UNIVERSITY OF CALIFORNIA, SAN DIEGO

Optical and Mechanical Behavior of the Optical Fiber Infrasound Sensor

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by

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The thesis of Scott DeWolf is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

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2009
DEDICATION

To Sarah and Delilah.
Only the mediocre are supremely confident of their ability.
— Sir Michael Atiyah
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I have come to understand that successful experimentation is, amongst other less flattering things, a combination of patience and experience. This has been amplified in my case thanks to Dr. Glenn Sasagawa, who has always taken the time to listen and provide useful suggestions for most of my laboratory work to date.

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ABSTRACT OF THE THESIS

Optical and Mechanical Behavior of the Optical Fiber Infrasound Sensor

by

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The Optical Fiber Infrasound Sensor (OFIS) is an interferometric pressure transducer consisting of a pair of optical fibers helically wrapped about a compliant tube. While the OFIS has been successfully deployed for nearly a decade, its performance has been plagued by downtime due to polarization fading (resulting from no interference fringe intensity) and a nearly continuous change in its pressure sensitivity. This thesis explores the use of very expensive polarization maintaining fiber and inexpensive Faraday mirrors, both of which solve the polarization fading issue. Laboratory measurements of the thermal behavior of the pressure sensitivity are found to confirm field tests, but measurements of the temperature dependence of the tube’s elastic modulus does not appear to adequately describe the observed behavior. Therefore, a crude analytic mechanical and optical model is explored to help investigate potential causes, however, the simple theory of compound thick cylinders was found to be problematic as a realistic model for the OFIS.
Chapter 1

Introduction

Sound can be defined as pressure fluctuations about some background level, as in the case of human speech at atmospheric pressure. The frequency content of sound can be broken into three categories roughly defined by the range of human hearing. This audible range is often taken to be 20-20,000 Hz, and sounds above and below this threshold are termed ultrasound and infrasound, respectively. In this thesis, the term infrasound is restricted to pressure fluctuations below 20 Hz with atmospheric pressure near the surface as the background level.

1.1 Infrasound and Infrasound Instrumentation

There are many examples of natural and anthropogenic sources of infrasound. While too numerous to describe individually, some naturally occurring sources include earthquakes (seismic-to-acoustic coupling), auroras, meteors, tsunamis, volcanoes (Reference [4] provides a review of the preceding phenomena), mountain-associated waves [7], atmospheric solitons [8], and convective storms [16]. The most ubiquitous and commonly studied source in coastal areas are microbaroms, which arise from the forcing of the lower atmosphere by nonlinear wave-wave interactions at the ocean surface [28]. Investigations of man-made sources range from shock waves from supersonic aircraft [12], wind turbines [22], and rocket launches [11], to mining blasts [1] and nuclear explosions [9, 10]. It is this last case that prompted the International Monitoring System (IMS) to in-
clude a global network of infrasound arrays along with seismic, hydroacoustic and radionuclide sensors for monitoring the Comprehensive Nuclear Test Ban Treaty.

The most significant limitation to measuring infrasound are pressure fluctuations due to turbulence, or “wind noise,” and while there exist several high quality infrasound microphones, or microbarometers, instrument development is largely focused on noise reduction technology. A comprehensive review of wind noise physics, abatement schemes and their limitations can be found in Reference [29], however, there appear to be two strategies that can be used individually or in combination. The first exploits the fact that these turbulent structures are spatially incoherent by mechanically averaging the pressure over an area. This is accomplished by connecting a microbarometer to an elaborate array of pipes whose inlets are distributed over a wide area (Figure 1.1), and is the current method used with IMS certified infrasound stations. There are an increasing number of studies centered around the concept of screening the turbulence by enclosing a microbarometer with or without a pipe array in a porous fabric tent. The principle drawbacks to either method are that they are expensive, labor intensive to build and maintain, and have a large ecological footprint, exacerbated by the fact that 23 of 60 IMS infrasound stations are on, or proposed for, remote oceanic islands [19].

1.2 The Optical Fiber Infrasound Sensor

Optical fibers are well known for their ability to measure strain [6], temperature and pressure [20], which motivated the development of a fundamentally different microbarometer that came to be known as the Optical Fiber Infrasound Sensor (OFIS) [34]. The OFIS consists of a compliant rubber tube helically wrapped with a pair of optical fibers, which form an equal-arm Mach-Zehnder interferometer. Laser light, typically at 1,310 nanometer wavelength, enters one end of both the double and single wrapping only to loop back where the other ends are combined to form interference fringes (Figure 1.2). Since the fibers are less elastic than the tube, they act as local reinforcement such that the tube near the double wrapped
fibers experiences less deformation than the single wrap for a given amount of strain in the tube. Therefore, the OFIS measures differential strain in the tube as a result of changes in the pressure field. Finally, both path lengths are modulated by a piezoelectric cylinder and the resulting interference fringe intensity is monitored by a photodetector. The necessity of modulating the paths allows for the discrimination between contractive and expansive fringes (negative and positive pressures, respectively), which is described in Appendix A.

There are several unique characteristics of the OFIS that can be exploited for measuring infrasound. Since the light experiences little attenuation in an optical fiber, the OFIS can be made almost arbitrarily long. When deployed, the OFIS is wrapped in fiberglass insulation and inserted into a perforated tube to allow burial beneath a porous medium. Therefore the OFIS employs both wind noise reduction technologies at once, but has the further advantage over mechanical filters in that it averages the turbulence at the speed of light rather than the speed of sound. The result is a lower limit on infrasonic noise in the 1 to 10 Hz band, as evidenced by Reference [34], with a much smaller footprint than a conventional pipe array. Given the flexibility of the OFIS, it can be deployed in different geometries, each with different amplitude and frequency responses that depend upon the direction of arrival. In the case of an array of linear OFISs, this has lead to the development of novel beamforming techniques to estimate the elevation, back azimuth, and phase velocity of infrasonic signals [31].

Unfortunately, temperature has a severe impact on the performance of the OFIS, and it is the purpose of this thesis to quantify what is happening and describe attempts to eliminate or mitigate these effects. Chapter 2 is concerned with the intensity of the interference fringes, which depends upon the polarization of the laser light as it emerges from each arm. As the temperature of the OFIS changes, the state of polarization in each of the two fibers changes with respect to the other. When the polarization states are orthogonal, the interference intensity is zero causing the sensor to go offline, and both the use of polarization maintaining (PM) fiber and Faraday mirrors (FM) are investigated. Finally, chapter 3 examines how temperature changes the sensitivity of the OFIS, which is quantified by the
optical phase difference per unit pressure. While temperature modestly changes the
elasticity of the tube, a crude analytic model is explored to understand possible
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Figure 1.1: Typical configuration for a high frequency 18 meter, 92-port and a low frequency 70 meter, 144-port pipe array (shown to scale). Each connects to a central microbarometer.
Figure 1.2: The Mach-Zehnder OFIS schematic, along with an exposed length shown next to a pencil for scale. Note that the OFIS is wrapped in fiberglass insulation and inserted into a 0.1 meter diameter corrugated drainage tube for both ground-lying and burial deployments.
Chapter 2

