Title
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EFFICIENCY OF DETECTING A $^8$Be WITH A $\Delta E-E$ COUNTER TELESCOPE

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ABSTRACT

A formalism is derived for calculating the probability $\varepsilon$ of detecting a $^8$Be event with a single $\Delta E-E$ counter telescope. Both the cases of a solid and of a gas target are treated. To evaluate $\varepsilon$ a FORTRAN program is presented. In addition, a simple formula for $\varepsilon$ is derived which holds in the limit of "small" geometric solid angles.

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I. INTRODUCTION

As described in Ref. 1, the two alpha-particles from the decay of a $^8\text{Be}$ in flight are identified as a $^7\text{Li}$ if they simultaneously traverse a counter telescope. If in addition the Q-value of the reaction leading to an outgoing $^8\text{Be}$ is more positive than the one for an outgoing $^7\text{Li}$, $^8\text{Be}$'s can be uniquely identified over a limited range of excitation by their total energy and particle identification. For example the alpha-particle transfer reaction $(^{12}\text{C},^8\text{Be})$ may be studied by this method. Figure 1 illustrates the differences between the Q-values of the $(^{12}\text{C},^8\text{Be})$ and $(^{12}\text{C},^7\text{Li})$ reactions on light nuclei.

The probability of detecting a $^8\text{Be}$ event is less than 1 because the "effective solid angle" for detection of the two breakup alphas is always smaller than the geometric solid angle of the counter telescope. This can be seen by observing that for a $^8\text{Be}$ whose center-of-mass (c.m.) velocity falls within the geometric solid angle, there is usually an appreciable probability that one or both of its breakup alpha-particles will fall outside the counter telescope, causing the event to be lost. In the present report we derive the necessary formalism and present a computer program that calculates the "effective solid angle" for a detector telescope with rectangular collimators. The problem is defined in Section II, and in Section III a very simple formula for the "effective solid angle" is derived using an approximation which is valid in the limit of "small" geometric solid angles. Section IV gives a discussion for the case of "large" solid angles while an evaluation of the resulting formula requires the computer program presented in Section V. Thus far all the results apply only in the case of a solid target whereas in Section VI the case of a gas target is treated. In Appendix B, an error in Ref. 1 is corrected: the absolute cross sections quoted there are based on incorrectly calculated detection efficiencies.
II. DEFINITIONS

The decay of the $^8$Be ground state into two alpha-particles is isotropic in the $^8$Be rest system. As illustrated in Fig. 2, $\vec{v}_0$ is the velocity of the $^8$Be in the laboratory (lab) system, while $\vec{c}$ is the velocity of one of the two alpha-particles in the $^8$Be rest system. Introducing the auxiliary quantities

$$\vec{v}_1 = \vec{v}_0 + \vec{c},$$

$$\vec{v}_2 = \vec{v}_0 - \vec{c},$$

and the angles $\alpha, \beta_1, \beta_2,$ and $\gamma$ which are defined in Fig. 2, we see from the laws of sines and cosines that the following relationships hold:

$$\sin \beta_1 = (c/v_1) \sin \gamma,$$

and

$$v_1^2 = v_0^2 + c^2 - 2v_0c \cos \gamma,$$

and

$$v_2^2 = v_0^2 + c^2 + 2v_0c \cos \gamma,$$

since

$$\alpha = \pi - \gamma.$$

From Fig. 2 it is evident that in $^8$Be decay, the alpha-particle with velocity vector $+\vec{c}$ is emitted into the forward hemisphere with respect to $\vec{v}_0$ while its companion alpha-particle with velocity vector $-\vec{c}$ is emitted into the backward hemisphere. A detector normal to $\vec{v}_0$ will register the forward alpha-particle as arriving a short time before the backward one. Thus the alpha-particle emitted into the forward hemisphere is called the "first" alpha-particle. Since there is a one to one correspondence between "first" alpha-particles and the $^8$Be decays, all $^8$Be events can be completely characterized by considering only the "first" alpha particles.
Throughout this report we assume that

\[ c \ll v_0, \quad (2.7) \]

which results in

\[ v_1 \approx v_2 \approx v_0 \quad (2.8) \]

and consequently

\[ \beta_1 \approx \beta_2. \quad (2.9) \]

Henceforth we shall drop the index on \( \beta \).

In the lab system the alpha-particles from a \(^{8}\text{Be}\) decay are confined to a cone with symmetry-axis \( \vec{v}_0 \) and a half angle \( \beta_{\text{max}} \) given by

\[ \beta_{\text{max}} = \frac{c}{v_0}. \quad (2.10) \]

Let us call \( E_\beta \) the kinetic energy of the \(^{8}\text{Be}\) in the lab system and \( Q \) its breakup Q-value. Then Eq. (2.10) may be rewritten as

\[ \beta_{\text{max}} = [Q/E_\beta]^{1/2}. \quad (2.11) \]

As an example, we note that for \( E_\beta = 40 \text{ MeV} \) and \( Q = 0.092 \text{ MeV} \), \( \beta_{\text{max}} = 2.8^\circ \).

