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Ex post inefficiency in a political agency model∗

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Abstract

We extend the model of Schultz (1996) to a dynamic setting with no policy commitment. Two parties that compete for election must choose the level of provision of a public good as well as the tax payment needed to finance it. The cost of producing the good may be high or low and this information is not known to the voters. We show that there exists an equilibrium in which the party that does not want much of the public good use the inefficient (high cost) technology even though the efficient one is available. Using the low cost technology would, by informing the voters about the cost parameter, force it to produce an excessively high level of the good. Interestingly, this equilibrium is not symmetric, suggesting that a party with a strong taste for the public good is less likely to adopt a wasteful policy.

Keywords: commitment, dynamic electoral competition, ratchet effect.

JEL classification: D72, D82, H41.

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1 Introduction

The central question addressed in this paper is whether democracies produce efficient results. According to the Chicago school of political economy, which ideas are summarized in Wittman (1989), the answer to this question is in the affirmative. The argument is simple and powerful: a politician adopting an inefficient policy would be voted out of office and replaced by a challenger.

While this argument is compelling, it misses an important point: the voters may not be perfectly informed about some characteristics of the policy and/or of the politicians. This view is associated with the Virginia school of political economy. Coate and Morris (1995) develop in an important paper a model along these lines. They show that a “bad” incumbent politician may adopt an inefficient policy because it consists in a disguised transfer to a special interest. Hidden transfers to special interest groups, although inefficient, are preferable for politicians who care about their reputation; a direct cash transfer to the special interest would indeed inform the citizens about the type of the incumbent politician.

Another issue that has been neglected until recently is the issue of policy commitment, as emphasized by Acemoglu (2002). When politicians are not able to commit to future policies, they may have some incentives to adopt inefficient policies today.\footnote{This line of reasoning is explored in Besley and Coate (1998).}

We propose in this paper a model characterized both by asymmetric information between the politicians and the voters and absence of policy commitment. The economy is the same as in Schultz (1996). A public good is produced at a constant marginal cost that can be high or low and is only observed by the politicians (not by the voters). The provision of this good is financed with a uniform tax. The voters/citizens differ only according to their valuation for the public good.

Schultz considers a standard static electoral competition model in which two parties compete for election and are able to commit to the policy announced during the campaign: if elected, they implement the platform announced during the campaign. Those parties are assumed to be policy motivated. They care about winning office but also about which policy is implemented. Finally the election is based on the majority rule: the party that receives more votes is elected.
Schultz obtains that the electoral equilibrium may be pooling: the parties’ platforms, that consist of quantities of the public good, are the same whether the cost is low or high. The basic intuition is that parties are unable to credibly convey the information to the voters. At equilibrium, the two parties propose the ideal policy of the (uninformed) median voter and are elected with probability 1/2. Because a small deviation from the equilibrium does not modify the beliefs of the voters, the deviating party loses with certainty. It follows that no party should deviate. The author deduces from this result that the equilibrium is *ex ante* inefficient: the expected utility of all individuals could be increased before the true value of the cost is known to the voters. However, the equilibrium is efficient *ex post*, in the sense that the elected party chooses a value of the tax rate that makes the budget constraint binding at the true cost value.

The assumption of commitment made in Schultz (1996) is reasonable when it is assumed that parties are purely opportunistic and only office motivated. Conversely, this assumption is somewhat more controversial when parties have policy preferences: once elected, a party has an incentive to deviate towards its preferred policy. A proper analysis with policy motivated parties should thus adopt the framework of repeated elections with no policy commitment. Whether a party will keep the promises made during the electoral campaign or not depends on the structure of rewards and punishments in future elections.

To address this point more precisely, we develop a model based on Duggan (2000), in the tradition of political agency models developed by Barro (1973) and Ferejohn (1986). We set up an *infinitely repeated elections model* and assume that there is no political campaigning at all, so that there is no possible policy commitment. In each period, a party has to choose a policy. At the beginning of the following period, this incumbent competes in an election against the challenger. The elected party is the one that receives the most votes (majority rule). The voters’ equilibrium strategy is both *retrospective* and *prospective*: it is retrospective in that they vote for the incumbent if and only if the utility generated by the last policy choice meets a given utility level; it is prospective in the sense that they vote for the politician yielding the highest continuation value. We consider two parties with policy preferences, these preferences being known to the voters. Party A is not favorable to a large provision of the public good whereas party B has a strong taste for it. The only source of asymmetric information is thus the marginal cost of production
of the public good.\(^2\)

We show that there exists a perfect Bayesian equilibrium in which \(A\) adopts a pooling strategy: he implements the same policy whether the cost is high or low. In other words, \(A\) offers the same quantity of public good at the same tax level, whatever the cost. When the cost is actually low, such a policy entails a waste of resources: more of the good could be produced at the same tax level. The underlying idea is that if producing efficiently, party \(A\) would inform the voters that the cost is low. It would then be induced to produce, in all future periods, a quantity of the good which is too high from its point of view. As soon as this party prefers to remain in power, it should follow this inefficient pooling strategy. In some sense, this mechanism is thus a *political ratchet effect*. We obtain the interesting additional result that this equilibrium is not symmetric: party \(B\) should not pretend that the cost is high when it is low. This follows from the fact that when the cost is low, party \(B\) benefits from a higher production at a lower cost. These results suggest that parties with a strong taste for the public good are less likely to adopt a wasteful behavior than others.

**Related literature** There are surprisingly few papers that address the question of inefficient policy-making. Apart from the above cited work by Coate and Morris (1995), Besley and Coate (1998) and Acemoglu (2002), two recent papers, although adopting very different settings, are relevant for our purpose. In Acemoglu and Robinson (2001), the political power of a group depends on its size. Adopting an inefficient policy (like a price distortion) can then be an effective way of expanding the size of this group in order to guarantee its future political power. The typical example of such a mechanism can be found in the distortive price support to farmers. Robinson and Verdier (2003) argue that employment in the public sector is an efficient way of redistributing income. However, it is politically attractive as it is credible (income transfers are not), excludable (public goods are not) and reversible (public investments are not).

The model is presented in the next section. Sections 3 and 4 are devoted to the analysis

\(^2\)This is a notable difference with the work of Coate and Morris in which the voters are imperfectly informed about both some characteristics of the policy and of the politicians. These authors argue that such a double uncertainty is necessary to generate some political inefficiency. Our study clearly contradicts their claim.
of equilibria.

