Title
ESSAYS ON MONETARY POLICY, CHINA'S ECONOMY AND EXCHANGE RATE

Permalink
https://escholarship.org/uc/item/84v3b7dz

Author
Wang, Ren

Publication Date
2012

Peer reviewed|Thesis/dissertation
UNIVERSITY OF CALIFORNIA
SANTA CRUZ

ESSAYS ON MONETARY POLICY, CHINA’S ECONOMY AND EXCHANGE RATE

A dissertation submitted in partial satisfaction
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Ren Wang

June 2012

The Dissertation of Ren Wang is approved:

Professor Carl E Walsh, Chair

Professor Joshua Aizenman

Professor Phillip McCalman

Tyrus Miller
Vice Provost and Dean of Graduate Studies
Abstract

ESSAYS ON MONETARY POLICY, CHINA’S ECONOMY AND EXCHANGE RATE

Ren Wang

University of California, Santa Cruz

June 2012

This dissertation consists of the three essays that allowed me to investigate three different economics phenomena. The discussion focuses on topics related to U.S. bank deregulation, China’s resource misallocation and currency carry-trade strategy. In particularly, the aim is to study how U.S. bank deregulation is related to Great Moderation in U.S. between the mid-1980s and the start of the subprime debt crisis in 2007, what the implication of monetary policy with China’s resource misallocation is and how a new carry trade method can be constructed.

The first paper models the effect of bank deregulation on the volatility of output and firms dynamics. By introducing a cost channel and bank sector into Bilbee, Ghironi and Melitz (2007)’s model, we test the volatility change under various shocks before and after bank deregulation. Simulations show that with the bank spread decreases by 1/3, the volatility of output and number of producers decrease significantly. This can provide a potential explanation for why the Great Moderation starts from the mid-1980s to mid-2000s.

The second paper constructs a dynamic stochastic general equilibrium model of China with heterogeneous sectors and resource misallocation. Recent literature has shown that there is significant resource misallocation between state-owned firms and
private firms in China. We find the presence of resource misallocation alters standard monetary policy conclusions in important ways. We show that there is an additional real effect of monetary policy—an allocation effect. This is different from the standard New Keynesian framework, where the real effect comes from the price rigidity. Monetary policy shocks also exert heterogeneous effects on private firms and state-owned firms. Given a common shock, the output volatility of private firms is higher than that of state-owned firms. The model can easily be extended to other developing countries with similar situations.

The third paper employs two methods to construct a portfolio for carry trade strategy. The first one is the minimal variance, subject to specific return and weights constraints. The other is the maximal R square, subject to the weights constraints. Using MATLAB, we can calculate the optimal weights, returns and Sharpe ratio for these two methods. By comparing the Sharpe ratio for different time intervals, we find that the overall maximal R square strategy is relatively more accurate than the minimal variances strategy. This provides an alternative method to construct a carry trade portfolio.
Dedication

I am extremely grateful to my advisor, Carl E. Walsh, for his comments, suggestions and inspirations throughout my graduate studies. Thanks for always taking the time to discuss problems with me. He was always eager to help, and constantly supported and motivated me during many moments of the Ph.D. program. I would also like to thank Joshua Aizenman, Phillip McCalman and Yin-Wong Cheung for their helpful comments and suggestions on preliminary drafts of my thesis. Furthermore, I am indebted to all my professors at the University of California, Santa Cruz for shaping me as an instructor and a researcher.

I would like to thank my colleagues and friends at the department provided insightful feedback at early stages of this thesis and read specific chapters. In particular, thank my fellow graduate students: Mariya Mileva, Aadil Nakhoda, Luba Pertersen, JeanPaul Rabanal, George Georgiou, Sergio Lago Alves, Orcan Çortuk and Julian Caballero for many useful discussions, comments and suggestions.

I am especially grateful to the University of California, Santa Cruz and Department of Economics at the University of California, Santa Cruz, for their financial support throughout my graduate studies.

Lastly I would like to thank my parents and my sister, for their help and motivation during the last five years.
## Contents

1 Introduction 1

2 Firm Dynamics and the Cost Channel: U.S bank deregulation 4
   2.1 Introduction ...................................................... 4
   2.2 The model ......................................................... 9
      2.2.1 Household .................................................. 10
      2.2.2 Firms ......................................................... 10
      2.2.3 Bank ......................................................... 13
      2.2.4 Symmetric Equilibrium .................................... 14
      2.2.5 Budget Constraint .......................................... 15
   2.3 Calibration and Simulation ..................................... 16
      2.3.1 Monetary Policy Shock .................................... 16
      2.3.2 Productivity Shock ......................................... 19
      2.3.3 Cost Shock ................................................ 21
   2.4 Conclusion ....................................................... 23

3 Resource Misallocation and Implication for Monetary Policy: the Case of China 24
   3.1 Introduction ..................................................... 24
   3.2 The Model ........................................................ 28
      3.2.1 Households ................................................. 28
      3.2.2 Entrepreneurs ............................................. 29
      3.2.3 State-Owned Firm ......................................... 30
      3.2.4 Retailers .................................................. 31
3.2.5 Equilibrium ......................................................... 32
3.3 Calibration and Simulation ................................. 36
  3.3.1 Monetary Policy Shock ................................. 36
  3.3.2 Technology Shock ................................. 38
  3.3.3 Output Volatility ................................. 39
3.4 Conclusion ......................................................... 40

4 Maximizing Predictability for Carry Trade 41
  4.1 Introduction ......................................................... 41
  4.2 Data ......................................................... 43
  4.3 Optimally weighted strategy ......................... 44
  4.4 Maximal R square strategy ......................... 48
    4.4.1 Step One: .............................................. 50
    4.4.2 Step Two .............................................. 50
    4.4.3 Step Three .............................................. 52
  4.5 Compare the Two Strategies ......................... 57
  4.6 Conclusion ......................................................... 59

5 Appendix: FOC 61

6 Appendix: Benchmark Model Summary 63

7 Appendix: Calibration 65

8 Appendix: Entrepreneur's problem 66

9 Appendix: Steady state 67
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Appendix: Log-linearize</td>
<td>75</td>
</tr>
<tr>
<td>11</td>
<td>Appendix: Calibration II</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>References</td>
<td>82</td>
</tr>
</tbody>
</table>
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Impulse Response Following a Monetary Policy Shock</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Impulse Response Following a Productivity shock</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>Impulse Response Following a Cost shock</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>Impulse Response Following a Monetary Policy Shock For Private Sector and State-owned Sector</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>Impulse Response Following a Monetary Policy Shock For Private Sector and State-owned Sector II</td>
<td>38</td>
</tr>
<tr>
<td>6</td>
<td>Positive Technology Shock in Private Sector</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>Realized Payoffs (12-month Moving Average)</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>Histogram for the six currencies’ return</td>
<td>48</td>
</tr>
</tbody>
</table>
List of Tables

1 Monetary Policy Shock .............................................. 19
2 Productivity Shock .................................................. 21
3 Cost Shock ............................................................. 23
4 Minimal Variance for 2008 .......................................... 46
5 Maximal R Square regression result based on equation (37) .... 53
6 Maximal R Square for 2008 based on equation (37) .......... 54
7 Maximal R Square regression result based on equation (42) .. 56
8 Maximal R Square for 2008 based on equation (42) ........ 57
9 One Year Time Forcast ............................................... 58
10 1-5 Year(s) Time Forcast ........................................... 59
1 Introduction

This dissertation consists of the three essays that allow me to investigate three different economics phenomena. The discussion focuses on topics related to U.S. bank deregulation, China’s resource misallocation and currency carry-trade strategy. In particularly, the aim is to study how U.S. bank deregulation in late 1970s and early 1980s is related to Great Moderation in U.S. between the mid-1980s and the start of the subprime debt crisis in 2007, what the implication of monetary policy with China’s resource misallocation is and how a new carry trade strategy can be constructed.

The first paper, “Firm Dynamics and the Cost Channel: U.S bank deregulation”, examines the relationship between U.S. bank deregulation in late 1970s and early 1980s and Great Moderation in U.S. from the mid-1980s to the subprime debt crisis in 2007. This paper is important for us to understand the Great Moderation. In the meantime, it can help us to study the subprime debt crisis since 2007 and differentiate the effects of various financial deregulations on macroeconomic stability and volatility. Bank deregulations happened in late 1970s that focus on phasing out a number of restrictions to increase bank industry competition, which significantly improves bank’s efficiency and decreases bank spread, are not quite related to the subprime debt crisis in 2007, which partly comes from mismatch between regulatory framework and financial innovation.

We introduce a cost channel and bank sector into Bilbiée, Ghironi and Melitz (2007)’s model and test the volatility change under various shocks before and after bank deregulation. This assumption is based on empirical evidences that the cost channel is an important and non-negligible channel for the transmission of exogenous
shocks, especially monetary policy shocks. The new model differs from previous literature in the Great Moderation by considering the effect of firms’ entry and exists. We not only consider the internal margin, which is the volatility of single firm’s output, but also analyze the external margin, which is the volatility of number of firms in the market. We calibrate our model and simulation results are consistent with empirical evidence. Simulations show that with the bank spread decreases by 1/3, the volatility of output and number of producers decrease significantly. This can provide a potential explanation for why the Great Moderation starts from the mid-1980s to mid-2000s.

The second paper, “Resource Misallocation and Implication for Monetary Policy: the Case of China”, studies China’s monetary policy with resource misallocation. The existence and importance of resource misallocation has been well documented in developing countries, especially in China. Current monetary models for developing countries abstract from the misallocation effect. Our augmented New Keynesian Model can help us understand the effect of monetary policy on developing countries better.

Empirical evidences show that monetary policy shocks exert heterogeneous effects on private firms and state-owned firms in China. The effect of monetary shocks on private firms is larger than state-owned firms. In this paper, we argue that the heterogeneity comes from resource misallocation. Recent literature has shown that there is significant resource misallocation between state-owned firms and private firms in China. State-owned firms continue to enjoy significantly more generous external finances than other types of Chinese firms. Private firms face more financial constraints. We construct a dynamic stochastic general equilibrium model of China with heterogeneous sectors and resource misallocation. Simulation results are consistent
with empirical evidence. We also find the presence of resource misallocation alters standard monetary policy conclusions in important ways. We show that there is an additional real effect of monetary policy—an allocation effect. Following a positive monetary policy shock, labor and capital will move from state-owned sector to private sector. This is different from the standard New Keynesian framework, where the real effect comes from the price rigidity. Our model can easily be extended to other developing countries with similar situations.

The third paper, “Maximizing Predictability for Carry Trade”, is based on the mismatch between uncovered interest rate parity theory (UIP) and exchange rate fluctuation. UIP states that high (low) interest rate currencies should depreciate (appreciate) compared to low (high) interest rate currencies. However, a large amount of empirical studies have documented that UIP doesn’t hold in reality. Market participants can take advantage of this failure, selling forward currencies that are at a forward premium and buying forward currencies that are at a forward discount—carry trade.

In this paper we employ two methods to construct a portfolio for carry trade strategy. The first one is most popular one for carry trade, the minimal variance, subject to specific return and weights constraints. The other is new in the literature of carry trade, the maximal R square, subject to the weights constraints. Using MATLAB, we can calculate the optimal weights, returns and Sharpe ratio for these two methods. By comparing the Sharpe ratio for different time intervals, we find that overall maximal R square strategy is relatively better than minimal variances strategy. This provides us an alternative method to construct a portfolio for carry trade.
2 Firm Dynamics and the Cost Channel: U.S bank deregulation

2.1 Introduction

There was a well-documented substantial decline in Macroeconomic volatility in the US economy between the mid-1980s and the start of the subprime debt crisis in 2007.\(^1\) This dramatic change, commonly referred to as the “Great Moderation”, has caused heated debate. Three main explanations have been suggested for the Great moderation: better monetary policy (Taylor 1999; Cogley and Sargent 2002; Anton Nakov and Andrea Pescatori 2009; Luca Benati and Paolo Surico 2009), structural changes (Margaret M. McConnell and Gabriel Perez-Quiros 2002; Chang-Jin Kim, Morley James and Piger Jermy 2008) and good luck (Ahmed, Levin and Wilson 2002; Stock and Watson 2003)\(^2\). There is no theoretic or empirical consensus on why macroeconomic volatility has declined\(^3\). This paper develops a DSGE model with a cost channel and firm dynamics to theoretically explain how U.S. bank deregulation has led to this declined phenomenon. In a speech, Bernanke (2004) argues that improvements in monetary policy have probably been an important source of the Great Moderation, but he also gives some credit to the deregulation that has occurred in many industries. Dynan, Elmendorf and Sichel (2006) empirically find that bank deregulation can explain the Great Moderation and argue that it should be added to

\(^1\)Empirical studies have shown the timing of the Great Moderation starts from mid-1980s (Chang-Jin Kim and Carles R. Nelson 1999; Margaret M. McConnell and Gabriel Perez-Quiros 2002; Stock and Watson 2003).

\(^2\)Alternative explanation includes demographic change (Nir Jaimovich and Henry E. Siu 2009).

\(^3\)see Stock and Watson 2003 for literature review.
the list of likely contributors. Stebunovs (2007) suggest that monopoly power implies more vigorous firm entry following a positive productivity shock. As a consequence, firm entry and output are less volatile after bank deregulation in 1980s. Our model shows that bank deregulation, together with the cost channel and firm dynamics, may be an important contributor to the Great Moderation.

Critics argue that financial deregulations and innovations are one of the reasons leading to the subprime debt crisis in 2007. However, the bank deregulations happened in late 1970s that focus on phasing out a number of restrictions to increase bank industry competition, which significantly improves bank’s efficiency and decreases bank spread, are not quite related to the subprime debt crisis in 2007, which partly comes from mismatch between regulatory framework and financial innovation. For some regulations after the late 1970s, new instruments of investment and speculation are introduced without adequate regulation and risk management controls around them. The most significant U.S. bank deregulation proceeded in the late 1970s is different. Empirical evidence shows that bank deregulation induced lower loan prices and smaller bank spreads. Along with deregulation, there was a substantial increase in small firms’ entry and exit. After deregulation, the volatility of small firms’ entry and exit as well as the aggregate volatility has declined significantly.

Prior to deregulation, U.S. banks faced multiple restrictions on geographic expansion both within and across state. The bank system was extremely fragmented. Banks are prohibited from branching into other states or even other cities. However, starting from the late 1970s, these restrictions eased substantially. These reforms are considered as the most significant deregulation in the U.S. history for the bank industry. Banks expanded their businesses to other cities and other states. They branched directly by opening new offices, or, indirectly, via mergers and acquisitions. The
reforms reduced monopoly power and improved operational efficiency. Jayaratne and Strahan (1998) find that after bank deregulation, operating costs and loan losses decreased sharply while most of the reduction in banks’ costs was passed along to bank borrowers in the form of lower loan rates. Dick (2006) and Stebunovs (2007) document similar evidence that bank spread decreased by 3% after bank deregulation and profit are unaffected. Correa (2008) used a firm-level data set consisting of U.S. manufacturing firms between 1976 and 1994 and bank data for the same period and found spreads for small firms decreased after deregulation about 30% relative to their mean value.

Along with deregulation, empirical evidence indicates that there has been significant increase in small firms’ rates of entry and exit. Black and Strahan (2002) find that the rate of new incorporations per capita increased by 3.8% following deregulation of branching restrictions and increased by 7.9% following deregulation of interstate banking restrictions. By using a state-level aggregate panel data set for the US between 1977 and 1994, they estimate how the rate of new business incorporations changed with enhanced competition and consolidation following deregulation in the banking industry. Reducing banking concentration (HerfindahlFindlay-Hirschmann Index) from 0.24 to 0.14 increases the number of firms by 4.6%. Kerr and Nanda (2009) also find that both business formation and failure grow substantially after interstate banking deregulation. Their analysis is based on micro-data for the period 1976-2001 from Longitudinal Business database, with approximately four million establishments. Estimates indicate that start-up entry for the first four years post-bank deregulation increased by 23%. Overall, for small firms, deregulation lead to enhanced competition from longer-term entrants and a reduction in market power.

