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Teacher Learning through Practices: How Mathematics Teachers Change in Practices with Innovative Curriculum Materials

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Teacher Learning through Practices: How Mathematics Teachers Change in Practices with Innovative Curriculum Materials

By

Hee-jeong Kim

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Education in the Graduate Division of the University of California, Berkeley

Committee in charge:
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Abstract

Teacher Learning through Practices: Supporting Mathematics Teachers to Change in Practices Providing by Innovative Curriculum Materials

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Professional learning and development in and from practices have focused either on how individuals learn from formally organized contexts or on how a group of teachers collectively develop within contexts. Thus there is little research on individual teacher professional learning through practices. This dissertation investigates the ways in which two experienced mathematics teachers develop the means of making sense of content and student thinking, and change their practices through their interactions with innovative curriculum materials. This dissertation also conceptualizes a new framework for teaching practices termed student thinking responsive teaching practices, which is grounded in the literature and this dissertation’s empirical investigations.

In my dissertation, I conducted case studies with two teachers, observing their classrooms over the course of a year to capture their everyday teaching practices during regular lessons, as well as their teaching practices during lessons involving innovative curriculum materials. I also conducted teacher interviews and observed professional development workshop sessions to inform my analysis of how the teachers changed their teaching practices in the classroom. I analyzed the collected data using a mixed methods approach. First, I conducted a qualitative analysis with an existing coding scheme from the literature and a new coding scheme I specifically developed. Second, I quantified my qualitative analysis findings to capture how and whether the two teachers changed toward becoming more responsive to student mathematical thinking in their everyday practices.

My dissertation findings clarify some of the mechanisms for change in teacher practices. First, when teachers are supported in, and make use of, instructional moves that focus on student mathematical thinking, not only is student learning advanced, but teacher learning is also advanced. Second, when teachers’ interactions with curriculum materials involve adapting and developing new pedagogical strategies for student thinking responsive teaching practices, they create opportunities to making sense of content and student thinking. In contrast, when they adapt curriculum materials to align with their existing practices, those opportunities are lost. Lastly, in addition to teachers’ beliefs and
knowledge, their professional identity is critical for understanding the ways in which teachers interact with and implement curriculum materials. It is important for teachers to see and make sense of themselves as learners and as professionals who are continuously reflecting on and through their practices. Such an identity may become a catalyst for changing their practices and need to be supported not only by innovative curriculum materials, but also at the school and district level. These results have implications for the design of professional development systems that support the positive aspects of curriculum-based learning.
This dissertation is dedicated to my mother, Eunja, and my daughter, Anna.
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Chapter 1: Introduction

“I thought I was trying to [have my] students talk a lot, I was trying to listen more to what the kids were saying, but when I watched my teaching video for National Board, it turned out that’s not true.”

- Ms. Janet, 7th grade mathematics teacher at the end-of-year interview (May 2014)

“I was a very traditional teacher for ten years...I feel like my regular teaching has changed a lot, because of the Common Core and because of the things that I’m doing [in the project]...I tried having them talk more and having them listen to each other.”

- Ms. Lee, 7th grade mathematics teacher at the end-of-year interview (May 2013)

Ms. Janet and Ms. Lee both taught seventh grade mathematics in the same urban school district. They wanted to learn new teaching practices aligned with the Common Core so that they can create more powerful learning environments in their classrooms. To do so, they volunteered to use curriculum support materials that have been designed to support teachers in learning those practices, specifically formative assessment. They participated in the same professional development (PD) workshops and participate in monthly PD meetings that support using the curriculum materials. Yet the results differed, as indicated by the reflections quoted above and as substantiated by classroom observations. Why did Ms. Janet not demonstrate change in her teaching practices, while Ms. Lee did? How and why did they learn differently through their implementation of the curriculum support materials?

Research on teacher decision-making (e.g., Schoenfeld, 2011) has clarified what teachers do and why, with a focus on teachers’ beliefs, knowledge, and goals. This perspective can naturally be applied to examine teachers’ use of innovative curriculum materials. However, little research has been conducted to investigate the ways in which teachers learn and develop through their interactions with rich curriculum materials, and how such materials can influence change in the teachers’ classroom practices. How does the interaction of teachers’ beliefs and knowledge with curriculum materials manifest in their classroom practices? How do teachers’ instructional moves, as supported by the materials, create learning opportunities not only for their students but also for themselves? How do teachers conceptualize their changes in classroom practices? These unanswered questions are the focus of my dissertation.

Making effective use of student mathematical thinking during instruction has been an issue of increasing importance in mathematics education (Carpenter, Fennema, & Franke, 1996; Sherin, Jacobs, & Philipp, 2011). Recent reform in school mathematics has emphasized that teachers need to pay more attention to students’ mathematical reasoning and justification in mathematics classroom, with an emphasis on helping students present, justify, critique, and build on each other’s ideas (National Council of Teachers of Mathematics (NCTM), 1989; 2000; National Research Council (NRC), 2001). This focus was further emphasized by the Common Core State Standards Initiatives (CCSSI-
Uncovering and assessing students’ ideas during instruction allow teachers to make more effective use of pedagogical strategies focusing on mathematical content, and to support and enhance students’ reasoning. Black and William (1998)’s literature review found that teachers’ use of assessment during instruction, particularly formative feedback, substantially improves students’ mathematical performance and achievement.

However, teachers face considerable challenges when they engage with student thinking during instruction because student thinking is often difficult to anticipate. Teachers, especially those who have little experience with handling student misunderstandings “in the moment,” need assistance with problems of practice such as how to orchestrate productive discussions using different student ideas. In practice, teachers have to make immediate decisions about which ideas would be most productive to discuss publicly, and how to provide better opportunities for students to reconceptualize their misunderstandings. One approach to support teachers who face such challenges is to provide educative curriculum materials that contain information about content, students, and curriculum with concrete examples (Ball & Cohen, 1996; Davis & Krajcik, 2005) so that teachers can better anticipate what kinds of misunderstandings students may demonstrate and make better decisions for students’ mathematical learning during instruction.

This dissertation examines how a particular kind of educative curriculum materials called Formative Assessment Lessons (FALs) supports teachers to engage with and respond to student mathematical thinking during instruction. FALs were designed to support teachers in becoming increasingly responsive to student mathematical thinking. All teachers volunteered to use FALs and were motivated to change their teaching practices. Investigating how such teachers learn and change in their teaching practices within the context of FAL usage allows us to better understand the interaction between teacher learning and innovative curriculum materials. In this dissertation, I conduct two strands of detailed analyses, with the following goals:

1. to understand how two teachers’ instructional moves, supported by FALs, did or did not create learning opportunities for themselves to learn about student mathematical thinking and how to respond to it;

2. to capture the changes in teachers’ everyday teaching practices.

I first discuss the rationale for this study and the review of prior research on teacher learning through practices in Chapter 2. The review situates this dissertation in the literature by addressing the importance of instruction focusing on student thinking, theoretical perspectives and frameworks regarding teacher learning through practice and interaction with curriculum materials, and how FALs support teacher learning. I then describe the methodology used to collect and analyze the data for my dissertation, as well as the broader context of this study, in Chapter 3. I specifically address the development and application of the new analytical frameworks used in this dissertation in Chapter 4.

Chapter 5-7 are analytic chapters examining different aspects of the two teachers’ practices. In Chapter 5, I use a preexisting framework from the literature to explore how two teachers used and interacted with the curriculum support materials from a bird’s-eye perspective. The chapter also describes how the two teachers’ use of curriculum support materials enhanced or limited their learning opportunities. The findings indicated that Ms.
Janet transformed and adapted the innovative curriculum support materials so that they aligned with her established teaching repertoire, rather than implementing or developing new practices that aligned with the curriculum support materials and PD workshops. In contrast, Ms. Lee attempted to implement new practices aligned with curriculum support materials and by PD workshops, which triggered a loss of balance in her classroom as her new practices emerged. Ultimately, the ways in which Ms. Lee worked with FALs created opportunities for her to learn about student thinking and deeper mathematical content. Doing so seemed to support her in developing new teaching practices. On the other hand, Ms. Janet’s approach to FALs seemed to limit learning opportunities for herself and hindered her attempts to develop new practices.

Building on these broader analytic insights, I conduct an in-depth analysis in Chapter 6 on how the two teachers’ instructional moves created learning opportunities for themselves to learn about student thinking, and how they responded differently to student mathematical thinking. Analyses of the two teachers’ teaching practice profiles demonstrate that they made use of student thinking to different degrees. Ms. Lee enacted particular pedagogical strategies that focused more on eliciting student reasoning and their thinking process, and made use of the elicited student thinking as resources for discussion. Her particular instructional moves present as a pattern with regard to learning about and making use of student mathematical thinking, which had the synergistic effect of creating more opportunities for her to learn about student thinking. This stands in contrast to Ms. Janet, whose pedagogical moves demonstrated lower levels of eliciting and using of student thinking.

Chapter 7 provides the results of capturing teacher change in teaching practices over time. I use mixed methods to demonstrate that Ms. Lee’s teaching practices changed significantly toward being more responsive to student mathematical thinking, while Ms. Janet’s practices did not. I also include teacher interviews to illustrate how the teachers themselves saw changes in their practice over time. I build on these insights to discuss the implications of these findings in Chapter 8.

In Chapter 8, I summarize and discuss the empirical findings. When teachers are supported in, and make use of, instructional moves that focus on student mathematical thinking, not only is student learning advanced, but teacher learning is also advanced. I argue that the right supports can enable teachers to change their teaching practice with sufficient engagement on part of the teachers. I also discuss the types of challenges encountered by teachers when they attempt to change and develop their practices, as well as the implications of those challenges for designing professional development that better supports teachers to improve their instructional practices. I conclude with the theoretical and methodological contributions this dissertation makes to our understanding of teacher learning through practices using innovative curriculum.
Chapter 2: Literature Review

Learning and development in and from professional practices, such as learning in the workplace, have been central and salient for any kind of professional development. The most important place for teachers to learn is their workplace, especially their own classrooms (Ball & Cohen, 1999; Putnam & Borko, 2000; Smylie, 1989). The classroom is where they actually implement their practices and become more knowledgeable about their students and their ways of teaching in everyday situations. However, professional learning through practice is often described as an unofficial or *ad hoc* process (Schön, 1983.) In addition, despite research on teachers’ professional learning and development has significantly contributed to our understanding on individual’s cognitive process and capacity (e.g., growth of teacher knowledge; or change of teacher beliefs) *from* formally organized contexts (e.g., teacher education program or PD workshops) or on collective development *within* communities (e.g., development of the ways of teacher talks within a community; or shift in participation within communities of practices), the field has tended to have a separated focus and there is little research on teacher learning *through* practice.

One promising strand of this research is to investigate teachers’ use of innovative or standards-based curriculum materials. Through investigating how teachers implement innovative curriculum materials in their own classroom, it provides an insight how teachers learn—make sense of subject-matter and curriculum materials, student learning, and general pedagogies—and change in their instructional practices through their own practices. Recent research on teachers’ use of standards-based curriculum materials has contributed to our understanding of factors that affect teachers’ decision-making in the use of curriculum materials (e.g., knowledge and resources, beliefs and “orientation toward curricular”¹, or teachers’ instructional goals) (Remillard, J. T., Herbel-Eisenmann, B. A., & Lloyd, 2009). However, the ways teachers make use of curricular materials in their instruction and how the ways influence teacher learning and changes in teacher practices remain largely unexplored.

This dissertation aims to fill this gap by investigating how mathematics teachers learn and change through practice as they use innovative curricular materials. In particular, my dissertation looks closely at how teachers create learning opportunities through their instructional practices in interaction with innovative curricular materials, and seek to extend current findings by contributing to our understanding of teacher development and change through their practice as a long-term process.

In this chapter, I first describe the rationale for my focus on student thinking centered instruction by summarizing prior research on powerful mathematics classrooms. Second, I summarize prior research on teacher learning through the use of curriculum materials, and theoretical perspectives that help situate and motivate my dissertation. Lastly, I briefly introduce Formative Assessment Lessons (FALs), which are innovative

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¹ The construct, “orientation toward curriculum,” is defined as “a set of perspectives and dispositions about mathematics, teaching, learning, and curriculum that together influence how a teacher engages and interacts with a particular set of curriculum materials (Remillard & Bryans, 2004, p. 364).”
educative curriculum support materials for teachers to learn and change their practices and which are a key element in my empirical studies.

**Why Should Teachers Be Responsive to Student Thinking?: Powerful Mathematics Classrooms**

Recent trends indicate that school mathematics require complex communication skills and have high expectations for mathematics learners. Students are expected to be powerful mathematical doers and thinkers instead of engaging in rote memorization of algorithms and solving relatively simple problems through the application of algorithms or formulas. Students are also expected to communicate to each other mathematically, where students present their ideas, analyze and critique other students’ mathematical thinking, and build on the ideas of others to refine their own thinking. To do that, students must have opportunities to experience and engage in these new practices in powerful mathematics classrooms.

Schoenfeld (2013, 2014) and colleagues have investigated what elements constitute a powerful mathematics classroom. The five dimensions for powerful mathematics classrooms consist of (1) the mathematics; (2) cognitive demand; (3) access to mathematical content; (4) agency, authority, and identity; and (5) formative assessment (Figure 2-1).

<table>
<thead>
<tr>
<th>The Mathematics</th>
<th>Cognitive Demand</th>
<th>Access to Mathematical Content</th>
<th>Agency, Authority, and Identity</th>
<th>Formative Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop</td>
<td>The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students’ mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so</td>
<td>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in</td>
<td>The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another’s ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically</td>
<td>The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction “meets students where they are” and</td>
</tr>
</tbody>
</table>
productive mathematical habits of mind.
large that students are lost at sea.
which a small number of students get most of the “air time” is not equitable.
solid), resulting in positive identities as doers of mathematics.
gives them opportunities to move forward.

Figure 2-1. TRU Math: Five Dimensions of Mathematically Powerful Classrooms

In short, the key features in each dimension are described as follows:

(1) *The mathematics:* students have opportunities to engage in high quality mathematics content and mathematical practices in meaningful and productive ways. The mathematics is more focused and coherent. Opportunities to make meaningful connections between, concepts, procedures, and contexts are provided.

(2) *Cognitive demand:* students have opportunities to engage in “productive struggles” that are neither easily overcome nor insurmountable.

(3) *Access to mathematical content:* classroom provides all students with equal opportunities to access mathematics.

(4) *Agency, authority, and identity:* students have opportunities to communicate and collaborate with each other mathematically, in ways that contribute to their development of agency, authority, and positive mathematical identity as they conceptualize themselves as people who can engage in mathematics meaningfully and productively.

(5) *Formative assessment:* teachers know what students understand and don’t understand, and provide formative feedback effectively based on this information.

While all five dimensions are intertwined, the *formative assessment* is the focus in my dissertation. In particular, the importance of teachers’ noticing and making effective use of student thinking for creating a discourse community has been addressed in many studies and literature (Black & Wiliam, 1998; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert, 1990; Sherin, Jacobs, & Philipp, 2011). In the next section, I elaborate on student thinking-centered instruction.

### The Role of Students’ Emergent Mathematical Thinking in the Classroom

Understanding and assessing students’ mathematical thinking have been considered the key starting point of instructional practices (Carpenter, Fennema, & Franke, 1996; Heaton, 2000; Sherin, 2011). By knowing their students’ current understanding, teachers can better support their students in connecting existing ideas to new ideas during instruction. Also, teachers can identify impediments to student learning and their students’ current understanding of a given topic so that teachers can begin their lessons with what students can do. Teachers can identify what the common errors and misconceptions are so that they can help students’ conceptual change using various pedagogical approaches. That is, knowing and understanding their students’ current mathematical thinking by eliciting students’ reasoning and making their mathematical

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2 This table is directly adapted from [http://ats.berkeley.edu/tools.html](http://ats.berkeley.edu/tools.html), “The Teaching for Robust Understanding (TRU) Framework”
thinking visible during the lesson is the first step for supporting students’ productive mathematical learning in the classroom.

These particular classroom practices involve teachers identifying individual students’ mathematical thinking for the purpose of using these assessments or diagnoses for improving individual students’ learning in the classroom. In these classroom practices, students’ ideas are valued and their voices are encouraged. Once teachers understand student mathematical thinking by eliciting and facilitating their students’ development of ideas, they promote productive discussion among students through argument and justification. Such practices are referred to in various ways, such as, “diagnostic teaching (Schoenfeld, 2011),” “formative assessment teaching (Black, Harrison, Lee, Marshall, & Wiliam, 2004),” “inquiry-oriented teaching (Ball & Cohen, 1999),” “discovery teaching (Hammer, 1997),” or “improvisational teaching (Heaton, 2000).” In this study, I refer these kinds of teaching practices—when instructional practices engage students in reasoning and prompt them to confront and resolve their own misunderstandings through discussion—as student thinking responsive teaching practices. These theoretical backgrounds inform my analytical framework, the development of which is described in detail in Chapter 4.

The key practices of student thinking responsive teaching are to understand students’ mathematical thinking and to make effective use of them in classroom instruction. By eliciting students’ mathematical thinking, teachers can begin to gain deeper knowledge of students’ mathematical thinking. After students’ mathematical thinking is elicited, it is important that teachers provide students with opportunities to deepen their mathematical understanding. Mathematics education researchers generally agree that developing and presenting students’ mathematical ideas in classroom discussions can provide rich opportunities for students to develop conceptual understanding and proficiency (CCSSI-M, 2010; Kazemi & Stipek, 2001; NRC, 2001). Therefore, it is a key practice of student thinking responsive teaching to establish a productive mathematical discourse community in the classroom where students are expected to explain their ideas, critique or build on their classmates’ ideas, and justify their own ideas. In order to establish productive mathematical discussion norms in the classroom, teachers change their roles from explaining concepts or lecturing to facilitating and orchestrating students’ mathematical talk. In the end, students can assume more central roles in their own learning with minimum assistance from the teacher to engage in mathematical discussions.

Several studies show that the practices of student thinking responsive teaching actually improve students’ learning. For example, Black and William (1998; Black et al., 2004) and others (de Lange, 1999; NRC, 2001; Shepard, 2000) report that formative assessment teaching—when assessment is an integral part of instruction—has a powerful effect on students’ substantial and long-term learning, as well as on students’ mathematical practices or habits of mind as student thinking takes on a more central role in classroom teaching and learning. Therefore, it is important for teachers to become more responsive to student mathematical thinking in order to support students’ mathematical learning in more powerful ways.
Educative Curriculum Materials for Teachers and How Teachers Learn through Using Curriculum Materials

In the previous section, I address the importance of student thinking responsive teaching practices and rationales for teachers to change their instructional practices toward becoming more responsive to student mathematical thinking. However, teachers face many challenges when they engage student thinking and reasoning in their instruction. Instruction based on student thinking is difficult to plan because it is hard to anticipate where the lesson will go, especially when unexpected students misunderstandings emerge (Cengiz, Kline, & Grant, 2011; P. Cobb, Wood, Yackel, & McNeal, 1992). It is also difficult for teachers to make immediate decisions on the fly when orchestrating discussions (Grant et al., 2009). In a very short period of time, they must consider which student ideas would be most productive to elicit and to discuss, which ideas would provide better opportunities to reconceptualize their (mis-)understandings, and which ideas would provide alternative ways of thinking. Teachers who attempt to engage in student thinking and to create a discourse community also experience tension between establishing discussion norms—how to talk—and coordinating mathematical content—what to talk (Sherin, 2002).

In order to support these teachers, especially those who are experiencing challenges, in becoming more responsive to student thinking and creating student thinking centered classrooms, curricular materials can play an important role in teachers’ learning through practice. When curriculum materials are designed to promote not only student learning but also teacher learning, they are called as educative curriculum materials (Davis & Krajcik, 2005b). The literature argues that such educative curriculum materials should be designed to pay more attention to their implementation and contain the following factors with concrete guidelines and examples in order to better support teachers’ professional learning (Ball & Cohen, 1996; Davis & Krajcik, 2005):

Educative curriculum materials can support teachers:

1. To gain knowledge of subject matter: curriculum materials can offer various ways of representing content and alternative ideas, with multiple ways of making connections among concepts.
2. To gain knowledge of students and their thinking: curriculum materials can provide rich examples of students’ (mis-)understandings so that teachers can anticipate their students’ understandings and prepare to enact their pedagogical strategies that enable better use of students’ thinking and offer formative feedback.
3. To gain curricular knowledge: curriculum materials can help teachers understand particular units with regard to the spectrum of the curriculum in both lateral and vertical ways, i.e., connections to other subjects as well as to the same subject area over the school year.
4. To help making visible pedagogical judgment and promoting pedagogical design capacity: teachers enact the curriculum materials having autonomy.

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3 Brown (2009) perceives teaching as a form of design activity in which teachers “perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies (p.18)” in an effort to achieve instructional goals; and curriculum materials as artifacts. In this context, he
but avoiding “lethal mutations.”

Understanding how teachers use curricular materials, particularly how teachers learn by using curricula, is a relatively new and ongoing challenge. Research on teachers’ use of curricular materials has been scattered; studies focus on either lesson designers or teachers. Research focusing on the perspectives of lesson designers emphasizes how teachers enact curriculum materials effectively by avoiding lethal mutations or lack of fidelity in curriculum use. On the other hand, research focusing on the perspectives of teachers emphasizes teachers’ expertise and autonomy when teachers create their own lesson materials. However, seeing curriculum as static materials or efficient curriculum use as a matter of fidelity is an oversimplification. Furthermore, these perspectives hinder research on teachers’ learning through curriculum use when the ultimate goal is to investigate ways to better support teachers to learn through curriculum use. Therefore, we may need an alternative perspective to conceptualize teachers’ use of curriculum as a more interactive and dynamic relationship. Remillard (2005)’s review argues that seeing this relationship as a mutual interaction between teachers and curriculum (or lesson designers), or participation of teachers with curriculum materials, may be critical to understand teachers’ learning through interaction—or participation—with curriculum materials.

Educative curriculum materials view teacher learning as teachers gaining relevant knowledge for teaching and the ability to flexibly apply that knowledge in new situations, and posit that these kinds of learning are situated in their everyday practices, including their use of curriculum materials. This perspective is complementary to Remillard’s where teacher professional learning occurs through their interaction with curriculum materials. That is, teachers can gain more knowledge for teaching, change their perceptions and beliefs, and/or improve their capacity and ability to flexibly use curriculum by interacting with curriculum materials in a participatory relationship.

One example of such an approach in research is Sherin and Drake’s curriculum strategy framework (2009, p. 471). They argue that simply providing pedagogical information with concrete examples is not enough for supporting teacher learning through curricular materials. By investigating how teachers read, evaluate, and adapt curricular materials before-, during-, and after-instruction, we can help teachers better access the curriculum materials that support teachers’ implementation and learning through use of curricular materials. As teachers adapt curriculum materials, they provide a “curriculum adaptation spectrum (p. 487).” The curriculum adaptation spectrum is a continuum of omit, replace, and create. Teachers omit when they simply delete lesson activities or components of a lesson from a curriculum. Teachers replace when they substitute a component of a lesson with different component. Teachers create when they make “significant changes (p. 486)” to their use of curriculum materials by creating new activities or lesson components. This framework provides a fruitful insight on the various ways of different teachers interact with curriculum materials.

My dissertation builds on and extends these investigations on teacher learning and development of practices by clarifying how teachers make use of innovative curriculum definitions of pedagogical design capacity as “a teacher’s ability to perceive and mobilize existing resources in order to craft instructional episodes (p. 29).”

Sherin and Drake conceptualize teacher’s curriculum use strategy as a 3 x 3 matrix.
materials in their instructional practices and, further investigation on how they change during and from instructional practices over time. Due to my dissertation’s focus on how teachers create learning opportunities for themselves when they adapt curriculum materials, my analysis draws on the curriculum adaptation spectrum of Sherin and Drake’s curriculum strategy framework (2009) with the added dimension of learning opportunities for teachers, which I call the matrix of curriculum adaptation strategies and learning opportunities (Table 2-1). I make one modification to Sherin and Drake’s framework, however: I add “as-is” as another type of adaptation, or non-adaptation, for instances when teachers use curriculum materials without making any modifications.

Table 2-1. Matrix of Curriculum Adaptation Strategies and Learning Opportunities

<table>
<thead>
<tr>
<th></th>
<th>Ways of interaction with curricular materials</th>
<th>Opportunities to learn about student thinking and content</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before-instruction</strong></td>
<td>Omit, replace, as-is, or create</td>
<td>Limit or create what kinds of opportunities</td>
</tr>
<tr>
<td><strong>During-instruction</strong></td>
<td>Omit, replace, as-is, or create</td>
<td>Limit or create what kinds of opportunities</td>
</tr>
<tr>
<td><strong>After-instruction</strong></td>
<td>Omit, replace, as-is, or create</td>
<td>Limit or create what kinds of opportunities</td>
</tr>
</tbody>
</table>

My dissertation aims to explain the process of teacher learning and change in the context of curriculum support materials called Formative Assessment Lessons (FALs) that were provided to teachers. In the next section, I will briefly describe how FALs function as effective educative curriculum materials for teachers to learn and change in their practices. More specific affordances and descriptions of FALs will be introduced in the methods chapter, Chapter 3.

Supporting Teachers with Innovative Curriculum Materials: Formative Assessment Lessons (FALs)

FALs, or Classroom Challenges, were developed by members of the Shell Centre at the University of Nottingham with the intent of supporting teachers in implementing a powerful mathematics classroom aligned with CCSSM values. In particular, FALs were developed to support teachers assess and improve students’ understanding of mathematical concepts and skills, as well as their ability to use mathematical practices in various lesson phases: before, during, and after lessons. In the pre-lesson phase, pre-assessment tasks are provided to students to reveal their current mathematical thinking and misconceptions to the teacher. Based on student thinking and common errors that emerge from the pre-assessment tasks, teachers can anticipate and plan the direction of the lesson. Teachers also provide students feedback in various ways, such as commenting on each pre-assessment response or discussing common misconceptions during the main lesson. In the post-lesson phase, post-assessment tasks are provided to students so that
they can reflect on how they’ve improved after the lesson. Teachers can also use the post-assessment tasks to reflect on their own teaching and student learning.

FALs are grounded in 25 years of rigorous research on changes in teacher beliefs, as well as formative assessment. For example, Malcom Swan (2006) explored shifts in teacher beliefs as a result of teachers using earlier versions of FALs that supported eliciting and building on student thinking. Formative assessment has been shown to improve students’ mathematical learning when implemented well (Black & William, 1998). In this line of research on formative assessment, FALs are designed to enable teachers to realize the potential of their students by qualitatively assessing their mathematical thinking through the use of rich tasks and pedagogical strategies. Furthermore, FALs are freely available from a website\(^5\); schools and school districts nationwide encourage teachers to use FALs, indicating recognition of FALs’ great potential as *educative curriculum materials* for teacher learning and change from their use of lessons. Their widespread use afford the investigation of many questions pertaining to research and practice, such as how teachers make sense of student thinking and mathematical content through their implementation of FALs; how teachers’ choices during FAL implementation create learning opportunities, not only for their students, but also for themselves; how teachers’ interactions with FAL materials empower them to change their everyday teaching practices and develop new teaching practices; and what kinds of challenges teachers face when they implement FALs and how to support them through that process.

In sum, FALs have great potential for teacher learning and change towards becoming more focused on student mathematical thinking. This potential promises (informs?) me to investigate how teachers learn and change in practices through implementation of FALs. I have shown how my dissertation is grounded in ongoing research on teachers’ use of curriculum materials and the process of teacher learning and change through their practices. My dissertation findings will contribute to this line of inquiry and provide further insights into how to better support teachers in using innovative curriculum materials.

The theoretical works that I have reviewed in this chapter provide a valuable perspective on teacher learning and change in interacting with innovative curriculum materials and is a first step to conduct empirical investigation. In the next chapter, I provide the broader contexts and methods that I used to conduct the empirical studies in this dissertation.

Chapter 3: Methods

The methodological goals of this study are (1) to capture the change of mathematics teachers’ teaching practices over a school year; (2) to document how the teachers learn through their teaching experience of implementing particular curriculum materials; and (3) to provide a detailed description of teaching practices with regard to student thinking responsive teaching or formative assessment.

This study employs two strands of analysis in terms of these goals. In each strand of analysis, several phases of data analysis are employed.

In this chapter, I describe the broad context of the study. This dissertation study was part of a larger research project in which a larger corpus of data was collected, with additional data being collected for this study in particular. Then, data collection and data sources are described. Last, the overview of analytical approaches is provided. The detailed analytical framework is described in Chapter 4.

The Broader Contexts of Research Settings

This study takes place in the context of the Mathematics Assessment Project (MAP), a collaboration between the University of California, Berkeley and the University of Nottingham. The central goal of the MAP project is to support US schools in implementing the Common Core State Standards for Mathematics (CCSSM; 2010) by designing and developing rich curriculum support materials and assessment tools. At Berkeley during the 2012-2013 school year, four middle school teachers participated in the field testing (including classroom observation) of “Formative Assessment Lessons” (FALs). In the 2013-2014 school year a total of eight teachers—four middle school teachers and four high school teachers—participated in the same way. All of the teachers volunteered to use FALs in their everyday instruction over the year.

Some parts of my dissertation data were drawn from this larger study. In line with the concerns of the MAP project, this study initially focuses on how teachers’ teaching practices changed over a year, given that they implemented a series of FALs and had professional development support for their implementation. The participating teachers were excited by the opportunity to implement the FALs and to work with their students, colleagues, and the research team. They also had a fair degree of flexibility with regard to lesson implementation, e.g., they could choose which lessons they would teach, and when, depending on the circumstances of their regular lesson schedule. In this context, this dissertation study is more like a naturalistic observational study rather than strictly designed. In what follows I provide an overview that explains the research setting and data collection, given that this study employs several data sources (see Figure 3-3).

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6 The MAP project is a 6-year long (2009-2015) GATEs-funded collaborative project between the University of California, Berkeley, with Alan Schoenfeld, PI, and the University of Nottingham, led by Hugh Burkhardt and Malcolm Swan. The main focus of the project is the creation, support, and study of 100 Formative Assessment Lessons, intended to support teachers in using formative assessment. For more detailed information about the project, see: [http://map.mathshell.org](http://map.mathshell.org).
**Curriculum Support Materials: FALs.** The curricular support materials, *Formative Assessment Lessons* (FALs) or Classroom Challenges, were developed to support teachers to engage productively with student’s emergent mathematical thinking. Because the implementation of the FALs is the core part both of the larger project and of this dissertation study in order to capture teacher change as a result of the implementation, here I describe FALs in more detail.

The FALs embody the designer’s specific pedagogical and content visions, and are complex in structure. FALs have two different types of lessons from 6th grade to high school: conceptual development lessons and problem solving lessons. Conceptual development lessons are intended to meet the requirements of Common Core State Standards ("proficient students expect mathematics to make sense;" CCSSI-M, 2010), to help students to develop the mathematical concepts underlying the lessons, and to connect those concepts to their other mathematical knowledge. Conceptual development lessons support teachers in making students’ mathematical thinking visible so that the teachers can provide formative feedback throughout each lesson with specific suggestions, sample questions, and examples. Conceptual development lessons suggest teachers to implement the following phases:

![Figure 3-2. Suggested Implementation of Conceptual Development Lessons](image)

Problem solving lessons are developed to meet another requirement of CCSSM, “they [students] take an active stance in solving mathematical problems (CCSSI-M, 2010),” and focus on the application of previous mathematical knowledge to non-routine, openly structured problems within mathematics and with real world situations. The problem solving lessons involve students critiquing sample student work, as a mechanism for identifying and being exposed to various strategies, and then learning from the consideration of those strategies. These problem solving lessons are also designed to assess students’ capacity to apply their mathematical knowledge so that teachers can provide formative feedback during instruction. The suggested implementation phases of problem solving lessons have a bit more variation in each phase, but they generally proceed as follows:
The general structure of FAL implementation has three phases: (1) students’ pre-assessment before the lesson, (2) activities during the lesson including launch, small group activities, and sharing of student work in whole class discussion and/or student presentations, and (3) students’ post-assessment (in concept development lessons), or a combination of two practices: (a) students revisiting and revising their work, and/or (b) students constructing a written final reflection on what they have learned (in problem solving lessons) after the lesson. The pre-assessment phase reveals students’ current mathematical thinking and misconceptions to the teacher. Based on student thinking and common errors that emerge during the pre-assessment phase, teachers can anticipate and plan the direction of the lesson. Also, teachers provide students feedback in various ways such as commenting on each pre-assessment paper or discussing common misconceptions during the main lesson. In the post-assessment or reflecting phase, teachers reflect on their own teaching and student learning from the lessons; students reflect on how they have improved after the lesson.

Even though the purpose of this study is not to validate these lessons, the FALs provide important context in this study for two reasons. The analysis in this study with regards to teachers’ use of FALs aims (1) to capture the change in teachers’ practices as a result of implementing FALs; and (2) to explore how the FALs provide affordances and opportunities for teachers to make sense of mathematics and student mathematical thinking.

**Professional Development.** The larger project provided two different types of professional development sessions (PD) for participant teachers in order to support teachers in implementing FALs and to learn teaching practices aligned with CCSSM: Summer intensive four full days long PD workshops and monthly PD meetings.

Both focal teachers engaged in the 2012 summer PD workshop with 12 other teachers (6th grade to 11th grade teachers) from the same school district. The PD was led by two facilitators from the larger project. The purpose of the PD was to introduce the teachers to the FALs as a tool for developing pedagogy grounded in and responsive to students’ thinking. The four-day PD workshop introduced the teachers to key strategies for formative assessment, focusing on the roles both students and teachers play in
producing and analyzing feedback data. Teachers also studied how to implement the CCSSM specifically with regard to FALs. They watched and discussed some teachers’ implementation of FALs, discussed students’ mathematical thinking revealed in the pre-assessments conducted by other teachers in previous years, learned pedagogical strategies to elicit and make use of students’ thinking, learned how to facilitate group activity, and learned how to orchestrate productive classroom discussions.

Teachers also had five monthly PD meetings over the school year. In these meetings, Ms. Lee engaged with three other mathematics teachers from different schools in the same district. Ms. Janet engaged with three other mathematics teachers from the same school, Morris Middle School. Basically, these monthly PD meetings offered similar opportunities. Teachers shared their reflections from conducting FALs, discussing the challenges and issues around implementing these lessons. They also studied FAL(s) that they would teach, planned and rehearsed teaching the FAL(s), analyzed their own students’ previous knowledge about the lesson revealed on any pre-assessments, and read research articles that supported their teaching. Both teachers implemented three FALs during the school year.

**Participants: Focal Teachers and School Contexts**

From the more than twenty teachers who participated in the project in two school districts over two school years (2012-2014) and eight teachers from my initial observations, I selected two teachers, Ms. Lee and Ms. Janet, to conduct a systematic comparison of why and how they made changes (or not) in their teaching practices and what they learned through FAL implementation. Here I first describe the context for each teacher and her schools. Then, I provide the rationales for why I choose these two teachers for the systematic comparison in my study.

One participating teacher, Ms. Lee, taught 7th grade mathematics at Bear Middle School in an urban school district located in the San Francisco bay area during the 2012-2013 school year, the year I conducted extensive observations of her teaching. She had 10 years of teaching experience in middle school when I observed in her classroom, and was moving toward coaching. She participated in implementing FALs twice and she was in the second year of implementing FALs when I observed her classroom.

The other participating teacher, Ms. Janet, taught 7th grade mathematics at Morris Middle School, in the same school district as Ms. Lee during the 2013-2014 school year. Her experience included 2 trimesters teaching all different levels of math at high school, 5 years teaching 6th grade mathematics and science, and she moved to teach 7th grade during the observation year.