2 The model

2.1 Description of the economy

The economic problem is exactly the same as in Schultz (1996). A quantity $x$ of a public good is produced. The cost of production of $x$ units of the good is $cx$ where $c$ can take on one of two values: $c^l$ or $c^h$ with $0 < c^l < c^h$. The government finances the provision of the public good through a uniform tax $\tau$.

The citizens are identical in all respects but their valuation for the public good, $\theta$. The preferences of a type $\theta$ individual are described by the following function:

$$v(x, \tau; \theta) = \theta u(x) - \tau,$$

where $u$ is increasing and strictly concave; $\theta$ is distributed on $[\underline{\theta}, \overline{\theta}]$ according to a density function $f$, the median of this distribution being denoted $\theta_m$.

We impose that the budget be balanced each period (no debt is allowed) so that the government budget constraint is

$$cx \leq \int_{\underline{\theta}}^{\overline{\theta}} \tau f(\theta) \, d\theta$$
$$\Rightarrow \quad cx \leq \tau.$$

When $cx < \tau$, there is obviously a waste of resources.

The optimal policy of individual $j$ in state $s \in \{l, h\}$ will be denoted $(x^s_j, \tau^s_j)$. This is given by the conditions

$$\theta_j u'(x^s_j) = c^s$$
$$\tau^s_j = c^s x^s_j.$$ (2) (3)

Equation (2) comes from the first-order conditions on $x$ and $\tau$. Equation (3) means that individual $j$’s optimal policy requires a binding budget constraint. Differentiating these equations, we obtain

$$\frac{dx^s_j}{dc^s} = \frac{1}{\theta_j u''(x^s_j)} < 0$$ (4)
\[
\frac{d\tau_j^s}{dc^s} = x_j^s + c^s \frac{dx_j^s}{dc^s} = x_j^s + \theta_j u'(x_j^s) \frac{1}{\theta_j u''(x_j^s)} < 0 \text{ iff } E(x_j^s) = -x_j^s \frac{u''(x_j^s)}{u'(x_j^s)} < 1.
\]

We will assume in the following that \( E(x) \), the elasticity of the marginal utility of consumption, is always lower than 1. This implies that the optimal tax level of any individual increases when the marginal cost decreases. We further make the simplifying assumption that this coefficient is constant, considering an isoelastic utility function:

\[
u(x) = x^{1-\varepsilon} \frac{1}{1-\varepsilon},
\]

where \( E(x) = \varepsilon \leq 1 \).

### 2.2 The political game

There are two parties/politicians \( A \) and \( B \) with policy preferences \( \theta_A \) and \( \theta_B \) such that \( \theta_A < \theta_m < \theta_B \). Party \( A \) represents individuals with a moderate taste for the public good whereas party \( B \)'s constituency favors a high provision of this good. The players of this game are thus the two parties and the voters. There is asymmetric information among these players. The parties learn the value of the marginal cost \( c \), which is constant across periods, at the beginning of the game whereas the voters are initially uninformed. Their prior belief is that the cost is high with probability \( \mu_0 \). They may or may not learn the true value of the cost later on, depending on the equilibrium path of play.

**Timing**

Building on Duggan (2000), we study a repeated elections model with infinite horizon. At the beginning of each period \( t \), an election takes place in which the voters decide whether to reelect the incumbent party or to appoint the challenger. The elected politician is the one that receives the most votes (majority rule).\(^3\) He then chooses a policy for the

\(^3\) We assume that when the two parties receive the same number of votes, the incumbent politician is reelected for sure. In the equilibria described below, the incumbent and the challenger will always be tying. This comes however from our assumption that there is a continuum of types. With a finite number of types, such a case would almost never occur.
current period \((x_t, \tau_t)\). In the following period \(t + 1\), the same sequence of events occurs again.

The history at date \(t\), \(h_t\), describes the publicly observed events in the first \(t\) periods, namely the party in power and the policy chosen in each period.

**Payoffs**

The voters

Type \(\theta\) voter’s payoff from the sequence \(\{x_t, \tau_t\}\) of policy outcomes is the discounted sum of per period utility levels:

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} v(x_t, \tau_t; \theta),
\]

where \(\delta < 1\) is the time discount factor.

The parties

Party \(i\)’s (\(i = \{A, B\}\)) payoff from the sequence \(\{x_t, \tau_t\}\) of outcomes is:

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} [v(x_t, \tau_t; \theta) + \beta \omega_i(i_t)],
\]

where \(\omega_i(i_t)\) is the indicator function taking the value 1 if and only if \(i_t = i\). The parties have therefore some preferences on the policy implemented in each period, but they also value the fact of holding office, deriving a utility level \(\beta\) in such a case.

**Strategies**

The parties

A (pure) strategy of party \(i\) specifies the policy chosen \(p_{i,t} = (x_{i,t}, \tau_{i,t})\) if elected in period \(t\). It is a function of history at date \(t - 1\), \(h_{t-1}\), and of the state of the world \(s \in \{l, h\}\).

The voters

A (pure) strategy of voter \(j\) specifies, for every possible history at date \(t - 1\), \(h_{t-1}\), the action chosen in period \(t\), \(a_{j,t} \in \{I, C\}\), where \(a_{j,t} = I\) (resp. \(C\)) means that this individual votes for the incumbent (resp. challenger). We consider voting strategies that are retrospective: each voter decides to vote for the incumbent if and only if the utility generated by its last policy choice is at least equal to a given threshold level \(\bar{v}\). Formally,

\[
a_{j,t} = I \text{ iff } v(x_{t-1}, \tau_{t-1}; \theta_j) \geq \bar{v}_{j,t}.
\]
Because we assume a continuum of voters, no voter will ever be pivotal in the election. This implies that any value of $V$ is consistent with equilibrium behavior: as he will not affect the election outcome, any voter is completely indifferent between voting for one candidate or the other. A natural restriction on voting behavior is however to assume that the voters act as though pivotal in each election. They will thus vote for the incumbent if the expected utility from re-electing him is at least as high as the expected utility of electing the challenger. This property is called \emph{prospective voting}. We will consider in the following strategies that are time invariant. More precisely, we will consider stationary Markov strategies with the beliefs of the voters as the state variable. In other words, as soon as the beliefs of the voters remain constant across time, the strategies of the parties and the voters remain unchanged as well. This implies, following the argument made in Banks and Duggan (2002), that \emph{the re-election utility level $V$ must be equal in any period to the expected continuation value of electing the challenger} if the strategy is to be both retrospective and prospective.\footnote{The argument runs as follows. Let $\sigma$ be a strategy profile and $\nu_t(\sigma)$ the continuation value of electing the challenger in period $t$. Define also $p$ as the policy choice in $t - 1$. Should the incumbent be reelected in period $t$, he would also choose $p$ (the beliefs of the voters on the cost parameter are unchanged). If $\sigma$ determines that the incumbent subsequently be replaced, the expected utility for a type $\theta$ voter from retaining the incumbent in period $t$ is

\[(1 - \delta) v(p; \theta) + \delta \nu_{t+1}(\sigma).\]