After deregulation, empirical evidence suggests that the volatility of small firms’
entry and exit and aggregate output declined sharply. Davis, Haltiwanger, Jarmin and Miranda (2007) find that the volatility of firm’s entry and exit in the U.S. economy has declined by more than 40% since 1982. Even though there is an opposite trend between publicly traded and privately held firms, the decrease in volatility of privately held firms’ entry and exit dominates the increase in volatility of publicly traded firms’ entry and exit. And much of the decline in GDP volatility can be explained by the declined volatility of privately held firms’ entry and exit. Stebunovs (2007) documents similar evidence. He also develops a dynamic, stochastic, general equilibrium model to connect bank deregulation to declined volatility. The model predicts an increase in the number of firms and a decrease in firm size after deregulation because of reduced bank local monopoly power. In this model, firms need to borrow from the bank to pay the entry cost necessary to start their business. Firm entry and output are less volatile after deregulation as higher competition implies more vigorous firm entry over the business cycle. However, Stebunovs (2007) fails to consider an important factor that can explain the Great Moderation: monetary policy. Luca Benati and Paolo Surico (2009) suggest that good monetary policy, as a key driving force, decreases in the variance of inflation and the output gap by using VAR analysis.

Our paper differs from previous literature in a few ways. The Great Moderation literature fails to consider the effect of firms’ entry and exist. As we have argued above, this is an important variable in explaining output volatility. We provide a DSGE model based on Bilbiiee, Ghironi and Melitz (2007)’s model. This model endogenizes firms’ entry decision by introducing an entry cost. Firms decide entry or not entry based on whether the present value of future profit is greater than the entry cost. This enables us to analyze the effects of various shocks on firms’ dynamics and
volatility of output.

The second contribution of this paper is to link the cost channel with the Great Moderation. The cost channel is an important and non-negligible channel for the transmission of exogenous shocks, especially monetary policy shocks. Several studies (Anton Nakov and Andrea Pescatori 2009; Luca Benati and Paolo Surico 2009) show that monetary policy is the most crucial determinant in the volatility decline. There is strong evidences showing the existence of the cost channel (Bath and Ramey (2001), Dedola and Lippi (2005), Chowdhury, Hoffmann and Schabert (2006), Ravenna and Walsh (2006), Tillmann (2008))4 and that the effect of interest rates on prices are strongly related to working capital (Dedola and Lippi (2005), Gaiotti and Secchi (2006), Ravenna and Walsh (2006)). To our knowledge, we are the first to use the cost channel to explain the Great Moderation. This enables us to give a full picture as to how monetary policy affects the overall economy. By introducing the cost channel into Bilbiie, Ghironi and Melitz (2007)’s model, simulation results show that the cost channel does play an important role in the Great moderation.

Last but not least, we introduce a bank sector with financial frictions into the Bilbiie, Ghironi and Melitz (2007)’s model. A decline in monopoly power for the bank industry will improve efficiency, and increased competition forces banks to decrease their loan price. In our model, we assume that bank deregulation will significantly

---

4 Rabanal (2003) finds that the cost channel is quantitatively insignificant in a Bayesian framework. There may be several reasons why Rabanal (2003) cannot detect the cost channel effect: firstly, the data is quarterly data and in a short period. Quarterly data over a short period may not be a good source when attempting to detect a cost channel. The second reason is that results from Bayesian estimation may be sensitive to the imposed prior for a small sample. In his model, Rabanal’s prior assumptions include 16 distributions for 16 parameters. Without robustness check, it is difficult to determine whether the result is reliable.
decrease financial frictions and the benefits will be passed on to the borrowers: the bank spread decreases significantly. This is consistent with the empirical literature (Dick (2006), Stebunovs (2007) and Correa (2008)). Also we assume that small firms need to borrow to finance their working capital. This is not an unreasonable assumption. Berger and Udell (1998) show that commercial banks alone provide over 30% of total equity plus debt for starting firms and more than 80% of lending in the credit line market. Moreover, privately held firms account for more than two-thirds of private business employment and produce half of U.S. economy output.

Based on our model, we can show that along with the bank deregulation there is a significant increase of firm’s entry. And after the deregulation, simulations indicate that the volatility of small firms’ entry and exit and the volatility of aggregate output decline significantly. Here we do not argue that this is the answer to the Great Moderation, but the model with firm dynamics and cost channel is able to explain at least part of the story.

This paper is organized as follows. Section 2.2 introduces the model. Section 2.3 shows the simulation result. Section 2.4 concludes.

2.2 The model

Our model is based on Bilbiee, Ghironi and Melitz (2007). We modify the model in two ways. The first modification is that firms need to borrow from the bank to pay wage before production, as in Ravenna and Walsh (2006). The second modification introduces the banking sector with financial frictions to model the cost channel and the effects of institutional banking sector changes on firms’ behavior.
2.2.1 Household

Household are identical and normalized to one. In each period, the household maximizes $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$, where utility in period $t$ is given by:

$$U(C_t, L_t) = \log C_t - \chi \frac{L_t^{1+1/\phi}}{1+1/\phi}$$  \hspace{1cm} (1)

where $\chi > 0$, $\varphi \geq 0$, $L_t$ is the labor supply in period $t$ and $C_t$ are composite goods that are defined over a continuum of good $\Omega$. In every period, depending on the number of producing firms, only a subset of goods $\Omega_t \subset \Omega$ is available.

$$C_t = \left[ \int_{\omega \in \Omega_t} c_t(\omega)^{\theta-1} d\omega \right]^{\theta/(\theta-1)}$$  \hspace{1cm} (2)

where $\theta > 1$. The consumption-based index can be defined below as:

$$P_t = \left[ \int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega \right]^{1/(1-\theta)}$$  \hspace{1cm} (3)

Finally the household’s demand for each individual good $\omega$ is given by:

$$c_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t = \rho_t(\omega)^{-\theta} C_t$$  \hspace{1cm} (4)

where $p_t(\omega)$ denote the nominal price of a good $\omega \in \Omega_t$.

2.2.2 Firms

As in Bilbiee, Ghironi and Melitz (2007), in this economy, there is a mass of
monopolistically competitive firms. Each firm produces only one variety \( y_t(\omega) \), \( \omega \in \Omega_t \), with the same production function. Labor is the only input and \( Z_t \) is labor productivity:

\[
y_t(\omega) = Z_t l_t(\omega)
\]  

(5)

Prices are sticky in this economy. Firms face Rotemberg (1998) nominal price rigidity in the form of a quadratic cost of adjusting prices. This cost can be expressed in real term:

\[
pac_t(\omega) \equiv \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \frac{p_t(\omega)}{P_t} y_t^D(\omega), \kappa > 0
\]  

(6)

Here \( y_t^D(\omega) \) is the total demand for the output of firm \( \omega \). \( p_t(\omega) \) is the producer \( \omega \)'s price. \( P_t \) is the consumer price index defined in equation (3).

Firms need to borrow from the bank at a gross nominal interest rate \((1 + \psi_{t-1})\) to pay wages in advance. \( \psi \) represents bank spread, which will be explained in details later. Firm \( \omega \)'s real profit in period \( t \) can then be written in real terms as:

\[
d_t(\omega) = \rho_t(\omega) y_t^D(\omega) - w_t l_t(\omega) \frac{1 + \psi_{t-1}}{1 + \pi_t^{CPI}} - \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \rho_t(\omega) y_t^D(\omega)
\]  

(7)

where \( \rho_t(\omega) = \frac{p_t(\omega)}{P_t} \) is the real price of firm \( \omega \)'s output, \( y_t^D(\omega) \) is the total demand for firm \( \omega \)'s output, \( \pi_t^{CPI} = \frac{P_t}{P_{t-1}} - 1 \) is the gross inflation rate and \( w_t = \frac{W_t}{P_{t-1}} \) is the real wage. So the real value of the firm \( \omega \) is the expected present discounted value of future profit from \( t + 1 \) on:

\[
v_t(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_t s d_s(\omega)
\]  

(8)
here

\[ \Lambda_{t,s} \equiv \frac{[\beta(1-\delta)]^{s-t} U_C(C_s,L_s)}{U_C(C_t,L_t)} = \frac{[\beta(1-\delta)]^{s-t} C_t}{C_s} \] (9)

where \( \delta \) is the probability of a death shock for each period, which is constant and exogenous.

The first order condition with respect to \( p_t(\omega) \) yields

\[ p_t(\omega) = \mu_t(\omega) \frac{W_t(1 + \psi_{t-1})}{Z_t} \] (10)

\( \mu_t(\omega) \) is the markup, which is defined as:

\[ \mu_t(\omega) \equiv \frac{\theta}{(\theta - 1) \left[ 1 - \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \right] + \kappa Y_t} \] (11)

where

\[ Y_t = \frac{p_t(\omega)}{p_{t-1}(\omega)} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right) \]

\[ -E_t \left[ \Lambda_{t+1} \frac{Y_{t+1}(\omega)}{Y_t(\omega)} \frac{P_t}{P_{t+1}} \left( \frac{p_{t+1}(\omega)}{p_t(\omega)} \right)^2 \left( \frac{p_{t+1}(\omega)}{p_t(\omega)} - 1 \right) \right] \]

As we can see, when prices are flexible (\( \kappa = 0 \)) or constant, the markup reduces to \( \frac{\theta}{\theta - 1} \).

Entrants are forward looking. Firms need to pay an entry cost in units of consumption \( f_E \), which is normalized to one for simplicity. There is a one-period time-to-build lag in the model, so entrants at time \( t \) only produce at time \( t + 1 \). Free entry
condition ensures that
\[ v(\omega) = f_E = 1 \] (12)

At the beginning of every period, there is a death shock with probability \( \delta > 0 \). The time of entry and production implies that the number of producing firms during period \( t \) is given by
\[ N_t = (1 - \delta)(N_{t-1} + N_{E,t-1}) \] (13)
where \( N_{t-1} \) is the number of firms producing in period \( t - 1 \), and \( N_{E,t-1} \) is the number of firms paying the entry cost in period \( t - 1 \), \( N_t \) is the number of firms producing in period \( t \).

2.2.3 Bank

The bank receives deposits \( D_t \) from households at the risk-free rate \( i_t \) and grant one-period riskless loans at a rate \( \psi i_t \). Here, \( (\psi - 1)i_t \) is the bank spread in each period where \( \psi \geq 1 \).

\[ \text{Max} \{X_t((1 + \psi i_t) - D_t(1 + i_t)) \} \] (14)

subject to
\[ X_t \leq D_t \]
then optimality requires:
\[ X_t = D_t \] (15)

Changes in \( \psi \) represent institutional change, specifically in this paper, bank dereg-
ulation. Bank deregulation reduced banks’ monopoly power and improved operational efficiency. Empirical evidence shows that operating costs and loan losses decreased sharply while most of the reduction in banks’ costs were passed along to bank borrowers in the form of lower loan rates. Following the bank deregulation, enhanced competition, improved operation efficiency and better risk management lead to a much lower bank spread. Here, a decrease in $\psi$ presents the decrease of bank spread. $\psi$ can also represent risk premium. More efficient and diversified bank sectors reduced the risk for each bank. A decline in $\psi$ means a lower risk premium. Small firms can borrow at a lower cost. Or we can consider $\psi$ as the markup for the bank. Higher competition in the bank industry improves efficiency and induces lower loan losses. Bank markup declines. For our model, $\psi - 1$ can be simply explained as the bank spread.

### 2.2.4 Symmetric Equilibrium

In a symmetric equilibrium, all firms make identical choices, so producer price is given by $p_t(\omega) = p_t$, and demand for each firm is $y_t^D = \rho_t^{-\theta} Y_t$. The consumer price index (equation (3)) can then be simplified to $p_t = p_t N_t^{-1/(\theta - 1)}$.

Let $\pi_t = \frac{p_t}{p_{t-1}} - 1$ denote inflation in producer prices. Let $PAC \equiv Npac = \kappa \frac{\pi_t^2}{2} Y_t$, which comes from equation (6). Equilibrium requires the nominal deposit $B = w_t L_t$, so the financial cost in terms of real output is $(\psi - 1) r_t w_t L_t$. We can write total output as: $Y_t = (\psi - 1) r_t w_t L_t + C_t + PAC$, so that $C_t = \left(1 - (\psi - 1) r_t \frac{w_t}{Z_t} - \frac{\kappa}{2} \pi_t^2\right) Y_t$.

We obtain the following markup equation from equation (11)

$$
\mu_t \equiv \frac{\theta}{(\theta - 1) \left(1 - \frac{\kappa}{2} \pi_t^2\right) + \kappa Y_t}
$$

\[\text{Equation (16)}\]

\[\text{5 see Jayaratne and Strahan (1998), Dick (2006), Stebunovs (2007) and Correa (2008).}\]
where

\[ \Upsilon_t = (1 + \pi_t) \pi_t - \beta (1 - \delta) E_t \left[ \frac{N_t}{N_{t+1}} \frac{1 - (\psi - 1) r_t \psi_t - \delta \pi_t^2}{1 - (\psi - 1) r_{t+1} \psi_{t+1} - \delta \pi_{t+1}^2} (1 + \pi_{t+1}) \pi_{t+1} \right] \]

2.2.5 Budget Constraint

Households enter period \( t \) with \( x_t \) shares in a mutual fund of firms, nominal deposit \( B_{N,t} \) in the commercial bank. In period \( t \), household receives dividend income from mutual shareholdings, the value of selling its initial share position, gross interest income on deposit and prepaid wage income \( W_t L_t \). The mutual fund pays a total profit \( D_t \) in the end of each period in units of currency that is equal to the total profit of all firms that produce in that period, \( P_t N_t d_t \). \( T_t \) is a lump-sum tax. The household allocates these resources between deposit and shares in mutual fund in the next period. The budget constraint in units of currency is:

\[ B_{N,t+1} + V_t N_H x_{t+1} + P_t C_t = (1 + i_{t-1}) B_{N,t} + (D_t + V_t) N_t x_t + W L_d + T_t \] (17)

Optimality requires:

\[ C_t^{-1} = \beta E_t \frac{P_t}{P_{t+1}} C_{t+1}^{-1} R_t \] (18)

\[ \chi (L_t)^{1/\varphi} C_t = \frac{W_t}{P_t} \] (19)
\[ v_t = \beta (1 - \delta) E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-1} (v_{t+1} + d_{t+1}) \right] \]  
(20)

Where \( v_t = \frac{V_t}{P_t} \)  
(21)

2.3 Calibration and Simulation

We calibrate the model according to Bilbiee, Ghironi and Melitz (2007). \( \beta \) is set at 0.99 and \( \delta \) is set at 0.025. We assign 3.8 to \( \theta \) and 77 to \( \kappa \). Entry cost \( f_E \), preference \( \chi \) and productivity \( Z \) are all set at 1. We close the model by applying the following monetary policy rule: \( i_t = 0.8i_{t-1} + 0.3\pi_{t+1} \). By comparing the volatility of output and firms’ entry under different shocks before and after deregulation, we can show the effect of bank deregulation on the volatility of output and firms’ entry. Here bank deregulation is represented by parameter \( \psi \). Following bank deregulation, the bank spread parameter \( \psi \) decreases from 1.8 to 1.2, which is consistent with the fact, the bank spread decreased about 30%.