I selected these two teachers, Ms. Lee and Ms. Janet, to conduct a systematic comparison of why and how they made changes (or not) in their teaching practices and how they learned through FAL implementation. Here is the rationale for my selection. First, both teachers were teaching same grade students, 7th grade, in the same urban school district when I observed their teaching. Second, both teachers volunteered to implement FALs and were eager to participate in this project: they were excited to learn to teach according to the CCSSM and the project recommendations, and eager to implement FALs. That is, both teachers were excited to learn about how to teach FALs in terms of becoming more responsive to student thinking. Third, they participated in the

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7 All names of teachers, students, and schools are pseudonyms.
same summer PD workshops and implemented the same number of FALs during the school year when I observed each of their teachings. Therefore, I hypothesize that if they illustrated different changes in their teaching practices while they had very similar contexts in terms of their participation in this study, it would be meaningful to investigate what and how they learned as they implement FALs and why they show changes differently.

**Data Sources and Data Collection**

This research employs a multi-level approach (Yin, 2008) using qualitative research methods. This study employs several data sources: classroom observations (regular teaching observations and FAL teaching observations), teacher interviews, PD meeting observations, and artifacts. The data for this study were collected over a period of two school years (2012-2013 and 2013-2014). I observed 2012 Summer PD workshops where both teachers participated. I observed Ms. Lee’s classroom and monthly PD meetings over a 2012-2013 school year and Ms. Janet’s classroom and monthly PD meetings over a 2013-2014 school year (see Figure 3-3).

![Figure 3-4. Overview of data collection](image)

**Classroom observations of regular teaching.** The primary purpose of these regular teaching observations was to document the teachers’ everyday classroom practices and to trace changes in their everyday teaching practices over the year. All classroom observations were video recorded and the video camera followed the teacher. Field notes were taken to supplement video data. I recorded Ms. Lee’s regular lesson teaching four times—twice at the beginning (October) and twice at the end (May) of the 2012-2013 school year. The third video of regular teaching observation of Ms. Lee was taped fragmentally (only the initial part of the lesson and small group work were recorded) because of technical issues with the video camera. I recorded Ms. Janet’s regular mathematics teaching six times: twice at the beginning (September), twice at the
middle (December and January), and twice at the end (April and May) of the 2013-2014 school year.

I analyzed the teachers’ discourse closely to identify patterns with regard to eliciting students’ thinking and reasoning and making use of elicited students’ mathematical thinking. I also analyzed students’ discourse over time to see the patterns how students respond to particular teachers’ questioning. The analysis of regular teaching observation video data is provided in Chapter 7.

**Classroom observations of teaching FALs.** The FAL teaching observations served three purposes: (1) to observe and document how the teachers implemented specific FALs with regard to dealing with students’ mathematical thinking (i.e., formative assessment teaching or diagnostic teaching); (2) to observe how the teachers learned from their teaching experiences, in particular their learning about and making use of their own students’ mathematical thinking as they implemented the lessons; and (3) to understand the challenges teachers faced as they implemented FALs, and the conditions for teacher change. Similar to the regular teaching observations, all FALs observations were video-recorded and the video camera followed the teacher. Field notes were also taken to supplement video data. Both Ms. Lee and Ms. Janet implemented three FALs throughout the school year (see Table 3-1). A global analysis of the teachers’ decision making to use the affordances of the FALs over the school year is provided in Chapter 5. A local analysis of how teachers’ instructional moves created opportunities to learn about mathematics and student thinking is provided in Chapter 6.

**Table 3-2.** FALs that two teachers implemented over the school year

<table>
<thead>
<tr>
<th>Taught FAL</th>
<th>Title &amp; Content Goals</th>
<th>Lesson Types</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ms. Lee</strong></td>
<td>“Developing a Sense of Scale”</td>
<td>Problem solving</td>
<td>Nov 28-30, 2012</td>
</tr>
<tr>
<td></td>
<td>- Using the relationship of direct proportion to solve proportional relationship problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>“Increasing and Decreasing Quantities by a Percent”</td>
<td>Concept development</td>
<td>Dec 13-14, 17, 2012</td>
</tr>
<tr>
<td></td>
<td>- Interpreting percent increase and decrease in proportional relationships</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>“Drawing to Scale: Designing a Garden”</td>
<td>Problem solving</td>
<td>May 29-30, 2012</td>
</tr>
<tr>
<td></td>
<td>- Interpreting and using proportional reasoning and metric units.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ms. Janet</strong></td>
<td>“Using Positive and Negative Numbers in Context”</td>
<td>Concept development</td>
<td>Sep 18-19, 2013</td>
</tr>
<tr>
<td></td>
<td>- Understanding directed numbers and operations in context</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>“Increasing and Decreasing Quantities by a Percent”</td>
<td>Concept development</td>
<td>Dec 3-4, 2013</td>
</tr>
<tr>
<td></td>
<td>- Interpreting percent increase and decrease in proportional relationships</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Teacher interviews. I conducted two types of teacher interviews. The first type consisted of two 60-minute semi-structured interviews at the beginning and the end of the school year. The main purpose of the first interviews was to learn about the teachers (their backgrounds, how they understand the mathematics they teach, their students’ mathematical thinking, their views on pedagogy, their teaching goals, their professional goals, and so on) at the beginning of the year, and to learn how they would reflect on their teaching experiences and challenges as they participated in this study (implementation of FALs and PD experiences). The interviews additionally served to document and supplement my analyses of classroom observation.

The second type of teacher interview consisted of brief 5- to 10-minute interviews, either in person or via e-mails before and after the regular and FAL lesson observation. All in-person interviews were audio-recorded and transcribed. The purpose of the pre-observation interviews was to clarify the teachers’ goals and plans for each lesson. The purpose of the post-observation interviews was to capture the teachers’ reflections on their teaching of each lesson. These short interviews provided additional information to better understand teaching goals and behaviors in each lesson.

PD Observations. As I described above, both teachers participated in two different types of PD workshops and meetings: four full-day workshops during 2012 summer and monthly PD meetings. Summer PD workshops were video-recorded. Monthly PD meetings were held five times over each observation year and these meetings were either video-recorded or documented with field notes. This PD observation data supplements my analyses of classroom observations.

Artifacts. I used several artifacts, such as FAL lessons, teachers’ lesson plans, students’ worksheets, textbooks, posters, teachers’ teaching manuals, and pre- and post-assessments of FALs as supplementary evidence to inform my analyses.

Overview of Analytical Approach

In this section, I briefly outline the analytical approaches used in each empirical chapter. A primary goal of this dissertation study is to better understand how teachers learn through their teaching practices when they are provided innovative curricular support materials and to understand teacher change in everyday practice as a one-year-longitudinal result of implementation of FALs and of PD supporting. To explore this topic, my analysis consists of two strands. Strand 1 captures the changes in two teachers’ everyday teaching practices towards becoming more responsive to student thinking and creating a productive mathematical discussion environment. Strand 2 examines how two teachers implement FALs differently, that is, how two teachers make decisions to use curricular materials in globally. This analytical strand also captures profiles of two teachers’ teaching practices in using a particular FAL, to understand in more detail how the teachers respond to student thinking and create opportunities to learn student thinking and mathematics.
The analysis of strand 1 employs regular classroom observations as a main data source; teacher interview data is supplemental. The analysis of strand 2 employs FAL teaching classroom observations and the affordances of FALs as main data sources; PD meeting observations and teacher interviews are supplemental. The second phase of the strand 2 employs one particular FAL teaching classroom observation as a main data source.

For all strands of analysis, all video data gathered in classroom observations were transcribed and iterative viewings of the data were made in an attempt to generate analytic categories that captured teachers’ teaching practices in various ways. The generation and refinement of the analytic categories involves a process of compiling the instances of a teaching practice aligned with FAL implementation goals, specifying inclusion and exclusion criteria, generating a hypothesis about the various teaching practices, revisiting the data corpus to support or reject the hypothesis, and checking inter-rater reliability. Therefore, it is consistent with the grounded theoretical approach and developing analytical category for open coding (i.e., coding scheme) discussed by Glaser & Strauss (2007), MacQueen, McLellan, Kay, & Milstein (1998), and Miles & Huberman (1994). Qualitative contexts and evidence are provided for quantified coding results.

All of the detailed processes of developing analytical categories and the analytical framework are elaborated in Chapter 4. The analyses of the first strand, teacher change, yield the findings in Chapter 7. The findings of the global analysis of the second strand, teacher decision-making in using FALs, are described in Chapter 5. The results of the profiles of teaching practices with one particular FAL with the second analytical framework and teacher learning opportunities through instructional practices are provided in Chapter 6.
Chapter 4: Analytical Framework and Analysis

In Chapter 2, I discuss what student thinking responsive teaching practices aims and why it is important for teachers to change their practices toward becoming more responsive to student mathematical thinking. I also characterized key aspects of student thinking responsive teaching practices grounded in research literature. Building on that theoretical work, in this chapter I describe in detail the phases of my analysis. I also describe how I developed analytical frameworks for the first and second strands of analysis using classroom observation video data. To recall the two strands of analysis: In the first strand I consider whether two teachers’ teaching practices changed over a year as a result of implementing the FALs, and if yes, in what ways the teacher’s teaching practices changed. The second analytical strand investigates how the two teachers responded to student mathematical thinking and what kinds of learning opportunities they created both for themselves and for their students by exploring how they implemented FALs differently. I first developed the analytical framework for the first strand to check and capture whether or not the two teachers’ teaching practices changed. Then, I closely looked at their ways of implementing new practices and new curricular support materials (the FALs) in more systematic ways. The second strand of analysis also involves the analysis, in more global ways, of how the two teachers made different use of FALs.

Strand 1. Whether and how do teachers change in their teaching practices?

This analytical strand identifies the kinds of teaching practices that are associated with being responsive to students’ mathematical thinking, and is used to capture the changes or lack of changes in Ms. Lee and Ms. Janet’s teaching practices. The development of the analytical framework for capturing changes in everyday teaching practices involves three main phases: (1) initial iterative viewings of classroom video data for capturing interesting moments and patterns; (2) categorizing emerging themes for creating a stable codebook; and (3) testing and refining the codebook. As described in Chapter 3, theses phases on developing the analytical framework from codebook are consistent with the theoretical and methodological approaches to qualitative methods discussed by Glaser & Strauss (2007), Miles & Huberman (1994), and MacQueen, McLellan, Kay, & Milstein (1998).

In the first phase, iterative watching of the two teachers’ classroom video data (both regular teaching and FALs teaching) occurred with transcriptions and with the StudioCode program. The goal was to capture interesting events involving each teacher’s particular discourse patterns, evidence of eliciting and making use of students’ mathematical thinking (a main goal of this study), and of changes in teaching practices. During this initial pass and the transcription of the classroom video data I flagged instances that seemed to involve interesting patterns of each teacher’s discourse and their interactions with students’ discourse. By establishing a coding scheme with this evidence, specific themes emerged and became components in the analytical schemes and this analytic scheme had several cycles of modifications associated with iterative observation with the emerged categories as the second and third phases. Through the iterative observation and modification, the analytical framework became increasingly robust.
Six kinds of teacher discourse were identified and framed as analytic categories. The categories capture pedagogical moves where teachers elicited students’ mathematical thinking and reasoning for productive mathematical discussions (see Table 4-1): (1) Explaining; (2) Restating, evaluating, and clarifying; (3) Questioning without eliciting thinking or reasoning (gathering answers with questions, a rather procedural process); (4) Soliciting mathematical ideas and strategies; (5) Eliciting reasoning (asking students to justify ideas); and (6) Promoting productive mathematical discussion (prompting students to have a discussion with each other, with little teacher assistance).

Table 4-1. Analytical categories of teaching practices towards making effective use of students’ thinking

<table>
<thead>
<tr>
<th>Teaching Practice</th>
<th>Description and Examples in the Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Explaining</td>
<td>Teacher explains concepts/methods directly without providing space for students to explain their thinking.</td>
</tr>
<tr>
<td></td>
<td>e.g., “Let’s do the multiplication on your paper: where 6% is .06 and 56 time .6. 6 times 6 is 36, 6 times 5 is 30, so we have 336.”</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>Teacher restates/evaluates/clarifies (or asks them to clarify) simply what students said without any further facilitation of their thinking.</td>
</tr>
<tr>
<td></td>
<td>e.g., “That was an excellent explanation.”</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Thinking or Reasoning (Procedural Processes)</td>
<td>Teacher asks students procedural process questions WITHOUT providing space for students to explain their reasoning.</td>
</tr>
<tr>
<td></td>
<td>e.g., “What’s the percent of 1/2?” “50 percent.”</td>
</tr>
<tr>
<td>D. Soliciting Mathematical Ideas and Strategies</td>
<td>Ask students what they did/do with the problems/tasks. The questions’ intention contains how students solve the problem/tasks.</td>
</tr>
<tr>
<td></td>
<td>e.g., “Tell me which problem you did and what you did in that problem”</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>Teacher elicits students’ reasoning and justification.</td>
</tr>
<tr>
<td></td>
<td>e.g., “Can you explain why this is up to 50?”</td>
</tr>
<tr>
<td>F. Promoting Productive Mathematical Discussion</td>
<td>Teacher asks students to elaborate/reframe someone else’s strategies including (dis)agreement, prompts students for further participation, gives students ownership of their ideas, or accepts students’ spontaneous ideas.</td>
</tr>
<tr>
<td></td>
<td>e.g., “Do you agree with him? Why?” “Can you expand on what Anna said or why it doesn’t make sense to you?”</td>
</tr>
</tbody>
</table>

The first code (code A) in teacher discourse, “Explaining,” captures instances during which a teacher explains concepts or strategies directly without providing a space for students to express their mathematical thinking. If this code happens frequently, it may indicate that the teaching episode was more didactic in nature. Code B “Restating/
Evaluating/Clarifying,” captures instances during which a teacher simply restates, evaluates or clarifies what students said without prompting further intellectual participation on part of the students, such as simply saying “excellent,” “okay, this is two plus $x$ [right after a student states, “two plus $x$’”]” or “say that again?” Code C, “Questioning without Eliciting Thinking or Reasoning,” captures instances during which a teacher asks a question that results in very short answers from students without the teacher eliciting students’ reasoning or asking how students solved the problems/tasks. The teacher seems to already have a specific procedural answer (such as, specific numbers) in her/his mind when (s)he asks these kinds of questions. The fourth code (code D), “Soliciting Mathematical Ideas and Strategies,” captures instances during which a teacher asks students how to solve a problem. In this case, the question is not asked for the purpose of publicly displaying (correct) knowledge, but to understand students’ mathematical thinking, which is an important aspect of formative assessment lessons. Code E, “Eliciting Reasoning,” captures instances during which a teacher asks students why they got a certain answer and students justify their answers. The last code (code F), “Promoting Productive Mathematical Discussion,” captures various strategies to promote mathematical discussion for having students develop accountability and authority, such as asking students to reframe, elaborate, and/or revise ideas. A teacher may ask a student to reframe what other students said about their mathematical thinking in his/her own language, or to elaborate on what other students said about their mathematical thinking. These pedagogical moves help foster accountability on part of students to understand each other’s ideas. Also, a teacher may publicly attribute ownership for the presented mathematical ideas by making statements such as, “Isiah’s method is to use the distributive property and Julian’s method is to use block models.” Furthermore, a student may spontaneously raise his/her hand and express his/her mathematical thinking to the class. If the teacher accepts these ideas in the classroom, the instance is coded as code F as well, because this pedagogical move serves to share intellectual authority with the student.

The focus of the documentation of teacher change in this study is to capture if their teaching practices change towards becoming responsive to students’ mathematical thinking. The fifth category, “eliciting reasoning,” is most closely aligned with the PD’s goal to promote productive mathematical discussions. This enables students to argue and justify their ideas that are then elicited and made public to others. The sixth category, “promoting productive mathematical discussion,” is the ultimate goal where teachers act as facilitators by using students’ mathematical thinking. This allows students to spontaneously engage in mathematical productive discussions for conceptual understanding with minimal prompting from teachers. Therefore, shifts in teaching practices where instances in the fifth and sixth analytic categories become more prevalent would indicate that the teacher is becoming more aligned with FAL implementation goals. The other four categories were also coded in order to determine the prevalence of the fifth and sixth categories relative to the whole.

Each code has several sub-codes, such as, for code A, “Explaining” involves “explaining concepts and strategies,” or “presenting problem.” Because code F, “Promoting Productive Mathematical Discussion,” captures more types of interactions involving accountability and authority, and more teacher pedagogical moves than other codes in this study, I provide a number of sub-codes. The sub-codes are “accepting
ideas,” “ascribing ownership of ideas,” “elaborating on someone else’s ideas,” “reframing someone else’s ideas,” and “devising different approaches to problem.” The detailed description of each code and examples are provided in Table 4-2.

Table 4-2. Sub-codes of “F. Promoting Productive Mathematical Discussion”

<table>
<thead>
<tr>
<th>Kinds</th>
<th>Description and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accepting ideas</td>
<td>When students raise their ideas spontaneously, teacher accepts the ideas. e.g., A student raises his hand and Teacher said, “Do you want to revise your answer? What is it?”</td>
</tr>
<tr>
<td>Ascribing ownership of ideas</td>
<td>Teacher ascribes students ownership of their ideas e.g., “This is Jason’s method and this is distributive property. You can choose either one to solve the problem.”</td>
</tr>
<tr>
<td>Elaborating on another students’ strategies</td>
<td>Teacher asks to elaborate someone else’s thinking, methods, strategies e.g., “Rosie, can you add on what she said?”</td>
</tr>
<tr>
<td>Reframing another student’s statement</td>
<td>Teacher asks to reframe someone else’s thinking e.g., “Philip, can you reframe what Eli said?”</td>
</tr>
<tr>
<td>Devise different approaches to problem</td>
<td>Teacher solicits different methods e.g., “That’s one way, anyone else show different method?”</td>
</tr>
</tbody>
</table>

Student discourse codes are presented similarly (see Table 4-3). Code A, “Answering the Teacher Without Reasoning” captures instances during which student(s) answer procedural processes or known answer questions. It may occur most frequently in response to the teacher posing procedural process questions (teacher discourse code B). Student discourse code B, “Answering What Students Do” captures instances during which student(s) discuss their solutions and strategies in procedural terms. It relates to teacher discourse code D and it reveals students’ what the students did but not their reasoning. Code C, “Explaining Own Reasoning to the Teacher,” captures instances during which student(s) explain their reasoning directly to the teacher. Code D, “Accountable Talk and Authority,” captures various instances seen as indications of students’ accountability and authority, such as, student(s) reframe another student’s methods or reasoning, student(s) build on other student(s)’ thinking, or student(s) express their agreement or disagreement. These are given as sub-codes (see Table 4-4). The ultimate goal for productive mathematical discussion is code D, which students engage in mathematical discussion with little teacher’s stimulating.

Table 4-3. Identifying classroom practices focusing on students’ discourse

<table>
<thead>
<tr>
<th>Student Talk</th>
<th>Description and Examples in the Data</th>
</tr>
</thead>
</table>

23
A. Answering the Teacher without Reasoning (Known Answer Questions, Procedures)

Students answer known answer questions, read problems, or say what they did WITHOUT providing their reasoning. e.g., “What’s the percent of 1/2?” “50 percent.” “Excellent.”

B. Answering the Teacher What They Do (Neutral between reasoning and known answer questions)

Students answer a question or say what they do/did WITHOUT reasoning but indicate how they figured out the problem/tasks. e.g., “I did number 1 and I multiply 2 and 3, which is 6.”

C. Explaining Own Reasoning to the Teacher

Students answer a question or talk about their own strategies and reasoning to the teacher. e.g., T: “How do we know that when we went from 100 to 150 there’s an increase of 50? S: “Okay, because 50% of 100 is 50, so you add that other 50% to the next one. Then, 100 plus 50 is 150.”

D. Accountable Talk and Authority

Students explain/reframe/revolve/build on what other students said. e.g., “What Aria said was, so 100% and when you have 50% of it, you have 150%.”

**Table 4-4. Sub-codes of “D. Accountable Talks and Authority”**

<table>
<thead>
<tr>
<th>Kinds</th>
<th>Description and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D. Accountable Talks and Authority</strong></td>
<td></td>
</tr>
<tr>
<td>Raising ideas on their own</td>
<td>Students raise their ideas spontaneously or show their willingness to share their ideas e.g., “Can I come up there and show you how I did?”</td>
</tr>
<tr>
<td>Building on another students’ strategies</td>
<td>Students elaborate someone else’s thinking, methods, strategies e.g., “I agree what Udo said the addition is correct, but I think it’s more than that. What I notice is 33.60 is exactly 6% because …”</td>
</tr>
<tr>
<td>Reframing another student’s statement</td>
<td>Students reframe someone else’s thinking e.g., “She said it was like 1/3 because 1/3 of it would be one 50…”</td>
</tr>
<tr>
<td>Agree/Disagree</td>
<td>Students show their agree or disagree on others’ ideas e.g., “I agree with all the people who have spoken because it’s just addition.”</td>
</tr>
</tbody>
</table>

The classroom observation data are chunked into episodes that indicate the lesson structure and specific activity structures such as, a Do Now episode, Small Group Work, Student Presentation, and Whole Group Discussion. Each episode will be coded with the analytical categories (i.e., coding scheme; 4-1 and Table 4-3). Qualitative descriptions will be also provided (such as, lesson/episode contexts, tasks, lesson flows, and narrative evidence for teacher/students discourse).

---

8 At the beginning of each lesson, both Ms. Lee and Ms. Janet present Do Now problems and students are working individually for several minutes and then have a whole class discussion about the Do Now problems. Here, Do Now episode refers to a whole class discussion structure with Do Now problems.
Strand 2-a. How do teachers respond to student thinking and create opportunities for themselves to learn about students’ mathematical thinking and mathematical content?

The purpose of developing this analytical framework is to capture profiles of instructional practices of how the two teachers responded to student mathematical thinking and how they created learning opportunities for themselves. This analytical strand focuses on how the two teachers implemented the FALs differently, in order to investigate in detail why the results of the strand 3 are so different – that is, why one teacher changes significantly while the other does not, as they implement the FALs. In order to test my hypothesis that they may create different learning opportunities for themselves while they implement FALs, the analytical framework for strand 2 is developed by refining from and zooming in on teacher discourse codes D, E, and F in Table 4-1 (as well as student discourse codes B, C, and D in Table 4-3).

In order to capture how teachers respond to student thinking, “eliciting student ideas” should be prior to the use of student thinking. It became Dimension 1 of this analytical framework. The process of how teachers made use of student ideas, particularly how they used elicited student ideas as resources for productive discussion, is Dimension 2. (The ultimate goal of responding to student thinking is to provide opportunities for students to reconceptualize their current understandings and to pursue alternatives through engaging in productive discussion.) Dimension 3 focuses on how the teachers supported students to engage in productive discussion. All three dimensions are distinguished but inter-related with regard to becoming more responsive to student thinking. Each dimension consists of four developmental levels—level 0 to level 3. Level 0 refers to a traditional teacher-centered classroom practice. The developmental trajectory from level 1 to level 3 represents gradual developmental changes of classroom practices towards better student-thinking-centered classroom practice (see Table 4-5).

<table>
<thead>
<tr>
<th>Level</th>
<th>Dimension 1. Eliciting Student Thinking</th>
<th>Dimension 2. Using Student Thinking as Resources for Discussion</th>
<th>Dimension 3. Promoting Mathematical Discussion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mostly teacher explains concepts and strategies</td>
<td>Student thinking is seldom elicited so that there is no opportunity to use student thinking for discussion. Teacher does not attempt to understand student thinking.</td>
<td>There is seldom mathematical discussion or teacher-student discourse. Students are mostly passive listeners.</td>
</tr>
<tr>
<td>Level 1</td>
<td>Teacher poses mostly known answer questions or procedural process questions. Questioning functions to keep students listening, paying attention, or classroom management rather than eliciting student thinking. Mostly IRE(^9) sequencing types of questions.</td>
<td>Students are expected to get only answers or applied algorithms (not using for the discussion). Their answers are evaluated right away (right or wrong) and/or students’ misunderstandings are corrected immediately. Teacher knows only superficial features of student thinking (e.g., answer is right or wrong).</td>
<td>Mathematical discussion is more teacher-student relationship. Teacher asks questions and students respond. When a student answers, other students still listen passively or wait until teacher asks questions.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Level 2</td>
<td>Teacher begins to probe students’ mathematical thinking and strategies less focusing on just answer-gathering. Students still need teacher’s assistance to articulate their thinking.</td>
<td>Teacher collects student thinking without immediately evaluating or correcting, allowing the teacher to understand student thinking qualitatively. BUT students do NOT have a room to build on ideas. Teacher addresses directly so that students do not have opportunities to build on ideas or change their ideas on their own (it does not allow to use student thinking for productive discussion).</td>
<td>Students begin building on others’ ideas by showing their (dis)agreement, reframing, or asking questions by teacher’s prompt. Teacher is still the main facilitator of this mathematical discussion. Teacher may explain how to participate in mathematical discussion explicitly (e.g., how to critique others’ ideas, justify their ideas, express their (dis)agreement.)</td>
</tr>
<tr>
<td>Level 3</td>
<td>Teacher solicits student mathematical reasoning with open-ended questions and students start to articulate their mathematical thinking.</td>
<td>Teacher uses student ideas and strategies as resources for discussion, in addition to collecting students’ thinking qualitatively. Students’ misunderstandings are also used as resources and opportunities to reconceptualize ideas and pursue alternatives.</td>
<td>Students have more central roles, with little teacher’s assistance, to engage in mathematical discussion and to build on other’s ideas by critiquing, justifying, and refining their ideas mostly on their own. Teacher supports this atmosphere.</td>
</tr>
</tbody>
</table>

Dimension 1, “eliciting student thinking”: In current US classroom context, many teachers pose questions. However, the actual function of the questioning may or may not elicit student thinking. The levels in this dimension distinguish the nature of the questioning by looking at students’ responses. Level 0 represents teacher-centered didactic teaching in which the teacher only explains without making any space for students to explain their thinking. Level 1 describes teacher questioning that is largely IRE sequencing; and the nature of students responses is typically procedural or focused.

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\(^9\) Cazden (1988) characterizes the classroom talks as Initiation-Response-Evaluation (IRE) sequence if the process consists of a teacher initiates (I), student responds (R) to the teacher’s initiated question, and the teacher offers some form of evaluation (E) for the response.
on answers rather than reasoning. In Level 2, the teacher begins to ask some questions that elicit student thinking concerning their strategies. Such discussions are not yet well established (cf. level 3) so students need to be supported by teacher to articulate their thinking. In level 3, the teacher mostly solicits student mathematical thinking and reasoning with open-ended questions and “why” questions. Students are able to articulate their thinking and reasoning.

Dimension 2, “using student thinking as resources for discussion,” captures the ways in which teachers use student mathematical thinking and formative assessment. When students’ current understandings are revealed, teachers who are using productive formative assessment use students’ understandings as resources, provide opportunities to reconceptualize ideas, or invite other students to participate in the discussion. On the other hand, teachers who do not use formative assessment productively typically identify students’ errors immediately and explicitly and try to address those misunderstandings directly. Level 0 reflects that there is no chance for the teacher to understand students’ thinking so that (s)he can not make any use of students’ mathematical thinking. Level 1 reflects that students are expected to provide procedural solutions rather than to talk about their reasoning, so that the teacher does not have opportunities to develop his/her understanding of students’ thinking. Level 2 reflects that students’ thinking is elicited and/or misunderstandings are revealed but that they are not used as resources for the productive discussion. Misunderstandings or errors are simply corrected by the teacher. Level 3 indicates that the teacher collects common students’ (mis)understandings and these students’ understandings become resources for productive mathematical discussions and opportunities for students to re-conceptualize their ideas and/or pursue alternatives.

Dimension 3, “promoting productive mathematics discussion,” captures how teachers support students to engage in discussion. Promoting student accountable discussion (see analytical strand 1 in Table 4-1) is involved in this dimension and students’ spontaneous engagement in productive mathematical discourse community is the final goal of this dimension. Productive mathematical discussion involves students taking into account others’ thinking, justifying their own reasoning, and elaborating on others’ ideas. Level 0 indicates that students’ discussion seldom happens or that teacher-student interaction does not happen. The developmental shift from level 1 to level 3 reflects the evolution of the discussion from teacher-student interaction (teacher prompts questions and students respond) to student-student discussions requiring little teacher’s guidance or facilitation. Students have more central roles engaging in mathematical discussion by arguing, justifying, refining, elaborating their ideas.

This analytical framework was developed with similar cycles of the first analytical categories: iterative watching video data and tagging instances of a proposed kind of teaching practices with the Studiocode program and transcript, compiling the instances, specifying inclusion and exclusion criteria, and returning to the data to support or reject the hypothesis (Glaser & Strauss, 2007; Miles & Huberman, 1994; and MacQueen, McLellan, Kay, & Milstein, 1998).

The FAL classroom observation data are also chunked into episodes that indicate the lesson structure and specific activity structures such as, a Do Now episode, Small Group Work, Student Presentation, and Whole Group Discussion. Each episode is coded using the analytical framework (Table 4-5). Qualitative description and evidence (such
as, lesson/episode contexts, tasks, lesson flows, and narrative evidence for teacher/students discourse) are also provided.

In this analytical strand, I focus on one particular FAL, Percent change lesson, in order to conduct a systematic comparison between the two teachers’ instructional moves. I chose this lesson because the Percent change lesson was implemented by both teachers as a second of three lesson implementations. The results of this analysis—profiles of two teachers’ teaching practices—is provided in Chapter 6. Analytical strand 2 also documents a pattern in the relationship between teacher instructional moves and teacher learning opportunities. This is given in Chapter 6 as well. This second round of analysis refines our understandings of the process of teacher learning about student thinking through their instructional practices.

**Strand 2-b. Global analysis how teachers make a decision to use FALs**

In this global analysis, first I provide an overview of which FALs the two teachers used over each school year, what the general affordances provided by the FALs are, and how the two teachers made use of those affordances differently over the year. In the section of description of general affordances of FALs, I discuss a particular lesson, Percent-change lesson, as a specific example of affordances of FALs. I chose that lesson because both teachers’ teaching of that lesson is analyzed in strand 2-a.

By way of background, the findings of this analysis of teacher choices with regard to the affordances of FALs are provided in Chapter 5. As described above, the results of first strand of analysis are provided in Chapter 7 and the findings from the second strand of analysis are illustrated in Chapter 6.
Chapter 5: How The Teachers’ Interactions with Materials Created Opportunities for Their Learning

In chapter 2 I demonstrated that the ways teachers use curricular materials plays a critical role in classroom instruction not only for student learning but also for the teachers’ learning from their instructional experiences. I also described a spectrum of teachers’ adaptation of curricular materials as including omit, replace, and create. Building on that theoretical work, in this chapter I focus on the empirical question of how the two teachers interact with the curricular materials (FALs), and how their choices and interactions with materials create or limit their learning opportunities to learn about student mathematical thinking and mathematical content which help or hinder teachers develop new teaching practices.

In order to explore how the teachers interact with the curricular materials and their impact on teacher change, first, I describe the affordances of FALs, illustrating them with concrete examples from a particular FAL. The particular FAL, the Percent-change lesson, was chosen because both teachers implemented it. This enables a systematic comparison of how the two teachers used the affordances of the lesson. Following this, I provide an analysis of how the teachers adapted the Percent-change lesson. I conclude with a discussion of how their different ways of adapting the lesson created or limited opportunities for the teachers to learn about student thinking and content.

5-1. Affordances and Lesson Designer’s Intention of FALs

The most important affordance of FALs is that the support teachers in engaging in learning about students’ mathematical thinking in three instructional phases: before, during, and after teaching lessons by conducting pre-assessment prior to the lesson, particular pedagogical strategies during the lesson, and post-assessment following the lesson. Teachers have opportunities to deepen their understanding of students thinking by conducting pre-assessments consisting of complex problems prior to implementing the lesson. Instead of scoring these assessments, teachers are encouraged to solicit and analyze students’ common (mis-)understandings and to provide formative feedback during instruction. For understanding and more soliciting students’ thinking during the lesson, the lesson calls for using various lesson structures (e.g., small group work and student presentations), and pedagogical strategies (e.g., using posters, card sorting and matching activities, and using sample student work to critique each other’s ideas). Post-assessments provide opportunities for both teachers and students to reflect on the students’ learning.

In order to support teachers to use FALs effectively, PD meetings were provided. At the beginning of the PD meetings, teachers usually reflected on their experience teaching of FALs. They shared the challenges of implementing FALs and what they had learned by implementing FALs, and they discussed how they would address the problems of practice for the next time. At the second part of PD meetings, teachers planned which FAL they would teach next, and how they would approach it. Because they taught different grades, teachers had a “buddy (or buddies)” to work on a FAL they planned to teach next. A buddy might be a researcher, a student teacher, and/or another teacher. Teacher and buddy planned and prepared to teach a lesson together by reading lesson guides, rehearsing lesson activities.
(doing mathematics tasks and small group activities), analyzing students’ current thinking revealed in the pre-assessments, and sharing pedagogical strategies for the lesson.

Mathematics education researchers generally agree that understanding student mathematical thinking is deeply intertwined with mathematical content and pedagogical strategies (Carpenter et al., 1996; Kim, 2012; Miriam G Sherin, 2002). The FALs and FAL-related PD meetings also provided opportunities for teachers to understand and learn mathematical contents and pedagogical strategies related to student mathematical thinking. Tables 5-1, 5-2, and 5-3 describe what FALs involving FAL-related PD meetings provide general affordances for teachers in each phase—before, during, and after instruction—in more detail.

Table 5-3. Affordances of FALs before Instruction

<table>
<thead>
<tr>
<th>Opportunities</th>
<th>Resources</th>
<th>To understand curriculum materials and content</th>
<th>To understand student mathematical thinking</th>
<th>To learn pedagogical strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD meetings</td>
<td>About the FALs: goals and affordance, general lesson structure (in the initial PD meeting)</td>
<td>Anticipate different strategies students may have</td>
<td>Ways of eliciting student thinking and reasoning with ‘sentence starters’</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematical goals</td>
<td>Anticipate struggles students may have</td>
<td>Ways of promoting discussion (e.g., small group work)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Specific mathematics content and mathematical practices</td>
<td></td>
<td>Organize lesson activities and plan lesson (e.g., launch, small group activities)</td>
<td></td>
</tr>
<tr>
<td>Lesson guides</td>
<td>Big mathematical ideas</td>
<td>Students’ common mistakes and misunderstandings</td>
<td>Suggested questions and prompts</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematical goals</td>
<td></td>
<td>Additional questions for use in the lesson</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Specific mathematics content and mathematical practices</td>
<td></td>
<td>Organize lesson activities and plan lesson (e.g., launch, small group activities)</td>
<td></td>
</tr>
<tr>
<td>Pre-assessments</td>
<td>Different mathematical strategies and approaches</td>
<td>Their own students’ current mathematical understandings and errors</td>
<td>Prepare additional questions and prompts based on analyzing own students’ tasks</td>
<td></td>
</tr>
</tbody>
</table>

At the first PD meetings, a general overview of FALs was introduced. The information provided in the first meeting provided opportunities for the teachers to learn about the FALs. They learned that the goal of implementing FALs is for teachers to become more responsive to student thinking. They became familiar with the general structure of
FALs, what kinds of mathematical goals are in each FAL, etc. In the general PD meetings, teachers engaged with mathematical tasks and student work as described above—reading lesson guides, rehearsing lesson activities (doing mathematics tasks and small group activities), analyzing students’ current thinking as revealed in pre-assessments, and sharing pedagogical strategies for the lesson. This provided opportunities to anticipate student strategies and challenges so that the teachers can also plan what pedagogical strategies would be useful for their students, as well as rehearsing the mathematics and mathematical practices in the lesson. Teachers also shared their teaching experiences, so that they had more ideas about pedagogical strategies in terms of how to elicit student ideas and promote student-to-student discussion.

