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STOPPING-POWER DIFFERENCES BETWEEN
POSITIVE AND NEGATIVE PARTICLES
AT LOW VELOCITIES

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For a symposium such as we are having this morning on the
"Penetration of Charged Particles in Matter," I doubt if I could present
a formula more fundamental to this subject than the following one:

\[-t = \frac{2m \cdot r_0^2 \cdot z^2}{\beta^2} \left[ \ln \left( \frac{2mc^2 \cdot \gamma^2 \cdot v_{\text{MAX}}}{I^2} \right) - 2\beta^2 - 2 \frac{C}{Z} - \delta \right] \text{mc}^2/\text{cm}\]

where

\[I = \text{mean excitation energy}\]
\[\frac{C}{Z} = \text{shell correction term}\]
\[\delta = \text{density correction term}\]

This is a very familiar expression to us all. It gives the rate at
which a charged particle loses energy by ionization in traversing matter.
For the experimentalist in particle and nuclear physics, it is used, for
example, as the basis for estimating particle velocities; for estimating
the charges carried by relativistic, heavy primary cosmic rays; and,
when augmented with total energy information, for estimating particle
masses, as is now being done so accurately with the solid-state particle
identifier. Certainly, it is basic to any theoretical or semi-empirical
computations on particle ranges in matter.

This expression describes well the energy loss process, as
attested to by the several experimental techniques I have just mentioned.
But, we are also aware of its limitations. The energy-loss formula is principally valid when particle velocities are large compared to the electron velocities in the stopping material. When the velocity becomes relativistic, the expression diverges logarithmically and must be corrected to account for the restriction in energy loss in condensed materials by the polarizability of the stopping medium. At low velocities, the tightly bound electrons are perturbed only adiabatically and therefore do not contribute to the stopping. For multi-charged particles, the mechanisms of charge exchange add yet another complication.

None the less, experiments and theoretical work on the mean excitation energies, $I$, the shell corrections, $C/Z$, and the relativistic density correction term, $\delta$, have led to a rather thorough comprehension of the energy-loss process. I would like to mention that the 1964 report on Studies in Penetration of Charged Particles in Matter published by the National Academy of Sciences - National Research Council exhibits well the "state-of-the-art" of this field.

These very general remarks serve to introduce the subject of my talk, which is on the "Stopping Power Differences Between Positive and Negative Particles at Low Velocities." Quite obviously, the energy-loss formula does not indicate that such a difference might exist. The $z^2$ term disallows any difference between the stopping of positive and negative particles. Of course, the reason for this lies in the fact that this expression for the rate of energy loss was derived using the lowest order Born approximation. This is essentially an impulse approximation, where the interaction time between the incident ion and the atomic
electron is short, \( \tau \approx \frac{\hbar}{E_n} \), where \( E_n \) is the energy transferred in the collision. In order that the sign of the charge of the incident particle enter into the problem one requires long collision times, and the Bethe-Bloch formula is not valid under these conditions.

What, then, is the experimental situation? First, it can be said that differences in the stopping power between positive and negative particles have been observed; and second, that these differences occur for velocities \( \beta < 0.18 \). It is my purpose, then, to review for you the experiments that have given us some information on this interesting effect. As you will become aware, the observations have been few, some of marginal statistical accuracy. All herald the need for more experiments.

The first unambiguous evidence that particles of opposite charge lose energy at different rates came from a series of nuclear emulsion experiments carried out between 1960 and 1963 by Barkas, Dyer, and Heckman. The purpose of their experiments was not to study stopping power differences, but to measure the momenta and masses of the \( \Sigma \) hyperons produced when \( K^- \) mesons are captured at rest by protons. In Table I we summarize results from these experiments that are relevant to our present discussion. When a \( K^- \) is captured by a proton at rest, two final states are possible: (a) \( \pi^- + \Sigma^+ \) and (b) \( \pi^+ + \Sigma^- \). By measuring the ranges and momenta of the \( \Sigma \) hyperons and pions in these reactions as well as the proton and pion decay products of the \( \Sigma \)'s, Barkas, et al., accurately determined the masses of the \( K \) meson and \( \Sigma \) hyperons. They also were able to verify energy and momentum balance for the \( \pi^- + \Sigma^+ \) final state, but, unexpectedly, not for \( \pi^+ + \Sigma^- \). In
reaction (a), the momentum of the $\pi^-$ and $\Sigma^+$ are equal within the quoted errors, whereas in (b) the momenta of the $\pi^+$ and $\Sigma^-$ appear to differ by $2.0 \pm 0.4$ MeV/c, a 5-standard deviation discrepancy. A number of possibilities were considered in order to explain the difference. The final conclusion Barkas, Dyer, and Heckman came to was that everything was all right, provided you neglect the range of the $\Sigma^-$. In other words, they concluded the range of the $\Sigma^-$ hyperon, at the pion momentum of 172.7 MeV/c, was some 25\% greater than expected ($\langle R \rangle = 708.9$ rather than 684\% ) because slow negative particles lose energy at a lower rate than do positive particles at the same velocity. It was at this time we realized that the well established range-energy relation for emulsion was based solely on positive particle ranges. It was now evident that this relation may not apply to slow, negative particles.

