approx 0-10 \text{ volts} \).
(b) Expanded view of the period 3 window in Fig. 2.4(a). The period doubling route to chaos is observed as $V_o$ is increased.

Fig. 2.5 (a) Return map, $I_{n+1}$ vs. $I_n$, made from the current signal peaks for diode 1N4004. The folding over on the right is evidence of higher dimensional character. $R = 100\,\Omega$, $L = 10\,mH$, $\omega = 150\,kHz$, $V_o = 0.75\,volt$. (b) Return map, $I_{n+1}$ vs. $I_n$, made from the current signal peaks for diode 1N953. $R = 100\,\Omega$, $L = 10\,mH$, $\omega = 100\,kHz$, $V_o \approx 7\,volts$.

Fig. 2.6 Bifurcation diagram made from the voltage signal across diode 1N953 showing thresholds $V_1$, $V_2$, and $V_3$ for periods 2, 4, and 8; threshold for chaos $V_c$; band merging $M_0$; and windows of periods 6, 5, 7, 3, 6, 12, 9, and 13.

Fig. 2.7 (a) Sinusoidal driver voltage. (b) Voltage across the diode $V$ at the period 5 window showing pattern of visitation RLRR. (c) Voltage $V$ at the period 6 window showing pattern of visitation RLRRR.

Fig. 2.8 Power spectral density vs. frequency showing subharmonics to $f/32$. The components agree with predictions shown in dashed bars to within 2 dB except for the peak at $f/16$. The small size of the peak at $f/16$ is due to a low frequency filter used in the measuring circuit.
Fig. 3.1 The fifth iterate of the logistic map at $\lambda = \lambda_5 = 3.73775$ showing the five fixed points $x_i$ at the period 5 window.

Fig. 3.2 Expanded view of Fig. 3.1 near one of the fixed points for $\lambda \leq \lambda_5$, showing a narrow channel through which successive iterates pass from point A to point B.

Fig. 3.3 Oscilloscope photo of $I(t + 5T)$ vs. $I(t)$ ($I_{n+5}$ vs. $I_n$) for the current, showing directly the fifth iterate map and a tangent bifurcation; the diagonal line is hand drawn. (a) For driving voltage $V_o < V_{oc}$, in the fully developed chaotic regime. (b) For $V_o \leq V_{oc}$, showing intermittency. (c) For $V_o = V_{oc}$, showing the five fixed points at the period 5 window. (d) For $V_o > V_{oc}$, in the chaotic region following the period 5 window.

Fig. 3.4 Oscilloscope photo of $I(t)$ vs. time at 15 milliseconds per sweep. (a) For the driving voltage $V_o = V_{oc}$ at the period 5 window. (b) and (c) For $V_o \leq V_{oc}$; showing intermittency.

Fig. 3.5 Top curve is dual-beam oscilloscope trace of the voltage across the p-n junction $V(t)$ for $\epsilon \approx 10^{-3}$, showing intermittency: periodic regions (and event dots P immediately below) and aperiodic regions. The schematically drawn pulse trains $P(t)$, $B(t)$, and $E(t)$ show, respectively, the pulses for periodic peaks, the beginning of a periodic region, and the end of a periodic region.
Fig. 3.6 Block diagram of the circuit used to measure the average length of a laminar region \( \langle \ell \rangle \), and the probability distribution \( P(\ell) \). TAC, MCA, and PHA are abbreviations for time-to-amplitude converter, multichannel analyzer, and pulse height analyzer, respectively.

Fig. 3.7 Experimental plot of \( \log_{10} \langle \ell \rangle \) vs. \( \log_{10} \epsilon \) for observed intermittency near the period 5 window. Dashed line through data has slope -0.43.