In Fig. 3 we introduce a cartesian coordinate system in the plane of the detector with its origin in the center. If \( \vec{v}_0 \) is directed towards the origin and if one of the breakup alpha-particles hits the detector at the point \((x,y)\), then the other alpha will hit the detector at \((-x,-y)\), due to Eq. (2.9). More generally (see Fig. 4), if \( \vec{v}_0 \) is directed towards the point \((\xi, \eta)\) and one of the alpha-particles hits the detector at \((\xi + x, \eta + y)\), then the other alpha will arrive at the point \((\xi - x, \eta - y)\). Both alpha-particles will be detected if one of them falls inside the rectangle \( R(\xi, \eta) \) delineated with heavy lines. Thus for every point \((\xi, \eta)\) there is a certain solid angle into which the "first" alpha-particle has to be emitted, if both are to be detected. The size of this solid angle—expressed in the c.m. system of the \(^{8}\text{Be}\)—divided by the solid angle...
of $2\pi$ steradians (into which the "first" alpha-particle can be emitted) is the probability $E(\xi,\eta)$ that a $^8$Be-particle with $\vec{v}_0$ directed towards the point $(\xi,\eta)$ will be detected.

The polar coordinates $\alpha$ and $\phi$ describe the c.m. motion of the "first" alpha-particle (see Fig. 3), where $\alpha$ is the polar angle between $\vec{c}$ and $\vec{v}_0$, and $\phi$ is the azimuthal angle between the x-axis and the projection of $\vec{c}$ on the detector plane. A $^8$Be-event with its velocity vector directed at the point $(\xi,\eta)$ has a detection probability $E(\xi,\eta)$ which is given by the expression;

$$E(\xi,\eta) = \frac{1}{2\pi} \int_{R(\xi,\eta)} \sin \alpha \, d\alpha \, d\phi.$$  \hspace{1cm} (2.12)

By folding $E(\xi,\eta)$ into the geometric solid angle, the "effective solid angle" $\Omega_{\text{eff}}$ is obtained

$$\Omega_{\text{eff}} = \frac{1}{D^2} \int_{-B/2}^{B/2} \, d\eta \int_{-A/2}^{A/2} E(\xi,\eta) \, d\xi.$$  \hspace{1cm} (2.13)

The geometric solid angle is given by

$$\Omega = \frac{AB}{D^2},$$  \hspace{1cm} (2.14)

where $D$ is the distance between the detector and the target and $A$ and $B$ are the detector width and height, respectively.

Designating the ratio of $\Omega_{\text{eff}}$ over $\Omega$ as the $^8$Be detection efficiency $\varepsilon$, we have

$$\varepsilon = \frac{1}{AB} \int_{-B/2}^{B/2} \int_{-A/2}^{A/2} E(\xi,\eta) \, d\xi \, d\eta.$$  \hspace{1cm} (2.15)
III. DETECTION EFFICIENCY FOR "SMALL" SOLID ANGLES

To calculate $E(\xi, \eta)$, one has to express the variables $\alpha$ and $\phi$ in terms of $x$ and $y$. From Fig. 3 one can easily verify the equation

$$\sin(\alpha - \beta) = \frac{v_0}{c} \sin \beta$$

(3.1)

or

$$\alpha = \beta + \arcsin \left( \frac{v_0}{c} \sin \beta \right).$$

(3.2)

Because of inequality (2.7) this reduces to

$$\alpha \approx \arcsin \left( \frac{v_0 \beta}{c} \right).$$

(3.3)

Since

$$\beta \approx \sqrt{\frac{x^2 + y^2}{D}},$$

(3.4)

we find the following transformation

$$\alpha = \arcsin \left( \frac{v_0}{c} \sqrt{\frac{x^2 + y^2}{D}} \right).$$

(3.5)

The corresponding relationship between $\phi$ and $(x, y)$ is

$$\phi = \arctan \frac{y}{x}.$$

(3.6)

In Section IV we shall describe the calculation of the detection efficiency $\epsilon$ using transformations (3.5) and (3.6). However, in the present section a simplified version of this calculation is given. By introducing the additional assumption that the angle $\alpha$ is "small enough", Eq. (3.5) can be replaced by

$$\alpha \approx \frac{v_0}{c} \sqrt{\frac{x^2 + y^2}{D}}.$$

(3.7)

If the dimensions of the detector are such that

$$\alpha_{\text{max}} = \arcsin \left( \frac{v_0}{c} \frac{\sqrt{A^2 + B^2}}{2D} \right) \leq 15^\circ,$$

(3.8)
approximation (3.7) holds within 1%. As an example, we note that for the experimental configuration given in Table I, \( \alpha_{\text{max}} = 27^\circ \).