The actual suggestion that negative particles lose energy at rates less than do positive particles came from Walter Barkas. The clue here, I'm sure, was that he, along with Wallace Birnbaum and Frances Smith had observed a similar discrepancy between the ranges of positive and negative pions in their pion mass experiments, some seven years earlier.\(^3\) Unexplained at that time was their result that the $\pi^-/\pi^+$ mass ratio was less than unity by $(0.31 \pm 0.14)$\%. A very small, but statistically significant anomaly. The data presented in Table II was obtained from their measurements. After correcting for emulsion distortion, Barkas, Birnbaum, and Smith found that the weighted mean of three measurements of the mass ratio was $0.9969 \pm 0.0014$ (p.e., internal consistency). Note that only one of the three measurements deviates significantly from unity. However, Barkas et al. could
not reveal any systematic errors that might throw this result in doubt, hence included it in the final result. In retrospect, this is precisely what they should have done. Experimenters less confident in their data may well have done differently.

At that time these authors called attention to the possible difference in the stopping cross sections for particles of opposite sign to account for this observation. A $\pi^-/\pi^+$ mass ratio less than unity could be explained if the average energy loss rate of the stopping $\pi^-$ was slightly less than that of the $\pi^+$.

It was Professor Fermi who, after noting this suggestion, pointed out that the Mott Theory of scattering may be applied to the scattering of electrons by both negative and positive pions in the coordinate frame in which the pion is at rest. He found in this way that, because of a sign dependent relativistic term in the Mott theory, the average impulse transmitted to the $\pi^-$ meson is less than that received by the $\pi^+$. The result of his calculations was to increase the apparent mass ratio by about 0.1%. Fermi's correction was included in the final estimate of the mass ratios. Although this correction was not sufficient to explain the range, hence mass, discrepancy, it marks the first instance of a theoretical estimate for the stopping power differences of oppositely charged particles. We point out, however, that the application of the Mott scattering formula is valid when the quantity $\frac{Z}{137B} \ll 1$; and hence, the Fermi correction is not generally applicable to slow, stopping particles.

To corroborate the conclusion that the ranges of the negative pions are greater than those of positive charge at the same velocity, Barkas, Osborne, Simon, and Smith performed an additional experiment to measure
the range difference at 1.6 MeV. In this experiment particular attention was given to the elimination of systematic errors inherent in this type of experiment. Unfortunately, the small number of events (178) seriously limited the statistical accuracy of their result. The difference between negative and positive pion ranges as determined by this experiment was $\Delta R = 3.1 \pm 1.1 \mu$ at a mean pion range of $\langle R \rangle = 96 \mu$.

So far I have only mentioned data on the anomalous ranges of negative particles in emulsion. There is, however, some evidence that stopping power differences also occur in hydrogen. Peter Schmidt has reported on the hyperon masses as obtained from measurements of the ranges and momentum of the products of $K^- + p$ interactions at rest in the Brookhaven 30-in. hydrogen bubble chamber. Here, as in the emulsion experiment described earlier, the masses of the $\Sigma^-$ hyperon determined by methods of range and momentum are not in accord with each other. Surprisingly, the range differences for the $\Sigma^-$ in hydrogen and emulsion appear to be comparable. For example, the anomalous $\Sigma^-$ mass obtained when range measurements are involved in its determination imply that the ranges of the $\Sigma^-$ produced in the $K^- + p$ reaction ($\beta_\Sigma = 0.143$) exceed those expected from the range-energy relation by $1.5 \pm 0.6\%$ in hydrogen, and by $3.6 \pm 0.8\%$ in emulsion.