If $\sigma$ determines that the incumbent be retained forever, the expected utility from retaining the incumbent in period $t$ is $v(p; \theta)$. Observing that with unchanged beliefs $\nu_{t+1}(\sigma) = \nu_t(\sigma)$, the voter prefers to retain the incumbent if $v(p; \theta)$ is greater than $\nu_t(\sigma)$ in both cases. Therefore retrospective and prospective behaviors by the voter are compatible only when $\overline{V}_t = \nu_t(\sigma)$.}

\textbf{Beliefs}

The parties are perfectly informed about the state of the world. Initial beliefs of the voters are given by $\mu_0$ (probability that the cost is high). These beliefs are updated at the end of each period following the policy choice of the incumbent politician: $\mu_t = \mu(h_{t-1})$. As usual, these beliefs are updated according to Bayes rule whenever possible, that is following an equilibrium play.

\emph{Out of equilibrium beliefs} are assumed to be the following. When the voters observe a policy choice on or below the high cost frontier, that is a choice which is feasible in both states of the world, they do not modify their beliefs. Conversely, a policy choice outside this frontier is necessarily informative because it is only feasible in the low cost state.
Whenever they observe such a play, the voters conclude that the cost of production of the public good is low.

3 Political equilibria

We will consider (perfect Bayesian) equilibria such that the party initially in power remains in office forever and always chooses the same policy \((x^e_i, \tau^e_i)\) on the equilibrium path. In these equilibria, called Equilibria with Policy Persistence (EPP), the following lemma holds.

**Lemma 1 Median decisiveness**

In any EPP, the incumbent politician is reelected if and only if the median type individual votes for him.

**Proof.** This lemma follows directly from the single crossing property of individuals’ preferences. From (1), the slope of an indifference curve

\[
\frac{dx}{d\tau} = \frac{1}{\theta u'(x)}
\]

is decreasing with \(\theta\). This implies that the indifference curves cross only once.

In any given period \(t\), the continuation value from electing party \(i\) for a type \(\theta\) individual is

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} v(x^e_i, \tau^e_i; \theta) = v(x^e_i, \tau^e_i; \theta).
\]

Let us denote with \(-i\) the alternative party to party \(i\). If \(v(x^e_i, \tau^e_i; \theta) > v(x^e_{-i}, \tau^e_{-i}; \theta)\), individual \(\theta\), who acts prospectively, should vote for party \(i\). In case of equality, consistency with the retrospective voting behavior requires that this individual votes for the incumbent.

Consider now the case \(x^e_A < x^e_B\). If \(v(x^e_A, \tau^e_A; \theta_m) > v(x^e_B, \tau^e_B; \theta_m)\), the median type votes for \(A\). The single crossing property implies that \(v(x^e_A, \tau^e_A; \theta) > v(x^e_B, \tau^e_B; \theta)\) \(\forall \theta < \theta_m\) and therefore that these individuals also vote for \(A\). Therefore if \(A\) is the incumbent, he is reelected. If \(B\) is the incumbent he is voted out of office. When \(v(x^e_A, \tau^e_A; \theta_m) = v(x^e_B, \tau^e_B; \theta_m)\), each party receives half of the votes and the incumbent is reelected by assumption (see footnote 3).

If \(v(x^e_A, \tau^e_A; \theta_m) < v(x^e_B, \tau^e_B; \theta_m)\), the median type votes for \(B\). The single crossing property implies that \(v(x^e_B, \tau^e_B; \theta) > v(x^e_A, \tau^e_A; \theta)\) \(\forall \theta > \theta_m\) and therefore that these individuals also vote
for \( B \). Therefore if \( B \) is the incumbent, he is reelected. If \( A \) is the incumbent, he is voted out of office. When \( v(x_A^a, \tau_A^a; \theta_m) = v(x_B^b, \tau_B^b; \theta_m) \), each party receives half of the votes and the incumbent is reelected by assumption.

The analysis in the case \( x_A^a > x_B^b \) is similar. When \( x_A^a = x_B^b \) and \( \tau_A^a \neq \tau_B^b \), all the citizens vote for the party proposing the lower tax rate. Finally, when \( x_A^a = x_B^b \) and \( \tau_A^a = \tau_B^b \), they are all indifferent between the two parties. ■

The median type individual will be called the median voter in the remainder of the paper. This lemma is very useful as it allows us, on the voters’ side, to consider only the strategy of the median voter. In other words, the game can be reduced to a game between three players: party \( A \), party \( B \) and the median voter.

3.1 Efficient equilibrium

We first discuss efficient equilibria in which both parties propose the ideal policy of the median voter.

**Proposition 1** There exists an equilibrium in which both parties always propose the ideal policy of the median voter

\[
p^e_{A,t}(c_h) = p^e_{B,t}(c_h) = (x^h_m, \tau^h_m = c^h x^h_m) \]

\[
p^e_{A,t}(c_l) = p^e_{B,t}(c_l) = (x^l_m, \tau^l_m = c^l x^l_m). \]

The voting strategy of the median voter is characterized by

\[
\nabla (\mu_t) = \mu_t v(x^h_m, \tau^h_m; \theta) + (1 - \mu_t) v(x^l_m, \tau^l_m; \theta).
\]

**Necessary and sufficient conditions for this equilibrium to exist are**

\[
v(x^l_m, \tau^l_m; \theta) \geq v(x^h_m, \tau^h_m; \theta_A) \tag{5}
\]

and

\[
\delta \geq \max \left\{ \frac{v(x_A^a, \tau_A^a; \theta_A) - v(x_m^l, \tau_m^l; \theta_A)}{v(x_A^a, \tau_A^a; \theta_A) - v(x_m^l, \tau_m^l; \theta_A) + \beta}, \frac{v(x_B^b, \tau_B^b; \theta_B) - v(x_m^l, \tau_m^l; \theta_B)}{v(x_B^b, \tau_B^b; \theta_B) - v(x_m^l, \tau_m^l; \theta_B) + \beta} \right\}. \tag{6}
\]