Temperature-Induced Polarization Drift

The first limitation of the OFIS concerns the size of the ellipse formed by plotting the interference fringe signal versus its derivative. A least squares fit to this ellipse is performed by a digital signal processing system called the Femtometer (Appendix A) to determine the optical phase difference in radians. When the light from each arm of the OFIS is combined, the resulting interference fringe intensity depends upon the relative phase of the two beams. However, the intensity contrast between the constructive and destructive interference fringes depends upon the relative state of polarization (SOP) of each arm with respect to the other. For example, if both arms emit linearly polarized light where their electric fields are both vertical, the resulting intensity will depend only on their relative phase. However, if the electric field of one arm is horizontal while the other remains vertical then no interference can take place. Therefore, the fitted ellipse changes size with the relative SOP, which can lead to inaccurate phase estimates when the SOP in the two arms are near orthogonal and the ellipse becomes very small, taking the sensor offline.
2.1 Observations of Polarization Drift

A common way to express the SOP of monochromatic light is in terms of the Stokes parameters, which can be represented graphically as a point on a sphere whose radius is determined by the intensity, known as the Poincaré sphere. Following the convention of Born and Wolf [5], the $x$ and $y$ components of an electromagnetic wave propagating in the $z$ direction are:

\begin{align*}
E_x &= a_1 \cos (\tau + \delta_1) \\
E_y &= a_2 \cos (\tau + \delta_2),
\end{align*}

where $a_{1,2}$ and $\delta_{1,2}$ are the amplitudes and phases, and $\tau = \vec{k} \cdot \vec{x} - \omega t$, the wave vector, spatial location, angular frequency, and time, respectively. The Stokes parameters are defined as:

\begin{align*}
s_0 &= a_1^2 + a_2^2 \\
s_1 &= a_1^2 - a_2^2 \\
s_2 &= 2a_1a_2 \cos (\delta) \\
s_3 &= 2a_1a_2 \sin (\delta),
\end{align*}

where $\delta = \delta_2 - \delta_1$. It is straightforward to show that:

\[ s_0^2 = s_1^2 + s_2^2 + s_3^2. \]

By eliminating $\tau$ from (2.1,2) and reducing the resulting quadratic to canonical form, it can be shown that the SOP itself can be represented by an ellipse. Finally, a stereographic projection of the SOP ellipse onto a sphere [23] yields:

\begin{align*}
s_1 &= s_0 \cos (2\chi) \cos (2\psi) \\
s_2 &= s_0 \cos (2\chi) \sin (2\psi) \\
s_3 &= s_0 \sin (2\chi).
\end{align*}

The results are the equations of a sphere of radius $s_0$, with longitude $2\psi$ and latitude $2\chi$. For example, $(2\psi, 2\chi)$ states of linearly polarized light fall along the equator starting with horizontal at $(0, 0)$ and vertical at $(\pi, 0)$, whereas right and
left circular polarizations occur at the north and south poles, respectively. All the states in between are generically referred to as elliptically polarized, with their handedness determined by the upper and lower hemispheres.

Measuring the SOP of each OFIS arm was the first step to determine what could be done to solve the polarization drift problem. This was done with a polarimeter, which typically operates by splitting the input beam into multiple paths and passing each through a different series of polarizers and retarders whose output intensities are proportional to the Stokes parameters [17]. In this case, a General Photonics POD-101A in-line polarimeter was used, whose accompanying software was able to collect all four Stokes parameters at a minimum of 1,000 samples per second for a maximum of four hours. Since the interference ellipse size changed most significantly with temperature, and the steepest temperature gradients occur at sunrise and sunset, SOP measurements were taken on each arm of a 10 meter ground-lying OFIS from 4:30 to 8:30 PM local time. The results (Figure 2.1) show that the SOP in the single wrap arm varies considerably, especially in comparison to the double wrap arm. Since the SOP perturbations are most likely caused by stress-induced birefringence imparted during thermal expansion and contraction, and given that the OFIS functions based on the strain differential between the single and double wrap fibers, it follows that the SOP variation would be greater for the single wrap.

2.2 The Polarization-Maintaining OFIS

2.2.1 Design, Construction and Deployment

Polarization can be maintained in an optical fiber by inducing a birefringence profile in the plane perpendicular to the axis of the fiber, thus creating a preferential orientation for the electric field to propagate. In PM fiber (not to be confused with polarizing fiber) this is accomplished by including a material with a different coefficient of thermal expansion, such that when the glass is heated and drawn into a fiber, thermal stress-induced birefringence occurs upon cooling. There are three main types of PM fiber (Figure 2.2), each with differing characteristics
such as birefringence contrast and uniformity over the length of the fiber. Only Corning PANDA and Fibercore HB1500 Bow-tie types were used for the prototype PM OFISs since PANDA is the most widely available and Bow-tie had the highest birefringence contrast making it less likely to experience loss of PM characteristics under helical winding.

Given that PM fiber was over one hundred times more expensive than the typical Corning SMF-28 fiber, it was decided to deploy three unequal arm Mach-Zehnder OFISs. Each OFIS consisted of two 30 meter, 900 micron jacketed fibers whose helical wraps were 5.5 cm apart, resulting in total lengths of approximately 18 meters. The availability of a NP Photonics “Rock” 1,533 nanometer wavelength laser source with a coherence length of several hundred kilometers (spectral linewidth of less than 1 kHz) allowed the second arm to be only 3 meters, which was deployed in an attached instrument box. Finally, the sensors were wrapped with fiberglass insulation and inserted into 0.1 meter diameter plastic perforated tube.

The three sensors were deployed on the ground at the Camp Elliott Field Station (Figure 2.3) located about 20 kilometers east of the SIO campus. Since there was only enough hardware to run one sensor at a time, each was run independently for several weeks. Temperature data was collected between the OFIS tube and insulation and on the outside of the perforated tube with two Onset HOBO sensors, which consist of a thermocouple and data logger. An OFIS calibration system (contained in the gray box in the lower left-hand corner of Figure 2.3), consisting of a commercial loudspeaker hermetically sealed to a plastic plate and a calibrated Setra 265 pressure transducer, was connected to the internal volume of each OFIS tube running a 0.1 Hz continuous wave at approximately 60 Pascals (peak-to-peak).

### 2.2.2 Results: Ellipse Stability and Optical Phase Noise

The PM fiber provided very good ellipse stability, resulting in no downtime due to polarization fading. This is quantified by the size of the interference ellipse, which can be calculated from the parameters determined by the Femtometer
system (Appendix B). Histograms of two days of data show that both PM OFIS ellipse areas show some variability, but are not in danger of approaching zero. This is in contrast to the SMF-28 OFIS whose ellipse area variance is over quadruple that of the PM OFISs, and also has a non-zero probability of collapse. Hence, the use of PM fiber does solve the polarization fading issue.

