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Angular Dependence of Optical Properties of Homogeneous Glasses

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ANGULAR DEPENDENCE OF OPTICAL PROPERTIES OF HOMOGENEOUS GLASSES

R.A. Furler, Ph.D.

ABSTRACT

This paper presents an algorithm to determine the angular dependence of the transmittance and reflectance of homogeneous glazing layers given the reflectance and transmittance at normal incidence, the wavelength, and the thickness of the layer. In the first subsection, the author addresses the development of an algorithm to calculate the reflectance and transmittance of a single homogeneous glazing layer as a function of the angle of incidence (including multiple reflections). Next, the complex refractive index used to approximate the angular properties is determined. The final section describes how the procedure outlined in the following section can be used to approximate angular solar and visible transmittance and reflectance, and the accuracy of the approximated data is evaluated.

INTRODUCTION

The purpose of this paper is to provide an algorithm to approximate the angular dependence of the transmittance and reflectance of homogeneous glazing layers. The only data required are the visible or solar transmittance and reflectance at normal incidence and the layer thickness. These two data sets are the most commonly used indices supplied by product manufacturers. The algorithm can be used to increase the accuracy of computer programs that calculate energy fluxes through glazing(s) based on the glazing thickness and optical properties at normal incidence.

The angular dependence of 3-mm clear glass is often given in percent mean the absolute error of transmittance or reflectance, if not noted otherwise. Even worse, this paper should be received at ASHRAE no later than July 3, 1991.
At normal incidence, Equation 1 reduces to
\[ r_o = \frac{(n-1)^2}{(n+1)^2}. \] (3)

The transmissivity, \( t \), is given by
\[ t(\theta) = 1 - r(\theta). \] (4)

The transmittance, \( T(\theta) \), and reflectance, \( R(\theta) \), counting multiple reflections, are obtained from:
\[ T(\theta) = \frac{t(\theta)^2 e^{-2kd\cos \theta}}{1 - r(\theta)^2 e^{-2kd\cos \theta}}, \] (5a)
\[ R(\theta) = r(\theta) (1 + e^{-kd\cos \theta} \cdot T(\theta)), \] (5b)

where \( d \) is the thickness of the plate and \( k \) is the imaginary part of the refractive index of medium 2, known as the extinction index, and the absorption coefficient \( \alpha = 4 \pi k/\lambda \). Equations 1 through 5 determine, for unpolarized light, transmittance \( T \) and reflectance \( R \) of a homogeneous glass as a function of the angle of incidence \( \theta \) for known thickness \( d \), wavelength \( \lambda \), and refractive index \( n \).

Figure 2 graphs reflectance and transmittance vs. the angle of incidence \( \theta \) for \( d = 4 \text{ mm} \), \( \lambda = 600 \text{ nm} \), \( n = 1.5 \), and \( k = 5 \cdot 10^{-8} \). This value of the real part of the refractive index is a good average for silica glasses in the wavelength range from the UV through the NIR. The extinction index chosen represents medium-absorbing glass (i.e., bronze or green). The reflectance and transmittance remain almost constant from normal incidence \( (\theta = 0^\circ) \) up to about \( 40^\circ \). For grazing incidence \( (\theta = 90^\circ) \), the reflectance is unity and the transmittance equals zero. At this angle, both are independent of \( n \) and polarization.

Note that the focus of this paper is to develop a procedure for characterizing the solar optical properties of glazings at specific angles of incidence, \( \theta \). Often, hemispherical properties are also of interest. They can be easily calculated from the angular-specific data and the algorithm presented in this paper using the following equations:
\[ R_h = \int_0^{\pi/2} R(\theta) \cos \theta \sin \theta \, d\theta \] (6a)
\[ T_h = \int_0^{\pi/2} T(\theta) \cos \theta \sin \theta \, d\theta. \] (6b)

Comparing the Angular Reflectance and Transmittance of Different Glazings

The angular optical properties of uncoated glazings were observed to behave in similar patterns; transmittances and reflectances are roughly constant for angles less than \( 40^\circ \), with transmittances dropping to 0 and reflectances increasing to 1.0 at \( 90^\circ \). This observation raises the question as to whether or not a single function can be used to represent the angular dependence of transmittance and reflectance for different values of \( n \) and \( k \) (different glazing materials).