**Proof.** Suppose first that \( A \) is in power at the beginning of period \( t \).

1) The cost is high.
If he plays the equilibrium policy, \(A\) is re-elected. Indeed, the voters believe that the cost is high and set a reelection utility level \(V(1) = v(x^h_m, \tau^h_m; \theta_m)\). If he deviates to another policy, he is not re-elected, whether the voters are informed or not. He will not want to deviate when:

\[
v(x^h_m, \tau^h_m; \theta_A) + \beta \geq (1 - \delta) [v(x^h_A, \tau^h_A; \theta_A) + \beta] + \delta v(x^h_m, \tau^h_m; \theta_A).
\]

The first term represents the payoff from sticking to the equilibrium strategy. In this case, \(A\) always implements \((x^h_A, \tau^h_A)\) and is re-elected forever. If \(A\) deviates, he is never re-elected. The best possible deviation is then to choose his optimal policy. In all the following periods, he obtains the utility associated with \(B\)'s equilibrium policy. Rearranging terms this condition becomes

\[
\delta \geq \frac{v(x^h_A, \tau^h_A; \theta_A) - v(x^h_m, \tau^h_m; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(x^h_m, \tau^h_m; \theta_A) + \beta}.
\]

2) The cost is low.

If \(A\) plays the equilibrium policy, the voters learn that the cost is low and impose a reelection threshold \(V(0) = v(x^l_m, \tau^l_m; \theta_m)\). Thus \(A\) is re-elected. If he deviates in the region above the high cost frontiers, he is not re-elected anymore as the voters are informed that the cost is low and ask for \(V(0)\). If \(A\) deviates under the high cost line, he is not re-elected unless he plays \((x^h_m, \tau^h_m)\). Condition (5) guarantees that such a deviation is not worthwhile.

We finally state the condition ensuring that \(A\) always prefers to be re-elected:

\[
\delta \geq \frac{v(x^l_A, \tau^l_A; \theta_A) - v(x^l_m, \tau^l_m; \theta_A)}{v(x^l_A, \tau^l_A; \theta_A) - v(x^l_m, \tau^l_m; \theta_A) + \beta}.
\]

We now prove that (8) implies (7). Our method of proof consists in showing that the function

\[
\Delta = v(x_A, \tau_A; \theta_A) - v(x_m, \tau_m; \theta_A) = \theta_A u(x_A) - cx_A - \theta_A u(x_m) + cx_m,
\]

is decreasing with \(c\), where we have dropped the superscripts pertaining to the state of the world.

\[
\frac{d\Delta}{dc} = \frac{dx_A}{dc} \theta_A u'(x_A) - c - x_A + \frac{dx_m}{dc} \theta_A u'(x_m) - c
\]

\[
= -x_A + x_m \frac{\theta_A u'(x_m)}{\theta_m u''(x_m)} + \frac{u'(x_m)}{u''(x_m)},
\]

11
where we have used the FOC on $x_A$ and $x_m$ (see (2)) and (4). Recalling that $u$ is isoelastic, we have $u'(x_m)/u''(x_m) = -x_m/\varepsilon$. This leads to

$$\frac{d\Delta}{dc} = -x_A + x_m[1 + \frac{\theta_A}{\theta_m} - \frac{1}{\varepsilon}].$$

From the first-order conditions on $x_A$ and $x_m$, we obtain $x_A/x_m = (\theta_A/\theta_m)^{1/\varepsilon}$ and thus

$$\frac{d\Delta}{dc} = x_m[1 - (\frac{\theta_A}{\theta_m})^{1/\varepsilon} - \frac{1}{\varepsilon} \left(1 - \frac{\theta_A}{\theta_m}\right)] = x_m \times \zeta(\varepsilon, \theta_A/\theta_m).$$

$$\frac{d\zeta}{d(\theta_A/\theta_m)} = 1 - \frac{1}{\varepsilon} \left(\frac{\theta_A}{\theta_m}\right)^{(1-\varepsilon)/\varepsilon} - 1],$$

which is positive. Observing that $\zeta(\varepsilon, 1) = 0$, we can conclude that $\zeta(\varepsilon, \theta_A/\theta_m) < 0$ and thus that $d\Delta/dc < 0$.

Suppose now that $B$ is in power at the beginning of period $t$.

The reasoning is the same as for party $A$. Note however that no condition like (5) is needed, as $B$ always prefers $(x_l^m, \tau_l^m)$ to $(x_h^m, \tau_h^m)$. The conditions analogous to (7) and (8) ensuring that $B$ prefers to be reelected are

$$\delta \geq \frac{v(x_h^B, \tau_h^B; \theta_B) - v(x_h^m, \tau_h^m; \theta_B)}{v(x_h^B, \tau_h^B; \theta_B) - v(x_h^m, \tau_h^m; \theta_B) + \beta} \quad (9)$$

and

$$\delta \geq \frac{v(x_l^B, \tau_l^B; \theta_B) - v(x_l^m, \tau_l^m; \theta_B)}{v(x_l^B, \tau_l^B; \theta_B) - v(x_l^m, \tau_l^m; \theta_B) + \beta} \quad (10)$$

We show that only (10) needs to be considered as it implies (9). The argument is the same as for $A$ except that we now need to differentiate $\zeta$ with respect to $\varepsilon$:

$$\frac{d\zeta}{d\varepsilon} = \frac{1}{\varepsilon^2} \ln \left(\frac{\theta_B}{\theta_m}\right) \left(\frac{\theta_B}{\theta_m}\right)^{1/\varepsilon} + \frac{1}{\varepsilon^2} \left(1 - \frac{\theta_B}{\theta_m}\right) > 0 \iff \ln \left(\frac{\theta_B}{\theta_m}\right) \left(\frac{\theta_B}{\theta_m}\right)^{1/\varepsilon} > \frac{\theta_B}{\theta_m} - 1,$$

which is true when $\varepsilon < 1$, as argued in the proof of lemma 3 in the appendix. Observing that $\zeta(1, \theta_B/\theta_m) = 0$, we can conclude that $\zeta(\varepsilon, \theta_B/\theta_m) < 0$. ■

In this equilibrium, represented in figure 1, both parties adopt the same strategy which is stationary. They propose the ideal policy of the median voter in both states of the world. On the equilibrium path, the party that is initially in power remains the
Figure 1: Efficient equilibrium
incumbent forever and the state of the world is revealed in the first period to the voters. The reelection rule is time invariant but depends on the beliefs of the voters. The median voter always asks for the highest possible utility level, which depends on whether he is informed about the state of the world and on its realization.