From transformations (3.6) and (3.7) the following differentials are obtained

\[
d\alpha = \frac{v_0}{cD} \frac{1}{\sqrt{x^2 + y^2}} (x \, \partial x + y \, \partial y) ,
\]

\[
d\phi = \frac{1}{2} \frac{1}{x + y} (-y \partial x + x \partial y)
\]

and the Jacobian is

\[
\frac{\partial (\alpha, \phi)}{\partial (x, y)} = \frac{v_0}{cD} \frac{1}{\sqrt{x^2 + y^2}} .
\]

The substitution of \((x, y)\) for \((\alpha, \phi)\) in Eq. (2.12) leads to the trivial integration over \(R(\xi, \eta)\)

\[
E(\xi, \eta) = \frac{1}{2\pi} \left( \frac{v_0}{cD} \right)^2 \int \int_{R(\xi, \eta)} dx dy
\]

\[
= \frac{1}{2\pi} \left( \frac{v_0}{cD} \right)^2 (A-2|\xi|)(B-2|\eta|) .
\]

Introducing Eq. (3.12) into Eq. (2.15) and integrating over \(\xi\) and \(\eta\), one obtains the following expression for the efficiency \(\varepsilon\)

\[
\varepsilon = \frac{1}{8\pi} \left( \frac{v_0}{cD} \right)^2 AB .
\]

With the help of expressions (2.10), (2.11), and (2.14) this may be rewritten as

\[
\varepsilon = \frac{\Omega}{8\pi} \frac{E_0}{Q} .
\]

It is interesting to note at this point that the efficiency calculated using Eq. (3.13) differs only a few percent from the value derived from the more exact expression [Eq. (4.5)] for the example given in Table I.
IV. DETECTION EFFICIENCY FOR "LARGE" SOLID ANGLES

In this section we no longer make use of approximation (3.7). It is then not possible to derive a closed formula for the efficiency and therefore numerical integration is necessary to obtain a value for \( \epsilon \).

From Eqs. (3.5) and (3.6) one derives the following Jacobian

\[
\frac{\partial (a,\phi)}{\partial (x,y)} = \frac{V_0}{cD} (x^2+y^2)^{-1/2} \left[ 1 - \left( \frac{V_0}{cD} \right)^2 (x^2+y^2) \right]^{-1/2} .
\]  

(4.1)

A transformation of variables in Eq. (2.12) yields

\[
E(\xi,\eta) = \frac{1}{2\pi} \left( \frac{V_0}{cD} \right)^2 \int_{-B/2+|\eta|}^{A/2-|\eta|} \int_{-B/2+|\xi|}^{A/2-|\xi|} dy \int_{-B/2+|\eta|}^{A/2-|\xi|} dx \left[ 1 - \left( \frac{V_0}{cD} \right)^2 (x^2+y^2) \right]^{-1/2} .
\]  

(4.2)

Integration over the variable \( y \) gives (see Ref. 2, p. 257, #19)

\[
E(\xi,\eta) = \frac{2}{\pi} \frac{V_0}{cD} \int_0^{A/2-|\xi|} dx \arcsin \left[ \frac{B/2 - |\eta|}{\left( \frac{cD}{V_0} \right)^2 - x^2} \right]^{1/2} .
\]  

(4.3)

To simplify the above integral, we have used the symmetry of Eq. (4.2) with respect to reflection of the \( x- \) and \( y- \) axes.

Introducing Eq. (4.3) into Eq. (2.15) yields

\[
\epsilon = \frac{4}{AB} \frac{V_0}{cD} \int_0^{A/2} \frac{1}{\sqrt{x}} \int_0^{B/2} dx \arcsin \left[ \frac{B/2 - \eta}{\left( \frac{cD}{V_0} \right)^2 - x^2} \right] .
\]  

(4.4)

where we have made use of the symmetry of the problem under reflection of the \( \xi- \) and \( \eta- \) axes. Integration over the variable \( \eta \) yields (see Ref. 2, p. 277. # 308)
\[ \epsilon = \frac{2}{\pi} \frac{v_0}{ABcD} \int_0^{A/2} \int_0^{A/2-\xi} \left\{ \frac{B}{2} \arcsin \left( \frac{B/2}{(cD/v_0)^2 - x^2} \right)^{1/2} \\
+ \left( \left( \frac{cD}{v_0} \right)^2 - x^2 - \frac{B^2}{4} \right)^{1/2} - \left( \left( \frac{cD}{v_0} \right)^2 - x^2 \right)^{1/2} \right\} \right\} \]

Further integrations over the first term of the integrand have to be done numerically. It is therefore convenient to perform the integrations of the second and third terms numerically too, although they can be done analytically. In Section V we describe the computer program EFFI which evaluates Eq. (4.5) by numerical integration.

As indicated by the example given in Table I, the \(^8\)Be detection efficiencies for the experimental setup of Ref. 1 are of the order of \(10^{-2}\). The \(^8\)Be detection efficiencies and absolute cross sections quoted in Ref. 1 are in error by approximately a factor of four. Appendix B of this present report gives corrected values for the absolute cross sections determined in the above experiment.