Given the functions \( T(\theta) \) and \( R(\theta) \), we can derive pairs of "normalized" functions--\( T_n(\theta) \) for the transmittance and \( R_n(\theta) \) for the reflectance--which permit a comparison of the angular dependence of reflectance and transmittance regardless of \( T(\theta = 0) \) and \( R(\theta = 0) \).

\[ T_n(\theta) = \frac{T(\theta)}{T(\theta = 0)} \] (7)
The graphs in Figures 3 and 4 show the functions $T_s(\theta)$ and $R_s(\theta)$ for different values of the extinction index ($k = 10^{-4}$, $10^{-5}$, $10^{-6}$, $10^{-7}$, and $10^{-8}$; $n = 1.5$; $d = 4$ mm; $\lambda = 600$ nm). For weak absorption ($k \leq 10^{-8}$), we observe only a small dependence of $T_s$ and $R_s$ on $k$, where absorption is strong ($k \geq 10^{-8}$), $T_s$ and $R_s$ are more sensitive to $k$. Furthermore, note that $k$ depends very much on $\lambda$. However, the dependence on $n$ (in the range $1.4 \leq n \leq 1.6$) and the direct wavelength $\lambda$ (within 0.3 $\mu$m to 4.6 $\mu$m) is small.

This dependence of $T_s$ and $R_s$ on $k$ indicates that approximations based on only two "universal" functions, $T_s(\theta)$ and $R_s(\theta)$, for any type of glass might show significant errors, especially for tinted glass exhibiting high values of the extinction index, $k$. This is demonstrated in Figure 5, which presents the normalized function $T_s$ for the solar transmittance of typical 3-mm clear and 6-mm bronze single glazing and a double glazing consisting of typical 6-mm clear and 6-mm bronze. The normalized transmittance of 3-mm clear glass is often used to approximate the angular transmittance for all glass types and thicknesses. Figure 5 shows this will result in significant errors. Compared to 3-mm clear glass, the other two samples shown in Figure 5 exhibit errors up to 11% and 28% at 70°, with 3% and 4.5% errors at 35°. This can result in very inaccurate calculations of the energy performance of windows, given that most of the time during the day the sun illuminates a window at angles of incidence greater than 45°.

We expect the most exact approximation if we use the specific pair $T_s(\theta)$ or $T(\theta)$ and $R_s(\theta)$ or $R(\theta)$ corresponding to the pair $n$ and $k$ for which $T(\theta) = T_s$ and $R(\theta) = R_s$ (Equations 1 through 5, $\theta = 0$). As this would result in an infinite number of "universal" functions, we turn to the development of an algorithm that allows $n$ and $k$ to be determined from $R_s$ and $T_s$, the reflectance and transmittance at normal incidence of a single glass pane known from measurements. In the following section, this algorithm for homogeneous glasses is described.
Using the Reflectance and Transmittance to Determine the Refractive Index

We can invert the procedure presented earlier and use the reflectance and transmittance at normal incidence for a given wavelength to determine the refractive index. This is easily done. The real part of the refractive index \( n \) is derived by inverting Equation 3

\[
\frac{n}{1 - n^2} = \frac{1 + \sqrt{R_0}}{1 - \sqrt{R_0}}. \tag{9}
\]

We use the relationship between \( T(\theta) \), \( R(\theta) \), and \( k \) given in Equation 5b to determine the extinction index, \( k \), considering normal incidence (\( \theta = 0 \)) with \( R(0) = R_0 \) and \( T(0) = T_0 \)

\[
k = -\frac{\lambda}{4\pi d} \ln \left( \frac{R_0 - R}{R_0 T_0} \right). \tag{10}
\]

We derive the reflectivity at normal incidence, \( r_0 \), from Equation 5a using Equation 5b to eliminate the exponential factor in Equation 5a:

\[
r_0 = \frac{\beta - \sqrt{\beta^2 - 4(2 - R_o)R_o}}{2(2 - R_o)} \tag{11}
\]

where

\[
\beta = \beta_0^2 - R_o^2 + 2R_o + 1. \tag{12}
\]