The condition in equation (6) ensures that both parties prefer to choose a policy that guarantees reelection rather than choosing their ideal policy and not being reelected thereafter. This condition is likely to be satisfied when the preference for the present of the parties is not too strong ($\delta$ high enough): if the parties care sufficiently about the future, they are not ready to forgo future benefits from holding office even if today they can implement their ideal policy. Observe that this condition cannot be satisfied when the benefit of holding office, $\beta$, is nil, that is when the parties are only policy motivated. There are indeed two reasons why a party may want to remain in power. The first one is the direct utility gain $\beta$. The second reason is that it represents a mean of influencing policy. In the equilibrium considered, both parties propose the same policy if elected. The second motive for staying in power is therefore absent. It follows that when $\beta = 0$, politicians have no interest whatsoever in being re-elected. They prefer to deviate and implement today their ideal policy.

Equation (5) states that $A$ has no incentive to pretend that the cost is high when it is low. Even though this would result in a waste of resources, it would lead to a lower public good provision which would better fit $A$’s preferences. Interestingly, no such condition is needed for $B$, who always favors more public good provision at a lower cost.

### 3.2 Inefficient equilibrium

We now turn to the main result of the paper, that is the possibility of obtaining an equilibrium in which a politician chooses a Pareto dominated policy. Let $(\tilde{x}_A, \tilde{\tau}_A)$ be the policy such that

$$\theta_m u(\tilde{x}_A) - \tilde{\tau}_A = \mu_0 v(x^h_B, \tau^h_B; \theta_m) + (1 - \mu_0) v(x^l_B, \tau^l_B; \theta_m)$$

$$\tilde{\tau}_A = c^h \tilde{x}_A$$

and $(\tilde{x}_A, \tilde{\tau}_A)$, $s = h, l$, the policies such that

$$v(x^s_B, \tau^s_B; \theta_m) = v(\tilde{x}_A, \tilde{\tau}_A; \theta_m)$$

(11)
\[ \tilde{\tau}_A^s = c^s \tilde{x}_A^s. \]

It should be noted that these policies do not exist for all possible values of the parameters of the model, \( \theta_m, \theta_B, c^h, c^l \) and \( \mu_0 \). However, one can always find some values for these parameters such that the policies defined above exist. In particular, for \((\tilde{x}_A, \tilde{\tau}_A)\) to exist, \( \mu_0 \), the initial probability that the cost is high, must be large enough.

**Proposition 2** The following strategies constitute a perfect Bayesian equilibrium.

If \( \mu_t = \mu_0 \),

\[
\begin{align*}
p^c_{A,t}(c_h) &= p^c_{A,t}(c_l) = (\tilde{x}_A, \tilde{\tau}_A) \\
p^e_{B,t}(c_h) &= (x^h_B, \tau^h_B); p^e_{B,t}(c_l) = (x^l_B, \tau^l_B) \\
\bar{V}_t &= \mu_0 v(x^h_B, \tau^h_B; \theta_m) + (1 - \mu_0) v(x^l_B, \tau^l_B; \theta_m) = v(\tilde{x}_A, \tilde{\tau}_A; \theta_m).
\end{align*}
\]

If \( \mu_t \neq \mu_0 \),

\[
\begin{align*}
p^c_{A,t}(c_h) &= (\tilde{x}_A, \tilde{\tau}_A); p^c_{A,t}(c_l) = (\tilde{x}_A, \tilde{\tau}_A) \\
p^e_{B,t}(c_h) &= (x^h_B, \tau^h_B); p^e_{B,t}(c_l) = (x^l_B, \tau^l_B) \\
\bar{V}_t &= \mu_t v(x^h_B, \tau^h_B; \theta_m) + (1 - \mu_t) v(x^l_B, \tau^l_B; \theta_m) \\
&= \mu_t v(\tilde{x}_A, \tilde{\tau}_A; \theta_m) + (1 - \mu_t) v(\tilde{x}_A, \tilde{\tau}_A; \theta_m).
\end{align*}
\]

**Necessary and sufficient conditions for this equilibrium to exist are**

\[
v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) \geq v(\tilde{x}_A^l, \tilde{\tau}_A^l; \theta_A) \tag{12}
\]

\[
\frac{\partial x}{\partial \tau} \bigg|_{v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)} \geq c^h \tag{13}
\]

and

\[
\delta \geq \max \left\{ \frac{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(x^l_A, \tau^l_A; \theta_A) + \beta v(x^l_A, \tau^l_A; \theta_A) - v(x^h_A, \tau^h_A; \theta_A)} \right\}. \tag{14}
\]

**Proof.** Suppose first that \( A \) is in power at the beginning of period \( t \).

1) The beliefs of the voters are the initial beliefs.
The equilibrium play of $A$ is pooling at $(\tilde{x}_A, \tilde{\tau}_A)$. The reelection rule of the (uninformed) median voter, $\nabla = v(\tilde{x}_A, \tilde{\tau}_A; \theta_m)$, is satisfied so that $A$ is reelected. The conditions ensuring that $A$ does not want to give up reelection in states $h$ and $l$, respectively are

$$v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) + \beta \geq (1 - \delta) [v(x^h_A, \tau^h_A; \theta_A) + \beta] + \delta v(x^l_B, \tau^l_B; \theta_A)$$

$$\Leftrightarrow \quad \delta \geq \frac{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(x^l_B, \tau^l_B; \theta_A) + \beta} \quad (15)$$

and

$$v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) + \beta \geq (1 - \delta) [v(x^l_A, \tau^l_A; \theta_A) + \beta] + \delta v(x^l_B, \tau^l_B; \theta_A)$$

$$\Leftrightarrow \quad \delta \geq \frac{v(x^l_A, \tau^l_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^l_A, \tau^l_A; \theta_A) - v(x^l_B, \tau^l_B; \theta_A) + \beta}. \quad (16)$$

A deviation by $A$ on or below the high cost frontier is not informative to the voters (following our specification of out-of-equilibrium beliefs). Therefore, if $A$ deviates “to the left” (lower $\tau$), he is not reelected anymore and accordingly obtains a lower payoff. If he deviates “to the right”, he is still reelected but with a less desirable policy if $\partial x / \partial \tau \big|_{v=v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)} \geq c^h$. One can easily verify that this condition is implied by (13), so that this deviation is not profitable.

