GENERAL EQUILIBRIUM IN VERTICAL MARKET STRUCTURES: MONOPOLY, MONOPSONY, PREDATORY BEHAVIOR AND THE LAW

by

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General Equilibrium in Vertical Market Structures: Monopoly, Monopsony, Predatory Behavior and the Law

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Introduction

Recent court decisions have drawn a sharp distinction between “predatory bidding” and “predatory selling.” In the case of “predatory bidding” the literature has drawn a distinction between overbuying and raising rivals’ cost. The former is intended to cause harm to input market competitors ultimately allowing the predatory firm to exercise monopsony power. Raising rivals’ cost is instead intended to raise input cost of the output market competitors and thus allow the predatory firm to exercise market power by raising or maintaining prices. The standard in pure “predatory selling” instances, after many decades of economic and legal debate, was resolved by the United States Supreme Court in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 US 209 (1993). This ruling found that suppliers in output markets are not predatory unless (1) the prices charged are below the seller’s cost and (2) the seller has a “dangerous probability” of recouping its lost profits once it has driven its competitors from the market. In this paper we analyze whether the standard for liability in “buy-side” or monopsony cases should be the same or as high as the standards for liability in “sell-side” or monopoly cases.

From a theoretical perspective, much of the debate among Kirkwood (2005), Salop (2005) and Zerbe (2005) is sourced with the distinction between consumer welfare and economic efficiency, the distinction between partial equilibrium and general equilibrium economic welfare analysis, and the distinction between substitutes and complements. In this paper we develop a model that allows us to isolate the implications of each of these distinctions. Moreover, we are able to specify the theoretical standards for predatory conduct and the fundamental forces that dictate violations of such standards. Some of the questions that can be answered by our
theoretical formulation include (a) Under what conditions does overbuying lead to consumer harm? (b) Should predatory buying be required to satisfy the below cost pricing test of Brooke Group? (c) Should allegations of “raising rivals’ cost” also be subject to the same below cost pricing test of Brooke Group?

After specifying the general equilibrium model and the competitive equilibrium benchmark, the first formal analysis evaluates market power in output markets. For this case we prove the proposition that if a concentrated industry has market power only in the output market and related sectors behave competitively, then overbuying in the input market is not profitable. Here the key to monopoly rents is restricting output, not driving up the prices of an input or equivalently overbuying an input. We also show that, under the specified conditions, monopolistic firms achieve greater rents or monopoly profits under general equilibrium than they would achieve under typical partial equilibrium models. One of the more interesting implications of the general equilibrium lens is that the existing Department of Justice Merger Guidelines will typically give inaccurate results in assessing the profitability of a firm raising its prices by 5 or 10 percent if the analysis is not performed in a general equilibrium framework.

After setting out three major propositions under monopoly power in the output market, we turn to distortions in the input market focusing on monopsonistic power. Here we find, contrary to the Ninth Circuit ruling in Ross Simmons v Weyerhauser matter, that if a concentrated industry does not have the ability to alter its output price through its input buying behavior, then the industry cannot increase its profits by overbuying the input. Instead, under the general equilibrium lens, the traditional monopsony result is obtained where the input market quantity is restricted. Under the same lens we also demonstrate that monopsonistic firms may gain more rent than conventional estimates based on partial equilibrium models would suggest, just as in the case of monopoly, but for more likely cases on the supply side monopsonistic firms will not gain as much as implied by carefully specified partial equilibrium models. In the latter
conditions, a firm has less market power and distorts the price in an input market less when equilibrium adjustments of a related industry are taken into account. We also show that a firm that has the ability to manipulate price by a given amount such as specified by the Department of Justice Merger Guidelines is invalid if done with ordinary or partial equilibrium input supplies.

We also consider the more general case where the vertical structure consists of a single firm or colluding firms that have market power in both their input and output markets. Here we are able to develop seven propositions that turn on characteristics of technologies of competing industries and the characteristics of input supplies and output demands including the degree of substitutability or complementarity in both supplies and demands. Finally we present the case of *naked overbuying* as a means of exercising market power.

We emphasize at the outset that our results are developed in a static model rather than a two-stage model where the firm with market power first drives out its competitors and then exercises greater market power than previously held in a subsequent recoupment stage. In contrast, much of the relevant legal literature considers the two-stage approach, and some even suggest such a two-stage framework is the only explanation for overbuying. In contrast, we show that such extreme behaviors are profitably sustainable on a continual basis using a static framework where general equilibrium adjustments are considered. Further, we suggest that such models offer a more practical explanation for the substantive impacts of overbuying or other predatory behavior because two-stage models do not explain well why firms do not re-enter markets just as easily as they leave unless other anticompetitive factors are present. These results also demonstrate a distinct difference in possibilities for overselling compared to overbuying, which prove, contrary to the arguments of Noll (2005), that the buy-side aspects of predatory behavior are not the mirror image of sell-side predatory behavior.

In the two-stage framework, if a competing firm’s best use of its resources is to produce a particular product under competitive pricing but finds switching to production of an alternative
to be optimal when a predatory buyer drives up its input price, then its optimal action is to return to its first best use of resources as soon as the predatory behavior is reversed in an attempt to recoup by driving down the input price. Thus, unless this competitive readjustment is artificially prevented, such as by buying up fixed production resources, two-stage predatory behavior cannot be optimal. Thus, proving two-stage predatory behavior should require identification of an artificial barrier to other firms' re-entry in the recoupment period. Alternatively, the conditions developed in this paper would be required for a temporal aggregation of the two-stage problem if predatory behavior were optimal for any firm.

**Equilibrium Analysis of Economic Welfare**

To address these issues, we use the approach advanced by Just, Hueth, and Schmitz (2004, pp. 355-361) for comparison of welfare effects where equilibrium adjustments occur across many markets as well as many types of consumers and producers. This approach permits an analysis of indirect equilibrium adjustments that determine the implications of monopolistic behavior in markets that are interdependent with other markets. Such a framework can explain seemingly extreme monopoly behavior including overbuying even in static models where recoupment periods are not necessary. Before developing specific results for the market structure considered in this paper, we summarize the underlying equilibrium measurement of welfare.

**Assumption 1.** Suppose each of \( J \) utility-maximizing consumers has exogenous income \( m_j \), and is endowed with a nonnegative \( N \)-vector of resources \( r_j \), has monotonically increasing, quasiconcave, and twice differentiable utility \( U_j(c_j) \), where \( c_j \) is a corresponding nonnegative \( N \)-vector of consumption quantities, the budget constraint is \( p(c_j - r_j) = m_j \), and \( p \) is a corresponding \( N \)-vector of prices faced by all consumers and firms in equilibrium.
Assumption 2. Suppose each of $K$ firms maximizes profit $pq_k$ given an implicit multivariate production function $f_k(q_k) = 0$ where $q_k$ is an $N$-vector of netputs ($q_{kn} > 0$ for outputs and $q_{kn} < 0$ for inputs) where each scalar function in $f_k$ is monotonically increasing, concave, and twice differentiable in the netput vector, other than for those netputs that have identically zero marginal effects in individual equations (allowing each production process to use a subset of all goods as inputs producing a different subset of all goods as outputs).

Proposition 1. Under Assumptions 1 and 2, the aggregate equilibrium welfare effect (sum of compensating or equivalent variations in the case of compensated demands evaluated at ex ante or ex post utility, respectively) of moving from competitive pricing to distorting use of market power in a single market $n$ is given by

$$\Delta W = \int_{\tilde{p}_n(0)}^{\tilde{p}_n(\delta)} q^n_s(p_n) dp_n - \int_{\tilde{p}_n(0)}^{\tilde{p}_n(\delta)} q^n_d(p_n) dp_n$$

where $q^n_s(\cdot)$ is the aggregate equilibrium quantity supplied of good $n$, and $q^n_d(\cdot)$ is the aggregate equilibrium quantity demanded of good $n$, $\delta = p^n_d(\delta) - p^n_s(\delta)$ is the effective price distortion introduced in market $n$, and $p^n_s(\delta)$ and $p^n_d(\delta)$ represent the respective marginal cost and marginal benefit of good $n$ considering all equilibrium adjustments in other markets in response to changes in $\delta$.


Proposition 1 allows an account of equilibrium adjustments that occur throughout an economy in response to the distortion in a single market. Further, the welfare effects (compensating or equivalent variation) of a change in $\delta$ can be measured for individual groups of producers using standard estimates of profit functions and for individual groups of consumers using standard estimates of expenditure or indirect utility functions by evaluation at the initial and subsequent equilibrium price vectors. If other markets are distorted, then this result can be

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1 In the case of indirect utility functions, the welfare effects are not measured by the change in the function. Rather, compensating variation, CV, is defined by $V(p^s, m_j^0 - CV) = V(p^o, m_j^0)$ and equivalent variation, EV, is defined by
modified accordingly (Just, Hueth, and Schmitz 2004, pp. 361-365) but, in effect, only the case of a single distortion is needed for results in this paper.

The graphical implications of Proposition 1 are presented in Figure 1. With no distortion, equilibrium in market $n$ is described by the intersection of ordinary supply, $\overline{q}_n^s(p_n, \overline{p}(0))$, and ordinary demand, $\overline{q}_n^d(p_n, \overline{p}(0))$, where $\overline{p}(0)$ denotes conditioning on all other equilibrium prices throughout the economy under no distortions, i.e., when $\delta = 0$. If the distortion $\delta = \delta_0$ is introduced in market $n$, then after equilibrium adjustments throughout the economy, ordinary supply shifts to $\overline{q}_n^s(p_n, \overline{p}(\delta_0))$ and ordinary demand shifts to $\overline{q}_n^d(p_n, \overline{p}(\delta_0))$, which are conditioned on prices throughout the economy with a specific distortion, $\delta = \delta_0$, in market $n$. The effective general equilibrium supply and demand relationships that implicitly include equilibrium adjustments throughout the economy in response to changes in the distortion $\delta$ are $q_n^s(p_n^s(\delta))$ and $q_n^d(p_n^d(\delta))$, respectively.

With monopoly pricing in market $n$, $p_n^d(\delta)$ represents the equilibrium market $n$ price, and $\delta = p_n^d(\delta) - p_n^s(\delta)$ represents the difference in price and general equilibrium marginal revenue, EMR. This marginal revenue is not the marginal revenue associated with either the ordinary demand relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopoly problem, it is the marginal revenue associated with the general equilibrium demand, $q_n^d(p_n^d(\delta))$, that describes how price responds with equilibrium adjustments throughout the economy in response to changes in the market $n$ distortion. In this case, $q_n^s(p_n^s(\delta))$ represents how marginal cost varies with equilibrium adjustments in other markets, so marginal cost is equated to EMR at $q_n^d(p_n^d(\delta_0))$.

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$V(p^1, m^1) = V(p^0, m^0 + EV)$ where $V$ is the indirect utility function and superscripts 0 and 1 represent initial and subsequent equilibrium conditions.

2 Throughout this paper, the terms "ordinary supply" and "ordinary demand" are taken to refer to partial equilibrium supplies and demands, respectively, which take as given certain conditions not directly involved in the relevant market.
With monopsony, \( p_n^e(\delta) \) represents the equilibrium market price and
\[
\delta = p_n^d(\delta) - p_n^e(\delta)
\]
represents the difference in the general equilibrium marginal outlay, EMO, and price. This marginal outlay is not the marginal outlay associated with either the ordinary supply relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopsony problem, it is the marginal outlay associated with the general equilibrium supply, \( q_n^d(p_n^e(\delta)) \), that describes how price responds with equilibrium adjustment throughout the economy to changes in the market distortion. In this case, \( q_n^d(p_n^e(\delta)) \) represents how marginal revenue varies with equilibrium adjustments in other markets, so marginal revenue is equated to EMO at \( q_n^d(p_n^e(\delta_o)) \).