2.3.1 Monetary Policy Shock

Following a transitory monetary policy shock, the central bank unexpectedly increases the nominal interest rate by 1% with zero exogenous persistence. Because the impulse response curves after the monetary policy shock before the bank deregulation \( \psi = 1.8 \) and after bank deregulation \( \psi = 1.2 \) are quite similar, we only show the one with \( \psi = 1.8 \). This is displayed in Figure 1. With sticky price, a temporary increase in the nominal interest rate \( i \) generates deflation in the consumer price index \( \Pi C \) and
higher real interest rates, which in turn depresses aggregate demand and supply. The household postpones its consumption $C$ (leading to a higher deposit $D$) because of higher real interest rate and firms produce less because of higher borrowing costs. Wage $W$ decreases and unemployment increase (employment $L$), consistent with the conventional wisdom associated with contractionary monetary policy in New Keynesian economy. The key feature in our model is that not only do firms reduce their production but also there is a fall in the number of entrants $Ne$ and a gradual decrease in the number of producers $N$. Decreasing aggregate demand and increasing borrowing costs reduces firms’ current and future profit and hence prevent new entrants into the market. Lewis (2006) and Bergin and Corsetti (2008) find empirical evidence that tight monetary policy has a contractionary effect on entry and the total number of producers. Our simulation results are consistent with their VAR studies.
With the same nominal interest rate shock, we can compare the standard deviation of output $Y$, entry Ne and producers N before bank deregulation ($\psi = 1.8$) and after bank deregulation ($\psi = 1.2$). Results are in Table 1. We found that the standard deviation of output decreases by 25.14% and the standard deviation of producers decreases by 27.74%. Bank deregulation increased competition and enhances operational efficiency. A more efficient banking system works as a buffer in the monetary transmission channel. Because of higher competition, banks are reluctant to pass all the costs associated with increases of the nominal interest rate on to firms. Also, economies of scale, the use of new technology and efficient operations boost banks’ profit margin and improve their ability to further alleviate the effect of monetary policy shock on firms.
Table 1: Monetary Policy Shock

<table>
<thead>
<tr>
<th></th>
<th>$b = 1.8$</th>
<th>$b = 1.2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>0.048081</td>
<td>0.034745</td>
<td>−27.74%</td>
</tr>
<tr>
<td>$Ne$</td>
<td>0.231444</td>
<td>0.220543</td>
<td>−4.71%</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.025898</td>
<td>0.019388</td>
<td>−25.14%</td>
</tr>
</tbody>
</table>

2.3.2 Productivity Shock

Following a positive productivity shock with standard deviation 0.01 and persistence parameter 0.9, we have figure 2.\(^6\) A positive productivity shock increases firms’ current and future profit and hence induces new entrants $Ne$ into the market. Output surges not only because firms produce more but also because more firms are producing. The positive productivity shock also generates a higher marginal productivity of labor, which increases the demand for labor. Both wage $W$ and labor supply $L$ rise.

\(^6\)Because the impulse response curves after the productivity shock before the bank deregulation $\psi = 1.8$ and after bank deregulation $\psi = 1.2$ are quite similar, we only show the one with $\psi = 1.8$. 
With the same shock, we can compare the standard deviation of standard deviation of output $Y$, entry $Ne$ and producers $N$ before bank deregulation ($\psi = 1.8$) and after bank deregulation ($\psi = 1.2$). In table 2, we can see that the standard deviation of output decreases by 10% and the standard deviation of producers decreases by 9.9%. Bank deregulation generates a positive effect on the economy. It facilitates borrowing, reduces borrowing cost, enhances management and provides information. Firms become more effective at resisting negative technology shocks and benefit more from positive technology shocks. Bank deregulation reduces the financial constraints of the firm. We find bank deregulation diminishes and shrinks shocks to the macroeconomy. This is consistent with Bernanke et al.(1999).

\[^7\]Bernanke et al. (1999) find that credit frictions amplify and propagate shocks to the macroecon-
2.3.3 Cost Shock

Following a cost shock with standard deviation 0.01 and persistence 0.9, we have figure 3. A cost shock generates inflation. The central bank reacts by raising the nominal interest rate, which leads to a higher real interest rate. The household postpones its consumption $C$ (higher deposit $D$) and firms hire less labor $L$ and produce less. Employment fall and the wage $W$ decreases. Higher borrowing cost reduces firms’ current and future profit and hence prevents new entrants $Ne$ into the market. The number of producers $N$ is falling.

---

Table 2: Productivity Shock

<table>
<thead>
<tr>
<th></th>
<th>$b = 1.8$</th>
<th>$b = 1.2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>0.045853</td>
<td>0.041315</td>
<td>$-9.90%$</td>
</tr>
<tr>
<td>$Ne$</td>
<td>0.115238</td>
<td>0.100977</td>
<td>$-12.38%$</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.030617</td>
<td>0.027541</td>
<td>$-10.00%$</td>
</tr>
</tbody>
</table>

---

8Because the impulse response curves after the cost shock before the bank deregulaion $\psi = 1.8$ and after bank deregulation $\psi = 1.2$ are quite similar, we only show the one with $\psi = 1.8$. 

None of the outputs are null.
With the same shock, we can compare the standard deviation of standard deviation of output $Y$, entry $Ne$ and producers $N$ before bank deregulation ($\psi = 1.8$) with after bank deregulation ($\psi = 1.2$). Results are in table 3. We can see that the standard deviation of output decreases by 7.27% and the standard deviation of producers decreases by 18.88%. The cost shock is quite similar to a nominal interest rate shock. Bank deregulation leads to a more efficient banking system. The effect of cost shock on firms through the interest rate channel (via borrowing cost) can be reduced by a more efficient banking system. A lower volatility of interest rate passed to firms generates a smaller volatility of output.
2.4 Conclusion

The standard deviation of main macroeconomic variables – especially the standard deviation of output and producers – in response to monetary, productivity, and cost shocks declines sharply after bank deregulation. This experiment suggests that bank deregulation in the presence of a cost channel is an important structural change which can partially account for the substantial decline in macroeconomic volatility in the US since the mid 1980s.
3 Resource Misallocation and Implication for Monetary Policy: the Case of China

3.1 Introduction

In the past three decades, China has undergone rapid economic growth and experienced growing importance in financial markets and private sector. Capital controls have allowed the People’s Bank of China to actively set short-term interest rate to maintain price stability and promote economic growth. Dickinson and Liu (2007) find there is an increasing influence of interest rates on aggregate output and private enterprises are increasingly reacting to monetary policy changes. However, it is puzzling that the reaction of state-owned firms to monetary policy shocks is not statistically significant. In this paper, we construct a dynamic stochastic general equilibrium model with heterogeneous sectors and resource misallocation that sheds light on the puzzle. There is a growing literature on the view that the extent of misallocation of resources in developing countries is quantitatively important. ⁹ We explore the model further trying to understand the implication of resource misallocation for monetary policy formulation.

The existence and importance of resource misallocation has been well documented. Banerjee and Duflo (2005) find evidence of enormous heterogeneity in rates of return to the same factors within a single economy, contradicting the assumption of optimal resource allocation within each economy. Bartelsman et al. (2008) test the relationship between productivity and size across countries. The underlining hy-

hypothesis is that more productive firms tend to be larger than less productive firms (Melitz (2003)). They find that less advanced economies exhibit a weaker relationship between size and productivity, implying that resource misallocation may be a non-negligible distortion in developing countries. Alfaro et al. (2008) calibrate a heterogeneous firm model for 79 countries. By comparing an artificial economy’s plant-size distribution with an undistorted benchmark economy (the US), they argue that output taxes and subsidies are needed for the model to match each real economy. They find that these distortion across plants are powerful explanatory factors of cross-country differences in income. Jeong and Townsend (2007) calibrate a growth model with micro underpinnings and show that 73% of the increase in TFP between 1976 and 1996 in Thailand can be explained by the effect of misallocation. Similar other papers (Banerjee and Duflo (2005), Restruccia and Rogerson (2008), Buera et al. (2008)) find that at least 50% of the TFP gap between developed and developing countries can be explained by resource misallocation. 10

China, as the largest developing country, started from a situation of severe resource misallocation but managed to ignite the engine of reallocation. Lardy (2004) argues that reforms in the financial markets have been much slower than those in product and labor markets. Interest rates are still subject to government intervention to a large degree. Brandt and Li (2003) provide direct evidence that private firms are less likely to obtain loan and more loan collateral is required for private firms. Hale and Long (2010) shows that state-owned firms continue to enjoy significantly more generous external finances than other types of Chinese firms, small firms face more financial constraints. Several empirical studies (Genevieve Boyreau-Debray and Wei (2005), Liu and Siu (2006), Dollar and Wei (2007)) document that aver-

10Banerjee and Moll (2009) did a excellent review of resource misallocation.
age private firms have significantly higher returns to capital than state-owned firms. Hsieh and Klenow (2009) shows that moving to “U.S. efficiency” would increase TFP by 30%–50% and may have boosted its TFP 2% per year over 1998–2005 by reducing its distortions.

Current monetary models for developing countries abstract from the misallocation effect. Because the misallocation distortion in developing countries is more severe than in developed countries, it may be inappropriate for developing countries to use monetary models that are aimed for developed countries’ analysis. The first contribution of this paper is the construction of a dynamic stochastic general equilibrium model for China with heterogeneous sectors and resource misallocation. State-owned firms enjoy generous external financing from bank but private firms are credit constrained through a Kiyotaki-Moore (1997)-type financial constraint. Following a positive monetary shock, the real interest rate falls. The value of private firms’ collateral increases allowing them to borrow more. Output in private sector rises. Marginal productivity of labor in private firms increases. Labor moves from state-owned firms to private firms, inducing a decrease in state-owned firms’ marginal productivity of capital. Even when the real interest rate decreases, state-owned firms may not be willing to invest. That’s the reason why output in state-owned sector is not statistically significant following a monetary policy shock, which is consistent with Dickinson and Liu (2007)’s empirical findings. Hence, the presence of resource misallocation alters standard monetary policy conclusions in important ways. Different from standard new Keynesian framework, where the real effect comes from the price rigidity, there is an additional real effect of monetary policy due to resource misallocation— we call this the allocation effect.

The second contribution of this paper is in illustrating how common shocks exert
heterogeneous effects on private firms and state-owned firms. The output volatility of private firms is higher than the output volatility of state-owned firms given a common shock. This is consistent with Bernanke et al. (1999). They find credit frictions amplify and propagate shocks to the macroeconomy. Considering that the Chinese private sector has access to the most productive technology and has experienced the fastest growth in the last three decades, the "financial accelerator" generates extra welfare cost. Higher volatility increases the uncertainty of private firms' investment, inducing underinvestment in the private sector.

A closely related paper is Song, Storesletten and Zilibotti (2011). They developed a two-sector overlap generation model with asymmetric financial imperfections, which accounts quantitatively for China’s economic transition: high output growth, sustained returns on capital, reallocation within the manufacturing sector and a large trade surplus. Our paper differs from this paper in a few ways. In Song, Storesletten and Zilibotti (2011)’ model, money plays no role and nominal shock has no real effect on macroeconomy. We introduce a Calvo-type sticky price into our model. We show that with presence of price rigidity, both nominal and real shocks have real effect on macroeconomy. Also, our model has permanent representative household and entrepreneurs, which is an augmented New Keynesian-type model. This enables us to compare our model with standard New Keynesian frameworks to gain greater insight from resource misallocation.

This paper is organized as follows. Section 3.2 introduces the model. Section 3.3 presents simulation results. Section 3.4 concludes.
3.2 The Model

3.2.1 Households

Households are identical and normalized to one. In each period, the household maximizes $E_0 \sum_{t=0}^{\infty} \beta^t U \left( c_t, \frac{M_t}{P_t}, L_t \right)$, where utility in period $t$ is given by:

$$U(c_{H,t}, L_t) = \log c_t + \gamma \log \frac{M_t}{P_t} - \chi \frac{L_t^{1+\eta}}{1+\eta}$$  \hspace{1cm} (22)

where $\chi > 0$, $\gamma > 0$, $\eta \geq 0$, $L_t$ is the labor supply in period $t$ and $c_t$ are final goods. Final goods are defined over a continuum of retailers’ goods. \textsuperscript{11}.

$$c_{H,t}^{\omega} = \left[ \int c_t(\omega) \frac{\hat{c}_t^{\omega}}{\hat{e}^{\omega}} d\omega \right]^{\frac{\hat{e}^{\omega}}{\hat{e}}}$$  \hspace{1cm} (23)

where $\theta > 1$. The consumption-based index $P_t$ can be defined below as:

$$P_t = \left[ \int p_t(\omega)^{1-e} d\omega \right]^{\frac{1}{1-e}}$$  \hspace{1cm} (24)

where $p_t(\omega)$ is the price of retailer $\omega$’s good. Finally the household’s demand for each retailer $\omega$’s good is given by:

$$c_{H,t}^{\omega} = \left( \frac{p_t(\omega)}{P_t} \right)^{-e} c_{H,t}$$  \hspace{1cm} (25)

Households enter period $t - 1$ with real deposits $d_t = D_t / P_t$ in the commercial bank. In period $t$, it receives gross interest income on deposit and real wage $w_t L_t$, where $w_t = W_t / P_t$. $T_t$ is a lump-sum tax (transfer). $F_t$ are lump-sum profits received

\textsuperscript{11}described below.
from the retailers. The budget constraint in real terms is given by:

\[ d_t + c_t^H = R_{t-1}d_{t-1}/\pi_t + w_tL_t - T_t - \frac{\Delta M_t}{P_t} + F_t \]  

(26)

where \( \pi_t = \frac{P_t}{P_{t-1}} \). Optimality requires:

\[ \frac{1}{c_t^H} = \beta_t E_t \frac{R_{t+1}}{c_{t+1}^H \pi_{t+1}} \]  

(27)

\[ \chi L_t^H c_t^H = w_t \]  

(28)

3.2.2 Entrepreneurs

Entrepreneurs use a Cobb-Douglas constant returns-to-scale technology to produce an intermediate good \( y_{E,t} \) in a competitive market. They use labor \( L_{E,t} \) and capital \( k_{E,t-1} \) as inputs:

\[ y_{E,t} = A_E k_{E,t-1}^{\gamma} L_{E,t}^{1-\gamma} \]

where \( A_E \) is the technology parameter. Following Bernanke et al. (1999) and Iacoviello (2005), we assume that if borrowers repudiate their debt obligations, the lenders can only repossess the borrowers’ assets in real term \( mE_t(k_{E,t} \pi_{t+1}/R_t) \), where \( m \leq 1 \) because of the transaction cost (Kiyotaki and Moore (1997) and Iacoviello (2005)). Thus, we have:

\[ b_t \leq mE_t(k_{E,t} \pi_{t+1}/R_t) \]
To fit China’s economy, we want a steady state in which the entrepreneurial return to savings is greater than the interest rate, which implies a binding borrowing constraint. We assume that entrepreneurs discount the future more heavily than households (Iacoviello (2005)). They maximize:

$$E_0 \sum_{t=0}^{\infty} \beta_t^E \left[ \log c_{E,t} \right]$$

where $\beta_E < \beta_H$, subject to the technology constraint, the borrowing constraint and the flow of funds in real term:

$$y_{E,t}/x_{E,t} + k_{E,t-1}(1 - \delta) + b_t = c_{E,t} + ab_{t-1}R_{t-1}/\pi_t + k_{E,t} + w_tL_{E,t}$$

where $x_{E,t} = \frac{p_t}{p_{E,t}}$ and $p_{E,t}$ is the selling price for good $y_{E,t}$ at period $t$. Optimality requires:

$$\frac{1}{c_{E,t}} - \gamma_t - E_t \frac{\beta_E R_t}{c_{E,t+1} \pi_{t+1}} = 0$$

$$- \frac{1}{c_{E,t}} + \gamma_t mE_t(\pi_{t+1}/R_t) + E_t\left[ \frac{\beta_E v y_{E,t+1}}{k_{E,t}x_{E,t+1} c_{E,t+1}} + \frac{\beta_E (1 - \delta)}{c_{E,t+1}} \right] = 0$$

$$\frac{(1 - v)y_{E,t}}{L_{E,t} x_{E,t}} - w_t = 0$$

where $\gamma_t$ is the Lagrange multiplier on the borrowing constraint.