Fig. 3.8 Relative probability distribution \( P(\ell) \) vs. periodic length \( \ell \) (in units of \( 5T = 62.5 \mu s \)) for intermittency near the period 5 window at \( \epsilon = 2.5 \times 10^{-4} \).
CHAPTER IV  FIGURE CAPTIONS

Fig. 4.1  Observed $\log_{10} P(V_n)$ vs. voltage peaks $V_n$ across varactor diode 1N953. (a) No added noise ($\sigma_m = 0$), showing period 16. (b) $\sigma_m = 1.4 \times 10^{-4}$. (c) $\sigma_m = 8.7 \times 10^{-4}$. (d) $\sigma_m = 5.5 \times 10^{-3}$.

Fig. 4.2  Observed power spectral density vs. frequency under the same conditions as Fig. 4.1. (a) No noise added, sharp subharmonics $f/2$ to $f/16$ displayed. (b) Added noise $\sigma_m = 1.4 \times 10^{-4}$ removes the sharp $f/16$ components. (c) $\sigma_m = 8.7 \times 10^{-4}$ removes the sharp $f/8$ components. (d) $\sigma_m = 5.5 \times 10^{-3}$ removes the sharp $f/4$ component.

Fig. 4.3  Observed upper half-branch of bifurcation diagram, $\{I_n\}$ vs. $V_o$, showing period $2 \rightarrow 4 \rightarrow 8$. (a) Added noise $\sigma_m = 1.4 \times 10^{-4}$. (b) $\sigma_m = 8.7 \times 10^{-4}$.

Fig. 4.4  Observed $\log_{10} P(V_n)$ vs. $V_n$. (a) At the onset of the period 5 window with no added noise. Only three out of five peaks are observed because two occur when the p-n junction is forward biased and are compressed towards the zero line. (b) With added noise $\sigma_m = 1.1 \times 10^{-4}$. (c) In a chaotic region ($\lambda \approx 3.7$) with no added noise. (d) With added noise $\sigma_m = 1 \times 10^{-3}$.

Fig. 4.5  Experimental plot of $\log_{10} \langle \varepsilon \rangle$ vs. $\log_{10}$ (noise voltage) for observed intermittency in the period 5 window with $\varepsilon = 0$. The dashed line through the data has slope $-0.65$. 
Fig. 4.6 Relative probability distribution $P(\lambda)$ vs. periodic length $\lambda$ (in units of $5T = 62.5$ µs) for intermittency in the period 5 window ($\varepsilon = 0$) induced by an additive random noise voltage with standard deviation $g = 10^{-4}$. 
CHAPTER V  FIGURE CAPTIONS

Fig. 5.1  Attractor \( \{ x_n \} \) vs. \( \lambda \) computed from Eq. (2.1), showing onset of chaos at \( \lambda_c \), and period 5 and period 3 windows. A crisis occurs at points near C and also at points near A and B where the unstable orbit drawn in dashed line intersects the chaotic attractor.

Fig. 5.2  Upper section of bifurcation diagram observed for driven p-n junction. At point A the driving voltage amplitude \( V_0 = V_3 \) is at the threshold value for the period 3 window. At point B a crisis of the attractor occurs, \( V_0 = V_{*3} \).

Fig. 5.3  Block diagram of experimental set-up used to measure the Poincaré section of the nonlinear oscillator.

Fig. 5.4  Oscilloscope picture of the Poincaré section observed for the driven p-n junction. (a) \( V_3 < V_0 < V_{*3} \), showing three separate bands. (b) \( V_0 = V_{*3} \), showing crisis as a sudden change to a one-band attractor.

Fig. 5.5  Oscilloscope picture of bifurcation diagram observed for the driven p-n junction at the period 3 window. (a) With driving voltage amplitude \( V_0 \) decreasing; there is a sudden jump down at point A. (b) With \( V_0 \) increasing; there is a sudden jump up at point B. (c) Drawing overlay of (a) and (b) together with an unstable orbit drawn in dashed line intersecting the chaotic attractor which gives rise to the hysteresis.
Fig. 5.6 Oscilloscope picture of two attractors observed for the driven p-n junction near the period 3 window. The three dots correspond to the period 3 window for $V_0 \geq V_3$. The solid line corresponds to the chaotic attractor for $V_0 \leq V_3$. For $V_0 = V_3$, the system jumps discontinuously between the two attractors in a hysteresis crisis.