The application of this result to the related market structure of this paper is illustrated simplistically for the case of perfect substitutes in demand for final products and perfect substitutes in supply of inputs in Figure 2. Suppose in Figure 2(a) that output demand jointly facing two products or industries is \( p(y + z) \) where \( y \) and \( z \) are the quantities sold of each of the products. Suppose also that both products or industries use the same input in production and thus the input supply jointly facing the two industries is \( w(x_y + x_z) \) where \( x_y \) and \( x_z \) are the respective quantities of the input used by the two industries (both the input and output market are represented on the same diagram, assuming for the graphical analysis that the production process transforms the input unit-for-unit into outputs). If the \( y \) industry consists of a single firm whereas industry \( z \) is a competitive industry, we have the dominant-firm-competitive-fringe structure as a special case of Figure 2. Figure 2(b) represents the competitive response of production activity in the \( z \) industry as a function of the difference in the input and output price. Specifically, supply at the origin of Figure 2(b), shown in reverse, is the point at which the corresponding difference in prices in (a) is just high enough that the \( z \) industry would start to produce. Suppose with increasing marginal cost for the \( z \) industry that at output price \( p^0 \) and input price \( w^0 \) the \( z \) industry uses input quantity \( x_z^0 \). The corresponding excess demand, \( ED \), and excess supply, \( ES \), to the \( y \)
industry are shown in Figure 2(a). Note that the two vertical dotted lines in Figure 1(a) sum to the vertical dotted line in Figure 2(b).

To maximize profits, the $y$ industry can use the input supply and excess supply relationships directly from Figure 2(a) as shown in Figure 2(c). For comparability, the output demand and excess demand relationships from Figure 2(a) must be transformed into input price equivalents by inversely applying the production technology of the $y$ industry for purposes of determining how much to produce. That is, where $y = y(x_y)$ is the production function of the $y$ industry and $x_y = y^{-1}(y)$ is the associated inverse function, the equivalent input demand $D^*$ in Figure 2(c) is found by substituting the demand relationship in Figure 2(a) into $y^{-1}(\cdot)$. The equivalent excess demand, $ED^*$, in Figure 2(c) is found similarly. Then the $y$ industry maximizes profit by equating the general equilibrium marginal revenue, $MR^*$, associated with $ED^*$, and the general equilibrium marginal outlay, $MO$, associated with the excess supply.

The core insights in this paper arise because the production technologies for the two industries may not be similar and may not be unit-for-unit technologies. In contrast to the traditional monopoly-monopsony result where market quantities are restricted to increase profits, equilibrium adjustments can cause displacement of the $z$ industry by the $y$ industry in the case of overbuying. Moreover, these results are modified when the outputs are not perfect substitutes in demand or the inputs are not purchased from the same market but in related markets.

**The Model**

Based on the general economy model, we are now in a position to evaluate a related market structure that exists within the general economy. To abstract from the complications where compensating variation does not coincide with equivalent variation (nor with consumer surplus), consumer demand will be presumed to originate from a representative consumer, and
that prices of all goods, other than two related goods of interest, are set by competitive conditions elsewhere in the economy. As a result, expenditures on other goods can be treated as a composite commodity, \( n \), which we call the numeraire. More concretely, suppose that demand is generated by maximization of a representative consumer utility that is quasilinear in the numeraire, \( u(y, z) + n \), where \( y \) and \( z \) are non-negative consumption quantities of the two goods of interest and standard assumptions imply \( u_y > 0, u_z > 0, u_{yy} < 0, u_{zz} < 0 \), and \( u_yu_{zz} - u_{yz}^2 \geq 0 \) where subscripts of \( u \) denote differentiation.\(^3\)

Suppose the consumer’s budget constraint is \( p_y y + p_z z + n = m \) where \( p_y \) and \( p_z \) are prices of the respective goods and \( m \) is income. Substituting the budget constraint, the consumer’s utility maximization problem becomes \( \max_{y,z} u(y, z) + m - p_y y - p_z z \). The resulting first-order conditions yield the consumer demands in implicit form,

1. \( p_y = u_y(y, z) \)
2. \( p_z = u_z(y, z) \).

Downward sloping demands follow from the concavity conditions, \( u_{yy} < 0 \) and \( u_{zz} < 0 \). The two goods are complements (substitutes) in demand if \( u_{yz} > (\leq) 0 \).

Suppose the two goods, \( y \) and \( z \), each has one major input. For simplicity and clarity, suppose the quantities of any other inputs are fixed. Thus, the respective production technologies can be represented by

3. \( y = y(x_y) \)
4. \( z = z(x_z) \)

\(^3\) While the weaker assumption of quasi-concavity can be assumed for consumer problems, we use the more restrictive assumption that \( u_yu_{zz} - u_{yz}^2 \geq 0 \) to attain symmetry of the mathematical analysis, which saves space and enhances intuition.
where $x_y$ and $x_z$ represent the respective input quantities and standard assumptions imply $y' > 0$, $y'' < 0$, $z' > 0$, and $z'' < 0$, where primes denote differentiation.

Suppose the inputs are related in supply so that the industries or products compete for inputs as well as sales of total output. To represent the related nature of supply, suppose the respective inputs are manufactured by a third competitive industry with cost function $c(x_y, x_z)$. Thus, input supplies in implicit form follow

\begin{align*}
(5) & \quad w_y = c_y(x_y, x_z) \\
(6) & \quad w_z = c_z(x_y, x_z)
\end{align*}

where $y$ and $z$ subscripts of $c$ represent differentiation with respect to $x_y$ and $x_z$, respectively, and standard assumptions imply $c_y > 0$, $c_z > 0$, $c_{yy} > 0$, $c_{zz} > 0$, and $c_{yy}c_{zz} - c_y^2 \geq 0$, where $c_{yz} > (>) 0$ if $x_y$ and $x_z$ are substitutes (complements) in supply. For convenience, we also define $x_z = \hat{c}(w_y, x_y)$ as the inverse function associated with $w_y = c_y(x_y, x_z)$, which implies

\begin{align*}
\hat{c}_w &= 1/c_{yz} > (>) 0 \quad & \text{and} \quad \hat{c}_x &= -c_{yy}/c_{yz} < (>) 0 \quad \text{if} \quad x_y \text{and} \quad x_z \text{are substitutes (complements)}.
\end{align*}

Suppose that the $z$ industry always operates competitively as if composed of many firms. The profit of the $z$ industry is $\pi_z = p_z \cdot z(x_z) - w z$. The first-order condition for profit maximization requires

\begin{equation}
(7) \quad w_z = p_z z'(x_z).
\end{equation}

The second-order condition for a maximum is satisfied because $z'' < 0$ and prices are regarded as uninfluenced by the firm’s actions.

Finally, suppose behavior of the $y$ industry is given by

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4 This industry may represent a hypothetical firm formed by aggregating the behavior of many producers under competitive conditions.

5 For the special case where $c_{yy}c_{zz} - c_y^2 = 0$, which is not normally admitted in standard convexity conditions, we introduce a concept of perfect substitutes in supply where, in effect, $c(x_y, x_z)$ becomes $c(x_y + x_z)$ and $c(\cdot)$ is a convex univariate function.
Equations (1)-(7) are sufficient to determine the general equilibrium supply and demand relationships facing the $y$ industry. A variety of cases emerge depending on market structure and the potential use of market power by the $y$ industry.

**Competitive Behavior**

If the $y$ industry is composed of many firms that do not collude, then the first-order condition for (8) requires

$$w_y = p_y, y'.$$

As for the $z$ industry, the second-order condition is satisfied because $y'' < 0$ and prices are regarded as uninfluenced by firm actions. This yields the case where $\delta = 0$ in Figure 1.

Focusing on the $y$ industry for given $x$, the system composed of (1)-(7) can be reduced to a two equation system that describes the general equilibrium input supply and output demand facing the $y$ industry, viz.,

$$p_y = u_y(y(x), z(\hat{c}(w_y, x_y)))$$

$$c_z(x, \hat{c}(w_y, x_y)) = u_z(y(x), z(\hat{c}(w_y, x_y)))z'(\hat{c}(w_y, x_y)).$$

Equations (10) and (11) define implicitly the general equilibrium supply and demand relationships for the $y$ industry. Because (10) and (11) are not in explicit form, comparative static methods can be used to determine

$$\frac{dp_y}{dx_y} = u_{yy}y' + \frac{u_{yz}z'}{c_{yz}} \left[ \frac{dw_y}{dx_y} - c_{yy} \right]$$

$$\frac{dw_y}{dx_y} = c_{yy} + \frac{(c_{yz} - u_{yz}y'z')c_{yz}}{\pi_z}$$
where throughout this paper we define for notational simplicity $\pi_{zz} = u_{zz}z'z'' + u_zz'' - c_{zz} < 0$, which is the marginal effect of $x_z$ on the first-order condition of the $z$ industry given demand for $z$ and supply of $x_z$. The relationships in (12) and (13) implicitly define the input and output prices for the $y$ industry as a function of its input level $x_y$, or equivalently in terms of its output level, $y = y(x_y)$.

The relationship in (12) is a critical effect in this paper that measures the effect of an increase in the purchased quantity of the $y$ industry’s input on the $y$ industry’s output demand through its indirect effect transmitted through the $z$ industry markets. If more of the $y$ industry’s input is purchased, then its input price is bid up, the supply of a competing input produced for the $z$ industry (which is a substitute output for input suppliers) is reduced, the production activity of the $z$ industry is then reduced, and the reduction in $z$ output causes the demand for $y$ to increase (decrease) if $y$ and $z$ are substitutes (complements) in demand. This effect can be compared to the direct effect on the price of the $y$ industry’s input in maximizing profit if the $y$ industry consists of a single firm with market power. However, with competitive behavior by the $y$ industry, condition (9) together with (10) and (11) defines the competitive equilibrium output price $p_y = \bar{p}_y$, input price $w_y = \bar{w}_y$, and input quantity $x_y = \bar{x}_y$, where other equilibrium quantities and prices follow from $\bar{y} = y(\bar{x}_y)$, $\bar{x}_z = c(\bar{w}_y, \bar{x}_y)$, $\bar{z} = z(\bar{x}_z)$, $\bar{w}_z = c_z(\bar{x}_y, \bar{x}_z)$, and $\bar{p}_z = u_z(\bar{y}, \bar{z})$.

**Market Power Only In the Output Market**

The first noncompetitive market structure that can be easily evaluated is the case with market power only in the output market. The $y$ industry would have market power only in the output market if many other industries or many firms in another industry also use the same input $x$, effectively rendering input price $w_y$ unaffected by $y$ industry activity. To preserve remaining
generality, suppose only one competitive industry produces $x_z$ with a supply represented implicitly by

$$(6') \quad w_z = c_z(x_z).$$

For this case, $w_y$ is fixed. Accordingly, equation (5) is dropped and equation (6) is replaced by (6') in the system that structures the equilibrium. The system in (1)-(4), (6') and (7) can be reduced to

$$(10') \quad p_y = u_y(y(x_y), z(x_y))$$

$$(11') \quad c_z(x_z) = u_z(y(x_y), z(x_y)) z'(x_z).$$

Equations (10') and (11') define implicitly the general equilibrium supply and demand relationships for the $y$ industry in this case.

