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Low-Rank Decomposition and Total Variation Regularization of Hyperspectral Video Sequences

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Abstract—Hyperspectral video sequences (HVSs) are well suited for gas plume detection. The high spectral resolution allows the detection of chemical clouds even when they are optically thin. Processing this new type of video sequences is challenging and requires advanced image and video analysis algorithms. In this paper, we propose a novel method for gas plume detection recorded in HVSs. Based on the assumption that the background is stationary and the gas plume is moving, the proposed method separates the background from the gas plume via a low-rank and sparse decomposition. Furthermore, taking into consideration that the gas plume is continuous in both spatial and temporal dimensions, we include total variation regularization in the constrained minimization problem, which we solve using the augmented Lagrangian multiplier method. After applying the above process to each extracted feature, a novel fusion strategy is proposed to combine the information into a final detection result. Experimental results using real data sets indicate that the proposed method achieves very promising gas plume detection performance.

Index Terms—Hyperspectral video sequences, detection, low-rank, sparse, total variation.

I. INTRODUCTION

HYPERSPECTRAL imaging has been used in a number of applications such as land-use or land-cover mapping, forest inventory, or urban-area monitoring [1–4]. However, hyperspectral video sequences (HVSs) are much less common due to their large data size. Thanks to the dramatic improvements of sensor technologies, HVSs are currently being developed, opening the door to new research avenues and application, involving critical new methodological developments [5–8].

Standoff detection of chemical gas plume is necessary to environmental monitoring, emergency response, chemical warfare threat mitigation and earth sciences [9]. However, this task still remains a challenging problem as conventional RGB images are not able to capture the potentially invisible gas plume. For example, a wide range of chemical gases have distinctive spectral signatures in the long-wave infrared (IR) (LWIR) region, requiring high resolution in the electromagnetic spectrum for detection. Another approach for plume detection utilizes temporal information by capturing multiple images. HVSs reveal the spectral properties of the scene and record its evolution over time at the cost of processing a large data hypercube [10-13].

Some prior work has been done on gas plume detection in recent years. The existing works can be classified into three categories. The first category is methods based on anomaly detection. In [14-16], the Automatic Matched Subspace Detector (AMSD) was used to detect gas plumes and the Clutter Matched Filter (CMF) was used to develop gas plume detection algorithms [17-18]. Both the AMSD and CMF detectors can detect gas plumes against a complex background, in which single pixel false alarms are eliminated by post processing techniques. Thus, these methods are widely used in hyperspectral image (HSI). But in HVS, these methods can only be applied frame by frame and the post processing technique

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might be different in different scenes. The second category performs clustering of spectral data to separate the plume from the background [8, 19-20]. These methods often exhibit more spatial continuity. In an earlier paper [8], it was shown that principal component analysis applied to each frame resulting in temporal flicker for the video, requiring a Midway equalization procedure. The later works [19,20] are able to perform graph-based clustering across several video frames, without this effect. The third category is to extend the traditional object tracking algorithms to HVS. The mean shift tracker is applied in [21]. In [22], the position of the plume is first estimated, using the temporal redundancy between two consecutive frames. Then a Binary Partition Tree (BPT) is built and pruned according to the previous estimation. By this way, the temporal continuity of HVS is discovered. However, the spectral characteristic of HVS is not fully used, because these methods work with only a few features of HVS.

In this paper, we propose a method to process HVS for the detection of a chemical gas plume diffusing in the atmosphere. We assume that the gas plume is the only moving object and the background of different frames is assumed to remain almost constant. Some change is allowed such as variation caused by illumination change or periodic motion of dynamic textures. Thus it is reasonable to assume the background information is low dimensional. Even in a static shot, there may be noise corrupting the background, but the proper low-dimensional approximation will remove this. Moreover, the gas plume is a moving object which we assume moves independently of the background. It includes spectral changes that cannot be incorporated into the low-dimensional model of background. These gas plume pixels are treated as outliers in the low-dimensional model. As the gas plume occupies only a small fraction of the image pixels, it can be treated as a sparse contribution. We assume finally that the gas plume forms a spatially and temporally connected set of pixels. Based on these assumptions, we use a low-rank plus sparse matrix decomposition to separate the background from the gas plume. In addition, the total variation (TV) regularization for video can explicitly describe the gas plume continuity in both the spatial and temporal directions. In traditional moving object detection methods [23-28], one column of the matrix is the vectorized image of a frame, hence only one 2-D matrix is required to represent the video. However, in HVS, there are hundreds of bands. And one 2-D matrix becomes unwieldy large in order to cover all the information contained in HVS.

To overcome this problem, we propose a novel fusion strategy called MaxD. After first performing detection in each band, it takes the maximum value of the detected gas plume part across different features. In this way, we utilize all the useful information contained in different features. The experiments on real gas plume diffusing hyperspectral videos demonstrate that our proposed model can achieve higher detection accuracies in gas plume detection compared with the methods which only use one principal component feature. Experimental results also show that, the proposed method particularly performs well for target detection in noisy HVS with changing object outline. Thus our method is well suited to gas plume detection in HVS since the imaging quality in long-wave infrared wavelength is lower than in other wavelength.

Although the combination of the TV regularization and low-rank decomposition has been explored in conventional video object detection [29-31], it is the first time to use the low-rank decomposition based method in hyperspectral video. The advantage of hyperspectral video is the abundant spectral information that can provide us more information about the gas plume. In this paper, the TV regularized low-rank decomposition method is used as an initial step to extract the gas plume’s information from one spectral component or combination. Then a further step is conducted which fuses the detection results of all the spectral information. In addition, both the sparse and TV constraints are imposed on the gas plume part according to the characteristic of gas plume. However, different regularization combinations are imposed on the foreground since the moving objects in [30, 31] are different from the gas plume.