These data give, I believe, ample evidence for the notion that the range of a particle in matter depends on the sign of the charge as well as mass and velocity. However, these range measurements tell us only that at some velocity, the difference in the ranges of a positive and negative particle is some quantity $\Delta R$. Lacking is information on the how and where this range difference comes about. An obvious next step in the pursuit of this problem is to observe directly the difference in the energy-loss
rates of stopping positive and negative particles. It was with this objective in mind that Peter Lindstrom and I undertook the experiment that I now want to describe. It was our feeling that, because range differences of several percent had been measured for positive and negative pions over ranges of the order of 100μ, the rate of ionization may well differ by some 10% at low velocities. If so, this could be observed as differences in the grain desnities of stopping pions in nuclear emulsion.

The data I shall discuss were obtained from a stack of Ilford G.5 emulsions that was exposed to beams of stopping π⁺ and π⁻ mesons. As some of you may well know, the tracks of stopping pions in G.5 emulsions are highly saturated. It was therefore necessary to limit the development of the emulsion so that ionization measurements were possible, yet would permit the pions to be unambiguously identified as to charge by the emulsion scanner. Figure 1 illustrates how the last 50μ of a stopping π⁻ and π⁺ appeared to the scanner. We divided the last 200μ of the stopping pion tracks into cells, nominally 5 to 50μ in length. The first five of these are super-imposed on the tracks in this illustration. The data we recorded for each cell of the charge-identified pions were (a) the number of blobs, B, (b) the linear fraction of the cell that consisted of gaps, L, and (c) the start and end coordinates of the cell. There was one operational procedure we had to establish, however, before any measurements could be made—namely, where to start the measurements. We have tried to demonstrate this problem in Fig. 1. In case of the π⁻, the last grain could easily be the first grain of a heavily ionizing star prong. On the other hand, if we use the μ-meson
decay track as a guide, there is no blob at all at the ending of the $\pi^+$. Therefore, it was decided that the grain density measurements had to begin at the first well defined blob of the stopping pion track that was separated from the end by a measurable gap. We eliminated, therefore, the ambiguous terminal blob of the track. The actual starting points of the measurements were distributed about an average 1.1\mu from the pion endings. The starting point distributions of the samples of the $\pi^+$ and $\pi^-$ mesons we used to intercompare the rates of ionization were identical.

Table III gives the results of our grain density measurements. Listed for each cell are the average range intervals over which the B and L measurements were made, the mean $\pi^+$ velocity and two estimates of the grain density ratio $g_+/g_-$. In Column (a), the grain density ratios are given by the ratios $(B/L)_+/(B/L)_-$ and in (b) by $\ln L_+ / \ln L_-$. These data are based upon a total of $1.85 \times 10^5$ blob and gap length measurements and are the compilation of the results of five scanners from eleven different emulsion plates. The two values of $g_+/g_-$ are not independent measurements, but do serve to check on the overall accuracy of the grain density measurements.

The data demonstrate quite clearly that there are grain density differences between the positive and negative pions. At the lowest velocity we were able to measure ($\beta = 0.05$), the grain density of the $\pi^+$ is greater than that of the $\pi^-$ by some 7 to 8 per cent. Our results indicate that the grain density ratios decrease monotonically with increasing velocity, becoming consistent with unity at the higher velocities.

To relate grain densities to rates of energy loss, we make the assumption that the grain structure of a particle track results from
energy loss in silver bromide only. This grain-producing ionization of silver bromide is known as the restricted rate of energy loss. In Fig. 2 we relate the observed grain density, $g$, to the restricted energy loss $\nu$ (in units $\text{MeV g}^{-1} \text{cm}^2$). Here we introduce the empirical two-parameter function of $\ln \left( \frac{n}{n-g} \right) = \lambda \ln \nu$ by plotting $\ln \left( \frac{n}{n-g} \right)$ vs $\ln \nu$. $n$ is the average number of silver bromide crystals penetrated by the ion and $\lambda$ is a constant dependent on the sensitivity of the emulsion. We obtain the best maximum likelihood fit of the $\pi^+$ grain density measurements to this function when $n = 2.20 \pm 0.05$ grains $\mu^{-1}$ and $\lambda = 0.0257 \text{ MeV}^{-1} \text{ g cm}^{-2}$.

