Title
Heavy ion beams for inertial confinement fusion

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Heavy Ion Beams for Inertial Confinement Fusion

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Heavy Ion Beams For Inertial Confinement Fusion

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UCSD Plasma Physics Seminar
Nov. 8th, 1999

Fusion Context & Parameters

Beam Currents & Accelerator

Beam Transport: Analogy with Photons

Quadrupole Magnet Transport

Role of Space Charge

Breathing Mode Instability
Power Plant

\[ \text{3 Plant Efficiency} \times \text{1000 MW Electric} \times \text{10 Hz Rep Rate} = \text{3000 MW Fusion Energy} \]

\[ \text{100 mTons x Gain of 75 = 75 mTons} \]

Typical High \( \geq \) Case

Ion Source

Stepper

X-Ray

Theoretical

PP and Pellet Fusion
Heavy Ions Have Short Range
In The Stopper at High Energy

\[ T = 4.0 \text{ GeV} \quad C^+ \]
\[ \rightarrow \text{Range} = 0.1 \text{ gm/}cm^2 \]

\[ \text{Total Ion} = \frac{4.0 \text{ MJ}}{4.0 \text{ GeV}} = 10^{-3} \text{ C} \]

\[ \text{Charge} \]

\[ \text{Ions into target in} \quad 10^{-8} \]
\[ \rightarrow 4 \times 10^{-4} \text{ Watts beam power} \]
\[ 100,000 \text{ Amperes beam current} \]

Relativistic Factor
\[ \beta^2 = \sqrt{\left(\frac{I}{mc^2}\right)^2 + 2\left(\frac{I}{mc^2}\right)} \]
\[ = 0.256 \]

- Only Slightly Relativistic -
Currents

100,000 Amperes is divided among many beams to reduce space charge effects

Say $N_{beam} = 50$

$\Rightarrow I_{beam} = \frac{100,000}{50} = 2000$ Amperes

At Source:
- $T = 1.6$ MeV
- $I_{beam} \approx 0.5$ Ampere

Accelerate to 4000 MeV

$\Rightarrow I$ increases by $\times \sqrt[1.6]{4000^2} = 50$

Compress during Acceleration by $\times 4$

End of Accelerator $I_{beam} = 4 \times 50 \times 0.5 = 100$ Amp

Compress another $\times 20$ from Accelerator to Target $\Rightarrow 2000$ Amp
Induction Linear Accelerator

N Beams in Acceleration Gap

Magnets to Transport Beams

Insulator

Ferro magnetic Torus

Switch + HV Supply
52 Beam (efficient) Bundle In Magnet

- cryostat

- structural support

- thermal insulation

- vacuum

- beam
Ions Are Transported by a Periodic System of Magnetic Lenses

Analogous With Photons

Focus Lens

\[ f = \text{focal length} \]

Defocus Lens

Periodic System Case of \( \frac{1}{2\pi} = L \)

Photon displacement \( x(\alpha) \)

Periodic Orbit

\[ x(\alpha) \sim \cos\left(2\pi \frac{\alpha}{6L}\right) \]
Case of $f = \lambda$ cont.

Define $\phi_0 = \text{phase advance} = \frac{2\pi}{\lambda} = 60^\circ$

Per lattice period.

Try $f = 4/\lambda$:

Orbit period $= 4\lambda \rightarrow \phi_0 = 90^\circ$

Try $f = 4/\lambda$:

Orbit period $= 2\lambda \rightarrow \phi_0 = 180^\circ$

General Formula:

\[
\cos(\phi_0) = 1 - \frac{4f}{\lambda}
\]

<table>
<thead>
<tr>
<th>$\frac{4f}{\lambda}$</th>
<th>$\phi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$1$</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>$2$</td>
<td>$90^\circ$</td>
</tr>
<tr>
<td>$4$</td>
<td>$180^\circ$</td>
</tr>
</tbody>
</table>
Photon Analogy - Cont.