For illustrative purposes, Fig. 1 shows the flowchart of the proposed detection algorithm using a diagram. The main contributions of this paper can be therefore summarized as follows:

1) We propose a new framework of gas plume detection in HVS different from the conventional approaches found in the literature. First, a low rank plus sparse decomposition with total variation regularization is employed to produce each feature’s detection result. We then use the MaxD fusion strategy to combine different features’ detection results into an effective and efficient algorithm to solve the problem.

2) Taking into consideration that, the gas plume is a small cluster and moves continuously between neighboring frames, we naturally incorporate the space and time connectivity by introducing total variation regularization.

3) HVS contain many bands which have important spectral information. The proposed fusion strategy in the decision process allows to better use the information of different features.

The rest of this paper is organized as follows. In Section II, the details of the proposed gas plume detection method based...
on low-rank, sparse and total variation model (GPD-LRSTV) is described. In Section III, experiments with real data are described and analyzed followed by conclusions in Section IV.

II. PROPOSED GAS PLUME DETECTION METHOD

A. Problem Formulation

In the HVS a gas plume arises in a given frame and moves during the sequence. Our goal is to separate the moving gas plume from the background. Background modeling or subtraction from video sequences is a popular approach to detect activity in a scene which finds its application in video surveillance from static cameras [28]. However, background estimation is complicated by the presence of a moving object, as well as variability in the background itself. Because background components in different frames are highly correlated, it is reasonable to assume that this variability is low-rank. In addition the foreground gas plume is spatially localized and sparse. Due to these properties, robust principal component analysis (RPCA) [32] is widely used for moving object detection. In traditional video, however, each frame is composed of one band which is vectorized as column of a matrix. Unlike these traditional videos, HVS have hundreds of bands in each frame, and it is very computational complex to put the whole frame in a column. Thus in the first step, a one-band image is extracted at each frame. The extracted one-band image will be detailed in Section II-C. Suppose we are given a HVS including $T$ frames, and each frame can be seen as a 3-D tensor of size $M \times N \times b$, where $M, N$ and $b$ are the number of rows, columns and bands of each frame, respectively. Then the vectorized one-band images are stacked as columns of a matrix $X \in \mathbb{R}^{MN \times T}$. This matrix can be expressed as the sum of a low-rank matrix modeling the background and a sparse error matrix representing the gas plume in the scene as follows:

\[
\min_{A,E} \text{rank}(A) + \lambda \parallel E \parallel_0 \quad \text{s.t.} \quad A + E = X, \tag{1}
\]

where $A \in \mathbb{R}^{MN \times T}$ represents the background matrix and $E \in \mathbb{R}^{MN \times T}$ represents the sparse error matrix. $\lambda$ is a positive parameter controlling sparsity. Unfortunately, (1) is a highly nonconvex optimization problem and no efficient solution is known. A tractable optimization problem can be obtained by relaxing (1), replacing the $\ell_0$-norm with $\ell_1$-norm and the rank norm with the nuclear norm $\parallel A \parallel_* = \sum \sigma_i(A)$ where $\sigma_i(A)$ is the $i$th singular value of $A$. This yields the following convex surrogate:

\[
\min_{A,E} \parallel A \parallel_1 + \lambda \parallel E \parallel_1 \quad \text{s.t.} \quad A + E = X. \tag{2}
\]

Candes et al. [33] proved that the underlying low-rank matrix $A$ and the underlying sparse matrix $E$ can be exactly recovered with high probability.

The original work of RPCA in [32] proposed iterative thresholding methods with low complexity, but their convergence is generally very slow. Then various algorithms have been developed for specific problems. Two of the most popular techniques are the Proximal Gradient (PG) method and the Augmented Lagrangian Method (ALM), both of which are applicable to a variety of convex problems. The accelerated proximal gradient (APG) methods proposed by Lin et al. [34] are faster and generally more accurate. The ALM [35] achieves state-of-the-art performance in terms of both speed and accuracy to the best of our knowledge. Moreover, the inexact ALM proposed in [35] requires significantly fewer partial singular value decomposition (SVD). Other methods [23-25] have also been proposed in the Bayesian setting. Improved models have been proposed by introducing regularizations such as adaptive graph term [36], structured sparse term [37]. Further, non-convex formulations of the original problem are introduced by using the Schatten- $p$ norm [38] or a non-convex rank approximation [39].

B. Total variation regularized low-rank sparse decomposition

In our HVS, the gas plume is continuously present in the scene in both spatial and temporal dimensions. Classical object detection methods use Markov dependencies to introduce this property by adding spatial and temporal priors [24-25]. The approximate solution to these problems can be obtained by either Markov chain Monte Carlo (MCMC) [40] analysis or with variational Bayesian (VB) methods [41]. In this paper, TV regularization over space-time is investigated to enforce spatial smoothness and temporal consistency. In particular, the following optimization problem is proposed:

\[
\min_{A,E} \parallel A \parallel_1 + \lambda \parallel E \parallel_1 + \beta \parallel E \parallel_{TV} \quad \text{s.t.} \quad A + E = X, \tag{3}
\]

where $\parallel \cdot \parallel_{TV}$ is the TV-norm and $\beta$ is a coefficient for the strength of that term. Before explaining the TV-norm, we first define the notation that will be used.

The sparse error matrix $E$ can be represented as a 3-D tensor $E^{Ten} \in \mathbb{R}^{M \times N \times T}$, and we use $E^{Ten}[m,n,t]$ to indicate the intensity of position $(m,n)$ at frame $t$. To simplify the numerical computation, we stack all the entries of $E^{Ten}$ into a column vector $E^{vec} = \text{vec}(E^{Ten}) \in \mathbb{R}^{MN \times T}$ according to lexicographic order, where $\text{vec}()$ represents the vectorization operator.