Before implementing a FAL, teachers were also expected to read the lesson guide carefully. The opportunities for teachers to learn across three domains (curricular and content, student thinking, and pedagogical strategies) in the lesson guides are similar to the ones described in PD meetings. By reading a lesson and lesson guide and doing the mathematics in the lessons—assessment tasks and small group activities—by themselves, the teachers had opportunities to understand mathematical ideas and mathematical strategies more deeply, to solve problems and to do small group activities, and to reflect on mathematical content and practice goals for their students. Lesson guides also had affordances for teachers to understand students’ typical misunderstandings and errors by providing “common issues” tables and “suggested questions and prompts” (e.g., see Figure 5-3 below). The lesson guide also suggests that teachers, based on the pre-assessment tasks conducted with their own students create their own “common issues” table and prepare additional questions for use during instruction. Teaching FALs is challenging, in that the teacher’s role now includes orchestrating often unexpected student ideas. In planning and visualizing the lessons collectively, teachers were supported in anticipating the big mathematical ideas underlying a lesson, student misunderstandings, errors, or problem approaches that might emerge during instruction. This enabled them to plan pedagogical strategies in response.

The FALs provided another strong affordances before instruction, by having the teachers conduct a pre-assessment and analyzing the student mathematical thinking revealed in it. Teachers had opportunities to deepen their understandings of their students’ current mathematical thinking, such as: students’ typical understandings, where students may struggle, what students understand easily, or what common errors students have. Through making sense of their students’ mathematical thinking, teachers also may learn different mathematical strategies to solve problems or develop mathematical concepts. This pre-lesson activity provided teachers an opportunity to prepare better pedagogical strategies for use during instruction—for example, what kinds of mathematical questions can provide opportunities for students to have better or different strategies, to pursue alternatives, or to promote student-student discussion.

During instruction, while they implement FALs and pedagogical strategies that the FALs (and PD) suggest, teachers have additional opportunities to grapple with emergent student mathematical thinking, for example seeing which ideas are easy and which are challenges for students (see Table 5-2). In particular, FAL lesson plans recommend that teachers spend a large amount of lesson time on small group work and student presentation, so that teachers have more opportunities to understand and assist individual student mathematical thinking. Each FAL includes small group activities and/or student presentation time, such as having student present their solutions in posters, sorting/matching cards, or
analyzing sample students’ works, and presenting groups’ work on posters. These activities have affordances for both teachers and students to monitor students thinking. Student discussions—generating hypotheses, critiquing, justifying, and modifying—provide multiple opportunities for formative feedback.

As described above, in deepening their knowledge about student mathematical thinking, teachers also develop their understanding of mathematical content and practices. Through eliciting student mathematical thinking and facilitating student-student discussion, teachers may gain additional mathematical strategies and approaches for the lesson unit. By performing pedagogical strategies learned from FALs and PD meetings, teachers have opportunities to develop their own pedagogical practices (see Table 5-2). The mechanism of how teachers’ instructional moves create learning opportunities for themselves is described in Chapter 6.

**Table 5-4. Affordances of FALs during Instruction**

<table>
<thead>
<tr>
<th>Opportunities</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>To understand curriculum materials and content</td>
<td>To understand student mathematical thinking</td>
</tr>
<tr>
<td>Lesson activities</td>
<td>Additional mathematical strategies and approaches</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After implementing FALs, teachers are encouraged to provide post-assessment tasks for students, so that teachers and students alike can reflect on the student learning. In concept development lessons the post-assessment tasks consist of problems similar to the pre-assessment tasks, so students can compare their pre- and post-lesson performance. In problem solving lessons, students revisit the pre-assessment tasks in order to improve their solutions. In both styles of post-assessments, teachers can reflect on what students learned and where they still have challenges or struggles. FALs also encourage teachers to reflect on their own teaching by asking questions such as, “What went well with the lesson?” “Did the lesson go as planned? If not, why?” “How did the student respond, in their attitudes and their discussion?” “What will you do differently next time, why?” “How might the structure and pedagogy of the lesson carry over to other lessons?”

The post-assessments provide teachers with opportunities to make sense of their teaching and student learning through the FALs, with a focus on the three domains identified in Table 5-3. Teachers may develop additional mathematical strategies and approaches through reviewing student post-assessments. Teachers have opportunities to understand what students learned and did not learn as they engaged in the lesson. They also have opportunities to reflect on their pedagogical strategies and to decide what strategies they will continue to use or use differently next time.
In the PD meetings, as described above, teachers reflected on their experience of teaching FALs and shared strategies or challenges they found as they engaged in the FALs. This gave teachers opportunities to gain additional mathematical strategies and approaches around the lessons, to understand additional student thinking as other teachers shared what they learned about their own student thinking, to reflect on where the lessons went well or did not, and to plan to modify their pedagogical strategies for the future (see Table 5-3).

Table 5-5. Affordances of FALs after Instruction

<table>
<thead>
<tr>
<th>Resources</th>
<th>To understand curriculum materials and content</th>
<th>To understand student mathematical thinking</th>
<th>To learn pedagogical strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-assessments</td>
<td>Additional mathematical strategies and approaches</td>
<td>What students learned or understood about the mathematics in the lesson</td>
<td>Reflect on the lesson and pedagogical strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Where students still remain struggled and challenged</td>
<td>Plan to use different strategies in the future</td>
</tr>
<tr>
<td>PD meetings</td>
<td>Additional mathematical strategies and approaches</td>
<td>Additional student thinking as sharing other students’ works</td>
<td>Reflect on their pedagogical strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Plan to use different/improved strategies in the future</td>
</tr>
</tbody>
</table>

In summary, the FALs and FAL-related PD meetings provided various affordances and opportunities for teachers to learn across three domains: curriculum materials and content, student mathematical thinking, and pedagogy. These opportunities were also provided in three phases: before, during, and after implementing FALs. Tables 5-1, 5-2, and 5-3 summarize respectively the general affordances of FALs across the three domains in each phase.

5-2. Global Analysis on How The Two Teachers Choose Affordances of FALs

In order to understand how the teachers which affordances they would use over the school year, I first provide in this section some information about which FALs they implemented. I closely look at affordances of a particular FAL, Percent-change lesson, as an example of general affordances of FALs as described in the previous section. Then, I analyze how the two teachers adapted the Percent-change lesson similarly or differently.

The FALs the two teachers implemented over the year
Both teachers, Ms. Lee and Ms. Janet, implemented three FALs over each course of the year. Table 5-4 and Table 5-5 describe which FAL each teacher taught and what the mathematical content goals of each FAL are.

**Table 5-6.** Ms. Lee’s Implementation of FALs over the school year (2012-2013)

<table>
<thead>
<tr>
<th>FAL1</th>
<th>FAL2</th>
<th>FAL3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson</strong></td>
<td>“Developing a Sense of Scale”</td>
<td>“Drawing to Scale: Designing a Garden”</td>
</tr>
<tr>
<td></td>
<td>(problem solving)</td>
<td>(concept development)</td>
</tr>
<tr>
<td><strong>Content goals</strong></td>
<td>Proportional relationships in real-world problems</td>
<td>Proportional relationships in percent-change; relationships between percents, decimals, and fractions</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>November, 28-30, 2012 (three days)</td>
<td>December, 13-17, 2012 (three days)</td>
</tr>
</tbody>
</table>

**Table 5-7.** Ms. Janet’s Implementation of FALs over the school year (2013-2014)

<table>
<thead>
<tr>
<th>FAL1</th>
<th>FAL2</th>
<th>FAL3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson</strong></td>
<td>“Using Positive and Negative Numbers in Context”</td>
<td>“Increasing and Decreasing Quantities by a Percent”</td>
</tr>
<tr>
<td></td>
<td>(concept development)</td>
<td>(concept development)</td>
</tr>
<tr>
<td><strong>Content goals</strong></td>
<td>Comparing, adding, and subtracting positive and negative integers</td>
<td>Proportional relationship in percent-change; relationship between percents, decimals, and fractions</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>September, 18-19, 2013 (two days)</td>
<td>December, 3-4, 2013 (two days)</td>
</tr>
</tbody>
</table>

In order to understand in what ways two teachers adapt FALs—how they interact with curricular FALs with regard to making use of the affordances of FALs—in three phases (before, during, and after instruction) in more detail, I provide an analysis of a particular FAL, “Increasing and Decreasing Quantities by a Percent (Percent-change lesson),” because both teachers conducted this lesson as their second of three FAL implementations of FALs among their total three FALs implementations in the same month, December. This allows for a systematic comparison of how the two teachers differently interact with the lesson in each phase.
I first present specific affordances of the lesson as an example of the general affordances in the first section. Then, I describe how two teachers *adapt* this particular lesson similarly and differently.

**Percent-change Lesson**

As with other FALs, the most important affordance of this lesson is that it helps teachers assess students’ thinking and provide formative feedback with regard to the content goals in the three lesson phases: prior to, during, and after actual instruction. In particular, this lesson unit is intended to support teachers in assessing “how well students are able to interpret percent increase and decrease,” and “to identify and help students who have the following difficulties: (1) translating between percents, decimals, and fractions; (2) representing percent increase and decrease as multiplications; and (3) recognizing the relationship between increases and decreases (p. T-1 in the lesson package, Appendix A).”

Prior to the actual instruction, this lesson intends teachers to have the students do pre-assessment tasks (see Appendix A for full tasks) for about fifteen minutes either in class or for homework, a day or more before the instruction. The pre-assessment tasks involve four problems with a sample problem showing how to use a calculator (see Figure 5-1 and Figure 5-2 as examples).

![Figure 5-5. Sample problem for pre-assessment of the Percent-change lesson](image)

One example of the tasks on pre-assessment is as following Figure 5-2:

2. Maria sees a dress in a sale. The dress is normally priced at $56.99. The ticket says that there is 45% off. She wants to use her calculator to work out how much the dress will cost. It does not have a percent button.

Which keys must she press on her calculator?
Write down the keys in the correct order. (You do not have to do the calculation.)

![Figure 5-6. Example of problems in the pre-assessment task](image)

As with other FALs, this Percent-change lesson package also provides teachers with a “common issue (p. T-3 in the lesson package, Appendix A)” sheet that contains students’ possible errors and misunderstandings and suggested prompts for addressing those issues. In this lesson, lesson designers anticipate that students may (1) incorrectly assume percent increase or decrease is computed by addition or subtraction instead of multiplication (e.g., 56.99 – 0.45 or 56.99 – 1.45); (2) make incorrect decimal conversions (e.g., 56.99 x 0.45); (3) use inefficient methods (e.g., 56.99 x 0.45 = answer, then 56.99 – answer); or that they (4) are

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10 The original title of this lesson is “Increasing and Decreasing Quantities by a Percent,” I call this lesson as “percent-change lesson” in brief in this dissertation.
not able to calculate percent change; (5) fail to use brackets in the calculation; or (6) misinterprets what needs to be included in the answer (e.g., the answer is just operator symbols.). For these anticipated errors or misunderstandings, suggested questions are provided. For example, if a student makes incorrect decimal conversions, the teacher may prompt, “how can you write 50% as a decimal? How can you write 5% as a decimal?”

<table>
<thead>
<tr>
<th>Common issues</th>
<th>Suggested questions and prompts</th>
</tr>
</thead>
</table>
| Student makes the incorrect assumption that a percentage decrease means the calculation must include a subtraction | • Does your answer make sense?  
• Can you check that it is correct?  
• In a sale, an item is marked “50% off.” What does this mean? Describe in words how you calculate the price of an item in the sale. Give me an example.  
• Can you express the decrease as a single multiplication? |
| For example: 56.99 – 0.45 or 56.99 – 1.45. (Q2.) \(^{11}\) A single multiplication by 0.55 is enough. |
| Student uses inefficient method | • Can you think of a method that reduces the number of calculator key presses?  
• How can you show your calculation with just one step? |
| For example: 56.99 × 0.45 = ANS, then 56.99 – ANS (Q2.)  
A single multiplication is enough. |

\(^{11}\) Question 2 is the problem in Figure 5-2.

Figure 5-7. Example of a common issue table in the lesson guide

As shown in both both the pre-assessment tasks and the common issue table, this lesson affords teachers enough time – if they take advantage of it – to analyze students’ mathematical thinking revealed through the pre-assessment and to prepare how to use these students’ ideas as resources for discussion and to support individual students in small group during actual instruction.

Table 5-8. Affordances of Percent-change lesson before instruction

<p>| To understand curriculum materials and content | To understand student mathematical thinking | To learn pedagogical strategies |</p>
<table>
<thead>
<tr>
<th>Mathematical ideas:</th>
<th>Mathematical goals:</th>
<th>Mathematical contents:</th>
<th>Mathematical practices:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional relationship in percent-change; relationship between percents, decimals, and fractions; relationship between increases and decreases</td>
<td>interpret percent increase and decrease in (a) translating between percents, decimals, and fractions; (b) representing percent increase and decrease as multiplication; and (c) recognizing the relationship between increases and decreases</td>
<td>Analyze proportional relationship and use them to solve real-world and mathematical problems; Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers</td>
<td>Reason abstractly and quantitatively; Look for and make use of structure</td>
</tr>
</tbody>
</table>

Anticipating students’ strategies and struggle points for card matching activities

Anticipating issues in pre-assessment tasks as students’ familiarity with using calculators (Ms. Lee)

Effective ways of organizing small group works

Effective ways of facilitating accountable participating in discussion and group works

Brainstorming some questions to elicit student thinking

Students’ common mistakes and misunderstandings in common issues table:
- incorrect assumption that a percentage increase means the calculation must include an addition;
- incorrect assumption that a percentage decrease means the calculation must include a subtraction;
- converting the percentage to a decimal incorrectly;
- using inefficient method to calculating percentage;
- unable to calculate percentage change; and
- calculation mistakes

Suggested questions and prompts in the common issues table: Does your answer make sense? What does your answer mean? Describe in words how you calculate the price of an item in the sale. Give me an example. How can you write 50% as a decimal? In your problem, what operation will the calculator carry out first?

Creating additional questions for use

Plan the lesson structure and activities

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12 This particular strategies or ideas are discussed in PD meetings where only Ms. Lee and other teachers attended (not Ms. Janet).
Before implementing this particular lesson, both teachers participated in a PD meeting. In this meeting, as other PD meetings, the teachers explored big mathematical ideas—proportional relationship in percent-change, relationship between three different representations (percent, fraction, and decimal), and relationship between increase and decrease—by rehearsing lesson activities (mostly card matching activities). The teachers shared and generated different strategies, anticipated students’ struggle points, and discussed possible ways of supporting students. The teachers also shared and discussed pedagogical ways to elicit student thinking and reasoning, to promote productive mathematical discussion, and to have accountable participation in small group works. The teachers were also expected to read the lesson guides carefully so that they could fully understand and clarify the mathematical goals of the percent-change lesson and its CCSSM-related goals. As with other FALs, the common issues tables also provided teachers opportunities to understand possible student misunderstandings and possible ways of supporting students to develop their understandings of the mathematical ideas (see Table 5-6).

During instruction, implementing the Percent-change lesson also provides opportunities for teachers to learn about the content and student thinking. Teachers practice their new pedagogical strategies as they implement the lesson. The opportunities during instruction depend on how teachers enact the lesson, in particular how they choose the affordances suggested in the lesson guides and PD meetings. The analysis of how teachers created learning opportunities related to student thinking and to content is presented in next section.

The affordances after implementing the Percent-change lesson are also similar to general affordances of post-assessment after instruction given in Table 5-3, though of course the reflection depend on what took place during instruction. During PD meetings, neither
teacher had the opportunity to reflect on her teaching in terms of the particular mathematical content. Instead, both teachers had opportunities to talk about general pedagogical strategies—both what goes well and what was problematic—and what they learned about student thinking when implementing the FALs. (See Table 5-7)

**Table 5-9. Affordances after instruction of Percent-change lesson**

<table>
<thead>
<tr>
<th>Resources</th>
<th>To understand curriculum materials and content</th>
<th>To understand student mathematical thinking</th>
<th>To learn pedagogical strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-assessments</td>
<td>Additional mathematical strategies and approaches</td>
<td>What students learned or understood about the mathematics in the lesson</td>
<td>Reflect on the lesson and pedagogical strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What students still struggled with and were challenged by</td>
<td>Plan to use different strategies in the future</td>
</tr>
<tr>
<td>PD meetings</td>
<td>Additional mathematical strategies and approaches <em>(did not happen in public space after teaching this particular lesson)</em></td>
<td>Additional student thinking as sharing other students’ works <em>(did not happen in public space after teaching this particular lesson)</em></td>
<td>Reflect on their pedagogical strategies: specifically, sentence stems for prompting discussion and conversations</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Reflect on challenges: timing issues, preparing supplies, multiple days attendance</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Plan to use different/better strategies in the future</td>
</tr>
</tbody>
</table>

**How the two teachers chose the affordances of Percent-change lesson**

Now I present whether or not and how the two teachers, Ms. Lee and Ms. Janet, choose which affordances of the Percent-change lesson to explore and how they interacted with the curricular support materials. Recalling the spectrum of curriculum adaptation in Chapter 2: teachers may *create* new activities in addition to suggested curriculum; teachers may *replace* a component of a curriculum with something different; or teachers may *omit*
components of a curriculum by deleting a part of a lesson. If teachers do not change the lesson, I refer the teachers using the curriculum as is.

In order to understand how the two teachers adapted the Percent-change lesson differently, I first present what the lesson suggests that teachers implement (See Table 5-8). The lesson suggests that teachers administer the pre-assessment either before or during the lesson (see Figure 5-1 and 5-2, for example of the assessment task; see Table 5-6, for example of affordances of the assessment tasks in terms of content and student thinking). This pre-assessment task is to be revisited at the end of the lesson so that the students can see what they learning from the lesson and improve their original responses.

During the instruction, the lesson suggests to have small groups begin by working on matching two cards sets: (a) Card set A: amount of money ($100, $150, $160, and $200; See Figure 5-4); and (b) Card set B: arrow representing “percent increases or decreases” (e.g., down by 50%, up by 33 1/3%) and blank (See Figure 5-5). As students are placing the percent change arrow cards (card set B) between money cards (card set A), the teacher makes sure each student takes turns to place the cards, students explain their thinking about why they placed the arrow cards where they did; and partners challenge card placements they disagree with. The teacher also supports students in making sense of the relationship rather than making specific suggestions toward particular answers. Teachers can ask questions that are in “common issue” tables in lesson guide or that they generate to help student clarify their thinking. A sample completed initial activity is shown as Figure 5-6. The lesson also suggests, if time allows, that each group of students be given the opportunity to share their work with students in other groups. After that, they may have a chance to discover if there were any errors in their matches and if so, re-think their own group’s work. The intended goal of this small group activity is for teachers to support students in refining their thinking and justifying their decisions, rather than suggesting a particular approach to the task. Students are encouraged to think through their misconceptions by discussion with each other rather than by the teacher’s direction.

Figure 5-8. Card set A: Money cards

Figure 5-9. Card set B: Percent increase and decreases
After this first round of activity is completed, the lesson plan suggests additional card matching activities with decimal multiplier increase/decrease arrow cards that include some blank arrows (see Figure 5-7). It is important to note that students are placing the arrow representing decimal multipliers adjacent to the arrows from the first card matching activity (percent change) cards, in order for them to explore the relationship between two different representations of proportional reasoning, percents and decimals. If time allows, students have the opportunity to share their work with other groups and, on reflection, make changes in their own group’s work. Another small group work is to match fraction multiplier cards (see Figure 5-8) in addition to two card matching activities so that students can explore the relationships between three different representations about proportional reasoning. A sample of completed final small group activity is shown as Figure 5-9.

**Figure 5-10.** A sample of completed initial small group activity

**Figure 5-11.** Card set C: Decimal multipliers  **Figure 5-12.** Card set D: Fraction multipliers
Figure 5-13. A sample of completed extension small group activity

There is additional card set, money cards representing with cents and decimals (See Figure 5-10) if there are students who need additional challenges. This is optional.

Figure 5-14. Card set E: Additional money cards set for students who need additional challenges

Toward the end of the lesson, whole group discussion is suggested to conclude the activity by discussing and generalizing what has been learned. The generalization involves extending what has been learned to new examples (e.g., suppose prices increase by 10%, how can I say that as a decimal multiplication, as a fraction multiplication? What is the fraction multiplication to get back to the original price? How can you write that as a percentage?), and then discussing what the students suggest.

After this whole group closing discussion, students return to their original pre-assessment. They are given the opportunity to think about what they have learned from the lesson and to improve their own work. This post-assessment (or revisit pre-assessment) can be done as homework if lesson time does not allow for it in the classroom. The full lesson which involves the suggested lesson structure, activity, materials, and guidance for teachers is given in appendix A. Table 5-8 summarizes the suggested lesson outline and activities, including the key points during the lesson at which the teacher should support students.
| **Table 5-10.** Suggested lesson implementation of Percent-change lesson |
|---|---|---|
| **Before** | **During** | **After** |
| **Pre-assessment** | **Lesson structure & activities** | **Teacher support for activities** | **Post-assessment** |
| | Launch: Introducing small group activities of matching card sets | Teacher makes sure students explain their thinking every time they place the cards; students take turns to place cards; partner agree/disagree and challenge him/her | |
| | Small group works for matching card sets: Money cards and percent increase/decrease cards | Teacher supports student problem solving (not making specific suggestions toward a particular approach, but ask questions to help student clarify their thinking) | |
| | Small group works for additional matching cards sets: Decimal multiplier increase/decrease cards in addition to the first card matching activity | Teacher encourages students to use each other as a resource for learning | |
| | Extension activity: Diagonals using blank cards (optional) | For decimal multiplier card sets, make sure students to make connections between different representations of an increase or decrease | |
| | Sharing student works & make changes in their own work (optional) | The generalization involves extending what has been learned to new examples, and then examining some of the conclusions | |
| | Small group works for matching additional matching cards (fraction multipliers) | Revisit pre-assessment tasks and improve their original responses | |
| | Whole group discussion: Conclude the lesson by discussing and generalizing what has been learned | | |

**How the two teachers adapted the percent-change lesson**

Now, I present how two teachers, Ms. Janet and Ms. Lee, adapted the percent-change lesson. First, Ms. Janet did not administer the pre-assessment, which would have helped her to understand her students’ mathematical thinking in more depth. Thus she omitted the essence of one of the FALs’ affordances. She conducted the lesson during two consecutive days. At the beginning of every day’s instruction, she created a new lesson structure called as “Do Now” (a common beginning lesson structure). This Do Now time gives both teacher and students the opportunity to settle into classroom routines, with the students working tasks
relevant to the lesson to come. Ms. Janet also used this quiet individual work time to do a homework check. After about 5 – 8 minutes of individual work time, there is a whole class discussion time in which the students share their work on the Do Now problems (about 10 – 15 minutes in all).

In her instruction on Percent-change lesson, Ms. Janet created problems for Do Now session shown as Figure 5-11 and Figure 5-12. The problems are mostly about recalling memorized the algorithms to figure out the percent change and practicing how to use this algorithm (e.g., what numbers to plug in the formula). This is not aligned with values the FAL suggested. FALs, particularly in this Percent-change lesson, suggest teachers not to make specific suggestions toward a particular approach but to help students navigate their reasoning and clarify their thinking. That is, even though she created a new structure, Ms. Janet replaced essence of pedagogical practices by substituting encouraging students to explore proportional reasoning into memorizing and using algorithms.

- What is the equation to find the percent of a number?
- What is 30% of 40?
- What is the equation to find the percent change (increase/decrease)?
- What is the percent decrease from 40 to 12?

**Figure 5-15.** Do Now problems in Day 1 (Ms. Janet)

- Look at your work from yesterday.
  1) Write down everything you remember.
  2) What did you learn?

**Figure 5-16.** Do Now problems in Day 2 (Ms. Janet)

While the FAL suggests all three different card matching activities (percentage, decimal, and fraction) for small group activity, Ms. Janet omitted two of the activities and conducted only one activity, matching percent increase and decrease cards (card sets A and B). One of the main goals of the percent-change lesson is to explore the relationship between percents, decimals, and fractions but Ms. Janet omitted the activities related to this goal. In addition, the way she supported students’ small group work was significantly different from the way suggested in the lesson plan. While the plan suggests that teachers not make specific suggestions about how to approaching the activities, and rather scaffold the students into thinking about the mathematical relationships involved, Ms. Janet did not support that kind of sense-making. Rather, she continuously directed students to use a formula (percent = change/original) that they had practiced in the Do Now session. That is, she transformed the lesson from an exploration of mathematical connections into the application of a procedure that the students had practiced.
At the end of each day’s instruction, Ms. Janet had student presentation time serve as a way of closing and concluding small group activities. It is different from what the lesson suggested. The lesson suggests that students share their work by rotating groups so that one student remains in his/her own group to justify and explain his/her group’s reasoning while students from other groups ask questions to him/her. For concluding lesson, the lesson suggests to conclude and generalize as a whole group discussion. Ms. Janet *replaced* this activity by combining student presentation as a sharing time and whole group discussion. However, student presentation was actually not sharing their works with other students. Rather, it functioned as a tool to conclude if the student used a formula correctly to figure out the relationship between two money cards. Only one student came to show his/her works in front of the class. Ms. Janet continuously asked other students if he/she correctly used the formula and the student sat in front of the classroom until the class ended. The lesson concluded with the students memorizing the formula and practicing how to plug the numbers from the problems into the formula. Therefore, the suggested ways of supporting students’ work in this phase are also *omitted*. Just as the pre-assessment was not administered, the post-assessment was not administered. The summary of the ways of Ms. Janet *adapted* the lesson is provided in Table 5-9.

**Table 5-11.** Ms. Janet’s implementation of Percent-change lesson

<table>
<thead>
<tr>
<th>Before</th>
<th>During</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-assessment</td>
<td>Lesson structure &amp; activities</td>
<td>Teacher support for activities</td>
</tr>
</tbody>
</table>

Create: Use a new structure, “Do Now” in the beginning of each lesson (Individual work on the problems and then whole class discussion about the problems)

Replace (Omit?) the suggestion: “Do Now” problems are not aligned with FALs’ suggestions of sense-making (formula for percent-change and recalling memorizing of formula)

Omit: Ms. Janet made specific suggestions toward a particular approach that is to use formula rather than supporting various ways of sense-making in the activity

Omit: Ms. Janet did not provide opportunities for students to make connections between different representations (percent, decimals, fractions)

Omit: Conclude lesson activity as “memorizing formula and how to plug in numbers to the formula”—percent change/original)

Did not administer

Omit: Did not administer

Omit: Small group works for matching only percent change cards (delete decimal multiplier cards activity)

Replace: Having student presentation time (sharing students works) but it is more teacher-centered rather than student-centered presentation

Ms. Lee conducted the Percent-change lesson over three consecutive days. She administered the pre-assessment one day before the instruction of the Percent-change lesson. She had some time to analyze her students’ mathematical thinking and ability to interpret percent increase and decrease with decimal and fractions. However, as she raised an issue in PD meeting about the pre-assessment task that the task assumes students are familiar with using a calculator but her students are actually not, she modified the problems. Ms. Lee deleted part of the pre-assessment that uses a calculator. Instead, she added another answer table to guide students to think about two different methods to figure out the problem (see Appendix B for the whole sheet of pre- & post-assessment problems that Ms. Lee modified). This modification still maintained the essence (in even better ways for her own students) of the original tasks, in that it was aimed at of sense-making in percent increase and decrease (for example, see Figure 5-13 comparing to Figure 5-2, the original problem).

Maria sees a dress on sale. It is normally priced at $56.00. Now it is on sale for 45% off. How much will she pay for the dress?

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
</table>

Figure 5-17. An example of pre-assessment problems that Ms. Lee modified
During instruction, Ms. Lee also created a new lesson structure, “Do Now” at the beginning of the first and last day of instruction, which has similar functions as Ms. Janet’s. While students worked individually on the Do Now problems that Ms. Lee created (see Figure 5-14 and Figure 5-15), the students settled down for class and she checked homework. However, the Do Now problems that Ms. Lee created are significantly different from the ones that Ms. Janet created. The problems that Ms. Lee created aimed to help students explore sense-making in percent increase and decrease and clarify their thinking so they could explain it, rather than focusing on using an algorithm or a formula. These underlying approach in the Do Now problems is aligned with the ones in the FALs.

For small group work, Ms. Lee provided all three arrow card sets (Card set B, C, and D) in the order the lesson suggested so that students could navigate the relationships among three different representations (percents, fractions, and decimals) of percent-change. In order for students to avoid cognitive overload, Ms. Lee firstly asked students to match only “increase” cards in the order of percents, fractions, and then decimals. Once students were mostly done with the “increase” cards of all three card sets, they were encouraged to place the “decrease” cards of all three card sets. This adaptation is considered as replace. As Ms. Janet did not have time to share students’ works by rotating each group, Ms. Lee also did not have time to share students’ work by rotating each group (omit the student sharing time). However, the ways of her supporting students’ works are similar to what the lesson suggested. Ms. Lee supported students in explaining their reasoning while they placed the cards, encouraged students to share their ideas with each other, and guided them to make connections between the three representations (percents, decimals, and fractions).

What is the total cost of a jacket that sells for $56 from the factory but the store charges a 6% markup?

The selling price of $89.60 does/does not make sense because ______________.

Figure 5-18. Do Now problems in Day 1 (Ms. Lee)
At the end of the last day’s lesson, Ms. Lee led a whole group discussion time aimed at filling out the blank arrows in Figure 5-4. Students also had partner talk time to explain their thinking before having a whole group discussion. The lesson concluded with the discussion of what they had explored in small groups, e.g., making sense of the relationships between the three representations, of the different relationship between increase and decrease, and identifying the increase and decrease in proportional relationships.

As the lesson suggested, the pre-assessment problems were provided as a post-assessment but Ms. Lee just asked the students to explain/show their work without using the two different methods format that she had used for the pre-assessment. This is because the two-method approach in the pre-assessment had not turned out to be valuable for the students. (see an example of post-assessment problems in Figure 5-16)

Table 5-12. Ms. Lee’s implementation of Percent-change lesson

<table>
<thead>
<tr>
<th>Before</th>
<th>During</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-assessment</td>
<td>Lesson structure &amp; activities</td>
<td>Teacher support for activities</td>
</tr>
</tbody>
</table>

Maria sees a dress on sale. It is normally priced at $56.00. Now it is on sale for 45% off. How much will she pay for the dress? Explain and show your work.

Figure 5-20. An example of a post-assessment problem (Ms. Lee)

The summary of the ways of Ms. Lee adapted the Percent-change lesson is provided in Table 5-10.
5-3. The Teachers’ Interactions with the FALs and the Resulting Learning Opportunities for the Teachers

I found that in making use of curricular materials (as exemplified by the discussion of the Percent-change lesson above), Ms. Janet and Ms. Lee made similar use of the lesson sequence and activity structures —but the way they supported the activities was significantly different, as shown in Table 5-9 and Table 5-10. Both teachers created Do Now problems and modified the lesson structure with similar functions, omitted some structures (e.g., small group work, student presentation time, etc) and activities (card matching), and replaced parts of the lesson structure (e.g., student presentation time).

However, there were significant differences in the ways the two teachers implemented the activities. Ms. Janet mostly omitted some important principles emphasized in the FALs. Although she created new structure and new problems in the Do Now session, the problems did not elicit student reasoning or make use of student thinking but emphasized recalling and memorizing formulas and algorithms. Ms. Janet also omitted some opportunities for students to explore deep mathematical concepts and relationships. She did not use the two other card-matching activities (decimal and fraction multiplier cards, see Figure 5-7 and Figure 5-8) that have a potential for students to explore the connections and

| Replace: Modified the tasks that enabled students to more focus on core mathematical ideas rather than use of calculator | Create: Having new structure, “Do Now” in the beginning of everyday lesson (Individual work on the problems and the whole class discussion about the problems) | Replace: Small group work for matching all three card sets (percents, decimals, and fractions) but only with increase cards first. Later, in Day 2, they discussed and matched decrease cards. |
| Omit: Did not having student sharing time | Create the Do Now problems aligned with the principles and goals that FALs suggest. Ms. Lee also guided students to make sense of the percent change relationship rather than rote calculations in whole group discussion. | As is: Ms. Lee supported students to explain their reasoning while they placed the cards; encouraged students to exchange their ideas each other |
| Replace: Whole group discussion by teacher’s guide to conclude the lesson | As is: Ms. Lee supported students to make connections among three representations (percents, decimals, and fractions) | Replace: Generalizing and concluding the activities with the existing posters (not with new examples) |
| Replace: Revisit the modified version of pre-assessment |
relationships between different mathematical representations of percent-change. Ms. Janet also omitted one of the important pedagogical strategies that FAL suggested: not pushing students to use a particular approach but supporting them in various ways of sense-making on the card-matching activities. Ms. Janet did not support students in exploring different ways of approaching the card-matching activities. Instead, she trained students to use a formula (percent = change/ original) and concluded the lesson by having students memorize the formula and practice its application. Ms. Janet’s ways of interacting with the curricular materials limits opportunities not only for her students to access mathematical content in richer ways but also for her to learn about additional mathematical content (including possible additional mathematical strategies and approaches to the content) and student thinking and reasoning.

On the other hand, Ms. Lee replaced or made use of FALs as is with regard to ways of supporting student mathematical activities, as illustrated by her implementation of the Percent-change lesson. She provided all three (percent, fraction, and decimal) card-matching activities step by step (guiding them to first focus on percent increase and then on fraction and decimal increases; all three representations in decrease relationship was provided last). She encouraged students to explore the relationships in different ways, and with sense-making approaches. Her interactions with the curricular materials created not only for her students to access the mathematical contents in richer ways but also opportunities for herself to develop a deeper understanding of her own students’ mathematical thinking and reasoning. Ms. Lee also supported students to explain their reasoning while they were placing the cards during small group. This created opportunities not only for students to discuss mathematics productively but also for the teacher, Ms. Lee, to understand their reasoning while she was supporting their work.

Whether or not they administered the pre-assessment and post-assessment also shows differences between two teachers:. Administering the pre- and post-assessment is one of the important affordances the FALs provide with regard to opportunities for teachers to understand student mathematical thinking—both before the actual lesson and reflecting on what happened after the lesson. However, Ms. Janet did not administer either the pre- or post-assessment in the Percent-change lesson, depriving her of opportunities to advance her knowledge of student mathematical thinking in general and her students’ thinking in particular. The pre- and post-assessments offer opportunities to gain deeper insights into student thinking, and top determine the efficacy of one’s teaching. Ms. Janet missed these opportunities by omitting (not administering) the assessment tasks.

In contrast, Ms. Lee administered both the pre- and post-assessments. As described in earlier, she actually modified the assessment tasks to fit her own students’ capacity. During the PD meeting prior to this Percent-change lesson implementation, Ms. Lee said that her students were not familiar with calculator. Thus the actual pre-assessment that she provided her students was a modified version—she deleted the part referring to calculator use and added tables that requested students to provide two different approaches to the problems. The post-assessment was supposed to revisit the pre-assessment. Ms. Lee again deleted the calculator part and the call for two different approaches since she noticed from pre-assessment that her students did not provide two meaningfully different approaches. This replacing way of using FALs provide her opportunities to understand her own students’ mathematical thinking and additional mathematical strategies on the percent-change and relationship among different representation (percent, fraction, and decimal). It also provided
her opportunities to advance her pedagogical content knowledge and content knowledge. Table 5-11 and Table 5-12 provide the summary of how these two teachers’ different ways of interacting with curricular materials (FALs, particularly, Percent-change lesson) created and limited opportunities for them to learn about student mathematical thinking and content.

