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Edward P. Kahn

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Assessment of the Potential for Full Co-ordination of the California Electric Utilities*

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*A Report to the California Energy Resources Conservation and Development Commission
ABSTRACT

The principles of power pooling are discussed in the context of the California electric utility industry. The management, planning and operating tasks involved are enumerated. An analytic approximation of the loss of load probability is developed and applied to California data. This gives estimates of the reserve sharing potential in the present system.
LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERCDC</td>
<td>California Energy Resources Conservation and Development Commission (also referred to as ERCDC)</td>
</tr>
<tr>
<td>ERDA</td>
<td>Energy Research and Development Administration</td>
</tr>
<tr>
<td>FPC</td>
<td>Federal Power Commission</td>
</tr>
<tr>
<td>LADWP</td>
<td>Los Angeles Department of Water and Power</td>
</tr>
<tr>
<td>LOLP</td>
<td>Loss of Load Probability</td>
</tr>
<tr>
<td>NEPOOL</td>
<td>New England Power Pool</td>
</tr>
<tr>
<td>PG&amp;E</td>
<td>Pacific Gas and Electric Company</td>
</tr>
<tr>
<td>PJM</td>
<td>Pennsylvania-New Jersey-Maryland Interconnection</td>
</tr>
<tr>
<td>SCE</td>
<td>Southern California Edison Company</td>
</tr>
<tr>
<td>SDG&amp;E</td>
<td>San Diego Gas and Electric Company</td>
</tr>
<tr>
<td>SMUD</td>
<td>Sacramento Municipal Utility District</td>
</tr>
</tbody>
</table>
I. Introduction

It is widely known that economies of operation and investment in the electric utility industry can be realized through the close co-ordination of several service areas. Public policy is difficult to make in this area, however, because it is difficult to evaluate these potential economies and their costs. Such an evaluation must address technical issues such as the feasibility of interconnection, the cost of relieving bottlenecks in the transmission network, and the necessary control technology to effect full co-ordination. In addition the institutional framework of a fully co-ordinated system must also be developed. This involves establishing a management structure that will serve as a decision making authority. A set of billing procedures must be created to account for services exchanged among member companies. In addition, interconnected electric utility arrangements also require a joint planning and analysis capability and an operations center for day to day practices.

It is impossible to make a comprehensive analysis of all these issues within the scope of this study. What can be done, however, is to sketch the outlines of the problem, develop estimates of the potential economies and summarize the experience of other regions which have developed fully co-ordinated power pooling arrangements. In section II, we discuss the current institutional structure within which the California electric utilities operate.
and how this effects the co-ordination issue. In section III, we address the institutional requirements of a fully co-ordinated power pool, drawing on the experience of such institutions in the eastern United States. In section IV, we make a technical assessment of the reserve sharing potential in the present California utility system. This analysis is based upon a model of power system reliability which is derived in section V.

II. The California Power Pool and ERCDC Energy Shortage Contingency Planning

The only formal arrangement currently in effect for co-ordinated operation of the electric utilities in California is the California Power Pool Agreement. This contract was signed in 1964 by the three private utilities PG&E, SCE and SDG&E. It has been informally criticized from several sources as more of an impediment to co-ordination than an aid. There are two principal points to this criticism. First, there are financial penalty provisions for reserves support lasting longer than two hours. This tends to create a disincentive for members relying upon one another in emergency situations. The concept of a fully co-ordinated power pool implies that members will have financial incentives to share reserve capacity. If California is to realize the benefits of power pooling, there must be an accounting mechanism that will tend to minimize redundant investment in reserve capacity. Interconnections such as PJM and NEPOOL use a split savings approach to this problem. With this method both parties to a power exchange transaction receive reasonable benefit. The method for calculating the split savings
will be discussed further in section III.

The second main criticism of the California Power Pool is the exclusion of municipal utility districts. For example, LADWP has large reserve requirements due to lack of interconnection. As a hedge against construction delays and other uncertainties, the Department plans for even larger margins than required by reliability. These investments could be substantially reduced if reserve help could be assured. Smaller municipal utilities have complained about excessive charges for transmission services over lines owned by the private companies. Although some of these charges have been dealt with by settlement agreements, there are still complaints about lack of access to alternate supply resources for municipals due to lack of transmission service.

Present conditions offer ERCDC an opportunity to address these issues under the energy shortage contingency planning authority. If present trends continue and 1976-77 proves to be another drought year, there will be energy shortages in the hydro-electric generation system. The impact of such shortages could be alleviated by integrating the excess capacity in the LADWP system into a statewide allocation program. The authority of ERCDC under AB1570 in the area of emergency planning is broad. Essentially ERCDC is authorized to plan for the provision of basic services and maintenance of a sound state economy. If adverse hydro conditions develop PG&E, SMUD and other northern California utilities may be unable to meet customer demand. The large reserves of LADWP are one potential resource ERCDC may wish to use in its plan to deal with this type of shortage. The California Power Pool
Agreement is one potential obstacle to the implementation of such a plan. ERCDC would have to recommend to the legislature that SCE wheel LADWP power to PG&E lines. The price for this service could be excessive if the Pool Agreement governed. Once ERCDC steps into this problem, it has opened the way to a more extensive role in capacity planning and operation of the electric utilities. The more comprehensive role implied by full co-ordination, however, would probably require legal and legislative action.

III. Institutional Requirements for Full Co-ordination

There are three generic tasks associated with fully co-ordinated power pooling, management structure, planning, and operations. The first area, management, presents no technical problems and so will be discussed from the point of view of institutional alternatives. Planning and operations are technical functions which must be given an institutional base to achieve full co-ordination. In these areas we will summarize the problems to be solved, the analytic capability needed and the state of the art.

A. Management structure

A power pool is an association of electric utility companies which typically differ greatly in service area, load and capacity resources. To operate such an organization a mechanism must be devised which allocates authority to the various members. Essentially two different kinds of management organizations have evolved. The New England Power Pool (NEPOOL) allocates voting authority to member utilities in proportion to each
system's peak load. For a given policy or practice to be adopted at least 75% of the total votes must be cast in favor. This corresponds to the sum of the votes of the five largest members. In effect the largest utility has a veto power since it has more than 25% of the votes. A negative vote of 15% by at least two participants can block any proposed action. The Pennsylvania-New Jersey-Maryland Interconnection (PJM) allows only the major utilities on its management committee. Action taken requires a unanimous vote of the representatives to this committee. This, in effect, is a more conservative arrangement than NEPOOL.