Problem: Show \( \cos(\alpha_0) = 1 - \frac{1}{2}\eta \)

Use
\[
\begin{pmatrix}
X(t) \\
X'(t)
\end{pmatrix} = M
\begin{pmatrix}
X(0) \\
X'(0)
\end{pmatrix}
\]

Find Eigenvalues of \( M \), etc...

What about \( \frac{1}{4} > \eta \rightarrow \cos(\alpha_0) < -1 \)

→ Unstable Orbit →

Return to \( \frac{5}{\Phi} = 4 \)

A different Orbit from Case on p. 5

Lens locations
\[ \frac{L}{r} = 4 \text{ cont.} \]

This orbit shows linear growth on the average!

\[ \rightarrow \text{We are at a half integer resonance} \]

\# of oscillations = \( \frac{1}{2} \# \) of lattice periods

\[ \rightarrow \text{We are at edge of step point} \]

For \( \sigma < 15^\circ \), stable oscillations but large local variations deviations from \( \cos \left( \frac{6\sigma}{L} \right) \) As \( \sigma \rightarrow 15^\circ \)

\[ \frac{d^2x}{dt^2} = -\left( \frac{\sigma_0}{L} \right)^2 x \]
We can also alternate focus & defocus lenses and still transport photons.

Case of $f = L$:

- Orbit
- Period = $12L$

Phase advance = $360^\circ$ in $12L$

But lattice period = $2L$ New

$\therefore \sigma_0 = \frac{360^\circ}{6} = 60^\circ$

General Formula:

$$\cos(\sigma_0) = 1 - \frac{L^2}{2f^2}$$

<table>
<thead>
<tr>
<th>$\frac{L}{f}$</th>
<th>$\sigma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>60$^\circ$</td>
</tr>
<tr>
<td>$\sqrt{2}$</td>
<td>90$^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>180$^\circ$</td>
</tr>
<tr>
<td>&gt;2</td>
<td>unstable</td>
</tr>
</tbody>
</table>
For Charge Particles

In principle lenses can be short solenoids.

\[ \frac{1}{l_s} = l_s \left( \frac{B_s}{2BR} \right)^2 \]

- \( l_s \) = solenoid length
- \( B_s \) = \( u \) field
- \( [BR] = \frac{momentum}{charge} = "Rigidity" \)
  
  \[ [BR] = 3.107 \times 10^7 \frac{A}{Z} \]

\( A = \) mass in amu
\( Z = \) charge state
Consider 4.0 GeV C\textsuperscript{5+} \( A = 133 \) \( Z = 1 \)

\[
[BR] = 3.107 \times 0.256 \times 133 = 106 \ T \cdot m
\]

Say \( B_1 = 5.0 \ T \)

\[ \Rightarrow \text{Radius of Curvature} \quad R = \frac{106}{5} = 21 \ m \]

Generally \( R \gg \text{Magnet Lens Size} \)

- Opposite of Magnetic Confinement of Plasmas

Say \( S = 1.0 \ m \)

\[ [BR] = 106 \ T \cdot m \quad B_5 = 10 \ T \]

\[ \Rightarrow \frac{1}{\gamma} = 1.0 \times \left( \frac{10}{2 \times 106} \right)^2 = \frac{1}{450} \ m \]

- Not very useful here

Good for Electrons!
**Big Idea:** Use Magnetic Quadrupoles

The solenoid is weak because $B$ is nearly parallel to beam.

For quadrupole $B$, it is normal.