Since the reflectance, \( R_o \), and the transmittance, \( T_o \), at normal incidence (\( \theta = 0 \)) are often known for a designated glazing layer (i.e., given thickness), the above procedure can be applied to determine the refractive indices. The refractive indices can then be used to calculate the angular properties (see subsection titled "Using the Refractive Index to Determine Reflectance and Transmittance"). This procedure was used to approximate the angular solar transmittance for 6-mm bronze single glazing. Figure 6 shows a comparison between this approximation and the accurate data. For all angles, the error is less than 0.3 %. Compared to the accuracy of measured data of transmittance and reflectance, the error of this approximation is negligible. Since measured data are not absolutely accurate, we require that \( 0 < R_o + T_o < 1 \) when using the above algorithm; otherwise Equations 10 through 12 will result in nonphysical values for \( k \). Even when \( R_o \) and \( T_o \) satisfy this requirement, the measurement error can cause unreasonable results of \( k \). In both cases, we obtain less accurate approximations of the angular dependence of the transmittance and reflectance (see subsection entitled "Comparing the Angular Reflectance and Transmittance of Different Glazings").

AN APPROXIMATION OF THE ANGULAR DEPENDENCE OF VISIBLE AND SOLAR TRANSMITTANCE AND REFLECTANCE

In the previous section, we derived relations between the optical properties of a single homogeneous glazing layer for a given wavelength. Very often the data available refer to visible or solar radiation at normal incidence. In this section, we adapt the procedure described in the previous section to approximate the angular dependence of these wavelength-integrated properties and define the accuracy of this approximation.

Usually, for a given designation and thickness of glazing, the solar and visible transmittance and reflectance at normal incidence are given by manufacturers in order to describe a glazing performance. These indices have been derived by weighting the spectra for normal incidence with a standard terrestrial solar spectrum (Mecherikunnel 1978) or the sensitivity of a standard human eye (IES 1984), respectively. Thus, they reflect a variety of different values of the wavelength \( \lambda \) and \( k \). Note that \( k \) is needed explicitly in Equation 5. It was found that the center wavelengths (area-weighted, not peak average) of the visible (575 nm) and solar spectrum (898 nm) produce effective extinction indices \( k \), which will yield reasonable approximations. With these "effective" wavelengths and the normal solar and visible transmittances and reflectances, we can use Equations 9 through 11 to calculate "effective solar" and "effective visible" \( n \) and \( k \) values. These, in turn, can be used with the "effective" wavelengths to calculate \( T(\theta) \) and \( R(\theta) \) from Equations 1 through 5, where \( T(\theta) \) and \( R(\theta) \) are approximations of the angular dependence. This procedure is coded in Fortran and is given in the appendix.

To determine the accuracy of our approximations for the visible and solar optical properties of homogeneous glass panes, we first used values of \( n \) and \( k \) as a function of the wavelength, \( \lambda \) (ranging from .31 \( \mu \)m to 4.6 \( \mu \)m), given by Rubin (1985). For fixed angles of incidence, \( \theta \) (ranging from 0° to 90°), and glass thickness, \( d \) (ranging from 3 mm to 10 mm), we then calculated the spectral reflectance and transmittance for five glass types (clear, bronze, green, grey, low-iron) using Equations 5a and 5b. From these spectra, we determined the wavelength-integrated visible
and solar reflectance and transmittance. The data corresponding to normal incidence (0°) were then used to calculate the approximations following the procedure described above (Equations 9 through 12, subsequently Equations 1 through 5).

Comparison of the accurate with the approximated data shows errors of less than 0.2% for the visible and less than 1.5% for the solar properties for all the glass types investigated. If we exclude the “green” glass type, which exhibits the most extreme variations in absorption by wavelength, and consequently in k, the error is even less than 1% for the solar properties. These errors did not vary significantly with thickness in the range between 3 mm and 10 mm but with the angle of incidence, increasing from zero at normal incidence and reaching the maximum at angles between 60° and 85°. For angles between 0° and 35° the error is less than 0.5%.

Furthermore, we evaluated an approximation of the angular dependence of the transmittance and reflectance using one pair of the functions T(θ) and R(θ) corresponding to a single value of n and k for all five glass types mentioned above and for thicknesses between 3 mm and 10 mm. This is a simple extension of the use of a polynomial approximation. For the visible data, the best choice for n and k we could find (n = 1.525, k = 1.2·10^-4) results in errors up to ±5.5%. The best choice of n and k for the solar data (n = 1.521, k = 2.2·10^-4) exhibits errors up to ±6.5%. The effective n and k derived from the optical properties of 3-mm clear single glazing do not correspond to this best choice. Therefore, the errors found previously are even higher, as we used the normalized functions of 3-mm clear single glazing as universal functions.