In California, where some planning and all siting authority is rested in the ERCDC, there would necessarily be a relationship between a fully co-ordinated pool and the regulatory authority. Several alternative arrangements are possible. ERCDC might become one representative on a pool management committee. In effect, since the Commission has veto power over siting decisions, it will have to agree to planning decisions made separately or jointly by the utilities. The only real question on this point is procedural; will ERCDC participate in the discussions which go into utility plans before they are formalized or will there be a iterative procedure of utility plans, Commission review, and then reformulation of plans. Operating procedures which influence reliability also come under a broad interpretation of ERCDC's emergency planning authority. The security of a statewide power pool can be planned for so that day to day operations will not constantly be
risking service interruptions. ERCDC will have to acquire expertise in the area of transmission system analysis to exercise this power.

Power pooling will entail wholesale transactions. The rate schedule for such transactions will have to be filed with the Federal Power Commission. The billing procedure for energy exchanges which has been worked out by NEPOOL and PJM is a possible model for California. It works in the following way. Suppose utility A needs an extra 50 MW of power in the next hour. The pool dispatcher calculates the incremental cost of that power first on a pool-wide basis and then on the basis that utility A is an isolated island. The difference between these two numbers is the potential savings. Typically the pool will have power available for a cost lower than utility A's incremental cost. In that case A will pay the incremental cost plus one half the savings. The extra payment will go to the supplying member. If A can generate the power more cheaply than the pool there is no transaction.

As in all phases of the electric utility industry, the transmission system is less transparent than the generation system. Charges for the use of pool transmission facilities are more difficult to assess than those for energy exchange. NEPOOL fixes a wheeling rate that is adjusted for the amount of outage service received as a percentage of the day's transfers. This second adjustment deals with the limits of transmission capacity in a crude way. Essentially there is a peak-loading
problem on high voltage lines which makes some transfers have greater impact than others. This depends on the geometry of the transaction and the system load pattern at the time of transfer. In principle, the type of analysis which is necessary to insure the stability and security of the transmission system could be extended to account for the allocation of costs associated with power transfers. This type of analysis is discussed in the next section.

B. Planning and Analysis

The benefits of pooling can only be captured if they are planned for. Moreover, full co-ordination requires investments to optimize a system which was not planned as a whole. Generically there is no difference between planning for a power pool or planning for a single service area. The same problems must be addressed in both cases. A practical difference does appear in the size of interconnected systems. An algorithm that is computationally tractable for small or moderate-sized systems, may become extremely expensive to implement for a large pool. Computational complexities only become limiting, however, for on-line operating applications and do not pose any serious problems for planning purposes.

A detailed account of the various planning and analysis tasks associated with a modern power system can be found in a recent ERDA Conference publication entitled, Systems Engineering for Power: Status and Prospects. This report enumerates the various planning and operating tasks and gives a state of the
art assessment of the technical capability in these areas. For the purposes of policy analysis, the current models of power system performance are typically too cumbersome for easy assessment of alternatives. This limits the ability of agencies like ERCDC, FPC and so on to evaluate the plans of the utility industry and supply the necessary public perspective. Even within the industry there is some recognition that the flexibility of closed form analytic models is preferable to elaborate and inaccessible computer techniques. An example of what can be done with analytic models is the LOLP model presented in section V and used in section IV. LOLP is the standard measure of power system reliability. To perform the estimate given in section IV by conventional means would require months of data preparation, coding, debugging, etc. and many dollars of computer time. This is an impediment to independent regulatory analysis.

Co-ordinated power pools plan capacity additions as a whole, but there is an institutional problem of allocating capacity expansion responsibility to member companies. PJM has an elaborate formula to make this allocation which is based on member peak loads, average company forced outage rates, a large unit adjustment, and other system characteristics. NEPOOL reports dissatisfaction with its initial capacity allocation formula because of wide variation in forecasting accuracy among the companies. NEPOOL now plans to set a pool wide reserve margin that each company will apply to its own forecasted peak in determining capacity requirements. Such a method assumes little
variation in average forced outage rates from one company to another. For a system like PJM this is unrealistic. California has a good deal of variation also, so the PJM formula would probably be more equitable than the NEPOOL formula.

For full co-ordination to become a reality in California it is likely that investments will have to be made in the transmission system. The potential benefits in reserve sharing which are calculated in section IV can only be realized if there is increased interchange of power from systems with extra capacity to those which are in need. By definition it would be impossible to tell which systems will be exchanging reserve power in advance of the event. Therefore, planning for co-ordination would involve an increase in transmission capacity across the entire state. Exactly where and how much extra capacity would have to be the subject of extensive study. ERCDC is not now in the position to make an independent study of these questions. Some preliminary remarks on the scope of such an analysis can be made, however.

Transmission systems in the Western United States are limited by stability problems. These are generically of two types. Transient stability refers to the ability of the network to maintain synchronization following a line fault or equipment malfunction. Typically transient conditions refer to events in the first few seconds following a disturbance. Dynamic instabilities refer to a number of phenomena including low frequency oscillations that appear following slow changes in load.
or generation. The Pacific Intertie (DC) exhibits 20 cycle/minute oscillations when the electrical angle between North and South becomes large. These problems can be dealt with using various control technologies. For example, d-c line modulation controls are being installed on the Pacific Intertie to increase its dynamic stability margin. ERCDC should be informed about the effect of such changes in a manner similar to its supply planning reports on generation. There are a variety of security enhancement techniques that could be employed to make better use of existing lines so that transient and dynamic stability limits can be reduced. Beyond using such control technologies additional transmission lines might be necessary. As in all cases of system optimization, a cost/benefit analysis should be performed to see whether the cost of transmission system improvements is justified by the savings in reserve capacity. From the estimates in section IV, it appears that the reserve savings are sufficiently great to justify the costs of full co-ordination. Of course, detailed analysis is necessary to prove this claim.

