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BRIEF REVIEW OF THE ALLOWED BETA TRANSITION
Tsuneyuki Kotani
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BRIEF REVIEW OF THE ALLOWED BETA TRANSITION* 

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ABSTRACT 

The results of theoretical calculations are listed and the 
experimental data are briefly reviewed. If the present experimental 
results are correct, we may have to abandon either the assumption of 
time reversal invariance or the present formalism of the two component 
theory of the neutrino, or both.

* According to the suggestions by Dr. R. D. Tripp and Dr. M. L. Good, 
this brief review is written to make clear the present situation of 
$\beta$-decay theory and to consider the experimental possibilities for 
testing time reversal invariance directly. Most parts of this review 
are due to the kind informations and suggestions of Dr. M. Morita at 
Columbia University. Most of the important theoretical results were 
obtained by Drs. J. D. Jackson, S. B. Treiman and H. W. Wyld 
(reference (11)).

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After Lee and Yang raised the question regarding the invariance of weak interaction under space inversion (P), charge conjugation (C) and time reversal (T), Wu and her coworkers confirmed that charge conjugation invariance is violated, and so is space inversion invariance. The theory of $\beta$-decay then becomes correspondingly more complicated. In order to consider future experiments which will give us useful information, it is desirable that the results of theoretical calculations be given in convenient form, along with the present experimental results.*

The contents of this review are restricted to only the allowed transitions, except for the energy spectrum of Ra E which is a famous 1st forbidden transition ($1 \rightarrow 0$ transition, with parity change).

The coupling constants ($C_j$ and $C'_j$) are given by their real parts ($g_j$ and $g'_j$) and their phase factors ($\phi_j$ and $\phi'_j$); for example,

$$C_j = g_j e^{i\phi_j}.$$  

Throughout this review, the following notations are used:

- $\vec{J}$: vector in the direction of spin of oriented nuclei,
- $\vec{P}$: momentum of electron,
- $E$: total energy of electron ($E^2 = P^2 + 1$),
- $\hat{n}_b$: unit vector in the direction of spin of polarized electron,
- $\vec{q}$: momentum of neutrino,
- $\vec{k}$: momentum of photon.

* Most of the experimental results cited in this review have not yet been confirmed accurately and we hope the reader will have the opportunity of bringing them up-to-date, when the experimental results will be completed.

** In this review, we assume that $M_F = \int \beta = \int \lambda$, $M_{GT} = \int \beta \bar{\lambda} = \int \bar{\beta}$ and in addition, all phases of nuclear matrix elements are the same ones. In the analysis of experimental results, especially $(\lambda_\alpha \lambda_\beta) \neq (\lambda_\beta \lambda_\alpha)$, we must take care of the phase difference between two nuclear matrix elements.
\[ g_F^2 = (g_S^2 + g_S^2 + g_V^2 + g_V^2) \]

\[ g_{GT}^2 = (g_T^2 + g_T^2 + g_A^2 + g_A^2) \]

\[ \chi^2 = \frac{(g_{GT} - M_{GT})^2}{g_F^2 M_F^2} \]

\[ G = \frac{g_A}{g_S} \]

All unexplained notations are identical with the standard notations in the \( \beta \)-decay theory.\(^1,2\)

The present experimental results are not so good, but some conclusions about the order of magnitude of coupling constants may be derived.

I. The absolute values of pseudovector type coupling constants (\( g_A \) and \( g_A' \)) are smaller than tensor type coupling constants (\( g_T \) and \( g_T' \)).\(^3,5\) We may say that

\[ 0.1 \geq \frac{g_A^2 + g_A'^2}{g_T^2 + g_T'^2} \geq 0 \]

and

\[ g_T \approx -g_T' \]

This may be concluded from the following types of experiments:

(Ia) The absence of Fierz interference term in the energy spectrum of pure Gamow-Teller transitions (A.2)*

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3. (A.2) means the second paragraph in Appendix. The experimental data are given there, along with the theoretical argument.
(Ib) The $\beta$-neutrino angular correlation in the decay of $\text{He}^6$ (A 3.1).

(Ic) The electron angular distribution with respect to the direction of orientation of nuclei, for example, the oriented $\text{Co}^{60}$-decay (A 4.1).

(Id) The longitudinal polarization of $\beta$-rays, (A 5.1).

We can not make any definite conclusions about the Fermi coupling constants; that is,

II. Dr. Deutsch finds that the vector coupling constants ($g^v = -g^v$) may be larger than the scalar coupling constants ($g^s = -g^s$) (A 5.2). On the other hand, the experimental results about the $\beta$-neutrino angular correlation shows us that $g^2_s + g^2_v$ may be larger than $g^2_v + b^2_V$ (A 3.3) (Fig. 1).

Therefore it is necessary to consider if these two experimental results can be understood consistently.

(Ila) First of all, we consider the possible interpretations under the assumption of the two component theory of the neutrino in which $g_1 = -g_1$. (It is assumed from the conclusion (I) that $g_A = -g_A = 0$.) If the $T$ invariance is correct, it can be said, as is well known, that absence of the Fierz interference term ($b_2$ (A 2)) means that $g_S = 0$ or $g_V = 0$. Therefore, the values of asymmetry coefficient $\lambda$ (A 3.2) in the $\beta$-neutrino angular correlation against the ratio $\chi^2$ (Eq. (3)) are given by extreme cases ($G = (g_V/g_S) = 0$ or $\infty$) in Fig. (1). As shown in Fig. (1), all experimental values of $\lambda$ for neutron and for $\text{Ne}^{19}$ (Eqs. (25) and (28)) are less than $1/3$. Then, $G$ must be nearly equal to zero. Even when we consider the maximum value for $b_F$ (Eq. (14)) which may not be equal to zero strictly within the present experimental uncertainty, this conclusion is not changed drastically.

$(0.15 > G \geq 0$ or $\infty \geq G > 6.8$ for $0.29 > b_F \geq 0$).
If \( g_s \gg g_v \), the longitudinal polarization of positrons in the pure Fermi transition has to be positive. (A5.2). From the experimental results for the angular distribution of positrons in the decay of oriented Co\(^{58}\) nuclei (A4.3) (Fig. 2), the ratio of \( X^2 \) (Eq. (3)), in the case of Co\(^{58}\) decay, must be larger than 100. These predictions do not agree even with the present rough data. (See (A5.2) and (A12)).