**Table 5-13.** Ms. Janet’s interaction with curricular materials and learning opportunities

<table>
<thead>
<tr>
<th>The ways of interaction with curricular materials</th>
<th>Opportunities to learn about student thinking and content</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before (pre-assessment)</strong></td>
<td></td>
</tr>
<tr>
<td>Omit: did not administer pre-assessment or delete essence of the FALs’ intention.</td>
<td>Limit opportunities to anticipate different mathematical strategies and challenges students may have.</td>
</tr>
<tr>
<td><strong>During (implementing FALs)</strong></td>
<td></td>
</tr>
<tr>
<td>Create: Do Now problems and discussion (but focusing on procedural process).</td>
<td>Create opportunities to check how many students recall the formula and use it, but limit opportunities to understand student thinking deeper and wider.</td>
</tr>
<tr>
<td>Omit: Core mathematical goal—exploring various ways of sense-making of the relationship of two quantities with multiple ways of representing the mathematical relationship (e.g., percent, fraction, decimal).</td>
<td>Limit opportunities to gain additional mathematical strategies and approaches related to the activity she omitted; to understand student mathematical (mis)understandings and strategies deeper and wider; to understand student sense-making in related to the mathematical content that she omitted.</td>
</tr>
<tr>
<td><strong>After (post-assessment)</strong></td>
<td></td>
</tr>
<tr>
<td>Omit: did not administer post-assessment.</td>
<td>Limit opportunities to know additional mathematical strategies and approaches; additional student’s mathematical (mis)understandings and strategies; to learn better ways of pedagogical strategies in the future.</td>
</tr>
</tbody>
</table>

**Table 5-14.** Ms. Lee’s interaction with curricular materials and learning opportunities

<table>
<thead>
<tr>
<th>The ways of interaction with curricular materials</th>
<th>Opportunities to learn about student thinking and content</th>
</tr>
</thead>
</table>
Summary

In summary, the fundamental ways they interact with curricular materials are critical for teachers to create important opportunities to learn about student mathematical thinking and mathematical content while the ways overall lesson structure may not be essential. Just as interacting with curriculum materials may result in “lethal mutations “(Brown & Campione, 1996, p. 292) when teachers merely focus on surface features, in this case, the lesson structure and what activities the teachers provided seem to be surface features of implementing FALs. Rather, the key principle underlying FALs is knowing and responding to student mathematical thinking—i.e., how teachers provide the lesson activities. Further, maintaining mathematical content goals and mathematical practice goals of the lessons is another major way to avoid “lethal mutations.” In the Percent-change lesson, the mathematical content goals are to develop an ability to interpret percent increase and decrease, in particular, (a) translating between percents, decimals, and fractions; (b) representing percent increase and decrease as multiplication; and (c) recognizing the
relationship between increase and decreases. Ms. Janet, however, omitted two of the main goals—relationship between percents, decimals, and fractions; and representing percent change as multiplication. In turn, this substantial change both in ways of supporting student learning and in lesson goals limited opportunities not only for her students to learn but also for the teacher, Ms. Janet, to learn and change. More issues of individual teachers’ cognitive and contextual factors that influence to their different ways of interacting with curricular materials are discussed in Chapter 8.
Chapter 6: How Two Teachers’ Instructional Moves Create or Limit Opportunities for Themselves to Learn

In the previous chapter, I provided a global analysis of how two teachers adapt FALs differently and how their interactions with FALs create or limit opportunities for them to learn about student mathematical thinking and content. In this chapter, I focus on a local analysis of how the teachers responded to student mathematical thinking during instruction and how their instructional moves created or limited opportunities for them to learn about student mathematical thinking and mathematical content.

In order to understand and capture how teachers respond to student mathematical thinking, I provided an analytical framework with three dimensions (Table 4-5) in Chapter 4. In this chapter, I first apply this analytical framework to present a “bird’s eye” view of profiles of the two teachers’ implementation/adaptation of the Percent-change lesson. Then, I discuss how these instructional moves created learning opportunities for Ms. Lee, and how Ms. Janet’s instructional moves did not create (sometimes, even limited) meaningful learning opportunities for her.

6-1. How Two Teachers Respond to Student Mathematical Thinking in Percent-change Lesson

A “bird’s eye” view of profiles of the two teachers’ teaching practices in the Percent-change lesson.

To recall the analytical framework for understanding how teachers respond to student mathematical thinking, there are three dimensions: (1) Eliciting student thinking, (2) Using student thinking as resources for discussion, and (3) Promoting mathematical discussion. Each dimension has four developmental levels (from level 0 to level 3, that is, from traditional teacher-centered to student-thinking-centered classroom practices).

In order to look closely at the two teachers’ pedagogical moves, the lesson is chunked into episodes depending on shifts of lesson structure and lesson content focus (e.g., whole class discussion for “Do Now” problems, small group work for percent card matching, or whole class discussion for concluding lesson). If an episode is longer than 8 minutes, it is chunked as sub-episodes depending on shifts of content focus. For example, the Do Now episode for Ms. Lee is chunked as 3 sub-episodes because it is longer than 10 minutes and every sub-episode discusses a different problem.

The “bird’s eye” profile of Ms. Janet’s teaching the Percent-change lesson is shown in Figure 6-1 and Figure 6-2. Overall, Ms. Janet’s profiles of teaching practices with regards to being responsive to student thinking are mostly represented as level 1 across three dimensions. Two episodes are represented as level 2, EP#SG3(W) and EP#W/P1. I will describe how this is coded and the relationship between teachers’ instructional moves and their learning opportunities in more detail in the later section. In this “bird’s eye” view section, I describe lesson structure and overview of lessons briefly.

On the first day of Percent-change lesson implementation, Ms. Janet started with a Do Now session (individual work time on Do Now problems for about 2 minutes and then led whole class discussion on the Do Now problems for about 8 minutes) as
described in Chapter 5. The whole discussion of the Do Now problems is chunked as two episodes (EP#W1 and EP#W2 in Figure 6-1) that each is segmented as a mathematical theme (EP#W1: Do Now problem #1 and #2; EP#W2: Do Now problem #3 and #4, see problems in Figures 5-11 in Chapter 5).

After she instructed the students about she expected them to do in small groups (not coded for this study), the students started to place the percent increase and decrease arrow cards between the four money cards ($100, $150, $160, and $200), starting to work with a partner and then sharing their work with another pair). This small group work lasted 22 minutes and 40 seconds, but there was a 7 minute intermission during which Ms. Janet brought an issue of common “misconceptions” that she noticed from observing small group activities to the whole class. This episode is chunked as EP#SG3(W) so that the remaining small group segment is chunked into three more episodes, EP#SG1, EP#SG2, and EP#SG4.

Ms. Janet concluded the lesson by bringing up one student and his idea to discuss with whole class for about 14 minutes. This episode is combined of whole class discussion led by the teacher and of student presentation so I named the episodes as EP#W/Ps. In the first part of this episode, named as EP#W/P1, a student presented his method on one card matching (from $150 to $200) and Ms. Janet asked three other students to discuss it, for about 6 minutes. In EP#W/P2, the student who presented his work was still in front of the classroom and Ms. Janet asked other students in whole group to conclude the problem and the day’s lesson.

![Figure 6-21. Profiles of Ms. Janet’s teaching practice in Percent-change lesson (Day 1)](image)

On the second day of this lesson, Ms. Janet also started with a brief guide to the lesson and about two minutes of individual work time on Do Now problems. The
discussion of the Do Now problems was brief because the problems were to recall the previous day’s lesson. After this quick check, Ms. Janet instructed the student to continue the small group work on what they had been doing the previous day (about four minutes).

During small group work time (for about twelve minutes), for the first episode (EP#SG1), students continued to match arrow cards (percent increase and decrease) from the previous day. In the second episode (EP#SG2), the students were asked to create a poster. One student kept matching the arrow cards and another student in the group glued them on a poster.

Ms. Janet concluded a lesson by bringing two students to the front of the classroom (she asked one student to come up, but the student was shy so she brought her partner to the front of the classroom) and leading a whole group discussion focusing on the student’s ideas. This episode lasted twenty four minutes, so it is chunked into four sub-episodes (EP#W/P1-EP#W/P4) with the boundaries depending on shifts on discussed mathematical topic, such as listening the student’s method, discussing how she could use the formula, discussing what the original number is, and concluding with the punch line of the day, which is “use the formula”.

![Figure 6-22. Profiles of Ms. Janet’s teaching practice in Percent-change lesson (Day 2)](image)

Ms. Lee’s “bird’s eye” teaching profile for the Percent-change lesson is shown as Figure 6-3 and Figure 6-4. Overall, Ms. Lee’s responses to student thinking is more varied, being, mostly level 2 and level 3.

On the first day of Percent-change lesson implementation, Ms. Lee started with individual work time on the Do Now problems that she created for her class (see Figure 5-14 in Chapter 5). After this 5 minutes of individual work time, she led a whole class discussion on the Do Now problems. It was about 13 minutes long, and it is chunked as
three episodes, EP#W1, EP#W2, and EP#W3, the break points representing a shift in mathematical topic.

As described in the Chapter 5, Ms. Lee provided all three arrow cards (percents, fraction, and decimals) for the card matching small group activity. After instruction of small group activity for about 6 minutes (not coded for this study), the students started to place percent increase cards between the four money cards. They were expected to match fraction and decimal increase cards after completing the placement of the percent increase cards. After about 20 minutes of this small group activity, the first day of class ended.

Figure 6-23. Profiles of Ms. Lee’s teaching practice in Percent-change lesson (Day 1)

The entire second day of Ms. Lee’s lesson (fifty minutes) was devoted to small group work continuing what the students had done the first day. The ways in which Ms. Lee responded to student thinking and supported students to work the activity were very similar to the first day.

On the third day, Ms. Lee’s students started to work on the Do Now problems (see Figure 5-14 in Chapter 5) individually. This quiet individual work time lasted for about seven minutes. Then, Ms. Lee led a whole class discussion of the Do Now problems for about fifteen minutes. In the first segment of this Do Now whole class discussion episode (EP#W1), students discussed about the first sample method in the problem. In the second episode (EP#W2), they discussed the second sample method in the problem and attempted to connect two different methods.

The rest of the lesson (about twenty-six minutes) was mostly led as a whole class discussion. This episode is divided into five sub-episodes. In EP#W3, students discussed percent increase and fraction representation for the percent from $150 to $200. This
episode includes partner talk time and whole class discussion. In EP#W4, students discussed the decimal representation of percent increase from $150 to $200, connecting to a sample student method in the Do Now problems. In EP#W5, students discussed percent increase and fraction representation of the increase from $160 to $200, and then the same relationship with a decimal representation. In EP#W6, students discussed percent decrease from $200 to $150 with fraction and decimal representations. In this episode, they also discussed the relationship between increase and decrease; and percent decrease from $200 to $160 and its relationship with fraction and decimal representations.

At the end of the lesson, they worked on post-assessment tasks individually for about ten minutes. This episode is not coded as this episode is individual work time.

Figure 6-24. Profiles of Ms. Lee’s teaching practice in Percent-change lesson (Day 3)

A comparison of the two teachers’ teaching practices in the Percent-change lesson.

This section provides a close look at the two teachers’ adaptation of the Percent-change lesson, applying analytical framework II. I have three sub-sections in this section: comparisons of (1) the Do Now whole discussion episode; (2) a Small group episode; and (3) the concluding whole discussion episode. In each section, I first provide some general observations and sample excerpts from the relevant episodes from each of the two teachers. Then I provide descriptions how to code.

Do Now episode: Ms. Janet.

In the first day of the Do Now whole class discussion episode, Ms. Janet kept encouraging students to recall the formula for percent change without any focus on conceptual understanding. She also asked the student to apply the formula to the
remaining three problems. She checked the students’ answers numerically and did not examine their thinking or reasoning.

In the second day Do Now whole class discussion, Ms. Janet quickly checked to see if the students could recall what they did the first day, asking “write down everything you remember and what did you learn?” When a student answered “percent change equals change over original,” Ms. Janet appraised his answer and asked a student if they remembered another way to express this. Another student answered, “percent change equals part over whole.” Then, they quickly moved to instruction of small group work.

I provide an excerpt of the Do Now whole class discussion from the first day of lesson below:

Excerpt 6-1. Ms. Janet’s Do Now episode (Day 1).

1. Ms. Janet: Number 1. What is the equation to find the percent of a number?
3. Ms. Janet: I think, the percent times whole equals part.
4. [Aron repeats his answer by Ms. Lee’s question, “say louder?”]
6. Students: Yes/Yeah. [Some students show their thumbs up]
7. Ms. Janet: Is that right? So, okay, write it that down. That’s the first equation.
8. [Ms. Janet writes “% x whole = part” on the board]
9. Okay, do you think this equation might help when you work in your small group?
10. Students: Yes.
11. Ms. Janet: Yes, it might. So, you should make sure you know it. Okay?
14. Ms. Janet: 4. Vote. What do you guys think? [some students show their thumbs up to show agreement]
15. Ms. Janet: Yes? Okay, how did you figure this out, Dan?
16. Dan: I multiply the part…
17. Ms. Janet: Not a part, the.. [She indicates % on the formula that she wrote on the board]
18. Dan: The percent.
19. Ms. Janet: So, in this case, 30 times?
20. Dan: 40.
21. Ms. Janet: [writes 30% x 40 = ,below the formula] and then the “part” in this case is?
22. Dan: 12.
23. Ms. Janet: Right, so 12 is the part of 40. Right?
The discussions of the two other remaining problems are similar. One student said the equation for the percent change is “\% = \text{change/original}” and Ms. Janet wrote it down. And for the last problem, one student said the answer to the problem as “28” and she asked other students “how did you figure it out?” And another student answered, “subtracted.” And then, she elaborated. She pointed out that 28 is the “change,” and “original” or “starting point of an arrow” is 40. She wrote down as “\% = 28/40” and pointed out that students would use this formula in their small group work.

The explanation of how to code each dimension is provided in the next section.

Do Now episode: Ms. Lee

In the first day Do Now whole class discussion, Ms. Lee led the discussion focusing on students’ conceptual understanding and sense-making. She kept pushing students to think and explain why the problem makes sense or does not make sense. At first, students were struggling to articulate their reasoning; they only pointed out that the calculation or decimal point was not correct. However, Ms. Lee kept emphasizing “sense-making” – why students think the solution does not make sense beyond mis-calculation or errors. After collecting several students’ explanations, Ms. Lee narrowed the mathematical focus down to the comparison between 6% and 60%. Finally, several students could articulate their reasoning and build on the idea.

In the third day Do Now whole class discussion (she did not have a Do Now the second day), Ms. Lee attempted to elicit student thinking by asking them what they noticed from two different methods for figuring out the selling price of 25% markup of a $120 sweater (see Figure 5-15 in Chapter 5). To recall the figure 5-15:

<table>
<thead>
<tr>
<th>Abdo’s work</th>
<th>Vinnita’s work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$120(0.25) = $30</td>
<td>$120(1.25) = $150</td>
</tr>
<tr>
<td>$120 + $30 = $150</td>
<td>Vinnita said, look at the graph</td>
</tr>
</tbody>
</table>

Selling price - $150

I notice …
I wonder …

4. ____________
5. ____________
6. ____________

Recalling Figure 5-15. Do Now Problems in Day 3 (Ms. Lee)

The students started by making some superficial observations, some non-mathematical or noting that they were expected to use different methods to figure out the problem. However, after Ms. Lee elicited their thinking focusing on each method, students began to articulate their thinking. She also let students have partner talk time so that they could talk about their ideas with their partners before making them public. Ms.
Lee also asked why questions and have other students rephrase what the previous students had explained. The way Ms Lee handled the Do Now problem in Day 3 was similar to what she did in Day 1.

Here is a sample excerpt from the Day 1:

Excerpt 6-2. Ms. Lee’s Do Now episode (Day 2).

Ms. Lee: So now let’s talk about why does it or does not it make sense? I'd like
to see other hands go up, hands that have not gone up before. I'm
going to go Rina, Nicole, Jamie. I’m going to be calling on somebody
else to kind of paraphrase what you’re hearing, summarize what
they’re hearing. Go ahead.

[Three students, Rina, Nicole, and Jamie, talked about mis-calculation of decimal of the problem]

Ms. Lee: I want you guys to think about this. Can... let’s stop right here on this
one. So, Nicole brought up a good point. Okay, and there’s confusion
about 6% versus 60%. How do you change that to a decimal? Before
we even talk about decimals I want you to think about... without even
thinking about decimals... if I looked at 33 dollars for 6%,
Victor, eyes on me please? Does that make sense? Why or why not?
I’m calling on people I haven’t heard from. Yancy and then Matt.

[Two students, Yancy and Matt, still talked about mis-calculation of decimal]

Ms. Lee: Sense-making. I want you to talk about sense making. What does
make sense and what does not make sense? You can ask the people
that went before you.

Felcia: Uh, well. I said this doesn’t make sense because if you were looking
for 6% you wouldn’t go for more than half of 56. You’d have to go for
a lot less.


[Felica rephrased what she talked and then another student showed his agreement on her explanation]

Ms. Lee: Then I want think about something as you still listen to other people if
I should leave or change. Okay? I'd look to go Udo and Daisy.

Udo: I see the error in what... in...his work because it makes sense but it
doesn’t make sense because the 33.60 is way over 6%. But it works
60% then it would make sense so then the person probably thought it
was 60 when they were maybe trying to change it to a decimal? Or
they made a mistake like that and then added up, the math of adding
up is right but they should notice how its 6% only and it equals almost
100 times more, a 100% more than they should know it's too much to
equal up to.

Ms. Lee: Go ahead Daisy.

Daisy: I agree Udo. They might have made a mistake when they were doing it
33.60 is way over 6% it’s not even close and um... what Udo said the
addition is correct but um I don’t think they... I think they
[indiscernible].

Ms. Lee: Go ahead Ian.

Ian: Going off of what Udo said, I guess you’re right it does make sense
because it doesn’t because um half... just 50% would be like 20
something so 6% would have to be a lot less. Maybe they didn’t put
the decimal over enough so maybe it's like 3.360 because that seems
closer to what may be an answer.

[Ms. Lee gestures to another student]

S: What I notice is 33.60 is exactly 6% because they probably got 3.36
and multiplied by 10 to get 33.60.

Ms. Lee: Okay, let’s kind of do the math right now guys on your paper. So even
if... let's do it... even if you already know how to do it so let’s kind of
work on it together.
Comparisons between two teachers in Do Now whole class discussion episode

Dimension 1. Eliciting student reasoning. The focus of the first dimension is on the ways teachers eliciting student thinking and what kinds of student thinking are elicited. To advance students’ mathematical thinking and reasoning, it is important to pose more open questions that allow students to articulate their own reasoning than to pose procedural questions.

The excerpts from Ms. Janet’s lesson given above indicate that her questioning serves mostly to help students recall the equation for percent-change and apply the equation. Students were asked to memorize and recall the formulas for percent (% = part/whole) and percent change (% = change/original) without addressing the conceptual understanding underlying the equations and without any connections even among procedures. They were also asked to simply apply this formula to the problems. Ms. Janet asked students to write another way of the percent, which represents % x whole = part. However, they were not asked why these two equations represent the same relationship and students were constrained to straightforward processes (e.g., plugging numbers into the equations). Student comments were brief, such as just stating an answer (line 4, 17, 24, 26, and 29).

Later, Ms. Janet asked a question that seems to elicit students’ thinking (line 20), “how did you figure this out?” However, she interrupted students’ talk (line 22-23) and the exchange took the form of be IRE sequences, without addressing conceptual understanding or reasoning underlying the why question. Therefore, Ms. Janet’s questioning is more close to level 1 (procedural process answerable questions, IRE sequencing types) even not to level 2.

In the first chunk of Ms. Lee’s Do now whole class episode (EP#W1), students mainly discussed whether the example solution was correct. Students explained where the sample solution was correct (the calculation, 56 plus 33.60 equals 89.60, was correct) and what was not correct (the decimal point of 33.60 was not correct). Then Ms. Lee switched the focus to why this was or was not making sense. The excerpts of Ms. Lee above demonstrate that she elicited students’ reasoning about why the solution did not make sense to them. For the first part (line 1-20), even though Ms. Lee pushed students’ reasoning, the students still did not articulate their sense-making. This pedagogical move seems to make a connection between procedures (the calculation—addition and multiplication of decimals) and concepts (percent increase (6%) of the original price ($56)). However, students’ answers did not seem to be fully connected. Students explained some procedural processes instead of making connections between procedures and concepts. Students expressed their conceptual confusion or partial understandings of the idea, why 33.60 does not make sense. It showed that they could not make a full connection but did try to make some connections. Therefore it is coded as level 2 with regard to the teacher’s efforts at eliciting student thinking and to what kinds of ideas are elicited.

In the second and third chunk of this episode (EP#W2 and EP#W3), students were encouraged to make connections between procedures (decimal place when they multiply decimals) and conceptual ideas (6% wouldn’t go for more than half of $56). Also, they made some connections between this problem and previous day’s problem that seemed to talk about decimal places. Furthermore, students were encouraged to articulate their reasoning concerning why 33.60 does not make sense; students finally reached consensus
on the idea, “6% of $56 should be less than a half (50%) of $56 which is $28 but $33.6 is way too much than $28” or “33.60 is exactly 6% of $56.” As shown in the last part of the discussion (line 22-60), students finally could explain their thinking why the solution does not make sense so that two sub-episodes are coded as level 3.

**Dimension 2. Using student thinking as resources for productive discussion.** This dimension focuses on capturing the ways of the teacher makes use of students’ mathematical thinking and (mis-)understandings. It is key for effective diagnostic teaching and for use of formative assessment. Teachers can come to understand students’ mathematical thinking – both what they do or do not understand – by having students talk through their reasoning. An effective strategy is to have students change their thinking or pursue alternatives using others’ thinking as resources, rather than having the teacher evaluate or fix every student’s misunderstandings directly.

There were big differences between two teachers in the way they used students’ thinking. In the first day of the Do Now episode, Ms. Janet asked short-answer questions that did not provide space for providing their reasoning and then asked other students to show their agreement quickly by showing thumbs-up gestures. This showing of agreement did not elaborate the student’s ideas but simply evaluated the student’s answer. This pedagogical move eliminates opportunities for students both to further their understandings and to build on other students’ ideas. When a student’s error comes up, she did not use this as a resource for productive discussion but fixed his answer immediately, interrupting while the student (Dan) was talking (line 21-26). This pattern shows up similarly in the second chunk of this episode. Therefore, both chunks of the episode are coded as level 1.

In contrast, Ms. Lee first collected several students’ thoughts with regard to one mathematical idea and listened carefully to what they talked without interrupting. She did not evaluate whether the students’ explanations were right or wrong. Instead, she directed the discussion towards key concepts, and attempted to make students build on other students’ ideas. She worked carefully on one mathematical idea before moving to another focal mathematical idea, at which point she once again collected students’ thinking about it. This pattern maintained for all three chunks of the episode.

When students did not explain why the sample solution does not make sense, she did not say whether their talk was correct or not. Instead, she described one student (Nicole)’s idea and then made space for students to think about the narrowly-focused idea (6% and 60%) by themselves (line 10-16). This provided opportunities for students to reconceptualize their ideas and to think differently towards the goal, articulating their “making-sense.” Also, when students still did not explain well, she provided opportunities for students to ask other students so that the elicited ideas became resources for students to engage in productive mathematical discussion (line 22-28). The discussion eventually got to the point where the students did change their thinking and articulate their reasoning (line 36-60). Therefore, the all three chunks of the episode are coded as level 3.

**Dimension 3. Promoting productive mathematical discussion.** The focus of this dimension is to capture the ways teachers promote productive mathematical discussion in which students elaborate, justify, and argue about their thinking each other. It also focuses on how the teacher scaffolded interactions among students and how students engaged in these mathematical conversations.
The interaction between Ms. Janet and her students in the Do Now whole class episode was mostly teacher-student one-to-one. Ms. Janet asked a question of one student and he/she answered. Other students just showed their agreement/disagreement collectively using gestures (e.g., thumbs up) by the teacher’s request. And then, another student briefly responded to what Ms. Janet asked. There was no space for students to elaborate other students’ ideas or justify their reasoning. Ms. Janet did not explain how to participate in productive mathematical talks explicitly either across two sub-episodes. Therefore, both chunks of this episode are coded as level 1.

On the other hand, Ms. Lee, as shown in the sample excerpt above, pushed students to rephrase what other students said (line 22-31), and to elaborate on another student’s thinking. Eventually, the students could build their ideas on what other students said, along with saying “agree or disagree (line 36-57).” Therefore, the later chunk of this episode is coded as level 3. The first chunk is coded level 2 because the discussion was mostly driven by teacher questions (what the example solution of the problem shows correct) to three students, with the teacher facilitating the discussion rather than students discussing with each other or building on each other’s ideas spontaneously. However, after this pattern of discussion went around once, two students rephrased what another student said with the teacher’s facilitation and one student added to what other students said, saying “I was going to agree with them.” Therefore, the second chunk is coded level 2. The levels of this dimension in Ms. Lee’s episode ranged from 1 to 3 as the discussion was going on, but it ended at level 3.

Small group episode: Ms. Janet’s case.

In small group of the first day, Ms. Janet asked students to work on only percent change (both increase and decrease) cards. While students worked on the activities, Ms. Janet started to ask why students placed an arrow card. However, while Ms. Lee waited until students finished their explaining and did not immediately evaluate whether their explanation was right or wrong, Ms. Janet interrupted students’ explanations frequently to correct them. When she found a different way of explanation from what she wanted to hear or incorrect answers, she tried to fix students’ answer right away. Furthermore, she kept saying, “use the formula!” without connecting ideas even though there were some elicited students’ ideas that could have been used.

As I mentioned before, there was a sub-episode that switched to whole group discussion structure. Ms. Janet found that there was a lot of confusion among students about percent decrease from $150 to $100. Many students placed arrow cards between $100 and $150 as “increase by 50%” and “decrease by 50%.” So, she said to the whole group while students were working as a pair, “one misconception, one of those arrows is correct and one of them is not. I won’t tell you which, but you need to think about what we were talking about on the Do Now. Where are you starting from? Because your denominator is going to be different from one of the arrows, right? Okay that's my hint. Okay, you have 5 more minutes”.

After 5 minutes, she asked to the whole group, “did anyone figure out which arrow was wrong” and it turned to the whole group discussion structure. This whole group discussion was similarly led to the Do Now episode. One student articulated his thinking not only using a formula but a cursory connection between fraction (1/3) and percent (33.3%) as his own way. However, Ms. Janet did not use his thinking and directly
went back to use the formula (% = change/original). After discussing “decrease 33.3%” from $150 to $100 using a formula, small group activities were resumed. The remaining small group activities demonstrated very similar patterns to those described above, with IRE sequences and a procedural focus on formulas.

One example excerpt of Ms. Janet’s episode is provided as below and discuss in more detail comparing with Ms. Lee’s episode.

**Excerpt 6-3. Ms. Janet’s Small Group episode (day 1)**

86 Ms. Janet: How did you figure this out? Down by 33 1/3%?
87 S1: 50 is one third of 150. 150 minus 50 is 100.
88 Ms. Janet: Yeah, but I want you think about...yeah, No, no, both are right. That’s right, because you said 33% of 150 is 50. Right? That’s a third, right, that’s 50. But, that’s correct, this is good. But I want you to show it with the formula, right? The change. What's the change? How much is it going down by, guys? from 100.
93 S1: By 50%.
94 Ms. Janet: Not percent. It’s going down by 50. Just 50, right? So, write it down. 95 50 is the change. Write it down. So, it's 50 over...
96 S1: 150.
97 Ms. Janet: Exactly. Good. Because that’s what you are starting, right? And then, I want you to prove to me. Simplify this and show me that that equals that, right? Because that’s what I wanted you to use formula.

[Moves to new group]

101 Ms. Janet: … So these ones are good, right? But then you gotta think about- which one do you want to start? I can help you with one. Which one do you want me to help you with?
103 S2: This one.
105 Ms. Janet: What’s that? This one? Between 200 and 150? So from 200 to 150, use the formula. How much is it changing?
107 S2: 200?
108 Ms. Janet: No, 200 to 150, how much is it changing?
109 S2: Oh, uhh...
110 Ms. Janet: It’s going down right? By how much? What’s 200 minus 150? [T to whole class] Okay, again I want to hear talking. Kay I don't hear some talking right now.
113 S2: 50?
114 Ms. Janet: Yeah 50, right? So that's gonna-write it down, that’s your numerator, right, 50? Yup good, 50, that's the numerator cus isn’t that how much it changes, right? Over your original. So if we’re going- you said 200 to 150, right? So if you’re starting here, the arrow’s gonna go in that direction, right? So you're starting with 200. So that's your denominator and then you just gotta simplify it and figure out what percent that is.
Small group episode: Ms. Lee

In the first sub-episode of small group work, Ms. Lee began by supporting group interaction explicitly, e.g., explaining how to collaborate in placing the arrow cards without one person dominating the discussion. She also asked a student in a group, “why did you put the arrow card there?” and asked another student in a group to rephrase what the students explained. And she encouraged students to ask these questions and to justify/rephrase/argue their explanations with each other. From these pedagogical moves, which elicited students’ reasoning, Ms. Lee seemed to collect students’ understandings of how they conceptualized the percent increase from $100 to $150. It did not show that she encouraged students to think of alternate strategies. This may be because this first she was focused on having the students engage in the activity or because the percent increase from $100 to $150 was not challenging to the students. Students were asked to rephrase what their partner said but the teacher still needed to facilitate in first two chunks of episodes.

However, as the small group activities went on, students discussed the mathematics each other without needing major facilitation. When Ms. Lee scaffolded students in other activities that were more challenging to students (placing other arrow cards, from $150 to $200 or from $160 to $200), she elicited various strategies to figure things out and encouraged their use.

One sample excerpt is below. I discuss this example in more detail, comparing it with Ms. Janet’s episode, later in this section. In this excerpt, they talked about the percent change from $150 to $200. In the second group, there were three students because one student remained after the students paired up.

Excerpt 6-4. Ms. Lee’s small group episode (day 1)

121  *Ms. Lee:* I’m going to ask you a question. So Kathryn and Yancy. How much is the increase, how much the amount?
123  *Kathryn (K):* 50.
124  *Ms. L:* 50 dollars. Okay, so when you increase by 50 dollars here, what fraction of 150 is 50 dollars?
126  *K:* Um... um... isn't it 1/2? Right?
127  *Ms. L:* Why do you say 3?
128  *Yancy (Y):* 3?
129  *Ms. L:* Why do you say 3?
130  *Y:* Two 50s is 100 and another 50 for 150.
131  *Ms. L:* [to Kathryn] What did she just say?
132  *K:* She said 150 for 50 and then another... I would have said... I would go like this though.
133  *Ms. L:* Can you explain?
135  *K:* I would be like... two 50s is 100. Oh, I see now. I’m stupid!
136  *Ms. L:* Can you tell me how many 50s are in 150?
137  *K:* 3.
138  *Ms. L:* Why?
139  *K:* Because there’s one 50 in 50 and two 50s which equals 100.
So the 50 is 1/3 of 150. So, how much does it increase by then, do you think?

Y: 1/3?

K: Oh, 1/3.

Ms. L: Okay, are we okay with that? Can you explain that in complete sentences please? I'm going to ask Yancy and then Kathryn. Go ahead Yancy, explain why this is up by 33 and a third.

Y: Because it's up by 50 and 1/3 of 150 would equal 50. Would equal 50% because there's 3 of them, and each third equals 50.

Ms. L: Kendall can you just explain what Yancy said?

K: Yancy said that it was like 1/3 because 1/3 of it would be one 50 because that's 1 out of 3 and then 33% because that's what you had to add for the other 50's on the 100.

Ms. L: Okay, listening to each other I want you to... I'm going to ask you to say 3 sentences. And I want you to say 2, 3 short sentences. Why is this up by 33%? Go [to Yancy], go [to Kathryn], go.

K: Wait, what?

Ms. L: 3 sentences why is this up by 33%?

Y: Okay, 3 sentences... number 1 reason because 1/3 of 150 equals 50%. And then...

Ms. L: 1/3 of 150 equals 50 dollars.

Y: 50 dollars, I’m sorry. And then if you wanna $50 plus, I mean, $150 plus $50 equals 200. So that means 150 is up by 1/3. And that’s it.

Ms. L: Go ahead [to Kathryn].

K: 1/3 represents 50 out of 150 and then 33%.

Ms. L: Okay, excellent. Now go into the decimals.

[New Group]

Ms. L: Okay, so we’re left with this. What’s going on here? Here’s the only one that’s left. Can you tell me why... what do you think? How come we’re uncertain about this?

Denny(D): Uh... I didn’t do the calculation.

S1: But I was... I was working on the calculation and doing the multiplication on this...

Ms. L: Okay, so... I’m sorry to interrupt. Nicole, eyes on me. What’s change here?

D: 50.

Ms. L: 50 what? [gesturing “explain more”]

D: Dollars.

Ms. L: 50 dollars. So you added 50 dollars to 150 to get to 200. So I want you to think about that 50 dollars. What % is that 50 dollars out of this 150?

S1: 1/3. Ohhh! Oh. So.

Ms. L: Talk to me. Talk to me.

S1: So the uh... if the 1/3 of it is 50 and so if you multiply 150 times .33 point third...

Ms. L: Stay away from multiplying. Nicole and Denny jump in.

D: Mainly if, same thing with this one, if you divided this one by 4, or get 1/4 of this...
Okay, you just said 50. You told me to increase by 50. So how many times does 50 go into 150?

3. [both students answered at the same time]

So what fraction of 150?

1/3.

What's 1/3 in decimal or percent? Okay, so Nicole, now you summarize why is this up by 33 and 1 third.

Because 15... I mean 150 is... 50 is 1/3 of 150 so uh.... it goes up, and then it goes up again to get 200.

Okay, you guys are doing good. Now, go to the decimals. The pink.
Comparisons between two teachers in small group episodes

Dimension 1. Eliciting student reasoning. Ms. Janet helped students place four arrows: (1) $100 \rightarrow $150, (2) $150 \rightarrow $100, (3) $200 \rightarrow $150, and (4) $160 \rightarrow $200. However, in all four problems, she pushed students to use the formula, “\%=change/original,” that she had introduced in the Do Now episode. Ms. Janet started by asking why students had placed the arrow cards where they did, which seems to be eliciting student reasoning. However, her students’ answers focused on “right” or “wrong” rather than on their reasoning. Students were encouraged to use only one simple strategy, “use the formula” through the all activities. This reduced the challenge to finding out what the “change” was and what the “original” number was, from the problem. These are skill-oriented processes: subtracting the original number from the final number and (find the “change”), this number as the numerator in the fraction “change”/original number, simplifying the fraction and expressing it as a percent. There was neither connections between procedures and concepts nor a focus on conceptual understandings.

Ms. Janet also interrupted students’ explanations frequently without allowing much wait time for her students to explain. The excerpt contains many IRE sequences and short-answer question. Even though the student’s answers showed his thinking (line 2) after she asked “why” questions, Ms. Janet just focused on using the formula (lines 5-8). Most of what she said focused on procedural aspects of using the formula, such as what the numerator or denominator is and on simplify the resulting fraction. For these reasons, all sub-episodes were coded as level 1 except whole group episode.