C. Operations Control Center

Power pooling is only possible if the complex transactions of energy interchange are co-ordinated in a centralized base of operations where generation can be scheduled for economy and system security. There are a large number of control centers operating throughout the world which handle day to day operations for large power systems. A thorough discussion of the state of the art in designing such centers is given in the article by
Dy Liacco appearing in ERDA's Systems Engineering for Power.

Without repeating this discussion, it would be useful to emphasize a few points. First, this is not a new and untried concept. Systems control centers have been performing well for many years with increasing sophistication. Second, some capability in this area already exists in California. Dy Liacco does not discuss the PG&E system, but does indicate in some detail what SCE is capable of doing in the way of system control.

According to Dy Liacco, SCE had a modern control center operating in Los Angeles early in 1976. This center is planned to have the following on-line capability:

1. Automatic Generation Control
2. Minimum NO\textsubscript{x} Emission Dispatch
3. Security Monitoring
4. Steady State Security Analysis
5. State Estimation (a mathematical procedure for estimating the bus voltage magnitudes and phase angles of the network)
6. On-Line Load Flow [real time analysis of power flows on main lines for use in functions (3) and (4)]
7. Supervisory Breaker Control (manual load control for load shedding and restoration in emergencies)

For the purposes of assessing the feasibility of full coordination, ERCDC needs to know the level of detail of system representation at this center, the degree and kind of coordination with PG&E, LADWP and SDG&E, and the costs of improvements in these areas.
IV. Technical Assessment

In this section the potential benefits of power pooling are estimated in terms of capacity deferrals that would be possible in a fully co-ordinated system. Taking the present capacity resources of the state as an aggregate, a calculation is performed to show the maximum co-incident peak load which could be served by the present resources if full co-ordination were achieved. This result is converted into the language of growth rates to show that under normal hydro conditions no new capacity would be needed until 1979 or 1980.

California faces the prospect of a power shortage in the summer of 1977 due to potential drought conditions. P.G.&E is the utility which will be most directly affected because of its heavy dependence on hydro generation. They have stated publicly that with the 1100 MW of Diablo Canyon 1 on line, the potential emergency could be avoided. Pooling represents an alternate solution to the problem. Excess capacity from other California utilities could supply the potential shortfall. LADWP is the major source of additional capacity. According to data presented in ERCDC Docket No. 76-FOR-3, the Department has 576 MW over and above the 30.4% reserve margin they require for reliability purposes. The results of section B below show that a 15% reserve margin would be sufficient for a statewide system. Thus from LADWP alone another 579 MW would be available (579 = .15 x 3860 MW, 3860 MW = LADWP forecasted 1976 peak). There are institutional problems associated with this solution that are beyond the range of the discussion here. For now it is sufficient to explore
the technical potentials. I begin with the California resources and then turn to the impact of the Pacific Northwest interties.

A. California controlled supply resources

In this section, the model of LOLP and reserve margin presented in section V is applied to the California utilities. This application considers only those capacity resources which are owned by the utilities. Thus, long term contracts for power from the Pacific Northwest are neglected by this first analysis. These resources are discussed in section B. The results presented here, therefore, represent a conservative estimate of the reserve sharing potential for the California electric power system.

There are two principal data requirements for the model:
1) an accurate list of the generators owned by the utilities specifying the capacity of each, and 2) a forced outage rate for each generator representing the reliability of the unit in question. In California, the complicated hydro-electric system presents a minor problem for the first requirement. For example, the PG&E Supply Planning Form 3 does not list the number of generators at each hydro site, only the capacity for a dry year "August". The number of generators was, therefore, tabulated for PG&E and the major water agencies from data published by the Cal Resources Agency. Table 1 gives a summary of all the hydro-electric resources in California.

Data on forced outage rate was derived from Supply Planning Form 7. Again there is some slight ambiguity in the definition of terms. It is not clear what is being represented by the numbers given. If by the phrase "percent of year" it is meant
calendar year, then this definition differs from that used by the Edison Electric Institute. The "calendar year" reading would make all numbers too low. To correct for this potential bias, I have increased all published data by about 1%. This means that if a unit is listed as having an FOR = 4%, I treat it as 5%. A more thorough investigation could resolve this problem.

Having collected the relevant data, it must be aggregated somewhat for convenient use in the LOLP model. It is typical in planning studies to group generators of similar size, fuel type and reliability for analytical purposes. One must be careful that the averaging techniques do not introduce biases in the analysis. Experience has shown that very large units have the most significant impact on the reliability of power systems. Therefore, all units over 300 MW have been averaged only with units of nearly the same size. In Table 2 the generator categories are summarized.

The California data necessary to apply the normal approximation model is summarized in Table 3. A fifth order correction term has been added. The use of this will be illustrated in the following calculations. We begin by using the parameter b from the normal model to calculate LOLP. We assume that the one day in 10 years criterion is the reliability objective. In probability terms, this means a value of from $2.74 \times 10^{-4}$ to $4 \times 10^{-4}$, depending upon whether weekends and holidays are counted. The influence on reserve margin of this range is small.
The first step is to calculate the first order normal approximation, LOLP₀. Recall that

\[ \text{LOLP}_0 = \frac{1}{2} \text{Erfc}(b), \]

where

\[ \text{Erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \]

and

\[ b = \frac{\bar{S} - W}{\sigma_s \sqrt{2}} \]

where

\[ \bar{S} = \text{average power available} \]

\[ = \sum \alpha_i (1 - L_i) \]

\[ \alpha_i = \text{capacity of } i^{\text{th}} \text{ generator} \]

\[ L_i = \text{forced outage rate of } i^{th} \text{ generator} \]

\[ W = \text{coincident peak load} \]

\[ \sigma_s = \text{the standard deviation of the random variable representing total power in the system.} \]

We calculate LOLP₀ for \( b = 2.8 \) and \( b = 2.9 \),

\[ b = 2.8, \text{LOLP}_0 = 3.75 \times 10^{-5} \]

\[ b = 2.9, \text{LOLP}_0 = 2.05 \times 10^{-5}. \]

The correction terms to LOLP₀ are as follows
where $a_3$, $a_4$, and $a_5$ are correction coefficients defined in terms of the moments of the probability distributions. They are defined in Table 3 more specifically.