A deviation above the high cost frontier (when feasible) informs the voters that the cost is low. To be reelected, $A$ must provide the median voter with a utility level at least as high as

$$\nabla = v(x^l_B, \tau^l_B; \theta_m) = v(\tilde{x}_A, \tilde{\tau}_A; \theta_A).$$

When condition (12) is satisfied, $A$ does not want to make such a deviation.

2) When the voters are informed, the pooling strategy is not optimal: $A$ should deviate in both states of the world. In state $l$, he is not reelected if he sticks to the pooling strategy. In state $h$, he is reelected but could obtain a higher utility level.

The separating strategy described in the proposition is optimal. Whether the cost is low or high, party $A$ is voted out of office if deviating “to the left” whereas, under condition (13), a deviation “to the right” allows party $A$ to be reelected but yields a lower utility level. Conditions ensuring that $A$ prefers to be reelected are

$$\delta \geq \frac{v(x^h_A, \tau^h_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^h_A, \tau^h_A; \theta_A) - v(x^l_B, \tau^l_B; \theta_A) + \beta} \quad (17)$$

and

$$\delta \geq \frac{v(x^l_A, \tau^l_A; \theta_A) - v(\tilde{x}_A, \tilde{\tau}_A; \theta_A)}{v(x^l_A, \tau^l_A; \theta_A) - v(x^l_B, \tau^l_B; \theta_A) + \beta}. \quad (18)$$
From (13), \( v(\tilde{x}_A^h, \tilde{\tau}_A^h; \theta_A) > v(\tilde{x}_A, \tilde{\tau}_A; \theta_A) \). Therefore (15) implies (17). Moreover from (12), (18) implies (16). Consequently the conditions ensuring that \( A \) always prefers to be re-elected are those in (14).

Suppose now that \( B \) is in power at the beginning of period \( t \). The strategy described in the proposition is clearly optimal as \( B \) obtains his optimal policy in both states of the world and is always reelected.

We finally have to show that condition (12) is possible. The following lemma, proved in the appendix, will be useful:

**Lemma 2** For a coefficient of relative risk aversion \( \varepsilon \) sufficiently close to 1, \( \tilde{\tau}_A \) is decreasing with \( c^s \).

Let us denote \( \tilde{A}^h \) (resp. \( \tilde{A}^l \) and \( \tilde{A} \)) the point \( (\tilde{x}_A^h, \tilde{\tau}_A^h) \) (resp. \( (\tilde{x}_A^l, \tilde{\tau}_A^l) \) and \( (\tilde{x}_A, \tilde{\tau}_A) \)). Let us also denote \( I_m^h \) (resp. \( I^l_m \) and \( I_m^l \)) the indifference curve for \( m \) through \( \tilde{A}^h \) (resp. \( \tilde{A}^l \) and \( \tilde{A} \)). Lemma 2 implies that \( \tilde{A}^l \) lies to the right of \( \tilde{A}^h \). Therefore \( I_m^l \) lies above \( I_m^h \). By construction, \( I_m \) lies between \( I_m^h \) and \( I_m^l \) so that \( \tilde{A} \) must be to the right of \( \tilde{A}^h \). Lemma 2 also implies that it is possible that \( \tilde{\tau}_A < \tilde{\tau}_A^l \). As indifference curves are increasing, this is a necessary condition for the indifference curve for \( A \) through \( \tilde{A} \) to be above the indifference curve for \( A \) through \( \tilde{A}^l \), which is equivalent to condition (12). This condition is more likely to hold the steeper the indifference curves for \( A \) (i.e. the smaller \( \theta_A \)) and the bigger \( \mu_0 \).

This equilibrium is represented in figure 2. The strategy of party \( B \) consists in proposing his ideal policy \( (x_{B}^s, \tau_{B}^s) \) in state \( s = h, l \). It is separating. Consequently, if \( B \) is in power at some point in time, voters learn the value of the cost. This is not true for \( A \) whose policy choice depends on the voters’ beliefs. When these latter are not informed (they hold the initial beliefs), \( A \) adopts a pooling strategy: he proposes the same policy in both states in the world. This implies that the policy choice is inefficient when the cost of production is low as more of the public good could be produced with the same tax receipts. The logic underlying this result is that if \( A \) adopts a Pareto efficient policy when the cost is low, he reveals this information to the voters. In such a case, the strategy of \( B \) implies that he would choose the policy \( (x_{B}^l, \tau_{B}^l) \) if elected. In order to give the
Figure 2: Inefficient equilibrium
median voter a utility level at least equal to \( v(x_B^l, \tau_B^l; \theta_m) \), A must select a policy “to the right” of \((\tilde{x}_A^l, \tilde{\tau}_A^l)\). However, under condition (12) he is not willing to do so. In other words, even though the good is produced more efficiently, the increase in production that A must ensure in order to be reelected is excessively high given his preferences for the good. Condition (14) precisely guarantees that A wants to be reelected. It should be noted that this may be the case even when the benefit of holding office is 0. The explanation is straightforward: if A is voted out of office, he will never be reelected again. Therefore the policy choice will be the optimal policy of B forever. Even though there is a short run gain for A due to the implementation of his ideal policy, there is a long run cost. When the preference for the present is not too strong, A prefers to remain in office.

We have shown that politician A who doesn’t want much of the public good may adopt a Pareto dominated policy at the political equilibrium. Is the converse true? Is it also possible that the party B displaying a high taste for the public good also adopts an inefficient behavior? The next section addresses this question.

4 Not all parties behave inefficiently

We show in the next proposition that the symmetric equilibrium in which A always proposes his optimal public good level and B may play pooling does not exist.