If all present experimental results about the \( \beta \)-neutrino angular correlation, (shown in Fig. 1) were wrong and \( \lambda > (1/3) \), we could avoid these present difficulties about the polarization of \( \beta \)-rays and the angular distribution of positron in the oriented Co\(^{58}\)-decay, because \( G = \infty \), that is, \( g_s \ll g_v \). However, we will meet with some difficulty to explain the energy spectrum of Ra E.\(^3\)

(IIb) Secondly, we consider the case that the two component theory of the neutrino is correct, but that T invariance is violated. If \( \lambda < (1/3) \), we get \( \sqrt{2} g_s > g_v \) for \( b_F = 0 \).\(^*\) This maximum value for the vector coupling constant tells us that the longitudinal polarization of positron must be larger than \(-0.35(P/E)\) in the pure Fermi transition; that is,

\[ \mathbf{(P/E)} > P_L > -0.35(P/E). \]

In order to be consistent with the present experimental results for \( \mathbf{(Fig. 1)} \), this polarization may have to be nearly equal to zero or

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\(^3\) M. Yamada, Prog. Theor. Phys. 2, 268 (1953). N. Fujita, private communication. The author should like to express his thanks to Professor M. Sasaki for informing him of Dr. Fujita's conclusion. S. Matsunaga, private communication.

\(^*\) For these calculations, it is assumed that \( g_s = 0 \).

If \( g_S > g_v \) and \( g_v \), it is necessary for the interpretation of spectrum in the Ra E decay that \( g_S g_T \cos(\phi_S - \phi_T) \) has the same sign as \( g'_S g'_T \cos(\phi'_S - \phi'_T) \). This condition is in conflict with the experimental data on Co\(^{60}\) and Co\(^{58}\) (A4.3), (A.5.1) and (A.12). Therefore we must introduce a contribution from the vector coupling. This means violation of T invariance, if the recoil experiments are correct (A.3.3).
a positive value. On the other hand, the asymmetry parameter \( \beta^3 \), Eq. (36) for the positron distribution in the oriented Co\(^{58}\) decay can take any values between two extreme cases \( G = 0 \) and \( G = \infty \) in Fig. 2, because there is an arbitrary parameter, \( \cos (\alpha_S - \alpha_T) \) in \( \beta^3 \).

(IIc) If the absolute value of the polarization of \( \beta^-\) rays is larger than 0.35 \( (P/E) \) for the pure Fermi transitions, and the recoil experiment about the neutron and the Ne\(^{19}\)-nuclei is correct, we must abandon the present form of the two component theory of the neutrino. If it is so, there are many possibilities to explain all experimental results described above. (For example, see the footnote (8) of reference (35).) One example of these possibilities is that \( g_S = g'_S, \ g_T = -g'_T \), \( |g_S| > |g_V| \) and \( |g'_V| \), and \( g_A = g'_A = 0 \). In order to explain all data cited above, except the Ra\( E \), we need not assume the violation of \( T \) invariance in this assignment. However, in this assignment, the energy spectrum of Ra\( E \) may suggest the abandonment of the assumption of \( T \) invariance.

But, now, we can not reach a definite conclusion about the Fermi coupling constants \( (g_S, g'_S, g_V, g'_V) \). Therefore, it is useful to classify the necessary experiments for determining the relative magnitudes and signs of \( C_S, C'_S, C_V, C'_V, C_T \) and \( C'_T \).

(a) First of all, it is desirable that many different kinds of experiments be performed to determine the ratio of Fermi coupling constants
\[
\frac{(g_V^2 + g'_V^2)}{(g_S^2 + g'_S^2)}.
\]
One of the simplest experiments for this determination is, of course, the measurement of polarization of \( \beta^-\) rays in 1) pure Fermi transitions (the \( 0 \rightarrow 0 \) transition, for example, \( C^{10}, C^{14}, \ C^{34}, A^{34}, Al^{26}, \) and \( K^{38} \)) or in 2) mixed \( \beta^-\) decay, which involves both allowed Fermi and Gamow-Teller interactions, but in cases for which
\[
|\frac{M_{GT}}{M_{F}}| < < |\frac{M_T}{M_F}|
\] (for example, \( A^{35} \)) or 3) the decay of neutron
\[
\frac{[A_{5,5}]}{[A_{5,5}]} = 29,31,5 \tag{Note: If the two component theory is not correct, there is ambiguity in the analysis}, except the recoil experiment.
\]
T.D. Lee, lecture at the 7th Rochester Conference, April 1957; and C. S. Wu, lecture at the Washington APS Meeting, April 1957.
(b) It becomes more important to reinvestigate the recoil experiments, especially for pure Fermi transitions and for neutron decay [A.3].

(c) The measurement of the asymmetry coefficient in the polarized neutron will shed much information on this question, as will the same type of experiment in polarized nuclei. [A.4].

(d) If \( (g_V^2 + g_s^2)/(g_V^2 + g_s^2) > 1 \), direct information for testing T invariance will be supplied by measuring the quantity \( \hat{J} \times \hat{P} \) (A.8) in the decay of mixed transition which involve both Fermi and Gamow-Teller interaction (\( \Delta T = 0 \), no). There may be another method for this purpose, by measuring the \( \beta - \gamma \) correlation for suitable nuclei.

(e) If \( (g_S^2 + g_s^2)/(g_V^2 + g_s^2) > 1 \), direct information for testing time reversal invariance will be given by measuring the quantity \( \hat{J} \times \hat{P} \times \hat{q} \) (A.6) the quantity \( \hat{J} \times \hat{P} \times \hat{k} \) (\( \hat{J} \times \hat{k} \)) with \( n = 1 \) and 3 (A.10) or \( \beta \)-circularly polarized \( \gamma \)-angular correlation from oriented nuclei (A.11).