V. THE COMPUTER PROGRAM EFFI

To calculate the detection efficiency \(\epsilon\) plus several related quantities, we used the FORTRAN program EFFI. A program listing is given in Appendix A along with an example of its output.

For each \(^8\)Be laboratory energy, one data input card is required. The first five parameters must be in floating point format (F10.3):

- \(ELAB\), the lab energy of the \(^8\)Be, in MeV (columns 1-10),
- \(Q\), the energy released by the breakup of the \(^8\)Be, in MeV (columns 11-20),
- \(AR\), the horizontal width of the rectangular collimator, in mm (columns 21-30)
AV, the vertical width of the rectangular collimator, in mm (columns 31-40), and

D, the distance between target and collimator, in mm (columns 41-50).

A sixth parameter, an integer K (column 60), indicates whether or not there are more cases to be evaluated. If \( K \leq 0 \), the program will go to END; if \( K > 0 \), the next case will be calculated.

The program's output contains a list of the input parameters plus calculated values for the following quantities:

\( \Omega \), the solid angle in sr according to Eq. (2.14),

\( \varepsilon \), the efficiency calculated from Eq. (4.5) and also from the simplified expression Eq. (3.13),

\( \Omega_{\text{eff}} \), the effective solid angle evaluated in the framework of Eq. (4.5),

\( d\Omega \), the angular resolution defined by the horizontal width of the collimator, and

\( \text{FWHM}(\Delta) \), the full width at half maximum of the differential efficiency (\( \Delta(\xi) \)).

This function is defined by the expression

\[
\Delta(\xi) = \frac{1}{AB} \int_{-B/2}^{B/2} E(\xi, \eta) d\eta
\]

(5.1)

and the quantity \( \Delta(\xi)d\xi \) can be interpreted as being the probability that a \(^8\)Be, with its velocity vector \( \vec{v}_0 \) directed into the interval \((\xi, \xi + d\xi)\), will be detected. The \( \xi \)-axis is the horizontal axis. This differential efficiency is at its maximum for a \(^8\)Be directed toward the centerline of the detector \((\xi = 0)\) and decreases to zero at the edges of the detector \((\xi = \pm A/2)\). Hence, the
kinematic spread is not proportional to dθ, but to the FWHM of the distribution Δ(ξ). A table of Δ(ξ) for ξ between 0 and A/2 completes the output of EFFI.

The precision with which ė is calculated should be better than the quantity ERR which is set to 10^{-3} in the main program (11th statement card). This has been verified by checking the dependence of the results of ERR.

Figure 5 gives ė as a function of E_0 for the experimental setup of Ref. 1 (see also Table I). On Fig. 6 an example of the differential efficiency Δ(ξ) is graphed. These curves are practically linear, which reflects the fact that for Ref. 1 Eq. (3.13) is a good approximation.

VI. EFFICIENCY OF DETECTING A ^8Be EMITTED FROM A GAS TARGET

For the case of a gas target, the previously developed formalism must be modified. Figure 7 shows schematically the experimental setup where now two collimators define the solid angle and target thickness. The front collimator is at a distance L_1 from the center of the gas cell and has a width A; its height is supposed to be such that it does not limit the solid angle. The rear collimator is at a distance

D = L_1 + L_2

and has width A and height B. For a laboratory scattering angle θ, the quantities t_0, t_1, x, δ, and τ are defined in Fig. 7.

We define the effective solid angle as

\[ \Omega_{\text{eff}} = \frac{1}{\tau} \int_{-t_1}^{t_1} \varepsilon(t) \Omega(t) dt , \]
where the parameter \( t \) is the coordinate of \( T \), the point where the reaction occurs, see Fig. 7. A particle emerging from point \( T \) sees a solid angle \( \Omega(t) \) which can be expressed as

\[
\begin{align*}
\Omega(t) &= \begin{cases} 
\frac{AB}{D^2} & \text{for } 0 \leq |t| \leq t_0 \\
\frac{x(t)B}{D^2} & \text{for } t_0 \leq |t| \leq t_1
\end{cases} 
\end{align*}
\] (6.3)

For the definition of \( x(t) \), see Fig. 7. In formulating expression (6.3) we assume that the difference between \( D \) and the distance \( XT \) may be neglected, i.e.

\[
D \gg t_1 \cos \theta 
\] (6.4)

and

\[
A,a \ll L_2 
\] (6.5)

Since \( \Omega \) is a function of \( t \), \( \epsilon \) is also. The quantity \( \tau \) defined in Fig. 7 is the usual "gas target thickness"; it may—with the assumptions (6.4) and (6.5)—be expressed as

\[
\tau = t_0 + t_1 
\] (6.6)