Fig. 5.7 Oscilloscope pictures of the attractor observed for the driven p-n junction as $V_0 \rightarrow V_{om}$, the voltage for the $4 \rightarrow 2$ band merge. (a) $V_0 < V_{om}$; (b) $V_0 < V_{om}$; (c) $V_0 \leq V_{om}$; (d) At $V_0 = V_{om}$ a crisis is observed as a transverse expansion of the attractor.
Fig. 6.1 Bifurcation diagram of the current for diode IN4004 made at large driving voltages. A period adding sequence of wide windows is observed in addition to local period doubling. The initial period doubling cascade shown in Fig. 2.3 is indicated by the double-pointed arrow. $R = 100\Omega$, $L = 10\, \text{mH}$, $\omega = 150\, \text{kHz}$, $V_0 \approx 0-10\, \text{volts}$.

Fig. 6.2 (a) The current signal vs. time at the wide period 5 window. The signal is composed of staircases with five ascending steps each. (b) The voltage across the p-n junction at the period 5 window. The voltage is nearly zero for the first four current peaks and has a sharp pulse at the highest peak of the current.

Fig. 6.3 The current signal at the narrow period 7 window between the wide period 3 and period 4 windows. The narrow window consists of a four staircase followed by a three staircase.

Fig. 6.4 The current signal when the system is chaotic between the wide period 2 and period 3 windows. The current consists of 1, 2, and 3 staircases of varying heights.

Fig. 6.5 Experimentally determined return map, $I_p(t)$ vs. $I_p(t+T)$ (or $I_n$ vs. $I_{n+1}$ in our notation). The map consists of four folds followed by a straight line. $V_0$ is set just before the period 5 window. $R = 100\Omega$, $L = 10\, \text{mH}$, $\omega = 150\, \text{kHz}$, and $V_0 \approx 8\, \text{volts}$. The line $I_n = I_{n+1}$ is also drawn by the measuring circuit.
Fig. 6.6 Experimentally determined return map, $I_{n+1}$ vs. $I_n$, at the wide period 5 window. Four of the points lie on the straight line part, while one lines on the parabolic part.
CHAPTER VII  FIGURE CAPTIONS

Fig. 7.1  (a) One-dimensional piecewise continuous (but not smooth) iterative map consisting of a parabola joined to a straight line. (b) Period 2 cycle with one iterate on the parabola and one iterate on the straight line. (c) Period 3 cycle with one iterate on the parabola and two iterates on the straight line part. The equation for the map is 
\[ x_{n+1} = Ax_n + F \] for \( x_n \geq K \), 
\[ x_{n+1} = AK + F + L\left( (x_n - B)^2 - (K - B)^2 \right) \] for \( x_n < K \).

Fig. 7.2  Bifurcation diagram computed for the map of Fig. 7.1. A period adding sequence very similar to the experimental data is computed. The parameters are \( A = 0.85 \), \( B = 8 \), \( F = 2.7 \), \( Q = 2 \), and \( K = 7 \). The control parameter is \( L \).

Fig. 7.3  Computed period 7 window between the wide period 3 and wide period 4 windows. The window consists of a three staircase (1,2,3) followed by a four staircase (1,2,3,4).

Fig. 7.4  Computed chaotic region between the wide period 3 and wide period 4 windows showing an intermittency between three staircases (1,2,3) and four staircases (1,2,3,4). The staircases appear at varying heights.

Fig. 7.5  Driven p-n junction circuit with p-n junction replaced by the parallel combination of current source 
\[ I_d(V) = I_o(e^{qV/kT} - 1) \] and capacitance 
\[ C(V) = C_j(V) + C_s(V) \] where \( C_j(V) \) is the junction capacitance and \( C_s(V) \) is the charge storage capacitance.
Fig. 7.6 Comparison of experimental data on the left with numerical solution of differential equations on the right. (a) Current at the period 2 window. (b) Current at the period 3 window. (c) Current at the period 4 window. (d) Current at the period 5 window. (e) Voltage across the p-n junction at the period 5 window.
Fig. 1.3
Fig. 1.5
Fig. 2.1
Fig. 2.2(a)
Fig. 2.2(b)
Fig. 2.4(a)
Fig. 2.4(b)
Fig. 2.5(a)
Fig. 2.7
Fig. 3.1
Fig. 3.2
Fig. 3.3
Fig. 3.4
stepper
motor
start pulse
end pulse
start pulse
end pulse
start end
TAC