(f) If the measurement of polarization of \( \beta \)-rays in the pure Fermi transition indicates that the polarization is quite small, then

\[ g_V g_s^* \cos(\phi_V - \phi_s^*) \]

is nearly equal to \( g_s g_s^* \cos(\phi_s - \phi_s^*) \). As long as experimental results about the \( \beta \)-neutrino angular correlation and the electron angular distribution in the decay of oriented \( {\nu}_{58} \) are correct, the assumption of T invariance must be abandoned. In this case, the measurement of the quantity \( \hat{J} \times \hat{P} \times \hat{q} \) (A.8) will give us direct information about the T invariance.\[ \hat{J} \times \hat{P} \times \hat{q} \]

(g) In order to eliminate the unknown parameters, that is, nuclear matrix elements, the useful experiments are the measurements of \( \gamma \)-anistropy, \( \beta - \gamma \)-correlation, or \( \beta - \gamma \) circular polarization correlation in the decay of unpolarized, aligned or oriented nuclei, for example \( {\nu}_{58} \) and \( {\nu}_{22} \) (A.9, A.12).
(h) The measurement of polarization of the electrons in Ra E decay will be important. If \((g_S^2 + g_S'^2)/(g_V^2 + g_V'^2) > 1\), we will be able to get information about the assumption of the T invariance. \(^{30}\) If \((g_S^2 + g_S'^2)/(g_V^2 + g_V'^2) < 1\), there may be some difficulty to explain the energy spectrum from the theoretical point of view; the polarization experiment in Ra E decay will give us complementary information about the nuclear matrix elements. \(^{6}\) It is interesting to note that the experiments for testing directly the assumption of T invariance will give also much information on questions such as the relative signs of \(S, V\) and \(T\).

(i) Now we believe that the tensor coupling constants \((g_T^2, g_T'^2)\) are larger than the pseudovector coupling constants \((g_A^2, g_A'^2)\). (See the conclusion (I).) However, we have only one direct proof for this conclusion, that is, the \(\beta\)-neutrino angular correlation in the He \(^6\) decay (A.3.1). It is desirable that more recoil experiments in the decay of pure Gamow-Teller transition be performed to confirm this conclusion.

The author would like to express his sincere thanks to many physicists for sending their preprints and Dr. R. Gatto for his helpful discussions.

\(^{6}\) We must take care of the fact that some coefficients are disappeared under special conditions for coupling constants, even when the T invariance is violated; for example, we can not observe the main term of the coefficients \(D(Eq. (50) of (A.6))\), if \(g_S = +g_S'\), \(g_T = -g_T'\) and \(\cos(\phi_S - \phi_T) = \cos(\phi_S' - \phi_T')\).
APPENDIX 7

The results of theoretical calculation are given in convenient form with the experimental results that have come to our attention. The general distribution function of \( \gamma \)-ray is given by

\[
W(J, p, q, \sigma) = \frac{1}{(2\pi)^5} F(\pm Z, E) p E (E_0 - E)^2 dE d\sigma d\sigma dC,
\]

where \( C \) means a correction term for the statistical shape factor and

\[
\Sigma = \Sigma_F + \Sigma_{GT},
\]

\[
\Sigma_F = |M_F|^2 g_F^2 , \quad g_F^2 = g_S^2 + g_T^2 + g_A^2 + g_V^2,
\]

\[
\Sigma_{GT} = |M_{GT}|^2 g_{GT}^2 , \quad g_{GT}^2 = g_T^2 + g_A^2 + g_A^2.
\]

(5)

The following abbreviations are used:

- In all formulas the upper signs refer to electron decay and the lower signs to positron decay.
- Fermi-transition: pure Fermi transition (0 \( \rightarrow \) 0; parity change no),
- Gamow-Teller-transition: pure Gamow-Teller transition (\( \Delta J = \pm 1 \); no),
- Mixed transition: both allowed Fermi and Gamow-Teller interactions are involved. (\( \Delta J = 0 \) except 0 \( \rightarrow \) 0 transition; no),
- 4-component theory: the conventional theory of the neutrino,
- 2-component theory: the two component theory of the neutrino \( g_i = -g'_i \).

\(*\) In this Appendix, the following approximations are used: \( \gamma Z^2 << 1 \), neglections of the finite deBroglie wave length correction and of the nuclear size effect. These correction terms are \( \sim (1/\text{100}) \) the main term.


(A.1) The ft value (B - X curve).

[Theory] \[1,2\]

\[ft = \frac{B}{(1 - x) \left| M_F \right|^2 + x \left| M_{GT} \right|^2} \]

\[x = \frac{2}{g_{GT}^2 + g_F^2} \]

\[B = \frac{2 \pi^2 \hbar^2 \log 2}{(g_F^2 + g_{GT}^2) \hbar^2 c^2} \]

[Experimental Result] \[3\] (These results are not complete for order \((aZ)^2\))

\[x = 0.560 \pm 0.012 \]

\[B = 2787 \pm 70 \]

\[\left| \frac{g_F}{g_{GT}} \right| \sim 0.90 \pm 0.05 \]

(A.2) Energy Spectrum (Fierz Interference Term).

[Theory] \[1,7\]

\[C = 1 + b \frac{1}{E_F} \]

\[\sum b = \sum_{GT} b_{GT} + \sum_{F} b_{F} \]

\[\sum_{GT} b_{GT} = \pm 2 \left| M_{GT} \right|^2 \left[ g_7 g_A \cos(\theta_T - \theta_A) + g'_T g'_A \cos(\theta'_T - \theta'_A) \right] \]

\[\sum_{F} b_{F} = \pm 2 \left| M_F \right|^2 \left[ g_S g'_V \cos(\theta_S - \theta'_V) + g'_S g_V \cos(\theta'_S - \theta_V) \right] \]

\[\text{See, for example, O. Kofoed-Hansen and A. Winther, Kgl. Danske Videnskab.}
\text{Selskab, Mat.-Fys. Medd. 30, No. 2(1956), or L. Michel, Proceedings of International Conference at Seattle (1956) (Rev. Mod. Phys. 29, 263, 1957))} \]
[Experimental Result] \(^9\star\)

\[
0.05 \geq b_{GT} > 0 , \quad (12)
\]
\[
0.29 \geq b_F \geq 0 , \quad (13)
\]
\[
0.15 \geq b_F \geq 0 . \quad (14)
\]

[Note]

For simplicity, let us assume that \( b = 0 \). If we use the two component theory of the neutrino, we have the following restrictions:

\[
\begin{align*}
\varepsilon_S &= 0 \quad (15a) \\
\varepsilon_V &= 0 \quad (15b) \\
\phi_S - \phi_V &= 90^\circ \quad (15c)
\end{align*}
\]

and

\[
\begin{align*}
\varepsilon_F &= 0 \quad (15d) \\
\varepsilon_A &= 0 \quad (15e) \\
\phi_T - \phi_A &= 90^\circ \quad (15f)
\end{align*}
\]

The conditions of (15c) and (15d) mean that the assumption of \( T \)-invariance must be abandoned.