In the whole group sub-episode between small group sub-episodes (EP#SG3), one student’s reasoning was well-elicited several times as I described above. However, the student’s explanation was not fully articulated and was fragmented by teacher’s probing. Therefore, it was coded as level 2.

In contrast, Ms. Lee mostly asked students why they placed the arrow cards or how they figured out the percent whenever she moved to a new group. When she found a wrong or uncertain card placement, she provided scaffolding questions based on students’ elicited thinking, asking “why” first. She usually did not interrupt while students explained their methods or reasoning but waited until they finished their explanation. Ms. Lee also tried to encourage students’ reasoning without evaluating their answer immediately, allowing students to articulate their thinking. This gave students room for re-visiting their reasoning.

Ms. Lee scaffolded the placement of three arrows throughout the small group activities: (1) $100 \rightarrow $150, (2) $150 \rightarrow $200, and (3) $160 \rightarrow $200. In the first activity, $100 \rightarrow $150, specific students’ reasoning was that “the amount of change from $100 to $150 is $50. And the $50 is a ‘half’ of 100, which is same as 50%.” Students did not seem to be challenged by this first activity.

The second activity, the percent increase from $150 \rightarrow $200, was very challenging to students. Some students revealed difficulties in figuring out the percent increase. Ms. Lee scaffolded them with specific questions, such as, “how much, the amount of dollars, is the increase,” “what fraction of 150 is 50?” and then let them figure out the relationship between 1/3 and 33.3%. If students revealed errors in figuring out the fraction, 50 of 150, she also scaffolded with a question, “how many 50s in 150?” instead of saying “simplify the fraction” or directly explaining to them. These scaffolding
questions were based on students’ conceptual understandings, supporting students in making sense and making connections between procedures and concepts (simplifying fractions). This conceptual understanding helped students to apply their understanding to another activity, $160 \rightarrow $200. Students figured out that the amount of change was $40 and Ms. Lee encouraged students to understand conceptually rather than simply calculate. They multiplied $160 by .25 (they seemed to use “guess and check” to figure this number out) and they got $40 (the amount of change). Ms. Lee asked students in the group to find another way, saying, “other than multiplying… how many 25s are in 100?” Also, Ms. Lee provided an example, “there are four 25s in 100,” and a question, “if you divide 160 by 4, how much are in a group?” And students answered 40, which is the amount of change from $160 to $200. These scaffoldings helped students to make a connection between fraction (1/4) and percent (25%). Ultimately, the students conceptually understood why this arrow ($160 \rightarrow $200) was a 25% increase. Ms. Lee’s pedagogical moves were aimed at students’ conceptual understanding. Furthermore, students were encouraged to make connections between concepts and procedures and to explain their reasoning.

Therefore, all five sub-episodes were coded as level 3. The sample excerpt above documents this. The first group facilitation represents how Ms. Lee elicited students’ mathematical thinking and helped them advance their thinking based on their revealed understanding. And in the second group, Ms. Lee asked why they could not place one arrow and students explained what they tried to do (line 170-172).

Lesson conclusion episode: Ms. Janet

Ms. Janet had a wrap-up at the end of each day’s lesson. At the first day’s lesson, she called one student who did not use a formula in his small group to the front of the classroom. The student presented his work on the percent increase from $150 to $200. He presented his thinking in one sentence, “33 and 1/3 % of 150 is 50, and 150 plus 50 equals 200.” Ms. Janet asked two other students to rephrase what he said. Then, she led the discussion toward “using a formula” by saying “class, we’re gonna help him out. I want you to use the formula. … He’s right, his thinking is awesome, but I want you to be able to use the formula.” And then, the whole class discussion turned to talk about the ways of using a formula, such as seeking numbers of change and original, and calculating the percent (change divided by original).

The lesson concluded as “how to use the formula, percent = change/original” by seeking numbers that represent “change” and “original” in the problems. She also concluded that students had to memorize 1/3 is 33.333%.

At the second day’s lesson wrap-up, two students came to the front of the room to discuss their way of figuring out the percent change from $100 to $150. They said, “we did up by 50% because 50% of 100 is 50, and 100 plus 50 equals 150.” However, Ms. Janet stated that she did not understand their approach because they did not use a formula. Similarly to the first day’s episode, she asked two other students to help them use the formula, “percent = change/original.” The discussion focused on finding appropriate numbers for the denominator and numerator in the formula, for example “original number is where it started from.” In the next discussion two student presenters explained how they figured out the decrease by 33 1/3 percent, from $150 to $100. Two presenters talked this time, they used the formula so that they got 50/150. They said it
equaled 1/3, which was 33 and 1/3 percent. The lesson concluded that the lesson’s punch line is “original is where you start from” in figuring out how to use the formula. See excerpt 6-5 as an example.

**Excerpt 6-5.** Ms. Janet’s lesson conclusion whole class discussion episode (day 2).

The excerpt is from EP#W/P1 as below, when two students discussed the 50% increase from $100 to $150.

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196 Ms. Janet: Okay, so, Jamie, you guys said this was up by 50%. How did you figure that out? Can you explain what you did? Um, so you know how it says change over original? Um, that one went up by 50% because this one is...
199 Okay, so I'm gonna have Megan explain it first and then Jamie, you re-explain it. Go ahead, Megan.
201 Megan (M): We did up by 50% because 50% of 100 is 50 and 100 plus 50 equals 150.

Ms. J: (…) So my question. Am I asking them about the down by 33 and 1/3 percent?

Sts: No.

Ms. J: No. I'm asking them, how did they calculate this percent. This 50%. How did they calculate that this was up by 50%? Okay, answer that. Megan, and then Jamie, re-explain. Answer it again, Megan, ’cause I didn't really understand what you said.

M: Since we're working with 100, 50% of 100 is 50 and 100 plus 50 equals 150.

Ms. J: Jamie.

Jamie (J): She said 100 over, um,

Ms. J: Okay someone from the audience. How did they figure out 50%? They didn't use the formula, did they?

Sts: No.

Ms. J: No, they did not. Only 2 people get it. 3 people get it. Kay, Megan, can you re-explain what you said? Because people are not understanding ’cause you didn't use the formula, but it still worked. We're gonna talk about why.

M: Since we're working with 100, 50% of 100 is 50 and 100 plus 50 equals 150.

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Ms. J: Okay, can someone rephrase what Megan just said? Nathan?

Nathan (N): She said that, that, that, half of 100 is 50 and that 50 plus 100 is 150.

Ms. J: And why did you say half? ’cause at first she said 50%. What do you know about 50%?

N: 50% is always half of a number.

Ms. J: And why did you say half? ’cause at first she said 50%. What do you know about 50%?

J: Um, she, I don't get it.

Ms. J: So do you understand that 50% of 100 is 50?

J: Yes.

Ms. J: Why?

J: Because it's half.

Ms. J: Yeah. And then if you, if it says up by 50% you're adding half of 100, right?

J: And so it's 50 more?

Ms. J: Yeah. Do you understand?

J: A little.

Ms. J: A little? Kay, so explain. Why is it up by 50%?

J: Because half of 100 is 50 and then you have to add it and then it equals 150.

Ms. J: Beautiful. Kay, is there a different way that they could have done it, guys? With using the formula? (...) How could they have used the formula, guys?

Gloria (G): By putting, by putting the 100 in the original and the change in, and put a 50 like that and you need to um, you need to show how you have to put.

Ms. J: Okay, so here's a pencil. I want you to write that down. Okay, explain it again, Jane, and then you guys are going to add this to their work. Okay, underneath the 50%. Explain it again.

G: By putting the 100 in the, in the original and then the change, put 50, and then you're gonna figure out how you're gonna get the 50%.

J: Like this?

G: Yeah.

Ms. J: And what does 1/2 equal, guys?

Sts: 50%.

Ms. J: Yeah, and then draw an arrow, Jamie, draw an arrow from that, your work, to 50%. That's how you figure it out.

Concluding the lesson whole class episode: Ms. Lee

Ms. Lee had her lesson wrap-up on the third day of implementation of the Percent-change lesson. Her lesson conclusion part is mostly whole class discussion integrated with several intervals of partner talk time. She reviewed two “increase” relationships (1) from $150 to $200 and (2) from $160 to $200 with three different representations (percent, fraction, and decimals); and two “decrease” relationships: (1) from $200 to $150 and (2) $200 to $160 with the three representations. For the first mathematical topic, “percent
increase from $150 to $200,” she started by asking how much the increase is; she then and led the discussion by asking, “what percent would that by and how would you find that out?” After students had two minutes of partner talk time, one student volunteered to discuss the answer, “33 and a third percent.” Then, Ms. Lee kept stressing students’ reasoning by asking “why.” At first, students were having hard time explaining their reasoning, but they started to talk about their thinking with Ms. Lee’s scaffolding questions. They discussed the relationship between fractions and percents and then, Ms. Lee rephrased their expressed ideas in her own words (Episode #W3).

In Episode #W4, Ms. Lee’s pedagogical patterns are similar to the previous whole group discussion episodes. She threw out a big question for students to think about with regard to decimal representation, “what do you multiply – 150 times what number is going to give you 200?” The students had partner talk time, and then Ms. Lee led a whole group discussion with “why” questions. After helping students to understand that increasing by 33 1/3% can be represented as 1.33 (decimal) and explaining it, Ms. Lee solicited several students’ thinking and she revoiced their expressed ideas in her own words. Ms. Lee also connected their work with the previous Do Now problems.

In Episode #W5, they discussed the increase from $160 to $200 with similar patterns to Episode #W3 and #W5. In Episode #W6, they discussed the decrease from $200 to $150 and from $200 to $160 similarly. The discussion was brief, with summaries from Ms. Lee.

I provide sample excerpt from EP#W5 as below:

**Excerpt 6-6.** Ms. Lee’s concluding whole class discussion episode.

254 Ms. Lee: Okay, think about what amount it increased by. (...) Anna, how much
255 increase by? The money. From 160 to 200.
256 Anna (A): 25%.
257 Ms. L: No, I'm talking about money. How much did the money increase by. 160...
258 A: Oh, 40.
259 Ms. L: $40. And how do you know it increased by 25%?
260 A: Because, I remember I did the math and forgot how I got it but um... I don't
261 know.
262 Ms. L: Okay so 40 is 25%. We're looking at... what comes to your mind when you
263 say 25%?
265 Ms. L: 1/4. So I need to think about 1/4. How is 40 and 160 related? I'm going to
266 stick with Anna so she can get it. So you're saying... I really like the
267 connection you made. It's 1/4. How do you make 40 and 160 related since you
268 just said 25%.
269 A: Can I do this one?
270 Ms. L: Mm-hmm.
271 A: Um... (think time) well. I don't know.
272 Ms. L: Well, no, no. You were right there. So you have 40 right? So how is 40 related
273 to 160 do you think? You just said that 25% is 1/4. 1/4 of what? Of 100,
274 somebody said. 100%. So you know that 25, Anna look at my paper, 25, 50,
275 75, 100. So there are 4 25's in 100.
276 So what about 40 in relation to 160?
Comparisons between the two teachers’ concluding lesson whole class episodes

Dimension 1. Eliciting student reasoning. Again, the focus of this dimension is how teachers elicit student reasoning and what kinds of students thinking are elicited. The excerpts from Ms. Janet above, EP#W/P1 in day 2, show that she started eliciting student thinking by
asking how two student presenters figured out that increase by 50% from $150 to $200. The
students explained what they did (lines 254-258). The student, Megan, explained their
methods to figure out 50% in non-algorithmic way (lines 259-260). This is another way that
student could think; it could provide an opportunity for the teacher to pursue student thinking.
However, Ms. Janet evaluated her answer by saying, “Megan, I didn’t really understand what
you said (lines 266-267, 274-276)” because Megan “didn’t use the formula (line 276).” Ms.
Janet also asked Nathan and Jamie to explain their reasoning about the relationship between
one half and 50% with “why” questions (line 280-281, 287, 293). Furthermore, Ms. Janet
attempted to make a connection from the students’ non-algorithmic thinking to how to use a
formula. Although the focus of the lesson was on solving the problems procedurally and
memorizing and utilizing algorithms and formulas, the questions in this episode evoked
student thinking and reasoning rather than focusing on answer-getting. Therefore, this short
excerpt is coded as level 2 in this dimension. However, after two students presentations and
checking non-presenters’ rephrasing, Ms. Janet once again turned to short-answer questioning.
She had the students write down the procedure for using the formula, and the lesson
concluded with a review of “using a formula, %=change/original” and “how to know what the
original number is: the number where you started from” So, the next episode, EP#W/P2, is
coded as level 1.

Episodes EP#W/P3 and EP#W/P4 have similar patterns, with the teacher asking how
to figure out the decrease from $200 to $150, but two presenters explained that they figured
out the answer by using the formula: “original number is 150 and change is 50 so 50/150 is
1/3, which equals to 33 1/3%.” Ms. Janet asked them why 1/3 is 33 1/3%. Ms. Janet’s
questions are not fully open-ended questions to pursue students’ reasoning; sometimes the
purpose of questioning is to clarify the procedures that will produce the desired answers..
Thus EP#W/P3 is coded as level 2. In the EP#W/P4, the questions were mostly procedural
process things, such as memorizing the formula and finding original number to use the
formula. The punch line of the lesson was “the original number is the number where you
started from.” Therefore, it is coded as level 1.

In contrast, Ms. Lee’s case, she started by ask a much broader question—“what
percent did it increase by, and how would you find that out?” The questions were more open
(e.g., lines 262-263, “what comes to your mind when you say 25%?” instead of asking “what
fraction is equal to 25%?”). These kinds of questions help students reflect on their reasoning
and conceptual understanding. When she asked, “how do you figure out” or “why,” Ms. Lee
gave enough time for students to think before explaining their thinking (lines 271, 277). She
also asked students not to answer with short phrases but to answer in a sentence (line 286), a
move that supports students in explaining their reasoning rather than focusing on answer
getting. Ms. Lee’s ways of eliciting reasoning were similar to those in other episodes, such as
the Do Now and small group episodes described earlier. However, in the lesson wrap-ups, Ms.
Lee sometimes led the discussions hastily because the students had already spent two days
exploring the mathematical concepts and ideas. In EP#W4 and EP#W6, Ms. Lee summarized
the mathematical concepts and ideas by revoicing only a few students’ elicited thinking and
she did not pursue students’ reasoning in depth. Nonetheless, Ms. Lee’s questioning still
focused on eliciting reasoning rather on issues of implementing procedures.

Dimension 2. Using student thinking as resources for productive discussion. The focus
of this dimension is how deeply teachers understand student thinking and how they use it as
resources for discussion. Ms. Janet’s lesson conclusion usually starts with one or two students
presenting their small group’s work. In this episode, two students started to explain their work, and Ms. Janet started to collect student presenters’ thinking as described in dimension 1. When the students presented their ideas, Ms. Janet did not, as she did in other episodes, evaluate their ideas right away with a word, “right” or “wrong.” Rather, she asked them to repeat what they said (line 274-276) but it contained some of her evaluation, that their work was not understandable because they did not use a formula (line 275-276). Ms. Janet did ask two other students, Nathan and Jamie, to rephrase what Megan said (line 278-280, 283-288). However, this rephrasing was only used for checking their understanding of Megan’s way of thinking. It was not fully used as a resource for productive mathematical discussion because the students did not build on each other’s ideas. Therefore, this episode is coded as level 2 in dimension 2. Other episodes show similar patterns, except for the last episode, EP#W/P4. In the very last episode, she mostly summarized the lesson with IRE or answer-getting types of questions focused on figure out what number would be denominator and numerator of the formula, percent = change/original. There were not opportunities for students to build on each other’s ideas. The last episode, therefore, is coded as level 1.

In contrast, Ms. Lee she usually solicited several students’ thinking without evaluating their contributions right away. Ms. Lee used a student, Anna’s partial understandings as resources for a conversation that explored the relationship between 25% of 160 and ¼ of 160, as they worked to represent the increase $160 to $200 using percent, fraction, and decimal forms. Anna said that she forget about algorithms or procedural process to figure out the relationship (lines 260-261), but Ms. Lee persevered and helped her advance her understanding (lines 265-268, 272-288). Anna did get the point, even though it was not fully internalized (line 289-290). And then, Ms. Lee invited other students to build on or elaborate on Anna’s idea by rephrasing, verifying, and repeating (lines 291-294, 300, and 316). Therefore, this episode is coded as level 3. However, Ms. Lee was sometimes in hurry to wrap up all relationships (among fractions, percents, and decimals; and of increase and decrease) in particular EP#W3 and in EP#W6 are coded as level 2.

Dimension 3. Promoting productive mathematical discussion. The focus of this dimension is how teachers support students in discussing mathematics in productive ways—by helping them build on each other’s ideas, and critique and justify their own and others’ ideas. Ms. Janet’s students had some opportunities to repeat and rephrase what other students talked. From the excerpt, lines 291-294, 300, and 316 are the examples of this. However, it was mostly caused by the teacher’s prompt, so this episode is coded as level 2. In the next episode, there were similar patterns of repeating, reframing, and elaborating on others’ talk supported by Ms. Janet. However, in the final two episodes, the discussion was mostly an IRE exchange between the teacher and one student. When Ms. Janet asked a (typically procedural) questions, one student provided a brief answer. Ms. Janet kept asking such questions until the student(s) got the answer.

In contrast, Ms. Lee typically encouraged students to rephrase, verify, or build on what other students said. As noted, however, in this lesson wrap-up, she was in hurry to cover a lot of content. Nonetheless, she still asked other students to rephrase or build on (lines 291-294, 300, and 316) elicited student’s ideas. For that reason this episode is coded as level 2.

Section Summary. In this section I provided analytic profiles of the two teachers’ instructional moves when they implemented the Percent-change FAL. As described in the “bird’s eye” view section, Ms. Janet’s pedagogical moves were coded mostly as level 1 and some were coded as level 2, particularly in student presentation episodes. In contrast, Ms.
Lee’s pedagogical moves were mostly coded as level 2 or level 3. This represents a change for Ms. Lee: at the beginning of the year her questions were typically procedural or teacher-centered moves not focusing on eliciting student thinking and promoting discussion. My comparisons of teaching practices zoomed into the two teachers’ ways of implementing FALs and ways of responding to student thinking. It also showed how the developed analytical framework was used to capture teachers’ elicitation and use of student thinking.

6-2. How The Two Teachers’ Instructional Moves Created or Limited Opportunities for Them to Learn about Student Mathematical Thinking and Mathematical Content

In the previous section, analysis provides characteristics of the two teachers’ different pedagogical moves with regards to responding to student mathematical thinking. In relation to their different profiles of teaching practices, in this section, I analyze how two teachers’ different pedagogical strategies created or limited learning opportunities to learn about student mathematical thinking and the mathematical content when they implemented the FALs. It is a zoom-in version of Chapter 5 with regards to the teachers’ learning opportunities teachers have when they implement innovative curricular support materials. The possible impact of these experiences on change in everyday teaching practices is discussed in Chapter 7.

As described in Chapter 5, Ms. Janet frequently omitted key features of the FALs while Ms. Lee tried to implement the FALs with some fidelity. In this chapter, I provide a more local analysis of how the teachers’ instructional moves afforded opportunities for making sense of student mathematical thinking and mathematical content. Due to the nature of fine-grained analysis, I provide each teacher’s pedagogical patterns in the Do Now episodes.

Ms. Janet’s pedagogical patterns and learning opportunities in the opening discussion (Do Now episode)

Ms. Janet usually started by asking students to recall a formula or algorithm. It allowed her to check if the student had memorized the formula or algorithm (no opportunities for developing conceptual understanding). Ms. Janet evaluated answers right away for correctness, and checked to see how many of the students knew the answer and/or thought it was correct. If it was, she typically moved to have students memorize it; if it was not correct, she either corrected it right away or asks other students to correct it and moved to another discussion cycle. She rarely tried to support discussions in which students built on each other’s ideas.

For example, in excerpt 6-1 above, Ms. Janet asked students to recall the equation/formula to find the percent of the number that they had learned in a prior lesson and would use in this lesson. Right after one student spoke his memorized equation, Ms. Janet asked whole class to indicate whether it is correct or not by using a thumbs-up or thumbs-down gesture. By doing so, Ms. Janet was able to see how many students had memorized it. These pedagogical moves limited opportunities for Ms. Janet to deepen and expand her understanding of students’ thinking. No student reasoning was elicited, thus, she did not and could not make use of student thinking in classroom. In the next problem, Ms. Janet asked students to use the formula revealed by a student, Aron. She did ask Dan about his thinking, saying “how did you figure this out”—but while Dan was talking, she interrupted and corrected his answer, using IRE sequences (see Table 6-1).
<table>
<thead>
<tr>
<th>Pedagogical Strategies</th>
<th>Learning about students’ thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Prompting students’ recall of algorithm or formula</td>
</tr>
<tr>
<td>2</td>
<td>Evaluating the answer right away (right or wrong) and making them write it down</td>
</tr>
<tr>
<td>3</td>
<td>Asking how students use the formula or algorithm</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Evaluating the answer right away</td>
</tr>
</tbody>
</table>

Ms. Lee’s pedagogical patterns and learning opportunities in the opening discussion (Do Now episode)

Ms. Lee’s pedagogical moves with respect to student thinking are more cyclical in nature. She started with big questions that elicited student mathematical thinking and reasoning. She then collected several students’ thinking and reasoning without immediately evaluating the answers. This gave Ms. Lee time to understand what students understood and where they had difficulties. This also provided students opportunities to push their thinking further, particularly for conceptual understanding and in articulating their thinking and reasoning. Then, Ms. Lee typically asked other students to rephrase or build on what previous students had said. In this process, Ms. Lee also collected their thinking without evaluation, occasionally offering scaffolding. These interactions provided Ms. Lee opportunities to understand additional students’ thinking and to see if other students followed the discussion. It also invited other students into the (productive) mathematical discussion. Once the key ideas had been elaborated in discussion, Ms. Lee narrowed down the mathematical content topic by asking more focused questions. After several such patterns, Ms. Lee brought things together by stating the punch line in her own words. She then began another pedagogical cycle with a big question.
### Table 6-16. Ms. Lee’s pedagogical patterns and learning opportunity (Do Now episode)

<table>
<thead>
<tr>
<th>Pedagogical Strategies</th>
<th>Learning about students’ thinking</th>
<th>Making use of students thinking (to productive mathematical discussion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Eliciting students’ mathematical thinking and reasoning with a bigger open-question</td>
<td>Teacher understands which parts of mathematics students understand or do not understand. e.g., Ms. Lee could understand three students’ thinking: Rina’s partial understanding of the problem, Nicole’s understanding the core idea of the problem, and Jamie’s partial understanding.</td>
<td>Students explain their thinking and reasoning qualitatively instead of answer-getting or procedural process types of ideas. e.g., Rina and Nicole explained their thinking.</td>
</tr>
<tr>
<td>2 Collecting students’ thinking without immediate evaluation</td>
<td>Students attempt to build on prior student’s explanation by talking after other students’ explanation. e.g., Rina tried to rephrase what Nicole said.</td>
<td></td>
</tr>
<tr>
<td>3 Asking (other) students to rephrase or to build on the elicited students’ talks</td>
<td>Students have an opportunity to understand the core idea that revealed by other student(s) and build on the ideas. e.g., Ms. Lee asked Nicole to repeat what she said about the core idea and allowed other students to ask to Nicole if they didn’t understand.</td>
<td></td>
</tr>
<tr>
<td>4 Collecting (other) students’ thinking without immediate evaluation</td>
<td>Students attempt to build on prior student’s ideas or expand the ideas. e.g.,</td>
<td></td>
</tr>
<tr>
<td>5 Narrowing the mathematical topic down or switch the discussion direction to the core idea for another cycle of discussion e.g., Ms. Lee asked students to think if “6% of $56 is $33.6” was making sense in terms of the dollar amount.</td>
<td>Teacher learning of students’ thinking of a particular idea maintains through next discussion about zoomed in the idea. e.g., Ms. Lee could still understand if students had sense-making the relationship between percent and decimals by asking if “6% of $56 is $33.6” made sense.</td>
<td>Teacher uses the revealed idea(s) to keep discussing the core mathematical idea. Students keep discuss on the core idea. e.g., Ms. Lee moves towards next question which is related to the idea that revealed by Nicole and related to the targeted mathematical goals in this lesson (the relationship between percent and decimal).</td>
</tr>
</tbody>
</table>
This pedagogical cycle creates multiple opportunities for Ms. Lee to learn about students’ thinking (see Table 6-2). For example, in the excerpt 6-7 below, Ms. Lee asked three students (Rina, Nicole, and Jamie) to explain why they thought the sample student work did or did not make sense. This created opportunities for Ms. Lee to understand and collect students’ reasoning. In this excerpt, Rina explanation of why the sample student work did not make sense was not clear. After that, Nicole’s explanation had a punch line why it did not make sense. Jamie tried to rephrase what Nicole talked but she seemed not to understand the mathematical ideas that they were discussing. From this conversation of three students, Ms. Lee could deepen her understanding of what her students understood and did not. Ms. Lee asked Nicole to repeat her idea that the sample student’s work contained confusion between 6% and 60%. This idea is correct and it was important for the other students to think about it more deeply. Ms. Lee narrowed down the discussion topic to whether it made sense for $33.60 to be 6% of $56. This intervention opened up another discussion cycle but deeply related to the core mathematical idea, the relationship between percent and decimal.

**Excerpt 6-7.** Ms. Lee: Do Now episode in percent-change lesson. They discuss the Day 1 Do Now problem in Chapter 5. Recall the problem,

```
What is the total cost of a jacket that sells for $56 from the factory but the store charges a 6% markup?

The selling price of $89.60 does/does not make sense because ____________.
```

In this excerpt, Ms. Lee elicited reasoning from three students without evaluating it right away. She narrowed the focus down to if $33 makes sense as 6% of $56, building on Nicole’s point that the sample student work referred not to 6% but 60%.

318 Ms. Lee: So now let’s talk about why does it or does not it make sense? I'd like to see other hands go up, hands that have not gone up before. I'm going to go Rina, Nicole, Jamie. I’m going to be calling on somebody else to kind of paraphrase what you’re hearing, summarize what they’re hearing. Go ahead.

322 R: I said that it doesn't make sense because they didn’t add correctly. They did the wrong problem. They just put the 0 plus 6 and they just dropped down the 6 instead of changing it to the tenth.

325 N: Um... they got it wrong because they thought 6 percent equals .6 instead of .06.

326 Ms. Lee: Jamie.
J: Oh, Nicole said that, um... instead of um... doing... like, they got the answer but they didn’t show any work. So then they just didn’t do the problem with multiply, the multiplication problem but the answers are similar.

Ms. Lee: Nicole, can you repeat what she just said I'd like everybody to listen to what Nicole said and if you don't understand then you can ask Nicole to explain some more. Go ahead, Nicole.

N: They got 6 percent equals .6 but it actually equals .06 because we switch .6 equals to 60%.

Ms. Lee: I want you guys to think about this. Can... let’s stop right here on this one. So, Nicole brought up a good point. Okay, and there’s confusion about 6% versus 60%. How do you change that to a decimal? Before we even talk about decimals I want you to think about... without even thinking about decimals... if I looked at 33 dollars for 6%, Does that make sense? Why or why not? I’m calling on people I haven’t heard from. Yancy and then Matt.

Summary

Through analysis on two teachers’ pedagogical patterns with regards to responding to student thinking, there are some pedagogical moves that create or limit teachers’ opportunities to understand student mathematical thinking. In general, the mathematical topics of this percent change lesson (e.g., a 50% increase from 100 to 150 represents a 33 1/3% decrease when one goes from 150 to 100; connections between decimals, fractions, and percents) are known to be challenging for students. In providing narrow algorithms, Ms. Janet deprived the students of the mathematical connections that are important, and deprived herself of opportunities to reflect on student thinking.

In contrast, Ms. Lee provided opportunities for the students to explore their understandings, thus making connections and grappling productively with misconceptions; in working through the various ways students had of thinking about the mathematics, Ms. Lee had the opportunity to refine her understanding of student thinking and, possibly, to make more connections herself regarding the mathematics.
Chapter 7. Teacher Change in Everyday Teaching Practices

In Chapters 5 and 6, I described how two teachers implemented FALs differently and how their interactions with curricular materials (Chapter 5) and their pedagogical strategies (Chapter 6) created opportunities for them to learn about student thinking and content. In this chapter, I focus on whether and how the two teachers’ regular teaching changed toward being more responsive to student mathematical thinking.

In order to capture changes in everyday teaching practices toward being responsive to student thinking, I developed a coding scheme as discussed in Chapter 4 (see Tables 4-1 and 4-3). In this chapter, I apply the coding schemes to the video observation data of the two teachers’ regular lessons at the beginning and end of the year. In addition to the quantitative coding analysis, I also conducted qualitative analyses of the video data to augment the quantitative results.

Together, the analyses capture whether there were changes in the two teachers’ teaching practices when they implemented regular, non-FAL lessons, such as whether the teachers shifted from teacher-centered moves to student thinking-centered moves. As described in Chapter 4, regular classroom observation video data were chunked into four major episodes: Do Now activity, Small Group work, Student presentation, and Lesson Conclusion. I compare and contrast each teacher’s two categories of episodes in their regular teaching: the Do Now episode and the Lesson Conclusion episode. First, I compare both teachers across the two years of the study to show that their pedagogical moves are similar at the beginning of the year, but show different trajectories of change over time. Then, I analyze how each of the teachers changes in practices over one year in more depth by providing concretes examples of their moves from classroom excerpts. Finally, I examine teacher interviews to illustrate how each teacher reflected on her own teaching in order to better understand the contexts for their change in practices.

To foreshadow the results, overall, Ms. Janet did not demonstrate significant changes in her regular teaching, while Ms. Lee showed significant changes toward being more responsive to student mathematical thinking.

7-1. Do Now Episode in Two Teachers’ Regular Teaching

The Do Now activities were created by each teacher and deeply integrated into their teaching practices. Each teacher’s Do Now activity served a similar function and purpose in their classroom, as described in Chapter 5. First, as a matter of classroom management, both teachers mentioned that having students work individually on Do Now problems served as a warm-up for students at the beginning of the lesson. It also allowed teachers time to go around the classroom and give students credit for homework while they worked on the problem. Second, mathematically, both teachers described Do Now problems as opportunities for students to recall previously learned concepts or procedural skills relevant to the upcoming lesson. Thus, student performance on these problems allowed both teachers to learn what kinds of skills or concepts students could or could not do before the lesson.

13 Lesson Conclusion episode consists of whole class discussion, sometimes combined with student presentation time.
began in earnest. Third, the Do Now activity includes a whole classroom discussion, which captures how the teachers elicit and make use of student thinking, as well as how they facilitate productive mathematical discussions. Fourth, the Do Now activity is integral to the teachers’ teaching practices throughout their career. Therefore, analyzing Do Now episodes allow for a systematic comparison, both between teachers (i.e., Ms. Lee vs. Ms. Janet) and between two points in time for each teacher (i.e., at the beginning of the year vs. at the end of the year). In addition, because Do Now activities are fairly routine in terms of content, variation in practices are less likely to be due to variance in content. That is, if a teacher showed changes in their practices during Do Now episodes, it may be a more powerful evidence of changes in their teaching practices.

A quantitative comparison of both teachers’ Do Now activity between the beginning and the end of the year reveals interesting differences between the teachers. Overall, the coded pedagogical moves of both teachers at the beginning of the year show some similarities. However, at the end of the year, the teaching practices demonstrated during the Do Now activity changed for Ms. Lee, but not for Ms. Janet.

At the beginning of the year, both Ms. Janet and Ms. Lee were observed to demonstrate similar patterns in their teaching practices in their Do Now activity (see Table 7-1 and Figure 7-1). Neither demonstrated much student-thinking centered practices, and neither elicited student thinking and reasoning or promoted productive discussion with much frequency. In Ms. Janet’s case, 82.61% of pedagogical moves were teacher-centered (codes A and B) or consisted of asking procedural questions (code C). I describe these codes A, B, and C as teacher-centered moves and codes C, D, and E are referred as student thinking responsive moves. In Ms. Lee’s case, 72.50% of pedagogical moves are coded as teacher-centered at the beginning of the year.

Table 7-17. Both teachers’ beginning of the year observation (Do Now episodes)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Ms. Janet: Do Now in September (Beginning of the year) (Total Duration: 05:29)</th>
<th>Ms. Lee: Do Now in October (Beginning of the year) (Total Duration: 13:19)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency&lt;sup&gt;15&lt;/sup&gt;)</td>
<td>Duration (% of time)&lt;sup&gt;16&lt;/sup&gt;</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>6 (13.04%)</td>
<td>1:00 (27.27%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>15 (32.61%)</td>
<td>0:55 (25.00%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>17 (36.96%)</td>
<td>1:14 (33.64%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and Strategies</td>
<td>3 (6.52 %)</td>
<td>00:11 (4.30%)</td>
</tr>
</tbody>
</table>

<sup>14</sup> The chi-square statistic is 3.52, df=5, and the p-value is 0.62. This is not significant difference at p < .05.
<sup>15</sup> “% of frequency” refers the ratio of the each code’s coded discourse to the whole coded teacher discourse.
<sup>16</sup> “% of time” refers the ratio of the each coded duration to the whole duration of each episode.
Figure 7-25. Comparison of Do Now activity teacher practices at the beginning of the year

However, at the end of the year, the trajectory of the two teachers’ teaching practices diverge\(^\text{17}\) (see Table 7-2 and Figure 7-2). Ms. Janet still demonstrated teacher-centered ways and asked procedural process questions (68%), while Ms. Lee’s teaching practices significantly changed towards being more student-thinking centered (47.37%), with teacher-centered ways and procedural process questions decreasing from 72.5% to 52.63%.

Table 7-18. Both teachers’ end of year observation (Do Now episodes)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Ms. Janet: Do Now in regular lesson in May (End of the year) (Total Duration: 5:24)</th>
<th>Ms. Lee: Do Now in regular lesson in May (End of the year) (Total Duration: 8:19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Duration</td>
<td>Frequency</td>
</tr>
<tr>
<td>(% of frequency)</td>
<td>(% of time)</td>
<td>(% of frequency)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>8 (16.00%)</td>
<td>1:00 (27.27%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>14 (28.00%)</td>
<td>0:55 (25.00%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>17 (34.00%)</td>
<td>1:14 (33.64%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and</td>
<td>5 00:11</td>
<td>10</td>
</tr>
</tbody>
</table>

\(^{17}\) The p-value is .065. df=5. The result is significant at p < .10.
I will now look closely at the coding results and provide qualitative contexts for each teacher and how individual teacher changed in their teaching practices.

**No Significant Change in Ms. Janet’s Do Now Episode**

**Broad contexts and lesson structure.** Ms. Janet’s school, James Middle school, had adopted the Springboard curriculum developed in the direction of Common Core Standards. Ms. Janet taught the Springboard curriculum three times a week on Mondays, Tuesdays, and Thursdays. The school had also adapted the Blended learning model, where students in a classroom split into two heterogeneous groups that rotated activities halfway through each lesson. One group participated in the lesson with the teacher while the other group worked independently on computational mathematical problems. However, every Wednesday, when the school was on a shortened period schedule, Ms. Janet had a problem solving day, where all students worked on a task consisting of contextual rich problems.