Finally, we investigated the influence of an inaccurate value of the thickness d on the approximation. Usually, a glass thickness declared as 4 mm may vary by a maximum of ±0.5 mm. If the data supplied for the visible and solar properties correspond to a 3.5-mm-thick sample, but we calculate our approximation based on a thickness of 4 mm, the deviation from the accurate data corresponding to a 3.5-mm sample shows no significant difference compared to the errors mentioned above.

CONCLUSIONS

Existing algorithms can grossly misrepresent the angular solar and visible transmittances and reflectances of common glazing materials. These errors, on the order of 3% to 30%, can lead to proportionately large errors when used in common computer codes that calculate the energy impacts of windows in buildings.

In this paper we present the theory behind, as well as the equations and computer code for, a new algorithm that approximates the angular solar or visible transmittances and reflectances of any uncoated glazing material. The only necessary inputs are the solar or visible properties at normal incidence and the glazing layer thickness, all commonly available properties. The theoretical error for this approximation is less than 1.5%. We are in the process of developing an experimental device to both validate this new algorithm and to allow us to develop similar algorithms for coated glazings.

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REFERENCES


APPENDIX

SUBROUTINE RT_th_approx(Rtot0,Ttot0,dmm,th,wlnm, Rtot_th,Ttot_th)

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INPUT

- total reflectance \( R_{tot0} \) at (angle of incidence) \( \theta=0 \)
- total transmittance \( T_{tot0} \) at (angle of incidence) \( \theta=0 \)
- thickness \( d_{mm} \) [mm]
- angle of incidence \( \theta \) [deg]
- wavelength \( w_{lnm} \) [nm]

\( R_{tot0} \) and \( T_{tot0} \) can correspond to one single wavelength or represent wavelength integrated reflectance and transmittance. In the latter case use adequate center-wavelength: 
- 575.0nm for VISIBLE
- 897.7nm for SOLAR

OUTPUT

- \( R_{tot\_th} \) total Reflectance and Transmittance of a homogeneous (uncoated) glass plate at the angle \( \theta \).
- \( T_{tot\_th} \) (uncoated) glass plate at the angle \( \theta \).

DESCRIPTION OF CALCULATION

1. calculate reflectivity \( R \) and transmissivity \( T \) at normal incidence
2. determine refractive index \( n, k \) assuming weak absorption \( k<<n \)
3. calculate \( R \) and \( T \) at new angle of incidence \( \theta \) for both polarizations
4. calculate total reflectance \( R_{tot\_th} \) and total transmittance \( T_{tot\_th} \) of a thick uncoated glass plate for both polarizations as a function of:
   - angle of incidence theta \( \theta \)
   - thickness \( d_{mm} \) in mm
   - refractive index \( n, k \)
   - wavelength \( w_{lnm} \) in nm

All reflectance and transmittance data must be in decimal form in every subroutine used!

```
real dmm,n,k,wlnm,Rtot0,Ttot0,Rtot_th,Ttot_th,th
save n,k

call NKfromRT(Rtot0,Ttot0,dmm,wlnm, n,k)

if (k .eq. 1000) then
  print*,"Ttot0 Laf",Rtot0, Rtot0='Rtot0'
  print*,Ttot0 (maybe Rtot0) give unreasonable results for k'
  print*,Ttot0 is probably to close to zero'
  print*,enter better Ttot0 and Rtot0'
  RETURN
end if

call RTapprox_th(th,dmm,wlnm,n,k, Rtot_th,Ttot_th)
RETURN
END
```
SUBROUTINE NKfromRT(Rtot,Ttot,dmm,wlnm, n,k)

This program calculates the complex refractive index (n,k) from the known (measured) total transmittance Ttot and reflectance Rtot at normal incidence of a homogeneous glass pane. The first step determines the reflectivity R and transmissivity T of one air-glass interface. R and T are then used to determine n and k.

No interference assumed.