### 3.2.3 State-Owned Firm

State-owned firms use similar production function to produce a heterogeneous intermediate good $y_{S,t}$ with technology parameter $A_S$ in a competitive market, where:
\[ y_{S,t} = A_S k_S^{v} L_{S,t}^{1-v} \]

State-owned firms can access funds from the bank at the same nominal interest rate as entrepreneurs but without constraints. Since the government backs the firms, there is no default risk. Their flow of capital is:

\[ k_S^{v} = k_{S,t-1}^{v} (1 - \delta) + i_t \]

Optimality requires:

\[ w_t = \frac{(1 - v) y_{S,t}}{L_{S,t} x_{S,t}} \]

\[ \frac{R_t}{\pi_t+1} = 1 + \frac{v y_{S,t}}{k_{S,t-1} x_{S,t}} - \delta \]

where \( x_{S,t} = \frac{p_t}{p_{S,t}} \) and \( p_{S,t} \) is the selling price for good \( y_{S,t} \) at period \( t \).

### 3.2.4 Retailers

There are a continuum of retailers of mass 1, indexed by \( \omega \), that buy intermediate goods \( y_{E,t} \) at a price \( p_{E,t} \) from entrepreneurs and \( y_{S,t} \) at a price \( p_{S,t} \) from state-owned firms. They employ the input goods at no cost to produce \( y_t(\omega) \) and sell it for a price \( p_t(\omega) \). Retailer \( \omega \)'s production functions is:

\[ y_t(\omega) = \left( \left( y_{E,t} \right)^{\frac{c-1}{c}} + \left( y_{S,t} \right)^{\frac{c-1}{c}} \right)^{\frac{c}{c-1}} \]
where $e$ is the elasticity of substitution. Consider the retailer’s cost minimization problem, which involves minimizing $y_{E,t} p_{E,t} + y_{S,t} p_{S,t}$ subject to producing $y_t(\omega)$.

Define:

$$p_t = [(p_{E,t})^{1-e} + (p_{S,t})^{1-e}]^{\frac{1}{1-e}}$$

Optimality requires:

$$y_{E,t}(\omega) = \left(\frac{p_{E,t}}{p_t}\right)^{-e} y_t(\omega), y_{S,t}(\omega) = \left(\frac{p_{S,t}}{p_t}\right)^{-e} y_t(\omega)$$

Each retailer chooses a sale price $p_t(\omega)$ taking $p_{E,t}$, $p_{S,t}$ and demand curve as given. The sale price can be changed in every period only with probability $1 - \theta$.

The optimal $p_t(\omega)$ solves:

$$\sum_{i=0}^{\infty} \theta^k E_t \{ \Lambda_{t,i} \left( \frac{p_t^*(\omega)}{P_{t+i}} \right) - \frac{X_t}{X_{t+i}} \} y_{t+i}^*(\omega) = 0$$

where $P_t$ is the CPI defined in equation (24) and $p_t^*(\omega)$ is the reset price. $y_{t+i}(\omega) = \left( \frac{p_t^*(\omega)}{R_{t+i}} \right)^{-e} Y_{t+i}$, $Y_t$ is the final good, $X_t = \frac{P_t}{p_t}$ and $\Lambda_{t,i} = \frac{\beta H_{t+i}}{C_{H_{t+i}}}$.

As a fraction $\theta$ of prices stays unchanged, the aggregate price level evolution is:

$$P_t = [\theta P_{t-1}^{1-\phi} + (1 - \theta)(P^*)_{t-1}]^{\frac{1}{1-\phi}}$$

### 3.2.5 Equilibrium

In equilibrium, all labor is employed either in entrepreneurial or state production:

$$L_t = L_{E,t} + L_{S,t}$$
Equilibrium in the output market requires that:

\[ Y_t = c_{H,t} + c_{E,t} + K_{E,t} - K_{E,t-1}(1 - \delta) + I_t \]

Banks are competitive. They receive deposits \( d_t \) from households at gross nominal interest rate \( R_t \) and loan \( b_t \) to private firms at the nominal interest rate \( R_t \) and \( i_t \) to state-owned firms at the nominal interest rate \( \tau R_t \). In equilibrium for the banking sector, the following condition holds:

\[ b_t + i_t = d_t \]

Appendix B describes the steady state. The model can be reduced to the following linearized system according to the algorithm developed by Uhlig (1999).

Endogenous variables are \( \{Y_t, c_{H,t}, c_{E,t}, k_{E,t}, k_{S,t}, b_t, d_t, L_{E,t}, L_{S,t}, L_t; R_t, \pi_t, x_{E,t}, x_{S,t}, X_t, w_t\} \):

\[ \hat{c}_{H,t} = E_t \hat{c}_{H,t+1} - \hat{r}_t \]

\[ \eta \hat{L}_t + \hat{c}_{H,t} = \hat{w}_t \]

\[ \hat{y}_{E,t} = v \hat{k}_{E,t-1} + (1 - v)\hat{L}_{E,t} \]

\[ \hat{b}_t = E_t \hat{k}_{E,t} - \hat{r}_t \]
\[
\frac{1}{x_E} (\hat{y}_{E,t} - \hat{x}_{E,t}) + \frac{k_E}{y_E} (1 - \delta) \hat{k}_{E,t-1} + \frac{b}{y_E} \hat{b}_t = \frac{c_E}{y_E} \hat{c}_t + \frac{b}{y_E} (\hat{b}_{t-1} + \hat{r}_{t-1}) + \frac{k}{y_E} \hat{k}_{E,t} + \frac{w_L E}{y_E} (\hat{w}_t + \hat{L}_{E,t})
\]

\[-m \hat{r}_t (\beta_H - \beta_E) - m \beta_E (\hat{r}_{t-1} + \hat{c}_{E,t} - E_t \hat{c}_{E,t+1}) + \beta_E (1 - \delta) [\hat{c}_{E,t} - E_t \hat{c}_{E,t+1}]
\]

\[+ \frac{\beta_E y_E}{k_E X_E} [E_t \hat{y}_{t+1} + \hat{c}_{E,t} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \hat{c}_{E,t+1}] = 0\]

\[\hat{y}_{E,t} - \hat{L}_{E,t} - \hat{x}_{E,t} = \hat{w}_t\]

\[\hat{y}_{S,t} = v \hat{k}_{S,t-1} + (1 - v) L \hat{x}_{S,t}\]

\[\hat{y}_{S,t} - \hat{L}_{S,t} - \hat{x}_{S,t} = \hat{w}_t\]

\[\frac{\hat{r}_{t-1}}{\beta_H} = \frac{v y_S}{k_S X_S} (\hat{y}_{S,t} - \hat{k}_{S,t-1} - \hat{x}_{S,t})\]

\[\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t\]

34
\[X^{e-1} = (ax_E)^{e-1} \hat{x}_{E,t} + ((1-a)x_S)^{e-1} \hat{x}_{S,t}\]

\[
\frac{b}{Y} \hat{b}_t + \frac{k_S}{Y} \hat{k}_{S,t} - \frac{k_S}{Y} \hat{k}_{S,t-1}(1 - \delta) = \frac{d}{Y} \hat{d}_t
\]

\[
\frac{\hat{L}_t}{L^{-(\eta+1)}} = \frac{L_S}{L^{1-\eta}} \hat{L}_{S,t} + \frac{L_E}{L^{1-\eta}} \hat{L}_{E,t}
\]

\[
\hat{Y}_t = \frac{cH}{Y} \hat{c}_{H,t} + \frac{cE}{Y} \hat{c}_{E,t} + \frac{k_E}{Y} \hat{k}_{E,t} - \frac{k_E}{Y} \hat{k}_{E,t-1}(1 - \delta) + \frac{k_S}{Y} \hat{k}_{S,t} - \frac{k_S}{Y} \hat{k}_{S,t-1}(1 - \delta)
\]

\[
\hat{y}_{E,t} = e \hat{x}_{E,t} - e \hat{X}_t + \hat{Y}_t
\]

\[
\hat{y}_{S,t} = e \hat{x}_{S,t} - e \hat{X}_t + \hat{Y}_t
\]

Central bank policy rule takes the form following Zhang (2009):

\[
\hat{R}_t = \gamma_1 \hat{R}_{t-1} + \gamma_2 (E_t \hat{\pi}_{t+1} - \pi_t) + \gamma_3 E_t \hat{\pi}_{t+1} + \gamma_4 \hat{Y}_t
\]
3.3 Calibration and Simulation

Because China’s data are relatively scarce and have undergone constant economic reform, we assign values to the parameters according to recent empirical works in the literature. Following the estimate of He et al. (2007), $\delta$ is set at 0.04 and $v$ is set at 0.6. According to Zhang’s (2009) estimation, $\theta = 0.84$, $e = 4.61$, $\gamma_1 = 0.75$, $\gamma_2 = 0.65$, $\gamma_3 = 0.1$, $\gamma_4 = 0.15$. Consistent with Iacoviello (2005), Zhang (2009) and Song, Storesletten and Zilibotti (2011), we assign 0.99 to $\beta_H$ and 0.98 to $\beta_E$. We set $\eta$ equal to 6.16 following Liu (2007). The share of capital private firms can pledge to repay is $m = 0.64$.

3.3.1 Monetary Policy Shock

The right figure comes from Dickison and Liu (2007). It is puzzling that the reaction of state-owned firms to monetary policy shocks is different from that of private firms. We show the simulation results following a monetary policy shock match empirical findings. Results are in Figure 4. Resource misallocation plays a key role in the puzzle of heterogeneous effect of monetary policy on private firms and state-owned firms’ output. With sticky price, a positive monetary shock reduces real interest rate. The ability of private firms to borrow more because of the rising value of collateral increases the marginal productivity of labor in private sector. However, there is a crowd-out effect on labor in state-owned sector—private sector firms are more attractive to workers. On one hand, the marginal productivity of capital in state-owned firms may decrease because labor shifts to the private sector. On the other hand, the borrowing cost, i.e. the real interest rate, decreases because of the monetary shock. As such, output in the state-owned sector may not be determined.
Figure 4: Impulse Response Following a Monetary Policy Shock For Private Sector and State-owned Sector

The left figure presents simulated results from our model following a monetary policy shock: HyE represents the output deviation in the private sector; HyS represents the output deviation in the state-owned sector. The right figure is an impulse response graph from Dickison and Liu (2007): YPE represents the output deviation in the private sector; YSOE represents the output deviation in the state-owned sector after an interest rate (CBLR) shock.

Figure 5 shows the additional real effect of monetary policy—the allocation effect. Simulated results in Figure II verify our argument on how labor share and capital moves in these two sectors. At the beginning of a monetary policy shock, labor share in the private sector increases and in the state-owned sector decrease. Capital shows a similar pattern. For state-owned firms, the benefit from the reduction of cost of capital has been offset by the loss from the drop of marginal productivity of capital.
3.3.2 Technology Shock

State-owned firms are, on average, less productive and have better access to external credit than do private firms. In the last three decades, the employment share and output share in the private sector are increasing while shares in state-owned firms are decreasing (Song, Storesletten and Zilibotti(2011)). The employment share in private sector increases from 4% in 1998 to 56% in 2007. Our model is not a growth model, but still we can do an experiment. If we consider a positive temporary technology...
shock with persistence parameter 0.5 in the private sector, we can see both shares significantly increases and last for about 15 periods. Results are in Figure 6. If we want to consider a permanent technology shock (larger $A_E$) in private sector, we will find another steady state with higher capital stock and higher labor share in private sector. Both results are consistent with China’s reality.

Figure 6: Positive Technology Shock in Private Sector

Labor and output deviations following a technology shock in private sector: HLE represents labor in the private sector; HLS represent labor in the state-owned sector; HyE represents output in the private sector; HyS represent output in the state-owned sector.

3.3.3 Output Volatility

Bernanke et al. (1999) find that credit frictions amplify and propagate shocks to the macroeconomy. Following a monetary policy shock, our model shows a similar pattern: the volatility of private sector is almost 50% higher than state-owned sector. The unanticipated decline in the real interest rate stimulates the demand for capital, which in turn raises investment and asset price. Raised asset prices in turn alleviate the borrow constraint and simulate investment further. We also simulate the result for
a common technology shock and we find that the volatility of private firm is higher than private firms but at less degree than a monetary policy shock. This is because state-owned firms enjoy significantly more generous external finances than private firms.

3.4 Conclusion

Resource misallocation is quantitatively important and alters standard monetary policy conclusion in important ways. In this paper, we construct a dynamic stochastic general equilibrium model with heterogeneous sectors for China and focus on the implication of resource misallocation on monetary policy. This model helps us to explain the puzzle found in Dickinson and Liu (2007)—heterogeneous response of state-owned sector and private sector. We identify an additional real effect of monetary policy—the allocation effect. Simulated results shows that credit frictions amplify the output volatility of private firms. However, this model is relatively simplified. At least one extension can be done to strengthen this study. Firm’s entry and exit in developing countries can be large, which may amplify the effect of resource misallocation on the economy. Bilbiiee, Ghironi and Melitz (2007) could add new insights to reinforce and complement our theory.
4 Maximizing Predictability for Carry Trade

4.1 Introduction

Uncovered interest rate parity (UIP) implies that high interest rate currencies should depreciate compared to low interest rate currencies. However, a large literature has shown it doesn’t hold in reality.\textsuperscript{12} In fact, they appreciate on average.\textsuperscript{13} Market participants can take advantage of this failure, selling forward currencies that are at a forward premium and buying forward currencies that are at a forward discount—carry trade.

In the literature, the hypothesis that UIP don’t hold in reality is mostly tested with the classic Fama (1984) model. Based on this model, Backus, Gregory, and Telmer (BGT) (1993), argue that the statistical properties of forward and spot exchange rates imply predictable returns from currency speculation. By using BGT strategy, Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006), shows that the currency speculation strategies yield high Sharpe ratio that are not a compensation for risk, and this strategy produce a higher Sharpe ratio than that of S&P 500. Burnside, Rebelo, and Kleshchelski (2008) expand their previous research and argue that there are large diversification gains from forming portfolios of currency strategies: diversification boosts the typical Sharpe ratio by over 50%. Darvas (2009) shows that carry trade is significantly profitable for most currency pairs and portfolios based on the analysis of 11 major currencies and 11 portfolios.

There are a number of methods to construct a portfolio for a carry trade. The most

\textsuperscript{12}See Engel (1996) for surveys of the literature.
\textsuperscript{13}Farhi and Gabix (2008) also present a comprehensive survey of the more recent literature and propose a alternative explanation for the fail of UIP.
popular one is the optimally weighted strategy, which computes the portfolio frontier and calculates the portfolio weights that maximize the Sharpe ratio. Burnside et al. (2006) shows that the Sharpe ratios of optimally-weighted portfolio strategy are substantially higher than those of the equally weighted portfolio strategy, another strategy that allocates equal weight to each currency. This paper proposes an alternative carry trade strategy by using the maximal $R^2$ method pioneered by Lo and Mackinglay (1997) and Cheung et al. (1997) to maximize the predictability by choosing portfolio weight optimally. Lo and Mackinglay (1997) constructed a portfolio that is maximally predictable with respect to a set of economics variables. And they showed the level of predictability is statically significant and economically significant. Cheung et al. (1997) employ a similar method to study the international stock market returns and found significantly increased adjusted $R^2$. Whereas the coefficients of the regression measure the sensitivity of the maximally predictable portfolio to various factors, it is the portfolio weights of the maximally predictable portfolios that tell us which assets are the most important sources of predictability. These portfolio weights give us an alternative method to form a carry trade strategy.