Ms. Janet’s regular teaching followed a consistent pattern. Her daily lesson started with making announcements, then presenting lesson objectives and Do Now problems on a board. Students wrote down lesson objectives and individually worked on Do Now problems for about 2 to 3 minutes. During that time, Ms. Janet checked her students’ homework. Then, she led a whole class discussion on the Do Now problems for about 5 to 10 minutes. What
followed after varied depending on whether it was a regular teaching day or a problem solving day.

On regular teaching days, students opened their textbooks, and Ms. Janet explained either a problem in the textbook or some concepts in the textbook. Students worked individually on problems for 4 to 5 minutes, and then shared their work with a partner for about 3 to 4 minutes. After that, each problem was presented by a different student in front of the classroom, followed by a whole class discussion led by Ms. Janet.

On problem solving days, Ms. Janet provided a lesson task, on which students worked individually for about 10 minutes. Students shared their work in groups of 3 to 4 students for about 10 minutes, then each problem was presented by 1 or 2 students in front of the classroom, followed by a whole class discussion led by Ms. Janet.

In this study, I analyzed two lessons on problem solving days—one at the beginning and the other at the end of the year—for several reasons. First, regular lessons covered content on a continual basis on a day to day basis due to the textbook structure, while a problem solving lesson covered content that was typically finished within a single day. In a given unit, the textbook contained about 20 problems meant to be completed in sequence, and each lesson picked up where the previous day’s lesson had ended. The self-contained problem solving lesson therefore facilitated the comparison of teaching practices across two time points. Second, problems assigned during problem solving days were richer than problems in their textbooks. As such, Ms. Janet had more leeway in terms of more independent decision-making or flexibility, making it more likely that any changes in her teaching practices would manifest. Third, Ms. Janet spent more time teaching students on problem solving days than regular teaching days since all students were participating at the same time, whereas on a regular teaching day, half of the students worked individually on computers at any given time. On regular lesson days, Ms. Janet taught the same content twice to two different groups of students for 30 minutes each. The students in each group consist differently in regular period of time, such as once a week or once a month. However, on a problem solving day, Ms. Janet teaches all of her students for 50 minutes. Therefore, it was more reasonable for the purposes of my study to observe her interaction with the same group of students for a longer period of time on problem solving days.

Lesson materials: Do Now problems and lesson tasks. The first problem solving lesson observation was in September and the final problem solving lesson observation was in May. Overall, the lesson materials were similar for both the beginning and end of year. Ms. Janet typically created Do Now problems that would give students opportunities to recall their concepts or procedures relevant for the day’s lesson. The Do Now problems of each lesson are shown in Figure 7-3 and Figure 7-4. The lesson tasks assigned to students for the problem solving days were taken from the Mathematics Assessment Resource Service (MARS)\(^{18}\) (see Figure 7-5 and Figure 7-6).

\(^{18}\) MARS is also a partnership between the Shell Center at University of Nottingham, UC Berkeley, and professional development providers as FALs. Some FALs are developed from MARS.
1. Solve: $-35 - (-15) = \ ?$
2. What is the average of -15, -5, and -10? 
3. What does change in temperature mean?

**Figure 7-27.** Do Now problems in September regular lesson (Ms. Janet, problem solving day)

1. What is the area formula for a circle? 
2. What is the area of a circle with a diameter of 6cm? 
3. What is the area of a circle with a radius of 6ft?

**Figure 7-28.** Do Now problems in May regular lesson (Ms. Janet, problem solving day)

<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-17°</td>
<td>-8°</td>
<td>10°</td>
<td>2°</td>
<td>-16°</td>
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<td>-15°</td>
<td>-4°</td>
<td>-28°</td>
<td>-6°</td>
<td>10°</td>
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</tbody>
</table>

1. Which date recorded the lowest daily temperature in the month? ________
   What was the temperature? ________ Degrees Fahrenheit.
2. What is the difference in temperature between Saturday Feb. 14 and Monday Feb. 23? ________
   Show how you determined your answer.

**Figure 7-29.** Sample lesson task from the regular lesson in September (Ms. Janet, problem solving day)
Pedagogical moves: coding results. In order to capture and understand whether or not and how Ms. Janet’s pedagogical moves changed, I coded the Do Now episodes and quantified the results using the coding scheme. Overall, Ms. Janet’s pedagogical moves did not show significant change between the two regular lesson teaching Do Now episodes (see Table 7-3 and Figure 7-7).

At the beginning of the year, Ms. Janet’s questioning patterns mostly consisted of asking short answerable questions to get the right answer (37%, see Table 7-3, Category C) and evaluating if the answers were correct (33%, Category B). Her students were not provided with many opportunities to advance their mathematical thinking through the use of various strategies and reasoning. Only 6.5% of total coded teacher discourse by Ms. Janet were soliciting ideas and strategies, and 4.4% were eliciting reasoning. Even though some students talked about their ideas beyond the description of procedural processes, Ms. Janet did not use those ideas as resources for discussion. Instead, she asked those students to use a formula, algorithms, or procedural process to get the right answers (see excerpt 7-1 for an example).

At the end of the year, Ms. Janet’s teaching practices did not exhibit significant change. The ways of questioning and the kinds of questions that Ms. Janet asked were similar to those at the beginning of the year (see excerpt 7-2 for an example). A large portion of questions she asked continued to consist of category C: questioning without eliciting reasoning or thinking (34%, see Table 7-3), rather than eliciting student reasoning (8%) and

---

Carlos bought a picture frame with a card insert to put a photo into. The insert she chose had a rectangular hole with a quarter-circle, radius 4 cm, in each corner. The measurements are shown in the diagram.

The area of a circle is \( \pi r^2 \), where \( r \) is the radius of the circle.

1. Find the area of a circle with a radius of 4 cm.
2. Find the area of a quarter-circle with a radius of 4 cm.
3. What is the area of the entire picture frame?
4. What is the area of the photo that shows through the hole in the insert. Show your calculations.
5. What percentage of the whole area of the picture frame is the part of the photo that can be seen? Show how you figured it out.

Figure 7-30. Sample lesson tasks from the regular lesson in May (Ms. Janet, problem solving day)
strategies (10%). Ms. Janet’s pedagogical moves did not seem to promote productive discussions.

Table 7-19. Frequencies and Duration of Coded Teacher Discourse (Ms. Janet’s case)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Do Now in regular lesson in September (Beginning of the year) (Total Duration: 05:29)</th>
<th>Do Now in regular Lesson in May (End of the year) (Total Duration: 5:24)</th>
<th>Differences between Two Episodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency)</td>
<td>Duration (% of time)</td>
<td>Frequency (% of frequency)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>6 (13.04%)</td>
<td>1:00 (27.27%)</td>
<td>8 (16.00%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>15 (32.61%)</td>
<td>0:55 (25.00%)</td>
<td>14 (28.00%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>17 (36.96%)</td>
<td>1:14 (33.64%)</td>
<td>17 (34.00%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and Strategies</td>
<td>3 (6.52 %)</td>
<td>00:11 (4.30%)</td>
<td>5 (10.00%)</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>2 (4.35%)</td>
<td>0:05 (1.21%)</td>
<td>4 (8.00%)</td>
</tr>
<tr>
<td>F. Promoting Productive Discussion</td>
<td>3 (6.52 %)</td>
<td>0:18 (8.18%)</td>
<td>2 (4.00%)</td>
</tr>
</tbody>
</table>

Figure 7-31. Comparison of Ms. Janet’s teaching practice profiles at the beginning and the end of the year

Excerpt 7-1. Ms. Janet: Do Now episode in September (beginning of the year)  
Ms. Janet’s questioning style is more like IRE sequencing types of questions and short answerable questions like procedural process, rather than eliciting student reasoning.  
In order to represent how I coded data, here I bring color coding in excerpts in this
Ms. Janet: Number three, Tom, what does change in temperature mean? Go ahead.

Tom (T): Um, the distance between a beginning number and ending number.

Ms. J: Okay, everyone, write that down if you had nothing. Change in temperature is the distance, okay, between, what did you say? It’s the distance between what?

T: Distance between a beginning number and ending number

Ms. J: Okay, it's the distance between the beginning number and the ending number. And when you're talking about distance, what are you talking about? Distance on what?

T: Number line

Ms. J: Yeah, distance on the number line, so that's one way to figure out change in temperature. Remember we talked about that? When you think about the distance between two numbers on a number line, isn't that the difference? Right, or how much it changes?

Sts: Yes

Ms. J: Yes, okay, this is the definition and this is how you can figure it out on the number line, right? For example, what's the change in temperature if it goes from 6 to 9, guys?

Sts: 3

Ms. J: Yeah, positive 3, positive 3. Right, it's going up three. But what if I said, what's the difference between 9 to 6? If it went from 9 to 6?

Sts: 3

Ms. J: Except for, from 9 to 6 instead of going up, it's going down, so it would be?

Sts: Down

Ms. J: Down, so it would be?

Sts: Negative 3

Ms. J: Negative 3 cus it's getting colder, right? So keep that in mind when you guys are working on this activity. Um, excellent job, those that participated.

Excerpt 7-2. Ms. Janet: Do Now episode in May (end of the year)

Ms. Janet’s questioning styles and types still demonstrate IRE sequencing. Questions are mostly like short answer gathering types such as memorizing formula and plugging numbers in formula. She pushes students to memorize and to use formula for district benchmark tests (see Figure 7-6 to see the problem discussed in excerpt).

Ms. Janet: Alright, so everyone should know the answer to number 1. Why?

Sts: It's on the board [the formula is written on the board in front].

Ms. J: Yes, it's on the board. Okay, so, let's see, Aron, Aron, what's the area formula for a circle?

Aron: The area for a circle formula is a, area equals pi times radius squared.
Ms. J: Yes, pi times radius squared. Raise your hand if you know that. Okay, about half of you. Okay, it's, do you think I want you to memorize that formula?

Ms. J: Yes, I do. But the good news is on the benchmark they actually give you the formula, but if you don't know how to use it, is that formula useful?

Ms. J: No, it's only useful if you know how to use it. Okay, let's see if you know how to use it. Number 2, what is the area of a circle with the diameter of 6? And if you've done nothing 'cause you're not sure, do you think you should be writing this down?

Ms. J: Yeah. Okay, only 2 people know how to do this. 3 people, 4 people, 5 people, Okay, Dan?

Dan (D): The answer or the equation?

Ms. J: Oh, explain to me what to do because some people don't know how to use the formula. Like what I realized in my other class is that a lot of people have this formula memorized, but they don't really understand what it means or like how you use it. Okay, so can you explain what you did exactly? From the beginning.

Dan (D): First I divided the diameter by 2, which gave me the radius, which is 3, and then multiplied 3 times 3 and got 9 and I multiplied 9 by pi which is 3.14 and I got 28.26.

Ms. J: Okay, 28.26 what?

Dan (D): Is the area of a circle.

Ms. J: Yeah, but what’s the units? What are the units?

Dan (D): Centimeters squared.

Ms. J: Good. Okay, so does anyone know- good explanation, Dan.
In sum, Ms. Janet’s Do Now episode does not change significantly in terms both of quantitative analysis (coding results) and of qualitative analysis (excerpt). The questioning moves still focus mostly on procedural process types of questions or recalling information such as formulas from memory. There are not many opportunities for students to develop their reasoning or promoting productive discussion.

**Significant Change in Ms. Lee’s Do Now Episodes.**

**Broad contexts and lesson structure.** Ms. Lee’s regular teaching also followed a consistent pattern. Her lessons typically began with “Do Now” problems projected on a board. Students worked individually on the Do Now problems for about 5-7 minutes while Ms. Lee checked the students’ homework. Then, Ms. Lee led a whole class discussion on the Do Now problems for about 10-15 minutes, followed by projecting a textbook task. Students worked individually on the textbook task for about 3-5 minutes. The lesson task usually required students to write in their journals to reflect on the day’s mathematical goals, expressing their confusions, questions, and some mathematical skills. The lesson itself then began in earnest. While the structure would vary depending on the tasks and activities, the lesson generally consisted of students working in small groups, followed by a whole class discussion to wrap up the small group work.

In this section, I analyze two regular lesson observations of Ms. Lee. Ms. Lee’s first regular lesson observation was in October, and the final regular lesson observation was in May. The two regular lessons’ structure generally followed the description above. However, student presentations were introduced in the May lesson at the end of the year. This new structure was not observed prior to May. It is possible that Ms. Lee introduced the new structure because FALs encourage teachers to use student presentations as opportunities for them to better understand student thinking and for students to articulate their ideas and to critique the ideas of others. Ms. Lee included student presentations in two out of the three FALs implemented over the year.

**Lesson materials: Do Now problems and lesson tasks.** The first observation of Ms. Lee’s regular lesson was also conducted in October and the final regular lesson observation was in May. Like Ms. Janet, Ms. Lee usually created Do Now problems that would help her students recall their previous knowledge and skills relevant to the topic of the day. The Do Now problems of each lesson (the first and the last observation) are shown in Figure 7-8 and Figure 7-9.

1. Place the following probabilities on the number line.
   \[ P(\text{red}) = \frac{1}{2} \quad P(\text{blue}) = \frac{1}{6} \quad P(\text{black}) = \frac{1}{4} \quad P(\text{green}) = 8.3\% \]

\[ \begin{array}{cccc}
0 & \quad & \quad & 1 \\
\hline
\end{array} \]

2. What is the probability of getting either a black or red marble? (use 2 different methods)

**Figure 7-32.** Do Now problems for the regular lesson in October (Ms. Lee)
1. Write the inequality and graph (underline or circle key words first)
A store only sells chocolates and candies in bulk. Chocolate costs $1.25 per pound and candy costs $0.50 per pound. You already bought 4 pounds of chocolate. If you want to spend at least $8.00 have many pounds of candy can you buy?
Inequality: __________________
Graph

2. Which expressions are equivalent to 2(x+3)? Explain using a model or words
   a) 2x+3       b) x+x+3+3       c) 2x+6       d) 2x+2(3)

**Figure 7-33.** Do Now problems for the regular lesson in May (Ms. Lee)

The two sets of Do Now problems were similar in a structural way and functions. On the other hand, the actual lesson materials differed between two observations. At the beginning of the year, Ms. Lee mostly followed the textbook. The textbook problems, like Ms. Janet’s textbook problems, involved basic concept development word problems (see Figure 7-10).

Lindsay has a paper bag full of Fruiti Tutti Chews in three different fruit flavors. She says, “If you reach into the bag, you have a \( \frac{1}{3} \) chances of pulling out a Killer Kiwi. There is \( \frac{3}{5} \) chance that you will get Crazy Coconut.”

a. If you reach into the bag, what is \( P(\text{coconut or kiwi}) \)?
b. Does there have to be another flavor in the bag? How can you tell? If so, assuming that there is only one other flavor, what is the probability of getting that flavor?
c. How many candies might Lindsay have in the bag? Is there more than one possibility? Assume that all candies in the bag are whole candies.

**Figure 7-34.** Lesson tasks for the regular lesson in October (Ms. Lee)

However, at the end of the year, Ms. Lee modified textbook problems and created a new type of lesson material (see Figure 7-11). This lesson material contained some sample work from her actual students, in effect incorporating her own students’ thinking into the task itself. In addition, the tasks required students not only to arrive at answers or understand simple concepts, but also to analyze other students’ reasoning and critique their ideas by asking for more elaboration or expressing (dis)agreement with the other students’ reasons. For example, in Figure 7-11, the tasks ask students not only to analyze the answers from the other students (Students A and B), but also to analyze their methods. This provided students with opportunities not only to figure out the answer, but also to develop their mathematical reasoning by analyzing the other students’ methods and arguing for or against these methods. This marks a significant shift between the materials used in the first and last regular lessons.
Lesson: Painting the Wall

How can you improve this work?

- Jose is asked to paint a wall that has a length of 80 feet and a width of 30 feet.

Jose says, “I painted the wall in one hour and forty minutes. So tell me, how many square feet per minute is that?

A. 24 square feet/minute  
B. 30 square feet/minute  
C. 40 square feet/minute  
D. 2400 square feet/minute

Student B: answer is B.

Student A: answer is A.

- Jabari invests $500 in his cousin’s advertising company. After only one month, he makes $125. He now has $625.

What is the percent profit he has made so far on his investment?

A. 10%  
B. 12.5%  
C. 25%  
D. 125%

Student A: answer is B.  
Student B: answer is C.

I agree/disagree because: ______________________________

Figure 7-35. Lesson tasks for the regular lesson in May (Ms. Lee)
Pedagogical moves: coding results. Ms. Lee’s teaching practice profile for Do Now episodes demonstrates significant change (see Table 7-4 and Figure 7-12).20 At the beginning of the year, as described above, Ms. Lee’s ways of questioning and the kinds of questions were similar to Ms. Janet’s. Ms. Lee asked more procedural process questions (35%) and evaluated right away after getting short answers from students (21%). Questions were not shown often as eliciting reasoning (only 3.8%). See Excerpt 7-3 for examples.

However, at the end of the year, Ms. Lee elicit student reasoning more frequently (16%, a 12% increase) and asked procedural process questions less frequently (22%, a 13% decrease). She asked more open questions prompting students to elaborate their thinking such as, “tell me how you got the answer” (see line 416 in Excerpt 7-4). She also promoted productive mathematical discussion more frequently (18%, an 8% increase). She attempted to support students in building on each other’s ideas with several strategies, such as asking to reframe what others say, to agree or disagree, and to add on what others say (see line 438-439 in Excerpt 7-4).

Table 7-20. Frequencies and duration of coded teacher discourse (Ms. Lee’s Do Now episode)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Regular Lesson in October (Beginning of the year)</th>
<th>Regular Lesson in May (End of the year)</th>
<th>Differences between Two Episodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency)</td>
<td>Duration (% of time)</td>
<td>Frequency (% of frequency)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>13 (16.25%)</td>
<td>1:11 (18.25%)</td>
<td>11 (14.47%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>17 (21.25%)</td>
<td>0:55 (14.14%)</td>
<td>12 (15.79%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>28 (35.00%)</td>
<td>1:38 (25.19%)</td>
<td>17 (22.37%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and Strategies</td>
<td>11 (13.75%)</td>
<td>0:47 (12.08%)</td>
<td>10 (13.16%)</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>3 (3.75%)</td>
<td>0:05 (1.29%)</td>
<td>12 (15.79%)</td>
</tr>
<tr>
<td>F. Promoting Productive Discussion</td>
<td>8 (10.00%)</td>
<td>0:53 (13.62%)</td>
<td>14 (18.42%)</td>
</tr>
</tbody>
</table>

The p-value is .058. df=5. The result is significant at p < .10.
**Excerpt 7-3. Ms. Lee: Do Now episode in October (Beginning of the year)**

This excerpt illustrates how Ms. Lee asked mostly known-answer questions or short-answerable procedural process questions after asking what students did with the problem, which the students answered without indicating their reasoning (see Figure 7-8). Ms. Lee continued to make evaluating statements, saying, “good” or “excellent” right after each student’s answer, and the discussion did not further advance students’ reasoning or accountability among students. As such, the discourse is indicative of an IRE sequence.

As in Ms. Janet’s excerpt, the following color coding is used below: A. Explaining (green); B. Restating/Evaluating/Clarifying (yellow); C. Questioning Procedural Process (purple); D. Soliciting Ideas and Strategies (Orange); E. Eliciting Reasoning (Blue); and F. Promoting Productive Discussion (Pink).

400  *Ms. Lee:* Okay, can you tell me what’s the percent for ¾?
401  *Student (S):* 75%.
402  *Ms. L:* Okay, now where do you think is 25%?
403  *S:* The 25…between 50% and 0.
404  *Ms. L:* Excellent. So, where do you think is the red then?
405  *S:* The red is in the middle, the 50%.
406  *Ms. L:* So, this is your red. So, where do you think is the black?
407  *S:* The black is between 5% and…
408  *Ms. L:* Where do you think is blue?
409  *S:* The blue is between 25 and 0.166…
410  *Ms. L:* Okay, so it’s going to be about… so this is going to be blue.
411  *S:* Where do you think is going to be green then?
412  *S:* Green is between blue and 0.
413  *Ms. L:* Good.
Excerpt 7-4. Ms. Lee: Do Now episode in May (End of the year)

This excerpt illustrates how Ms. Lee elicited students’ thinking, helped the students to elaborate their thinking, and promoted discussion. After asking a student, Chris, what he did, Ms. Lee pushed him to think about and articulate why and how he got the answer. Once he revealed that he did not know, Ms. Lee invited other students to join the discussion by asking other students what they thought about the student’s answer. She pushed other students to elaborate their thinking when their answers were terse. She also tried to promote discussion when students had different answers.

In the excerpt, they mainly discuss the second problem from Figure 7-9:

<table>
<thead>
<tr>
<th></th>
<th>2. Which expressions are equivalent to 2(x+3)? Explain using a model or words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>2x+3</td>
</tr>
<tr>
<td>b)</td>
<td>x+x+3+3</td>
</tr>
<tr>
<td>c)</td>
<td>2x+6</td>
</tr>
<tr>
<td>d)</td>
<td>2x+2(3)</td>
</tr>
</tbody>
</table>
Ms. Lee: Tell me which problem you did and what did with that problem.

Chris (C): I did number 2 and then I had it $2x+3$.

Ms. L: Tell me how you got that.

C: I don’t know.

Ms. L: So, do you think this is the answer [pointing and making a circle on a. $2x+3$]? What do you guys think?

[Several students raise hands and asking what they did.]

Rosie (R): I got d.

Ms. L: How did you get that?

R: Distributive property.

Ms. L: What do you mean by that?

R: I got $2x$ plus, $2$ times $3$, because.. I don’t know how to explain it.

Ms. L: I’m not saying that your answer is wrong, or correct. You could be vary. [Rosie finished her explanation with Ms. Lee’s guides]

Ms. L: That’s one way. Anyone else shows something else?

[several students raise hands]

Ms. L: Go ahead Yancy.

Yancy (Y): I chose c.

Ms. L: Can you tell me how?

Y: Well, $2$ times $3$ equals $6$, so if you already had that as $6$, then you could just do $2x+6$.

Ms. L: [to whole class] So, then, c and a, are they the same? $2x+6$ and $2x+3$ are same?

[students say yes and no together]

Ms. L: Okay why do you think c [pointing one student] and why do you think a [pointing another student]?

Comparison of Ms. Lee and Ms. Janet

Overall, Ms. Lee’s pedagogical moves in Do Now episode changed significantly toward being more student-thinking centered and promoting productive discussions, while Ms. Janet’s did not. At the beginning of the year, both teachers’ pedagogical practices were very teacher-oriented explanations and tended toward more procedural process types of questions. However, at the end of the year, Ms. Lee’s pedagogical moves demonstrated a significant shift toward being more responsive student thinking, with more instructional moves eliciting student reasoning and promoting productive discussions. In contrast, Ms. Janet’s instructional moves did not change significantly.

7-2. Small Group Episode in Two Teachers’ Regular Teaching

Small group episodes exhibit similar patterns of change and no change for the two teachers as did the Do Now episodes. Small group episodes provide an opportunity to closely examine how the teachers supported individual and group student work through their interactions with students, particularly through their formative feedback to students. Both teachers structured small group work or partner work time in their everyday teaching, which provided teachers with opportunities to observe and understand their students’ progress and challenges. They also provided opportunities for teachers to
orchestrate whole class discussions to conclude the lessons based on their observations of students during small group work.

Due to technological difficulties, Ms. Janet’s small group episodes were not codable. Although a systematic coding of Ms. Janet’s small group episodes was not possible, classroom observation data indicate that there were no significant changes in her pedagogical moves during small group episodes.

On the other hand, Ms. Lee’s questioning styles shifted notably between the two small group episodes as they did for the two Do Now episodes. In the October small group episode, Ms. Lee’s questions were more directive, and students talked more about procedures. For example, in the following exchange, students discuss how to calculate addition of fractions by focusing on common denominator, and Ms. Lee directs students to use one particular common denominator.

Ms. Lee: Are you guys sharing different methods?
S1: Yeah. How’d you get that?
S2: Me? I got the common denominator, and it’s 8. So it’s 4 eighths plus 4 eighths plus 8 eighths and 1 whole.
S3: How come it’s not a fourth? There’s 2 fourths.
S2: Oh, yeah. That, too. What? But then it’d still be a whole.
S1: Yeah.
Ms. Lee: So, it looked like you used a 4 as a common denominator and you used an 8. [to S4] What’d you use?
S4: I used 8, too.
Ms. Lee: Why don’t you try it with 4 then, what S3 said. See if you can use 4 as a common denominator, okay?

Ms. Lee also directed students in another instance:

Ms. Lee: Let’s work on right here. Okay, what common denominator did you use? 4? Did you use 4, too?
S5: I used 8.

Ms. Lee: So you multiply instead of addition so I’m going to change it to multiplication. [writing].

In the May small group episode, however, Ms. Lee’s questions were less directive in nature. Instead, she solicited students’ reasoning. For example, in the following excerpt, Ms. Lee asked “why” questions to have students explain their thinking and reasoning, and students also tried to make sense of other students’ reasoning on the by articulating their thinking.

Ms. Lee: Which one is correct? And why? Why?
S6: I would say A.
Ms. Lee: Okay, go ahead, why do you think it’s correct?
S6: I would say A because it covers the whole thing.

Ms. Lee: Okay, then why are they dividing by that 100?

S7: I don’t know, ’cause like eighty plus thirty does not equal 100.

S6: I thought B was wrong ’cause, how is it gonna do, how is it like 100 minutes
but then like he's gonna do 24 square feet per minute and then you finished
it in an hour and forty minutes but it's still...it doesn’t really make sense, it
doesn't like, yeah.

Ms. Lee walks to the new group.

Ms. Lee: Kay, which student is correct, A or B?

Students: A.

Ms. Lee: Explain in two sentences.

S8: On the paper?

Ms. Lee: Yeah, on the paper. But tell me, why do you think it should be...so I am
going to ask you guys, while some of you guys are writing, why do you
think it's A is correct?

S9: It's A cus um for student B, what they did, is they didn't multiply the two
numbers, they added them together and then I don' even understand how
they went back...so it seemed like they just guessed on it.

Ms. Lee: Okay, so then what can you do to A to improve him?

S8: A? For A?

Ms. Lee: Yeah, so then people can understand it better

In sum, although it was not possible to systematically compare the two teachers’ small group episodes, classroom observation data for Ms. Janet’s small group episodes and qualitative analyses of Ms. Lee’s small group episodes indicate that Ms. Lee’s pedagogical moves changed to student thinking responsive styles while Ms. Janet’s did not.

7-3. Lesson Conclusion Episode in Two Teachers’ Regular Teaching

Both teachers usually had lesson conclusion session as leading a whole class discussion at the end of each regular lesson. Ms. Lee’s lesson conclusion episode is similar to her Do Now episode in terms of her pedagogical moves—similar shifts were observed for her lesson conclusion episodes at the beginning and end of the year as for her Do Now episodes. At the beginning of the year, the lesson conclusion discussion was driven by Ms. Lee as was the case in the Do Now episode. At the end of the year, the lesson conclusion episode consisted of student presentations. Ms. Lee used student presentations as an opportunity not only for the presenters to expand their own thinking, but also for other students to have a productive discussion by asking other students to build on the presenters’ thinking.

In comparison to Ms. Lee’s pedagogical moves, Ms. Janet’s moves in the whole class discussion episodes are different from her own Do Now episodes because her lesson conclusion episodes typically consisted of student presentations. Usually, one or two students came to the front of the classroom and showed his/her work from his/her small group. Ms. Janet usually picked the student presenters if his/her work showed some possibility to be improved so that other students could correct his/her work by discussion led by Ms. Janet.
At the beginning of the year, both teachers tended toward teacher-centered or procedural process questioning styles of pedagogical moves. 80.70% of Ms. Janet’s moves and 66.67% of Ms. Lee’s moves consisted of codes A, B, and C (see Table 7-5 and Figure 7-13). Ms. Lee’s instructional moves were more teacher-oriented (teacher mainly explained and/or concluded the lesson, code A). Ms. Lee’s questioning styles show some procedural process or answer-getting (15.38%) and some soliciting students’ thinking and ideas (28.21%). While soliciting students’ thinking and strategies are student-thinking centered moves, Ms. Lee did not elicit student reasoning at all (0%). Ms. Janet asked some questions that were more than teacher-oriented explaining but the questions were mostly procedural process questions and right-away evaluations of students’ answers (35.09% for code B and 27.19% for code C). Ms. Lee’s solicitation of student ideas and student reasoning were only 3.51% and 8.77%, respectively. Both teachers did little to promote discussion (7.02% for Ms. Janet, and 5.15% for Ms. Lee).

Table 7-21. Both teachers’ beginning of the year observation (Lesson Conclusion episodes in regular lesson observations)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Ms. Janet: Lesson Conclusion in September (regular lesson) (Total Duration: 13:23)</th>
<th>Ms. Lee: Lesson Conclusion in October (regular lesson) (Total Duration: 09:07)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency)</td>
<td>Duration (% of time)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>21 (18.42%)</td>
<td>1:02 (14.76%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>40 (35.09%)</td>
<td>2:20 (33.33%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>31 (27.19%)</td>
<td>1:55 (27.38%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and Strategies</td>
<td>4 (3.51%)</td>
<td>0:18 (4.29%)</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>10 (8.77%)</td>
<td>0:42 (10.00%)</td>
</tr>
<tr>
<td>F. Promoting Productive Discussion</td>
<td>8 (7.02%)</td>
<td>0:43 (10.24%)</td>
</tr>
</tbody>
</table>
However, at the end of the year, the trajectory of two teachers’ teaching practices diverges significantly (see Table 7-6 and Figure 7-14). Ms. Janet’s pedagogical moves show a slight shift towards more student thinking responsive moves (-15.13% of code A, B, and C; +14.95% of code D, E, and F), but it is not statistically significant. On the other hand, Ms. Lee’s pedagogical moves show significant differences. Ms. Lee’s teacher-centered moves (code A, B, and C) decreased by 27.65% and her student thinking-centered (code D, E, and F) moves increased by 27.65%. I will now discuss each teacher in depth.

Table 7-22. Both teachers’ end of the year observation (Lesson Conclusion episode in a regular lesson)

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Ms. Janet: Lesson Conclusion in May (regular lesson) (Total Duration: 15:50)</th>
<th>Ms. Lee: Lesson Conclusion in May (regular lesson) (Total Duration: 11:25)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency)</td>
<td>Duration (% of total coded duration)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>10 (13.70%)</td>
<td>1:13 (28.85%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>23 (31.51%)</td>
<td>0:29 (11.46%)</td>
</tr>
</tbody>
</table>

21 Grouped codes as two: teacher-centered and student thinking responsive, that is, df = 1. Yates’ chi-square statistic is 6.571. Yates’ p-value is .010. It is significant at p < .05.
No Significant Change in Ms. Janet’s Lesson Conclusion Episode

The broader context of Ms. Janet’s school and general lesson structure of her teaching, as well as a description of the lesson materials, can be found in the previous Do Now episode section. Students worked in small groups on the provided lesson tasks (Figure 7-5 in September lesson; Figure 7-6 in May lesson) after the Do Now activity. After small group work, student(s) presented her or his works at the front and Ms. Janet led whole class discussion based on their presentation. I now provide Ms. Janet’s particular pedagogical moves and the coding results of her Lesson Conclusion episode.

**Pedagogical moves: coding results.** As indicated in the previous section, to trace and capture the changes in practices, I coded Lesson Conclusion episodes both at the beginning and at the end of the year and quantified the results using the coding scheme. As shown in Table 7-7 and Figure 7-15, Ms. Janet’s pedagogical moves make a minor
shift towards being more student thinking-centered. However, it is *not* a statistically significant change\(^{22}\).

Ms. Janet’s questioning styles and kinds of questions did not change significantly over the year. They still align with IRE sequencing or focus on procedural processes rather than attempting to elicit reasoning (see Table 7-7). Her eliciting reasoning types of questions were mostly when she asked a student presenter to explain his or her work. Classroom interactions mostly consisted of Ms. Janet asking procedural process questions and students responding with very short answerable questions or simple answers (see excerpt 7-5 for an example from the beginning of the year; see excerpt 7-6 for an example from the end of the year).

Table 7-23. Frequencies and duration of coded teacher discourse in Ms. Janet’s Lesson Conclusion episode

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Regular Lesson in September (Beginning of the year) (Total Duration: 13:23)</th>
<th>Regular Lesson in May (End of the year) (Total Duration: 15:50)</th>
<th>Differences between Two Episodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency (% of frequency)</td>
<td>Duration (% of time)</td>
<td>Frequency (% of frequency)</td>
</tr>
<tr>
<td>A. Explaining</td>
<td>21 (18.42%)</td>
<td>1:02 (14.76%)</td>
<td>10 (13.70%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/Clarifying</td>
<td>40 (35.09%)</td>
<td>2:20 (33.33%)</td>
<td>23 (31.51%)</td>
</tr>
<tr>
<td>C. Questioning without Eliciting Reasoning or Thinking</td>
<td>31 (27.19%)</td>
<td>1:55 (27.38%)</td>
<td>15 (20.55%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas and Strategies</td>
<td>4 (3.51%)</td>
<td>0:18 (4.29%)</td>
<td>4 (5.48%)</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>10 (8.77%)</td>
<td>0:42 (10.00%)</td>
<td>11 (15.07%)</td>
</tr>
<tr>
<td>F. Promoting Productive Discussion</td>
<td>8 (7.02%)</td>
<td>0:43 (10.24%)</td>
<td>10 (13.70%)</td>
</tr>
</tbody>
</table>

---

\(^{22}\) p-value = .34, df=5. This result is *not* significant at p < .05.
Excerpt 7-5. Ms. Janet: Lesson Conclusion episode in September. One student, Tom, came at the front of the classroom and showed his work, but he just reported his answer instead of explaining how he reasoned his way through the problem. However, Ms. Janet did not wait until Tom explained his thinking. Rather, she explained how she thought he got the answer (line 10-14). His answer did not seem to meet Ms. Janet’s goal, which was to have students use the equation to calculate temperature change, so the discussion became an IRE sequence where students were prompted to recalling and memorize the algorithm. In addition, Ms. Janet’s instruction mostly purely procedural.

To recall the color coding: A. Explaining (green); B. Restating/Evaluating/Clarifying (yellow); C. Questioning Procedural Process (purple); D. Soliciting Ideas and Strategies (Orange); E. Eliciting Reasoning (Blue); and F. Promoting Productive Discussion (Pink).

They discuss number 3 of the lesson task from the earlier Figure 7-5 as follows:

![Chart indicating the daily lowest temperature for each day of February in Fargo, ND. Temperatures are in Fahrenheit.](chart.png)
3. What is the greatest change in temperature between two consecutive days? Show how you figured it out.

Ms. Janet: Okay, Tom, come on up I want you to show yours. Okay, for number three. Okay, we're almost done, so hang in there. I want a few more people. Please stop talking- I want a few more people to talk about this. [Ms. Janet asked students what consecutive means and checked their understanding of the term]

So, Abel, explain what you did here.

Tom (T): For number three, I chose February 8th through February 9th.

Ms. J: Okay, revise that, Grace, on your paper. Go ahead.

T: And I got the temperature change was 29 degrees.

Ms. J: Yes because you are subtracting, right? Between these two. So put a subtraction sign. Just add a subtraction. February- now, remember, in the very beginning of class, guys, I talked about something. It got erased, but change in temperature. It's the distance between the two numbers, correct?

Does anyone remember the equation for change in temperature? Does anyone remember the equation for change in temperature? Ron?

Ron (R): Temperature minus the ending.

Ms. J: Yes, he's on the right track. What temperature?