\((A.3)\) \( \langle p,q \rangle \) : \( \beta \) -neutrino angular correlation

[Definition] \(10,1,11,12\)

\[
W(p,q) \sim 1 + A(P,W) \cos \theta_{8\nu} , \quad (16)
\]


where
\[ \lambda = \frac{F_a}{\frac{1}{s} + \frac{1}{s} b(1/E)} \]  

(A.3.1) Gamow-Teller transition

[Theory]
\[ a_{GT} \hat{F}_{GT} = \frac{1}{3} |M_{GT}|^2 \left\{ g_T^2 + g_T^2 - g_A^2 - g_A^2 \right\} \]
\[ \pm (2 \alpha Z/F) (g_T g_A \sin(\phi_T - \phi_A) + g_T^2 g_A \sin(\phi_T - \phi_A')) \]  

(18)

[Experiment]
\[ \text{He}^6 (0^+ \rightarrow 1^+) \]
\[ \lambda = \begin{cases} 
0.31 \pm 0.14 & \text{for } E = 2.0 \text{ MeV} \\
0.36 \pm 0.10 & \text{for } E = 1.25 \text{ MeV.} 
\end{cases} \]

[Note]
\[ \frac{1}{3} > \frac{(g_A^2 + g_A^2)}{(g_T^2 + g_T^2)} \Rightarrow 0 \]  

(19)


13 Note: In this review, terms including $\sin(\phi_1 - \phi_j)$ disappear, if the assumption of T-invariance is correct.
(A.3.2) Fermi Transition

[Theory]

\[ a_F \xi_F = \left| M_F \right|^2 \left\{ -g_s^2 - g_S^2 + g_V^2 + g'_V^2 + (2\alpha \frac{Z}{P}) \left( g_s g_V \sin(\phi_S - \phi_V) + g'_S g'_V \sin(\phi'_S - \phi'_V) \right) \right\} \] (20)

[Experiment]

(A.3.3) Mixed-transition

[Theory]

4-component theory

\[ \lambda = \frac{\xi_{GT} a_{GT} + \xi_{F} a_{F}}{(\xi_{F} + \xi_{GT}) + (\xi_{GT} b_{GT} + \xi_{F} b_{F})(1/E)} \] (21)

2-component theory \(^{(15)}\) (Fig. 1)

\[ \lambda = \frac{-1 \pm (2\alpha \frac{Z}{P}) G \sin(\phi - \phi') + G^2 + \frac{1}{2}(1 + G^2)X^2}{(1 + G^2)(1 + X^2) \pm (2 /E)G \cos(\phi_S - \phi'_V)} \] (22)

\[ G = 0 \ (g_V = 0) \] \(^{(15)}\)

\[ \frac{1}{3} \geq \lambda = \frac{-1 + \frac{1}{3}X^2}{1 + X^2} \geq -1 \] \(^{(23)}\)

\[ G = \infty \ (g_S = 0) \] \(^{(15)}\)

\[ 1 \geq \lambda = \frac{1 + \frac{1}{3}X^2}{1 + X^2} \geq \frac{1}{3} \] \(^{(24)}\)


\(^{(15)}\) To simplify the expression, the assumption that \( g_A = g'_A = 0 \) and \( b_F = (\text{12}) \) is used, if necessary.
Ne$^{19}$ \( \frac{1}{2}^+ \rightarrow \frac{1}{2}^- \) \( \frac{1}{2}^+ \rightarrow \frac{1}{2}^- \)

\[ \begin{cases} -0.21 \pm 0.08, \\ +0.14 \pm 0.13, \\ -0.15 \pm 0.2 \end{cases} \]

\( \lambda = \begin{cases} 17 \\ 18 \\ 19 \end{cases} \)

Neutron \( \frac{1}{2}^- \rightarrow \frac{1}{2}^+ \) \( A^{15} \)

\[ \lambda = 0.089 \pm 0.108 \]

(A.4) \((p, J)\): Angular distribution of \( \beta^- \) -ray from the oriented nuclei.

[Theory] \( l, 11, 12, 26 \)

\[ W(J, P) \sim 1 + \gamma \cos \Theta_{JP} , \]

\[ \gamma = \frac{P}{E} \frac{\langle J_z \rangle}{J} \beta' \]

(A.4.1) Gamow-Teller-transition

[Theory]

4-component theory

\[ \beta'_{GT} \oint_{GT} (1 + b_{GT}(1/E)) \]

\[ = 2 \left| M_{GT} \right|^2 \lambda_{JJ} \left[ \pm (g_T g_T' \cos(\phi_T - \phi_T')) \\ - g_A g'_A \cos(\phi_A - \phi_A') \right] \\
+ (\gamma Z/P)(g_T g'_A \sin(\phi_T - \phi_A') + g'_T g_A \sin(\phi_T' - \phi_A')) \]

(30)\(^{13}\)
The theoretical or semi-empirical estimation for the ratio of nuclear matrix elements are as follows

\[ \frac{\left| M_{GT} \right|^2}{\left| M_{F} \right|^2} = 1.6 \quad 21, 22 \]
\[ = 2.59 \quad 23, 24 \]

and

\[ x^2 = 2.43 \pm 0.13 \quad 18 \]
\[ = \frac{1}{(0.69 \pm 0.17)} \quad 17 \]
\[ = 2.08 \quad 22 \]