From the data in Table 3, we calculate

\[
\frac{a_3}{6} \left( \frac{1}{\sigma_s^2} \right)^{3/2} = .0118
\]

\[
\frac{a_4}{24} \left( \frac{1}{\sigma_s^2} \right)^2 = .0157
\]

\[
\frac{a_5}{120} \left( \frac{1}{\sigma_s^2} \right)^{5/2} = .0009.
\]

Using (2) and (4), we calculate LOLP, for $b = 2.8$

\[
\text{LOLP} = (1 + 5.92 + .312 + .84) 3.75 \times 10^{-5}
\]

\[
= 4.08 \times 10^{-4}
\]

for $b = 2.9$

\[
\text{LOLP} = (1 + 6.60 + 3.64 + 1.01) 2.05 \times 10^{-5}
\]

\[
= 2.51 \times 10^{-4}.
\]
These calculations show that the LOLP objective is reached with either parameter value, depending only in the final analysis on precise definition of the goal.

Let us now translate these results into values for reserve margin. All that is needed for this is the value of $\bar{S}$ for the California system. Recall that $\bar{S} = \sum \alpha_i (1 - L_i)$. This works out to 33.92 GW as can be checked from Table 3. Now look at the definition of $b$ from (1),

$$b = \frac{\bar{S} - W}{\sigma_s \sqrt{2}}$$

Substituting our data for $b = 2.9$, we get

$$W = \bar{S} - \sqrt{2} \cdot b \sigma_s$$

$$= 33.92 - 2.95$$

$$= 30.97$$

This says that full co-ordination of the California utilities would allow a co-incident peak load of 30.97 GW. Given a total capacity of 35.69 GW, this means a reserve margin of 14.25%. Using the parameter $b = 2.8$, we get a maximum load of 31.08 GW and a reserve margin of 14.83%. Notice that the difference in LOLP in (5) amounts to less than 0.5% in reserve margin.

B. Northwest Interconnection Benefit

The calculations given above omit the considerable amount of power which California imports under contract from the Pacific Northwest. To factor in these capacity resources they must be
treated as if they were generators controlled by California utilities. Clearly this is an idealization. Nonetheless, LADWP seems to treat its firm capacity from the Northwest in this manner. The 525 MW is listed in Supply Form 7 as a single hydro resource with a forced outage rate of 6%. It is reasonable to aggregate the out of state power and treat it as several large units because of the limited transmission capacity (three lines) through which it is available.

We now supplement the calculations in (A) as follows. The firm power imports into California are given in Reference 1 as 2892 MW. Since there are only three major transmission lines to import this and there is no data on the flows in these lines, we will assume each line carries an equal amount of power. Thus, Table 3 should be augmented by a new category, call it IT for intertie with \( \bar{\alpha}_i = 963.3 \) MW. We will use the LADWP forced outage rate of 6%, although it is not clear how this value was determined. With these assumptions, we calculate the following increments to system parameters:

\[
\langle (X_i - \bar{X}_i)^2 \rangle = .1570 \\
\langle (X_i - \bar{X}_i)^3 \rangle = .1331 \\
a_{4i} = .1210 \\
a_{5i} = .0399.
\]

Adding these to the values in Table 3 gives new system parameters as follows:
Next, we calculate correction factors to replace the values in (4),

\[
\frac{a_3}{6} \left( \frac{1}{\sigma_s^2} \right)^{3/2} = 0.1084
\]

\[
\frac{a_4}{24} \left( \frac{1}{\sigma_s^2} \right)^2 = 0.0203
\]

\[
\frac{a_5}{120} \left( \frac{1}{\sigma_s^2} \right)^{5/2} = 0.0014.
\]  

(6)

Now we re-iterate the calculation of LOLP. For the parameter \( b = 2.8 \), we calculate

\[
LOLP = (1 + 6.30 + 4.03 + 1.30) \times 3.75 \times 10^{-5}
\]

\[
= 4.74 \times 10^{-4}.
\]

With the value \( b = 2.9 \), we calculate

\[
LOLP = (1 + 7.02 + 4.70 + 1.57) \times 2.05 \times 10^{-5}
\]

\[
= 2.93 \times 10^{-4}.
\]  

(7)
Now it appears that \( b = 2.9 \) is the correct parameter value for reliability of one day in 10 years. So we proceed to compute the maximum allowable co-incidental peak load, \( W \).

\[
W = \bar{S} - \sqrt{2} \sigma_S b,
\]

where

\[
\bar{S} = 36.61 \text{ for the system with intertie, } \sigma_S = .8219, \text{ and } b = 2.9.
\]

Solving we get \( W = 33.24 \text{ GW} \) as the maximum load consistent with reliability.

This result, as we would expect, is considerably stronger than the result in part (A). It says that there is enough slack in the existing system so that full co-ordination would eliminate the need for new capacity for several years. Exactly how long a hiatus is possible depends upon two variables, the 1975 co-incident peak load (which is unknown at present) and the growth in peak demand. We know that the 1975 non-coincident peak was 28.77 GW. To be conservative we assume this was also the coincident peak. Now we can calculate what growth rate is implied by a four year or five year hiatus in new generator additions.

Suppose 1975 peak = 28.77 GW

\[ \text{1979 peak} = 33.24 \text{ GW} \]

Then growth rate = 3.68%

If we assume 33.24 GW is not reached until 1980, we are assuming a growth rate of 2.93%.
C. Conclusions

The preceding analysis can easily be extended to consider various expansion scenarios. For example, it is likely that Diablo Canyon units 1 and 2 will come on line in the next few years. This 2200 MW of new capacity will alter the system parameters we have calculated. The impact of these new units should be assessed. The methodology introduced here provides a more realistic assessment of the reliability impact of new units upon capital requirements than the simple derating procedure used in the SGEM model. The effective capacity of a large generator is not simply \((1 - \text{FOR}) \times \text{Rated Capacity}\). When a big unit goes down there must be an equal amount of reserve capacity to replace it. This is reflected in our parameter \(O_s^2\), but is not included in the SGEM model. Least cost expansion analysis must be supplemented by a more sophisticated reliability constraint than is presently used. The model we have introduced is capable of application to this problem. It is sufficiently flexible so that analysis of individual unit additions or entire supply plans are easily treated. This can be an important aid in supply planning.
TABLE 1