Because equations (10') and (11') are not in explicit form, once again comparative static methods must be applied to determine properties of the general equilibrium supply and demand facing the $y$ industry,

$$(12') \quad \frac{dp_y}{dx_y} = \frac{(u_{z z}u_{y y} - u^2_{z y} + (c_{z z} - u_z z')u_{y y}' - (c_{z y} - u_y z')u_{y y}')}{\pi_{z z}} = u_{y y}' + u_{y z} z' \frac{dx_z}{dx_y} = p_{y y}' < 0$$

$$(13') \quad \frac{dx_z}{dx_y} = -\frac{u_{y z} y'}{\pi_{z z}} > (0) \text{ if } u_{y z} > (0)$$

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6 Three different approaches can be used for this case. First, the industry that produces $x_z$ can be considered an industry independent of the one that produces $x_y$, as suggested directly by equation (6'). Second, if the same industry produces both $x_y$ and $x_z$, as maintained thus far, then the two equation system that describes its supplies in (5) and (6) can be replaced by supplies generated from a standard profit function, $\tilde{\pi}(w_y, w_z)$, where input prices are suppressed because they are regarded as fixed. Thus, supplies in explicit form are $x_y = \partial \tilde{\pi} / \partial w_y = \tilde{\pi}_y(w_y, w_z)$ and $x_z = \partial \tilde{\pi} / \partial w_z = \tilde{\pi}_z(w_y, w_z)$. If $x_y$ faces a perfectly elastic demand because many other industries or firms use it, then only the latter condition is relevant for the $y$ industry because $w_z$ is fixed. Further, because $w_y$ is fixed, the latter condition may be represented simply as a monotonic functional relationship between $x_y$ and $w_y$, which for continuity can be represented by (6'). Thus, both of these first two approaches yield the same results as derived in this section. As a third approach, intuition may suggest that if the industry that produces $x_z$ has a perfectly elastic supply of $x_y$, then it also has a perfectly elastic supply of its related product $x_z$. In this case, both equations (5) and (6) are simply dropped from the system that determines equilibrium supply and demand for the $y$ industry. With this approach, all qualitative results obtained in this section are unaltered. The only quantitative difference is that all $c_{z z}$ terms vanish.
where \( p_{yy} \) is defined as the slope of the general equilibrium demand for \( y \) after all equilibrium adjustments in the \( z \) sector, which in this case is derived as

\[
(14) \quad p_{yy} = u_{yy} - \frac{u_{yz} z^2}{\pi_{zz}} < 0.
\]

Negativity of \( p_{yy} \) is evident from the first right-hand expression of (12').

In this case, the first-order condition for maximizing \( \pi_y = p_y \cdot y(x_y) - w_y x_y \) using (12') is

\[
(9') \quad w_y = p_{yy} y' + p_y y'.
\]

This condition requires the equilibrium output price, \( p_y \), to be greater than ordinary marginal cost, \( w_y / y' \), for the \( y \) industry because \( p_{yy} y' < 0 \). This case corresponds to Figure 3 where market \( n \) represents the \( y \) market and

\[
\delta = -p_{yy} y' > 0.
\]

Because the input price is fixed, the general equilibrium marginal outlay coincides with the general equilibrium input supply and all partial equilibrium input supplies which are all perfectly elastic.

As is typical of monopoly problems, the second-order condition for this problem involves complicating third derivatives of the utility function. Accordingly, some conditions are possible where the second-order condition fails. However, the second-order condition can clearly be satisfied both for some cases where \( dx_y / dx_z \) is positive and some cases where \( dx_y / dx_z \) is negative.

For example, suppose third derivatives of the utility function vanish. Then the second-order condition becomes

\[
p_{yy} (2y'^2 + yy'') - \frac{(2u_{zz} - 2u_{yz} z')u_{yz} y' y'' x' d_x y' + p_y yy''}{\pi_{zz}} < 0.
\]

Because the first and last terms are negative under the plausible assumption that the \( y \) technology is not sharply downward bending, \( 2y'^2 + yy'' > 0 \), this condition can possibly be satisfied in some
circumstances when $dx/dx$ is positive and in some circumstances when $dx/dx$ is negative. In particular, the second-order condition holds when $2u_zz'' > 2c_{zz} + u_zz'$ and $dx/dx > 0$, or when $2u_zz'' < 2c_{zz} + u_zz'$ and $dx/dx < 0$.

Not surprisingly, these results show that if the $y$ industry does not have the ability to reduce industry $z$ activity by driving up the price of the input, then the $y$ industry cannot profitably increase its output price by overbuying the input. The general equilibrium demand for its output is downward sloping in its input quantity or, equivalently, by dividing (12') by $y'$, in its output quantity. Thus, a rather traditional monopoly result is obtained where the market quantity is restricted.

**Proposition 2.** With the market structure in (1)-(8), if the concentrated industry has market power only in the output market then neither input overbuying nor output overselling are profitably sustainable. Output is restricted to increase the output price.

Even though Proposition 2 and its intuition is similar to the typical monopoly pricing result, the same equilibrium does not arise if the $y$ industry optimizes its profit in a conventional partial equilibrium sense. To see this, note that the traditional partial equilibrium monopoly pricing rule equates the monopolist's marginal cost and marginal revenue based on the ordinary output demand. To compare with partial equilibrium optimization, we consider two alternative approaches to specification of the ordinary partial equilibrium demand. With the approach suggested by (1), the ordinary demand is conditioned on $z$ market activity as represented by the quantity $z$. We call this the *quantity-dependent ordinary demand*, meaning that it is conditioned on quantities in related markets. In this case, the first-order condition for maximizing the monopolist's profit, $\pi_y = p_y \cdot y(x_y) - wx_y$, requires

$$w_y = u_yyy' + p_y y'.$$
The only difference in this first-order condition and (9') is that $u_{yy}$ replaces $p_{yy}$. Equation (14) implies that $u_{yy} < p_{yy} < 0$ because the numerator of the right-hand fraction is positive while the denominator is negative. Thus, the general equilibrium demand is more elastic or less steep than the ordinary demand. Intuitively, the quantity-dependent ordinary demand does not allow the consumer to shift consumption to the $z$ market as the price of $y$ is increased, which accounts for the less elastic nature of the ordinary demand compared to the general equilibrium demand.

Because $u_{yy}$ negatively exceeds $p_{yy}$, the $y$ market has a smaller distortion under general equilibrium or, hereafter, informed monopoly behavior than under partial equilibrium monopoly behavior based on a quantity-dependent specification. That is, partial equilibrium monopoly pricing based on the quantity-dependent ordinary demand yields the first-order condition

$$w_y = u_{yy} + p_{yy},$$

which compares to the general equilibrium condition in (9'). Comparing the respective first-order conditions, the difference in the $y$ industry output price and marginal cost,

$$p_y - w_y / y',$

is less under informed monopoly behavior. This implies that monopolistic firms could not gain monopoly profits as great as traditional estimates with quantity-dependent partial equilibrium models would suggest. The reason is that general equilibrium demands that account for adjustments in other markets are more elastic than ordinary demands that hold quantities in related markets constant. These results prove:

**Proposition 3 (Quantity Conditioning).** *With the market structure in (1)-(8), if the concentrated industry has market power only in the output market then the concentrated industry maximizes profit by introducing a smaller monopoly distortion in price than associated with partial equilibrium monopoly analysis conditioned on quantities in the related market, regardless of whether the output are complements or substitutes in demand.*

7 Throughout this paper, the term "informed monopoly behavior" is defined as monopoly behavior that takes account of equilibrium adjustments that occur in related sectors and the effects of those adjustments on the general equilibrium supply and demand facing the $y$ industry. This is in contrast to partial equilibrium monopoly behavior, which does not.
In contrast, ordinary partial equilibrium demands are typically specified and estimated as conditioned on prices rather than quantities in other markets. We call the ordinary demand conditioned on prices rather than quantities in related markets the *price-dependent ordinary demand*. Properties of the relationship of \( p_y \) and \( y \) given \( p_z \) rather than given \( z \) can be derived by comparative static analysis of the two-equation system representing consumer demand in (1) and (2) holding \( dp_z = 0 \). That is, \( dp_z = 0 \) in (2) implies \( dz / dy = -u_{yz} / u_{zz} \). Using this result to totally differentiate (1) implies that the slope of the typical price-dependent ordinary demand for \( y \) is

\[
(15) \quad p_{yy}^* = dp_y / dy = u_{yy} + u_{yz} dz / dy = (u_{yy} u_{zz} - u_{yz}^2) / u_{zz} < 0.
\]

The first-order condition for maximizing the monopolist’s profit, \( \pi_y = p_y \cdot y(x_y) - wx_y \), given this ordinary demand specification requires

\[
(16) \quad w_y = p_{yy}^* y' + p_y y'.
\]

Comparing to (14) reveals that \( u_{yy} < p_{yy} < p_{yy}^* \).

This yields the interesting result that general equilibrium demand is less elastic or steeper than the typical price-dependent ordinary demand conditioned on other market prices as depicted in Figure 3, even though it is more elastic than the quantity-dependent ordinary demand. Intuitively, the price-dependent ordinary demand allows the consumer to shift consumption to the \( z \) market as the price of \( y \) is increased, which accounts for the more elastic nature of the ordinary demand compared to the quantity-dependent case. However, it ignores the upward movement of the price of \( z \) that occurs in general equilibrium, which is why the general equilibrium demand for \( y \) is less elastic than the price-dependent ordinary demand.

Because \( p_{yy} \) negatively exceeds \( p_{yy}^* \), the \( y \) market has a larger price distortion under general equilibrium monopoly behavior, which we hereafter call informed monopoly behavior, than under partial equilibrium monopoly behavior. That is, comparing the first-order conditions

\footnote{Throughout this paper, the term “informed monopoly behavior” is defined as monopoly behavior that takes account of equilibrium adjustments that occur in related sectors and the effects of those adjustments on the general}
in (16) and (9') reveals that the difference in the \( y \) industry output price and marginal cost, 
\[ p_y - w_y / y' \], is greater under informed monopoly behavior. This implies that monopolistic firms can gain greater monopoly profits than traditional estimates with price-dependent partial equilibrium models would suggest. The reason is that general equilibrium demands that embody price adjustments in other markets are less elastic than ordinary demands holding prices in related markets constant suggest. These results prove:

**Proposition 3' (Price Conditioning).** *With the market structure in (1)-(8), if the concentrated industry has market power only in the output market then the concentrated industry maximizes profit by introducing a larger monopoly distortion in price than associated with conventional price-dependent partial equilibrium monopoly analysis conditioned on prices in the related market, regardless of whether the outputs are complements or substitutes in demand.*

Not surprisingly, from (13'), the effect of monopoly behavior by the \( y \) industry that reduces \( y \) and \( x_y \) from the competitive equilibrium, after equilibrium adjustments, is either to reduce demand for \( z \) (which reduces both input and output levels for \( z \)) if \( y \) and \( z \) are complements, or to increase demand for \( z \) if \( y \) and \( z \) are substitutes. In either case, the feedback effect is to increase demand for \( y \), which according to (12') makes the general equilibrium demand facing the \( y \) industry less elastic than if no adjustment occurred in the price of the competing good \( z \) (as in the typical partial equilibrium case). That is, in the case of complements, raising the price of \( y \) by restricting sales reduces the demand and price for \( z \), which increases the price-dependent ordinary demand for \( y \), thus making the general equilibrium demand facing the \( y \) industry less elastic than the ordinary demand that holds the price of \( z \) constant. In the case of substitutes, raising the price of \( y \) by restricting sales increases the demand and price for \( z \), which increases the price-dependent ordinary demand for \( y \), thus also making the general equilibrium supply and demand facing the \( y \) industry. This is in contrast to partial equilibrium monopoly behavior, which does not.
demand facing the $y$ industry less elastic than the ordinary demand that holds the price of $z$
constant. This is why the $y$ industry has more market power and distorts the price in the $y$ market
more given equilibrium adjustments of the related industry than typical price-dependent partial
equilibrium analysis implies.