INPUT : Rtot, Ttot, dmm, wlnm
OUTPUT : n, k

wavelength wlnm in nm
thickness dmm in mm

real n,k,num1,num2

pi=2.*asin(1.)
d=dmm/1000

if (Ttot .lt. 0) Ttot=0
if (Rtot+Ttot .gt. 1) then
Rmax=max(Rtot,Ttot)
print*, 'Rtot or Ttot have changed from :',Rtot,Ttot
if (Rmax .eq. Rtot) Rtot=1-Ttot
if (Rmax .eq. Ttot) Ttot=1-Rtot
print*, ' to :',Rtot,Ttot
end if

if (Ttot .gt. 0) then
num1 = Ttot**2-Rtot**2+2*Rtot+1
num2 = sqrt(num1**2-4*(2-Rtot)*Rtot)
deno = 2*(2-Rtot)
Rmin = (num1-num2)/deno
Rplus = (num1+num2)/deno
Rplus >= Rtot ! This is unphysical, therefore Rplus is no solution.
else
Rmin=Rtot
end if

if (Rmin .lt. 0) then
print*, 'unphysical result for Rtot, Ttot, wlnm=',Rtot,Ttot,wlnm
elseif (Rmin .le. Rtot) then
R=Rmin
T=1-R
if (Ttot .ne. 0) then
n=(1+sqrt(R))/(1-sqrt(R))
a=(Rtot-R)/R/Ttot
alpha=-log(a)/2/d
k=alpha/2/pi*wlnm/1e9
end if
if ((Ttot .eq. 0) .or. (k .gt. 1e-2)) k=1000
end if

if (k .gt. 1e-4) print*, 'Warning. Out of range of Approximation'

RETURN
END
SUBROUTINE RTapprox_th(th_deg,dmm,wlnm,n,k, Rtot_th,Ttot_th)
  This program calculates the total reflectance Rtot_th and the total
  transmittance Ttot_th of a thick (no interference) homogeneous glass
  plate at the angle-of incidence th_deg for unpolarized light.
  INPUT : th_deg, dmm, wlnm, n, k
  OUTPUT : Rtot_th, Ttot_th
  thickness dmm in mm
  wavelength wlnm in nm
  angle of incidence th_deg in deg
  real dmm,wlnm,n,k,th_deg

  pi=2.*asin(1.)
  wl=wlnm/1.e9
  th=th_deg/180*pi

  call Rtheta(n,th, Rp,Rs,Tp,Ts)
  call RTtot_homog(Rp,Tp,n,k,th,dmm/1000, wl, Rtotp_th,Ttotp_th)
  call RTtot_homog(Rs,Ts,n,k,th,dmm/1000, wl, Rtots_th,Ttots_th)
  Rtot_th=(Rtots_th+Rtotp_th)/2
  Ttot_th=(Ttots_th+Ttotp_th)/2

  RETURN
END

SUBROUTINE Rtheta(n,th, Rip,Ris,Tip,Tis)
  This subroutine calculates the reflectivity Rip,Ris and transmissivity
  Tis,Tip for both TE ('s') and TM ('p') polarization of a single
  air-glass-interface of a homogeneous glass for a given refractive
  index (n,k) in function of the angle of incidence th.
  INPUT : n, th
  OUTPUT : Rip, Ris, Tip, Tis
  thickness dmm in mm
  wavelength wlnm in nm
  angle of incidence th_deg in deg
  real n

  Snell's law (for negligible absorptance resp. extinction index) :
  ph=asin(1/n*sin(th))

  Reflectance- and Transmittance-Intensities (->Fresnel's eq.):
  Ris=((cos(th)-n*cos(ph))/(cos(th)+n*cos(ph)))**2
  Rip=((n*cos(th)-cos(ph))/(n*cos(th)+cos(ph)))**2
  Tis=1-Ris
  Tip=1-Rip

  RETURN
END
SUBROUTINE RTtot_homoq(R,T,n,k,th,d,wl, Rtot,Ttot)

   c this subroutine calculates the total Reflectance Rtot and Transmittance
   c Ttot of a homogeneous glass pane in function of the angle of
   c incidence th. No interference assumed. R and T are the reflectivity
   c and transmissivity, respectively, at this given angle of incidence th.
   c
   c INPUT : R, T, n, k, th, d, wl
   c OUTPUT : Rtot, Ttot
   c
   c thickness          dmm   in m
   c wavelength         wlnm in m
   c angle of incidence  th   in rad

   real n,k

   pi=2.*asin(1.)

   c Snell’s law (for negligible absorptance resp. extinction index) :
   c
   ph=asin(1/n*sin(th))

   c extenuation within the glass pane for a single path:
   c
   alpha=k*2*pi/wl
   a=exp(-2*alpha*d*cos(ph))

   c Total Reflectance and Transmittance including multiple reflections:
   c
   Ttot=a*T**2/(1-a**2*R**2)
   if (Ttot .le. 0.) Ttot=0.
   Rtot=(1+a*Ttot)*R

   RETURN
   END