<table>
<thead>
<tr>
<th>Hydro Resources Utility Control</th>
<th>Dry Year Aug. Capacity</th>
<th># of Units</th>
</tr>
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<tbody>
<tr>
<td>PG&amp;E (a)</td>
<td>3628</td>
<td>143</td>
</tr>
<tr>
<td>SMUD</td>
<td>649</td>
<td>9</td>
</tr>
<tr>
<td>LADWP</td>
<td>1281</td>
<td>31</td>
</tr>
<tr>
<td>SCE (b)</td>
<td>1173</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>6731</td>
<td>270</td>
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<tr>
<td>Major Water Agencies (c)</td>
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<tr>
<td>DWR</td>
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<td>USBR</td>
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<td>28</td>
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<td>MWD</td>
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<td>Small Companies</td>
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<tr>
<td>Totals</td>
<td>10024</td>
<td>345</td>
</tr>
</tbody>
</table>

a) includes City and County of San Francisco, East Bay MUD, Merced ID, Nevada ID, Oroville-Wyandotte ID, Placer Co. Water Agency, Oakdale and S. San Joaquin ID, Modesto and Turlock ID, Yuba Co. Water Agency.

b) includes 331 MW from Hoover delivered by USBR.

| Thermal (a) | SDG&E: Silver Gate 1-4, South Bay 1 & 2, Encina 1, 2, & 3 Station B—all units |
| Thermal (b) | SCE: Coolwater 1 & 2, Highgrove 1-4, Redondo 1-4, San Bernadino 1 & 2, Etiwanda 1 & 2 |
| Thermal (c) | SCE: Encina 4, South Bay 4 |
| Thermal (d) | SDG&E: Redondo 5 & 6, Alamitos 1 & 2, El Segundo 1 & 2, Hunting Beach 1 & 2, Mandalay 1 & 2 |
| Thermal (e) | SCE: Alamitos 3 & 4, El Segundo 3 & 4, Etiwanda 3 & 4, San Onofre 1 |
| Thermal (f) | LADWP: Harbor 1-5, Valley 1 & 2 |
| Thermal (g) | SCE: Alamitos 7, Etiwanda 5, Huntington Beach 5, Mandalay 5, Long Beach 10 & 11 |
| Thermal (h) | SDG&E: South Bay GT-1, San Onofre 1 |
| Thermal (i) | SCE: Valley 3 & 4, Scattergood 1 & 2, Haynes 1-4, Huntington Beach 3 & 4, Ellwood 1 |
| Thermal (j) | LADWP: Scattergood 3, Haynes 5 & 6 |
| Thermal (k) | SCE: Alamitos 5 & 6, Redondo 7 & 8 |
| Thermal (l) | LADWP: Mohave 1 & 2, Navaho 1 & 2 |
| Thermal (m) | PG&E: Avon 1, Contra Costa 1-5, Hunters Point 2-4, Kern 1 & 2, Martinez 1, Morro Bay 1 & 2, Moss Landing 1-5, Oleum 1 & 2, Pittsburg 1-4, Potrero 1-3 |
| Thermal (n) | SCE: Contra Costa 6 & 7, Morro Bay 3 & 4, Pittsburg 5 & 6 |
| Thermal (o) | PG&E: Moss Landing 6 & 7, Pittsburg 7 |
| Thermal (p) | SCE: Ormond Beach 1 & 2 |
| Thermal (q) | PG&E: Geysers 1-11 |
| Thermal (r) | SCE: Mohave 1 & 2, Four Corners 4 & 5 |
| Thermal (s) | SDG&E: All Gas Turbines (17 units) |
| Thermal (t) | PG&E - SMUD: Rancho Seco 1 |
### TABLE 3

<table>
<thead>
<tr>
<th>Category</th>
<th>(n_i)</th>
<th>(\bar{\alpha}_i)</th>
<th>(L_i)</th>
<th>(\langle (X_i - \bar{X}_i)^2 \rangle)</th>
<th>(\langle (X_i - \bar{X}_i)^3 \rangle)</th>
<th>(a_{4i})</th>
<th>(a_{5i})</th>
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<tbody>
<tr>
<td>H</td>
<td>345</td>
<td>29</td>
<td>0.02</td>
<td>0.0057</td>
<td>0.0002</td>
<td>0.0000</td>
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<tr>
<td>T(a)</td>
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<td>69</td>
<td>0.02</td>
<td>0.0025</td>
<td>0.0002</td>
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<tr>
<td>T(b)</td>
<td>12</td>
<td>201.4</td>
<td>0.03</td>
<td>0.0142</td>
<td>0.0028</td>
<td>0.0005</td>
<td>0.0001</td>
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<tr>
<td>T(c)</td>
<td>7</td>
<td>327.7</td>
<td>0.035</td>
<td>0.0252</td>
<td>0.0077</td>
<td>0.0022</td>
<td>0.0005</td>
</tr>
<tr>
<td>T(d)</td>
<td>15</td>
<td>94.6</td>
<td>0.045</td>
<td>0.0058</td>
<td>0.0005</td>
<td>0.0000</td>
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<tr>
<td>T(e)</td>
<td>11</td>
<td>189.4</td>
<td>0.055</td>
<td>0.0205</td>
<td>0.0035</td>
<td>0.0005</td>
<td>0.0000</td>
</tr>
<tr>
<td>T(f)</td>
<td>7</td>
<td>416.7</td>
<td>0.07</td>
<td>0.0791</td>
<td>0.0284</td>
<td>0.0084</td>
<td>0.0011</td>
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<tr>
<td>T(g)</td>
<td>32</td>
<td>114</td>
<td>0.08</td>
<td>0.0306</td>
<td>0.0029</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>T(h)</td>
<td>6</td>
<td>334</td>
<td>0.08</td>
<td>0.0494</td>
<td>0.0139</td>
<td>0.0031</td>
<td>0.0002</td>
</tr>
<tr>
<td>T(i)</td>
<td>3</td>
<td>732</td>
<td>0.08</td>
<td>0.1183</td>
<td>0.0727</td>
<td>0.0354</td>
<td>0.0046</td>
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<td>T(j)</td>
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<td>750</td>
<td>0.05</td>
<td>0.0534</td>
<td>0.0560</td>
<td>0.0215</td>
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<td>T(k)</td>
<td>11</td>
<td>45.6</td>
<td>0.06</td>
<td>0.0013</td>
<td>0.0001</td>
<td>0.0000</td>
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<td>T(l)</td>
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<td>413</td>
<td>0.10</td>
<td>0.0614</td>
<td>0.0203</td>
<td>0.0048</td>
<td>0.0003</td>
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<tr>
<td>T(m)</td>
<td>17</td>
<td>18</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>T(n)</td>
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<td>0.07</td>
<td>0.0511</td>
<td>0.0389</td>
<td>0.0244</td>
<td>0.0067</td>
</tr>
</tbody>
</table>