In this paper, we construct the optimally weighted strategy and maximal R square strategy and calculate the Sharpe ratio respectively. Comparing the result, we find that the maximal $R^2$ strategy outperforms the optimally weighted strategy. The reason may be that the optimally weighted strategy assumes that the variance and mean of the return is constant for each currency, which may be not true in reality. The exchange rate gets more volatile in recent years, especially during the subprime lending crisis. On the other side, the maximal R square adjusts the weights to maximize the prediction of future variables, giving us a relatively better result when the market is volatile.
The paper is organized as follows. Section 4.2 briefly describes the data. Section 4.3 and 4.4 present the methodologies and empirical results: optimally weighted strategy and maximal R square strategy. Section 4.5 compares the two methods based on different time interval analysis. Section 4.6 concludes.

### 4.2 Data

The data set for the analysis covers a panel of six countries: Australia, Japan, New Zealand, Norway, Sweden and Switzerland. These currencies are usually selected to apply the carry trade strategy based on the following criteria: selling the currency with a weaker economy and buying a currency with a stronger currency; selling the currency with the lower interest rate and buying the currency with the higher interest rate; lining trader’s position with the currency expectation. The sample period covers January 1986 to December 2008. Overall, the sample includes 276 observations for each of the six countries.

The forward exchange rate is obtained from DATASTREAM, originally sourced from Barclay’s Bank International, observed at a monthly frequency. The data on spot exchange is obtained from Federal Reserve Bank of St. Louis, also observed at a monthly frequency. All the forward and spot exchange rates are the first working date rates for each month, quoted as foreign currency per U.S. dollar. We assume U.S dollar is the home currency and all the returns are based on U.S. dollar. The data for consumer price index is obtained from the IFS database.
4.3 Optimally weighted strategy

Because of the failure of UIP, the well-known parity condition doesn’t hold:

\[ S_t (1 + R^*_t) E_t S_{t+1} \neq 1 + R_t \]  \hspace{1cm} (29)

Here \( R^*_t \) is the monthly foreign interest rate denominated in foreign currency. \( R_t \) is the monthly domestic interest rate denominated in domestic currency. \( S_t \) and \( S_{t+1} \) are the spot exchange rate in \( t \) and \( t + 1 \) respectively, defined as foreign currency per U.S. dollar. The carry trade strategy consists of borrowing low interest rate currency and lending the high interest rate currency. Assume \( y_t \) is the amount of U.S. dollars borrowed. The payoff to this strategy denominated in U.S dollars is:

\[ y_t [S_t (1 + R^*_t)/S_{t+1} - (1 + R_t)] \]  \hspace{1cm} (30)

If the covered interest parity condition holds:

\[ S_t (1 + R^*_t)/F_t = 1 + R_t \]  \hspace{1cm} (31)

Here \( F_t \) the monthly forward exchange rate is the first working date rate for each month. Combining equation (30) and (31) we have:

\[ x_t [F_t/S_{t+1} - 1] \]  \hspace{1cm} (32)

Where

\[ x_t = y_t (1 + R_t) \]
$x_t$ is the number of U.S. dollars sold forward. The reason we use forward and spot exchange rates to calculate the return is that compared with lending high-yielding currencies and borrowing low-yielding method, this calculation has lower transaction cost.

With equation (32), I set $x_t = 1$, and calculate the return for every month for each currency.

Figure 7: Realized Payoffs (12-month Moving Average)

Figure 7 displays realized payoffs (in U.S. dollar) for each currency. Since the payoffs are volatile, we graph the 12-month moving average payoff for each currency. On average, we can see relative long periods of positive or negative payoffs for each currency. There should be a diversified gain if rational investors use diversification to optimize their portfolios. Based on the classical finance theory, we can find the efficient frontier. At each time $t$, the optimization problem is:

$$\min\{w_t \sum w_t\}$$  \hspace{1cm} (33)
Subject to:

\[ w'_t r_t = r_p, w'_t 1 = 1, -1 \leq w \leq 1 \]

We assume the \( w_t \) is a weight vector for the currencies at time \( t \). \( \Sigma \) is the variance and covariance matrix for the six currencies calculated based on the historical return (from time 0 to time \( t \)). \( r_t \) is a vector of expected return at time \( t \). \( r_p \) is given as the portfolio return. Here we assume \( w_t \) is between negative one and positive one. Buying or selling forward exchange rate is an option. Solving this problem we can get the optimal weight for each currency. Using these weights we can calculate the realized return for \( t + 1 \). For time \( t + 1 \), we calculate \( \Sigma \) based on the historical return from time 0 to \( t + 1 \). Then solving (33) again, we obtain the optimal weight and the realized return for \( t + 2 \). Iterating this process, we get a matrix of optimal weights and a vector of realized returns from time \( t + 1 \) to \( T \).

Table 4: Minimal Variance for 2008

<table>
<thead>
<tr>
<th>Weight(sum up to 100%)</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td></td>
</tr>
<tr>
<td>1/1/2008</td>
<td>-17.89%</td>
</tr>
<tr>
<td>2/1/2008</td>
<td>-17.49%</td>
</tr>
<tr>
<td>3/1/2008</td>
<td>-19.64%</td>
</tr>
<tr>
<td>4/1/2008</td>
<td>-21.55%</td>
</tr>
<tr>
<td>5/1/2008</td>
<td>-21.54%</td>
</tr>
<tr>
<td>6/1/2008</td>
<td>-18.31%</td>
</tr>
<tr>
<td>7/1/2008</td>
<td>-15.91%</td>
</tr>
<tr>
<td>8/1/2008</td>
<td>-19.06%</td>
</tr>
<tr>
<td>9/1/2008</td>
<td>-20.41%</td>
</tr>
<tr>
<td>10/1/2008</td>
<td>-28.91%</td>
</tr>
<tr>
<td>11/1/2008</td>
<td>-31.99%</td>
</tr>
<tr>
<td>12/1/2008</td>
<td>-30.54%</td>
</tr>
<tr>
<td>Mean</td>
<td>-21.94%</td>
</tr>
</tbody>
</table>

Mean Sharpe Ratio: -0.480131

46
The result in table 4 shows the optimal weights and returns for each month in 2008. For example, for January, 2008, we should short sell Australian dollar (17.89%), Japanese Yen (1.94%), Swedish Krona (3.45%) and Switzerland Franc (24.57%), and long New Zealand dollar (100%) and Norwegian Krone (47.86%). Each dollar will generate a portfolio return of 3.83% for January 2008. The mean weight represent the arithmetic mean weight for the year of 2008. We can see that the average return is negative in 2008, which is -2.78%. This is not surprising because of the subprime lending crisis and the very volatile change of forward and spot exchange rates. This can also be found in figure 7; the returns jump around in 2007 and 2008. In section 5, we will show the returns for the recent years and the Sharpe ratios. Here we emphasized the mean and variance of the payoffs to the currency speculation. These statistics are sufficient to characterize the distributions of the payoffs only if they are normal. Figure 8 shows the histogram (blue bar) of the return based on U.S. dollar payoff for each currency. We also plot a normal distribution (red line) with the same mean and variance as the empirical distribution for each currency. From the figure we can see some skewness and excess kurtosis, but the Jarque-Bera normality test indicates that the null hypothesis (“the data are normally distributed”) cannot be rejected at the 5% significant level (all h with value zero).

\[14\] For Matlab 2011: Jarque-Bera test: h = jbtest(x) performs a Jarque-Bera test of the null hypothesis that the sample in vector x comes from a normal distribution with unknown mean and variance, against the alternative that it does not come from a normal distribution. The test returns the value h = 1 if it rejects the null hypothesis at the 5% significance level, and h = 0 if it cannot. Source: http://www.mathworks.com/help/toolbox/stats/jbtest.html
4.4 Maximal R square strategy

To apply this strategy, the first thing we need to figure out is how the exchange rate is predicted. However, it’s difficult to find such a model. The research under this area is still undergoing. It’s not our intention to select the best exchange rate forecasting combination. Our objective in this paper is to compare the optimal weight strategy with a maximal $R^2$ strategy. So our choice of variables is mainly drawn from the BGT model, which is a revisited version of Fama (1984)’s model. This model has been tested in the literature and shown to yield good results in the field of carry trade.

Generally, the Fama (1984)’s model argues that the common evidence for pre-

---

15Meese and Rogoff (1983) questioned the standard exchange rate models: “do they beat the random-walk model for forecasting changes in exchange rates?” Cheung et al (2005) test a wide set of models, like PPP, UIP, productivity based model, composite specification model and sticky-price monetary model, find that it’s very difficult to find forecasts from a structural model that can consistently beat the random walk model. However, Engel and West (2004) argue that the actually the expectation of the future exchange rate matters a lot, and the innovation of current fundamentals may not have a large effect on the exchange rate.

dictability of the return comes from regressions of the form:

\[
\frac{(F_t - S_{t+1})}{S_t} = a + b\frac{(F_t - S_t)}{S_t} + u_{t+1}
\]  

(34)

\(S_t(F_t)\) denotes the spot(forward) exchange rate defined as foreign currency units
per unit of U.S. dollar at time \(t\). The logic is the same as borrowing low-interest-rate
currency and lending the high-interest-rate currency shown in section 3. The way to
execute the trade is that: selling (buying) forward when the payoff predicted by the
regression is positive (negative). The advantage of this method is that we can have
longer data set available and using futures execute lower transaction cost. Suppose
\(1/S_{t+1}\) is a martingale, then:

\[
E_t\left(\frac{1}{S_{t+1}}\right) = \frac{1}{S_t}
\]

(35)

The equation (34) can be roughly changed to the BGT model:

\[
\frac{(F_t - S_{t+1})}{S_{t+1}} = a + b\frac{(F_t - S_t)}{S_t} + u_{t+1}
\]  

(36)

The left hand side is equal to the return when selling (buying) forward when the
payoff predicted by the regression is positive(negative), which is the same in equation
(32).

Burnside et al (2006) based on this model to form carry trade strategy, comparing
minimal variance strategy and equally weighted strategy, find that a minimal variance
strategy significantly outperformed an equally weighted strategy and the total return
of a minimal variance strategy has a very similar return to that of the S&P 500 but
much smaller volatility. For this paper, we are going to revise the model to apply the
method of maximal R square. The notation here is similar to Cheung et al. (1997).

### 4.4.1 Step One:

For each country $j$ we have the following regression equation, which is an augmented BGT model:

$$\frac{(F_j^t - S_{jt+1})}{S_{jt+1}} = \alpha_j + \sum_{i=1}^{6} \beta_{ji} \frac{(F_{it}^j - S_{it}^j)}{S_{it}^j} + u_{jt+1} \quad (37)$$

Instead of just using one country’s previous exchange rate variable as independent variable, I use all the countries’ previous exchange rates as independent variables to explain each country’s next period exchange rate, because the maximal $R^2$ method maximizes a portfolio return (here we have six exchange rates) systematically. The portfolio return should be explained by the six country’s previous exchange, since, based on BGT model, each one of them is a good independent variable for the related dependent variable. What is more, the purpose of maximizing the $R^2$ is to find a diversified portfolio that gives us the most predictable return. The higher the currency correlation is, the higher prediction. It’s reasonable to include other exchange rates as independent variables. Also, because the world capital market is getting integrated closer and closer, for a single country, like Japan, the exchange rate with one country should be related to the exchange rate with another one, although not complete related. We will found that other countries’ previous exchange rates are also good independent variable. All the coefficients are significant.

### 4.4.2 Step Two

After we estimate equation (37) based on the time interval $[0, t]$, we obtained the
coefficients for each country. Then we have the following equation:\(^{17}\)

\[
Z_t = B'X_{t-1} + \epsilon_t
\]  

(38)

Where

\[
Z_t = (Z_t^1, Z_t^2, Z_t^3, Z_t^4, Z_t^5, Z_t^6)
\]

\[
Z_t^j = (F_t^j - S_t^j) / S_t^j, j = 1, 2, 3, 4, 5, 6
\]

\[
X_{t-1} = (X_{t-1}^1, X_{t-1}^2, X_{t-1}^3, X_{t-1}^4, X_{t-1}^5, X_{t-1}^6)
\]

\[
X_{t-1}^j = (F_{t-1}^j - S_{t-1}^j) / S_{t-1}^j, j = 1, 2, 3, 4, 5, 6
\]

Let \( \gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6)' \), representing the weight vector for the portfolio. Then the model can be described as:

\[
\gamma'Z_t = \gamma'B'X_{t-1} + \gamma'\epsilon_t
\]  

(39)

When regressing \( \gamma'Z_t \) on \( X_{t-1} \), the \( R^2 \) is given by

\[
R^2 = \frac{\gamma'\Gamma_x \gamma}{\gamma'\Gamma_z \gamma}
\]  

(18)

Where

\[
\Gamma_x = \text{var}(B'X_{t-1})
\]  

\(^{17}\)Move one period backward for empirical analysis. 

\(^{18}\)Here a constant independent variable is included.
\[ \Gamma_z = \text{var}(Z_t) \]

Maximizing \( R^2(\gamma) \) we can find the optimal weight \( \gamma' \). With these optimal weights, we can calculate the realized return at time \( t + 1 \).

**4.4.3 Step Three**

Redo step one and step two based on the time interval \([0, t + 1]\). We can get the optimal weights and realized return at time \( t + 2 \). Iterating this process, we can get a matrix of optimal weight and a vector of realized return from time \( t + 1 \) to \( T \).