Then we denote $D_h, D_v$ and $D_d \in \mathbb{R}^{MN \times MN}$ as the first-order forward finite-difference operators along the horizontal, vertical and temporal directions, respectively. The definitions of each operator are $D_h E^{vec} = \text{vec}(E^{Ten}[m+1,n,t] - E^{Ten}[m,n,t])$.
\[ \mathbf{D}_E^{E_{\text{vec}}} = \text{vec}(E_{\text{vec}}^T[m,n+1,t] - E_{\text{vec}}^T[m,n,t]) \] and
\[ \mathbf{D}_E^{E_{\text{vec}}} = \text{vec}(E_{\text{vec}}^T[m,n,t+1] - E_{\text{vec}}^T[m,n,t]) \] with periodic boundary conditions [42].

The TV-norm \( \| E \|_{TV} \) can either be defined as the anisotropic TV-norm [43]
\[ \| E \|_{TV} = \sum_i (\gamma_h \left| \mathbf{D}_h E_{\text{vec}}^j \right| + \gamma_v \left| \mathbf{D}_v E_{\text{vec}}^j \right| + \gamma_t \left| \mathbf{D}_t E_{\text{vec}}^j \right|) \] (4)
or the isotropic TV-norm
\[ \| E \|_{TV} = \sum_i \sqrt{(\gamma_h^2 \left| \mathbf{D}_h E_{\text{vec}}^j \right|^2 + \gamma_v^2 \left| \mathbf{D}_v E_{\text{vec}}^j \right|^2 + \gamma_t^2 \left| \mathbf{D}_t E_{\text{vec}}^j \right|^2)} \] (5)
where \( [\cdot] \) denotes the \( i \)th element of the argument and \( (\gamma_h, \gamma_v, \gamma_t) \) are constants used for greater flexibility in controlling the forward difference along each direction. The anisotropic TV-norm will lead to piecewise smooth solutions with defined horizontal and vertical edges in the sparse error image. The isotropic TV-norm tends to produce defined and smooth edges in all directions.

Due to excellent performance suppressing discontinuous changes generally regarded as noises, TV regularization has been widely used for image and video denoising [44-46].

Denoting \( \mathbf{D} = [\gamma_h \mathbf{D}_h^T, \gamma_v \mathbf{D}_v^T, \gamma_t \mathbf{D}_t^T] \in \mathbb{R}^{3N \times N^2} \), \( \| E \|_{TV} \) is the \( \ell_1 \) norm of vector \( \mathbf{D} E_{\text{vec}} \). Thus we can use \( \| \mathbf{D} E_{\text{vec}} \|_1 \) and \( \| \mathbf{D} E_{\text{vec}} \|_{2,1} \) to represent \( \| E \|_{TV} \) and \( \| E \|_{TV} \), respectively.

### C. Algorithm

To solve problem (3), the alternating direction method (ADM) [34] is applied. We first introduce an auxiliary variable \( J \) to split the energy:
\[
\min_{A,E,J} \| A \|_1 + \lambda \| E \|_1 + \beta \| J \|_{TV}
\text{s.t. } A + E = X, J = E. \tag{6}
\]
The augmented Lagrangian function of problem (6) is
\[
L(A,E,J,Y_1,Y_2) = \| A \|_1 + \lambda \| E \|_1 + \beta \| J \|_{TV} + \langle Y_1, X - A - E \rangle + \langle Y_2, E - J \rangle + \frac{\mu}{2} (\|X - A - E\|_F^2 + \|E - J\|_F^2)
\] (7)
\[
= \| A \|_1 + \lambda \| E \|_1 + \beta \| J \|_{TV} - \frac{\mu}{2} (\| Y_1 \|_F^2 + \| Y_2 \|_F^2)
\] (7)
\[
+ \frac{\mu}{2} (\| X - A - E + \frac{Y_1}{\mu} \|_F^2 + \| E - J + \frac{Y_2}{\mu} \|_F^2),
\]
where \( Y_1 \) and \( Y_2 \) are the Lagrange multiplier matrices, \( \mu \) is a positive penalty scalar. \( \{ , , \} \) denotes the matrix or Frobenius inner product, and \( \| \cdot \|_F \) represents the Frobenius norm. The proposed algorithms consists in updating the variables \( A, E, \) and \( J \) repeatedly, by minimizing \( L \) with other variables fixed as well as updating the dual variable \( Y_1 \) and \( Y_2 \). With some algebra, the updating schemes can be stated as follows.

\[
A_{k+1} \leftarrow \arg \min_A \frac{1}{\mu} \| A \|_1 + \frac{1}{2} \| X - A - E_k + \frac{Y_1}{\mu} \|_F^2, \tag{8}
\]
\[
E_{k+1} \leftarrow \arg \min_E \frac{\lambda}{2 \mu} \| E \|_1 + \frac{1}{2} \| E - \frac{1}{2} (J_k - \frac{Y_2}{\mu} + X - A_{k+1} + \frac{Y_1}{\mu}) \|_F^2, \tag{9}
\]
\[
J_{k+1} \leftarrow \arg \min_J \frac{\beta}{\mu} \| J \|_{TV} + \frac{1}{2} \| E - (E_{k+1} + \frac{Y_1}{\mu}) \|_2^2, \tag{10}
\]
where (8) and (9) can be efficiently computed via the singular value thresholding (for \( A \)) and the soft thresholding operator (for \( E \)).

Then the Lagrange multipliers are updated by \( Y_1 \leftarrow Y_1 + \mu (X - A_{k+1} - E_{k+1}) \) and \( Y_2 \leftarrow Y_2 + \mu (E_{k+1} - J_{k+1}) \).