Normalization of \( \Omega_{\text{eff}} \) in Eq. (6.2) is chosen such that \( \Omega_{\text{eff}} \) becomes equal to \( \frac{AB}{D^2} \), if \( \epsilon \) is unity everywhere. To verify this one makes use of the relationships

\[
x(t) = \delta L_2 
\] (6.7)

and

\[
\delta = \frac{(t_1 - |t|) \sin \theta}{L_1 - t_1 \cos \theta} 
\] (6.8)
and hence

$$\Omega(t) = \begin{cases} \frac{AB/D^2}{BL_2(t - |t|)\sin\theta} & \text{for } 0 \leq |t| \leq t_0 \\
\frac{B}{D^2(L_1 - t_1\cos\theta)} & \text{for } t_0 \leq |t| \leq t_1 \end{cases} \quad (6.9)$$

To calculate $\Omega_{\text{eff}}$, one has to make an ansatz for the variation of $\varepsilon$ with $t$. Under the condition that expression (3.13) is valid, the detection efficiency is simply proportional to the solid angle. This will be used throughout the following and leads to the result

$$\varepsilon(t) = \frac{\varepsilon(0)\Omega(t)}{\Omega(0)} \quad (6.10)$$

where

$$\varepsilon(0) = \frac{1}{8\pi} (v_0/c)^2 \Omega(0) \quad (6.11)$$

EFFI may be used to check the ansatz (6.10). It is of course possible to introduce a more complicated dependence of $\varepsilon$ on $\Omega$ and to extend the following equations accordingly.

Insertion of Eqs. (6.6), (6.9), and (6.10) into Eq. (6.2), yields the following expression for the effective solid angle

$$\Omega_{\text{eff}} = \frac{2\varepsilon(0)}{D^2} \frac{B}{t_0 + t_1} \left\{ A t_0 + \frac{L_2^2(t_1 - t_0)^3 \sin^2 \theta}{3A(L_1 - t_1 \cos \theta)^2} \right\} \quad (6.12)$$

To evaluate $\Omega_{\text{eff}}$, one has to express $t_0$ and $t_1$ in terms of $A, \overline{A}, L_1, L_2$, and $\theta$. This is straightforward and the results are

$$t_0 = \frac{-AL_1 + \overline{AD}}{2L_2 \sin \theta - (A - \overline{A}) \cos \theta} \quad (6.13)$$
and

\[ t_1 = \frac{AL_1 + AD}{2L_2 \sin \theta + (A+A) \cos \theta} \]  \hspace{1cm} (6.14)

As an example, the parameters of the gas target experiment described in Ref. 1 are summarized in Table I.
REFERENCES


Table I. Summary of the parameters relating to the experimental setup described in Ref. 1, along with calculated values for the detection efficiency and the "effective solid angle" for a 40 MeV ^8Be event. See text for definitions of variables.

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<th>Value</th>
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</tr>
<tr>
<td>B</td>
<td>5.08 mm</td>
</tr>
<tr>
<td>C</td>
<td>120.6 mm</td>
</tr>
<tr>
<td>c/ν₀</td>
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<tr>
<td>α_max</td>
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</tr>
<tr>
<td>Ω</td>
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</tr>
<tr>
<td>Ω_eff</td>
<td>6.8 x 10⁻⁶ sr</td>
</tr>
<tr>
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</tr>
<tr>
<td>ε(Eq. (3.13))⁺</td>
<td>1.08 x 10⁻²</td>
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</tbody>
</table>

<table>
<thead>
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</tr>
<tr>
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<td>t₁</td>
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<td>Ω_eff</td>
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</tr>
<tr>
<td>Ω_eff/Ω(0)</td>
<td>ε(0) 0.84</td>
</tr>
</tbody>
</table>

* for E₈ = 40 MeV, Q = 0.092 MeV
FIGURE CAPTIONS

Fig. 1. A plot of the difference between the Q-values of the \((^{12}\text{C}, ^{8}\text{Be})\) and \((^{12}\text{C}, ^{7}\text{Li})\) reactions on the same target nuclei, illustrates the range of excitation over which one can uniquely identify a \(^8\text{Be}\) with the method of Ref. 1. If the charge of the target is even, the range of \(^8\text{Be}\) spectra free of \(^7\text{Li}\) contamination is \(\geq 8\) MeV for \(T_Z = 1\), \(\geq 10\) MeV for \(T_Z = 1/2\), and \(\geq 14\) MeV for \(T_Z = 0\). In the case of \(T_Z = 1/2\) targets with odd \(Z\), \(^7\text{Li}\) states begin to contaminate the \(^8\text{Be}\) spectra at a much lower excitation energy varying from a minimum of \(4\) MeV to a maximum of \(8\) MeV.

Fig. 2. Velocity vector diagram resulting from the decay in flight of a \(^8\text{Be}\) into two alpha-particles as seen in the lab system. See discussion in text.

Fig. 3. The breakup alpha-particles can conveniently be described by a polar coordinate system in the \(^8\text{Be}\) rest-system and also by a cartesian coordinate system in the plane of the detector. See discussion in the text.

Fig. 4. For the case of a \(^8\text{Be}\) event directed toward the point \((\xi,\eta)\) in the plane of the detector, the rectangle \(R(\xi,\eta)\) defines the solid angle into which both alpha-particles must be emitted in order that the \(^8\text{Be}\) event will be detected. The coordinates of the vertices of \(R(\xi,\eta)\) are given in the \((x,y)\) system.