16

20 J. M. Robson, Phys. Rev. 100, 933 (1955). For neutron decay,
\[ x^2 = 3 \frac{g_{GT}^2}{g_F^2} \]
22 M. Morita, private communication.
26 Ambler, Hayward, Hoppes, Hudson and Wu, preprint, "Further Experiments on $\beta$ -decay of Polarized Nuclei."
27 Postma, Huiskamp, Miedma, Steenland, Tolholk and Gorter, preprint.
2-component theory

\[ \beta'_{\text{GT}} = \lambda_{JJ'} \frac{(g_T^2 - g_A^2) - (2\gamma E/P)g_T g_A \sin(\phi_T - \phi_A)}{(g_T^2 + g_A^2) \pm (2 /E)g_T g_A \cos(\phi_T - \phi_A)} \]

where

\[ \lambda_{J,J'} = \begin{cases} 1 & \text{for } J \rightarrow J' = J - 1 , \\ - \frac{J}{(J + 1)} & \text{for } J \rightarrow J' = J + 1 . \end{cases} \]  

Experiment

\[ \text{Co}^{60} \quad (5^+ \xrightarrow{\beta^-} 4^+ \xrightarrow{\gamma} 2^+ \xrightarrow{\beta^+} 0^+) \]

\[ \beta' = -1 \pm 25, 26 \]

\[ \text{Na}^{22} \quad (3^+ \xrightarrow{\beta^+} 2^+) \]

\[ \beta' = \]

(A.4.2) Fermi-transition

None

(A.4.3) Mixed-transition

[Theory]

4-component theory\[ (g_A = b_{\text{GT}} = 0) \]

\[ \gamma (1 + b_{\frac{E}{S}}) \beta' = \frac{\pm 2}{(J + 1)} \left[ g_T g_T' \cos(\phi_T - \phi_T') \right] \left| W_{\text{GT}} \right|^2 \]

\[ + 2 \left[ \frac{J}{J + 1} \right] \left\{ g_S g_T' \cos(\phi_S - \phi_T') + g_S g_T \cos(\phi_S' - \phi_T') \right\} \]

\[ \pm \left( \frac{\gamma}{2} \right) \left\{ -g_T \sin(\phi_T - \phi_T') - g_T' \sin(\phi_T' - \phi_T) \right\} \]

(35)
2-component theory\textsuperscript{15} \ (G_A = G_{GT} = 0) \quad \text{(Fig. 2)}

\[
\beta^* = \frac{1}{(J + 1)} \frac{\sqrt{1 + G^2} X^2}{1 + \frac{1 - J(J+1)}{2J} G \cos(\theta_S - \theta_T)} \sqrt{1 + G^2} \times \frac{(J(J+1))}{(1 + G^2)(1 + X^2) \pm (2J \pm 1) G \cos(\theta_S - \theta_T)}
\]

(36)\textsuperscript{13}

where

\[
X = \frac{\frac{\varepsilon_T}{\varepsilon_S} \left| \frac{M_{GT}}{M_G} \right|}{\gamma(1 + G^2)}
\]

(37)

Experiment

Co\textsuperscript{58} \quad (2^+ \rightarrow 2^+, 0^+) \quad 26,27

\[
\beta' \sim \frac{1}{3}
\]

(38)

Na\textsuperscript{24} \quad (4^+ \rightarrow 4^+ )

\[
\beta' = -0.20 \pm 0.14 \quad 28
\]

(A.5) \ (\vec{p}, \vec{e}^-) \ : \ longitudinal \ polarization \ of \ \beta^- \ ray \ in \ ordinary \ \beta^- \ decay.

[Definition] \textsuperscript{8,6,11,12,29,30,31}

The longitudinal polarization in the direction \vec{n} is defined by

\[
P_L = \frac{W(\vec{e}, \gamma \vec{n}) - W(\vec{p}, -\vec{n})}{W(\vec{p}, +\vec{n}) + W(\vec{p}, -\vec{n})}
\]

(39)

\[
= (P/E)\left( \frac{\varepsilon G}{\varepsilon + \varepsilon b(1/E)} \right)
\]

(A.5.1) Gamow-Teller-transition

[Theory]

\(L\)-component theory \quad (b_{GT} = 0. \quad \text{Eq.(12)})
\[
G_{GT} \propto g_T = \pm 2 \left[ g_T g'_T \cos(\theta_T - \theta'_T) - g_A g'_A \cos(\theta_A - \theta'_A) \right] \\
+ 2 \frac{x Z}{P} \left[ g_T g'_A \sin(\theta_T - \theta'_A) + g'_T g_A \sin(\theta'_T - \theta_A) \right]
\]

\[\text{(40)}\]

2-component theory

\[
P_L = \frac{Z}{P/E} \frac{(g_T^2 - g_A^2) - (2 x Z/P) g_T g_A \sin(\theta_T - \theta_A)}{(g_T^2 + g_A^2) \pm \frac{2}{E} g_T g_A \cos(\theta_T - \theta_A)}
\]

\[\text{(41)}\]

[Experiment]

\[
\text{Go }^{60} \quad (5^- \rightarrow 4^+ \rightarrow 2^- \rightarrow 0^-)
\]

\[
P_L = -(P/E) \pm 20\%
\]

\[\text{(42)}\]

\[
\text{Na }^{22} \quad (3^- \rightarrow 2^- \rightarrow 0^-)
\]

\[
P_L = +
\]

\[\text{(43)}\]

---


30 K. Alder, B. Stech and A. Winther, preprint, "On Parity Non-Conservation in $\beta$-Decay."

31 T. Kotani, UCRL-3724, "A Possible Test of Time-Reversal Invariance in Beta-Decay."


(A.5.2) Fermi-transition

[Theory]

4-component theory

\[ G_F^F = \pm 2(g_S^S g_V^V \cos(\phi - \phi_S) - g_S^V g_V^S \cos(\phi_V - \phi'_V)) \]

\[ \pm 2 \frac{Z}{F} \left( g_S^S g_V^V \sin(\phi - \phi'_V) \mp g_S^V g_V^S \sin(\phi'_S - \phi'_V) \right) \]

(44)

2-component theory

\[ P_L = \frac{(P/E) \left( (g_S^2 - g_V^2) - (2Z/F)g_S g_V \sin(\phi - \phi_V) \right)}{(g_S^2 + g_V^2) \mp \frac{2}{E} g_S g_V \cos(\phi - \phi_V)} \]