Notes: \(\bar{\alpha}_i\) is in units of MW.

\[\langle (X_i - \bar{X}_i)^n \rangle\] uses units of GW

\[
a_{4i} = \langle (X_i - \bar{X}_i)^4 \rangle - 3\langle (X_i - \bar{X}_i)^2 \rangle^2
\]

\[
a_{5i} = \bar{\alpha}_i^5 L_i (1 - L_i) (1 - 2L_i) (1 - 12L_i (1 - L_i))
\]

\[
\sigma_s^2 = 0.5185
\]

\[
a_3 = \sum_i \langle (X_i - \bar{X}_i)^3 \rangle = 0.2281
\]

\[
a_4 = \sum_i a_{4i} = 0.1010
\]

\[
a_5 = \sum_i a_{5i} = 0.0216
\]
V. Normal Approximation to LOLP

A. In this section we derive analytic results concerning the reliability of electric power systems. It is tedious and cumbersome to calculate manually and exactly the various probability measures of interest in practical cases. The amount of data typically considered in such calculations is large. As a result, the main emphasis in reliability analysis has shifted toward numerical simulation in recent decades. If all that is needed is a very specific answer to a very specific question, this is a perfectly reasonable procedure. But the flexibility required for policy analysis is difficult to achieve with numerical simulation. The sensitivity of a particular result to changes in parameter values is hidden from easy observation. The nature of reliability itself is not illuminated by these methods. Analytic models, on the other hand, provide a conceptual picture of the factors affecting reliability. This provides a reasonable framework for choosing a particular planning policy.

We begin by defining the loss of load probability (LOLP). LOLP is simply the probability that the instantaneous load cannot be met by the available generating capacity. We fix the following notation:

\[ \alpha_i = \text{capacity of the } i^{\text{th}} \text{ generator} \]
\[ L_i = \text{forced outage rate of the } i^{\text{th}} \text{ generator} \]
\[ Z_i = \text{random variable (Bernoulli) whose value is 1 when } i^{\text{th}} \text{ generator running and 0 when it is out.} \]
\[ W = \text{instantaneous load} \]
\[ N = \text{the number of generators} \]
\[ T = \text{total capacity}. \]

We can write the expression for LOLP very simply,

\[ \text{LOLP} = P\left[ \sum_{i=1}^{N} \alpha_i Z_i < W \right]. \tag{1} \]

*This work is based on results of Kahn, Levy, Davidson and Kaplan "Approximations to LOLP" LBL-5942.
It will be convenient to introduce a quantity $R$, defined by (2), which is related to the usual reserve margin,

$$R = (T - W)/T.$$  \hspace{1cm} (2)

The reserve margin usually discussed we denote as $R_m$. $R_m$ is related to $R$ by the simple expression (3). It will be easier to work with $R$ and then translate to $R_m$, than otherwise,

$$R_m = R/(1 - R).$$  \hspace{1cm} (3)

Rewriting (1) in terms of $R$, we have

$$\text{LOLP} = P\left[\sum \alpha_i \tilde{z}_i < (1 - R)T\right].$$  \hspace{1cm} (4)

We will approximate (4) using the well known Central Limit Theorem of probability theory. Having made this approximation, we then solve for $R$ to produce a simple expression which will tell us what reserve margin is necessary for a given system and LOLP. We will then provide analytic corrections to the approximation. These corrections do not change the basic form of our expression for $R$.

B. It will be convenient to transform (4) so that we can work with a standardized variable whose mean is zero and whose variance is one. We introduce a little more notation.
Let \( S = \sum \alpha_i^2 z_i \) be the random variable corresponding to the total power of the system;

\[
S = \sum \alpha_i (1 - L_i)
\]

is the mean of \( S \), and

\[
\sigma_S^2 = (S - \bar{S})^2 = \sum \alpha_i^2 L_i (1 - L_i)
\]

is the variance of \( S \).

Now we can rewrite (4) in terms of the standard variable

\[
LOLP = \Phi \left[ \frac{\sum \alpha_i z_i - \sum \alpha_i (1 - L_i)}{(\sum \alpha_i^2 L_i (1 - L_i))^{1/2}} \right] < \frac{(1 - R) T - \sum \alpha_i (1 - L_i)}{(\sum \alpha_i^2 L_i (1 - L_i))^{1/2}}.
\]

We have already argued that the probability (5) can be approximated by a Gaussian in the case where \( \alpha_i = \alpha_j \) for all \( i \) and \( j \) (Reference 0). The Central Limit Theorem states that a sum of \( N \) independent random variables is distributed normally in the limit \( N \rightarrow \infty \) under quite general conditions. In most practical situations the \( L_i \) are uniformly bounded away from 0 and 1, then it is easy to show that a sufficient condition for the Central Limit Theorem to hold is

\[
\lim_{N \rightarrow \infty} \frac{\alpha_{\text{max}}}{\sqrt{\sum_{i=1}^{N} \alpha_i^2}} = 0,
\]

where \( \alpha_{\text{max}} \) is the largest generator in the mix. This condition roughly states that the largest generator should not get too big with respect to system size. Because the Central Limit Theorem applies only in the limit, care must be taken when using it approximate to finite systems.

In Section C, we derive corrections to our basic approximation. For now it will suffice to study the first order application of the Central Limit
Theorem and derive from it an expression for the variable $R$.