**Proposition 3** There does not exist an equilibrium symmetric to the inefficient one in the previous section.

**Proof.** See appendix ■

Such an equilibrium would require that party A chose his preferred policy in the two states of nature. On the other side, party B would pick up a pooling policy until voters are informed about the cost. Symmetrically to what happened with proposition 2, we would have the following equilibrium strategies:

If \( \mu_t = \mu_0 \),

\[
\begin{align*}
p_{B,t}^e (c_h) &= p_{B,t}^e (c_l) = (\tilde{x}_B, \tilde{\tau}_B) \\
p_{A,t}^e (c_h) &= (x_A^h, \tau_A^h); p_{A,t}^e (c_l) = (x_A^l, \tau_A^l) \\
\nabla_t &= \mu_0 v(x_A^h, \tau_A^h; \theta_m) + (1 - \mu_0) v(x_A^l, \tau_A^l; \theta_m) = v(\tilde{x}_B, \tilde{\tau}_B; \theta_m) \end{align*}
\]
If $\mu_t \neq \mu_0$,

$$
p_{B,t}^e(c_h) = (x_B^h, \tau_B^h); \quad p_{B,t}^c(c_l) = (x_B^l, \tau_B^l)
$$

$$
p_{A,t}^e(c_h) = (x_A^h, \tau_A^h); \quad p_{A,t}^c(c_l) = (x_A^l, \tau_A^l)
$$

$$
\nabla_t = \mu_t v(x_A^h, \tau_A^h; \theta_m) + (1 - \mu_t) v(x_A^l, \tau_A^l; \theta_m)
$$

$$
= \mu_t v(x_B^h, \tau_B^h; \theta_m) + (1 - \mu_t) v(x_B^l, \tau_B^l; \theta_m).
$$

These strategies are represented on figure 3.

We show that it is never optimal for $B$ to offer the pooling policy when the cost is low so that the previous strategies cannot be sustained in equilibrium. The idea underlying the result is that when $A$ plays his optimal policy in each state of the world, $B$ should deviate from the pooling strategy in the low cost state, proposing a policy that entails more public good provision at a lower cost and leaving the median voter indifferent between this policy and $A$’s optimal policy, therefore guaranteeing reelection for $B$.

This result means that there is a fundamental asymmetry between the parties in the sense that the party with a low taste for the public good is more likely to select inefficient policies. However, this does not mean that the party with a strong taste for the public good never wastes resources. The existence of an equilibrium with $B$ adopting a pooling strategy is still an open possibility.

5 Concluding comments

We have shown that a party in power may prefer to supply a low quantity of a public good when more could be produced at the same tax level. This is clearly a wasteful behavior. The reason for such a behavior is that if the median voter were informed that the cost of production is low, he would ask for a larger provision of the good, against the interest of the party with a low taste for the public good. This suggests that when a party with a strong taste for the public good is in power, the policy adopted is less likely to involve a waste of resources. Proposition 3 proves that this intuition is correct. However it relies very much on a technological asymmetry of the model: because the budget is balanced each period, the set of feasible policies expands when the cost decreases. This implies that a party can pretend that the cost is high when it is low but not the converse. Allowing
Figure 3: The symmetric configuration with B wasting resources is not an equilibrium.
some debt would restore some symmetry. However, as soon as debt is observable, it still
does not allow a party to pretend that the cost is lower than it really is.

Lastly, a few words concerning the nature of the political inefficiency are in order.
A possible interpretation of this inefficiency is that the politician in power collects more
money than needed and “burns” the unused amount. This is of course an unrealistic
outcome of the model but it captures the incentives faced by elected policy-makers. A
proper framework should allow the politicians to “steal” public money and use it for
their own personal consumption, which is not the case in our modeling. We guess that
introducing such a possibility would in fact reinforce our main result as it represents an
additional motive for the politician to pretend that the technology is inefficient.
Technical appendix

A Proof of lemma 2

We remove in this proof the superscript denoting the state of the world. From (11), $\tilde{x}_A$ is implicitly defined by the following equation:

$$\theta_m u(\tilde{x}_A) - c\tilde{x}_A - \theta_m u(x_B) + cx_B = 0$$  \hspace{1cm} (19)

where $x_B$ satisfies the first-order condition $c = \theta_B u'(x_B)$.

Differentiating equation (19) with respect to $c$, we obtain

$$\frac{d\tilde{x}_A}{dc} = \frac{\tilde{x}_A + \theta_m \frac{dx_B}{dc} u'(x_B) - x_B - c \theta_m}{\theta_m u'(\tilde{x}_A) - c}.$$  \hspace{1cm} (20)

Differentiating $\tilde{\tau}_A = c\tilde{x}_A$ and recalling that $u(x) = x^{1-\varepsilon}/(1 - \varepsilon)$, we get

$$\frac{d\tilde{\tau}_A}{dc} = \tilde{x}_A + c \frac{d\tilde{x}_A}{dc} = \frac{\tilde{x}_A \theta_m u'(\tilde{x}_A) - x_B \theta_B u'(x_B) - \frac{x_B}{\varepsilon} u'(x_B)(\theta_m - \theta_B)}{\theta_m u'(\tilde{x}_A) - c}.$$  \hspace{1cm} (21)

The denominator is positive. We thus have to check that the numerator, denoted $N$, is negative.

$$N = x_B u'(x_B) \left[ \theta_m \frac{\tilde{x}_A u'(\tilde{x}_A)}{x_B u'(x_B)} - \theta_B - \frac{\theta_m - \theta_B}{\varepsilon} \right].$$  \hspace{1cm} (22)

Observing that

$$\frac{\tilde{x}_A u'(\tilde{x}_A)}{x_B u'(x_B)} = \frac{u(\tilde{x}_A)}{u(x_B)},$$

we have that $N < 0$ if and only if

$$\frac{u(\tilde{x}_A)}{u(x_B)} < \frac{\theta_B (\varepsilon - 1) + \theta_m}{\varepsilon \theta_m}.$$  \hspace{1cm} (23)

From (19),

$$\frac{u(\tilde{x}_A)}{u(x_B)} = \frac{\theta_B}{\theta_m} (1 - \varepsilon) \left( \frac{\tilde{x}_A}{x_B} - 1 \right) + 1.$$  \hspace{1cm} (24)

Therefore the previous inequality is satisfied if and only if

$$\frac{\tilde{x}_A}{x_B} < \frac{\theta_m - \theta_B}{\varepsilon \theta_B} + 1.$$  \hspace{1cm} (25)
From the strict concavity of $u$, $u(x_B) - u(\tilde{x}_A) < u'(\tilde{x}_A)(x_B - \tilde{x}_A)$. Rearranging (19), we obtain

$$u(x_B) - u(\tilde{x}_A) = \frac{c(x_B - \tilde{x}_A)}{\theta_m} < u'(\tilde{x}_A)(x_B - \tilde{x}_A)$$

$$\iff \frac{u'(\tilde{x}_A)}{u'(x_B)} > \frac{\theta_B}{\theta_m}$$

$$\iff \tilde{x}_A < \left(\frac{\theta_m}{\theta_B}\right)^{1/\varepsilon}.$$ 

Observing that

$$\lim_{\varepsilon \to 1} \frac{\theta_m - \theta_B}{\varepsilon \theta_B} + 1 = \lim_{\varepsilon \to 1} \left(\frac{\theta_m}{\theta_B}\right)^{1/\varepsilon} = \frac{\theta_m}{\theta_B},$$

and since $(\theta_m - \theta_B)/\varepsilon \theta_B$ is continuous in $\varepsilon$, we have that $\tilde{x}_A/x_B < (\theta_m - \theta_B)/\varepsilon \theta_B + 1$ for $\varepsilon$ sufficiently close to 1 and therefore that $d\tilde{x}_A/dc < 0$.