(45)

[Experiment]

\[ C_{34}^{34} \] \[ 0 \rightarrow 0 \]

(46)

\[ P_L = \frac{33}{\text{33}} \]

(A.5.3) Mixed-transition

[Theory]

4-component theory

\[ P_L = \frac{(P/E) \left( \sum F^GF^F \sum GT^GT^G \right)}{\left( \sum F \sum GT \right) \mp (1/E) \left( \sum F b_F \sum GT b_GT \right)} \]

(47)

33 M. Deutsch, private communication. The author should like to express his thanks to Dr. M. Morita and Dr. S. B. Treiman for informing him of this result.
2-component theory\textsuperscript{15} \( (g_A = b_{GT} = 0) \)

\[
P_L = \frac{\gamma \left[ \left( g_S^2 - g_V^2 \right) \left| M_F \right|^2 + g_T^2 \left| M_{GT} \right|^2 \right] - \alpha \gamma \left[ g_S g_V \sin(\theta_S - \theta_V) \right] \left| M_T \right|^2}{\left( g_S^2 + g_V^2 \right) \left| M_F \right|^2 + \left| M_{GT} \right|^2 \left( 2/\beta \right) g_S g_V \cos(\theta_S - \theta_V) \}
\]

(48)\textsuperscript{13}

**[Experiment]**

\( \gamma > 0 \) (first unique forbidden; yes)\textsuperscript{34}

\[
\text{Sc}^{46} \ (\gamma^+\beta^- \to \gamma^+ \to 2^+ \to 0^+) \quad 34^f
\]

\[
P_L \approx 0
\]

(A.6) \( J \cdot [\hat{p} \times \hat{q}] \) : Recoil Experiments with Oriented Nuclei.

**[Theory]**\textsuperscript{11,12}

\[
P_{Jpq} = \frac{W(\bar{J} \cdot [\hat{p} \times \hat{q}]) - W(\bar{J} \cdot [\hat{-p} \times \hat{q}])}{W(\bar{J} \cdot [\hat{p} \times \hat{q}]) + W(\bar{J} \cdot [\hat{-p} \times \hat{q}])}
\]

(49)

\[
= \frac{D}{1 + b \frac{\alpha}{E}} \frac{\langle J \cdot [\hat{p} \times \hat{q}] \rangle}{J \cdot E \cdot q}
\]

Note: If \( \bar{J}, \hat{p} \) and \( \hat{q} \) are not perpendicular to each other, the other term with \( (\hat{p} \cdot \hat{q}) \), (A.3), and similar terms must be added.


\textsuperscript{34} Taenfelder, private communication to Dr. A.H. Rosenfeld.
\[ \int D = 2 \sqrt{\frac{J}{J+1}} \left\{ g_S g_T \sin(\phi_S - \phi_T) + g_S' g_T' \sin(\phi_S' - \phi_T') \right\} \]

\[ \sim \frac{\alpha Z}{P} \left\{ -g_V g_T \cos(\phi_V - \phi_T) - g_V' g_T' \cos(\phi_V' - \phi_T') \right\} \left| \frac{m_P}{m_{GT}} \right| \]

using the assumption that \( g_A = g_A' = 0 \).

\[ (A.7) \quad (\vec{p} \times \vec{q}) \cdot \text{Electron Polarization in Recoil Experiment} \]

\[ \text{[Theory]} \quad 11,12 \quad (\text{The definition is given by Eq. (5) of Inchen--} \]

\[ \text{TaUMAN--WYLDAS paper (Phys. Rev. 66, 517 (1931)).} \]

\[ P_{pq} = \frac{L}{(1 + a \frac{(\vec{p} \cdot \vec{q})_Z}{E_q} + b \frac{1}{E}) \cdot E_q} \]

where

\[ L = \left| \frac{M_P}{M} \right| \left\{ 2(g_S g_V \sin(\phi_S - \phi_V) + g_S' g_V' \sin(\phi_S' - \phi_V')) \right\} \]

\[ + \frac{1}{3} \left| \frac{M}{M_{GT}} \right| \left\{ -\frac{\alpha Z}{P} (g_T^2 + g_T'^2) \right\} \]

using the assumption that \( g_A = g_A' = 0 \).

\[ \begin{align*}
\text{a} &= a_P \frac{2}{3} \psi = \psi_0, \text{Eqs. (15) and (20)}, \\
\text{b} &= \text{Eq. (9)}.
\end{align*} \]
(A.8) \((\overrightarrow{S} \cdot [\langle \vec{J} \rangle \times \vec{p}])\): Electron Polarization in Decay of Oriented Nuclei

[Theory] \textsuperscript{11,12}

\[
P_{\sigma Jp} = \frac{R}{1 + b(1/E)} \left( \frac{\langle \vec{J} \rangle \times \vec{p}}{J E} \right)
\]

where

\[
R = \left| M_{GT} \right| ^2 \lambda_{JJ'} \left[ -\alpha Z/P \right] 2 g_T g'_T \cos(\theta_T - \theta'_T)
\]

\[
+ \delta_{JJ} \left| M_{\pi} \right| \left| M_{GT} \right| \sqrt{\frac{J}{J+1}} \left[ -2(g_V g'_T \sin(\theta_V - \theta'_T) + g'_V g_T \sin(\theta'_V - \theta_T)) \right]
\]

\[
= 2 \frac{\alpha Z}{P} (g_S g'_T \cos(\theta_S - \theta'_T) + g'_S g_T \cos(\theta'_S - \theta_T))
\]

using the assumption that \(g_A = g'_A = 0\). Here \(\delta_{JJ'}\) is the Kronecker delta symbol and

\[
\lambda_{J'J} = \begin{cases} 
1 & \text{for } J \rightarrow J' = J - 1 \\
\frac{1}{J+1} & \text{for } J \rightarrow J' = J \\
-\frac{J}{J+1} & \text{for } J \rightarrow J' = J + 1
\end{cases}
\]

(A.9) \(\tau(\vec{p} \cdot \vec{k})\) Angular Correlation between \(\beta\) and \(\gamma\) from Unoriented Nuclei (with or without observing circular polarization of \(\gamma\) ray).