$B_x = G y$

$B_y = G x$

$D_+ \cdot B_+ = 0$

$D_- \times B_- = 0$

$G = \text{Quadrupole Gradient}$

$$\frac{dP_x}{dt} = -2e \frac{\sqrt{2}}{2} B_y$$

$$\frac{dP_y}{dt} = +2e \frac{\sqrt{2}}{2} B_x$$

cont
Quadrupole - Cont.

\[ \frac{d^2 X}{d z^2} = - \frac{G}{[CBR]} \times \]

\[ \frac{d^2 y}{d z^2} = + \frac{G}{[CBR]} \quad y \]

\[ \Rightarrow \quad \frac{1}{r} = \frac{G \cdot \text{quad}}{[CBR]} \]

Focus in One Plane + Defocus In The Other \rightarrow Alternate Polarity To Get Transport In Both Planes (Recall Photons)

Say \( G = 50 \text{ T/m} \)

\[ \text{quad} = 0.5 \text{ m} \]

\[ [CBR] = 106 \text{ T-m} \]

\[ \Rightarrow \quad \frac{1}{r} = \frac{50 \times 0.5}{106} = \frac{1}{4.25} \text{ m} \]

\( \Rightarrow \quad r \approx 0.23 \text{ m} \)
Summary So Far

1. Use Alternating Quad Polarities
   \[ \frac{d^2 X}{dt^2} = -\left(\frac{\sigma_0}{2l}\right)^2 X \quad \text{same for } y \]

2. \[ \cos \sigma_0 = 1 - \frac{2L^2}{\pi^2} \quad \text{Thin long} \]

3. Keep \( \sigma_0 \) < 180°

4. \[ \frac{1}{T} = \frac{G \text{ quad}}{CBR} \]

5. \( \text{quad} \ll L = \text{spacing of lattice (half period)} \)

6. \[ G \leq \frac{3 \text{ Tesla}}{\text{Beam Radius}} \quad \text{Gradient} \]

7. \[ [BR] = 3.107 \geq A/\rho \quad \text{Rigidity} \]

* \( \sigma_0 \) To be reduced later
Why Hold The Beam Together?

1. Transverse Thermal Pressure
   - Small -

2. Transverse Space Charge Force
   - Large -

Slug of charge moving at \( v_x \approx \beta c \)

Roughly

\[
\frac{1}{r} \, \sin^{-1} E_r = \frac{r}{\varepsilon c}
\]

Assume uniform \( s \) \( \alpha < r < a = \varepsilon dy c \)

\[
E_r \approx \frac{E r}{\varepsilon c} \quad s = \frac{A}{\pi a^2}
\]

\[
\frac{dP_x}{dt} = ze \left( E_x - \frac{v_x}{\beta c} B_y \right)
\]

\[= \frac{ze}{r} E_r \times \frac{L}{r^2} \times \text{From} \frac{v_x}{\beta c} B_y
\]

\[= \frac{ze}{\varepsilon c} \frac{L}{r^2} \times \frac{ze A}{2 \pi c a^2 y^2}
\]

\[= \left( \frac{ze A}{2 \pi c a^2 y^2} \right) \times \]
Recall \( I = \frac{1}{\beta c} \), \( P_x = \frac{\gamma m \frac{dx}{dt}}{dt} \)

\[
\frac{d^2 x}{d\tau^2} = \frac{g}{a^2} x
\]

\[
G = \frac{2 \pi e I}{(3\gamma)^3 \frac{e^2}{4\pi \varepsilon_0} M c^5} = \text{"Dimensionless Pervance"}
\]

\[
G = \frac{2 \tau (I / 31,07 \times 10^6)}{(3\gamma)^3 A}
\]

Say\( I = 100 \) Amperes

\[
\begin{align*}
\beta \gamma &= 0.256 \\
A &= 15.3 \\
\tau &= 1
\end{align*}
\]

\[
G = 2.88 \times 10^{-6}
\]

Looks small but it is important

With no quadrupoles

\[
Z_{\text{blow up}} \approx \frac{u}{10} \approx 18 \text{ m}
\]

\[
\alpha = 2.9 \times 10^{-6}
\]
Combine Equations of Motion

\[ \frac{d^2 x}{d \tau^2} = -\left(\frac{\alpha}{2L}\right)^2 x + \frac{a}{a^2} x \]