Shortly after the initial setup it became apparent that fluctuations in the laser wavelength caused by the unequal arms were contributing a significant amount of noise to the optical phase data. This was partially remedied by subtracting the signal from a second unequal arm “null” interferometer consisting of a 30 meter and 1 meter SMF-28 fiber housed in the adjacent building. Since the lengths of the null interferometer arms were not identical to the OFISs, the null signal needed to be scaled by a constant trivially determined by least squares. Figure 2.5 is an example of this differencing on the Bow-tie OFIS, and shows a residual noise of about 5 to 6 Hz not found in a Brüel and Kjær (B&K) 4193 reference microphone located 10 meters west of the OFISs. Investigating the causes of this residual noise and/or constructing an identical null interferometer, in addition to other problems such as the large amounts of scatter in the fitted ellipse and oscillations in the ellipse area caused by the calibration signal were not investigated due to the results of the following section.

2.3 Faraday Mirrors and the Michelson OFIS

Compensating for unwanted birefringence in the path of an interferometer by creating a phase conjugate (or time reversed) beam was first investigated by Martinelli in 1989 [27], and successfully implemented shortly thereafter in a fiber optic Michelson interferometer by Kersey [24]. This was done by using a Faraday rotator followed by a mirror, which has come to be known as a Faraday rotator mirror, or Faraday mirror (FM). To understand why this particular combination works, one must be familiar with the Jones calculus for modeling optical components.

The Jones calculus is a common way to mathematically express how fully
polarized light behaves as it passes through optical components such as polarizers, rotators, and retarders or wave plates. Similar to Section 2.1, an electromagnetic wave can be expressed as a complex vector [18]:

\[
\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} a_1 e^{i(\tau + \delta_1)} \\ a_2 e^{i(\tau + \delta_2)} \end{pmatrix}
\] (2.11)

For example, horizontally polarized light implies \(a_2 = 0\), and normalization reduces the Jones vector to:

\[
\vec{E}_h = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

whereas right circularly polarized light implies that the component amplitudes are equal \((a_1 = a_2)\), and the phase of \(E_y\) is shifted by \(-\pi/2\) with respect to \(E_x\) \((\delta_1 = 0\) and \(\delta_2 = -\pi/2\)):

\[
\vec{E}_{RCP} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}.
\]

Using this formalism, optical components are elementary matrices \((A)\) that operate on the Jones vectors (note that it is also common to use \(4 \times 4\) Mueller matrices to operate on a vector containing the Stokes parameters in the case of partially or unpolarized light, but this added generality is not necessary in this context). The incident beam is transformed into the output beam (denoted by a prime):

\[
\vec{E}' = A\vec{E},
\] (2.12)

and can logically be extended to a system of \(n\) components where \(A = A_n A_{n-1} \ldots A_2 A_1\). An intuitive example of such a matrix is that of a horizontal linear polarizer:

\[
A_{HLP} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},
\]
which applied to the previous Jones vector:

\[
\vec{E}' = A_{HLP} \vec{E}_{RCP} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -i \end{pmatrix} \]

\[
= \begin{pmatrix} 1 \\ 0 \end{pmatrix} 
= \vec{E}_h,
\]

where renormalization has eliminated the factor of \(1/\sqrt{2}\). Hecht [18] provides many more examples of Jones matrices for common optical components.

To show how the Faraday mirror operates, consider modeling the optical fiber by the Jones matrix for a general elliptical retarder (following the convention of Kersey [24], but disregarding losses):

\[
\vec{R} = \frac{1}{d} \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix} \tag{2.13}
\]

where \(a\) and \(b\) denote the complex birefringence of the fiber, \(d = \det(\vec{R})\), and the harpoon indicates the forward propagating direction, while a beam propagating through the same retarder in the reverse direction:

\[
\vec{R} = \frac{1}{d} \begin{pmatrix} a & -b \\ b^* & a^* \end{pmatrix}. \tag{2.14}
\]

The Faraday rotator imparts a \(\pi/4\) rotation before and after the reflection, and the total effect of the rotator \(A_{RO}t(\theta)\) and mirror \(A_M\) system is:

\[
A_{FM} = A_M A_{RO}t\left(\frac{\pi}{4}\right) A_{RO}t\left(\frac{\pi}{4}\right) \\
= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & \sin\left(\frac{\pi}{4}\right) \\ -\sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & \sin\left(\frac{\pi}{4}\right) \\ -\sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix} \\
= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{2.15}
\]
Therefore, the Jones matrix for the fiber and FM is:

\[ A = \vec{R} A_{FM} \vec{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \]  

(2.16)

which is independent of the birefringence of the fiber. Qualitatively this implies that the perturbations in the SOP by the fiber are “undone” by the reflected beam, provided that such perturbations do not occur faster than the time of flight of the beam within the system.

To apply FM to the OFIS required the use of a Michelson rather than a Mach-Zehnder interferometer (Figure 2.6). This was a trivial and very inexpensive modification, which simultaneously doubled the pressure sensitivity since the laser is reflected back through the OFIS fibers. Back reflections of the fringe signal to the laser were eliminated by the a built-in isolator, or optical diode. However, in the case where one laser is split into multiple paths to illuminate multiple OFISs, there was concern that the backward reflecting fringe signal would also propagate in the neighboring sensors thereby optically coupling each sensor. Fortunately the use of fiber connectors with angled faces, or FC/APC connectors, are designed to reduce such reflections, which resulted in no detectable fringe signal on adjacent sensors. Finally, the FM equipped Michelson OFISs have worked very well (Figure 2.4), and are being used for all current and future sensors.
Figure 2.1: Two views of the same Poincaré sphere, one rotated $\pi$ in azimuth with respect to the other. The evolution of the SOP in the single wrap arm is shown by circles, and the double wrap arm by dark squares.
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Figure 2.6: The Michelson OFIS schematic, showing that after traveling the length of the sensor, the phase conjugate beam from both arms are reflected back to the original splitter. It is at this point that the interference fringes are formed and sent to the photodetector.
Chapter 3

Temperature-Induced Pressure Sensitivity Change

Pressure sensitivity of the OFIS is quantified by the optical phase change in radians, per unit pressure, and normalized by the total length of the sensor ($rad \cdot Pa^{-1} \cdot m^{-1}$). The implementation of a continuous calibration system revealed that the pressure sensitivity of a 30 meter, ground lying OFIS changed by more than a factor of two over a range of about 20 degrees Celsius [30]. Since the IMS requires all microbarometer sensitivities vary less than five percent annually, it is essential to understand and take steps to mitigate this behavior.

3.1 Laboratory Observations

To confirm previous field measurements, a 1.5 meter long heating and cooling chamber was constructed from a 5 centimeter diameter copper pipe, helically wrapped with 6 millimeter diameter copper tubing with a wrap spacing of 5 centimeters that was periodically soldered to the larger pipe (Figure 3.1). This unit was enclosed in 5 centimeters of foam insulation and end caps constructed to yield a uniform temperature along its length. The temperature was controlled by pumping a water and ethylene glycol mixture through the helical tubing using a Solid State Cooling Systems 400 Watt ThermoCUBE thermoelectric temperature control system. This unit is capable of automatically cycling between two temperature...
set points, but does not allow for the control of the time it takes to obtain a given set point. Therefore, two Lytron CP15G05 aluminum heat exchangers submerged in 20 liters of water were added to increase the thermal inertia of the system, thereby reducing the cycle time. Finally, the temperature inside of the OFIS was measured using a calibrated Fluke 80TK thermistor module, whose output voltage was logged by the Femtometer system (Figure 3.2).