Planning electric power systems for reliability means fixing some small LOLP. If there is insufficient reserve capacity, LOLP will be greater than the reliability objective. In terms of Equation (5), fixing LOLP means letting the right hand side of the inequality be assumed equal to a constant $C$.

\[
\frac{(1 - R)T - \sum \alpha_i (1 - L_i)}{(\sum \alpha_i^2 L_i (1 - L_i))^{1/2}} = C
\]

(7)

The value of $C$ will be determined from the normal distribution tables in a manner described in Section III. For now, we will derive an expression for $R$ from (7). It will be convenient to introduce a little more notation.

Let $L_0 = 1/T \Sigma L_i \alpha_i$, $\bar{\alpha} = T/N$, $\sigma^2 = \sigma_s^2/N$, and $\mu = \bar{S}/N$.

Dividing the numerator and denominator of the left hand side of (7) by $N$, we get

\[
\frac{(1 - R)T/N - \mu}{1/\sqrt{N} \sigma} = C.
\]

(8)
From (8) and the definitions above, we can solve for $R$ and get the following relation

$$R = \bar{L}_0 + \frac{C \sigma}{\sqrt{\alpha \sqrt{T}}}.$$  \hspace{1cm} (9)

Equation (9) has a natural interpretation. Among other things it says that for a fixed generator mix, reserves will decrease for increasing capacity. This relation expresses the advantage of power pooling. The lower limit on reserves for large pools is determined by $\bar{L}_0$, the average forced outage rate of the system, weighted by the size of each unit. Equation (9) also allows easy numerical assessment of increasing unit size. Units which are large with respect to the average size will tend to increase the standard deviation $\sigma_S$ of $S$ faster than $\sqrt{\alpha}$ and $\sqrt{T}$ so that the second term will grow. This quantifies the impact of large units on reserve requirements.

Practical application of Equation (9) requires selection of the appropriate constant $C$. This is the subject to which we now turn.

C. Selection of the constant $C$ appropriate to a given system depends upon the reliability criterion as well as system characteristics. The first order normal approximation varies in accuracy with the standard deviation $\sigma_S$. For larger $\sigma_S$ the normal is better than for smaller $\sigma_S$. We will start with the first order case.

In Equation (5) we wrote down a standardized variable for the total power of a system. Let us call that variable $Y$; then

$$Y = \left[\Sigma \alpha_1 z_j - \Sigma \alpha_1 (1 - L_i)\right] / \left(\Sigma \alpha_1^2 L_i (1 - L_i)\right)^{1/2}. \hspace{1cm} (10)$$
Since $Y$ is standardized, we can write the density $P(Y)$ as follows

$$P(Y) = \frac{1}{\sqrt{2\pi}} \exp(-Y^2/2).$$  \hspace{1cm} (11)$$

Now $LOLP$ is the probability that $S$, total power, will be less than $W$, the system load. Thus,

$$LOLP = \int_0^W P(S) dS. \hspace{1cm} (12)$$

We can rewrite (12) in terms of $P(Y)$ as follows

$$LOLP = \int_{-\infty}^{W - \bar{S}} \frac{P(Y)}{\sigma_S} dY. \hspace{1cm} (13)$$

In the large $N$ limit, the lower limit of this integral may be taken to $-\infty$ without introducing appreciable error, so

$$LOLP = \int_{-\infty}^{W - \bar{S}} \frac{P(Y)}{\sigma_S} dY = \frac{1}{2\pi} \int_{-\infty}^{W - \bar{S}} \frac{P(Y)}{\sigma_S} \exp(-Y^2/2) dY. \hspace{1cm} (14)$$

This may be evaluated yielding

$$LOLP = 1/2 \text{Erfc} \left( \frac{\bar{S} - W}{\sqrt{2} \sigma_S} \right). \hspace{1cm} (15)$$
Thus the constant \( C \) in Equation (9) is determined in this case directly from the argument in the \( \text{Erfc} \) function of Equation (15). That is we let \( C \) be the value of \( (\bar{S} - W)/\sqrt{Z} \) os which will give us the LOLP which we desire.

To consider the general case, let us return to Equation (12). The probability that the generators in a system will produce total power \( S \), \( P(S) \), can be evaluated using the Fourier transform by writing

\[
P(S) = \int_{-\infty}^{\infty} \frac{e^{i\omega(S - \bar{S})}}{2\pi} G(\omega) d\omega,
\]

where

\[
G(\omega) = \int_{-\infty}^{\infty} e^{i\omega(\bar{X}_1 - X_1)} p_1(X_1) dX_1 \int_{-\infty}^{\infty} e^{i\omega(\bar{X}_2 - X_2)} p_2(X_2) dX_2
\]

\[
= f_1(\omega)f_2(\omega) \ldots f_N(\omega).
\]

In (17) we write \( \bar{X}_i \) for \( \alpha_i(1 - L_i) \). In general, \( P(X_i) \) is the probability that generator \( i \) produces power \( X_i \), that is

\[
\bar{X}_i = \int X_i p_i(X_i) dX_i.
\]

The derivatives of \( f_i \), written \( f_i^n(\omega) \), are given by

\[
f_i^n(\omega) = (-i)^n \langle (X_i - \bar{X}_i)^n \rangle,
\]

where

\[
\langle (X_i - \bar{X}_i)^n \rangle = \int (X_i - \bar{X}_i)^n p_i(X_i) dX_i.
\]
The first order normal approximation discussed above amounts to writing \( G(\omega) \) as a Gaussian. In this notation that is expressed by

\[
G(\omega) = e^{-\frac{\omega^2 \sigma_s^2}{2}}. 
\]  

(20)

 Corrections to the normal density (19) can be written in the following form

\[
G(\omega) = e^{-\frac{\omega^2 \sigma_s^2}{2}} \left\{ 1 + a_1 \omega + a_2 \omega^2 + a_3 \omega^3 + \ldots \right\},
\]  

(21)

Now \( G(\omega) \) can also be expressed in terms of a Taylor series expansion about the point \( \omega = 0 \). Namely,

\[
G(\omega) = G(0) + G'(0)\omega + \frac{G''(0)\omega^2}{2!} + \frac{G'''(0)\omega^3}{3!} + \ldots,
\]  