Because ordinary demands are typically estimated as depending on the prices of all
relevant goods (rather than quantities of other goods), we argue that the case of Proposition 3' is
typically relevant compared to Proposition 3. Accordingly, the remainder of the discussion in
this section is based on this assumption, although the results other than Proposition 3 are similar
in the quantity-dependent case.

While the price distortion is greater with informed monopoly, another issue is whether
the market quantity with informed monopoly is less than suggested by the conventional partial
equilibrium monopoly outcome. Where competitive equilibrium is denoted by overbars, the
competitive equilibrium satisfies $\bar{w} = \bar{p}_y y'$. Subtracting this relationship from (16) yields
$w - \bar{w} = p^*_{yy} y' + (p_y - \bar{p}_y) y' = 0$ where the latter equality follows from perfectly elastic input
supply. Thus, $p^*_{yy} y + (p_y - \bar{p}_y) = 0$. Following the partial equilibrium monopoly calculus,
which holds $p_z$ fixed in (15), yields $p_y - \bar{p}_y = -\int_{y}^{y'} p^*_y dy$. Thus, the partial equilibrium
monopoly solution has $y$ satisfying $p^*_{yy} y - \int_{y}^{y'} p^*_y dy = 0$. Under linearity ($p_{yy}$ is constant), this
condition becomes $p^*_{yy} y + p^*_y (y - \bar{y}) = 0$, which yields the familiar result, $y = \bar{y} / 2$.

Comparing the informed equilibrium monopoly case with the competitive equilibrium
condition by subtracting (9') instead of (16) yields $w - \bar{w} = p_{yy}, yy' + (p_y - \bar{p}_y) y' = 0$ and thus
requires $p_{yy} y + (p_y - \bar{p}_y) = 0$. In this case, $p_y - \bar{p}_y = -\int_{y}^{y'} p_{yy} dy$. Thus, the informed
equilibrium monopoly output satisfies $p_{yy} y - \int_{y}^{y'} p_{yy} dy = 0$. Under linearity of demand and

9To simplify notation, when inside an integral, $p_y^*$ is assumed to vary with $y$ along the path of integration. But, when outside an integral, $p_{yy}^*$ is assumed to be evaluated at the optimal partial equilibrium monopoly solution.

10To simplify, when inside an integral, $p_{yy}$ is assumed to vary with $y$ along the path of integration. But, when outside an integral, $p_{yy}$ is assumed to be evaluated at the optimal informed monopoly solution.
production technologies \((u_{yy}, u_{zz}, u_{y z}, z'\) are constants and \(z^* = 0\), this condition also yields \(y = \bar{y}/2\). Thus, with linearity, both approaches restrict the market quantity to the same degree while the price distortion is greater under informed equilibrium monopoly behavior. Thus, the deadweight loss is greater under informed equilibrium monopoly behavior.

More generally, these results show that both the conventional partial equilibrium monopoly quantity and the informed monopoly quantity can be greater (less) than half of the competitive market quantity as the corresponding demand is downward (upward) bending. Such analysis also reveals that the general equilibrium demand is more upward bending or less downward bending than the ordinary demand if \(p_{yyy} > p_{yyy}^*\), in which case the informed equilibrium monopoly quantity is less than the ordinary monopoly quantity, while the opposite is true if \(p_{yyy} < p_{yyy}^*\). These results prove:

**Proposition 4.** With the market structure in (1)-(8) where the concentrated industry has market power only in the output market, the informed concentrated industry restricts the output market quantity more (less) than suggested by the traditional partial equilibrium monopoly case if the general equilibrium demand is more convex than the partial equilibrium demand.

Certainly in the case of linearity or where the market quantity is smaller with informed monopoly, the deadweight loss will be larger than in the conventional partial equilibrium monopoly case or, equivalently, without a related sector. Also, as in conventional monopoly models, both consumer welfare and overall social efficiency are harmed by monopoly behavior.

Perhaps surprisingly, one of the most interesting implications of the general equilibrium lens is that the ability to exploit a market is increased by having a related sector regardless of whether the related good is a complement or a substitute product. The Department of Justice

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\(^{11}\) Such results depend heavily on third derivatives. For example, one can show that \(\text{sign}(p_{yyy} - u_{yyy}) = (-)\text{sign}(u_{y z})\) if \(2u_{y z}\) is smaller (greater) than both \(u_{y z}u_{y y} / u_{zz}\) and \(u_{y z}^2 / u_{y y}^2\). Similar relationships arise in relating \(p_{yyy}\) to \(p_{yyy}^*\).

Proposition 4 applies for partial equilibrium demands conditioned on either prices or quantities.
Guidelines provide a rule for determining the relevant market that depends on the ability of a firm to profit from raising price by 5 percent or 10 percent. Propositions 2 through 4 show that this ability may be possible given equilibrium adjustments in related markets even though it is not present under the price-dependent ordinary partial equilibrium elasticity of the immediate demand facing the firm. Thus, many more cases may pass the Guidelines rule if equilibrium adjustments in other markets are considered.

**Market Power Only In the Input Market**

A single firm using input $x_y$ to produce output $y$ would have market power only in the input market if many firms that do not use input $x_y$ employ alternative production technologies to produce output $y$. This might be the case if only one firm, either by patent or trade secret, has a process that uses input $x_y$ to produce $y$. In this case, the price $p_y$ would be unaffected by $y$ industry activity and the demand for $z$ can be represented effectively by\(^{12}\)

\[(2'') \quad p_z = u_z(z).\]

Accordingly, equation (1) is dropped and equation (2) is replaced by (2'') in the system that describes equilibrium. The system in (2''), (3)-(7) can be reduced to

\[(10'') \quad w_y = c_y(x_y, x_z)\]

\[(11'') \quad c_z(x_y, x_z) = u_z(z(x_z))z'(x_z)\]

\(^{12}\) If only $p_z$ is fixed, the representative consumer's optimized utility can be represented by an indirect utility function, $V(p_y, p_z, m)$. By Roy's identity, the demands are $y = -\left(\frac{\partial V}{\partial p_y}\right)\left(\frac{\partial V}{\partial m}\right) = d_y(p_y, p_z, m)$ and $z = -\left(\frac{\partial V}{\partial p_z}\right)\left(\frac{\partial V}{\partial m}\right) = d_z(p_y, p_z, m)$. If $y$ is available in perfectly elastic supply because other industries produce it, then only the latter condition is relevant for the single firm that produces $y$ using $x_y$. Because $p_y$ and $m$ are predetermined, the latter condition can be regarded simply as a monotonic functional relationship between $z$ and $p_z$, which for continuity we simply represent as $p_z = u_z(z)$. Note alternatively that if intuition suggests that the price of output $z$ should be unaffected by $y$ industry activity if the price of output $y$ is unaffected, then the only difference in results in this section is that both equations (1) and (2) are dropped from the system that determines equilibrium supply and demand for the $y$ industry. Under such circumstances, all qualitative results obtained in this section are unaltered. The only quantitative difference is that all terms involving $u_{zy}$ vanish and $u_y$ is simply replaced by $p_z$. 

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which define implicitly the relevant general equilibrium supply and demand relationships facing the firm producing $y$.

Comparative static analysis of (10") and (11") yield

$\frac{dw_y}{dx_y} = (u_{zz}z'^2 + u_{zz}z^2)c_{yy} - (c_{zz}c_{yy} - c_{zy}^2) = c_{yy} + c_{zy} \frac{dw}{dx_y} = s_{yy} > 0$

where, for later notational simplicity, $s_{yy}$ is defined as the slope of the general equilibrium supply of $y$, considering equilibrium adjustments in the $z$ sector, which in this case is

$s_{yy} = c_{yy} + \frac{c_{zy}^2}{\pi_{zz}} > 0$.

Positivity of $s_{yy}$ is evident from the first right-hand side expression of (12").

In this case, the first-order condition for (8) using (12") is

$w_y = p_yy' - s_{yy}x_y$.

This condition requires that the equilibrium input price, $w_y$, must be less than the ordinary value marginal product, $p_y'$, for the $y$ industry because $s_{yy}x_y > 0$. This result is depicted in Figure 4 where market $n$ represents the $y$ market and

$\delta = s_{yy}x_y > 0$.

In this case, the general equilibrium marginal revenue coincides with the general equilibrium output demand and all partial equilibrium output demands, which are all perfectly elastic.

Again, the second-order condition involves complicating third derivatives, in this case of the cost function and $z$ production function. Some local conditions are possible where the second-order condition fails. But ignoring third derivatives, the condition reduces to

$p_yy'' = -c_{yy} + c_{zy}^2 + c_{zy}^2 + \frac{2u_{zz}z'' + u_{zz}z'}{\pi_{zz}^2} dx_y < 0$.

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The first and second terms are negative and the third term is positive. The sign of the fourth term is opposite that of $u_{yz}$, and the sign of the fifth term is opposite that of $c_{yz}$. It suffices to note that this condition can possibly be satisfied in some circumstances when $c_{yz} > 0$ and in some circumstances when $c_{yz} < 0$, although the quantitative possibilities for $c_{yz}$ are broader when $u_{yz}$ is smaller negatively or larger positively.

These results show that if the $y$ industry does not have the ability to alter its output price by indirectly affecting industry $z$ activity through input buying behavior, then the $y$ industry cannot increase profits by overbuying the input. Because the general equilibrium supply of its input is upward sloping in its input quantity, a rather traditional monopsony result is obtained where the input market quantity is restricted.

**Proposition 5.** With the market structure in (1)-(8), if the concentrated industry has market power only in the input market then neither input overbuying nor output overselling are profitably sustainable. Input market purchases are restricted to reduce the input price.

Even though this result and its intuition is similar to the typical monopsony pricing result, the same equilibrium does not occur if the $y$ industry optimizes its profit in the conventional partial equilibrium sense. To see this, note that the traditional partial equilibrium monopsony pricing rule equates the monopsonist's value marginal product and marginal outlay where the marginal outlay is based on the ordinary input supply. As in the case of demand, two alternative approaches can be used to specify the ordinary partial equilibrium supply. With the quantity-dependent ordinary demand defined by (5), the relationship of $p_y$ and $x_y$ in the $y$ market is conditioned on activity in the $z$ market as represented by quantity $z$. We call this the *quantity-dependent ordinary supply*. If this specification of the ordinary supply is used, then the first-order condition for maximizing the monopsonist's profit, $\pi_y = p_y \cdot y(x_y) - w_y x_y$, requires

$$w_y = p_y y' - c_{xy} x_y.$$
The only difference in this first-order condition and \((9')\) is that \(c_{yy}\) replaces \(s_{yy}\). Equation (17) implies that \(c_{yy} > s_{yy} > 0\) because both the numerator and denominator of the right-hand fraction are positive. Thus, the general equilibrium supply is more elastic or less steep than the quantity-dependent ordinary supply as in Figure 4.

In this case, because \(s_{yy}\) is smaller than \(c_{yy}\), the \(y\) market has a smaller distortion with informed monopsony behavior (accounting for equilibrium adjustments in other markets) than with quantity-dependent partial equilibrium monopsony behavior. Accordingly, as shown by comparing the respective first-order conditions in \((9')\) and (18), the distortion measured by the difference in the marginal revenue product and input price, \(p_{y}y' - w_{y}\), is less under informed monopsony behavior.