## B Proof of proposition 3

In order to prove proposition 3, we use the following lemma:

**Lemma 3** $\tilde{\tau}_B^s = c_s \tilde{x}_B^s$ decreases when $c_s$ increases, where $\tilde{x}_B^s$ is implicitly defined by the equation

$$\theta_m u(\tilde{x}_B^s) - c_s \tilde{x}_B^s - \theta_m u(x_A^s) + c_s x_A^s = 0,$$

where $c_s = \theta_A u'(x_A^s)$.

**Proof.** Differentiating (20) and dropping superscripts, we have

$$\frac{d\tilde{\tau}_B}{dc} = \frac{\tilde{x}_B \theta_m u'(\tilde{x}_B) - x_A \theta_A u'(x_A) - \frac{x_A}{\varepsilon} u'(x_A) (\theta_m - \theta_A)}{\theta_m u'(\tilde{x}_B) - c}.$$ 

The numerator being negative, we want to show that the numerator $N$ is positive. After some manipulations we obtain

$$N > 0 \iff \frac{u(\tilde{x}_B)}{u(x_A)} < \frac{\theta_A (\varepsilon - 1) + \theta_m}{\varepsilon \theta_m}$$

$$\iff \frac{\tilde{x}_B}{x_A} > \frac{\theta_m - \theta_A}{\varepsilon \theta_A} + 1.$$ 

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From the concavity of \( u \),
\[
\frac{x_B}{x_A} > \left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon}.
\]
To achieve the proof, we only need to show that
\[
\left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon} > \frac{\theta_m - \theta_A}{\varepsilon \theta_A} + 1, \forall \varepsilon < 1.
\]
Observing that these two functions are decreasing with \( \varepsilon \) and equal when \( \varepsilon = 1 \), this will be the case if the slope of \( \left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon} \) is lower than the slope of \( \frac{\theta_m - \theta_A}{\varepsilon \theta_A} + 1 \), that is if
\[
-\frac{1}{\varepsilon^2} \ln \left( \frac{\theta_m}{\theta_A} \right) \left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon} < -\frac{1}{\varepsilon^2} \left( \frac{\theta_m}{\theta_A} - 1 \right)
\]
\[
\Leftrightarrow \ln \left( \frac{\theta_m}{\theta_A} \right) \left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon} > \frac{\theta_m}{\theta_A} - 1.
\]
As \( \left( \frac{\theta_m}{\theta_A} \right)^{1/\varepsilon} > \frac{\theta_m}{\theta_A}, \forall \varepsilon < 1 \), it is sufficient to show that
\[
\ln \left( \frac{\theta_m}{\theta_A} \right) \frac{\theta_m}{\theta_A} > \frac{\theta_m}{\theta_A} - 1.
\]
One can easily verify that \( x \ln x > x - 1, \forall x > 0 \). Hence the result.

We are now ready to prove proposition 3.

Let us denote \( \tilde{B} \) (resp. \( \tilde{B}^l \) and \( \tilde{B} \)) the point \( (\tilde{\tau}_B, \tilde{x}_B) \) (resp. \( (\tilde{\tau}_B^l, \tilde{x}_B^l) \) and \( (\tilde{\tau}_B, \tilde{x}_B) \)). Let us also denote \( I_m^h \) (resp. \( I_m^l \)) the indifference curve for \( m \) through \( \tilde{B} \) (resp. \( \tilde{B} \)). Similarly, denote \( \tilde{I}_B^m \) and \( \tilde{I}_B \) the indifference curves for \( m \) and \( B \) through \( \tilde{B} \). Finally, denote \( I_B \) the indifference curve for \( B \) through \( \tilde{B} \).

We want to prove that \( B \) always prefers the policy \( \tilde{B}^l \) to the policy \( \tilde{B} \). As indifference curves never intersect, it is enough to show that \( \tilde{I}_B \) crosses the low cost equation above the point \( \tilde{B}^l \).

Let \( A^l \) and \( A^h \) denote the optimal policies for party \( A \) in each state. From our assumption that \( \varepsilon \leq 1 \), we know that \( A^l \) lies above and to the right of \( A^h \). We also know that the median voter prefers more public good than party \( A \) in each state. Therefore the indifference curve for \( m \) through \( A^l \) lies above the indifference curve for \( m \) through \( A^h \). In other words, \( I_m^l \) lies above \( I_m^h \). By definition of \( \tilde{B} \), \( \tilde{I}_m \) lies between \( I_m^h \) (lower bound) and \( I_m^l \) (upper bound), depending on the value of \( \mu_0 \). Lemma 4 states that \( \tilde{B}^l \) lies to the right of \( \tilde{B}^h \). This implies that \( \tilde{I}_m \) crosses the low cost frontier to the right of \( \tilde{B}^l \). Consider now \( \tilde{I}_B \), the indifference curve for \( B \) through \( \tilde{B} \). We know from the fact that \( \theta_m \leq \theta_B \) that \( \tilde{I}_B \) is flatter than \( \tilde{I}_m \) at \( \tilde{B} \). Moreover, \( \tilde{I}_B \) and \( \tilde{I}_m \)
cross only once, precisely at $\tilde{B}$. This implies that $\tilde{I}_B$ crosses the low cost frontier to the right of the point where $\tilde{I}_m$ crosses the same frontier. By transitivity, $\tilde{I}_B$ crosses the low cost frontier to the right of $\tilde{B}^t$ so that $\tilde{I}_B$ lies below $I_B^t$. Hence the result.
References