A calibration system was constructed to continuously determine the sensitivity of the OFIS inside the heating and cooling chamber. This was a prototype system for the unit deployed in the PM OFIS experiments, which also consisted of a subwoofer glued to a back volume whose outlet was a standard barbed pipe fitting to accept flexible tubing. The subwoofer was driven by a Tenma TG120 function generator and a Techron 5515 DC-coupled amplifier, which ran a two-second sinusoid at a 2 Volt peak-to-peak amplitude. This was used to pressurize the inner volume of the OFIS tube as well as a Setra 265 calibrated pressure transducer.

The pressure sensitivity of two 1.4 meter OFISs was measured continuously over three, eight-hour heating and cooling cycles from approximately 5° to 50° Celsius. After converting the Setra signal to Pascals, both the OFIS and Setra data were narrowly bandpass filtered about the calibration frequency and their envelope functions obtained by computing the magnitude of the signal with its Hilbert transform. Therefore, the instantaneous sensitivity was calculated by dividing the Setra envelope by the OFIS envelope and the length of the sensor. To examine the potential significance of the environmental conditions under which the fiber was wrapped about the silicone tube, one OFIS was wrapped at room temperature (about 20°C) while the other in a walk-in cooler at about 8°C. While there were no significant differences in the nominal sensitivities or temperature dependences between the warm and cold wrapped OFISs, both exhibited near linear behavior (an example of which can be found in Figure 3.3). The average trend between all cycles for both OFISs was found to be:

\[
\frac{1}{\Phi(T)} \frac{d\Phi}{dT} = (10 \pm 1) \times 10^{-3} \, ^\circ\text{C}^{-1},
\]
relative to 20°C. For comparison, consider the recent results from four, 30 meter buried OFISs at the Camp Elliott Field station:

$$\frac{1}{\Phi(T)} \frac{d\Phi}{dT} = (30 \pm 7) \times 10^{-3} \degree C^{-1}.$$  

These two results suggest that the temperature dependence of the pressure sensitivity may also depend on the total length of the OFIS, however, more data is required to establish a formal trend.

### 3.2 Temperature Dependence of the Elastic Modulus

Since Hooke’s law states that the stress in a linear elastic material is directly proportional to the strain and the material’s elastic (aka: Young’s) modulus, it could be supposed that the observed changes in OFIS pressure sensitivity would be inversely proportional to the temperature changes in the elastic modulus of the silicone tubing. For example, at low temperatures one would expect the silicone tube to become more rigid, corresponding to an increase in the elastic modulus, requiring a greater amount of pressure to strain the tubing (as monitored by the optical fiber) by the same amount, therefore lowering the sensitivity. It was this reasoning that led to the measurement of the temperature dependence of the silicone tubing’s elastic modulus.

The elastic modulus of the silicone tube can be easily measured as the ratio of the tensile stress to tensile strain by hanging a length of the tubing from one end and measuring the extension caused by a known mass suspended from the other end. To show this, consider the following form of Hooke’s law:

$$\sigma = E\epsilon,$$

where $\sigma$ is the tensile stress, $\epsilon$ is the tensile strain, and $E$ the elastic modulus. Since the tensile stress is the force per unit of cross-sectional area ($A$) of the tube, and (courtesy of Newton’s second law) the force is the applied mass ($m$) and the
acceleration due to local gravity \( (g) \):

\[
\frac{mg}{A} = E\epsilon \\
m = \frac{\pi (b^2 - a^2) E}{gL} \Delta L,
\]

(3.1)

where \( a \) and \( b \) are the inner and outer radii of the tubing, and the axial strain, \( \epsilon = \Delta L/L \), is simply the change in length over the total length. Therefore, the slope of a linear least squares fit to the applied mass versus change in length is proportional to the elastic modulus.

The elastic modulus of the silicone tubing was measured for ten different temperatures from 5°C to 50°C using the same 1.5 meter long heating and cooling chamber, but mounted vertically and without the added thermal inertia. Repeated heating and cooling measurements of the temperature inside and outside the silicone tube revealed that the time needed for the temperature of the silicone to equilibrate was approximately two hours: the e-folding time was found to be \( \tau = 1,280 \pm 220 \) seconds, therefore \( 5\tau \approx 6,400 \) seconds, or about 1.8 hours. At each temperature, a series of ten masses were used to stretch the silicone tube by a distance measured with a traveling microscope. All measurements were repeated four times: twice for increasing and decreasing temperature, respectively.

The results reveal a very modest change in the elastic modulus with temperature. In order to account for the thermal changes in the length of the silicone tube, the coefficient of linear expansion was measured using the same setup, and was found to be (relative to 20°C):

\[
\frac{1}{L(T)} \frac{dL}{dT} = (4.00 \pm 0.06) \times 10^{-4} \, ^\circ C^{-1}.
\]

As shown earlier, the elastic modulus for each temperature was computed from the slope of the applied mass versus extension. The temperature dependence of the elastic modulus was very linear with a best fit slope of:

\[
\frac{1}{E(T)} \frac{dE}{dT} = (-2.06 \pm 0.08) \times 10^{-3} \, ^\circ C^{-1},
\]

relative to \( 3.48 \pm 0.01 MPa \) at 20°C. This amounts to a 0.2% change per degree Celsius compared to the 1.0% change shown in the laboratory measurements of
the previous section, and therefore further investigation is needed to understand this discrepancy.
Figure 3.1: The 1.5 meter long OFIS heating and cooling chamber connected to the thermoelectric temperature control system. This unit was subsequently enclosed by insulation and mounted horizontally for the calibration experiments and vertically for the elastic modulus and linear expansion experiments.
Figure 3.2: Schematic of the experimental setup for measuring the temperature dependence of the pressure sensitivity. The two heat exchangers (plumbed in series) submerged in water provided sufficient thermal inertia to slow the cycle time to three hours. Temperature data was taken from the thermistor located near the center of the OFIS interior.
Figure 3.3: Results for one 24 hour period, or three-cycles of temperature and sensitivity measurements. Each error bar corresponds to the standard error within each 1°C temperature bin, however, the linear fit was performed on the unbinned data.
Chapter 4

Analytic Opto-Mechanical Model

It was the inability for the temperature dependence of the elastic modulus to explain the observed temperature dependent sensitivity that lead to exploring a mechanical model for the OFIS. The first step in this direction was to model the OFIS as a single loop of fiber wrapped under tension about a silicone rubber tube as a pre-stressed, double-walled thick cylinder, the basis of which is a well known problem in elasticity dating back to Lamé and Clapeyron in 1833 [26]. Derivations similar to the one that follows can be found in nearly any introductory text on elasticity or strength of materials.