Quad Space Charge

For \( \alpha = \text{constant} = \alpha_0 \)

\[ x = \cos\left(\frac{\alpha}{2L} \right) \]

where \( \left(\frac{\alpha}{2L}\right)^2 = \left(\frac{\alpha_0}{2L}\right)^2 - \frac{G}{\alpha_0^2} \)

\( \sigma = "\text{Depressed Tune}" \)

Typical Fusion \( \alpha_0 \gtrsim 7 \sigma_0 \)

\( \sigma \) by \( 7 \sigma_0 \)

\( \sigma \ll \sigma_0 \) Because Thermal Pressure Is Low

But Thermal Pressure Becomes Important When Beam is Focused onto the Fusion Target.
Transport Limit

\[
\sigma \to 0 \quad (\text{Cold Beam})
\]

\[
\Rightarrow \mathcal{G} \sim \left( \frac{\sigma_0}{2L} \right)^2 \alpha \theta_0
\]

Recall \( \mathcal{G} \sim \frac{Z}{(\beta \gamma)^3 A} \)

\[
\Rightarrow \quad I_{\text{max}} \sim \sigma_0^2 \left( \frac{\alpha}{L} \right)^2 (\beta \gamma)^3 \frac{A}{Z} \]

Application of this formula to cost optimization is not obvious.

\[
I_{\text{max}} \approx (1465 \text{ Amps}) \left( \frac{3}{.256 \text{ rad}} \right) \left( \frac{2}{1.5 \text{ rad}} \right) \frac{A}{15.3} \]

At low energy \( \beta \gamma \approx 0.005 \) (1.6 MeV)

\[
\alpha / L \approx 0.07 \text{ Typical}
\]

\[
\Rightarrow \quad I_{\text{max}} \approx 0.53 \text{ Amps} \quad C_0^+ \]

But \( I_{\text{max}} \) rises rapidly with \( \beta \gamma \).
Breathing Mode Instability

Recall \( \frac{d^2 x}{d^2 t} = -\left(\frac{\sigma_0}{2L}\right)^2 x + \frac{Q}{a^2} x \)

For cold Beam \((\sigma \to 0)\) An Ion
At The Beam Edge Stays There

\( x \to a \)

Let \( a = a(t) \)

\( \Rightarrow \frac{d^2 a}{d^2 t} = -\left(\frac{\sigma_0}{2L}\right)^2 a + \frac{Q}{a} \)

Equilibrium \( a = a_0 \)

\( \Rightarrow \frac{\sigma}{a} = -\left(\frac{\sigma_0}{2L}\right)^2 a_0 + \frac{Q}{a_0} \) \( \{ \text{Previous Result} \} \)

Perturb \( a = a_0 + sa(t) \)

\( \frac{d^2 sa}{d^2 t} = -\left(\frac{\sigma_0}{2L}\right)^2 sa - \frac{Q}{a_0^2} sa \)

\( = -2 \left(\frac{\sigma_0}{2L}\right)^2 sa \)
Breathing Mode - Cont

\[ 
\sigma = e^{-i \Omega z} \]

\[ \rightarrow \sigma^2 = z \left( \frac{\sigma_0}{zL} \right)^2 \]

Phase Advance of Breathing

\[ \sigma_{\text{breath}} = \sqrt{2} \sigma_0 \]

Per Lattice Period (2L)

- I have cheated -

The equilibrium and perturbed equations have modulations of period 2L from quadrupoles

\[ \Rightarrow \text{Half-Integer Resonance and Stop Band At} \]

\[ \sigma_{\text{breath}} \geq 15^\circ \]

\[ \Rightarrow \text{Need } \sigma_0 \leq \frac{15^\circ}{\sqrt{2}} = 17.7^\circ \]

Experiments and PIC Simulations Show Trouble For \( \sigma_0 \geq 8.5^\circ \)

- Unresolved Mystery -