For the sake of simplicity, the subscript \( k+1 \) is omitted and problem (10) can be rewritten as follows:
\[
J_{\text{vec}} = \arg \min_{J_{\text{vec}}} \frac{\beta}{\mu} \| \mathbf{D} J_{\text{vec}} \|_q + \frac{1}{2} \| J_{\text{vec}} - \text{vec}(E + \frac{Y_1}{\mu}) \|_2^2, \tag{11}
\]
where \( q \) represents either the \( \ell_1 \) or \( \ell_{2,1} \) norm. Here we have used the fact that the Frobenius norm of \( J - (E + \frac{Y_2}{\mu}) \) is the 2-norm of \( J_{\text{vec}} - \text{vec}(E + \frac{Y_2}{\mu}) \), i.e.,
\[
\| J - (E + \frac{Y_2}{\mu}) \|_2^2 = \| J_{\text{vec}} - \text{vec}(E + \frac{Y_2}{\mu}) \|_2^2.
\]
To solve the subproblem (11), another auxiliary variable \( Q \in \mathbb{R}^{3N \times N^2} \) is introduced to split this energy. Thus we have
\[
J_{\text{vec}} = \arg \min_{J_{\text{vec}}} \frac{\beta}{\mu} \| Q \|_q + \frac{1}{2} \| J_{\text{vec}} - \text{vec}(E + \frac{Y_2}{\mu}) \|_2^2 \tag{12}
\text{s.t. } Q = \mathbf{D} J_{\text{vec}}.
\]
Denote \( B = \text{vec}(E + \frac{Y_2}{\mu}) \). Then the augmented Lagrangian function of (12) is
\[
L_{\text{vec}} (J_{\text{vec}}, B, Y_2) = \frac{\beta}{\mu} \| Q \|_q + \frac{1}{2} \| J_{\text{vec}} - B \|_2^2 + Y_2^T (Q - \mathbf{D} J_{\text{vec}}) + \frac{\eta}{2} \| Q - \mathbf{D} J_{\text{vec}} \|_2^2, \tag{13}
\]
iteratively solve the problem. Multiplier. Again, we use the alternating direction method to where

\[ \mu_k = 1.25 \left\| X \right\|_2, \quad \mu_{\max} = \mu_k 10^{-7}, \quad \rho_i = 1.5, \quad k = 0, \quad \text{tol}_i = 10^{-7}, \quad \text{tol}_2 = 10^{-4}. \]

1. While \[ X - A_k E_k \left/ \right. \text{tol}_1 \] do
2. Update variable \[ A_k \] :
\[ A_k = \Theta_{\mu_k}(X - E_k Y + (1 / \mu_k) Y_i) \]
where \( \Theta \) is the singular value thresholding operator[47].
3. Update variable \[ E_k \] :
\[ E_k = S_{(2 \mu_k)} \left( \frac{f_{j} - \left( 1 / \mu_k \right) Y_i + X - A_k E_k + (1 / \mu_k) Y_i}{2} \right) \]
4. Update variable \[ J_k \] :
\[ \text{Solve problem:} \quad \arg \min_{J_k} \frac{\beta}{\mu} \left\| D J_k \right\|_q + \left( \frac{1}{2} \right) \left\| J_k - \text{vec}(E_k + \frac{Y_i}{\mu}) \right\|_2 \]
5. Update \[ \eta_{k+1} \text{ via (16) (17)} \]
5. Update \[ Q_{i+1} \] via either (18) for the anisotropic total variation or (19) for the isotropic one
3. Update multiplier via (20)
4. Update \[ \eta \] via (21)
5. Check convergence:
\[ \text{If} \left\| J_{k+1} - J_k \right\|_2 / \| J_k \|_2 < \text{tol}_2 \text{ then break} \]
6. \[ l \leftarrow l + 1 \]
Output: optimal solution \( J_{k+1}, Q \)
5. Update the out loop Lagrange multipliers as follows:
\[ Y_i \leftarrow Y_i + \mu_k (X - A_k E_k - E_k) \]
\[ Y_2 \leftarrow Y_2 + \mu_k (E_k - J_k) \]
6. Update \( \mu \) as follows:
\[ \mu_{\min} = \min(\mu_{\max}, \rho_i \mu_i) \]
7. \[ k \leftarrow k + 1 \]
8. End while
Output: an optimal solution \( (A, E, J) \)

where \( \eta \) is the penalty parameter associated with the quadratic penalty term \( \left\| Q - D J_{k+1} \right\|_2^2 \) and \( Y_i \in \mathbb{R}^{2M(N-1)} \) is the Lagrange multiplier. Again, we use the alternating direction method to iteratively solve the problem.

Updating \( J_{k+1} \) with other terms fixed
\[ J_{k+1} \leftarrow \arg \min_{J_k} \frac{1}{2} \left\| J_k - B \right\|_2^2 + Y_i^T (Q_i - D J_{k+1}) + \frac{\eta_{k+1}}{2} \left\| Q_i - D J_{k+1} \right\|_2^2 \] (14)

Setting the derivation of the objective function to zero, we have
\[ (J_{k+2} - B) - D Y_i - \eta_{k+2} D^T (Q_i - D J_{k+2}) = 0. \] (15)

Then, the solution of (14) is found by considering the normal equation:
\[ (I + \eta_{k+2} D^T D) J_{k+2} = D Y_i + \eta_{k+2} D^T Q_i + B. \] (16)

Traditionally, the optimal solution can be simply obtained by computing the Moore-Penrose pseudo-inverse of \( (I + \eta_{k+2} D^T D) \). However, this approach is computationally expensive due to the large matrix size. Due to its block-circulant structure, it can be diagonalized by the 3-D-DFT matrix [43]. Hence, (16) has the following solution
\[ F^{-1} \left( \frac{F(D Y_i + \eta_{k+2} D^T Q_i + B)}{1 + \eta_{k+2} \left( \left| F(D Y_i) \right|^2 + \left| F(D Q_i) \right|^2 + \left| F(D G_i) \right|^2 \right)} \right) \]
(17)

where \( F(\cdot) \) denotes the 3-D Fourier transform operator, \( | \cdot |^2 \) is the element-wise square and the division is element-wise. The denominator can be precalculated as it remains constant.