4.1 Lamé Equations for a Thick-Walled Cylinder

Assuming the cylindrical coordinate convention \((r, \theta, z)\), consider a thick-walled cylinder of inner radius \(a\) and outer radius \(b\) subjected to both internal and external pressure, \(p_a\) and \(p_b\), respectively (Figure 4.1). Balancing the radial and tangential (aka: “hoop”) forces for an element of unit thickness leads to:

\[
(\sigma_r + d\sigma_r) (r + dr) d\theta = \sigma_r r d\theta + \sigma_\theta d\theta dr
\]

where \(\sigma_r\) and \(\sigma_\theta\) are the radial and tangential stresses, which in this case are the only two non-zero elements of the stress tensor. By assuming that \(drd\sigma_r \ll r\sigma_r, rd\sigma_r, \sigma_r dr\), and therefore disregarding \(drd\sigma_r\), canceling \(d\theta\), and dividing by \(dr\),
the equation of equilibrium becomes:

\[ r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0. \quad (4.1) \]

For an isotropic elastic material with Poisson’s ratio \( \nu \) and elastic modulus \( E \), the stress-strain relationship can be expressed as:

\[ \epsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{\alpha\alpha} \delta_{ij}, \quad (4.2) \]

where \( \epsilon_{ij} \) are the elements of the strain tensor, \( \delta_{ij} \) is the Kronecker delta, and the repeated indices indicate summation. Without shear or axial stresses, the radial and tangential strains are:

\[ \epsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta) \quad (4.3) \]
\[ \epsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r) . \quad (4.4) \]

Rearranging (4.3,4), it follows that the two elements of the stress tensor are:

\[ \sigma_r = \frac{E}{1 - \nu^2} (\epsilon_r + \nu \epsilon_\theta) \quad (4.5) \]
\[ \sigma_\theta = \frac{E}{1 - \nu^2} (\epsilon_\theta + \nu \epsilon_r) . \quad (4.6) \]

As proven by Housner and Vreeland [21], the two strain components in cylindrical coordinates can be expressed in terms of the radial displacement \( u_r \):

\[ \epsilon_r = \frac{du_r}{dr} \quad (4.7) \]
\[ \epsilon_\theta = \frac{u_r}{r} . \quad (4.8) \]

These results can be used to rewrite equations (4.5,6):

\[ \sigma_r = \frac{E}{1 - \nu^2} \left( \frac{du_r}{dr} + \nu \frac{u_r}{r} \right) \quad (4.9) \]
\[ \sigma_\theta = \frac{E}{1 - \nu^2} \left( \frac{u_r}{r} + \nu \frac{du_r}{dr} \right) . \quad (4.10) \]

By differentiating (4.9) with respect to \( r \):

\[ \frac{d\sigma_r}{dr} = \frac{E}{1 - \nu^2} \left[ \frac{d^2u_r}{dr^2} + \nu \left( \frac{1}{r} \frac{du_r}{dr} - \frac{u_r}{r^2} \right) \right] , \]
and using (4.9,10), the equation of equilibrium (4.1) can be expressed solely in terms of the radial displacement $u_r$:

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( ru_r \right) \right] = 0.$$ 

Integrating this separable equation twice yields:

$$u_r (r) = C_1 r + \frac{C_2}{r}, \quad (4.11)$$

where $C_1$ and $C_2$ are constants of integration. This is the general solution for the radial displacement of a thick-walled cylinder, however, it must be re-written in terms of stress to apply the boundary conditions (assuming the convention where compressive stresses are negative):

$$\sigma_r (r = a) = -p_a \quad (4.12)$$

$$\sigma_r (r = b) = -p_b. \quad (4.13)$$

It may appear to be unnecessary to express the stress in terms of the radial displacement only to solve for the stress, but it is required since $\sigma_r$ is a function of $u_r$ and $du_r/dr$. Defining $A = \frac{E}{1-\nu} C_1$ and $B = \frac{E}{1+\nu} C_2$, the general solutions for the stresses are:

$$\sigma_r (r) = A - \frac{B}{r^2} \quad (4.14)$$

$$\sigma_\theta (r) = A + \frac{B}{r^2}. \quad (4.15)$$

Application of the boundary conditions (4.12,13) determines $A$ and $B$ as:

$$A = \frac{a^2 p_a - b^2 p_b}{b^2 - a^2} \quad (4.16)$$

$$B = \frac{a^2 b^2 (p_a - p_b)}{b^2 - a^2}. \quad (4.17)$$

and:

$$C_1 = \frac{(1-\nu) (a^2 p_a - b^2 p_b)}{E (b^2 - a^2)} \quad (4.18)$$

$$C_2 = \frac{(1+\nu) a^2 b^2 (p_a - p_b)}{E (b^2 - a^2)}. \quad (4.19)$$
Finally, the desired form for the radial displacement can be found with (4.11) and (4.18,19):

\[
 u_r (r) = \left[ \frac{(1 + \nu) a^2 b^2 (p_a - p_b)}{E (b^2 - a^2)} \right] \frac{1}{r} + \left[ \frac{(1 - \nu) (a^2 p_a - b^2 p_b)}{E (b^2 - a^2)} \right] r, \tag{4.20}
\]

which is equation (10.46) of Faupel [14].

### 4.2 Pre-Stressed, Double-Walled Thick Cylinder

This section expands upon the previous one by adding a second cylinder about the first with an outer radius \( c \), subjected to an external pressure \( p_c \), with elastic modulus \( E_2 \) and Poisson’s ratio \( \nu_2 \). The radial displacements of the two cylinders are:

\[
 u_{r_1} (r) = \left[ \frac{(1 + \nu_1) a^2 b^2 (p_a - p_b)}{E_1 (b^2 - a^2)} \right] \frac{1}{r} + \left[ \frac{(1 - \nu_1) (a^2 p_a - b^2 p_b)}{E_1 (b^2 - a^2)} \right] r, \tag{4.21}
\]

for \( r \in [a, b] \)

\[
 u_{r_2} (r) = \left[ \frac{(1 + \nu_2) b^2 c^2 (p_b - p_c)}{E_2 (c^2 - b^2)} \right] \frac{1}{r} + \left[ \frac{(1 - \nu_2) (b^2 p_b - c^2 p_c)}{E_2 (c^2 - b^2)} \right] r, \tag{4.22}
\]

for \( r \in [b, c] \).

Since the second cylinder is force-fit about the first (this is analogous to either a shrink-fit or press-fit construction), an “interference distance” \( \delta \) can be defined as:

\[
 \delta = u_{r_2} (r = b) - u_{r_1} (r = b). \tag{4.23}
\]

Note that the case of \( \delta = 0 \) states that the radial displacement is continuous across the interface. It is essential to understand that \( \delta \) is the combination of the shrinkage of the inner radius of the second cylinder \( (u_{r_2} (b) > 0) \) and the outer radius of the first \( (u_{r_1} (b) < 0) \). Texts on contact mechanics often use \( \delta \) as the “distance of mutual approach” [15], however, in this context \( \delta \) is defined as the indentation. This leads to the constraint:

\[
 p_b = \frac{2 E_1 b c^2 (b^2 - a^2) p_c + E_2 (c^2 - b^2) \left[ 2 a^2 b p_a + \delta E_1 (b^2 - a^2) \right]}{b \left( E_1 (b^2 - a^2) \left[ b^2 + c^2 + \nu_2 (c^2 - b^2) \right] + E_2 (c^2 - b^2) \left[ a^2 + b^2 - \nu_1 (b^2 - a^2) \right] \right)}, \tag{4.24}
\]
(Note that setting $p_a = p_c = 0$ results in equations (10.51,2) of Faupel [14].) However, to model the second cylinder as a loop of optical fiber the radial displacement $u_{r_2}$ must be related to optical path length difference, which is governed by the strain-optic effect.