Fig. 5. A plot illustrating that for the parameters of Table I the dependence of the detection efficiency \(\epsilon\) on the incident \(^8\text{Be}\) event's laboratory energy is almost linear.

Fig. 6. Graph of the differential efficiency \(\Delta(\xi)\) showing that the detection probability is largest for \(^8\text{Be}\) velocity vectors directed towards the centerline of the detector. The parameters are those of Table I.
Fig. 7. Gas target and collimator diagram showing the dependence of the solid angle on T, the point where the reaction occurs.
The diagram shows a graph with the mass number of the target on the x-axis and the quantity $Q(\ell_2C, \ell_8Be) - Q(\ell_2C, 7Li)$ on the y-axis. Different symbols represent different values of $T_z$: 
- Circles for $T_z = 0$, even $Z$.
- Squares for $T_z = 1/2$, even $Z$.
- Triangles for $T_z = 1$, even $Z$.
- Boxes for $T_z = 1/2$, odd $Z$.
$^8$Be decay: velocity vector diagram

\[ \beta_1, \beta_2, v_1, v_0, v_2, +C, -C \]

Fig. 2
Transformation of c.m. \((\alpha, \phi)\) to lab \((x, y)\) coordinates
(A/2 - |η|, B/2 - |η|)

(A/2 + |η|, B/2 - |η|)

(-A/2 - |η|, B/2 - |η|)

(-A/2 + |η|, B/2 - |η|)

Definition of R(ξ, η)

Fig. 4
Detection efficiency ($\epsilon$) vs $^6$Be lab energy

$\epsilon$ (10^{-2})

$E_8$ (MeV)

30 40 50 60 70
Differential efficiency $\Delta (\xi)$

$\Delta (\xi) \left[ 10^{-3} \text{ mm}^{-1} \right]$

FWHM($\Delta$)

$\xi \left[ \text{mm} \right]$

Fig. 6
Schematic of gas target collimators

Beam

Rear collimator
width A

Front collimator
width \bar{A}

\( X \) \( x \)

L_2

\( \delta \)

L_1
APPENDIX A

PROGRAM EFFI (INPUT, OUTPUT, TAPE10=INPUT, TAPE9=OUTPUT)
THIS PROGRAM CALCULATES THE DETECTION EFFICIENCY OF BBE. IT IS DESCRIB
ED IN LBL-1214.
M. L. HARNEY AND G. J. WOZNIAK MAY 1972

COMMON STAM, FH, B1, B2, HINIT, BSQ

DIMENSION STAM(500)

READ(10, 101) ELAB, C, AR, AV, D, KK
WRITE(9, 102)
WRITE(9, 103) ELAB, Q, AR, AV, D
VRATIO = SQRT(ELAB/Q)
R1 = (1/VRATIO)**2
R2 = AV/2.
BSQ = B2*B2
A2 = AR/2.
IF (GEOM <= 0.) GOTO 20
ERR = 0.001
G = 2.5464790*VRATIO/(AR*AV*D)
OMEGA = AR*AV/(10*G)
HINIT = SQRT(ERR*ERR/OMEGA*2.356194*AV*AV/AR/VRATIO)
TOL = ERR*3.1415926/AR/VRATIO
ICC = 0

2 LMAX = INT(AR/2*HINIT)+1
IF (LMAX/2*2 - LMAX).GE.0 GOTO 5
LMAX = LMAX+1
5 IF (LMAX.GT.500) GOTO 30
IF (LMAX-9) 3, 3, 4
3 LMAX = 10

4 CALL ITG(0., AR/2., LMAX, TOL)
DZ = AR*S(STAM(LMAX-2)-2.*STAM(LMAX-1)+STAM(LMAX))/3.)*A2
IF (DZ.LE. ERR) GOTO 10
IF (ICC.GE. 2) GOTO 9
HINIT = HINIT*SQRT(ERR/DZ)
ICC = ICC + 1
GOTO 2

9 WRITE(9, 107)

10 ICC = 0
DO 11 I = 1, LMAX
STAM(I) = STAM(I)*G
11 CONTINUE
S = (1.333333333*STAM(1)+0.333333333*STAM(2))*HINIT
LM1 = LMAX-1
DC 12 I = 3, LM1, 2

12 S = (1.333333333*STAM(1)+0.333333333*STAM(1-1)+STAM(1+1))*HINIT
FH = STAM(LMAX)/2.
K = 1

14 IF (STAM(K).GE.FH) GOTO 15
K = K+1
GOTO 14

15 FWHM = 2.*(LMAX-K+1-(FH-STAM(K-1))/(STAM(K)-STAM(K-1)))*HINIT
FWHM = FWHM/D5*7.29579
RASP = AR/D5*7.29579
OMEFF = OMEGA*S
SS = 0.0397887*AR*AV/B
WRITE(9, 108) OMEGA, S, SS, OMEFF, RASP, FWHM
WRITE(9, 116)
DC 40 V = 1, LMAX
LL = LMAX+1-K
X = (K-1)*HINIT
WRITE(9, 117) X, STAM(LL)
CONTINUE