**Proposition 6 (Quantity Conditioning).** *With the market structure in \((1)-(8)\), if the concentrated industry has market power only in the input market then the concentrated industry maximizes profit by introducing a smaller monopsony distortion in price than associated with conventional partial equilibrium monopsony analysis conditioned on quantities in the related market, regardless of whether the inputs are complements or substitutes in supply.*

Proposition 6 implies that monopsonistic firms cannot gain as much monopsony profit as conventional estimates based on partial equilibrium models would suggest. The reason is that general equilibrium supplies that account for adjustments in other markets are more elastic than ordinary supplies that hold quantities constant in related markets. Intuitively, when input suppliers can switch to or from supplying other input markets, then their response in supplying the \(y\) industry is greater.

Alternatively, the ordinary partial equilibrium supply can be specified as conditioned on the price \(w_{y}\) rather than the quantity \(x_{y}\). We call this the *price-dependent ordinary supply*. In this case, the properties of the supply of \(x_{y}\) are found by comparative static analysis of (5) and (6).
Holding the price $w_z$ constant implies $dw_z = 0$, which according to (6) yields $dx_z / dx_y = -c_{yz} / c_{zz}$.

Using this result to totally differentiate (5) reveals the slope of the price-dependent ordinary supply that is conditioned on $w_z$ as

\begin{equation}
    s_{yy}^* = dw_y / dx_y = c_{yy} + c_{yz} dx_z / dx_y = c_{yy} - c_{yz}^2 / c_{zz}.
\end{equation}

If this specification of the ordinary supply is used in the monopsonist's calculus, then the first-order condition for maximizing profit, $\pi_y = p_y y(x_y) - w_y x_y$, is

\begin{equation}
    w_y = p_y y' - s_{yy}^* x_y.
\end{equation}

Comparing to (17) reveals that $c_{yy} > s_{yy} > s_{yy}^*$, which yields the interesting result that the general equilibrium supply is less elastic or steeper than the price-dependent ordinary demand, even though it is more elastic than the quantity-dependent ordinary supply.

In this case, because $s_{yy}^*$ is smaller than $s_{yy}$, the $y$ market has a larger distortion with informed monopsony behavior (which accounts for equilibrium adjustments in other markets) than price-dependent partial equilibrium monopsony analysis would suggest. Accordingly, as shown by comparing the respective first-order conditions in (9") and (20), the difference in the marginal revenue product and input price, $p_y y' - w_y$, is more under informed monopsony behavior, i.e., monopsonistic firms gain more monopsony profit than price-dependent partial equilibrium estimates would suggest. The reason is that general equilibrium supplies that embody price adjustments in other markets are more elastic than ordinary supplies that hold prices constant in related markets.

**Proposition 6' (Price Conditioning).** With the market structure in (1)-(8), if the concentrated industry has market power only in the input market then the concentrated industry maximizes profit by introducing a larger monopsony distortion in price than associated with partial equilibrium monopsony analysis based on price data from the related market, regardless of whether the inputs are complements or substitutes in supply.
Intuitively, much like the monopoly case, the price-dependent ordinary supply allows input suppliers to shift toward supplying inputs to the \( z \) industry as the price of \( x_y \) is reduced, which accounts for the more elastic nature of the ordinary supply compared to the quantity-dependent case. However, it ignores the upward downward movement of the price of \( x_z \) that occurs in general equilibrium, which is why the general equilibrium supply of \( x_y \) is less elastic than the price-dependent ordinary supply.

The critical question is which specification of the ordinary supply is appropriate for comparison. We suggest that the answer to this question depends on the circumstances of application. Because our purpose is to contrast the implications of general equilibrium analysis with typical partial equilibrium analysis, the question comes down to how a business manager assesses his input supply, or how economists, lawyers, and the courts estimate supply relationships in analyzing monopsony behavior. While typical specifications of supply systems derived with the profit function approach depend on prices rather than quantities of other outputs, such analyses are typically infeasible because of data limitations. Price-dependent analysis on the demand side can be conditioned on prices because final goods price data are relatively observable and abundant. However, supply side analysis is often severely hampered by unavailability of proprietary price data even though trade organizations often publish some form of quantity data.\(^\text{13}\) For this reason, a supply specification used for practical purposes may tend to control for the conditions in related markets with quantities rather than prices.

If this is the case, then the appropriate results for comparing general and partial equilibrium monopsony are reflected by (18), while if such analyses are based on price data from related input markets then the corresponding results are reflected by (20). For the purposes of this paper, the results based on (20) are basically the mirror image of the monopoly comparison\(^\text{13}\)

\(^\text{13}\) While lawyers and expert witnesses may have access to the proprietary data of their clients or legal opponents in legal proceedings, access to the proprietary data of indirectly related industries is unlikely.
of the previous section based on price-dependent specifications. For this reason, we focus the remainder of this section on the more interesting case where business managers and legal analysts can observe only quantities in related input markets (although the results other than Proposition 6 are similar in the price-dependent case). Thus, the appropriate specification of the ordinary supply of $x_y$ is conditioned on quantities rather than prices in the $x_z$ market.

Not surprisingly, from (13''), the effect of monopsony behavior by the $y$ industry (which reduces $x_y$ from the competitive equilibrium after equilibrium adjustments) is either to reduce $z$ industry activity (input and output levels) if $y$ and $z$ are complements, or increase $z$ industry activity if $y$ and $z$ are substitutes. In either case, according to (12''), this response makes the general equilibrium supply of $x_y$ facing the $y$ industry more elastic than if no adjustment occurred in the $z$ industry (i.e., as in the partial equilibrium case).

In the case of complements in supply, reducing the price of $x_y$ by restricting purchases reduces the supply and increases price $w_z$ for the $z$ industry. In turn, in general equilibrium, the $z$ industry reduces purchases of $x_z$, which reduces the ordinary supply of $x_y$ to the $y$ industry, thus making the general equilibrium supply facing the $y$ industry more elastic than the ordinary supply that holds $z$ industry quantity constant. In the case of substitutes, reducing the price of $x_y$ by restricting purchases increases the supply and reduces price $w_z$ for the $z$ industry. As a result, in general equilibrium, the $z$ industry increases purchases of $x_z$, which reduces the ordinary supply of $x_y$ to the $y$ industry, thus making the general equilibrium demand facing the $y$ industry more elastic than the ordinary supply that holds $z$ industry quantity constant. This is why the $y$ industry has less market power and distorts the price in the $x_y$ market less considering equilibrium adjustments of the related industry than in the case of partial equilibrium optimization.

While the price distortion is less with informed monopsony, another issue is whether the market quantity with informed monopsony is less than suggested by conventional partial
equilibrium monopsony pricing. Where competitive equilibrium is denoted by overbars, the competitive equilibrium satisfies $\bar{w}_y = \bar{p}_y y'$. Subtracting this relationship from (18) yields $w_y - \bar{w}_y = (p_y - \bar{p}_y) y' - c_{yy} x_y = -c_{yy} x_y$, where the latter equality follows from perfectly elastic output demand, which implies $p_y - \bar{p}_y = 0$. Following the partial equilibrium monopsony calculus, which holds $x_z$ fixed in (5), $\bar{w}_y - w_y = \frac{\partial^2}{\partial y^2} c_{yy} dx_y$. Thus, the partial equilibrium monopsony solution has $x_y$ satisfying $c_{yy} = \frac{\partial^2}{\partial y^2} c_{yy} dx_y$. Under linearity ($c_{yy}$ is constant), this condition becomes $c_{yy} x_y = (\bar{x}_y - x_y) c_{yy}$, which yields the familiar result, $x_y = \bar{x}_y / 2$.

Comparing to the informed equilibrium monopsony case, subtracting the competitive equilibrium condition from (9") rather than from (20) yields $w - \bar{w} = (p_y - \bar{p}_y) y' - s_{yy} x_y = -s_{yy} x_y$, where the latter equality follows from perfectly elastic output demand. In this case, $\bar{w}_y - w_y = \frac{\partial^2}{\partial y^2} s_{yy} dx_y$. Thus, the informed equilibrium monopsony input satisfies $s_{yy} = \frac{\partial^2}{\partial y^2} s_{yy} dx_y$. Under linearity of both supply and industry technology ($c_{yy}, c_{zy}, c_{yz}, z'$ are constants and $z'' = 0$), this condition also yields $x_y = \bar{x}_y / 2$. Thus, with linearity, both approaches restrict the market quantity to the same degree while the price distortion is smaller under informed monopsony behavior. Thus, the deadweight loss is smaller under informed monopsony behavior.

More generally, these results show that both the conventional partial equilibrium monopsony quantity and the informed monopsony quantity can be greater (less) than half of the competitive market quantity as the corresponding supply is upward (downward) bending. Such analysis also reveals that the general equilibrium supply is more upward bending or less downward bending than the ordinary supply if $s_{yy} y > c_{yy}$, in which case the informed monopsony

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14 For simplicity, when inside an integral, $c_{yy}$ is assumed to vary with $x_y$ along the path of integration. But, when appearing outside an integral, $c_{yy}$ is assumed to be evaluated at the optimal partial equilibrium monopsony solution.

15 For simplicity, when inside an integral, $s_{yy}$ is assumed to vary with $x_y$ along the path of integration. But, when outside an integral, $s_{yy}$ is assumed to be evaluated at the optimal informed monopsony solution.
quantity is greater than the conventional partial equilibrium monopsony quantity, while the opposite is true in the converse case.\textsuperscript{16}

**Proposition 7.** With the market structure in (1)-(8) where the concentrated industry has market power only in the input market, the concentrated industry restricts the market quantity less (more) than suggested by the conventional partial equilibrium monopsony case if the general equilibrium supply is more (less) convex than the partial equilibrium supply.

Certainly in the case of linearity or where the market quantity is greater with informed monopsony, the deadweight loss will be smaller than in the conventional partial equilibrium monopsony case or, equivalently, without a related sector. In this case, because the price of the y industry output if fixed, whether consumers are better off or worse off depends on the indirect effects on the price of the z industry output. If \( c_{yz} > (\) 0 then \( dx_z/dx_y < (\) 0 implying that industry z output increases (decreases) as the y industry moves from competitive to monopsonistic behavior, which following (2") can only occur if \( p_z \) decreases (increases). Thus, consumers gain if the inputs are substitutes \( (c_{yz} > 0) \) and lose if the inputs are complements \( (c_{yz} < 0) \) even though social welfare is harmed in either case.

The results of this paper thus demonstrate an interesting contrast between the monopoly and monopsony cases when price data on related consumer markets are available but only quantity data on related input markets are available. Partial equilibrium analysis overestimates the actual ability of a firm to exploit an input market and underestimates the actual ability of a firm to exploit an output market when there is a related sector. These results demonstrate that showing a firm has the ability to manipulate price by a given amount, such as specified by the Department of Justice Guidelines, is not valid in either case if done with ordinary partial equilibrium analysis.

\textsuperscript{16} Such results depend heavily on third derivatives. For example, one can show that \( s_{yy} - c_{yy} > (\) 0 if \( c_{yy} < (\) 0 and \( c_{yy} > (\) 0 implying that \( u_{yy} z^2 + u_{yx} z^+ \). Proposition 7 applies for partial equilibrium supplies conditioned on either prices or quantities.
Market Power in Both Input and Output Markets

Finally, we consider the more general case where the $y$ industry consists of a single firm or colluding firms that have market power in both their input and output markets. In this case, equilibrium is described by (1)-(7). For the purpose of deriving the core results, we introduce the following definition, which facilitates a shorthand notation representing the strength of substitution in input markets versus complementarity in output markets, upon which many results depend.