[Theory] \textsuperscript{30,35}

The daughter nucleus in the decay of unpolarized nuclei will be partly polarized along the electron direction. If the \(\beta\)-decay is followed by a \(\gamma\)-transition, this polarization can be studied by means of the \(\beta\) - \(\gamma\) angular correlation when at the same time the circular polarization \(\epsilon_q\) (or \(\gamma\)-quantum \(\tau\)) is detected.
These types of experiment may be used for a determination of the coupling constants. Especially it should be possible to determine relative sign of the various coupling constants. But these experiments do not offer a clear-cut experiment for testing T invariance, because the terms which appear due to the violation of T-invariance are of the order of \((\gamma Z/P)\) smaller than main asymmetry terms. The theoretical details for the asymmetry parameter \(a\) are given in reference (30) and (35).

\[ W(\theta, \gamma) \sim 1 + a \frac{P}{E} \cos \theta \]  

A measurement of \(a\) may be used to obtain data about nuclear spins. 36

\[ \text{[Experiment]} \]

\[ \text{Co}^{60} \hspace{1cm} (5^+ \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0) \]

\[ a = -0.40 \pm 0.09 \] 36

\[ \text{Na}^{22} \hspace{1cm} (3^+ \rightarrow 2 \rightarrow 0) \]

\[ a = +0.4 \] 36

\[ \text{Na}^{24} \hspace{1cm} (4^+ \rightarrow 4 \rightarrow 2 \rightarrow 0) \]

\[ S_{\text{C}}(4^+ \rightarrow 4^+ \rightarrow 2 \rightarrow 0^+ \rightarrow 0^+) \]

\[ a = -0.07 \pm 0.04 \] 36

35 M. Morita and R. S. Morita, preprint, "Time Reversal Invariance and Beta-Gamma Angular Correlation."

36 F. Boehm and A. H. Wapstra, preprint, "Beta-Gamma Circular Polarization Correlation Measurements I and II." For Co\(^{60}\) and Na\(^{22}\), the theoretically expected value for \(a\) is \(a = 0.33\), if we assume, that \(\varepsilon_A = \varepsilon_A' = 0\).

37 H. Schopper, preprint, Phil. Mag. (to be published).

36' A. H. Wapstra, private communication to Dr. H. E. Rose.
(A.10) $(\vec{J} \cdot [\vec{p} \times \vec{k}])\vec{J}^n$ (n = 1 and 3): $\beta - \gamma$ Correlation from Aligned or Oriented Nuclei.

[Theory] \(^{35}\)

Assuming the Coulomb correction very small, asymmetry of the angular correlation function can be expected only if the $\beta$ interaction is non-invariant under time reversal. It has the order of magnitude

$$(P/E)(g_T g_S \sin(\theta_S - \theta_T) + g_T g_S \sin(\theta_T - \theta_S))$$

Both aligned and polarized nuclei may be used for this experiment. The theoretical results for $2 \rightarrow 2 \rightarrow 0$ are given by M. Morita and R. Saito Morita (Reference 35).

[Experiment]

(A.11) $\vec{\gamma}(\vec{J} \cdot [\vec{p} \times \vec{k}])$: $\beta$ - Circularly Polarized $\gamma$ Angular Correlation from Oriented Nuclei.

[Theory] \(^{35}\)

When $\vec{J}$ and $\vec{k}$ are perpendicular, the asymmetry given in (A.10) vanishes. In this case, however, the $\beta - \gamma$ angular correlation function can still show (by observing the circular polarization of $\gamma$-ray (\(\vec{\gamma}\)) and using polarized nuclei), whether $T$ invariance is violated in $\beta$ - decay.

This asymmetry changes its sign for opposite circular polarization of $\gamma$ - ray (\(\vec{\gamma} \rightarrow \vec{\gamma}\)) and its order of magnitude is also

$$\ldots (\vec{g} \cdot \vec{\gamma} - \vec{g}_p \cdot \vec{\gamma}_p) \sim \vec{g} \cdot \vec{\gamma}.$$
(A.12) \( (k', J): \) \( \gamma \)-ray Angular Distribution from Oriented or Aligned Nuclei.

[Theory] \(^{38,39,35}\)

\[ W(\theta) \sim 1 - \frac{15}{14} (1 + \lambda) N_2 f_2 P_2(\cos \theta) - \frac{5}{3} (-2 + 5\lambda) N_4 f_4 P_4(\cos \theta) \]

where \( N_2 \) and \( N_4 \) are constants and \( f_2 \) and \( f_4 \) are the orientation parameters.

[Experimental Result]\(^{58}\)

\[ \text{Co}^{58} \quad (2^+ \beta^+ \rightarrow 2^+ \rightarrow 0^+) \]

\[ k = 0.8 \text{ Mev.} \]

\[ 14 \geq X^2 \geq 6. \]


\(^{41}\) M. Morita concluded that we might explain this experimental data by using a wider range for \( X^2 \), \( (65 \geq X^2 \geq 6) \). See Footnote (8) of reference (35).
FIGURE CAPTIONS

Figure 1: Theoretical curve of \( \lambda \) (Eq. (22)) as a function of \( X^2 \) (Eq. (31)) for various values of \( G = \frac{g_V}{g_S} \), with the experimental data for the \( ^{19}\text{Ne} \) nuclei and the neutron. \( \text{Assumption: } b = \mathcal{C} (\varepsilon_S (r)) \text{ and } \varepsilon_S (r) \).

Figure 2: Theoretical curve of the asymmetry parameter \( \beta' \) (Eq. (36)) as a function of \( X \) (Eq. (37)) for \( J = 2 \) and two extreme values of \( G \). \( \text{Assumption: } b = 0 (\varepsilon_S (r)) \text{ and } \varepsilon_S (r) \).
Note Added in Proof:

(I.) While this review was being typed, the author heard the new data about the Cs^136 decay.

(a) The longitudinal polarization of electron: \( P_L \sim 0 \) (A.5.3) (by Dr. Frauenfelder and his coworker).

(b) The \( \beta - \gamma \) circular polarization correlation: \( A = 0.45 \pm 0.10 \) (A.9). (by Dr. Wapstra and his coworker).