### 4.3 The Strain-Optic Effect

The optical phase, $\phi$, of light passing through an optical fiber can be written as [6]:

$$\phi = \beta L = \frac{2\pi n L}{\lambda}$$  \hspace{1cm} (4.25)

where $\beta$ is called the propagation constant, $n$ is the index of refraction of the fiber core, $\lambda$ is the wavelength of the light, and $L$ is the length of the fiber. The change in phase is then:

$$\Delta \phi = \beta \Delta L + L \Delta \beta = \Delta \phi_1 + \Delta \phi_2.$$  \hspace{1cm} (4.26)

The first term represents the optical phase difference due to the axial stretching of the fiber:

$$\Delta \phi_1 = \frac{2\pi n L}{\lambda} \epsilon,$$  \hspace{1cm} (4.27)

where $\epsilon = \Delta L/L$ is the axial strain in the fiber. The second term describes the change in the index of refraction as well as the waveguide mode dispersion caused by the reduction in diameter of the fiber core under axial stress. Making the frequent assumption that the core diameter remains constant [6, 20, 32, 3], the change in the index of refraction due to axial strain alone can be expressed as [3]:

$$\Delta \phi_2 = -\frac{\pi n^3 L}{\lambda} \left[ p_{12} - \nu_c (p_{11} + p_{12}) \right] \epsilon,$$  \hspace{1cm} (4.28)

where $\nu_c$ is the Poisson’s ratio of the fiber core and $p_{11}$ and $p_{12}$ are photoelastic constants dependent upon the material used. Finally, the formula for the total optical phase difference per unit strain can be formed from equations (4.32-34):

$$\Psi = \frac{\Delta \phi}{\epsilon} = \frac{\pi n L}{\lambda} \left[ 2 - \epsilon^2 \left[ p_{12} - \nu_c (p_{11} + p_{12}) \right] \right].$$  \hspace{1cm} (4.29)
While this theory has been successfully applied innumerable times for un-jacketed fiber, most relevantly in the case of a hydrophone whose design is similar to the OFIS [32], it is not clear how jacketing affects the stresses resolved on the core. Therefore, an empirical measurement of the optical phase difference per unit strain was performed on a 1.115 meter length of 900 micron jacketed Corning SMF-28 fiber using a pair of fiber vices, a precision translation stage and a 1,310 nanometer wavelength laser. The result was found to be:

\[ \Psi_{\text{empirical}} = (6.111 \pm 0.009) \times 10^6 \text{ rad} \cdot \text{strain}^{-1}. \]

Using published values for the constants [20]:

\[
\begin{align*}
  n & = 1.456 \\
  \nu_c & = 0.17 \\
  p_{11} & = 0.121 \\
  p_{12} & = 0.270,
\end{align*}
\]

and equation (4.29), the theoretical value was calculated as:

\[ \Psi_{\text{theoretical}} = 6.107 \times 10^6 \text{ rad} \cdot \text{strain}^{-1}, \]

which is in agreement with the measured value.

### 4.4 Single Fiber about a Thick Cylinder Subjected to Internal Pressure

In this section, the results of the previous two sections are used to model a single loop of optical fiber wrapped under tension about a silicone rubber tube as a pre-stressed, double-walled thick cylinder. The strain-optic behavior is included to better model the outer layer of the cylinder as an optical fiber and to convert its strain to an optical phase. After simplifying the previous results, an estimate of the theoretical pressure sensitivity will be made and compared to experimental data, followed by suggestions for future model improvements.
4.4.1 Theoretical Results

Assuming that the tube material is incompressible ($\nu_1 = 1/2$), and discarding the external pressure $p_c$, the radial displacements (4.21,22) and interface pressure (4.24) simplify to:

$$u_{r_1}(r) = \frac{3a^2b^2 (p_a - p_b)}{2E_1 (b^2 - a^2)} \frac{1}{r} + \left[ \frac{(a^2 p_a - b^2 p_b)}{2E_1 (b^2 - a^2)} \right] r, \text{ for } r \in [a, b]$$

(4.30)

$$u_{r_2}(r) = \frac{(1 + \nu_2) b^2 c^2 p_b}{E_2 (c^2 - b^2)} \frac{1}{r} + \left[ \frac{(1 - \nu_2) b^2 p_b}{E_2 (c^2 - b^2)} \right] r, \text{ for } r \in [b, c]$$

(4.31)

$$p_b = \frac{2E_2 (c^2 - b^2) [2a^2 b p_a + \delta E_1 (b^2 - a^2)]}{b \{2E_1 (b^2 - a^2) [b^2 + c^2 + \nu_2 (c^2 - b^2)] + E_2 (3a^2 + b^2) (c^2 - b^2) \}}.$$  

(4.32)

The strain in the optical fiber at $r = c$ simplifies with the use of (4.8) and (4.31,32):

$$\epsilon_{\theta_2} = \frac{u_{r_2} (r = c)}{c} = \frac{4b [2a^2 b p_a + \delta E_1 (b^2 - a^2)]}{2E_1 (b^2 - a^2) [b^2 + c^2 + \nu_2 (c^2 - b^2)] + E_2 (3a^2 + b^2) (c^2 - b^2)},$$

(4.33)

and the sensitivity:

$$\Phi = \frac{\epsilon_{\theta_2}}{p_a} \Psi = \frac{8\pi^2 nbc [2a^2 b p_a + \delta E_1 (b^2 - a^2)] - 2 - n^2 [p_{12} - \nu_c (p_{11} + p_{12})]}{2E_1 (b^2 - a^2) [b^2 + c^2 + \nu_2 (c^2 - b^2)] + E_2 (3a^2 + b^2) (c^2 - b^2)},$$

(4.34)

where $L = 2\pi c$.