**Definition.** Define $S = c_{yz} - u_{yx}y'z'$ as the measure of input substitution relative to output complementarity where complementary is represented by the additive inverse of substitution as measured by the cross derivative of consumer utility or input industry cost. If $S > (\prec) 0$ then inputs are more (less) substitutes in supply than outputs are complements in demand (which also includes the case where outputs are substitutes in demand), or inputs are less (more) complements in supply than outputs are substitutes in demand. Similarly, if $S < (\succ) 0$ then outputs are more (less) complements in demand than inputs are substitutes in supply (which also includes the case where inputs are complements in supply), or outputs are less (more) substitutes in demand than inputs are complements in supply. All cases where $S > (\prec) 0$ will be described as having input substitution greater (less) than output complementarity. If both inputs and outputs are substitutes (complements), then $S > (\prec) 0$.

The intuition of this definition follows from noting that $c_{yz}$ is the cross derivative of the cost function of the supplying industry with respect to the two input quantities, while $u_{yx}y'z'$ is the cross derivative of consumer utility with respect to the two input quantities after substituting the production technologies, $u(y, z) = u(y(x_y), z(x_z))$. For simplicity, the relationship of inputs
will always refer to input supply and the relationship of outputs will always refer to output demand.

Again, for given $x_y$ the system composed of (1)-(7) can be reduced to the two equation system in (10) and (11), which generates (12) and (13), for which further manipulation reveals

\[
(12'') \quad \frac{dp_y}{dx_y} = u_{yy}y' + \frac{u_{yz}z'S}{\pi_{zz}} = p_{yy}y' > (>) 0 \text{ as } u_{yz}S < (>)-u_{yy}\pi_{zz}y'/z' \\
\]

\[
(13'') \quad \frac{dw_y}{dx_y} = c_{yy} + \frac{c_{yz}S}{\pi_{zz}} = s_{yy} > (>) 0 \text{ as } c_{yz}S < (>)-c_{yy}\pi_{zz}. \\
\]

An interesting aspect of these results is that the general equilibrium demand is not necessarily more or less elastic than the ordinary demand. From (12''), $p_{yy}$ differs from $u_{yy}$ by

\[
(21) \quad p_{yy} - u_{yy} = \frac{u_{yz}z'S}{\pi_{zz}}y' > (>) 0 \text{ as } u_{yz}S < (>)-0.
\]

**Proposition 8 (Quantity Conditioning).** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium demand relationship facing the concentrated industry is less (more) elastic than the ordinary demand conditioned on quantity in the related output market if outputs are complements and input substitution is greater (less) than output complementarity, or outputs are substitutes and input substitution is less (greater) than output complementarity. In particular, the general equilibrium demand relationship is more elastic than the quantity-dependent ordinary demand if either both inputs and outputs are substitutes or both are complements.

Intuitively, when inputs are substitutes, monopsonizing the $x_y$ input market by reducing purchases causes an increase in supply of inputs to the $z$ industry and thus an increase in $z$ industry output, which, if $y$ and $z$ are complements, causes an increase in demand for $y$ that permits further exploitation by the $y$ industry in its output market. Thus, the general equilibrium demand is less elastic than where these adjustments are ignored. Conversely, when inputs are complements, monopsonizing the $x_y$ input market by reducing purchases causes a reduction in
supply of inputs to the \( z \) industry and thus a decrease in \( z \) industry output, which, if \( y \) and \( z \) are substitutes, causes an increase in demand for \( y \) that permits further exploitation. Thus, the general equilibrium demand is less elastic than where these adjustments are ignored just as in the case where inputs are substitutes and outputs are complements.

Alternatively, the elasticity of the general equilibrium demand can be compared to the elasticity of the more common ordinary demand conditioned on price in the related market, which is characterized by (15) and implies

\[
p_{yy} - p_{yy}^* = \frac{u_{yz}^2 \pi_{zz} + u_{zy}^z y' S}{\pi_{zz} u_{zz} y'} \begin{cases} > ( < \) 0 as \( u_{yz}^z \) \( < ( > \) \) as \( u_{zy}^z \) \( > ( < \) \)
\end{cases}
\]

or equivalently as \( c_{yz} u_{yz} < ( > \) \( (u_{yy} - p_{yy}^*)((c_{zz} - u_z z')(y'/z')).\)

**Proposition 8' (Price Conditioning).** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium demand relationship facing the concentrated industry is less elastic than the price-conditioned ordinary demand in every case where it is less elastic than the quantity-conditioned ordinary demand, and is more likely to be so as (i) the difference in the quantity-conditioned and price-conditioned demand elasticities is greater, (ii) the marginal productivity of the competitive industry is smaller compared to the concentrated industry, (iii) the marginal productivity of the competitive industry is more rapidly diminishing, and (iv) the competitive sector’s marginal input cost is more rapidly increasing. In particular, the general equilibrium demand relationship is less elastic than the price-dependent ordinary demand if either inputs are substitutes while outputs are complements or outputs are substitutes while inputs are complements.

The intuition of additional conditions in this proposition is as follows. The indirect price effects through the \( z \) industry of a reduction in purchasing of \( x_y \) in the case of input substitutes tends to cause a larger increase in the price of \( x_z \) when the marginal cost of \( x_z \) is increasing more rapidly (for a given effect of \( x_y \) on that marginal cost). Further, the increase in the price of \( x_z \)
tends to be translated into a larger increase in the price of z if the marginal productivity in the z industry is diminishing more rapidly. Also, the transmission of effects of changing $x_y$ through the z industry tends to be relatively greater than through the y industry as the marginal productivity in the z industry is relatively greater than the marginal productivity in the y industry. Finally, as the difference in quantity-conditioned and price-conditioned demand elasticities given by $u_{yy} - p_{yy}^* = u_{yx}/u_{zz}$ is greater, the cross-price effects on the y market arising from the z industry are greater making the general equilibrium demand less elastic. Similar reasoning applies to the case where inputs are complements and outputs are substitutes except that the z industry declines.

A further interesting and peculiar nature of the equilibrium relationship in $(12'')$ is that the general equilibrium demand facing the y industry is not necessarily downward sloping. In fact, comparing to $(21)$ as $\pi_{zz}$ approaches zero, the condition for $p_{yy} > 0$ becomes the same as for $p_{yy} > u_{yy}$. Recalling that $\pi_{zz} = u_{zz}\sigma^2 + u_{z}z^\sigma - c_{zz}$, this is the case where the technologies that produce z and $x_z$ approximate linearity ($z^\sigma \approx 0$, $c_{zz} \approx 0$) and consumer demand for z approximates linearity ($u_{zz} \approx 0$).

**Proposition 9.** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and production and demand in the competitive industry are approximately linear, the general equilibrium demand becomes upward sloping if outputs are substitutes and input substitution is greater than output complementarity, or outputs are complements and input substitution is less than output complementarity.

While upward sloping demands are generally counterintuitive according to accepted economic wisdom, the possibility exists with general equilibrium adjustment when the effects of adjustment are transmitted more effectively through the competitive industry than the concentrated industry. Consider the case where the y industry increases production and input use. Intuitively, when inputs are substitutes, increasing input purchases causes a reduction in supply.
of inputs to the \( z \) industry and thus a reduction in \( z \) industry output, which, if \( y \) and \( z \) are substitutes, causes an increase in demand for \( y \). If this transmission of effects through the \( z \) industry is sufficiently effective, e.g., because marginal productivity in the \( y \) industry is relatively low, then this upward pressure on the demand for \( y \) can be greater than the downward pressure on \( p_y \) caused by the increase in \( y \) output. If so, then the general equilibrium demand for \( y \) is upward sloping.

To examine plausibility of the conditions in Proposition 9 and later results, we will consider extreme but plausible cases of substitution and complementarity. For perfect substitutes in demand, we consider the specific case where the utility function takes the form \( u(y,z) = u(y + z) \). Thus, demand in implicit form satisfies \( p_y = p_z = p = u'(y + z) \) in which case \( u_y = u_z = u' > 0 \) and \( u_{yy} = u_{zz} = u_{yz} = u'' < 0 \). In this case, \( u_{yz} < 0 \) and the assumption \( u_{yy}u_{zz} - u_{yz}^2 \geq 0 \) is satisfied with strict equality. With perfect substitutes in demand, both industries effectively sell into the same market. Conversely, we define perfect complements in demand as the case where \( u_{yz} > 0 \) and \( u_{yy}u_{zz} - u_{yz}^2 \geq 0 \) is satisfied with strict equality. For simplicity in this case we assume \( u_{yy} = u_{zz} = -u_{yz} = u'' < 0 \). While various other definitions of perfect complements are used in standard consumer theory, this case is sufficient to demonstrate plausibility of certain possibilities, and this terminology simplifies subsequent discussion.\(^ {17} \)

Nevertheless, more general results are also indicated parenthetically for cases where \( u_{yy} \neq u_{zz} \).

Similarly, let the case where \( x_y \) and \( x_z \) are perfect substitutes in supply be defined by the case where \( c(x_y,x_z) = c(x_y + x_z) \). With perfect substitutes in supply, both industries effectively use the same input in their respective production processes. Thus, supply of the input in implicit

\(^ {17} \) A typical approach defines perfect complements as the case where consumption must occur in fixed-proportions, in which case derivatives of the utility function are discontinuous (or more generally where a unique combination of goods is consumed at each indifference level). We use a weaker definition of perfect complements that maintains continuity of second-order derivatives of the utility function and limits the degree of complementarity by concavity of the utility function, which permits deriving the cases of complements and substitutes simultaneously. However, identical qualitative results can be derived for separate analyses of the more typical case of perfect complements involving fixed-proportions consumption.
form becomes \( w_y = w_z = w = c'(x_y + x_z) \) in which case \( c_y = c_z = c' > 0, \ c_{yy} = c_{zz} = c_{yz} = c^* > 0, \)
\( \hat{c}_w = 1/c^* \), and \( \hat{c}_x = -1 \). This is the extreme case of positive \( c_{yz} \) where the assumption
\( c_{yy} c_{zz} - c_{yz}^2 \geq 0 \) is satisfied with strict equality. Similarly, we define as perfect complements in
supply the case where \( c_{yz} \) is negative and \( c_{yy} c_{zz} - c_{yz}^2 \geq 0 \) is satisfied with strict equality. For
simplicity in stating results, we assume in this case that \( c_y = c_z = c' > 0 \) and
\( c_{yy} = c_{zz} = -c_{yz} = c^* > 0 \). But more general results are also indicated parenthetically for cases
where \( c_{yy} \neq c_{zz} \).

To see that each of the conditions of Proposition 9 are plausible, suppose that the \( z \)
industry technology is linear and both inputs and outputs are perfect complements or both are
perfect substitutes. Then the condition in (12") can be expressed as

\[(u_{yy} u_{zz} - u_{yz}^2) y'(z' - z) + u_{yy} y'(u_{zz} z - c_{zz}) + c_{yz} u_{yz} z' = u^* c^*(z' - y') < (>) 0 \text{ as } z' > (z') y'. \]

While the first left-hand term is generally non-negative, it vanishes with perfect substitutes or
perfect complements in demand. The third left-hand term is negative if both inputs and outputs
are complements or both are substitutes and dominates the second term in the case where both
inputs and outputs are perfect substitutes or perfect complements (or more generally where
\( c_{zz} u_{yy} \geq c_{yz} u_{yz} \)) if \( z'' = 0 \) and \( z' > y' \). In this case, from (12")
\[ p_{yy} = (z'/y' - 1)u^* c^*/(u^* z'^2 - c^*) > 0 \text{ if } z' > y'. \]Thus, the general equilibrium demand is upward
sloping if the marginal productivity in the \( z \) industry is higher than in the \( y \) industry. With these
results, Proposition 9 can be restated.

\[ \text{---} \]

18 Various definitions of perfect complements are used in the production literature as well. A typical approach is to
represent perfect complements by fixed-proportions production, in which case the cost function has discontinuous
derivatives. Another approach considers production of a composite good, say \( x \), such that the production of \( x_y \) and \( x_z \)
are each monotonically increasing functions of \( x \). Here we maintain continuity of second-order derivatives of the
cost function by limiting complementarity by convexity of the cost function so the results for complements and
substitutes can be derived simultaneously. However, identical qualitative results can be derived from a separate
analysis of fixed-proportions production of inputs as long as conflicting fixed proportions are not imposed on
consumption.
Proposition 9'. With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, and the competitive industry technology is approximately linear, the general equilibrium demand becomes upward sloping if both inputs and outputs are sufficiently strong substitutes or both are sufficiently strong complements, and marginal productivity in the competitive industry is sufficiently higher than in the concentrated industry.