To obtain the total rates of energy loss from the grain densities is now straightforward. For each observed $g_+/g_-$ ratio, we evaluate the ratio of the restricted energy loss $\nu_+/\nu_-$, from which the ratio and difference of the total rate of energy loss $\nu$ (in units of $\text{MeV per cm}$) is computed. Presented in Fig. 3 are our results on the differences between the rate of energy loss for positive and negative pions vs the pion range. The two data points for each cell (indicated by the hatched areas over the range scale) were obtained from the two estimates of $g_+/g_-$. The dark points with the accompanying error bars are the $\ln L_+/\ln L_-$ data. At $\beta = 0.051$, the stopping power of the $\pi^+$ meson is about 60 MeV/cm greater than that of the $\pi^-$. This corresponds to about a 14% difference in the stopping powers. For velocities $\beta > 0.14$, the energy-loss rates for the positive and negative pions are equal within the 1% statistical errors.

These data on energy loss differences can be also presented in terms of differences in range. In this form we can compare directly the results of this experiment with the range data I have cited.
Figure 4 gives the differences in range between the $\pi^-$ and $\pi^+$ mesons, $R(\pi^-) - R(\pi^+) = \Delta R$, as a function of $R(\pi^+)$. The data points are values of the range differences for pion ranges between 1.1 and 200$\mu$m range, and correspond to velocity interval $\beta = 0.035$ to 0.183. The data are fitted quite well by an exponential function, asymptotic to $\Delta R = 6\mu$m, having a characteristic length of $45 \pm 10\mu$m. This curve is drawn through the data points.

Because of our inability to measure grain densities for ranges between 0 and 1.1$\mu$m, we have no information on the differences in the energy losses of the pions in this range interval. It is, in fact, the unknown behavior of the energy losses for the positive and negative pions for ranges less than 1.1$\mu$m that introduces the largest uncertainty in the estimate of the total range difference. This is illustrated here by the dashed curves above and below the data points. The top curve is the range difference expected if the energy lost by the $\pi^+$ meson in the first micron exceeds that of the $\pi^-$ by an amount $\Delta E = 15$ keV, an arbitrary, but perhaps not unrealistic, value. The lower curve applies if $\Delta E = -15$ keV. We also have included in this figure the three measurements of range differences in emulsion. These are: (a) the $\Sigma^-$ data, where $R$ and $\Delta R$ are normalized by the factor $m_{\pi}/m_{\Sigma}$, under the assumption that the energy loss rates of the negative and positive particles depend on velocity only; (b) The range difference of 1.6 MeV $\pi^\pm$ mesons; and, (c) at 725$\mu$m range the $5.5 \pm 3.2$ range difference between the negative and positive pion observed in the mass-ratio experiment.

Figure 4 presents the current status of emulsion data on the stopping power differences between positive and negative particles.
Clearly demonstrated here is the conclusion that the reported range differences can be fully accounted for by the results of this latest experiment. The sign and magnitude of the differences in the energy-loss rates we have observed are sufficient to reproduce satisfactorily the range difference data. The pion data are within their respective experimental errors. The $\Sigma^-$ data appear to be low, but there may still be unknowns in the behavior of stopping $\Sigma^-$ hyperons. Possible evidence for this is the excessive range straggles of the sample of $\Sigma^-$ hyperons used by Barkas, et al. to reveal the $\Sigma^-$-range anomaly.

In summary, the experimental evidence decisively shows that the rates of energy loss for positive particles exceed those for negative particles at equal velocities, when these velocities are comparable to those of the atomic electrons of the stopping medium. Experiments to determine the energy-loss differences at very low velocities are clearly needed. How the stopping powers of positive and negative particles depend on the atomic number of the stopping material is another problem that should be examined. Theoretical guide lines are conspicuously absent and are urgently needed.

A promising direction for theory is to examine how the Bethe-Bloch formula can be extended by using higher Born approximations. Barkas, who carried out some preliminary investigations toward this end, found that the second-order Born approximations introduces a term in the energy loss expression that is proportional to $z^3$ of the incident particle.\textsuperscript{7} Such a term is of the correct nature to account for the observations, but no estimate was made as to its magnitude.
The proposition that energy loss by ionization in matter is dependent on the sign of the incident particle is new. It adds another dimension to stopping power theory. To understand it offers us a formidable problem.
REFERENCES


FIGURE CAPTIONS

Figure 1. Illustration of stopping $\pi^-$ and $\pi^+$ meson in underdeveloped G.5 emulsion. The first five cells (5 to 25 $\mu$ in length) in which grain density measurements were made are superimposed on the tracks in this figure. Grain-density measurements were made between 1.1 and 200$\mu$ from the pion endings.

Figure 2. Restricted rate of energy loss $\iota'$ (in units MeV g$^{-1}$ cm$^2$) versus $\ln\left(\frac{n}{n-g}\right)$. $\iota'$ is computed for a 2 keV $\delta$-ray energy cut-off.