The result in table 6 shows the regression result based on the sample 1986-2008. All the coefficients passed the t-test at 5% significant level and the whole model also passed the F-test at 5% significant level. The adjusted \( R^2 \) from maximal \( R^2 \), which is 0.6803, is higher than that of any individual regression. \( \alpha \) represents the average forward premium that can’t be explained by independent variable. \( \beta \) shows the effect of each unit change of independent currency’s forward premium in period \( t \) on dependent currency’s forward premium in period \( t + 1 \). For example, \( \beta_4 \) for Australia means that, if the forward premium for Norwegian Krone increases one unit in period \( t \), the forward premium for Australian dollar will increase \( \beta_4 \) units in period \( t + 1 \), given no change for other independent variables. The row of optimal weight shows the weight for each currency, with which we can achieve the highest \( R^2 \) for equation (39).
Table 5: Maximal R Square regression result based on equation (37)

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Japan</th>
<th>New Zealand</th>
<th>Norway</th>
<th>Sweden</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.000611</td>
<td>0.004069</td>
<td>-0.004039</td>
<td>0.000574</td>
<td>-0.00038</td>
<td>0.0029664</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1.555577</td>
<td>0.042601</td>
<td>0.160962</td>
<td>0.110968</td>
<td>0.090341</td>
<td>0.0536258</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.0935264</td>
<td>1.489833</td>
<td>0.121884</td>
<td>0.032609</td>
<td>-0.10094</td>
<td>-0.063155</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.048722</td>
<td>0.049459</td>
<td>1.410200</td>
<td>-0.07195</td>
<td>-0.05024</td>
<td>0.0011188</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.2329834</td>
<td>0.01251</td>
<td>0.167621</td>
<td>1.640245</td>
<td>0.092896</td>
<td>0.2495274</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>-0.054622</td>
<td>-0.26574</td>
<td>0.005301</td>
<td>0.042558</td>
<td>1.652243</td>
<td>-0.107906</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>-0.179491</td>
<td>0.337111</td>
<td>-0.107779</td>
<td>-0.01242</td>
<td>0.078736</td>
<td>1.5584179</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.620761</td>
<td>0.600302</td>
<td>0.659433</td>
<td>0.636555</td>
<td>0.649219</td>
<td>0.5782503</td>
</tr>
<tr>
<td>Optimal weight</td>
<td>-20.56%</td>
<td>49.82%</td>
<td>100.00%</td>
<td>54.58%</td>
<td>-56.55%</td>
<td>-27.30%</td>
</tr>
<tr>
<td>Maximal $R^2$</td>
<td>0.6803</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

all $\alpha$ and $\beta$ are significant at 5%

Table 6 shows the optimal weight and return for 2008 by using the maximum R square method. We can see that the return is average return from this strategy is also negative in 2008, but with a better result, a much smaller negative return than minimal variance strategy. In section 5, we will show more details analysis of return and Sharpe ratio based on these two methods— minimal variance and maximum $R^2$. 
Table 6: Maximal R Square for 2008 based on equation (37)

<table>
<thead>
<tr>
<th></th>
<th>Weight(sum up to 100%)</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Australia</td>
<td>Japan</td>
</tr>
<tr>
<td>1/1/2008</td>
<td>-33.80%</td>
<td>65.36%</td>
</tr>
<tr>
<td>2/1/2008</td>
<td>-31.78%</td>
<td>64.72%</td>
</tr>
<tr>
<td>3/1/2008</td>
<td>-28.71%</td>
<td>64.52%</td>
</tr>
<tr>
<td>4/1/2008</td>
<td>-21.21%</td>
<td>63.20%</td>
</tr>
<tr>
<td>5/1/2008</td>
<td>-18.47%</td>
<td>65.72%</td>
</tr>
<tr>
<td>6/1/2008</td>
<td>-15.10%</td>
<td>66.13%</td>
</tr>
<tr>
<td>7/1/2008</td>
<td>-20.07%</td>
<td>65.80%</td>
</tr>
<tr>
<td>8/1/2008</td>
<td>-26.17%</td>
<td>68.75%</td>
</tr>
<tr>
<td>9/1/2008</td>
<td>-25.17%</td>
<td>65.83%</td>
</tr>
<tr>
<td>10/1/2008</td>
<td>-23.70%</td>
<td>63.63%</td>
</tr>
<tr>
<td>11/1/2008</td>
<td>-24.95%</td>
<td>52.20%</td>
</tr>
<tr>
<td>12/1/2008</td>
<td>-29.32%</td>
<td>46.42%</td>
</tr>
<tr>
<td>Mean</td>
<td>-24.87%</td>
<td>62.69%</td>
</tr>
</tbody>
</table>

Sharpe Ratio: -0.14753

Until now, this model fails to consider the fundamentals—the real exchange rate. Jorda and Taylor (2009) argue that the deviation from the fundamental equilibrium exchange rate (FEER) is an important predictor of exchange rate movements. All else equal, expected return is lower when the target currency is overvalued. However, Engel and West (2005) argue that the new present value models of exchange rates highlights the role of expectations in determining exchange rate movements. If we consider the forward exchange rate is a representative that including the expectation in determining exchange rate movements, we fail to consider the fundamentals.

For the following, we are going to incorporate the fundamental variables into the augmented BGT model. We define a new variable:

$$\Delta_t^j = \ln(S_t^j) + [\ln(CPI_t^j) - \ln(CPI_t^{US})]$$ (40)
Under the weak relative version of the purchasing power parity, $\Delta^j_t$ is the deviation from its long-run fundamental equilibrium.

In order to apply the maximal R square method, we need to find the common predictable components for the currency speculative return. As we did above, we incorporate six countries’ $\Delta^j_t$ as independent variables for each currency. The argument is similar. We assume that the world is integrating and any fundamental changes will affect all other countries. For the regression, it’s simple. We just need to change the independent variables, the new independent variables is that:

$$X_t = \left( \begin{array}{c} X_{t-1} \\ \Delta^j_{t-1} \end{array} \right)$$

(41)

Where

$$\Delta_{t-1} = (\Delta^1_{t-1}, \Delta^2_{t-1}, \Delta^3_{t-1}, \Delta^4_{t-1}, \Delta^5_{t-1}, \Delta^6_{t-1})'$$

For easy reading, we write out the regression equation for each currency:

$$\frac{(F^j_{t+1} - S^j_{t+1})}{S^j_{t+1}} = \alpha^j + \sum_{i=1}^{6} \beta^j_i \frac{(F^i_{t+1} - S^i_{t+1})}{S^i_{t+1}} + \sum_{i=1}^{6} \phi^j_i S \Delta^i_{t+1} + u^j_{t+1}$$

(42)

Then the steps follow. We can get a matrix of optimal weights and a vector of realized returns from time $t + 1$ to $T$.

Table 7 reports the regression result based on the sample 1986-2008. All the coefficients passed the t-test at 5% significant level and the whole model also passed the F-test at 5% significant level. The adjusted $R^2$ from maximal $R^2$, which is 0.6987, a little bit higher than previous one and higher than individual regression. The meaning of $\alpha$ and $\beta$ is similar to those in table 7. $\phi$ here represent the effect of independent currency’s fundamentals on dependent currency’s forward premium. For example, $\phi_4$
shows the effect of one unit change of Norwegian Krone’s fundamental on Australian dollar’s forward premium.

Table 7: Maximal R Square regression result based on equation (42)

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Japan</th>
<th>New Zealand</th>
<th>Norway</th>
<th>Sweden</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.08221</td>
<td>-0.32218</td>
<td>-0.08385</td>
<td>-0.25021</td>
<td>-0.23134</td>
<td>0.010794</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.612433</td>
<td>0.070538</td>
<td>0.154773</td>
<td>0.094537</td>
<td>0.120014</td>
<td>0.111408</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.10183</td>
<td>1.486902</td>
<td>0.134313</td>
<td>0.051035</td>
<td>-0.08668</td>
<td>-0.05901</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.12008</td>
<td>-0.01056</td>
<td>1.457888</td>
<td>-0.06438</td>
<td>-0.11205</td>
<td>-0.08737</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>0.219999</td>
<td>0.026211</td>
<td>0.161009</td>
<td>1.664968</td>
<td>0.103671</td>
<td>0.209323</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>-0.04366</td>
<td>-0.30567</td>
<td>0.014108</td>
<td>0.019209</td>
<td>1.624481</td>
<td>-0.10272</td>
</tr>
<tr>
<td>( \beta_6 )</td>
<td>-0.18648</td>
<td>0.329038</td>
<td>-0.16219</td>
<td>-0.02716</td>
<td>0.08489</td>
<td>1.590306</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>0.107817</td>
<td>-0.02212</td>
<td>0.033111</td>
<td>-0.01229</td>
<td>0.026487</td>
<td>0.05535</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>0.026333</td>
<td>0.05318</td>
<td>0.017248</td>
<td>0.026329</td>
<td>0.034057</td>
<td>0.008118</td>
</tr>
<tr>
<td>( \phi_3 )</td>
<td>-0.05364</td>
<td>0.001877</td>
<td>0.013511</td>
<td>-0.02409</td>
<td>-0.049</td>
<td>-0.04668</td>
</tr>
<tr>
<td>( \phi_4 )</td>
<td>-0.0233</td>
<td>-0.00572</td>
<td>-0.051</td>
<td>0.063947</td>
<td>0.026084</td>
<td>-0.04331</td>
</tr>
<tr>
<td>( \phi_5 )</td>
<td>-0.00092</td>
<td>0.054013</td>
<td>0.041606</td>
<td>0.017059</td>
<td>0.022081</td>
<td>0.014555</td>
</tr>
<tr>
<td>( \phi_6 )</td>
<td>-0.01991</td>
<td>-0.05699</td>
<td>-0.02331</td>
<td>-0.0453</td>
<td>-0.02747</td>
<td>0.039627</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.647276</td>
<td>0.612624</td>
<td>0.680548</td>
<td>0.646663</td>
<td>0.662254</td>
<td>0.591168</td>
</tr>
<tr>
<td>Optimal weight</td>
<td>-14.61%</td>
<td>48.89%</td>
<td>100.00%</td>
<td>46.46%</td>
<td>-58.62%</td>
<td>-22.63%</td>
</tr>
<tr>
<td>Maximal ( R^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.6987</td>
</tr>
</tbody>
</table>

Table 8 shows the optimal weights and returns for 2008 by using maximum \( R^2 \) method. We can see that the return is average return from this strategy is still negative in 2008. However, Sharpe ratio is lower than previous. If we consider the volatility of new variables, the fundamental—real exchange rate in the subprime lending crisis, the result is not surprising. In section 5, we will show more details analysis of return and Sharpe ratio based on these three methods.
<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Japan</th>
<th>New Zealand</th>
<th>Norway</th>
<th>Sweden</th>
<th>Swiss</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/2008</td>
<td>-18.50%</td>
<td>55.02%</td>
<td>100.00%</td>
<td>34.78%</td>
<td>-47.99%</td>
<td>-0.23315</td>
<td>0.051402</td>
</tr>
<tr>
<td>2/1/2008</td>
<td>-17.79%</td>
<td>54.72%</td>
<td>100.00%</td>
<td>32.06%</td>
<td>-47.57%</td>
<td>-0.21419</td>
<td>0.026044</td>
</tr>
<tr>
<td>3/1/2008</td>
<td>-17.50%</td>
<td>54.50%</td>
<td>100.00%</td>
<td>32.32%</td>
<td>-47.81%</td>
<td>-0.2151</td>
<td>-0.01705</td>
</tr>
<tr>
<td>4/1/2008</td>
<td>-12.54%</td>
<td>52.48%</td>
<td>100.00%</td>
<td>38.97%</td>
<td>-55.70%</td>
<td>-0.23212</td>
<td>0.00614</td>
</tr>
<tr>
<td>5/1/2008</td>
<td>-10.51%</td>
<td>54.74%</td>
<td>100.00%</td>
<td>41.43%</td>
<td>-59.65%</td>
<td>-0.26018</td>
<td>-0.02666</td>
</tr>
<tr>
<td>6/1/2008</td>
<td>-7.88%</td>
<td>54.76%</td>
<td>100.00%</td>
<td>30.40%</td>
<td>-59.87%</td>
<td>-0.17412</td>
<td>-0.04225</td>
</tr>
<tr>
<td>7/1/2008</td>
<td>-11.11%</td>
<td>55.47%</td>
<td>100.00%</td>
<td>28.80%</td>
<td>-53.87%</td>
<td>-0.19291</td>
<td>-0.03834</td>
</tr>
<tr>
<td>8/1/2008</td>
<td>-8.38%</td>
<td>56.02%</td>
<td>100.00%</td>
<td>19.06%</td>
<td>-78.87%</td>
<td>0.121701</td>
<td>-0.00051</td>
</tr>
<tr>
<td>9/1/2008</td>
<td>-20.71%</td>
<td>59.69%</td>
<td>100.00%</td>
<td>28.00%</td>
<td>-45.24%</td>
<td>-0.21741</td>
<td>-0.00726</td>
</tr>
<tr>
<td>10/1/2008</td>
<td>-20.90%</td>
<td>58.89%</td>
<td>100.00%</td>
<td>28.07%</td>
<td>-43.90%</td>
<td>-0.22152</td>
<td>-0.03809</td>
</tr>
<tr>
<td>11/1/2008</td>
<td>-20.19%</td>
<td>47.91%</td>
<td>100.00%</td>
<td>32.16%</td>
<td>-34.99%</td>
<td>-0.24887</td>
<td>-0.01853</td>
</tr>
<tr>
<td>12/1/2008</td>
<td>-20.16%</td>
<td>47.62%</td>
<td>100.00%</td>
<td>33.95%</td>
<td>-39.35%</td>
<td>-0.22053</td>
<td>0.028545</td>
</tr>
<tr>
<td>Mean</td>
<td>-15.52%</td>
<td>54.32%</td>
<td>100.00%</td>
<td>31.67%</td>
<td>-51.24%</td>
<td>-19.24%</td>
<td>-0.64%</td>
</tr>
</tbody>
</table>

4.5 Compare the Two Strategies.

In this section, we will consider different time intervals, and calculate the related Sharpe ratio for different strategies.

For the minimal variance strategy, we calculate the Sharpe ratio for each year (2004-2008). For example, in order to calculate the return for January 2004, we calculate the variance and mean for each currency based on data set from 1986.1-2003.12. Then we minimal variance subject to the expected return and the weight constraints. Solving this model we can get the optimal weights for January 2004. Using these weights we can calculate the optimal return. Iterating this we can solve all the return for 2004. Applying the same method, we can calculate the Sharpe ratio for 2004, also 2005, 2006, 2007 and 2008.
For the maximal $R^2$ strategy, we also calculate the Sharpe ratios for each year (2004-2008). For example, in order to calculate the return for January 2004, we regress equation (37) and equation (42) separately based on the data 1986.1-2003.12. Then we maximize the $R^2$ subject to the weight constraints respectively to get the optimal weights for January 2004. Using these weights we can calculate the return. Iterating this we can solve all the returns for 2004. Applying the same steps, we can calculate the Sharpe ratio for 2004, also 2005, 2006, 2007 and 2008.

The results are reported in table 9. We can see that the Maximal R square based on equation (42) is relative better than Maximal $R^2$ based on Equation (37). Out of the five years, for three years maximal $R^2$ based on (42) has a higher Sharpe ratio. Also, our results shows that maximal $R^2$ strategy can get relatively higher Sharpe ratio than minimal variance strategy, even though some of the results are negative when the market is volatile.

If we compare the overall Sharpe ratio, we can see this result more clearly. The table 10 calculated Sharpe ratio for different time interval: five year time interval (2004-2008), four year time interval (2005-2008), three year interval (2006-2008), two year time interval (2007-2008) and one year time interval (2008). For different time periods, we can see a higher Sharpe ratio based on maximal $R^2$ strategy.
example, for the time period from 2004 to 2008, maximal $R^2$ based on equation (42) got a Sharpe ratio of 0.213289, higher than maximal $R^2$ based on equation (37), 0.170066, also higher than the minimal variance strategy, 0.088509. The reason may be that the optimally weighted strategy assumes that the variance and mean of the return is constant for each currency, which may be not true in reality. The exchange rate gets more volatile in recent years. One the other side, the maximal $R^2$ adjusts the weights to maximize the prediction of future variables, giving us a better result when the market is volatile. So based on the dataset we have and the analysis we have done, overall, maximal $R^2$ based on equation (42) is better than minimal variance based on equation (37)—maximal $R^2$ strategy is relatively better than minimal variance strategy. This also tells that deviation from fundamental equilibrium exchange rate do explains the exchange rate change.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Min Variance</th>
<th>Maximal R2 (37)</th>
<th>Maximal R2 (42)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004-2008</td>
<td>0.088509</td>
<td>0.170066</td>
<td>0.213289</td>
</tr>
<tr>
<td>2005-2008</td>
<td>0.00384</td>
<td>0.104127</td>
<td>0.14112</td>
</tr>
<tr>
<td>2006-2008</td>
<td>-0.02329</td>
<td>0.114386</td>
<td>0.11554</td>
</tr>
<tr>
<td>2007-2008</td>
<td>-0.07059</td>
<td>0.213454</td>
<td>0.207942</td>
</tr>
<tr>
<td>2008</td>
<td>-0.48013</td>
<td>-0.14753</td>
<td>-0.21635</td>
</tr>
</tbody>
</table>

### 4.6 Conclusion

This paper employs two methods to construct a portfolio for a carry trade strategy. The first one is based on minimal variance, subject to return and weights constraints. The other is based on maximal $R^2$, subject to the weights constraints. Applying these two methods, we calculate the optimal weights, returns and Sharpe ratio. By
comparing the Sharpe ratio for different time intervals, we find that both carry trade strategies also suffer in the subprime lending crisis. But overall, the maximal $R^2$ strategy is better than the minimal variances strategy based on our dataset and analysis. This provides us with an alternative method to construct a portfolio for carry trade.
5 Appendix: FOC