If we assume time reversal invariance and the two component theory, the data (b) is consistent with \((T+S)\), but the data (a) is inconsistent. The data (a) can be explained by \((T+V)\), but the data (b) can not.

Therefore, we may have an additional information for abandonment of either the assumption of T-invariance or the present formalism of the two component theory of the neutrino, or both.

If we assume the two component theory and the abandonment of the T-invariance, we can get the condition

\[
\sqrt{2} > \frac{g_y}{g_S} \gtrsim 1
\]

from the data (a) and the recoil experiments for Ne^{19} (Good's data) and neutron (Robson's data). Then the longitudinal polarization of \( \beta \)-ray in the pure Fermi transition must be restricted as follows:

\[
0.33 \left( \frac{P}{E} \right) \geq |P_L| \geq 0
\]

There is a rumor that \( V > S \) in the recoil experiment of \( A^{35} \) decay. If \( \frac{1}{2} > \lambda > 0 \) in \( A^{35} \), it is consistent with the above assignment. If \( \lambda \sim 1 \) in \( A^{35} \) \( (g_y > g_S) \), we must abandon the present \( \beta \)-decay theory for the Fermi transition, as long as all recoil experimental results in \( n, \) Ne^{19} and \( A^{35} \) are correct.
II. On May 31, 1957 the author received the following letter from Dr. M. Deutsch. "We have no definitive results on Cl\(^{34}\). Preliminary results indeed indicate at least a small polarization in the direction expected from the two-component theory with conservation of leptons and vector interaction, but much work remains to be done before we can be sure."
### Table 2: Component Theory ($C_j = -C_j'$)

<table>
<thead>
<tr>
<th>$T$-invariance</th>
<th>Conserve</th>
<th>Non-conserve</th>
<th>Conserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Coupling</td>
<td>$(S, T)$ ($g_2 = 0$)</td>
<td>$(S, T)$ ($g_2 = 0$)</td>
<td>$(S, V, T)$ ($g_2 = 0$, $g_3 = 0$)</td>
</tr>
<tr>
<td>$(P, 3)$ (A.3)</td>
<td>$GT$</td>
<td>$\sqrt{3}$</td>
<td>$(-1 + \frac{x^2}{1 + x^2})$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mixed</td>
<td>$-1$</td>
<td>$\frac{1}{1 + x^2}$</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>Mixed</td>
<td>$\frac{x^2 - 2\cos \Theta \sqrt{x(1 + x^2)}}{x^2 - 2\cos \Theta \sqrt{x(1 + x^2)}}$</td>
<td>$\frac{x^2 - 2\cos \Theta \sqrt{x(1 + x^2)}}{x^2 - 2\cos \Theta \sqrt{x(1 + x^2)}}$</td>
</tr>
<tr>
<td>$(P, 5)$ (A.5)</td>
<td>$GT$</td>
<td>$\mp \lambda$</td>
<td>$\mp \lambda$</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Mixed</td>
<td>$\mp (\frac{P}{E})$</td>
<td>$\mp (\frac{P}{E})$</td>
</tr>
<tr>
<td>$(\vec{3}, [P, 3])$ (A.6)</td>
<td>$D$ ($\lambda = 0$)</td>
<td>$0$</td>
<td>$\frac{\sin \Theta}{\sqrt{x^2 - 1 + x^2}}$</td>
</tr>
<tr>
<td>$(\vec{3}, [P, 3])$ (A.7)</td>
<td>$L$ ($\lambda = 0$)</td>
<td>$0$</td>
<td>$\frac{2G}{(1 + x^2)(1 + x^2)}$</td>
</tr>
<tr>
<td>$(\vec{3}, [P, 3])$ (A.8)</td>
<td>$R$ ($\lambda = 0$)</td>
<td>$0$</td>
<td>$\frac{2G}{(1 + x^2)(1 + x^2)}$</td>
</tr>
</tbody>
</table>

**Assumption:**

1. $C_A \sim C'_A \sim 0$
2. $V_{E_3}$ term ($b$) = 0
3. $\alpha \beta \ll 1$

**Definition:**

- $C_j = g_j e^{i\phi_j}$, $C'_j = g'_j e^{i\phi'_j}$, $G = (g_3 / g_3)$
- $X^2 = \frac{2G_{T \lambda} / M_{W,T}^2}{\frac{G_{T \lambda}^2}{M_{W,T}^2}}$
- $G_T = \phi_3 - \phi_{T_3}$
- $G_T = \phi_3 - \phi_{T_3}$

**Upper limits:**

- $\beta^-$ decay
<table>
<thead>
<tr>
<th>Symbol/Operation</th>
<th>Expression</th>
<th>Experimental Results</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>$\lambda_{J J'}$</td>
<td>$C_6^{10} (\beta)$: 1</td>
<td>$g^2 \rightarrow h'$</td>
</tr>
<tr>
<td>$\max$</td>
<td>$\frac{x^2}{(J+1)(J+2)}$</td>
<td>$C_6^{10} (\beta)$: $\frac{1}{3}$ ( $J=2$)</td>
<td>$6 \leq x^2 \leq 6$</td>
</tr>
<tr>
<td>$\max$</td>
<td>$\pm \frac{(p-E)}{E}$</td>
<td>$\lambda_{E}^{26} (\beta)$: 0</td>
<td>$x^2: 0 \text{ (L.B.)}$</td>
</tr>
<tr>
<td>$\gamma (F, \delta)$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$\frac{2Gx \sin \theta \sqrt{J+1}}{\sqrt{1+G^2 (1+X^2)^2 J+1}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{\mathrm{FR}}$</td>
<td>$bad$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \lambda = \frac{(-1 + G^2) + \frac{1}{3} (1 + G^2) X^2}{(1 + G^2) (1 + X^2)} \]

\[ X^2 = \frac{\frac{g_{2F}}{g_s} |M_{2F}|^2}{\frac{g_{2F}}{g_s} |M_{2F}|^2} \]

\[ G = \frac{g_{2V}}{g_s} \]

\( G = 0 \)

\( G = 1 \)

\( G = \sqrt{3} \)

\( G = \infty \)

\( q_v < q_s \)

\( q_v > q_s \)

Neutron → Masson → Good → Robson