To calculate the pressure sensitivity of the single loop, the material parameters of the fiber must be known. As a first estimate, the elastic modulus and Poisson’s ratio of the jacketed fiber were determined by averaging the parameters of the constitutive materials, weighted by their respective cross-sectional areas. For example, the core and cladding of the fiber is 125 microns in diameter and composed of fused silica. This is coated with UV cured polyacrylate to a total diameter of 250 microns, which is coated again up to 900 microns with polyvinyl...
Table 4.1: Material Parameters for 900 micron SMF-28 Fiber

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Modulus $E\ (GPa)$</th>
<th>Poisson’s Ratio $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused Silica</td>
<td>71.7</td>
<td>0.17</td>
</tr>
<tr>
<td>Polyacrylate</td>
<td>3.2</td>
<td>0.35-0.42</td>
</tr>
<tr>
<td>PVC</td>
<td>2.5-4.1</td>
<td>0.33-0.46</td>
</tr>
</tbody>
</table>

chloride (PVC). Using published values for these materials in Table 4.1 [25, 20], the average values were found to be:

\[
E_2 = 4.6 \pm 0.7 \ GPa
\]

\[
\nu_2 = 0.39 \pm 0.06.
\]

The measured values of the elastic modulus of the silicone from the previous experiments (taken at 20°C) and the three radii were measured as:

\[
E_1 = 3.48 \pm 0.01 \ MPa
\]

\[
a = 0.0091 \pm 0.0002 \ m
\]

\[
b = 0.0118 \pm 0.0001 \ m
\]

\[
c = 0.0126 \pm 0.0001 \ m.
\]

The final step to predict the sensitivity of the single loop from (4.34) is to estimate the interference distance, or indentation $\delta$. Assuming no pre-stressing ($\delta = 0$):

\[
\Phi_{\text{theoretical}} (\delta = 0) = (1.0 \pm 0.3) \times 10^{-3} \ rad \cdot Pa^{-1}.
\]

The analytical evaluation of the indentation about an elastic cylinder due to an elastic ring of circular cross-section appears to be an unsolved problem, except in the case where the ring is rigid (ie: $E_2 \to \infty$) [2]. Unfortunately, the complexity of its solution prevents it from being useful in this thesis, however, another estimate of $\delta$ may be arrived at using the results of an elastic spherical indenter in an elastic half-space [15]:

\[
\delta^3 = \frac{1}{R} \left( \frac{3T}{4E^*} \right)^2,
\]

where:

\[
E^* = \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1}
\]
is the combined elastic modulus of the silicone tube and fiber, \( R = (c - b) / 2 \) is the radius of the fiber, and \( T \) is the indenter load, which is the tension of the fiber. Using a value of \( T = 1.62 \pm 0.03 \) Newtons:

\[
\delta = (5.4 \pm 0.4) \times 10^{-4} \text{ m},
\]

and the resulting sensitivity:

\[
\Phi_{\text{theoretical}} (\delta = 5.4 \times 10^{-4}) = 55 \pm 16 \text{ rad} \cdot \text{Pa}^{-1}.
\]

This illustrates the remarkable dependence the pressure sensitivity on \( \delta \) and the necessity for an accurate estimate, however, it may be useful as a free parameter.

### 4.4.2 Experimental Results

An experiment was conducted on a single loop of fiber about a silicone tube, and its sensitivity measured with a similar calibration system to all the previous experiments. The loop of fiber was wrapped at a tension of \( 1.62 \pm 0.03 \) Newtons and the PVC jacket welded over a length of 2-3 millimeters. After pressurizing the interior of the tube at five different amplitudes at a frequency of 0.5 Hz, the magnitudes of the calibrated pressure transducer and optical phase were determined using least squares. A linear fit to the optical phase difference versus the applied pressure resulted in a sensitivity of:

\[
\Phi_{\text{experimental}} = (11.9 \pm 0.6) \times 10^{-3} \text{ rad} \cdot \text{Pa}^{-1}.
\]

While the experiment has been repeated on several other loops of varying tension with similar results, it is clearly out of agreement with either of the above predicted values. Inverting (4.34) to find the indentation yields:

\[
\delta = (1.1 \pm 0.3) \times 10^{-7} \text{ m}.
\]

### 4.4.3 Suggestions for Future Work

It is difficult to assess whether the pre-stressed, double-walled thick cylinder can be useful to understand the temperature dependence of the pressure sensitivity
of the OFIS. The striking dependence on the interference distance, and the difficulty in obtaining its estimate severely limits the predictive power of the model. Inverting for \( \delta \) could be useful for obtaining an initial estimate of the pressure sensitivity that could be held constant to examine the effects of temperature change on the other model parameters. However, this approach would be highly problematic since the formulations of \( \delta \) depend upon these same parameters.

While there is some literature on helically wrapped composite tubes (Evans [13] provides a review), most reduce the problem to a bulk cylinder whose material properties are found by what amounts to an elaborate weighted-averaging scheme. The other extreme is the highly complex contact problem as in Reference [2]. However, the strong dependence upon the contact conditions shown above indicate that a realistic model for the OFIS should focus on the temperature dependence of the contact strain field, which could be addressed using a numerical method such as finite differences or finite elements. In this way the composite nature of the fiber could also be taken into consideration and better predict the strains resolved on the fiber’s core.
Figure 4.1: The left diagram shows a thick-walled cylinder with inner radius $a$ and outer radius $b$, subjected to internal pressure $p_a$ and external pressure $p_b$. The right side shows the forces on the area segment from the left diagram, both highlighted in gray.
Appendix A

Optical Path Length Modulation and the Femtometer System

While the primary focus of this thesis is the mechanical and optical behavior of the OFIS as an interferometric pressure transducer, it is important to understand how an estimate of the optical phase difference is calculated by measuring the intensity of the interference fringes. This will explain why the fiber path length is modulated, as mentioned in Chapter 1, and where interference ellipse originates, since its area was used as a metric for assessing the uptime of the sensor in Chapter 2. Furthermore, it is essential to note that the following technique is not unique to the OFIS, but can be applied to any Mach-Zehnder or Michelson type interferometer [33]. What follows is the description of the components needed to obtain an estimate of optical phase for the current Michelson interferometer OFIS design as shown in Figure 2.6.

As previously stated, there are two helical wraps of fiber such that there are four connectors at the end of an OFIS, which interface with one end of a device termed the modulator box (Figure A.1). At the other end of the modulator box is the laser input for illuminating the OFIS, which is split equally among two paths that wrap about a piezoelectric cylinder (piezo). A 156 kHz voltage signal supplied to the piezo causes its diameter to change by a small amount, thereby stretching the fiber a fraction of a wavelength. The piezo is split and wired such that one fiber is strained $\pi$ out of phase with the other. The modulated light is sent through both
arms where it is reflected off the Faraday mirrors and recombined at the splitter to form interference fringes. Therefore, the purpose of the modulator box is to supply the illumination to each OFIS arm, modulate the optical path length, and return the modulated fringe signal.

A collection of electronics known as a “Hanada rack” (Figure A.2) bridges the gap between the modulator box and the data acquisition system. The first module contains the oscillator and amplifier to drive the piezo, while the last consists of an amplified photodetector to receive the modulated fringe signal. In between these two units is a phase-sensitive detector, or lock-in amplifier. The output of the photodetector, \( x \), is fed into the lock-in amplifier to demodulate the fringe signal, producing an output, \( y \), proportional to the spatial derivative of the original fringe signal [33]. There is a secondary amplifier module to adjust the \( x \) and \( y \) voltage amplitudes and their DC offsets, conditioning them for input to the data acquisition system.

Recall that the point of modulating the optical path length is to discriminate between contractive and expansive fringes that respectively correspond to positive and negative external pressure acting on the OFIS. For example, if the fringe intensity is a maximum, corresponding to perfect constructive interference, it is impossible to determine if a subsequent decrease in intensity is the result of the optical path length increasing or decreasing. However, the plot of a changing fringe signal versus its electronically generated derivative traces out an ellipse whose instantaneous optical phase difference depends upon the \((x, y)\) location on the ellipse. Therefore, the next step is to obtain estimates of the ellipse location, size, and shape to obtain an estimate of the optical phase.