For section 2

\[
\begin{align*}
d_t(\omega) + v_t(\omega) &= \rho_t(\omega) y_t^D(\omega) - w_t l_t(\omega) \frac{1 + \psi_{t-1}}{1 + \pi_t^{CPI}} - \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_t - 1(\omega)} - 1 \right) \frac{p_t(\omega)}{p_t} y_t^D(\omega) \\
&\quad + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s(\omega) + \lambda_t \{ Z_t l_t(\omega) - y_t^D(\omega) \}
\end{align*}
\]

FOC:

\[
-w_t \frac{1 + \psi_{t-1}}{1 + \pi_t^{CPI}} + \lambda Z_t = 0
\]

\[
\begin{align*}
\frac{1}{P_t} y_t^D(\omega) - \kappa \left( \frac{p_t(\omega)}{p_t - 1(\omega)} - 1 \right) \frac{p_t(\omega)}{P_t} \frac{1}{\pi_t^{CPI}} y_t^D(\omega) - \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_t - 1(\omega)} - 1 \right) \frac{1}{P_t} y_t^D(\omega) \\
&\quad + E_t \Lambda_{t,t+1} \{ \kappa \left( \frac{p_{t+1}(\omega)}{p_t(\omega)} - 1 \right) \frac{1}{P_{t+1}} \frac{p_{t+1}^2(\omega)}{p_t^2(\omega)} y_{t+1}^D(\omega) \} - \theta \rho(\omega) \frac{1}{P_t} \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta - 1} Y_t \\
&\quad + \theta \kappa \frac{1}{2} \left( \frac{p_t(\omega)}{p_t - 1(\omega)} - 1 \right)^2 \frac{p_t(\omega)}{P_t} \frac{1}{P_t} \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta - 1} Y_t + \lambda_t \theta \frac{1}{P_t} \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta - 1} Y_t = 0
\end{align*}
\]

\[
\Rightarrow \quad \lambda_t = \frac{w_t \frac{1 + \psi_{t-1}}{1 + \pi_t^{CPI}}}{Z_t}
\]

\[
\mu_t(\omega) \equiv \theta \left( \theta - 1 \right) \left[ 1 - \frac{\kappa}{2} \left( \frac{p_t(\omega)}{p_t - 1(\omega)} - 1 \right)^2 \right] + \kappa Y_t
\]
where

\[ Y_t = \frac{p_t(\omega)}{p_{t-1}(\omega)} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right) \]

\[-E_t \left[ \Lambda_{t,t+1} \frac{y_{t+1}(\omega)}{y_t(\omega)} \frac{P_t}{P_{t+1}} \left( \frac{p_{t+1}(\omega)}{p_t(\omega)} \right)^2 \left( \frac{p_{t+1}(\omega)}{p_t(\omega)} - 1 \right) \right] \]
6 Appendix: Benchmark Model Summary

For Section 2

<table>
<thead>
<tr>
<th>Pricing</th>
<th>( \rho_t = \mu_t w_t (1 + b r_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markup</td>
<td>( \mu_t = \frac{\theta}{(\theta - 1)(1 - \frac{\xi}{2} \pi_t^2) + \kappa ((1 + \pi_t) \pi_t - \Gamma)} )</td>
</tr>
<tr>
<td>Variety effect</td>
<td>( \rho_t = (N_t)^{\frac{1}{1-\pi}} )</td>
</tr>
<tr>
<td>Profits</td>
<td>( d_t = (1 - \frac{1}{\mu_t} - \frac{\xi}{2} (\pi_t)^2) \frac{\rho_t}{N_t} )</td>
</tr>
<tr>
<td>Free Entry</td>
<td>( v_t = f_E = 1 )</td>
</tr>
<tr>
<td>Number of Firms</td>
<td>( N_t = (1 - \delta) (N_{t-1} + N_{E,t-1}) )</td>
</tr>
<tr>
<td>Intratemporal Optimality</td>
<td>( \chi (L_t)^{1/\varphi} C_t = (1 + \tau_t) w_t )</td>
</tr>
<tr>
<td>Euler Equation (shares)</td>
<td>( v_t = \beta (1 - \delta) E_t \frac{C_{t+1}}{C_t}^{-1} (v_{t+1} + d_{t+1}) )</td>
</tr>
<tr>
<td>Euler Equation (deposit)</td>
<td>( C_t^{-1} = \beta E_t \frac{1+ \pi_t}{1+ \pi_{t+1}} C_{t+1}^{-1} )</td>
</tr>
<tr>
<td>Output &amp; Consumption</td>
<td>( Y_t (1 - \frac{\psi}{2} (b - 1) r_t - \frac{\xi}{2} (\pi_t)^2) = C_t )</td>
</tr>
<tr>
<td>Aggregate Accounting</td>
<td>( C + N_{E,t} v_t = B_t r + N_t d_t + w_t L_t )</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>( \frac{1+ \pi_t}{1+ \pi_{t+1}} = \frac{\rho_t}{\rho_{t-1}} )</td>
</tr>
<tr>
<td>Bank</td>
<td>( B = w_t L_t )</td>
</tr>
</tbody>
</table>

where \( \Gamma = E_t \left[ \frac{C_{t+1}}{C_{t+1}} \frac{N_{t+1}}{N_{t+1}} \frac{Y_{t+1}}{Y_t} (1 + \pi_{t+1}) \pi_{t+1} \right] \)
<table>
<thead>
<tr>
<th>Linearized benchmark model summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pricing</td>
</tr>
<tr>
<td>Markup</td>
</tr>
<tr>
<td>Variety effect</td>
</tr>
<tr>
<td>Profits</td>
</tr>
<tr>
<td>Number of Firms</td>
</tr>
<tr>
<td>Intratemporal Optimality</td>
</tr>
<tr>
<td>Euler Equation (shares)</td>
</tr>
<tr>
<td>Euler Equation (deposit)</td>
</tr>
<tr>
<td>Output &amp; Consumption</td>
</tr>
<tr>
<td>Aggregate Accounting</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>CPI inflation</td>
</tr>
<tr>
<td>Bank</td>
</tr>
</tbody>
</table>
7 Appendix: Calibration

For Section 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3.8</td>
</tr>
<tr>
<td>$Z$</td>
<td>1</td>
</tr>
<tr>
<td>$\Psi_{\text{after deregulation}}$</td>
<td>1.2</td>
</tr>
<tr>
<td>$r$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>77</td>
</tr>
<tr>
<td>$f_E$</td>
<td>1</td>
</tr>
<tr>
<td>$\Psi_{\text{before deregulation}}$</td>
<td>1.8</td>
</tr>
</tbody>
</table>
8 Appendix: Entrepreneur’s problem

For Section 3

\begin{align*}
L &= E_0 \sum_{t=0}^{\infty} \beta_t \left\{ \log c_{E,t} + \lambda_t \left[ A_E k_{E,t-1} L_{E,t-1}^{1-v} / x_{E,t} \right] \\
&\quad + k_{E,t-1} (1 - \delta) + b_t - c_{E,t} - ab_{t-1} R_{t-1} / \pi_t - k_{E,t} - w_t L_t \right\} \\
&\quad + \gamma_t \left[ m_{E,t} (k_{E,t} \pi_{t+1} / R_t) - b_t \right] \\
\end{align*}

FOC:

\begin{align*}
\frac{1}{c_{E,t}} - \gamma_t - \frac{\beta_t R_t}{\pi_{t+1}} &= 0 \\
-\frac{1}{c_{E,t}} + \gamma_t m_{E,t} (\pi_{t+1} / R_t) + E_t \left[ \frac{\beta_E y_{E,t+1}}{k_{E,t} x_{E,t+1} c_{E,t+1}} + \frac{\beta_E (1 - \delta)}{c_{E,t+1}} \right] &= 0 \\
\frac{(1 - v) y_{E,t}}{L_{E,t} x_{E,t}} - w_t &= 0
\end{align*}
9 Appendix: Steady state

For Section 3

\[
\frac{1}{c_{H,t}} = \beta_H E_t \frac{R_{t+1}}{c_{H,t+1} \pi_{t+1}}
\]

\[
\frac{1}{\beta_H} = R
\]

\[
\chi L_i \eta_{cH,t} = w_t
\]

\[
\chi L_i \eta_{cH} = w
\]

\[
\frac{1}{c_{E,t}} - \gamma_t - E_t \frac{\beta_E R_t}{c_{E,t+1} \pi_{t+1}} = 0
\]

\[
\frac{1 - \beta_E / \beta_H}{c_E} = \gamma
\]

\[
- \frac{1}{c_E} + \gamma m E_t (\pi_{t+1} / R_t) + E_t \left[ \frac{\beta_E \nu_E \pi_{t+1}}{k_{E,E} c_{E,t+1} c_{E,t+1}} + \frac{\beta_E (1 - \delta)}{c_{E,t+1}} \right] = 0
\]

\[
- \frac{1}{c_E} + \frac{1 - \beta_E / \beta_H}{c_E} m \beta_H + \frac{\beta_E \nu_E}{k_{E,E} c_E} + \frac{\beta_E (1 - \delta)}{c_E} = 0
\]

\[
(\beta_H - \beta_E) m + \frac{\beta_E \nu_E}{k_{E,E}} + \beta_E (1 - \delta) = 1
\]

\[
\frac{\beta_E \nu}{\chi E (1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m)} = \frac{k_E}{y_E}
\]
\begin{align*}
w_t &= \frac{(1-v)y_{E,t}}{L_{E,t}x_{E,t}} \\
w &= \frac{(1-v)y_E}{L_Ex_E} \\
wL_E &= \frac{(1-v)}{x_E} \\
\chi L^n c_H &= \frac{(1-v)y_E}{L_E x_E c_H} \\
\frac{L_E}{L^{-\eta}} &= \frac{x_E \chi c_H}{(1-v)y_E} \\
y_{E,t}/x_{E,t} + k_{E,t-1}(1-\delta) + b_t &= c_{E,t} + b_{t-1}R_{t-1}/\pi_t + k_{E,t} + w_L E_{t,t} \\
v y_E/x_E - k_{E,t}\delta + m k_E \beta_H &= c_E + m k_E \\
v y_E/x_E &= c_E + k_E(m(1-\beta_H) + \delta) \\
c_E &= v y_E/x_E - k_E(m(1-\beta_H) + \delta) \\
y_E &= \frac{v}{x_E} - \frac{k_E(m(1-\beta_H) + \delta)}{y_E} \\
c_E &= \frac{v}{x_E} - \frac{\beta_E y_E(m(1-\beta_H) + \delta)}{x_E [1 - \beta_E (1-\delta) - (\beta_H - \beta_E) m]} \\
c_E &= \frac{v}{x_E} \left(1 - \frac{\beta_E(m(1-\beta_H) + \delta)}{1 - \beta_E(1-\delta) - (\beta_H - \beta_E)m}\right) \\
k_{S,t-1} &= k_{S,t-2}(1-\delta) + i_t \\
68
\end{align*}
\[ k_S \delta = \iota \]

\[
w_t = \frac{(1 - v)y_{S,t}}{L_{S,t}x_t^S}
\]

\[
w = \frac{(1 - v)y_S}{L_S x_S}
\]

\[
\frac{y_{E,t}}{L_{E,t}x_{E,t}} = \frac{y_{S,t}}{L_{S,t}x_{S,t}}
\]

\[
y_E = \frac{y_S}{x_S}
\]

\[
\frac{L_{E}x_{E}}{L_S x} = \frac{A_S S^v}{x_S}
\]

\[
A_E E^v = A_S S^v
\]

\[
x_E = \frac{A_S x_E}{A_{E}x_S}
\]

\[
\left(\frac{E}{S}\right)^v = \frac{A_S x_E}{A_{E}x_S}
\]

\[
E \frac{S}{S} = \left(\frac{A_S x_E}{A_{E}x_S}\right)^{\frac{1}{v}}
\]

\[
k_E = \frac{y_E}{Y} \frac{\beta_{E} v}{x_E \left[1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m\right]}
\]

\[
y_E = \frac{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{\beta_{E} v}
\]

\[
A_E E^{v-1} = \frac{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{\beta_{E} v}
\]

\[
E^{v-1} = \frac{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{A_E \beta_{E} v}
\]
\[ \frac{k_S}{Y} = \frac{y_S}{Y x_S (1 - (1 - \delta) \beta_H)} \]
\[ \frac{y_S}{k_S} = \frac{x_S (1 - (1 - \delta) \beta_H)}{v \beta_H} \]
\[ S^{v-1} = \frac{x_S (1 - (1 - \delta) \beta_H)}{A_S v \beta_H} \]

\[
\left( \frac{E}{S} \right)^{v-1} = \frac{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m] A_S v \beta_H}{A_E \beta_E x_S (1 - (1 - \delta) \beta_H)}
\]
\[
E = \frac{x_E A_S \beta_H [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{x_S A_E \beta_E (1 - (1 - \delta) \beta_H)}
\]

\[
\left( \frac{A_S}{A_E} \right)^{\frac{1}{v}} \left( \frac{x_E}{x_S} \right)^{\frac{1}{v}} = \left( \frac{x_E}{x_S} \right)^{\frac{1}{v}} \left( \frac{A_S \beta_H [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{A_E \beta_E (1 - (1 - \delta) \beta_H)} \right)^{\frac{1}{v}}
\]
\[
\left( \frac{x_E}{x_S} \right)^{\frac{1}{v}} = \left( \frac{A_E}{A_S} \right)^{\frac{1}{v-1}} \left( \frac{\beta_H [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}{\beta_E (1 - (1 - \delta) \beta_H)} \right)^{\frac{1}{v}}
\]
\[
x_E = \frac{A_E}{A_S} \left( \frac{\beta_E (1 - (1 - \delta) \beta_H)}{\beta_H [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]} \right)^v = z
\]
\((x_S)^{e-1} = \frac{((\rho - 1)/\rho)^{1-e}}{1 + (z)^{e-1}}\)

\[x_S = \frac{\rho}{(\rho - 1)(1 + (z)^{e-1})}\]

\(\chi L^n c_H = \frac{(1 - v) y_S}{L_S x_S}\)

\[L_S = \frac{(1 - v) y_S}{\chi x_S L^{n+1} x_S c_H}\]

\[L_S = \frac{(1 - v) y_S}{\chi L^{n+1} x_S Y c_H / Y}\]

\[\frac{R_t}{\pi_{t+1}} = 1 + \frac{vy_{S,t}}{k_{S,t-1} x_{S,t}} - \delta\]

\[1/\beta_H + \delta - 1 = \frac{vy_S}{k_S x_S}\]

\[\frac{1 - \beta_H(1 - \delta)}{\beta_H} = \frac{vy_S}{k_S x_S}\]

\[\frac{v \beta_H}{x^3(1 - (1 - \delta) \beta_H)} = \frac{k_S}{y_S}\]

\[Y_t = c_{H,t} + c_{E,t} + k_{E,t} - k_{E,t-1}(1 - \delta) + i_t\]

\[1 = \frac{c_H}{Y} + \frac{c_E y_E}{y_E Y} + \frac{\delta y_E k_E}{Y y^E} + \frac{k_S y_S \delta}{Y y}\]

\[1 = \frac{c_H}{Y} + \left(\frac{v}{x_E} - \frac{k_E m (1 - \beta_H) + \delta}{y^E}\right)\frac{y_E}{Y y^E} + \frac{\delta y_E k_E}{Y y^E} + \frac{k_S y_S \delta}{Y y}\]

\[1 = \frac{c_H}{Y} + \frac{v}{x_E} - \frac{y_E}{Y x_E (1 - \beta_E(1 - \delta) - (\beta_H - \beta_E) m)}\]