Updating \( Q_{i+1} \) with the other terms fixed
\[ Q_{i+1} = \arg \min_{Q} \frac{\beta}{\mu} \left\| Q \right\|_q + Y_i^T (Q - D J_{i+1}) + \frac{\eta_{i+1}}{2} \left\| Q - D J_{i+1} \right\|_2^2 \]. (18)

For \( q = 1 \), the solution is
\[ Q_{i+1} = S_{\beta / \mu q} (D J_{i+1} - Y_i / \eta_{i+1}) \]
where \( S_{\cdot}(\cdot) \) is the shrinkage operator whose definition on scalars is \( S_{\cdot}(\cdot) = \text{sgn}(x) \max(\left| x \right| - \varepsilon, 0) \). To extend the shrinkage operator to vectors and matrices, it is applied element-wise.

For \( q = (2, 1) \), we first denote \( Q_{i+1} = [Q_{i+1,b}, Q_{i+1,\tau}, Q_{i+1,\tau}^T, Q_{i+1,\tau}^T] \), \( Y_i = [Y_{i,b}, Y_{i,\tau}, Y_{i,\tau}^T, Y_{i,\tau}^T] \), and \( W_i = \gamma_i^b D_{i+1} J_{i+1} - (1 / \eta_{i+1}) Y_{i,b} \)
(analogous definitions for \( W_e \) and \( W_f \)). The solution in this case is given by
\[ Q_{i+1,b} = \max(W - \beta / \mu \eta_{i+1} W_e) - W_e \]

where \( W = \max\left( \sqrt{W_e^2 + W_f^2 + W_t^2} \right) \), \( \varepsilon \) is a small constant. Here the multiplication and division are component-wise operations.
Algorithm 2 Algorithm for GPD-LRSTV

Input: hyperspectral video sequence, parameters \( \beta > 0 \), \( \lambda > 0 \), \( p \)

1. Using PCA to extract the first \( p \) principal components in every frame

2. For \( i = 1 : p \)
   
   2.1. Form matrix \( X' \) where the \( j \)th column is the vectorized \( i \)th principal component of the \( j \)th frame.

   2.2. Normalize \( X' \) as follows
   
   \[ X' = \frac{1}{\max(||X'||)} X' \]

   2.3. solve the following problem using Algorithm 1
   
   \[
   \begin{align*}
   \min_{j, E'} & \quad \|A' + E'\|_1 + \beta\|E'\|_2^2 \\
   \text{subject to} & \quad \|A' + E'\|_F^2 \\
   \end{align*}
   \]

   End for

5. Compute the final moving object part \( \hat{E} \), subject to:

\[ \hat{E}_j = \max\{|E_j', \ldots, E_{j+N}^1, \ldots, E_{j+N}^M\}, \quad i = 1, 2, \ldots, M, j = 1, 2, \ldots, N \]

Output: an optimal solution \( \hat{E} \)

After solving each sub-problem, we update the dual variable

\[ Y_i \leftarrow Y_i + \eta_i (Q_{i+1} - DJ_{i+1}). \quad (21) \]

As mentioned in [42], the method of multipliers can achieve a faster rate of convergence by adapting the following parameter \( \eta \) update scheme:

\[ \eta_{i+1} = \begin{cases} \rho_2 \eta_i & \text{if } \|Q_{i+1} - DJ_{i+1}\| \geq \alpha \|Q_{i+1} - DJ_{i+1}\| \\ \eta_i & \text{otherwise.} \end{cases} \quad (22) \]

Here we set \( \rho_2 = 2 \). After obtaining the optimal \( J^{new} \), the solution of (10) is obtained by reshaping vector \( J^{new} \) back into the 2-D matrix variable \( J_{i+1} \).

The complete algorithm being proposed to solve (3) is summarized in Algorithm 1.

D. MaxD strategy for HVS gas plume

In HVS, there are potentially hundreds of spectral bands. However, in the LRSTV model only one-band video sequence is processed to separate the gas plume from the background. In order to make full use of the abundant spectral information, a preprocess step is conducted to produce the one-band video sequence before detection. The one-band video sequence should contain the sufficient information from the HVS. Band selection and feature extraction are two common technologies for preprocessing. In gas plume detection, the corresponding bands that reveal the most information of a specific kind of gas are different, so the band selection method is applied when the gas’s prior information is known. The selected band should be determined by the detected gas, but in this paper, the prior information of the detected gas is unknown, thus band selection method cannot be treated as a common way in preprocess in regard of different gases. Besides, due to the existence of Earth’s surface emission, atmospheric absorbance, and sensor noise, the captured HVS have limited quality. The band selection method is not appropriate as it directly selects the one-band video sequence from the lower quality raw HVS data. To get a high-quality video sequence and design a common method, a feature extraction method--principal component analysis (PCA) is applied in this paper. PCA is the most popular linear feature extraction method, which can keep the most information in the few components in terms of variance. Due to the low complexity and the absence of parameters, PCA is competitive for the purpose of classification and target detection of hyperspectral data [48].

Exploring multiple features in remote sensing images has been well studied for image classification or scene classification [49-52]. In this paper, for each frame of the HVS, PCA is performed and the first \( p \) components are kept. By performing PCA, the principal components act as a type of spectral filter where each component represents a particular chemical spectrum in the image. To obtain a \( p \)-dimensional representation of the data, each pixel is projected onto these \( p \) principal components. A natural strategy of gas plume detection is to extract one feature which provides the best contrast
between plume and background in each frame, then the extracted feature can be used as the input data matrix \( X \) of Algorithm 1. However, it is hard to decide which component has the best contrast between gas plume and background. Moreover, some parts of the gas plume are obvious in one principal component and some other plume parts will be obvious in other components. For example, Fig.2 (b)-(f) shows the resulted gas plume part using different principal components from frame 14, HVS-aa12. Details of HVS aa12 are presented in Section III-A. Compared to the ground truth map Fig.2 (a), the 1st and 5th principal components have better detection results than the other 3 components. In Fig.3 (b)-(f), the gas plume part using different principal components of frame 22 HVS aa12 are shown. The 3rd principal component is the best among the first five principal components. For different frames, the best feature for detection may not be the same. Thus it is unreasonable to use only one feature to represent the hyperspectral image for gas plume detection. To detect the gas plume with high probability in each frame, we propose a more robust method by combining all the features. In the first step, PCA is performed frame by frame to extract the first \( p \) principal components. Then the \( i \)th principal component of all frames are combined to form a matrix \( X' \) where each column is each frame’s \( i \)th feature. For fair comparison, all matrices