buffer discriminator
MCA
PHA
in advance
in

Fig. 3.6
Fig. 4.2
Fig. 4.6
Fig. 5.1
Fig. 5.3
Fig. 5.4
Fig. 5.5
Fig. 5.6
Fig. 5.7
Fig. 6.1
Fig. 6.2
Fig. 6.3
Fig. 6.5
Fig. 6.6
Fig. 7.1
Fig. 7.6
APPENDIX A

Figure A.1 shows a schematic of the nonlinear oscillator circuit. Buffers (LM310H) are used to measure the current I and voltage across the p-n junction V.
Fig. A.1

- LM310H
- IN953 or IN4004
- 10K
- 100
- 10mH
- EXACT 121B
- to diff. amp. of scope
- to scope
APPENDIX B

Figure B.1 shows a schematic of the circuit used to generate a bifurcation diagram from the current signal peaks $I_n$. TTL pulses in sync with the driving oscillator are delayed such that the hp 214B pulser strobos the peaks of the current signal $I(t)$. An AD536 true rms voltmeter is used to determine the driving amplitude voltage which is fed to the horizontal input of the oscilloscope. The current signal which is strobed is determined by measuring the voltage drop across the resistor $R$ with buffers (LM310H) and a differential oscilloscope plug-in.

Figure B.2 shows a schematic of the circuit used to determine the first and fifth iterate return maps. TTL pulses in sync with the driving oscillator are divided by two (for the first iterate) or by ten (for the fifth iterate) and delayed such that the current peaks are strobed by the hp 214B pulser. The divided pulses are also inverted, delayed, and fed to the hold input of an AD582 sample and hold. The pulses are delayed such that the sample and hold holds the value of a current peak. The hold time is made slightly longer than one period for the first iterate and slightly longer than five periods for the fifth iterate. The output of the sample and hold is fed to the horizontal input of the oscilloscope. The voltage signal $IR$ is fed to the vertical input of the oscilloscope.
EXACT 121B to nonlinear circuit

50K delay

1000 pf

10K

74123

1A 1B 2Q 6 7

in PULSER out hp 214B

to strobe of scope

to horizontal input of scope

5V

AD 536

LM310H

nonlinear circuit to differential amp of scope

Fig. B.1
Fig. B.2
Figure C.1 shows a schematic of the circuit used to determine the start and end (stop) of a periodic region during intermittency. Pulses from a discriminator set 1% below the highest voltage pulse across the p-n junction V are fed to a retriggerable flip-flop. The highest voltage pulse occurs every 5T at the period 5 window where T is the driving period. By setting the pulse width of the retriggerable flip-flop slightly greater than 5T, the flip-flop stays high throughout a periodic region and goes low during an aperiodic region when all the pulses V are less than the discriminator setting.

Figure C.2 shows a schematic of the circuit used to determine the length of a periodic region. The circuit produces a momentary (10 μs) dc voltage proportional to the time between the "start" signal and the "stop" signal which are taken from the circuit shown in Fig. C.1. The initial TTL circuitry ensures that a start pulse only occurs after a stop pulse. The start pulse opens a D-MOS switch which allows an integrator made from an LF356 op-amp to charge up. The stop pulse samples the integrator voltage with an AD582 sample and hold, holds the value for 10 μs, and closes the D-MOS switch, bringing the integrator voltage to zero.
Fig. C.1
Fig. C.2
APPENDIX D

Figure D.1 shows a schematic of the circuit used to produce the noise voltage. Shot noise produced by a reverse biased 6V Zener diode is amplified by a factor of 2500 by a two-stage ac-coupled amplifier made with LF356 op-amps.
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.