(22)

where \( G^i \) is the \( i \)th derivative of \( G \). We can rewrite (21) by expanding the exponential in a power series. In this case, we have

\[
G(\omega) = (1 - \sigma_s^2 \omega^2/2 + \sigma_s^4 \omega^4/8 + \ldots)(1 + a_1 \omega + a_2 \omega^2 + a_3 \omega^3 + \ldots). 
\]  

(23)

Gathering the co-efficients of like powers together yields

\[
G(\omega) = 1 + a_1 \omega + (a_2 - \sigma_s^2/2)\omega^2 + (a_3 - a_1 \sigma_s^2/2)\omega^3 + 
\]

\[
(a_4 - a_2 \sigma_s^2/2 + \sigma_s^4/8)\omega^4 + \ldots .
\]  

(24)
Now equate the co-efficients of like powers in (22) and (24) and we get

$$a_1 = G^4(0)$$
$$a_2 = \sigma_s^2/2 + G^2(0)/2$$
$$a_3 = G^4(0)\sigma_s^2/2 + G^3(0)/6$$
$$a_4 = (\sigma_s^2/2 + G^2(0)/2)\sigma_s^2/2 - \sigma_s^4/8 + G^4(0)/24$$
$$a_5 \ldots$$  \hspace{1cm} (25)

For the derivatives of $G$, we have

$$G^1(0) = \sum_{i=1}^{N} f_i^2(0) = (-i) \sum_{i=1}^{N} \langle (x_i - \overline{x}_i) \rangle = 0$$

$$G^2(0) = \sum_{i=1}^{N} f_i^2(0) = (-i)^2 \sum_{i=1}^{N} \langle (x_i - \overline{x}_i)^2 \rangle$$

$$G^3(0) = \sum_{i=1}^{N} f_i^3(0) = (-i)^3 \sum_{i=1}^{N} \langle (x_i - \overline{x}_i)^3 \rangle$$

$$G^4(0) = \sum_{i=1}^{N} f_i^4(0) = (-i)^4 \sum_{i=1}^{N} \langle (x_i - \overline{x}_i)^4 \rangle - (-i)^4 \sum_{i=1}^{N} \langle (x_i - \overline{x}_i)^2 \rangle^2$$  \hspace{1cm} (26)

Now we must evaluate these expressions in terms of the particular probabilities associated with power systems. Notice that

$$G^2(0) = -\sum_{i=1}^{N} \langle (x_i - \overline{x}_i)^2 \rangle$$

$$= -\sigma_s^2$$
as we would expect from equation (20). Further, when we evaluate
\[ \sum_i \langle (X_i - \bar{X}_i)^2 \rangle \] we get \[ \sum \alpha_i L_i (1 - L_i) \] which is consistent with our earlier notational conventions.

As an example of calculating the higher order moments, we derive the basic expression for \( \langle (X_i - \bar{X}_i)^3 \rangle \). By definition, we have

\[ \langle (X_i - \bar{X}_i)^3 \rangle = \int (X_i - \bar{X}_i)^3 P_i(X_i) \, dX_i \]

where

\[ P_i(X_i) = \delta(X_i) L_i + (1 - L_i) \delta(X_i - \alpha_i). \] (27)

Expanding the polynomial in the integral and evaluating, we get

\[ \langle (X_i - \bar{X}_i)^3 \rangle = \int \left( X_i^3 - 3X_i^2 \bar{X}_i + 3X_i \bar{X}_i^2 - \bar{X}_i^3 \right) P_i(X_i) \, dX_i \]

\[ = \alpha_i^3 (1 - L_i) - 3\alpha_i^3 (1 - L_i)^2 + 3\alpha_i^3 (1 - L_i)^3 - \]

\[ \alpha_i^3 (1 - L_i)^3 \]

\[ = -\alpha_i^3 (1 - L_i) L_i (1 - 2L_i). \] (28)

Equation (28) is the basic building block of the correction co-efficient \( a_5 \).

We have calculated the co-efficients \( a_3 \) and \( a_4 \) in this case, they are given below:

\[ a_3 = \sum_j (-1)^3 \langle (X_j - \bar{X}_j)^3 \rangle \]

\[ a_4 = \sum_j \langle (X_j - \bar{X}_j)^4 \rangle - 2\langle (X_j - \bar{X}_j)^2 \rangle^2 \] (29)
Evaluating $a_4$ we get

$$a_4 = \sum \alpha_i^4 L_i(1 - L_i)[1 - 6L_i(1 - L_i)].$$  \hspace{1cm} (30)

Using Equation (24) for $G(\omega)$ and including up to fourth order contributions, we have for LOLP.

$$\text{LOLP} = \frac{1}{2} \text{Erfc}\left(\frac{S - W}{\sigma_S \sqrt{2}}\right)$$

$$+ \frac{ia_3}{G} \left(\frac{(W - S)^2}{\sigma_s^2} - \frac{1}{\sigma_s^2}\right) e^{-\frac{(W - S)^2}{2\sigma_s^2}}$$

$$+ \frac{a_4}{24} \left(-\frac{(W - S)^3}{\sigma_s^6} + \frac{3(W - S)}{\sigma_s^4}\right) e^{-\frac{(W - S)^2}{2\sigma_s^2}}.$$  \hspace{1cm} (31)

These results are also established in Reference 7, in the general case.

Finally we turn to applying (31) for use in Equation (9). We can simplify the expression of (31) as follows.

Let $\text{LOLP}_0 = 1/2 \text{Erfc}(b)$,

$$\frac{\text{LOLP}}{\text{LOLP}_0} = 1 + \frac{a_3}{6} \left(\frac{1}{\sigma_s^3}\right) \left(2^{3/2} b^{3/2} - 2^{1/2} b\right)$$

$$+ \frac{a_4}{24} \left(\frac{1}{\sigma_s^4}\right) \left(4b^4 - 6b^2\right),$$

where

$$b = \frac{S - W}{\sqrt{2}(\sigma_s)}. \hspace{1cm} (32)$$
The appropriate constant $C$ to go into Equation (9) is $b \sqrt{2}$. This can be seen by algebraic manipulation of the definition of $b$. 
References


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