---

23 The question itself seems to be very similar to the one Ms. Lee asked, “tell me what you did” and this question in Ms. Lee’s case is mostly coded as “D. Soliciting Ideas and Strategies.” However, the reason why I coded it as “C. Questioning Procedural Process” rather than “D. Soliciting Ideas and Strategies” is the function of the question is more to get a short answer or focuses on procedural process types when we look at students’ responses to the questions.
Sts: Beginning temperature minus...

S2: Ending temperature.

Ms. J: Okay, close, beginning temperature minus ending temperature, is that right?

Sts: No.

Ms. J: It's the other way around.

S2: Beginning temperature minus the change.

Ms. J: No, other way around.

S2: Beginning temperature minus the ending temperature

Ms. J: Close. It's not beginning temperature minus ending temperature. It's...

Sts: Ending temperature minus beginning temperature.

Ms. J: No, this is the change in temperature, right? We're figuring out the change in temperature. The change in temperature is 29 degrees, right? Hello?

Sts: Mm hmm.

Ms. J: So it's the ending, so everybody on your paper write- again, you should be writing this down, okay, this is the ending temperature, right? It ends at negative 21, doesn't it?

Sts: Yeah.

Ms. J: So write underneath the negative 21, write ending temperature. Everybody write this down. Ending temperature. Minus 8 degrees. Isn't 8 degrees on February 8th? Yeah that's where it began, so minus the beginning temperature. So label 8 degrees, that's your beginning temperature. Ending temperature minus beginning temperature. So, one person, what is the formula for the change in temperature? I want you to get this out of this class. What is the formula for change in temperature? Only two people are raising their hand and you all just wrote it down. Jane.

Jane (J): Ending temperature minus beginning temperature.

Ms. J: Okay, is the formula for what?

J: For the temperature change.

Ms. J: Yes, okay. I will ask you that tomorrow on your Do Now. Do not forget it.

Excerpt 7-6. Ms. Janet: Lesson Conclusion episode in May.

In this excerpt, one student, Megan, projected her work and explained how she got the answer. She was able to articulate her thinking (lines #). After Megan repeated her line of reasoning, Ms. Janet asked another student, A, to repeat what Megan said. This seemed to promote discussion and the mathematical content that revealed through asking “why” seemed to be student reasoning. However, the way in which Ms. Janet supported students’ reasoning was very procedural in that she simply prompted both the presenter and other students repeat their reasoning instead of trying to reframe or re-explain using their own words.

To recall the color coding: A. Explaining (green); B. Restating/Evaluating/Clarifying (yellow); C. Questioning Procedural Process (purple); D. Soliciting Ideas and Strategies (Orange); E. Eliciting Reasoning (Blue); and F. Promoting Productive Discussion (Pink).
In the following excerpt, they discuss number two in the lesson task below:

Carlos bought a picture frame with a card insert to put a photo into. The insert she chose had a rectangular hole with a quarter-circle, radius 4 cm, in each corner. The measurements are shown in the diagram.

The area of a circle is $= \pi r^2$, where $r$ is the radius of the circle.

1. Find the area of a circle with a radius of 4 cm.
2. Find the area of a quarter-circle with a radius of 4 cm.

---

Ms. J: Megan, you can come up for number 2. I'm not sure what your method was, but we'll see.

[One student asked to go to bathroom and then they talked about off topic]

Megan (M): So first, they said, I had read the problem, it said find the area of a quarter circle with a radius of 4 centimeters. So first I used my work from the first problem and it’s 50.24, so I did 50.24 divided by 4 because it's looking for a quarter of it and I got 12.56 centimeters squared.

Ms. J: Okay, explain it one more time and then I’m gonna have someone else explain it. Someone, go ahead.

M: I read the problem and it said find the area of a quarter circle, which means 1/4 with radius of 4 centimeters so I used my work from the first problem and I did 50.24 divided by 4 and I got 12.56 centimeters squared.

Ms. J: Mm okay, so, questions before I have someone else explain it, are there any questions on that? No? Okay, so Ron, I know your group did it differently, who else was in your group? Okay, Austin, I know your group did it a different way, but can you re-explain Megan’s way? What did she do?

Austin (A): She multiplied 3.14 by 16 and then she got 50.24 and she divided it by 4 and got 12.56.

Ms. J: Yup, and then why did she divide it by 4?

A: Because the radius is 4 centimeters?
Ms. J: Okay, the radius is 4 centimeters, but that’s not why she divided by 4. Megan, why did you divide it by 4?24
M: Because it was a quarter circle, which means 1/4 of a circle.
Ms. J: Why did she divide it by 4?
A: Because the area of a circle?
Ms. J: No, say it again Maria, why did you divide by 4?
M: Because it's a quarter circle which means 1/4 of the circle.
A: Oh, it's a quarter circle.
Ms. J: Which means...
A: Which is 1/4 of the circle.
Ms. J: Yeah.

24 This “why” question functions not much as eliciting reasoning but as controlling the discussion to get answers directly what Ms. Janet thinks counts. The first “why” question in line 20 may has a function of eliciting reasoning so the codes are different between the first question and the later questions.
In sum, Ms. Janet’s pedagogical moves in the Lesson Conclusion episode did not change significantly over the year [stats?]. Qualitatively, there are some pedagogical moments that qualitatively different between two episodes. In the Lesson Conclusion episode at the end of the year, Ms. Janet solicited more student opinions and gave the students more opportunities to explain their work. However, Ms. Janet was still very directive, gave right away evaluations, and focused on the right answer (see lines 22 and 27). She also focused students on procedures (see lines 25-32). Thus, while students had more opportunities to express their ideas, Ms. Janet remained answer-oriented and focused the students on what she felt was important, the procedures.

**Significant Change in Ms. Lee’s Lesson Conclusion Episode.**

The broader context of Ms. Lee’s school and general lesson structure of her teaching, as well as a description of the lesson materials, can be found in the previous Do Now episode section (see Figure 7-10 for October lesson; and Figure 7-11 for May lesson above). In this section, I provide coding results of Ms. Lee’s Lesson Conclusion episode to capture the differences and changes of her pedagogical moves, particularly the shifts toward being more responsive to student mathematical reasoning and promoting productive mathematical discussions.

**Pedagogical moves: coding results.** Ms. Lee’s teaching practice profiles in Lesson Conclusion episodes demonstrate significant change [stats here, not in footnote, ()]. At the beginning of the year, a large percentage of her instructional moves was teacher-oriented explanations (38.46%, see Table 7-8). Her questioning styles also focused on procedural processes, which resulted in her students also focusing on procedures in their responses (see excerpt 7-7 for example).

However, at the end of the year, the teacher-oriented explanation decreased by 26.27% to only 12.20%. Instead, student-thinking centered instructional moves, such as, eliciting reasoning and promoting productive mathematical discussion, increased. Code E, Eliciting student reasoning, increased by 21.95%, and code F, promoting productive mathematical discussion, increased by 16.82%. For example, in excerpt 7-10, Ms. Lee asked open-ended questions, which provided students with opportunities to explain their thinking and reasoning. When a student wanted to use an algorithm, Ms. Lee pushed his reasoning by asking “why.” Ms. Lee also respected non-algorithmic thinking expressed by another student, and she used it as an opportunity for other students to further their discussion.

The summary of this change is represented well in Table 7-8 and Figure 7-16.

**Table 7-24.** Frequencies and duration of coded teacher discourse in Ms. Lee’s lesson conclusion episode

<table>
<thead>
<tr>
<th>Type of Teaching Practices</th>
<th>Regular Lesson in October (Beginning of the year) (Total Duration: 09:07)</th>
<th>Regular Lesson in May (End of the year) (Total Duration: 11:25)</th>
<th>Differences between Two Episodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Duration</td>
<td>Frequency</td>
</tr>
</tbody>
</table>

---

25 To note, I will discuss Ms. Janet also started to learn and change in her practices even though it is not statistically significant in Chapter 8.

26 p-value = .002. The results is significant at p < .05.
<table>
<thead>
<tr>
<th></th>
<th>(% of total coded moves)</th>
<th>(% of total coded duration)</th>
<th>(% of total coded moves)</th>
<th>(% of total coded duration)</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Explaining</td>
<td>15 (38.46%)</td>
<td>1:57 (53.42%)</td>
<td>5 (12.20%)</td>
<td>1:13 (28.85%)</td>
<td>-10 (-26.27%)</td>
</tr>
<tr>
<td>B. Restating/Evaluating/</td>
<td>6 (12.82%)</td>
<td>0:24 (10.96%)</td>
<td>6 (12.20%)</td>
<td>0:29 (11.46%)</td>
<td>-1 (-3.19%)</td>
</tr>
<tr>
<td>Clarifying</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0:05 (.50%)</td>
</tr>
<tr>
<td>C. Questioning without</td>
<td>6 (15.38%)</td>
<td>0:28 (12.79%)</td>
<td>5 (12.20%)</td>
<td>0:15 (5.93%)</td>
<td>-1 (-3.19%)</td>
</tr>
<tr>
<td>Eliciting Reasoning or</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0:13 (-6.86%)</td>
</tr>
<tr>
<td>Thinking</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0:01 (-2.85%)</td>
</tr>
<tr>
<td>D. Soliciting Ideas</td>
<td>3 (15.38%)</td>
<td>0:40 (18.26%)</td>
<td>7 (17.07%)</td>
<td>0:39 (15.42%)</td>
<td>-4 (-11.13%)</td>
</tr>
<tr>
<td>and Strategies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0:01 (-2.85%)</td>
</tr>
<tr>
<td>E. Eliciting Reasoning</td>
<td>0 (0%)</td>
<td>0:00 (0%)</td>
<td>9 (21.95%)</td>
<td>0:41 (16.21%)</td>
<td>9 (21.95%)</td>
</tr>
<tr>
<td>F. Promoting Productive</td>
<td>2 (5.13%)</td>
<td>0:10 (4.57%)</td>
<td>9 (21.95%)</td>
<td>0:56 (22.13%)</td>
<td>7 (16.82%)</td>
</tr>
<tr>
<td>Discussion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0:46 (17.57%)</td>
</tr>
</tbody>
</table>

**Figure 7-40.** Comparisons of Ms. Lee’s teaching practice profiles at the beginning and the end of the year

**Excerpt 7-7. Ms. Lee: Lesson Conclusion episode in October**

Ms. Lee’s questioning tended to focus on procedural processes, which in turn focused students on procedural processes in their response. Some of her instructional moves were to solicit student thinking by asking, “how did you get the answer?” but the subsequent discussion became IRE sequences. She also invited another student, Denny, to add on the ideas expressed by a previous student, Tim, but such attempts to promote productive discussions only occurred twice in this episode.
As described in a previous section, the lesson task was as follows:

Lindsay has a paper bag full of Fruiti Tutti Chews in three different fruit flavors. She says, “If you reach into the bag, you have a \( \frac{1}{3} \) chances of pulling out a Killer Kiwi. There is \( \frac{3}{5} \) chance that you will get Crazy Coconut.”

a. If you reach into the bag, what is P(coconut or kiwi)?

b. Does there have to be another flavor in the bag? How can you tell? If so, assuming that there is only one other flavor, what is the probability of getting that flavor?

c. How many candies might Lindsay have in the bag? Is there more than one possibility? Assume that all candies in the bag are whole candies.

---

558 Ms. Lee: So tell me what to do, Ann? (to another student) Put your hand down. Go ahead.
559
560 Ann (A): Um… There’s…
561 Ms. L: Okay, tell me what I’m going to need.
562 A: One third. [The teacher is writing the numbers on the screen]
563 A: And three fifths.
564 [students raise their hands]
565 Ms. L: Ann can do it, so we’re going to be patient with each other.
566 A: Equals 14 over fifteen.
567 Ms. L: How’d you get that?
568 A: Because I used a common denominator with 15. Then I multiplied it to get 9 over 15 and by five to get five over fifteen. 14 over 15.
569 Ms. L: Okay, excellent work Ann, thank you for being patient and for doing this.
570 Tim, could you read B please?
571 Tim (T): [reading the problem b] Does they have to be another flavor in the bag?
572 How can you tell? If so, assuming there is only one other flavor, what is the probability of getting that flavor.
573 Ms. L: Okay, tell me now, what do you think?
114

T: Um, I don’t know but, it says at the top right there are three different flavors, I just wrote that.

Ms. L: Okay, because there are three different flavors, you think that there should be another one. Yes. That’s good, I’m glad you’re paying attention. That was a big hint. Yes, there is another flavor, okay? Now, Denny you want to add on?

Denny: Yeah. For instance, 14 out of 15 had to be two of those flavors. So there can only be one. [bell rings]

Ms. L: Okay, since 14 out of 15 flavors…. That’s not the whole bag. So there’s going to be another one. So what’s the probability of the other one?

Denny: 1 out of 15.

Ms. L: 1 out of 15 [writes], and you get?

Denny: 15 out of 15.

Ms. L: That’s because 14 out of 15 is not a whole bag.

Excerpt 7-10. Ms. Lee: Lesson Conclusion episode in May
In this excerpt, a student, Ian, presented his ideas at the front of the classroom. Ms. Lee asked more open-ended questions, which allowed students more opportunities to explain their thinking. After Ian expressed his confusion with a concept, Marty built on Ian’s thinking when explaining his own thinking. Marty’s explanation was not a typical mathematics way of thinking, but Ms. Lee understood and accepted his non-algorithmic thinking, and it became an opportunity for promoting a rich and productive mathematical discussion. Ms. Lee invited Felicia and Ian to the discussion by asking them if they agreed with Marty’s reasoning, which was an explicit guidance for productive discussion. While Ms. Lee’s conclusion statement for the class was not clear enough for students to understand the relationship among all the different numbers, Ms. Lee provided many opportunities for students to develop their conceptual understanding of the percent profit, and to expand their thinking of the proportional relationship instead of memorizing algorithms. Particularly, when Ian wanted to use an algorithm (saying “should I do math? Line 30), Ms. Lee pushed his reasoning instead.

To recall the color coding: A. Explaining (green); B. Restating/Evaluating/Clarifying (yellow); C. Questioning Procedural Process (purple); D. Soliciting Ideas and Strategies (Orange); E. Eliciting Reasoning (Blue); and F. Promoting Productive Discussion (Pink).

As described in a previous section, the lesson task was as follows:

Jabari invests $500 in his cousin’s advertising company. After only one month, he makes $125. He now has $625.

What is the percent profit he has made so far on his investment?
A. 10%  B. 12.5%  C. 25%  D. 125%
Ian presented his thinking and works but expressed some confusions or partial understanding with little connection between the increased amount, $125, and percent profit 25%. $125 comes to his partial understanding as 1.25% or 25%. And another student, Marty, wants to share his ideas.

Ms. L: Okay, I’m gonna ask Marty and then I’m gonna ask somebody else this question so please listen up. Okay, go ahead, Marty.

Marty: Alright so since 500, I compared that to 100 and then I knew that was 5 times more, but I saw 125 and that can go into 500. 5 times so I decided...

Ms. L: 4 times. Oh, 5 times, you’re right.

M: Yeah, so I decided to go backwards and then I did uh I got 125 divided by 5 and then I got 25. So that’s 25% and if you do it 4 times it’s 1/4 of 500.

Ms. L: Okay thank you Marty, that was really, actually I really liked the way you explained that. Can you sit down now please?

F: Okay, so what I did was I looked at the answers. I looked at the answers and then I decided that it was that one the 25 because I multiply it, 5 times and 25, so it’s 125.

Ms. L: So, what is the answer then?

F: It’s the same as that.

Ms. L: A, B, C, D et cetera.

F: It’s B [12.5%].

Ms. L: Okay so, Ian, Felicia is saying the answer is B. Can you respond to what Felicia has said?

I: Um, how?

Ms. L: No, no can you explain what you agree or disagree, why?

I: Oh, I disagree because it’s not exactly, like you have the same number, but he decimal is in the wrong place. Like it’s you move the decimal over to the

---

27 This statement is coded as B. Evaluation, but qualitatively it is very different than just evaluating as right or wrong.

28 This statement is also different from what Ms. Janet usually asked to other students to re-phrase or Ms. Lee did earlier. Those functioned more likely either simply repeat what other students did while this has a room for the student to join in the productive discussion.
right, I mean to the left over one, then it would be correct, but 12.5 is like for
500, should I do the math?

Ms. L: No let’s uh, let’s take a look, uh, Ian, you ended up saying that C is the correct
answer. Can you explain why the C is the correct answer? Why is the C the
correct answer?

I: Alright, so, 25% of 500 is 125. If 125 was the increase, then it’s gonna be.
And since its 625, it would be...

Ms. L: So increase of 125. So, look at right here. So 25% of 500, so what Ian is
saying is 25% is 1/4. So 1/4 of 500. If you divide 500 by 4, you get 125. Since
from 500 to 625 we increase by 125, you’re gonna get an increase of 25%.
Kay, so increase of 25%. That's what profit, percent profit is about. They're
asking you how much did it increase by.

In sum, Ms. Lee’s pedagogical moves changed significantly over the year. Ms.
Lee had students discuss the meaning of what other students said, rather than merely
assert what the students did as being correct (see lines 12-16 in excerpt 7-10). Also, she
attempted to promote more discussions by inviting other students to build on the
mathematical ideas initially presented by a student (see lines 14-16, 24-25, 27 in excerpt
7-10) at the end of the year, whereas at the beginning of the year, she mostly dominated
the discussion by focusing on procedural processes.

Overall, in as other episodes, Ms. Lee’s pedagogical moves in Lesson Conclusion
episodes changed significantly toward being more student-thinking centered and
promoting productive discussions while Ms. Janet’s did not. At the beginning of the year,
their pedagogical practices were different in some ways, where Ms. Janet had more
questioning, albeit of the procedural process variety and Ms. Lee had more teacher-
oriented explanations. Overall, both teachers exhibited mostly teacher-centered moves or
questionings for procedural processes. However, at the end of the year, both teachers’
pedagogical moves demonstrate shifts toward being more student thinking-centered
ways, although Ms. Janet’s shifts were not significant.

**Teachers’ Reflections on Their Everyday Teaching Practices**

I conducted interviews with both teachers at the end of the year. Throughout the
interview, Ms. Janet expressed her belief that her regular teaching practices shifted after
implementing FALs, although she was not entirely certain. Because the interview was
conducted during an FAL workshop, she participated in workshop activities after the
interview. After the workshop, Ms. Janet came back to relay her realization that she
actually did not promote discussion between students when she watched her own
teaching video during the workshop. She reported that she had wanted to make students
explain their thinking, but she was not sure if she had. She then realized that student
discussions had not happened when she watched her own teaching videos. The interview
excerpts are as follows:

Ms. Janet: I feel like my structure has stayed the same, but I think probably my
questioning and like letting the kids explain more has shifted. Like, I feel like the
kids explain more and I talk less, I don’t know. But I don’t know if that's true, but
I have been trying to go that way. What do you, yeah, what do you think? Do you
think it's changed? Like I don't know, like it's hard for me to like see. […]

I think for me, the one thing that I’ve talked about is like, I feel like I'm a good facilitator of conversation, but it's more like student, teacher, student, teacher, and I want to get better at facilitating student to student conversation and I want students asking each other question.

[After end of the interview and workshop, Ms. Janet came back, and said]

Ms. J: I thought I was trying to [have my] students talk a lot, but one thing that I’ve noticed when I was watching the videotapes from my National Boards is I don’t give enough wait time. Because when I do give enough wait time, students are able to ask each other questions. So, I think next year what I think I’m gonna do is like count in my head and try to like give more wait time for the kids to like have that discussion between each other.

In contrast, Ms. Lee was more certain in her reflections that her regular teaching had changed. She explained that she used to be a very traditional teacher, with teacher-centric pedagogical moves such as explaining concepts and showing how to solve multiple-choice problems. However, she reflected that her pedagogical moves had changed toward making students talk a lot, after implementing FALs and participating in FAL PD. The interview excerpt is as follows:

Ms. Lee: I was a traditional teacher for ten years and my students were good test-takers. They knew how to analyze problems in multiple-choice situations and how to look at it. However, I found that my students couldn’t solve the new styles of problems. And that make me think, “wait a minute, there must be something wrong with the way I teach.” Because if I give the problem in the very traditional, like CST format, they knew how to do it. But if I would say, well, okay, “give me three solutions.” They would look like, “huh?” [laugh] you know? Or if I change the problem, like a word problem, they would look like that as well. […] I feel like my regular teaching has changed a lot through participating in this project [PD meetings and implementing FALs] as well as Common Core. I tried having them talk more and having them listen to each other.

7-4. Summary and Conclusion

Overall, Ms. Janet’s teaching practices did not demonstrate significant change in both the Do Now and the Lesson Conclusion episodes. Ms. Janet’s lesson structure was still rigid at the end of the year. Her pedagogical moves continued to focus on procedural processes and IRE sequences with short-answerable questions, rather than on eliciting student reasoning or soliciting ideas. Pedagogical moves related to promoting productive discussion were very infrequent, even though her personal pedagogical goal for the year had been to promote student discussion as revealed in her interview.
On the other hand, the changes in Ms. Lee’s teaching practices showed how she became more responsive to students’ mathematical thinking in both episodes. Lesson materials were also changed to be more responsive to student thinking. Ms. Lee incorporated her own student thinking into lesson materials that she modified from the textbook. The lesson structure also changed. Ms. Lee integrated and accepted her students’ mathematical thinking more actively using student presentations and various pedagogical strategies. Ms. Lee’s questions changed from being about procedural processes to “why” and “how” questions. She attempted to promote productive mathematical discussion more frequently than at the beginning of the year.

In this chapter, I explored how two teachers, Ms. Janet and Ms. Lee, demonstrate different degrees of change in their everyday teaching practices as a result of making use of curricular support materials (FALs) and professional development designed to support their teaching aligned with the values of CCSSM, specifically with a focus on formative assessment. Classroom observations and interviews shed light on potential issues that may arise when teachers use curricular support materials and on challenges that teachers face when they are prompted to change their teaching practices.

In the next chapter, Chapter 8, I build on these findings and findings from Chapters 5, 6, and 7 to discuss challenges that teachers face at the individual and policy levels in more depth.
Chapter 8. Discussion

This dissertation investigates how two mathematics teachers learned and their teaching practices changed when they were provided with innovative curricular support materials (FALs), and PD support for implementing them. I conducted a detailed analysis of two experienced teachers, Ms. Janet and Ms. Lee. Both teachers volunteered to implement FALs and to participate in FAL PD sessions, and both implemented three FALs over the school year. However, my analysis showed that one teacher, Ms. Lee, was able to exploit the curricular materials in ways that elicited and responded to student mathematical thinking. Doing so afforded her and her students opportunities to delve more deeply into the mathematics. This has provided a mechanism for changes in her own pedagogy. In contrast, the other teacher, Ms. Janet, whose initial stance did not focus on student mathematical thinking, made curricular choices that deprived her students of opportunities to demonstrate their understandings, which in turn deprived her of opportunities to work with their understandings. At the end of the year, Ms. Janet did not demonstrate significant change in her own pedagogical practices.

In this chapter I will briefly summarize my analytical findings and discuss key findings. Then I will discuss general issues and my research’s contributions to the field on how teachers’ interact with curricular materials and on how teachers learn and change through their practice. Finally, I describe several issues raised by this study that merit future research.

Summary and Discussion

In my study, I explore how two teachers who implemented the same curricular materials and participated similarly in PD opportunities interacted with and leveraged the materials in different ways. In Chapter 5, I focus on the relationship between this interaction of teachers with the curricular materials and their opportunities to learn. In Chapter 6, I provide two analyses: a broad, bird’s-eye view of the two teachers’ pedagogical strategies, and a fine-grained, detailed analysis of the teachers’ pedagogical patterns when they implemented one FAL. The analyses also describe the relationship between pedagogical strategies and the teachers’ learning opportunities. In Chapter 7, I analyze whether and how two teachers’ own pedagogical practices change in regular, non-FAL, lessons. Ms. Janet shows minimal changes in her everyday teaching practices while Ms. Lee shows significant changes towards being more responsive to student mathematical thinking in her everyday teaching practices. The following constitutes the summary of the two cases with regard to those three analytical topics.

Case 1. Ms. Janet: Minimal change in pedagogical practices after implementing three FALs over the year.

In her curriculum use, Ms. Janet omitted key principles underlying the curricular materials and eliminated some of the important mathematical goals. Both the broad and fine-grained analyses of her pedagogical strategies revealed that Ms. Janet’s ways of implementing the innovative curricular materials heavily relied on her existing teaching repertories instead of adapting or developing new practices. In providing narrow
algorithms in an easy way (e.g., driving students to use a formula), Ms. Janet deprived students of important mathematical connections. In turn, this deprived Ms. Janet of opportunities to reflect on and make sense of student mathematical thinking. Ms. Janet eliminated some essences and affordances of the curriculum materials in her implementation of FALs and this “lethal mutation” and adherence to her existing practices indicate a minimal change in her own pedagogy, as reflected by the non-statistically significant changes in terms of student thinking responsive teaching practices.

Case 2. Ms. Lee: Significant change in pedagogical practices after implementing three FALs over the year.

Ms. Lee adapted and used as is the affordances of the curricular materials, in alignment with the lesson designers’ goals and intentions. Both the broad and fine-grained analyses of her pedagogical strategies demonstrated that Ms. Lee developed new pedagogical strategies in ways that elicit and respond to student mathematical thinking. She provided opportunities for the students to explore their understandings, which enabled students to make connections and grapple productively with misconceptions. By working through the various ways students thought about the mathematical content, Ms. Lee had opportunities to refine her understanding of student thinking and, to make more connections herself regarding the mathematics. These instructional moves created opportunities not only for her students but also for herself to advance her understandings of student mathematical thinking and mathematical content. By the end of the year, although the curricular materials she used spanned less than two weeks of actual classroom time, her teaching had changed to the point where she was doing significantly less “demonstrate and practice” and more asking complex questions and responding to student thinking.

These two cases exemplify the core empirical observations of this study. Ms. Janet’s instructional moves and interactions with curricular materials were indicative of her existing practices: focusing on procedural processes rather than advancing conceptual understanding. They did not afford the means of her making sense of the content and student thinking of the content. Ms. Lee’s instructional moves and interactions with materials were indicative of her adapting and developing new pedagogical strategies: focusing on eliciting and responding student mathematical thinking. They afforded her learning opportunities of student thinking and contents. The analyses demonstrate how teachers learn and change through practice. The next question is why. Why did Ms. Janet, in contrast to Ms. Lee, not show significant change in practices, even though her pedagogical goals were to respond to student mathematical thinking and to promote student mathematical discussions? What challenges did both teachers experience when they learn and change in and from practices? To answer these questions, I go back to the data and research literature.

Individual Cognitive and Contextual Factors Affecting Changes in Teacher Practice

Research on general teacher decision-making explains what teachers do and why, with a focus on teachers’ goals, beliefs, and knowledge (e.g., Schoenfeld, 2011), and it can be naturally applied to explain teachers’ decision-making in curriculum use. A
growing number of studies on teachers’ use of curriculum materials also offers insights on how cognitive and contextual factors influence teachers’ use of curriculum materials (Davis & Krajcik, 2005; Remillard & Bryans, 2004; and see Remillard (2005)’s review). Therefore, individual teacher characteristics and resources, such as goals, knowledge and resources, belief and orientation may help explain why two teachers in this study made decisions in curriculum use (omitting, replacing, as-is, and creating), and the impact of those decisions on changes in their practices. That is, the teacher’s knowledge, resources, and ability to use new curriculum—pedagogical design capacity—and beliefs and orientations toward curriculum can constrain or facilitate the development of new approaches in curriculum use and the creation of learning opportunities through curriculum use. My dissertation extends current research findings by providing a more detailed account of the mechanisms of teachers’ interaction with innovative curriculum materials, with a particular focus on their adaptation of the materials.

For example, Ms. Janet’s narrative beliefs and orientation towards curriculum materials have a deep impact on how she interacts with curriculum materials. In my analysis chapters, Ms. Janet extensively omitted some components of FALs, such as pre-/post-assessments and important mathematical goals. In the interview (see Excerpt 8-1 below), when she was asked why she did not administer assessment tasks, she responded that she did not enough time. Administering FAL assessment tasks before- and after-instruction requires more time and effort on part of the teachers to qualitatively analyze their students’ responses. Instead, Ms. Janet quantitatively assessed her students’ understanding at a very surface level by asking them whether or not they already knew certain concepts at the beginning of the lessons. She also created and provided exit-tickets, which mostly consisted of memorizing formulae or algorithms. The exit-tickets were designed by Ms. Janet to align with district benchmark tests and to assess students’ understanding at the end of each lesson. In sum, her beliefs that revealed from interview valued district benchmark tests more than administering FAL assessments tasks, and her beliefs impacted her decision-making with regard to her use of curricular materials.

Excerpt 8-1. Interview with Ms. Janet at the year’s end29 (beliefs and orientation toward curriculum): Ms. Janet’s orientation towards curriculum materials and views about mathematics teaching and learning seem to value tests-oriented teaching rather than spending more time on innovative curriculum materials. This view influenced how Ms. Janet engaged in FALs, where she did not administer full assessment tasks, omitted several components and important mathematical goals of FALs, and provided her students formula and algorithms in a very easy way.

*Interviewer(I)*: So, I realized that, you taught the three Formative Assessment Lessons and the second one and third one, you didn’t do pre-assessment and post-assessment. Do you have any specific reasons, like timing?

*Ms. J*: Yeah, it was mostly timing, but I think, yeah, I think next year I definitely want to do the pre-test and the post-test. Because I think I did the pre-test for all of them, but I didn’t pass it back as a post-test. But part of that is just ’cause I have my own assessment stuff. Do you know what I mean?

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29 The interview was conducted in June, 19, 2014 at the end of the school year with closing classroom observations.
I: Yeah. I saw that the exit tickets and some similar things.
Ms. J: So yeah, I built kind of my own that were like a little bit more aligned to what I knew they were gonna take on the district benchmark.

In addition to the evidence that teachers’ beliefs and orientation towards curriculum impacts how teachers adapt curriculum materials in their instruction, I found that teachers’ beliefs are deeply intertwined with their professional identity—how teachers come to understand themselves as mathematics teachers and as professionals who continuously learn in and around practice—and that their identity plays an important role in their curriculum use. That is, understanding teachers’ professional identity is also key to understanding how and why teachers use curriculum materials in certain ways. For example, the year’s end interview with Ms. Janet revealed some aspects of her professional identity as following Excerpt 8-2. In her narration, Ms. Janet viewed herself as a mathematics teacher depending on her students’ outcomes in district benchmark tests. She valued her students’ performance in standardized testing as a metric for the quality of her teaching. This proved to be the key to understanding why Ms. Janet interacted with FALs in the ways she did: skipping assessment tasks, and providing formulas and algorithms rather than engaging her students in productive struggles to explore mathematical ideas and their connections.

Excerpt 8-2. Interview with Ms. Janet at year’s end (professional identity)

I: … I just wanted to ask you about yourself as a teacher for career or as a profession. So, um, do you feel that you are becoming an effective teacher as time goes by and what makes you feel that way? And then the plans for your professional growth and those kinds of things.
Ms. J: Yeah, I feel like, I'm becoming an effective teacher. I still feel like I have a long way to go, but like I had, like this year, I had 70% of my kids proficient and advanced on the final benchmark, which was a lot harder than like 3 or 4 years ago, like the benchmark that they gave us, so I feel like my achievement is improving and the curriculum is getting harder, so I feel like I am definitely, I feel like I'm definitely becoming more effective because of the data. The data shows me that.

(…) I want to become like a perfect teacher, which will never happen, but I think trying to like improve, continue to improve my own class but then also thinking about bigger picture of. (…)

I: What do you mean by, like, perfect or effective teacher?
Ms. J: I think my vision of a perfect teacher, like me? Like would be 100% of my students score proficient or advanced at the end of the year. That would be, that would be like a quote-end-quote perfect teacher. But I mean, I mean this year I had 95% of students that grew from the mid-year to the final, so, yeah, they’re definitely growing, but I think that, it just, I don’t know. So I had some kids that were still in the below category, but they grew by like 20% so it's like they had a huge growth over the year. But I
still want to push myself to that, thinking about like okay, even those kids that are coming in like super low, what can I do to get them to grade level by the end of my class? So, yeah.

The dissertation study also provides theoretical insights for aforementioned empirical findings. Not only are teachers’ cognitive perspectives (knowledge, beliefs), but also their professional identity are important in order to understand the ways in which teachers interact or participate with curriculum materials. That is, teacher knowledge, beliefs, and identity around contexts intertwine and systematically influence the ways teachers interact with curriculum materials. This aligns with Drake and Sherin (2006)’s perspective that teachers’ previous experience as learners of mathematics and teachers of mathematics is critical in order to understand the ways in which they adapt curriculum materials. My dissertation shows that it is also important for them to see and make sense of themselves as learners and as professionals who are continuously reflecting on and through their practices and their identities may be the catalysts for change in their practices. This finding is adding to Menanix (2015)’s dissertation.

**Challenges in the Process of Professional Learning and Change in Practices**

In the previous section, teachers’ narratives reveal that individual teachers’ cognitive perspectives (e.g., knowledge, beliefs) and professional identity shape their ways of adapting curriculum materials and their potential for change in practices. The teachers’ narration also reveals challenges that they faced. The challenges both teachers experienced were a part of their professional learning trajectory. This has implications for the roles of research and practice (e.g., policy) for teacher professional learning and development.

In the analyses of this study and the summary of the cases above, Ms. Lee’s everyday teaching practices changes toward being more responsive to student mathematical thinking as a result of implementing several FALs over the school year. The interview also revealed how she reflected on her pedagogy in Chapter 7. However, Ms. Lee encountered a dilemma:

*Ms. Lee:* I was a traditional teacher for ten years. […] I feel like my regular teaching has changed a lot through participating in this project as well as Common Core. I tried having them talk more and having them listen to each other. And then, that brought up another struggle that I should that I noticed that. Because I’m letting go more of like, teachers out of control and giving more of the voices of the kids. I feel like my management was so...more suffering. That’s the part that I really kind of have to think about. And, you know, I think about that with our current teachers too. Because Common Core, or the way we interpret in a school is about letting the kids struggle, letting the kids discuss and discover. And I guess, not a push back to the question that I have is when we let go like that, you know, what about the authority of the teacher. […] There should be some balance there that I feel like needs to be worth that.

In the interview, Ms. Lee references the loss of control of her classroom, which occurred when her teaching was becoming more responsive to student thinking. As she
increased her attempts to elicit students’ mathematical reasoning, wait for them to explain their thinking, and promote spontaneous discussions, classroom management issues began to emerge toward the end of the school year. In classroom observations during that time, Ms. Lee’s practices were temporarily those of novice teacher developing new pedagogical practices, rather than relying on her existing routinized pedagogical practices. This finding is not new. Similar cases have been documented in previous studies (e.g., Cohen, 1990; Lee & Wiliam, 2005), which suggest that the process of teacher change is a complex system with tensions between old or existing pedagogical routines and new instructional strategies. The findings of this dissertation supports and contributes to this area of research by explaining some of the mechanisms of teacher change in practices.

“Loss of balance” in the process of developing new practices: Toward a theory of teacher change and development in practices.

The mechanisms of developing new practices by the two teachers can be characterized using Piaget (1937)’s terminology for processes of human cognitive development: assimilation and accommodation. When people face new information, they may use their existing schema to make sense of the new information, where new information is modified to or assimilated with old or existing ideas in a process called assimilation. On the other hand, in the process of accommodation, when new information was not fitted into existing schema, there is cognitive disequilibrium, and existing schema are altered to achieve the status of balance, equilibrium. If the two cases are explained in these terms, Ms. Lee’s teaching practices are evidence for the process of accommodation with active disequilibrium, where the loss of balance is imperative for developing new practices. In contrast, Ms. Janet’s case presents more of the process of assimilation, where Ms. Janet relied more on her existing practices, rather than shifting her old practices towards new practices FALs sought to promote.