Similarly, the general equilibrium supply is not necessarily more or less elastic than the quantity-dependent ordinary supply. From (13''), $s_{yy}$ differs from $c_{yy}$ by

\[(23) \quad s_{yy} - c_{yy} = \frac{c_{yy}S}{\pi_{zz}} \quad (> ) 0 \text{ as } c_{yy}S < (>) 0.\]

Proposition 10. With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium supply relationship facing the concentrated industry is more (less) elastic than the ordinary supply conditioned on quantity in the related output market if inputs are substitutes and input substitution is greater (less) than output complementarity, or inputs are complements and input substitution is less (greater) than output complementarity. In particular, the general equilibrium supply relationship is more elastic than the quantity-dependent ordinary supply if either both inputs and outputs are substitutes or both are complements.

Intuitively, when outputs are substitutes, monopolizing the $y$ output market by reducing the quantity sold causes an increase in demand for the output of the $z$ industry and thus an increase in $z$ industry output and input use, which, if $x_z$ and $x_y$ are substitutes, causes a reduction in supply of $x_y$ that reduces the benefit of monopsonistic exploitation by the $y$ industry in its input market. Thus, the general equilibrium supply is more elastic than where these adjustments are ignored. Conversely, when inputs are complements, monopolizing the $y$ output market by reducing the quantity sold causes a reduction in demand for the output of the $z$ industry and thus
a decrease in z industry output and input use, which, if \( x_z \) and \( x_y \) are complements, causes a reduction in supply of \( x_y \) that also reduces the benefit of monopsonistic exploitation by the \( y \) industry in its input market. Thus, the general equilibrium supply is more elastic than where these adjustments are ignored in this case as well.

Alternatively, the elasticity of the general equilibrium supply can be compared to the elasticity of the price-dependent ordinary supply, which is characterized by (19) and implies

\[
S_{yy} - S_{yy}^* = \frac{c_{yy}S}{\pi_{zz}} + \frac{c_{yy}^2}{c_{zz}} (\langle \rangle) 0 \text{ as } c_{yy}S < (>) - \frac{c_{yy}^2 \pi_{zz}}{c_{zz}}
\]

or equivalently as \( c_{yy}u_y \rangle > (\langle) (c_{yy} - s_{yy}^*) (u_{zz}z^2 + u_z z^*)/(y^2z^*) \).

**Proposition 10' (Price Conditioning).** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, the general equilibrium supply relationship facing the concentrated industry is less elastic than the price-conditioned ordinary supply in every case where it is less elastic than the quantity-conditioned ordinary supply, and is more likely to be so as (i) the difference in the quantity-conditioned and price-conditioned supply elasticities is greater, (ii) the marginal productivity of the concentrated industry is relatively smaller, (iii) the marginal productivity of the competitive industry is more rapidly diminishing, and (iv) consumers have more rapidly diminishing marginal utility of the competitive good. In particular, the general equilibrium supply relationship is less elastic than the price-dependent ordinary demand if either inputs are substitutes while outputs are complements, or outputs are substitutes while inputs are complements.

The intuition of the additional conditions in Proposition 10' is as follows. The indirect price effects on the \( z \) industry price of a reduction the quantity of \( y \) sold tend to be greater when consumers have more rapidly diminishing marginal utility in the competitive good. Further, if the marginal productivity in the \( z \) industry is more rapidly diminishing, then the increase in \( z \) industry activity widens the margin between input and output prices. Again, the transmission of
effects through the $y$ industry tend to be relatively less as marginal productivity in the $y$ industry is relatively less, and this is particularly true relative to marginal productivity in the $z$ industry when $z^* = 0$. Finally, as the difference in quantity-conditioned and price-conditioned supply elasticities given by $c_{xy} - s_{yy}^* = c_{yz}^2/c_{zz}$ is greater, the cross-price effects on the $x_y$ market arising from the $z$ industry are greater making the general equilibrium supply less elastic. Similar reasoning applies to the case where outputs are complements and inputs are substitutes except $z$ industry activity declines.

Further, the general equilibrium supply facing the $y$ industry in $(13'')$ is not necessarily upward sloping. In fact, comparing to $(22)$, as $n_{zz}$ approaches zero, the condition for $s_{yy} < 0$ becomes the same as for $s_{yy} < c_{yy}$. This is the case where the technologies that produce $z$ and $x_z$ are nearly linear ($z^* \approx 0, c_{zz} \approx 0$) and consumer demand for $z$ is nearly linear ($u_{zz} \approx 0$).

**Proposition 11.** With the market structure in $(1)-(8)$ where the concentrated industry has market power in both its input and output markets, and production and demand in the competitive industry are approximately linear, the general equilibrium supply becomes downward sloping if inputs are complements and input substitution is greater than output complementarity, or inputs are substitutes and input substitution is less than output complementarity.

While downward sloping supplies are also generally counterintuitive according to accepted economic wisdom, this possibility also exists with general equilibrium adjustment when the effects of adjustment are transmitted more effectively through the competitive industry than the concentrated industry. Consider the case where the $y$ industry increases production and input use. Intuitively, when outputs are substitutes, increasing the output quantity causes a reduction in demand for the output of the $z$ industry and thus a reduction in $z$ industry input use, which, if $x_y$ and $x_z$ are substitutes, causes an increase in supply of $x_y$. If this transmission of effects through the $z$ industry is sufficiently effective, then this upward pressure on the supply of $x_y$ can be greater than the downward pressure on $w_y$ caused by the increase in the quantity of input use by
the \( y \) industry. If so, then the general equilibrium supply of \( x_y \) is downward sloping. In the case of indirect effects from output markets to input markets, a low marginal productivity causes the effects of a given output market change to be more dramatic in the input market, and therefore a low marginal productivity in the \( z \) industry relative to the \( y \) industry makes the indirect effects through the \( z \) sector more likely to dominate the direct effects of increasing production and input use in the \( y \) industry.

To see that the conditions of Proposition 11 are plausible, suppose the \( z \) technology is linear \((z^* = 0)\) and either inputs are perfect substitutes while outputs are perfect complements, or inputs are perfect complements while outputs are perfect substitutes. Then the condition in (13") can be expressed as

\[
c_{yy} u_{zz} z'^2 - c_{yx} y' z' - (c_{yx} c_{zz} - c_{zy}^2) = c^* u^* z'(z' - y') < (>) 0 \text{ as } z' > (<) y'.
\]

The last left-hand term vanishes with perfect substitutes or perfect complements in supply while the first left-hand term is negative. The second left-hand term including its sign is positive if both inputs and outputs are substitutes or both are complements. Further, the second term dominates the first if both inputs and outputs are perfect substitutes or perfect complements (or more generally if \( c_{yy} u_{zz} \geq c_{yx} u_{yz} \)) and \( y' > z' \). In this case, from (13"),

\[
s_{yy} = c^* u^* z'(z' - y') / (u^* z'^2 - c^*) < 0 \text{ if } z' < y'.
\]

Thus, the general equilibrium supply is downward sloping if the marginal productivity in the \( y \) industry is higher than in the \( z \) industry. With these results, Proposition 11 can be restated.

**Proposition 11'.** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and the competitive industry technology is approximately linear, the general equilibrium supply becomes downward sloping if both inputs and outputs are sufficiently strong substitutes or both are sufficiently strong complements and
marginal productivity in the competitive industry is sufficiently lower than in the concentrated industry.

Propositions 9' and 11' suggest that negative sloping general equilibrium supply cannot occur simultaneously with positively sloping general equilibrium demand because the conditions on marginal productivity comparisons between the two industries are mutually exclusive even with extreme cases of substitution and complementarity. Adding concavity in the z technology only makes the conditions more stringent. More generally, the slopes of the general equilibrium output demand and input supply can be compared using (12") and (13") to show that supply always cuts demand from below regardless of unconventional slopes. To do this, either the slope of the output demand must be converted by multiplying by \( y' \) for comparability because the marginal output price effect of a marginal change in input use pertains to \( y' \) units of the output at the margin, or the slope of the input supply must be converted equivalently by dividing by \( y' \). The former reveals

\[
\left[ \frac{dw_y}{dx_y} - \frac{dp_y}{dx_y} \right] \pi_z = (c_{yy} - u_{yy})u_zz' - (c_{yy} - u_{yy} y')(c_{zz} - u_{zz}z'^2) + (c_{yz} - u_{yz} z')(c_{yz} - u_{yz} y'z').
\]

To see that this expression is positive, suppose consumer utility is expressed as a function of the inputs, \( u^*(x_y, x_z) = u(y(x_y), z(x_z)) \). Then \( u^*(x_y, x_z) \) must be concave in \( x_y \) and \( x_z \) because \( u(y, z) \), \( y(x_y) \), and \( z(x_z) \) are each concave. Further, \( c(x_y, x_z) - u^*(x_y, x_z) \) must then be convex, which implies

\[
C = (c_{yy} - u_{yy})(c_{zz} - u_{zz}^2) - (c_{yz} - u_{yz})^2
\]

\[
= (c_{yy} - u_{yy} y' + u_{yy} y')(c_{zz} - u_{zz}z' + u_{zz}z'^2) - (c_{yz} - u_{yz} y'z')^2 > 0.
\]

Comparing (24) and (25) thus proves

\[
\left[ \frac{dw_y}{dx_y} - \frac{dp_y}{dx_y} y' \right] \pi_z = C - u_y y'(c_{zz} - u_{zz}z' + u_{zz}z'^2) > C > 0.
\]

**Proposition 12.** With the market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets then the general equilibrium supply relationship
facing the concentrated industry after transformation by its production technology always intersects the general equilibrium demand relationship from below.

Proposition 12 is worded in terms of the general equilibrium relationships as a function of the output \( y \), rather than the input \( x_y \). The result is proven above in terms of the input but holds for output as well because the production transformation is monotonic. With Proposition 12, analyzing the sign of \( \delta \) is sufficient to determine whether the equilibrium input use of the concentrated sector is larger or smaller than in the competitive equilibrium. Because the input quantity and output quantity of the concentrated industry have a monotonic relationship, both will be above the competitive level if either is, and both will be below the competitive level if either is. But by Proposition 12, the conditions for overbuying and overselling are mutually exclusive.