\[
X'_i, i = 1, 2, \ldots, p
\]

are normalized by \( X' \leftarrow \frac{1}{\max([X'])} X' \),

where \( \max([X']) \) represents the maximum entry of the matrix \( X' \). Each matrix \( X' \) is then decomposed into the background part \( A' \) and the plume part \( E' \) using (3) with the same parameters \( \lambda \) and \( \beta \). As different principal components are different projections of the original features, the resulting moving objects \( E' \) of different principal components have different ability to reveal the gas plume. To combine all the information, the plume detection is \( \hat{E} \) where each entry is the maximum value of all \( E' \) at the same coordinate, which can be written as

\[
\hat{E}_{i,j} = \max \{E'_{i,j}, E'_{i,j}, \ldots, E'_{i,j}\}, i = 1, 2, \ldots, M, j = 1, 2, \ldots, N (23)
\]

Here \( \hat{E}_{i,j} \) represents the absolute value of the \( i \)th row and \( j \)th column entry of matrix \( \hat{E} \). In this way, gas plume detected by different principal components is kept. For principal components which have low contrast between background and gas plume, the corresponding moving object part \( E' \) will be small, and it will not affect the final result as only the maximum value will be stored. Moreover, although this strategy is sensitive to noise, the TV regularization has removed the noise in prior steps. In [10], a Midway method for histogram equalization is used to reduce flicker between frames. Whereas in our method, the flicker between frames will not affect the final detection because we normalize all the principal components matrix \( X' \) and the low rank matrix is able to describe the flicker between frames. Thus the sparse error matrix will not affect by the flicker which only shows the gas plume.

The maximum value detection strategy is noted as MaxD and the main steps for the GPD-LRSTV algorithm are shown in Algorithm 2.

## III. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, we provide experimental results for our gas plume detection algorithm with real data sets. From different perspectives, we adopt two evaluation criteria. Because the gas plume can be seen as the anomaly in each frame, we can use the receiver operating characteristic curve (ROC) to measure the performance of the methods. A better detector would lie nearer to the upper-leftmost corner and result in a larger area under the curve [53]. As more than one frame has to be compared, the area under the ROC curve (AUC) of each frame is calculated for comparison. A bigger AUC statistic represents a better performance. For quantitative evaluation, we measure the accuracy of gas plume detection by comparing our result with the ground truth [54]. We can evaluate the results using precision and recall, which are defined as

\[
\text{precision} = \frac{TP}{TP + FP}, \quad \text{recall} = \frac{TP}{TP + FN}. (24)
\]

where TP, FP, TN, and FN are the numbers of true positives,
false positives, true negatives, and false negatives, respectively. To analyze the performance, we use the F-measure

\[
F\text{-measure} = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}
\]

(25)

The higher the F-measure, the more accurate the detection. In all the experiments, the final detection map is obtained by thresholding the sparse matrix. The threshold is not known in prior, and we only compare the mentioned methods. For fair

<table>
<thead>
<tr>
<th>Frame Number</th>
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<th>12</th>
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<td>0.8809</td>
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<td>0.2154</td>
<td>0.8026</td>
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Fig. 6. Detection results of GPD-IALM, GPD-DECOLOR, GPD-NRA, and GPD-LRSTV on the HVS-aa13 sequence. First column is the false color images of frame (9, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28), respectively. Second column is the ground truth. Third column and fifth column display the results obtained by GPD-IALM and GPD-NRA, which did not consider the spatial and temporal continuity. The fourth column shows the results of GPD-DECOLOR. There still exists noise in these images calculated by GPD-DECOLOR. The sixth column is the results of GPD-LRSTV which achieves the best results especially in the last row.
comparison, we take the threshold that gives the maximal F-measure with the ground-truth in all methods, thus comparing all methods’ best F-measures.

The parameters \( (\gamma', \gamma, \gamma) \) are set to \((1, 1, 0.1)\) empirically. For the parameters \( \lambda \) and \( \beta \), detailed discussions are presented in Part C, Section III. In our experiments, we adopt the isotropic TV, that is to say, \( q = (2, 1) \). The number of features extracted by PCA is \( p=5 \) which we find keeps most of the information in the original HVS.
from videos captured by three long wave infrared (LWIR) spectrometers, each covering the range from 7,820 nm to 11,700 nm, spaced 30 nm apart. The sets of images were taken during the release of chemical plumes at the Dugway Proving Ground. The videos were captured by three different long wave infrared spectrometers placed at different locations to track the release of known chemicals. The cameras were about 1.25 miles away from each release at an elevation of about 1300 feet. The sensors captured one data cube every five seconds consisting of measurements at wavelengths in the long wave infrared (LWIR) portion of the electromagnetic spectrum.

Fig. 4 features 12 consecutive frames of a hyperspectral sequence, provided by the Applied Physics Laboratory at Johns Hopkins University. They consist of video sequences recorded during the release of chemical plumes at the Dugway Proving Ground. The videos were captured by three different long wave infrared spectrometers placed at different locations to track the release of known chemicals. The cameras were about 1.25 miles away from each release at an elevation of about 1300 feet. The sensors captured one data cube every five seconds consisting of measurements at wavelengths in the long wave infrared (LWIR) portion of the electromagnetic spectrum.