71
\[
\frac{c_H}{Y} = 1 - \frac{v}{x^E} \frac{y_E}{Y} \frac{y}{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E)m]}
\]
\[
- \frac{v \beta_H \delta}{x_S (1 - (1 - \delta) \beta_H)} \frac{y_S}{Y}
\]

\[
L_t = L_{S,t} + L_{E,t}
\]

\[
\chi L^\eta + c_H
\]
\[
= \frac{(1 - v) y_S}{x_S} + \frac{(1 - v) y_E}{x_E}
\]

\[
\chi L^\eta + c_H
\]
\[
= \frac{(1 - v) y_S}{x_S Y} + \frac{(1 - v) y_E}{x_E Y}
\]

\[
L = \left( \frac{(1-v) y_S}{x_S Y} + \frac{(1-v) y_E}{x_E Y} \right) \frac{1}{\chi^\eta Y}
\]

\[
b_t = m_E (k_{E,t} \pi_{t+1}/R_t)
\]

\[
b = m_E \beta_H
\]

\[
\frac{b}{Y} = \frac{m \beta_H k_E}{Y}
\]

\[
\frac{b}{Y} = \frac{y_E}{Y} \frac{m \beta_H \beta_E v}{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E)m]}
\]

\[
b_t + i_t = d_t
\]

\[
b + k_S \delta = d
\]

\[
\frac{b + k_S \delta}{Y} = \frac{d}{Y}
\]

72
$y_{E,t} = \left( \frac{p_{E,t}}{p} \right)^{-e} Y_t$

$y_E = \left( \frac{p_E}{P/X} \right)^{-e} Y$

$= \left( \frac{X}{x_E} \right)^{-e} Y$

$y_E \over Y = \left( \frac{x_E}{X} \right)^e$

$y_E \over Y x_E = X^{-e} (x_E)^{e-1}$

$y_S = \left( \frac{p_S}{p} \right)^{-e} Y$

$= \left( \frac{p_S}{P/X} \right)^{-e} Y$

$= \left( \frac{X}{x_S} \right)^{-e} Y$

$y_S \over Y x_S = X^{-e} (x_S)^{e-1}$

Assume zero inflation, the steady state will be described by:

$$c_H \over Y = 1 - \frac{v}{x_E} y_E + \frac{y_E}{Y} \frac{\beta_{Ev} m (1 - \beta_H)}{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]} - \frac{v \beta_H \delta}{x_S (1 - (1 - \delta) \beta_H) Y} y_S$$

$$b = \frac{y_E}{Y} \frac{m \beta_H \beta_{Ev}}{x_E [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}$$

$$c_E \over Y = \frac{y_{Ev}}{x_E Y} \frac{1 - \beta_E (m (1 - \beta_H) + \delta)}{1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m}$$
\[
\frac{k_E}{Y} = \frac{\beta_{E_{VE}}}{Y_{X_E} [1 - \beta_E (1 - \delta) - (\beta_H - \beta_E) m]}
\]

\[
\frac{k_S}{Y} = \frac{Y_S \beta_H}{Y_{X_S} (1 - (1 - \delta) \beta_H)}
\]
10 Appendix: Log-linearize

\[
\frac{1}{c_{H,t}} = \beta H E_t \frac{R_t}{c_{H,t+1} \pi_{t+1}}
\]

\[
1 - \hat{c}_{H,t} = 1 + E_t \hat{R}_t - E_t \hat{c}_{H,t+1} - E_t \hat{\pi}_{t+1}
\]

\[
\hat{c}_{H,t} = E_t \hat{c}_{H,t+1} - \hat{r}_t
\]

\[
\chi \xi_t c_{H,t} = w_t
\]

\[
\eta \hat{L}_t + \hat{c}_{H,t} = \hat{w}_t
\]

\[
y_{E,t} = A_E k_{E,t-1} L_{E,t}^{1-v}
\]

\[
\hat{y}_{E,t} = v \hat{k}_{E,t-1} + (1 - v) \hat{L}_{E,t}
\]

\[
b_t = m E_t (k_{E,t} \pi_{t+1} / R_t)
\]

\[
\hat{b}_t = E_t \hat{k}_{E,t} - \hat{r}_t
\]

\[
y_{E,t}/x_{E,t} + k_{E,t-1}(1 - \delta) + b_t = c_{E,t} + b_{t-1} R_{t-1}/\pi_t
\]

\[+ k_{E,t} + w_t L_{E,t} \]

75
\[
\frac{y_E}{x_E} (1 + \hat{y}_{E,t} - \hat{x}_{E,t}) + \\
k_E (1 - \delta)(1 + \hat{k}_{E,t-1}) + b(1 + \hat{b}_t) = c_E (1 + \hat{c}_{E,t}) \\
+ b(1 + \hat{b}_{t-1} + \hat{r}_{t-1}) \\
+ k_E (1 + \hat{k}_{E,t}) \\
+ wL_E (1 + \hat{w}_t + \hat{L}_{E,t}) \\
\]
\[
\frac{y_E}{x_E} (\hat{y}_{E,t} - \hat{x}_{E,t}) + \\
k_E (1 - \delta) \hat{k}_{E,t-1} + b \hat{b}_t = c_E \hat{c}_{E,t} + b (\hat{b}_{t-1} + \hat{r}_{t-1}) \\
+ k_E \hat{k}_{E,t} + wL_E (\hat{w}_t + \hat{L}_{E,t}) \\
\]
\[
\frac{1}{x_E} (\hat{y}_{E,t} - \hat{x}_{E,t}) + \\
k_E (1 - \delta) \hat{k}_{E,t-1} + b \frac{\hat{b}_t}{y_E} = c_E \frac{\hat{c}_{E,t}}{y_E} + b \frac{\hat{b}_{t-1} + \hat{r}_{t-1}}{y_E} \\
+ k_E \frac{\hat{k}_{E,t}}{y_E} + wL_E \frac{\hat{w}_t + \hat{L}_{E,t}}{y_E} \\
\]
\[
\frac{1}{c_{E,t}} - \gamma - E_t \frac{\beta_E R_t}{c_{E,t+1}\pi_{t+1}} = 0 \\
\]
\[
- \frac{1}{c_{E,t}} + \left( \frac{1}{c_{E,t}} - E_t \frac{\beta_E R_t}{c_{E,t+1}\pi_{t+1}} \right) \\
mE_t (\pi_{t+1}/R_t) + E_t \left[ \frac{\beta_{E\nu E,t+1}}{k_{E,t}x_{E,t+1}\pi_{t+1}} + \frac{\beta_E (1 - \delta)}{c_{E,t+1}} \right] = 0 \\
-1 + (1 - E_t) \frac{\beta_E R_t c_{E,t}}{c_{E,t+1}\pi_{t+1}} \\
mE_t (\pi_{t+1}/R_t) + E_t \left[ \frac{\beta_{E\nu E,t+1} c_{E,t}}{k_{E,t}x_{E,t+1}\pi_{t+1}} + \frac{\beta_E (1 - \delta) c_{E,t}}{c_{E,t+1}} \right] = 0 \\
\]
76
\[
mE_t(\pi_{t+1}/R_t) + E_t[\frac{\beta E \nu y_{E,t+1} c_{E,t}}{k_{E,t} x_{E,t+1} c_{E,t+1}} + \frac{\beta E (1 - \delta) c_{E,t}}{c_{E,t+1}}] = 1
\]

\[
[1 - \frac{\beta E R c_E}{c_E \pi}] (1 + r\hat{r}_t - E_t \Delta \hat{c}_{E,t+1})] m \frac{\pi}{R} (1 - r\hat{r}_t) + \frac{\beta E \nu y_{E,t+1} c_{E,t}}{k_{E,t} x_{E,t+1} c_{E,t+1}} [1 + E_t \hat{y}_{t+1} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \Delta \hat{c}_{E,t+1}]
\]

\[
+ \frac{\beta E (1 - \delta) c_{E,t}}{c_E} [1 - E_t \Delta \hat{c}_{E,t+1}] = 1
\]

\[
[1 - \frac{\beta E R c_E}{c_E \pi}] (r\hat{r}_t - E_t \Delta \hat{c}_{E,t+1})] m \frac{\pi}{R} (1 - r\hat{r}_t)
\]

\[
- \frac{\beta E R c_E}{c_E \pi} (r\hat{r}_t - E_t \Delta \hat{c}_{E,t+1})] m \frac{\pi}{R} (1 - r\hat{r}_t)
\]

\[
+ \frac{\beta E \nu y_{E,t+1} c_{E,t}}{k_{E,t} x_{E,t+1} c_{E,t+1}} [1 + E_t \hat{y}_{t+1} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \Delta \hat{c}_{E,t+1}]
\]

\[
+ \frac{\beta E (1 - \delta) c_{E,t}}{c_E} [1 - E_t \Delta \hat{c}_{E,t+1}] = 1
\]

\[
- [1 - \frac{\beta E R c_E}{c_E \pi}] m \frac{\pi}{R} r\hat{r}_t
\]

\[
- \frac{\beta E R c_E}{c_E \pi} (r\hat{r}_t - E_t \Delta \hat{c}_{E,t+1})] m \frac{\pi}{R} + \frac{\beta E \nu y_{E,t+1} c_{E,t}}{k_{E,t} x_{E,t+1} c_{E,t+1}} [E_t \hat{y}_{E,t+1} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \Delta \hat{c}_{E,t+1}]
\]

\[
- \frac{\beta E (1 - \delta) c_{E,t}}{c_E} [E_t \Delta \hat{c}_{E,t+1}] = 0
\]

\[
-m r\hat{r}_t (\beta E - \beta E) - m \beta E (r\hat{r}_t - E_t \Delta \hat{c}_{E,t+1})
\]

\[
+ \frac{\beta E \nu y_{E,t+1} c_{E,t}}{k_{E,t} x_{E,t+1} c_{E,t+1}} [E_t \hat{y}_{E,t+1} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \Delta \hat{c}_{E,t+1}]
\]

\[
- \beta E (1 - \delta) E_t \Delta \hat{c}_{E,t+1} = 0
\]
\[-mr_t (\beta_H - \beta_E) - m\beta_E (r_t - E_t \Delta \hat{c}_{E,t+1})
\]
\[+ \beta_E v^E \left[ E_t \hat{y}_{E,t+1} - E_t \hat{k}_{E,t} - E_t \hat{x}_{E,t+1} - E_t \Delta \hat{c}_{E,t+1} \right]
\]
\[-\beta_E (1 - \delta) E_t \Delta \hat{c}_{E,t+1} = 0 \]

\[
\frac{(1 - v)y_{E,t}}{L_{E,t} x_{E,t}} - w_t = 0
\]
\[
\hat{y}_{E,t} - \hat{L}_{E,t} - \hat{x}_{E,t} = \hat{w}_t
\]

\[
y_{S,t} = A_S k_{S,t-1}^v L_{S,t}^{1-v}
\]
\[
\hat{y}_{S,t} = v \hat{k}_{S,t-1} + (1 - v) \hat{L}_{S,t}
\]

\[
\frac{(1 - v)y_{S,t}}{L_{S,t} x_{S,t}} - w_t = 0
\]
\[
\hat{y}_{S,t} - \hat{L}_{S,t} - \hat{x}_{S,t} = \hat{w}_t
\]

\[
\frac{R_t}{\pi_{t+1}} = 1 + \frac{v y_{S,t}}{k_{S,t-1} x_{S,t}} - \delta
\]
\[
R(1 + r_t) = 1 + \frac{v y_{S,t}}{k_{S,t} x_{S,t}} (1 + \hat{y}_{S,t} - \hat{k}_{S,t-1} - \hat{x}_{S,t}) - \delta
\]
\[
\frac{r_t}{\beta_H} = \frac{v y_{S,t}}{k_{S,t} x_{S,t}} (\hat{y}_{S,t} - \hat{k}_{S,t-1} - \hat{x}_{S,t})
\]

78
\[
\sum_{i=0}^{\infty} \theta^k E_t \{ A_t, i \left( \frac{P_t(\omega)}{P_{t+i}} - \frac{X}{X_{t+i}} \right) y_{t+i}(\omega) \} = 0
\]

\[
P_t = \left[ \theta P_{t-1}^{1-\phi} + (1 - \theta) P_{t-1}^{1-\phi} \right]^{1/\phi}
\]

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t
\]

\[
\kappa = (1 - \theta)(1 - \beta \theta)/\theta
\]

\[
\frac{1}{X_t} = \left[ \left( \frac{1}{x_{E,t}} \right)^{1-e} + \left( \frac{1}{x_{S,t}} \right)^{1-e} \right]^{1/\tau}
\]

\[
X_t^{e-1} = (x_{E,t})^{e-1} + (x_{S,t})^{e-1}
\]

\[
X^{e-1}(1 + (e - 1) \hat{X}_t) = (x_E)^{e-1}(1 + (e - 1) \hat{x}_{E,t})
\]

\[
+ (x_S)^{e-1}(1 + (e - 1) \hat{x}_{S,t})
\]

\[
X^{e-1} \hat{X}_t = (x_E)^{e-1} \hat{x}_{E,t} + (x_S)^{e-1} \hat{x}_{S,t}
\]

\[
b_t + i_t = d_t
\]

\[
b \hat{b}_{t} + k_s \hat{k}_{S,t} - k_s \hat{k}_{S,t-1}(1 - \delta) = d \hat{d}_t
\]

\[
b \frac{y}{Y} \hat{b}_t + k_s \frac{y}{Y} \hat{k}_{S,t} - k_s \frac{y}{Y} \hat{k}_{S,t-1}(1 - \delta) = d \frac{y}{Y} \hat{d}_t
\]

\[
L_t = L_{S,t} + L_{E,t}
\]
\[ L \hat{L}_t = L_S \hat{L}_{S,t} + L_E \hat{L}_{E,t} \]
\[ \hat{L}_t = \frac{L_S}{L} \hat{L}_{S,t} + \frac{L_E}{L} \hat{L}_{E,t} \]

\[ Y_t = c_{H,t} + c_{E,t} + k_{E,t} - k_{E,t-1} (1 - \delta) + i_t \]
\[ \bar{Y}_t = c_{H,t} + c_{E,t} + k_{E,t} - k_{E,t-1} (1 - \delta) + k_{S,t-1}^v - k_{S,t-2}^v (1 - \delta) \]
\[ \hat{Y}_t = \frac{c_{H}}{Y} \hat{c}_{H,t} + \frac{c_{E}}{Y} \hat{c}_{E,t} + \frac{k_{E}}{Y} \hat{k}_{E,t} - \frac{k_{E}}{Y} \hat{k}_{E,t-1} (1 - \delta) \]
\[ \hat{Y}_t = \frac{k_{S}}{Y} \hat{k}_{S,t} - \frac{k_{S}}{Y} \hat{k}_{S,t-1} (1 - \delta) \]

\[ y_{E,t} = (\frac{P_{E,t}}{P_t/X_t})^{-\varepsilon} Y_t \]
\[ \hat{y}_{E,t} = \varepsilon \hat{x}_{E,t} - e \hat{X}_t + \hat{Y}_t \]

\[ y_{S,t} = (\frac{P_{S,t}}{P_t/X_t})^{-\varepsilon} Y_t \]
\[ \hat{y}_{S,t} = \varepsilon \hat{x}_{S,t} - e \hat{X}_t + \hat{Y}_t \]

centralbankpolicy
11 Appendix: Calibration II

For Section 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>$\beta_H$</td>
<td>0.99</td>
<td>Iacoviello (2005)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.84</td>
<td>Zhang (2009)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>He et al. (2007)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.6</td>
<td>He et al. (2007)</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>0.98</td>
<td>Iacoviello (2005)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>6.16</td>
<td>Liu (2007)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>4.61</td>
<td>Zhang (2009)</td>
</tr>
</tbody>
</table>
12 References


51. Liu Qiao and Siu Alan(2006):" Institutions, financial development, and corporate investment: evidence from an implied return on capital in China," working paper, University of Hong Kong.