Z = 0.
WRITE(9,117)A2,ZZ
IF(KK)25,25,1
20 WRITE(9,109)
GOTO 25
30 WRITE(9,111)
25 CONTINUE
101 FORMAT(5F10.3,9X,111)
102 FORMAT(1111,///,/* EFFICIENCY OF DETECTION OF 88e***////* INPUT DATA/**
1/* LAB ENERG* 88e DECAY ENERG* RADIAL APERTURE VER
2tical APERTURE DIST DET TO TARGET**/* MEV MEV
3*/ MM MM*/)
103 FORMAT(1X,5(1X,E8.3,11X),///)
107 FORMAT(1X,///,/* REQUIRED PRECISION HAS NOT BEEN MET*)
108 FORMAT(1X,///,/* RESULTS**,///,/* SOLID ANGLE = *E10.3,///,/* EFFICIENCY
1= *E10.3,///,/* EFFICIENCY ACCORDING TO SIMPLIFIED FORMULA = *E10.3
2,///,/* EFFECTIVE SOLID ANGLE = *E10.3,///,/* OTHETA = *E10.3,/* DEGREE
3S */,/* FWHM = *E10.3,/* DEGREES*)
109 FORMAT(///,/* THIS PROGRAM CANNOT HANDLE A CASE, WHERE THE SOLID ANGLE
1= EXTENDS OVER THE EDGE OF THE CONE INTO WHICH THE TWO ALPHAS ARE
2* Emitted*)
1= EXTENDS OVER THE CONE INTO WHICH THE TWO ALPHAS ARE Emitted*)
110 FORMAT(1111)
111 FORMAT(1X,/,///,/* NUMBER OF POINTS OF DIFF EFFICIENCY LARGER THAN 500*)
115 FORMAT(1X,3(E10.3,5X))
116 FORMAT(1X,///,/* XI DIFFERENTIAL** EFFIC
1ENCY*//* MM */1/MM*)
117 FORMAT(1X,E10.3,5X,E10.3)
END

FUNCTION FUNC(X)
COMMON STAM,FH,B1,B2,HINIT,BSQ
DIMENSION STAM(500)
T=B1-X*X
TT=SQR(T)
Y=B2/TT
FUNC=(1.5707288- (SQR(1-Y))*(1.5707289+(-0.2121144+(0.0742610-0.018
17293)*Y)*Y)
FUNC=FUNC*B2 + SQR(T-RSQ) - TT
RETURN
END
SUBROUTINE ITG(XL,XU,LMAX,TOL)
C THIS SUBROUTINE INTEGRATES THE ANALYTIC FUNCTION FUNC(X) FROM XL TO XU
C AND AUTOMATICALLY ADJUSTS THE STEPWIDTH H SO THAT THE RESULT IS PRECISE
C AT LEAST WITHIN ±TOL. ITG STORES THE INTEGRAL IN STEPS OF
C (U-XL)/LMAX IN THE ARRAY STAM.
COMMON STAM,FH,B1,B2,HINIT,BSQ
DIMENSION STAM(500)
HINIT=(XU-XL)/LMAX
H=HINIT/2.
S=0.
M=0.
XL1=XL
K1=1
ICON1=0
ICON2=0
10 X=XL1+H
FZ=FUNC(X)
FM1=FUNC(X-H)
FP1=FUNC(X+H)
34 DZ=(FM1-2.*FZ+FP1)/3.*
HP1=SQRT(ABS(DZ)/TOL)/H
K2=INT(HINIT*HP1/2.)*1
IF(K2-K1)14,16,12
12 IF(ICON2-2)18,20,20
18 ICON2=ICON2+1
26 H=HINIT/K2/2.
K1=K2
GOTO 10
14 IF((5*K2-K1)22,22,16
22 IF(ICON124,24,16
24 ICON1=1
GOTO 26
20 WRITE(9,90)XL1,H
90 FORMAT(25H TOL NOT RESPECTED AT X= ,E10.3,8H + H= ,E10.3)
16 S=S+(2.*FZ*DZ)*H
XL1=XL1+2.*H
X=X+2.*H
L=INT((XL1-XL+0.001*H)/HINIT)
ICON1=0
ICON2=0
IF(LL-M128,28,30
30 M=M+1
HH=(XL1-M*HINIT)/2.
XX=XL1-HH
STAM(M)=S-(1.333333333*FUNC(XX)+0.333333333*(FUNC(XX-HH)+FP1))*HH
IF(M=LMAX)28,32,32
28 FM1=FP1
FZ=FUNC(X)
FP1=FUNC(X+H)
GOTO 34
32 CONTINUE
RETURN
END
EFFICIENCY OF DETECTION OF 8BE