Figure 3. The difference between the total rates of energy loss for positive and negative pions, $\iota_+ - \iota_-$, vs range. The energy-loss differences evaluated from the grain density ratios given in columns (a) and (b), Table III, are denoted by the symbols $\times$ and $\circ$, respectively. The hatched areas above the range scale indicate the interval of range over which the ionization measurements were made.

Figure 4. The differences between the $\pi^-$ and $\pi^+$ ranges, $\Delta R = R(\pi^-) - R(\pi^+)$ vs the pion range, as derived from the energy loss differences, Fig. 3. For ranges greater than 1.1$\mu$, $\Delta R$ can be represented by the function

$$\Delta R = 6\left[1 - \exp\left(\frac{R-1.1}{45}\right)\right].$$

This curve is drawn through the data points. The dashed curves above and below the data illustrate how $\Delta R$ depends on the (unknown) difference in energy loss between 0 and 1.1$\mu$. The top curve applies if the total energy lost by the $\pi^+$ in the first micron exceeds that of the $\pi^-$ by $\Delta E = 15$ keV.
The lower curve applies if $\Delta E = -15$ keV. The range differences reported in References 2, 3, and 4 are also shown.
TABLE CAPTIONS

Table I. Ranges of $\pi^+$ and $\Sigma^+$ from $K^- + p$ reaction (at rest), from Ref. 2.

Table II. $\pi^-/\pi^+$ mass ratios, from Ref. 3.

Table III. Grain-density ratios $g_+/g_-$ as evaluated from (a) $(B/L)_+(B/L)_-^{-1}$ and (b) $\ln L_+/\ln L_-$.

Fig. 2

- \lambda = 0.0257
- n = 2.20 \mu^{-1}
Fig. 3
Fig. 4
$K^- + p \rightarrow \begin{cases} 
\pi^- + \Sigma^+ (1189.4 \text{ MeV}) & \text{(a)} \\
\pi^+ + \Sigma^- (1197.6 \text{ MeV}) & \text{(b)} 
\end{cases}$

<table>
<thead>
<tr>
<th></th>
<th>$\pi^-$</th>
<th>$\Sigma^+$</th>
<th>$\pi^+$</th>
<th>$\Sigma^-$</th>
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<tbody>
<tr>
<td>Range</td>
<td>88.6 ± 0.5mm</td>
<td>818.8 ± 1.7μ</td>
<td>78.4 ± 0.2mm</td>
<td>684 ± 5μ*</td>
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<tr>
<td>Momentum</td>
<td>181.2 ± 0.4</td>
<td>181.3 ± 0.1</td>
<td>172.7 ± 0.4</td>
<td>172.7 ± 0.4</td>
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* Expected from range-momentum relation
### Pion Mass-Ratio Experiment: $m_\pi^-/m_\pi^+$

<table>
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<tr>
<th>Plate</th>
<th>Mass ratio (obs.)</th>
<th>Mass ratio (corrected for distortion)</th>
<th>Probable error</th>
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</thead>
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<tr>
<td>a</td>
<td>0.9908</td>
<td>0.9919</td>
<td>0.0021</td>
</tr>
<tr>
<td>b</td>
<td>0.9993</td>
<td>0.9992</td>
<td>0.0023</td>
</tr>
<tr>
<td>c</td>
<td>1.0009</td>
<td>1.0006</td>
<td>0.0023</td>
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</table>

**mean:** 0.9969 0.0014

*with Fermi correction:* 0.9978

Table II
<table>
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<tr>
<th>Cell</th>
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<th>( \langle \beta \rangle )</th>
<th>( \frac{g_+}{g_-} )</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(a)</td>
</tr>
<tr>
<td>1</td>
<td>1.1 - 5.1</td>
<td>0.051</td>
<td>1.078 ± 0.022</td>
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<tr>
<td>2</td>
<td>-10.1</td>
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<td>1.030 ± 0.016</td>
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<tr>
<td>3</td>
<td>-15.2</td>
<td>0.084</td>
<td>1.018 ± 0.015</td>
</tr>
<tr>
<td>4</td>
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<td>0.097</td>
<td>1.050 ± 0.013</td>
</tr>
<tr>
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<td>0.117</td>
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</tr>
<tr>
<td>7</td>
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<tr>
<td>8</td>
<td>-199.9</td>
<td>0.178</td>
<td>1.006 ± 0.014</td>
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Table III
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