A. Data sets description

The first two hyperspectral data sets used in our experiments were provided by the Applied Physics Laboratory at Johns Hopkins University. They consist of video sequences recorded during the release of chemical plumes at the Dugway Proving Ground. The videos were captured by three different long wave infrared spectrometers placed at different locations to track the release of known chemicals. The cameras were about 1.25 miles away from each release at an elevation of about 1300 feet. The sensors captured one data cube every five seconds consisting of measurements at wavelengths in the long wave infrared (LWIR) portion of the electromagnetic spectrum.

Fig. 4 features 12 consecutive frames of a hyperspectral sequence, provided by the Applied Physics Laboratory at Johns Hopkins University. They consist of video sequences recorded during the release of chemical plumes at the Dugway Proving Ground. The videos were captured by three different long wave infrared spectrometers placed at different locations to track the release of known chemicals. The cameras were about 1.25 miles away from each release at an elevation of about 1300 feet. The sensors captured one data cube every five seconds consisting of measurements at wavelengths in the long wave infrared (LWIR) portion of the electromagnetic spectrum.

Furthermore, in order to evaluate the extensibility and effectiveness of our proposed detection method on traditional moving objects, we perform the experiments on a third data set, which is a sequence of 30 frame hyperspectral video of a static scene with a moving box sliding from the right side of the image to the middle of the image. In each frame, 33 bands are sensed covering the range from 430 to 720 nm at 10 nm resolution, so the size of each frame is $480 \times 752 \times 33$. It is originally used to testify the restoration accuracy of a hyperspectral video of dynamic scenes from a few measured multispectral bands per frame [55].

B. Detection Results

The first data set studied is the HVS-aa13 map. It has a total of 31, with 20 containing the gas plume. Along with the HVS data, 11 frames of gas plume ground truth are provided to evaluate the performance. Qualitative and quantitative results of our methods on this data set are given.

First, to demonstrate the superiority of the introduced TV regularization, we compare our results with inexact Augmented Lagrange Multiplier (IALM) [35], DECOLOR [26] and nonconvex rank approximation (NRA)[39] methods. The IALM and NRA separate the background and gas plumes without considering the spatial and temporal smoothness. DECOLOR adopts the nonconvex sparsity penalty and Markov random field (MRF) to detect outliers which prefers the regions that are relatively dense and contiguous. As the DECOLOR directly gives the foreground mask without tuning thresholds, it cannot be used to compute the AUC statistics and applied in the MaxD progress. We employ the difference between the original video and the background part as the detection result. Fig. 5. (a) shows the 18th frame’s first principal component and Fig. 5. (b) Fig. 5. (e) show the foreground part of the IALM, DECOLOR, NRA, and LRSTV methods, respectively. To better show the differences between these different methods, we normalize the foreground part dividing by their corresponding maximum value as shown in Fig. 5. (b)-(d). The detected gas placed at a different location about 2 km away from the release of plume at an elevation of around 1300 feet. One hyperspectral image is captured every five seconds. The size of each frame is reduced to $128 \times 320 \times 5$ after PCA dimensional reduction.

The images are of dimension $128 \times 320 \times 129$, where the last dimension indicates the number of channels, each depicting a particular frequency from 7,820 nm to 11,700 nm, spaced 30 nm apart. The sets of images were taken from videos captured by three long wave infrared (LWIR) spectrometers, each...
plume of LRSTV is obvious and smooth, and there is no noise in the background positions as shown in Fig. 5. (d). This indicates the denoising effect of the introduced TV regularization.

Next, we use the above three methods to detect the foreground part in Algorithm 2. We denote the resulting methods using IALM, DECOLOR, NRA, and LRSTV as GPD-IALM, GPD-DECOLOR, GPD-NRA, and GPD-LRSTV, respectively. The final results are shown in Fig. 6. We see that the results are noisy except for GPD-LRSTV which is clean and similar to the ground truth. Since the gas plume dissipates, it is hard to detect by the 28th frame shown in the last row. Here, the results of GPD-IALM, GPD-DECOLOR, and GPD-NRA are rather poor. Because of the spatial and temporal continuity constraint enforced by TV regularization, our method can still recover the faded plume. GPD-LRSTV enforces continuity along the temporal dimension which penalizes rapid change between consecutive frames. To provide an objective comparison, the AUC and F-measure values of these frames are given in TABLE I and TABLE II. The anisotropic TV-norm is also considered for comparison. We use GPD-LRSTV(a) to denote the method using anisotropic norm. We can see that the proposed GPD-LRSTV achieves the best numerical results of every frame in terms of both AUC and F-measure statistics. GPD-LRSTV(a) achieves a similar result with GPD-LRSTV which indicates the anisotropic TV-norm and isotropic TV-
measure statistics of the methods using different principal components. Fig. 8. (a) Principal component with analogous LRSTV to represent the method with p=1, taking the first compared with the proposed GPD detected results using a single principal component are proposed GPD in Table III detected gas plume is continuous. The numerical results shown pixels. The GPD DECOLOR algorithm produces fewer components and the full GPD-LRSTV method using five components. It can be seen that in frame 11, 12 and 14, PC1-LRSTV (LRSTV using the first PC) achieves good results, and in the following frames PC3-LRSTV (LRSTV using the third PC) achieves good results. However, GPD-LRSTV generally achieves the best results in every frame. This demonstrates that the gas plume may not be detected by one single principal component, but by taking the maximum value of the principal components’ results we can obtain a robust detection result in every frame.