INPUT DATA

<table>
<thead>
<tr>
<th>LAB ENERGY OF 8BE MEV</th>
<th>DECAY ENERGY MEV</th>
<th>RADIAL APERTURE MM</th>
<th>VERTICAL APERTURE MM</th>
<th>DIST DET TO TARGET MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.000</td>
<td>0.0?</td>
<td>1.780</td>
<td>5.080</td>
<td>120.600</td>
</tr>
</tbody>
</table>

RESULTS

SOLID ANGLE = 8.27E-04
EFFICIENCY = 1.096E-02
EFFICIENCY ACCORDING TO SIMPLIFIED FORMULA = 1.076E-02
EFFECTIVE SOLID ANGLE = 8.374E-06
DTHETA = 8.457E-01 DEGREES
FWHM = 4.375E-01 DEGREES

<table>
<thead>
<tr>
<th>XI</th>
<th>DIFFERENTIAL EFFICIENCY 1/MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.468E-02</td>
</tr>
<tr>
<td>8.900E-02</td>
<td>2.270E-02</td>
</tr>
<tr>
<td>1.780E-01</td>
<td>1.972E-02</td>
</tr>
<tr>
<td>2.670E-01</td>
<td>1.774E-02</td>
</tr>
<tr>
<td>3.560E-01</td>
<td>1.577E-02</td>
</tr>
<tr>
<td>4.450E-01</td>
<td>1.380E-02</td>
</tr>
<tr>
<td>5.340E-01</td>
<td>9.839E-03</td>
</tr>
<tr>
<td>6.230E-01</td>
<td>7.277E-03</td>
</tr>
<tr>
<td>7.120E-01</td>
<td>4.977E-03</td>
</tr>
<tr>
<td>8.010E-01</td>
<td>2.658E-03</td>
</tr>
<tr>
<td>8.900E-01</td>
<td>0.8</td>
</tr>
</tbody>
</table>

IBL-1214
APPENDIX B

The absolute cross sections given in the last columns of Table I of Ref. 1 are in error, since they are based on incorrect values for the detection efficiency. A corrected table is given below. The references mentioned in the heading and the footnotes are those of Ref. 1.
Table BI. Results for the Reaction \(^{12}\text{C},^{8}\text{Be}\) on \(^{12}\text{C}\) and \(^{16}\text{O}\) Targets.

<table>
<thead>
<tr>
<th>Energies and (J^\pi) values of known levels (Refs. 7 and 1)</th>
<th>Energies (this work) (MeV)(^b)</th>
<th>((d\sigma/d\Omega)_{\text{obs}}^a) ((\mu\text{b}/\text{sr}))</th>
<th>((d\sigma/d\Omega)_{\text{abs}}^a) ((\mu\text{b}/\text{sr}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{16}\text{O}) g.s. 0(^+)</td>
<td>-0.03</td>
<td>1.5</td>
<td>100</td>
</tr>
<tr>
<td>6.050 0(^+)</td>
<td>6.07</td>
<td>8.2</td>
<td>600</td>
</tr>
<tr>
<td>6.919 2(^+)</td>
<td>6.92</td>
<td>6.6</td>
<td>480</td>
</tr>
<tr>
<td>10.353 4(^+)</td>
<td>10.34</td>
<td>16.0</td>
<td>1300</td>
</tr>
<tr>
<td>11.096 4(^+)</td>
<td>11.10</td>
<td>7.6</td>
<td>640</td>
</tr>
<tr>
<td>14.82 6(^+)</td>
<td>14.67</td>
<td>18.0</td>
<td>1700</td>
</tr>
<tr>
<td>16.304(^d) 6(^+)</td>
<td>16.27</td>
<td>13.0</td>
<td>1300</td>
</tr>
<tr>
<td>(^{20}\text{Ne}) g.s. 0(^+)</td>
<td>-</td>
<td>0.4</td>
<td>30</td>
</tr>
<tr>
<td>1.63 2(^+)</td>
<td>1.62</td>
<td>2.5</td>
<td>200</td>
</tr>
<tr>
<td>4.25 4(^+)</td>
<td>4.26</td>
<td>6.3</td>
<td>560</td>
</tr>
<tr>
<td>5.80 1(^-)</td>
<td>5.78</td>
<td>1.5</td>
<td>130</td>
</tr>
<tr>
<td>7.17 3(^-)</td>
<td>7.16</td>
<td>3.5</td>
<td>330</td>
</tr>
<tr>
<td>8.79 6(^+)</td>
<td>8.79</td>
<td>7.0</td>
<td>730</td>
</tr>
<tr>
<td>10.30 5(^-)</td>
<td>10.35</td>
<td>11.0</td>
<td>1,200</td>
</tr>
</tbody>
</table>

\(^a\) Cross sections for populating \(^{16}\text{O}\) and \(^{20}\text{Ne}\) final states are given in the c.m. system and are averages of several measurements at \(\theta(\text{lab}) = 14^\circ\) and \(17^\circ\), respectively.

\(^b\) Errors are quoted in the text.

\(^c\) The cross sections could be uniformly in error as much as 50%.

\(^d\) Reference 10.
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