The previously described components are essential for preparing the original fringe signal for estimating the optical phase, but it is the data acquisition and signal processing system called the Femtometer that yields the estimate. These \((x, y)\) data pairs are sampled at a rate of either 100 kHz or 50 kHz, and a user determined number of pairs are stored in a circular buffer to obtain least squares
estimates of the constants in the following parametric equations \[33\]:

\[
x = x_0 + a \sin (p + p_0) \tag{A.1}
\]

\[
y = y_0 + b \cos (p) , \tag{A.2}
\]

where \(x_0\) and \(y_0\) are DC offsets on the \((x, y)\) plane, \(a\) and \(b\) are the \(x\) and \(y\) ellipse amplitudes, \(p_0\) is the ellipse shape factor, and \(p\) is the optical phase difference. As new data replaces the old in the buffer, the ellipse parameters are updated and the optical phase is determined from the most recent \((x, y)\) pair by solving both equations \((A1,2)\) for \(p\) and taking the arctangent of their ratio. It is important to note that under extreme conditions, it is possible for the subsequent \((x, y)\) point to advance more than \(2\pi\) around the ellipse despite the high sample rate. With the previous limitation properly accounted for, the phase and ellipse parameter data are decimated and archived following the CSS 3.0 convention.
Figure A.1: Schematic of the modulator box, showing the OFIS connections on the left. While there are four connections to the OFIS, two send light into the OFIS and receives the reflected light from the FM found at the remaining two connections. Note that the single and double wrap arms can be interchanged.

Figure A.2: Block diagram of the fringe modulation and Femtometer system for estimating the optical phase difference between two OFIS arms. The Femtometer output is the five ellipse parameters (usually at a much lower sample rate) along with the \((x, y)\) inputs used to compute \(p\).
Appendix B

Derivation of Ellipse Parameters

Standard formulas for ellipse properties such as area, eccentricity, foci, etc. are usually given in terms of the semimajor and semiminor axes, therefore it is beneficial to find how these are related to the five ellipse parameters \((x_0, y_0, a, b\) and \(p_0\)) put out by the Femtometer. One approach is to reduce the parametric equations (1,2) of Zumberge, et al. [33] to canonical form, which are:

\[
x = x_0 + a \sin (p + p_0) \tag{B.1}
\]
\[
y = y_0 + b \cos (p). \tag{B.2}
\]

It can immediately be assumed that \(x_0 = y_0 = 0\) since the ellipse properties should not depend on its position. Solving both equations for \(p\) and taking the cosine of both sides:

\[
\frac{y}{b} = \cos \left[ \arcsin \left( \frac{x}{a} \right) - p_0 \right]. \tag{B.3}
\]

Splitting the argument of the cosine with \(\cos (\theta \pm \phi) = \cos (\theta) \cos (\phi) \mp \sin (\theta) \sin (\phi)\) and using the inverse identity \(\cos [\arcsin (\theta)] = \sqrt{1 - \theta^2}\), we arrive at:

\[
\frac{y}{b} = \sqrt{1 - \left( \frac{x}{a} \right)^2} \cos (p_0) + \frac{x}{a} \sin (p_0). \tag{B.4}
\]

Isolating the square root, squaring both sides and collecting like terms results in a quadratic in standard form:

\[
Ax^2 + Bxy + Cy^2 - D = 0, \tag{B.5}
\]
where:

\[ A = a^{-2} \]  \hspace{1cm} (B.6)
\[ B = -\frac{2}{ab} \sin (p_0) \]  \hspace{1cm} (B.7)
\[ C = b^{-2} \]  \hspace{1cm} (B.8)
\[ D = \cos^2(p_0). \]  \hspace{1cm} (B.9)

The next step is to rotate the quadratic to its principle axes by the transformation (Figure B.1):

\[ x = \xi \cos (\psi) - \eta \sin (\psi) \]  \hspace{1cm} (B.10)
\[ y = \xi \sin (\psi) - \eta \cos (\psi). \]  \hspace{1cm} (B.11)

Expanding and collecting like terms:

\[ A' \xi^2 + B' \xi \eta + C' \eta^2 - D = 0, \]  \hspace{1cm} (B.12)

where:

\[ A' = A \cos^2 (\psi) + B \cos (\psi) \sin (\psi) + C \sin^2 (\psi) \]  \hspace{1cm} (B.13)
\[ B' = -2A \cos (\psi) \sin (\psi) + B \left[ \cos^2 (\psi) - \sin^2 (\psi) \right] + 2C \cos (\psi) \sin (\psi) \]  \hspace{1cm} (B.14)
\[ C' = A \sin^2 (\psi) - B \cos (\psi) \sin (\psi) + C \cos^2 (\psi) \]  \hspace{1cm} (B.16)

and \( D \) remains unchanged from (B.9). For the rotation to arrive at the principle axis, \( B' \) must be zero, leading to a definition of \( \psi \):

\[ \tan (2\psi) = \frac{B}{A - C} = \frac{2ab \sin (p_0)}{a^2 - b^2}, \]  \hspace{1cm} (B.17)

with the aid of the trigonometric relationships:

\[ \cos^2 (\psi) - \sin^2 (\psi) = \cos (2\psi) \]
\[ \cos (\psi) \sin (\psi) = \sin (2\psi). \]

Note that since \( \tan (2\psi) \in (-\infty, \infty) \), there always exists a \( \psi \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \) such that \( B' = 0 \). Finally, by dividing by \( D \) we arrive at the canonical equation:

\[ \left( \frac{\xi}{u} \right)^2 + \left( \frac{\eta}{v} \right)^2 = 1. \]  \hspace{1cm} (B.18)
With two more inverse trigonometric relationships:

\[
\cos[\arctan(\theta)] = \frac{1}{\sqrt{1 + \theta^2}}
\]

\[
\sin[\arctan(\theta)] = \frac{\theta}{\sqrt{1 + \theta^2}}.
\]

and a fantastic amount of algebra we arrive at the semimajor and semiminor axes in terms of the Femtometer parameters:

\[
u = \sqrt{\frac{1}{2} \left[ a^2 + b^2 + \sqrt{a^4 + b^4 - 2a^2b^2 \cos(2p_0)} \right] } \quad (B.19)
\]

\[
v = \sqrt{\frac{1}{2} \left[ a^2 + b^2 - \sqrt{a^4 + b^4 - 2a^2b^2 \cos(2p_0)} \right] } \quad (B.20)
\]

where a quick test shows that when \( p_0 = 0 \), \( u = a \) and \( v = b \). Given that the area of an ellipse is \( \pi uv \), it is easy to show that:

\[
\text{Area} = \pi ab \cos(p_0) . \quad (B.21)
\]

Figure B.1: Geometry of ellipse rotation, where \( \eta \) and \( \xi \) correspond to its principle axes.
Bibliography