However, teaching is more than an individual cognitive activity. It is also situated in a participatory context with other individuals, particularly students, and with a specific subject matter. As found in research elsewhere (Kim, 2014), Ms. Lee’s teacher discourse changed towards being more responsive to student thinking, but the student discourse did not follow her shift. Teacher discourse analysis showed that Ms. Lee attempted to promote student-student discussion more frequently at the end of the year, while student discourse analysis showed that there was no significant change in the frequency of productive discussion among students. The qualitative analysis findings suggest that the lack of observed change in student discourse may have been at least in part due to problematic student engagement in classroom activities at the end of the year. A new theoretical framework may therefore be necessary to investigate this interpersonal mechanism of development of teaching practices as recent trends and expectations for teaching changed towards more student thinking-centered ways, such as student thinking.

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30 My previous article addresses this issue by supporting existing claims of this study is presented (Kim, 2014).
31 The original code name of this in the article was “accountable talks” but I refer it as “productive discussion among students” in order to match with the code in this dissertation study.
responsive teaching, in which students actively participate in the activities that all members of a classroom organize and reproduce.

One possible theoretical framework for future research on mechanisms of teacher development and change in practices may be a socio-cultural framework, such as Saxe (2012)’s form-function relationship. Investigating how particular forms of instructional practices serve different functions in two teachers’ implemented classroom practices, and how Ms. Lee’s form-function relationship shifts over time while Ms. Janet’s does not, may yield powerful insights that help explain teachers’ development and change in practices in the context of student thinking responsive teaching where students’ participation is critical.

Reflective Learning: Video-reflection Is Critical in Teachers’ Learning through Practice

Interviews with Ms. Janet raise another critical issue in teachers’ professional learning (quotes in Chapter 1, where analysis indicates a great potential for Ms. Janet to learn from videotapes of her own teaching and to change in her practices. During the interview, Ms. Janet was confident of her role in classroom as a facilitator, as the following excerpt demonstrates:

Ms. J: I think for me, the one thing that I've talked about is like, I feel like I'm a good facilitator of conversation, but it's more like student, teacher, student, teacher, and I want to get better at facilitating student to student conversation and I want students asking each other question.

After the interview ended, Ms. Janet returned to a PD meeting because the interview was conducted during a break time between PD meetings. However, soon after talking with other teachers, she came back and said the following:

Ms. J: I thought I was trying to [have my] students talk a lot, but one thing that I've noticed when I was watching the videotapes from my National Boards is I don't give enough wait time. Because when I do give enough wait time, students are able to ask each other questions. So, I think next year what I think I'm gonna do is like count in my head and try to like give more wait time for the kids to like have that discussion between each other.

I: Oh, yeah.

Ms. J: ‘Cause they do it in the small group, but in the whole class, I want that to happen.

[...] So, next year, I am going in a videotaping and then watching some video clips of classrooms and using that as our [department meeting] observation.

This provides a critical implication that video reflection is a valuable opportunity for teachers’ professional learning. All FAL teachers had been provided with some opportunities to verbally reflect on their teaching, and FAL implementation in particular during PD meetings. As shown in Chapter 5, while this allowed teachers to reflect on
their teaching, their reflection relied on their recollections of their own teaching, which were filtered through their subjective perspectives. This meant that not all teachers could take full advantage of these opportunities for their professional learning. Some teachers, like Ms. Janet, may benefit from different opportunities that would better allow them to notice problems in their implementation from a more objective perspective so that they can improve their instructional practices. This aligns with existing findings that video-club PD meetings are powerful and fruitful means of supporting teachers’ professional learning (Sherin & Han, 2004; van Es & Sherin, 2008).

Of particular note is how Ms. Janet was sufficiently motivated by her above observations that she made plans to incorporate video reflection in her department meetings next year. However, such endeavors by teachers need support from school leaders and districts in order to create and maintain a productive teacher learning community. In the interview at the beginning of the year, Ms. Janet had an ambitious plan for department meetings to serve as a means of supporting mathematics teachers’ professional development. Department meetings would be occasions for teachers to discuss mathematics and pedagogy, observe each other’s teaching, and share feedback, as the following interview excerpt from the beginning of the year demonstrates:

*Ms. J:* This year, we’re gonna start by like doing a problem solving problem together and just discussing like how we solve it. I think it does help because it gets to like how different people are solving it and then thinking about our students and how they would solve it, because not all the teachers in my department do what I do when I do problems. And then like think about where my students struggle, where they need practice, and like plan from there, so we’re trying to kind of like get people to see the value of actually doing the math ahead of time.[…]

And then like, we observe other teachers, so,

*I:* Oh, wow.

*Ms. J:* So there’s a protocol for, we debrief on like what we observed and, yeah. Yeah, and then, like this month we’re observing two teachers and then there's certain things that they want us to observe for, and so then we share the feedback and notes and then we kind of debrief.

*I:* Wow, how often do you observe other teachers?

*Ms. J:* We’re supposed to observe two a month.

However, at the end of the year, Ms. Janet revealed that the department meetings turned not to be as productive for teachers’ professional development as planned. The meetings were interrupted by logistics and unexpected impediments that were not about math teaching. Furthermore, observing other teachers’ classrooms presented a significant logistical challenge. The challenges she experienced as a department chair in this endeavor are illuminated in the following interview excerpt:

*Ms. J:* I think, overall, I wasn’t very pleased with my department meetings this year. It’s just like, cause a lot of it, I feel like we have to do just like business stuff that isn’t really about the math. […]

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32 The interview was conducted in September 8, 2013 at the beginning of the school year.
I: I think, we just talked about that [at the beginning of the year], in your school, you have to observe other teachers?

Ms. J: Yeah, yeah. So we try to build in the culture of like observe two teachers a month, but it’s just, it’s hard because the only free period you have is prep, and like, two of them I had office hours, and then the like, one was like my team meeting, so really I only had two free prep periods, but it ended up usually being filled with something like a parent coming or like the office needed me to fill out something. Do you know what I mean? I never really, it’s just logistically really hard so that's what [co-department chair] and I talked about doing next year is like me going in a videotaping and then like as a department, like watching some video clips of classrooms and using that as our observation, because I’ll only be teaching the two periods, so like in the afternoon I can go and like observe and like videotape other teachers and stuff like that.

The above excerpt underscores how we need to seek ways in which school leaders and districts can support teachers’ professional development and learning, such as adjusting teaching schedules to allow not only time for these activities, but also compensation and motivation for their professional growth.

Policy challenges and policy as a support system

The case of Ms. Janet above describes how her beliefs and orientation towards curriculum materials ultimately constrained her and did not allow her to develop new instructional practices. Ms. Janet’s narrative interviews also reveal that her professional identity is deeply related to her beliefs, which impacted her decisions and deprived her of learning opportunities. Ms. Janet also expressed challenges that can be mitigated by frequent reflection on her practices, which require research and policy can support. As described in the previous section, Ms. Janet’s pursuit of her professional goals were stymied by challenges on the policy side. Schools need to provide more support to teachers so that department meetings can serve as professional learning communities.

Ms. Janet had a tension between existing goals and beliefs of her and new ways of pedagogies from FALs. Her existing beliefs were established as a learner when she was a student. As mentioned in an earlier excerpt, her instructional goal was for to motivate her students to become independent learners by helping them gain higher achievement scores. Aligned with this pedagogical goal, Ms. Janet’s beliefs valued district benchmark tests. However, she encountered challenges when she implemented new instructional strategies that were not aligned with her beliefs, even though she wanted to learn them. See the following interview excerpts:

Ms. Janet: I think my main goal as a teacher is for my students to believe that they can be independent learners, that they don’t need me, basically, that they can just carry on on their own, because I feel like that’s the thing that gets them ready for college.

Um, I think a lot of like goal-setting, like reflection, um, and just like following up with them. And like you saw, like the Monday motivational thing. […] like reflecting after every unit, I would analyze the data and I
would say these are the percents. These are the students who mastered it and like thinking, then I would think about what they did well and what they could improve on, and from that I would make the next steps for myself as a teacher. Like this is what I’m gonna change, and this is why I’m gonna change it and like they would do the same process, like looking at their own [performance] data and thinking about what they could change, so I think me modeling that really helped and also just having them do it a lot.

I: How was your teaching Formative Assessment Lesson experience for you? Like, what did you enjoy the most or what was the challenging for you?

Ms. J: Hmm, I think it got better each time I taught one. I think the most challenging for me was not leading them so much, like not leading them to the answer kind of thing and just like that part where they’re working in small groups, just trying to listen and ask questions, and not like ask questions so that I can get them to say what I want them to say. […] Like, if I already did the math and I kind of have it in mind like how I want them to solve it, like leading them to a certain way of solving it versus like trying to understand how they’re doing it.

From this interview, her self-perception is changing while the analysis of her teaching observation data shows not significant change in her pedagogical moves. This is somewhat similar to the case of “Mrs. Oublier (Cohen, 1990),” in which the teacher’s perceptions do not match the actual observation data. Mrs. Oublier believed that her instructional practices were changed toward a reformed way while actual observation revealed that her teaching still remained traditional, with a belief about mathematics as a fixed body of right answers. Mrs. Oublier did not have opportunities to learn new views of mathematics, to reflect on her own teaching in terms of the new framework of mathematics, or to get any other resources for improving her teaching practices. Learning from Mrs. Oublier’s case, we need to have support systems in place that support teachers in changing in the ways they hope to change.

Furthermore, Ms. Janet’s values and goals as a mathematics teacher are strongly tied to policy (e.g., district benchmark tests) from the data. Learning from these two cases of Mrs. Oublier and Ms. Janet, policy may need to change first before teachers like Ms. Janet can change their practice. For example, the Common Core standards require teachers to change their mathematical teaching practices in order to create better learning environments that enable students to become powerful mathematical thinkers. Assessment approaches will need to change to align with those goals for teachers like Ms. Janet, whose professional identity is heavily dependent on student performance on district benchmark tests. Policy exists to and should support the learning of both teachers and students. Prior research suggests concrete ways in which policy stakeholders and schools can support teacher learning. For example, Cobb and Jackson (2011) argue that the following five components are essential for supporting teachers in improving their instructional practices:

1. A coherent instructional system consisting of explicit goals, visions, practices, and school-based professional learning communities;
(2) *Teacher networks* as professional relations (e.g., school-wide professional learning community);

(3) *Mathematics coaches’ practices* in providing job-embedded support for teachers’ professional development;

(4) *School leaders’ practices* as instructional leaders in providing feedback on high-quality mathematics instructional practices; and

(5) *District leaders’ practices* in supporting the development of school-level capacity for improving instructional practices.

These components give concrete examples and insights in what ways policy can be changed in order to support improvements of teachers’ instructional practices and it aligns with this dissertation’s empirical findings.

**Conclusion, Implication, and Future Research**

A major empirical finding of this dissertation is that when teachers are supported in, and make use of, instructional moves that focus on student mathematical thinking, not only is student learning advanced, but teacher learning is advanced as well. The right supports can enable teachers to change their teaching practices. This research also explored the ways teachers interact with curriculum materials and the relationship between those interactions and the teachers’ learning opportunities. I found that not only teachers’ cognitive characteristics (e.g., goals, beliefs, and knowledge), but also their professional identity in contexts impact teacher interactions with curriculum materials and how they create opportunities to learn and grow as professionals.

Theoretically, this dissertation contributes to our understanding of how mathematics teachers learn through their interactions with innovative curriculum materials and from their instructional moves. This research provides insights on the mechanisms of teacher change and development in practices, and suggests fruitful directions for future research. It also shows that productive change is far from guaranteed – that there are various ways that curriculum-based learning can be undermined. Hence, one must be concerned with putting systems in place that support the positive aspects of curriculum-based learning, and inhibit the use of counterproductive teaching strategies and methods.

This dissertation also makes several methodological contributions with regard to (1) how to measure teacher change in teaching practices using mixed methods, and (2) how to capture profiles of teaching practices, particularly with regards to formative assessment or student responsive teaching. Capturing classroom practices is an ongoing challenge in the field, and the frameworks I developed are promising first steps to capture teacher’s instructional practices or teaching styles with a bird eye’s view and to measure changes in classroom practices. While this dissertation focused on two case studies, the frameworks can be used to analyze a larger sample of FAL teachers to capture the range of trajectories in teachers’ development and change in practices so that we can better support individual teachers. Lastly, the frameworks can be modified for teacher use so that teachers can leverage the frameworks as tools that help them consider how to elicit and make use of student thinking for creating a productive mathematical discussion community. I hope that these frameworks can be applicable in improving other subjects (e.g., science)’ classroom practices.
This dissertation research has several implications. First, my empirical findings have implications for policy by identifying catalysts for teachers’ development and change through their interactions with curriculum materials. Individual teacher knowledge, beliefs, and professional identity are critical factors to understand the mechanisms of change, and concrete and appropriate support on the policy side are needed to support teachers’ instructional practices during this process. Schools and districts should support teachers to improve their instructional practices as a system not only for student learning but also for teacher learning by providing concrete, formative feedback about instructional practices to teachers.

Second, my findings also have implications for the design of teacher professional development and pre-service teacher education programs. My interview analysis indicates that video-reflection is critical for teachers to reflect on their own teaching and has great potential for teachers to develop and change their teaching practices. That is, structured video-based reflective practices can provide valuable opportunities both for in-service and pre-service teachers to improve their instructional practices.

Last, my dissertation has implications for teacher educators. Teachers should be considered active and serious learners. My analysis showed that student thinking focused pedagogy improves not only student learning, but also teaching practices and teachers’ professional identity, both of which have a great impact on their learning. That is, teachers should see themselves as active and serious learners who can learn through their own practice. The ways in which we address the importance of student thinking to teachers should be framed differently, where student thinking responsive teaching is helpful not only for their student learning but also for their “own” learning. To do so, policy stakeholders and teacher educators need to change the ways they see teachers, and how teachers see themselves, in addition to individual efforts by teachers to change their views and conceptions. We all must consider that teachers are preforming practices as professions and they are active and serious learners as learning through their professional practices.
References


Appendix A: Percent-change Lesson (full version)

CONCEPT DEVELOPMENT

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Increasing and Decreasing Quantities by a Percent

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: http://map.mathshell.org
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This lesson is directly retrieved from http://map.mathshell.org/lessons.php.
Increasing and Decreasing Quantities by a Percent

MATHEMATICAL GOALS
This lesson unit is intended to help you assess how well students are able to interpret percent increase and decrease, and in particular, to identify and help students who have the following difficulties:

- Translating between percents, decimals, and fractions.
- Representing percent increase and decrease as multiplication.
- Recognizing the relationship between increases and decreases.

COMMON CORE STATE STANDARDS
This lesson relates to the following Standards for Mathematical Content in the Common Core State Standards for Mathematics:

- 7.RP Use proportional relationships to solve multistep ratio and percent problems.
- 7.NS Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- 7.NS Solve real-world and mathematical problems involving the four operations with rational numbers.

This lesson also relates to the following Standards for Mathematical Practice in the Common Core State Standards for Mathematics:

- 2. Reason abstractly and quantitatively.
- 7. Look and make use of structure.

INTRODUCTION
This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- Students work in small groups on collaborative discussion tasks, to organize percent, decimal and fraction cards. As they do this, they interpret the cards’ meanings and begin to link them together. They also try to find relationships between percent changes. Throughout their work, students justify and explain their decisions to their peers.
- Students return to their original assessment tasks, and try to improve their own responses.

MATERIALS REQUIRED

- Each student will need, two copies of the assessment task Percent Changes, a calculator, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need copies of Card Sets: A, B, C, D, and E. All cards should be cut up before the lesson. Optional materials are a large sheet of card on which to make a poster, and some glue sticks and/or the poster template Percents, Decimals, and Fractions (1).
- You will also need copies of the extension material: Percents, Decimals, and Fractions (2).

THERE IS also a projector resource to help with whole-class discussions.

TIME NEEDED
15 minutes before the lesson, one 90-minute lesson (or two 45-minute lessons), and 10 minutes in a follow-up lesson (or for homework). Timings are approximate and will depend on the needs of the class.
BEFORE THE LESSON

Assessment task: Percent Changes (15 minutes)

Have the students do this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task Percent Changes.

Read through the questions and try to answer them as carefully as you can. The example at the top of the page should help you understand how to write out your answers.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students’ responses

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches.

We suggest that you do not score students’ work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students’ work, using the ideas that follow. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

The solution to all these difficulties is not to teach algorithms by rote, but rather to work meaningfully on the powerful idea that all percent changes are just multiplications by a scale factor.

---

Assessment task: Percent Changes

1. Tom usually earns $40.85 per hour. He has just heard that he has had a 6% pay raise. He wants to work out his new pay on this calculator. It does not have a percent button. Which keys must he press on his calculator? Write down the keys in the correct order. (You do not have to do the calculation.)

2. Maria sees a dress in a sale. The dress is normally priced at $56.99. The ticket says that there is 45% off. She wants to use her calculator to work out how much the dress will cost. It does not have a percent button. Which keys must she press on her calculator? Write down the keys in the correct order. (You do not have to do the calculation.)

3. Last year, the price of an item was $350. This year it is $450. Lena wants to know what the percentage change is. Write down the calculation she will need to do to get the correct answer. (You do not have to do the calculation.)

4. In a sale, the prices in a shop were all decreased by 20%. After the sale they were all increased by 25%. What was the overall effect on the shop prices? Explain how you know.

---

Percent Changes

1. Give each student a copy of the assessment task Percent Changes.

2. Maria sees a dress in a sale. The dress is normally priced at $56.99. The ticket says that there is 45% off. She wants to use her calculator to work out how much the dress will cost. It does not have a percent button. Which keys must she press on her calculator? Write down the keys in the correct order. (You do not have to do the calculation.)

3. Last year, the price of an item was $350. This year it is $450. Lena wants to know what the percentage change is. Write down the calculation she will need to do to get the correct answer. (You do not have to do the calculation.)

4. In a sale, the prices in a shop were all decreased by 20%. After the sale they were all increased by 25%. What was the overall effect on the shop prices? Explain how you know.
<table>
<thead>
<tr>
<th>Common issues:</th>
<th>Suggested questions and prompts:</th>
</tr>
</thead>
</table>
| Student makes the incorrect assumption that a percentage increase means the calculation must include an addition For example: 40.85 + 0.6 or 40.85 + 1.6. (Q1.) A single multiplication by 1.06 is enough. | • Does your answer make sense? Can you check that it is correct?  
• “Compared to last year 50% more people attended the festival.” What does this mean? Describe in words how you can work out how many people attended the festival this year. Give me an example.  
• Can you express the increase as a single multiplication? |
| Student makes the incorrect assumption that a percentage decrease means the calculation must include a subtraction For example: 56.99 − 0.45 or 56.99 − 1.45. (Q2.) A single multiplication by 0.55 is enough. | • Does your answer make sense? Can you check that it is correct?  
• In a sale, an item is marked “50% off.” What does this mean? Describe in words how you calculate the price of an item in the sale. Give me an example.  
• Can you express the decrease as a single multiplication? |
| Student converts the percentage to a decimal incorrectly For example: 40.85 × 0.6. (Q1.) | • How can you write 50% as a decimal? How can you write 5% as a decimal?  
• Can you think of a method that reduces the number of calculator key presses?  
• How can you show your calculation with just one step? |
| Student uses inefficient method For example: First the student calculates 1%, then multiplies by 6 to find 6%, and then adds this answer on: (40.85 ÷ 100) × 6 + 40.85. (Q1.) Or: 56.99 ÷ 0.45 = ANS, then 56.99 − ANS (Q2.) A single multiplication is enough. | • Are you calculating the percentage change to the amount $350 or to the amount $450?  
• If the price of a t-shirt increased by $6, describe in words how you could calculate the percentage change. Give me an example. Use the same method in Q3.  
• Make up the price of an item and check to see if your answer is correct. |
| Student is unable to calculate percentage change For example: 450 − 350 = 100% (Q3.) Or: The difference is calculated, then the student does not know how to proceed or he/she divides by 450. (Q3.) The calculation (450 − 350) ÷ 350 × 100 is correct. | • In your problem, what operation will the calculator carry out first?  
• If you just entered these symbols into your calculator would you get the correct answer? |
| Student subtracts percentages For example: 25 − 20 = 5%. (Q4.) Because we are combining multipliers: 0.8 × 1.25 = 1, there is no overall change in prices. |  
• Student fails to use brackets in the calculation For example: 450 − 350 ÷ 350 × 100. (Q4.) |
**SUGGESTED LESSON OUTLINE**

If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, then we suggest you end the lesson after the first collaborative activity and continue in a second lesson.

**Collaborative activity 1: matching Card Sets A, B, and C (30 minutes)**

Organize the class into groups of two or three students. With larger groups some students may not fully engage in the task.

Give each group Card Sets A and B.

Use the projector resource to show students how to place Card Set A.

Introduce the lesson carefully:

> I want you to work as a team. Take it in turns to place a percentage card between each pair of money cards.

> Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

> There is a lot of work to do today, and it doesn’t matter if you don’t all finish. The important thing is to learn something new, so take your time.

Pairs of money cards may be considered horizontally or vertically.

Your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

**Make a note of student approaches to the task**

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may make the mistake of pairing an increase of 50% with a decrease of 50%.

**Support student problem solving**

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors once the decimal cards are added.

For students struggling to get started:

> There are two ways to tackle this task. Can you think what they are? [Working out the percentage difference between the two money cards or taking a percentage card and using guess and check to work out where to place it.]

> How can you figure out the percentage difference between these two cards?

> This percentage card states the money goes up by 25%. If this money card (say $160) increases by 25% what would be its new value? Does your answer match any of the money cards on the table?
When one student has placed a particular percentage card, challenge their partner to provide an explanation.

*Maria placed this percentage card here. Martin, why does Maria place it here?*

If you find students have difficulty articulating their decisions, then you may want to use the questions from the Common issues table to support your questioning.

Students often assume that if an amount is increased and then decreased by the same percent, the amount remains unchanged.

*The price of a blouse is $20. It increases by ½. What is the new price? [$30]*
*The price of the blouse now decreases by ½. What is the final price? [$15]*
*Now let’s apply this to percentages. What happens if the $20 blouse increases by 50%?*
*What happens now when this new price decreases by 50%?*
*What percentage does the price need to decrease by to get it back to $20? [33⅓%]*

What does this show?

If the whole class is struggling on the same issue, you may want to write a couple of questions on the board and organize a whole-class discussion. The projector resource may be useful when doing this.

It may help some students to imagine that the money cards represent the cost of an item, for example, the price of an MP3 player at four different stores.

### Placing Card Set C: Decimal Multipliers

As students finish placing the percentage cards hand out Card Set C: Decimal Multipliers. These provide students with a different way of interpreting the situation.

Do not collect Card Set B. An important part of this task is for students to make connections between different representations of an increase or decrease.

Encourage students to use their calculators to check the arithmetic. Students may need help with interpreting the notation used for recurring decimals, and in entering $1.3$ as $1.33333333$ on the calculator.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. The following patterns may be noticed:

- An increase of, say, 33% is equivalent to multiplying by $1.\frac{1}{3}$.
- (An increase of 5% is not equivalent to multiplying by 1.5!)
- A decrease of, say, 33% is equivalent to multiplying by $(1-\frac{1}{3})=0.\frac{2}{3}$.
- The inverse of an increase by a percent is not a decrease by the same percent.

When the decimal multipliers are considered in pairs, the calculator will show that each pair multiplies to give 1, subject to rounding by the calculator.

<table>
<thead>
<tr>
<th>2</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.6</td>
</tr>
<tr>
<td>1.25</td>
<td>0.8</td>
</tr>
<tr>
<td>1.6</td>
<td>0.625</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.5</td>
</tr>
<tr>
<td>0.75</td>
<td>1.3</td>
</tr>
<tr>
<td>0.8</td>
<td>1.25</td>
</tr>
<tr>
<td>0.625</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The inverse of a percent is not a decrease by the same percent.
Extension activity
Ask students who finish quickly to try to find the percent changes and decimal multipliers that lie between the diagonals $150/160$ and $100/200$. Students will need to use the blank cards for the diagonals $150/160$.

Taking two lessons to complete all activities
You may decide to extend the lesson over two periods. Ten minutes before the end of the first lesson ask one student from each group to visit another group’s work. Students remaining at their seats should explain their reasoning for the position of the cards on their own desk (see the section on Sharing Work for further details.)

When students are completely satisfied with their own work, hand out the poster template Percents, Decimals, and Fractions (1). Students should use it to record the position of their cards. At this stage, one pair of arrows between each money card will be left blank.

At the start of the second lesson spend a few minutes reminding the class about the activity.

Try to remember what we were working on in the last lesson.

A mobile phone is reduced by 60% in the sale. Give me an example of what the phone could have originally cost and what it costs now. And another, and another...

[Take one of the examples given above.]
The mobile phone is not sold. It returns to its original price. What is the percent increase?

Return to each group their Percents, Decimals, and Fractions (1) sheet and the Card Sets A, B and C. Ask students to use their sheet to position their cards on the desk. Working with the cards instead of the sheet means students can easily make changes to their work and encourages collaboration between students.

Then move the class on to the second collaborative activity.

Sharing work (10 minutes)
When students get as far as they can with placing Card Set C, ask one student from each group to visit another group’s work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

If you are staying at your desk, be ready to explain the reasons for your group’s matches.
If you are visiting another group, write your card placements on a piece of paper. Go to another group’s desk and check to see which matches are different from your own.
If there are differences, ask for an explanation. If you still don’t agree, explain your own thinking. When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.

Students may now want to make changes.

Collaborative activity 2: matching Card Set D (30 minutes)
Give out Card Set D: Fraction Multipliers. These may help students to understand why the pattern of decimal multipliers works as it does.

Support the students as you did in the first collaborative activity.
The following pairings appear:

\[
\frac{1}{2} \quad \frac{3}{2} \quad \frac{2}{3} \\
\frac{4}{3} \quad \frac{3}{4} \quad \frac{5}{4} \quad \frac{4}{5} \\
\frac{8}{5} \quad \frac{7}{8}
\]

**Sharing work (10 minutes)**

When students get as far as they can placing Card Set D, ask the student who has not already visited another group to go check their answers against that of another group’s work. As in the previous sharing activity, students remaining at their desk are to explain their reasoning for the matched cards on their own desk.

Students may now want to make some final changes to their own work. After they have done this, they can make a poster.

Either:
- Give each group a large sheet of paper and a glue stick, and ask students to stick their final arrangement onto a large sheet of paper
- or:
- Give each group the poster template *Percents, Decimals, and Fractions (1)* and ask students to record the position of their cards.

The poster template allows students to record their finished work. It should not replace the cards during the main activities of this lesson as students can more easily make changes when working with the cards, and they encourage collaboration.

**Extension activities**

Ask students who finish quickly to try to find the fraction multipliers that lie between the diagonals $\frac{150}{160}$ and $\frac{100}{200}$.

*Card Set E: Money Cards (2)* may be given to students who need an additional challenge. *Card Sets B–D* can again be used with these *Money Cards*. Students can record their results on the poster template *Percents, Decimals, and Fractions (2)*.

In addition, you could ask some students to devise their own sets of cards.

**Whole-class discussion (10 minutes)**

Give each student a mini-whiteboard, pen, and eraser.

Conclude the lesson by discussing and generalizing what has been learned. The generalization involves first extending what has been learned to new examples, and then examining some of the conclusions listed above. As you ask students questions like the following, they should respond using mini-whiteboards.

*Suppose prices increase by 10%. How can I say that as a decimal multiplication?*

*How can I write that as a fraction multiplication?*
What is the fraction multiplication to get back to the original price?
How can you write that as a decimal multiplication?
How can you write that as a percentage?

**Improving individual solutions to the assessment task (10 minutes)**

Return the original assessment, *Percentage Change*, to the students together with a second blank copy of the task.

*Look at your original responses and think about what you have learned this lesson.*

*Using what you have learned, try to improve your work.*

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

**SOLUTIONS**

**Assessment Task: Percent Changes**

Students may answer Questions 1 - 3 in several ways. Here are some possible answers:

1. \[ 40.85 \times 1.06 = \]
   \[ \text{or } (40.85 \times 0.06) + 40.85 = \]
   \[ \text{or } 40.85 \times 0.06 = \text{ANS, ANS} + 40.85 = \]

2. \[ 56.99 \times 0.55 = \]
   \[ \text{or } 56.99 \times (0.45) = \]
   \[ \text{or } 56.99 \times 0.45 = \text{ANS, 56.99} - \text{ANS} = \]

3. \[ (450 - 350) + 350 \times 100 = \]
   \[ \text{or } 450 - 350 = \text{ANS, ANS} + 350 \times 100 = \]

4. *There is no overall change in the price:*

   *cost of product \times 0.8 \times 1.25 = cost of product* or

   *cost of product \times \frac{3}{4} \times \frac{5}{4} = cost of product*
Collaborative activity
Percent Changes

One month Rob spent $8.02 on his phone. The next month he spent $6.00. To work out the average amount Rob spends over the two months, you could press the calculator keys:

\[
\begin{array}{c}
\text{Calculator Keys:} \\
( \quad + \quad 0 \quad 2 \quad + \quad 6 \quad ) \quad + \quad 2 \quad = \\
\end{array}
\]

1. Tom usually earns $40.85 per hour. He has just heard that he has had a 6% pay raise. He wants to work out his new pay on this calculator. It does not have a percent button.
Which keys must he press on his calculator?
Write down the keys in the correct order.
(You do not have to do the calculation.)

2. Maria sees a dress in a sale. The dress is normally priced at $56.99. The ticket says that there is 45% off. She wants to use her calculator to work out how much the dress will cost. It does not have a percent button.
Which keys must she press on her calculator?
Write down the keys in the correct order.
(You do not have to do the calculation.)

3. Last year, the price of an item was $350. This year it is $450. Lena wants to know what the percentage change is.
Write down the calculation she will need to do to get the correct answer.
(You do not have to do the calculation.)

4. In a sale, the prices in a shop were all decreased by 20%. After the sale they were all increased by 25%.
What was the overall effect on the shop prices?
Explain how you know.
Card Set A: Money Cards (1)

$100 $150

$200 $160

Card Set E: Money Cards (2)

80¢ $1.20

$1.60 $1.28

Card Set B: Percent Increases and Decreases

Down By 50% 
Down by 20%

Up by 25%

Up by 60%

Down By 33⅓%

Down by 37½%

Down By 25%

Up by 50%

Up by 33⅓%

Up by 100%

Card Set C: Decimal Multipliers

× 1.6

× 0.6

× 0.75

× 2

× 1.5

× 0.625

× 0.8

× 1.3

× 0.5

× 1.25
Card Set D: Fraction Multipliers

$100 \times \frac{2}{1}$

$150 \times \frac{3}{2}$

$200 \times \frac{4}{5}$

$160 \times \frac{4}{3}$

$100 \times \frac{2}{3}$

$150 \times \frac{5}{8}$

$200 \times \frac{3}{4}$

$160 \times \frac{1}{2}$

Percents, Decimals, and Fractions (1)

Money Cards

$100$

$150$

$160$

$200$

Percents, Decimals, and Fractions (2)

$80\text{¢}$

$1.20$

$1.60$

$1.28$

Money Cards

$100$

$150$

$160$

$200$
Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the Shell Center Team at the University of Nottingham
Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans with Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by David Foster, Mary Bouck, and Diane Schaefer based on their observation of trials in US classrooms along with comments from teachers and other users.

This project was conceived and directed for MARS: Mathematics Assessment Resource Service by Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan and based at the University of California, Berkeley

We are grateful to the many teachers, in the UK and the US, who trialed earlier versions of these materials in their classrooms, to their students, and to Judith Mills, Carol Hill, and Alvaro Villanueva who contributed to the design.

This development would not have been possible without the support of Bill & Melinda Gates Foundation
We are particularly grateful to Carina Wong, Melissa Chabran, and Jamie McKee

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Appendix B1: Ms. Lee’s Modified Version of Pre-assessment for Percent-change Lesson

One month, Rob spent $8.00 on his cell phone. The next month he spent $6.00. To find the average amount Rob spends over the two months, he did the following.

\[(8.00 + 6.00) \div 2 = \]

<table>
<thead>
<tr>
<th>Explain Rob’s method in your own words.</th>
<th>What could be another method?</th>
</tr>
</thead>
<tbody>
<tr>
<td>He added the prices, then divided by how many prices there were.</td>
<td>6.00 + 7.00 = 8.00</td>
</tr>
</tbody>
</table>

1. Tom usually earns $40.00 per hour. He just heard that he had a 5% pay raise. What would be his new pay?

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer: $80.00</td>
<td>$80.00 \div 10% = $8.00</td>
</tr>
</tbody>
</table>

<p>| 2) Maria sees a dress on sale. It is normally priced at $56.00. Now it is on sale for 45% off. How much will she pay for the dress? |</p>
<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$56 \div 2 = 28</td>
<td>$56 \div 45% = 8</td>
</tr>
<tr>
<td>(\frac{56}{2} = 28)</td>
<td>(\frac{56}{45%} = 8)</td>
</tr>
</tbody>
</table>

<p>| 3) Last year, the price of an item was $350. This year it is $450. How would you find the percent change? Write the steps down in order. (You do not have to do the calculation) |</p>
<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>450 - 350 = 100</td>
<td>350 + 100 = 450</td>
</tr>
</tbody>
</table>

| 4) In a sale, the prices in a shop were all decreased by 20%. After the sale, they were all increased by 25%. What was the overall effect on the shop prices? Explain how you know. |
|----------|----------|
| Use prices where raised by 5%. |
| \(-20\% + 25\% = 5\%\) |
Appendix B2: Ms. Lee’s Modified Version of Post-assessment for Percent-change Lesson

1. Tom usually earns $40.00 per hour. He just heard that he had 5% pay raise. What would be his new pay? Explain or show work.

   \[
   \frac{40 \times 5}{40} = 200 \\
   \frac{40}{40} = 100 \\
   \frac{40}{40} = 100 \\
   \frac{40}{100} = 42 \\
   \]

2) Maria sees a dress on sale. It is normally priced at $56.00. Now it is on sale for 45% off. How much will she pay for the dress? Explain or show work.

   \[
   \frac{56.00 \times 0.55}{40} = 31.00 \\
   \frac{31.00}{42} = 42 \\
   \frac{31.00}{42} = 42 \\
   \frac{31.00}{42} = 42 \\
   \]

3) Last year, the price of an item was $350. This year it is $450. How would you find the percent change? Write the steps down in order. \(\text{You do not have to do the calculation}\).

   \[
   450 - 350 \quad \text{it grew} \\
   \text{by 100} \quad \text{by 100}\% \\
   \text{I would subtract} \\
   \text{450 from 350} \\
   \text{to get it and to} \\
   \text{double check I} \\
   \text{would add my} \\
   \text{answer to 350.} \\
   \]

4) In a sale, the prices in a shop were all decreased by 20%. After the sale, they were all increased by 25%. What was the overall effect on the shop prices? Explain how you know.

   \[
   \text{The shoes cost } \$51 \text{ if you mark down} \\
   \text{it by } 25 \text{ what's your } \text{new cost.} \\
   \]

   \[
   \frac{51}{25} = 3 \frac{1}{4} \quad \text{\$50} = 2 \frac{1}{4} \\
   \frac{50}{25} = 2 \frac{1}{2} \quad \text{\$50} \quad \text{\$50.} \\
   \frac{50}{100} = 1 \quad \text{\$50.} \\
   \]