Then, we apply our proposed method on the third data set. The results are shown in Fig. 9. In this video, the moving object has fixed shape and size in all frames and the differences between moving object and background is clearer than the gas plume videos. Thus the detection task is easier in this hyperspectral video sequence. From Fig. 9, it can be seen that all methods can detect the moving box and give a complete outline. This is because the scene is simpler and the original RPCA model can detect the moving object very well. TABLE V and VI shows the AUC and F-measure statistics for HVS-MB. GPD-DECOLOR gained the best performance as DECOLOR had been proven to be effective in object detection where the object keeps the same outline and has obvious differences with the background. Besides, in HVS-MB dataset, there is less noise in the scene due to the high imaging quality in 430nm-710nm wavelength. It is reasonable that GPD-DECOLOR achieved the best results in HVS-MB. However, we can still see that the result of GPD-LRSTV is contiguous in spatial direction and has less background part being detected as moving object.

We concluded that our proposed GPD-LRSTV method achieves very competitive detection results on all the tested HVS datasets. Moreover, the TV regularization can eliminate the noise errors, thus leading to the best detection accuracy while processing the HVS with low signal to noise ratio.

### Table V. AUC Statistics for Different Frames for HVS-MB

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<tr>
<th>Frame Number</th>
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<th>7</th>
<th>10</th>
<th>13</th>
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<th>22</th>
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</thead>
<tbody>
<tr>
<td>GPD-IALM</td>
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### Table VI. F-measure Statistics for Different Frames for HVS-MB

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![Fig. 10 Joint consideration of β and λ for HVS-aa12 (a) AUC statistics (b) F-measure statistics](image1)

![Fig. 11 Joint consideration of β and λ for HVS-aa13 (a) AUC statistics (b) F-measure statistics](image2)

norm have the similar effect in the gas plume detection.

The second data is the HVS-aa12 sequence. As with HVS-aa13 sequence, it has 31 frames and 20 frames with a gas plume present. The results are shown in Fig. 7. We find that GPD-IALM and GPD-NRA detect most of the gas plume, but many isolated background pixels are detected as well. The GPD-DECOLOR algorithm produces fewer incorrectly detected pixels. The GPD-LRSTV does not have this problem and the detected gas plume is continuous. The numerical results shown in Table III and Table IV prove the effectiveness of the proposed GPD-LRSTV.

To verify the efficiency of the proposed MaxD strategy, the detected results using a single principal component are compared with the proposed GPD-LRSTV. We use the PC1-LRSTV to represent the method with p=1, taking the first principal component with analogous definitions for other principal components. Fig. 8. (a)-(b) show the AUC and F-measure statistics of the methods using different principal parameters: λ and β. Ideally, we desire a robust method to
select them using MAP estimation. Due to the computational complexity, we do not use a robust estimation method here. Fig. 10 illustrates the obtained AUC and F-measure of HVS-aa12 when jointly taking the two regularization parameters into consideration. To add to this, for each pair we sum the F-measure of the 11 frames for which we have ground truth in the considered data set. \( \lambda \) is chosen from \{0, 0.0005, 0.0001, 0.0015, 0.002, 0.0025, 0.003, 0.0035, 0.004\} and \( \beta \) is chosen from \{0.01, 0.02, 0.03, 0.04, 0.05, 0.06\}. When \( \beta = 0 \), (3) degenerates to the RPCA problem which can be solved by IALM. The previous experiment has shown that the proposed GPD-LRSTV is superior to GPD-IALM which advocates including the advance of TV regularization. From Fig. 10-11, we can see that for \( \lambda > 0 \) the detection results are inferior to the results obtained for \( \lambda = 0 \). This confirms that gas plume takes a small part of the whole image scene, hence the foreground matrix is sparse. For the aa12 sequence, GPD-LRSTV achieves a high F-measure when \( \lambda \in [0.0005, 0.002] \) and \( \beta \in [0.03, 0.05] \). In our experiments, we set \( \lambda = 0.001 \), \( \beta = 0.04 \). Analogous to HVS-aa12, the best sets for HVS-aa13 is \( \lambda \in [0.001, 0.002] \) and \( \beta \in [0.01, 0.02] \) as shown in Fig. 10. For this HVS, we set \( \lambda = 0.002 \), \( \beta = 0.02 \).

D. Convergence analysis

In this section, we evaluate the convergence of LRSTV, as it is an iterative algorithm. LRSTV is applied on the first principal component of HVS-aa13 data set. The residual \( \| X - A - E \|_F \) and objective function value at each iteration are computed. The curves of residual and objective function value versus the number of iterations are plotted in Fig. 12, where we can clearly see that LRSTV convergences quickly.

E. Computational Complexity

The computation costs of the gas plume detection methods have also been compared. Detailed results are presented in Table VII. The algorithms are tested on a computer with a 64-b quad-core Intel Xeon CPU 3.33-GHz processor under Windows 7. GPD-IALM, GPD-NRA and the proposed method are implemented in MATLAB while the core part of GPD-DECOLOR is implemented in C++. So we can infer that the methods with temporal-spatial regularization (GPD-DECOLOR, GPD-LRSTV) cost more execution time than original RPCA methods (GPD-IALM, GPD-NRA). In the future, we will explore parallel implementations of our proposed method on GPUs to accelerate the execution of the algorithm, which is important for the application of our proposed method.

IV. CONCLUSIONS AND FUTURE RESEARCH LINES

In this paper, a novel hyperspectral video sequence for gas plume detection framework named GPD-LRSTV is proposed. It is based on the assumption that the background in different frames is reasonably stable and the moving object changes gradually in both space and time. We have formulated the variational gas plume detection problem and introduced spatial and temporal continuity with total variation regularization. Because there may exist hundreds of bands in a HVS, it is hard to extract a single band or feature to use for the LRSTV model. Thus we adopt a novel strategy which takes the maximum value over each principal component’s detection result. This way, we obtain a robust detection for every frame. Our results on three hyperspectral video sequences demonstrate the advantage of GPD-LRSTV.

The proposed method works well when the background is static. The case of dynamic background is for future work. Another important aspect deserving future research is the computational complexity of the proposed method. In this regard, we are currently developing efficient implementations using high performance computing architectures such as commodity graphics processing units (GPUs).
REFERENCES


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