To consider the net effect of the results above, the first-order condition for maximizing \( y \) industry profit, \( \pi_y = p_y \cdot y(x_y) - w x_y \), is

\[
(9'') \quad w_y = \frac{dp_y}{dx_y} y + p_y y' - \frac{dw_y}{dx_y} x_y.
\]

Equations (12'') and (13'') imply, in the notation of Figure 1, that

\[
(26) \quad \delta = -\frac{dp_y}{dx_y} y + \frac{dw_y}{dx_y} x_y = \left[ u_{yy} y' + \frac{u_{yz} z'}{\pi_z} \right] y + \left[ c_{yy} + \frac{c_{yz} y z'}{\pi_z} \right] x_y
\]

\[
= c_{yy} x_y - u_{yy} y y' + (c_{yz} x_y - u_{yz} y z')(S / \pi_z).
\]

Because either \( dp_y/dx_y \) can be positive or \( dw_y/dx_y \) can be negative, the result in (26) raises the question of whether the distortion \( \delta \) can be negative. If \( \delta > 0 \), as in the cases of either monopoly or monopsony alone, then the \( y \) industry reduces its production to exercise market power most profitably. However, if \( \delta < 0 \), then the \( y \) industry finds expanding production and input use beyond the competitive equilibrium increases profit. If this occurs because the general equilibrium demand is upward sloping as in Propositions 9 and 9', then the firm with market
power in both its input and output markets finds bidding up the price of its input, by buying more than in the competitive equilibrium, increases its demand sufficiently that the increase its revenue from monopoly pricing more than offsets the cost of buying its input (and more of it) at a higher input price. Thus, if \( \delta < 0 \) and \( z' > y' \), then overbuying of the input occurs in the conditions of Propositions 9 and 9', which is motivated by the increased ability to exploit market power in the output market.

On the other hand, if \( \delta < 0 \) occurs because the general equilibrium supply is downward sloping as in Propositions 11 and 11', then the firm with market power in both its input and output markets finds bidding down the price of its output and selling more than in the competitive equilibrium to increase its input supply sufficiently that the reduction in its cost with monopsony pricing more than offsets the loss of revenue from selling its output at lower prices. Thus, if \( \delta < 0 \) and \( z' < y' \), then overselling of the output would occur under the conditions of Propositions 11 and 11', as motivated by the increased ability to exploit market power in the input market.

To clarify the outcomes that are possible under (26), we consider special cases involving either perfect substitutes or perfect complements in input supply and output demand. With perfect substitutes, both industries use the same input and effectively sell into the same output market using different technologies for production. If the \( z \) technology is linear and \( y' \) is represented as \( y' = z'e \) where both inputs and outputs are perfect substitutes or both are perfect complements, then

\[
\bar{\delta} = c^r x_y - u^r y z' e + \frac{\left(c^s x_y - u^s y z'\right)\left(c^r - u^r z'^2 e\right)}{u^r z'^2 + u' z^2 - c^r} = 0 \text{ if } z^* = 0 \text{ and } e = 1.
\]

Differentiating \( \delta \) with respect to \( e \) to determine the sign of \( \delta \) by whether \( z' > (\leq) y' \) or, equivalently, by whether \( e < (\geq) 1 \) when \( z^* = 0 \) obtains

\[
\frac{\partial \delta}{\partial e} = \frac{c^r u^r z' (y - x_y z')}{\pi_{zz}} > (\leq) 0 \text{ as } y / x_y - z' > (\leq) 0.
\]
Thus, if \( z'' = 0 \), then \( \delta > (\leq 0) \) as \( (y' - z')(y/x_y - z') > (\leq 0) \), which implies that overbuying occurs if \( z' > y' \) and \( z' < y/x_y \), while overselling occurs if \( z' < y' \) and \( z' > y/x_y \). If marginal productivity in the \( z \) sector is diminishing \( (z'' < 0) \), then the denominator of the latter right-hand term in (26) is increased (negatively) in magnitude so the strength of the latter term that generates the possibility of overbuying or overselling \((\delta < 0)\) is reduced. Thus, the conditions leading to overbuying or overselling become more stringent.

**Proposition 13.** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and either both inputs and outputs are perfect substitutes or both are perfect complements with those of a competitive sector, overbuying of the input relative to the competitive equilibrium is profitably sustainable if the marginal productivity of the competitive sector is both greater than marginal productivity of the concentrated industry and less than the average productivity of the concentrated industry, and the competitive sector has a sufficiently linear technology. Relaxing linearity of the technology of the competitive sector further restricts the conditions for overbuying.

The intuition of Propositions 8, 9 and 9' suggests why the case of overbuying occurs only when either both inputs and outputs are substitutes or both are complements. If a firm with market power in both its input and output markets bids up the price of its input by overbuying in the case of substitutes in supply, then the supply of inputs to the competitive sector contracts and accordingly the supply of the competitive sector output declines. This can enhance output market conditions for the concentrated industry only when the outputs are substitutes so that reduced output supply and higher output price in the competitive sector increases demand for the concentrated industry. If outputs were complements when inputs are substitutes, then the reduced output of the competitive sector would drive up price for the competitive output causing demand for the concentrated industry to contract so that no benefits could be gained by overbuying the input.
Next consider the potential for overselling. Interestingly, the condition for overselling under linearity of production in the competitive sector is not symmetric with the case of overbuying.

**Proposition 14.** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets and either both inputs and outputs are perfect substitutes or both are perfect complements with those of a competitive sector, overselling of the output relative to the competitive equilibrium is profitably sustainable if the marginal productivity of the competitive sector is both less than marginal productivity of the concentrated industry and greater than the average productivity of the concentrated industry, and the competitive sector has a sufficiently linear technology. Relaxing linearity of the technology of the competitive sector further restricts the conditions for overbuying.

While Proposition 14 suggests the case for overselling the output is a mirror image of the case of overbuying, further analysis reveals that this is not the case. Positive profit requires \( p_y y - w_y x_y > 0 \) or, equivalently, \( y/x_y > w_y/p_y \). However, the first-order condition in \((9')\) implies that \( w_y / p_y = y' - (\delta / p_y) \). Combining these two conditions implies \( y/x_y > y' - (\delta / p_y) \). Because overselling requires \( \delta < 0 \), positive profit requires \( y/x_y > y' - (\delta / p_y) > y' \), which is contrary to the conditions of Proposition 14 where \( z' < y' \) and \( z' > y/x_y \) jointly imply \( y/x_y < z' < y' \). Thus, neither the case of perfect substitutes or perfect complements where \( u_{yy} u_{xz} - u_{xy}^2 = 0 \) is sufficient to generate overselling. In contrast, a similar analysis guarantees the marginal productivity condition of Proposition 13, which requires \( y/x_y < y' \) when \( \delta < 0 \).

**Proposition 15.** With the market structure in (1)-(8) where the concentrated industry has market power in both its input and output markets, sustained overselling cannot occur profitably as can the case of overbuying.
As for previous cases, the second-order conditions for Propositions 13 and 14 involve complicating third derivatives for the utility and cost functions as well as the z technology. Again, some local conditions are clearly possible where the second-order condition fails, but distinct conditions exist where the second-order condition holds for each of the special cases of Propositions 13 and 14, including cases where δ is positive and δ is negative. If third derivatives of the utility and cost functions vanish and the z production technology is linear, then the second-order condition is

\[
p_{yy} y' + p_y y^* - c_{yy} + \frac{(c_{yy} c_{zz} - c_{yz}^2) + (u_{yy} u_{zz} - u_{yz}^2)(y'' + yy^*)z'^2}{\pi_{zz}} - \frac{c_{zz} u_{yy} (y'' + yy^* + c_{yz} u_{zz} z'^2 - c_{yz} u_{yz} z'(2y' + x_y y^* < 0}{\pi_{zz}} < 0
\]

The first four left-hand terms are clearly negative if the y production technology is not too sharply downward bending, \( y'' + yy^* > 0 \). To evaluate the last term, without loss of generality, suppose similar to the approach used above for (24) that (27) is evaluated at arbitrary values of y, z, x_y, and x_z and that the units of measurement for y and z are chosen so that \( u_{yy} = u_{zz} \) and the units of measurement for x_y and x_z are chosen so that \( c_{yy} = c_{zz} \) at these arbitrary values. Then the numerator of the last term can be written as

\[
(28) \quad c_{yy} u_{yy} (y' - z')^2 + (2y'z' + yy^*) (c_{yy} u_{yy} - c_{yz} u_{yz}) + c_{yz} u_{yz} y^* (y - x_y z')
\]

where the first term is clearly negative and the second term is negative if the y production technology is not too sharply downward bending, in this case \( 2y'z' + yy^* > 0 \), because \( c_{yy} u_{yy} < c_{yz} u_{yz} \) follows from \( c_{yy} c_{zz} - c_{yz}^2 > 0 \) and \( u_{yy} u_{zz} - u_{yz}^2 > 0 \). Thus, (27) is negative as long as the last term of (28) does not dominate all other terms in (27). Two practical conditions make the last term small. First, as the y technology approaches linearity, the latter term vanishes. Second, the latter term vanishes as the average productivity of \( x_y \) in \( y \) approaches the marginal productivity of \( x_z \) in \( z \). Thus, practical cases satisfying the second-order condition can possibly
hold for all qualitative combinations of $u_{yz}$ and $c_{yz}$. In particular, when third-order terms and concavity of $z$ production are unimportant, the second-order condition holds for all cases where both inputs and outputs are substitutes or both are complements if the average productivity of $x_y$ in $y$ is less than the marginal productivity of $x_z$ in $z$, which it must be in the case of overbuying in Proposition 13 but cannot be in the case of overselling in Proposition 14.

Naked Overbuying as a Means of Exercising Market Power

Another form of predatory behavior that can be examined in a general equilibrium framework is naked overbuying where the firm with market power buys amounts either of its own input or that of its competitor that are simply discarded. To analyze this case, we consider only buying amounts of the competitor's input, which is equivalent to buying additional amounts of its own input in the case of perfect substitutes, and is a more efficient way to influence the market in the case of less-than-perfect substitutes. In this case, equation (6) is replaced by

\[(6^*) \quad w_z = c_z(x_y, x_z + x_0)\]

where $x_0$ is the amount of the competitor's input bought and discarded by the firm with market power. For this case, the system composed of (1)-(5), (6*), and (7) can be solved for

\[(10^*) \quad p_y = u_y(y(x_y), z(\hat{c}(w_y, x_y) - x_0))\]

\[(11^*) \quad c_z(x_y, \hat{c}(w_y, x_y)) = u_z(y(x_y), z(\hat{c}(w_y, x_y) - x_0))z'(\hat{c}(w_y, x_y) - x_0),\]

which define the general equilibrium supply and demand.

Comparative static analysis of (10*) and (11*) yields

\[(12^*) \quad \frac{dp_y}{dx_0} = \frac{c_{zz}u_{zz}z'(u_{zz}z'^2 + u_zz') - (u_{zz}z'^2 + u_zz')(u_{zz} - u_{yz})z'}{\pi_zz}\]

\[(13^*) \quad \frac{dw_y}{dx_0} = \frac{(u_{zz}z'^2 + u_zz')c_{yz}}{\pi_zz} > (\langle) 0 \text{ as } c_{yz} > (\langle) 0\]
as well as the same results in (12") and (13"). Further, writing (6\*') as \( w_z = c_z (x, \hat{c}(w_y, x_y)) \) yields

(29) \[
\frac{d w_z}{d x_y} = \frac{c_{yz} (u_{zz} x^z + u_{z} x^z) - c_{zz} u_{yz} y'z'}{\pi_{zz}} > (>) 0 \text{ as } c_{yz} > (>) 0 \text{ and } u_{yz} > (>) 0
\]

(30) \[
\frac{d w_z}{d x_0} = \frac{u_{zz} x^z + u_{z} x^z}{\pi_{zz}} c_{zz} > 0.
\]

**Proposition 16.** With the market structure in (1)-(5), (6\*'), and (7)-(8) where the concentrated industry has market power in both its input and output market, naked overbuying of the related industry’s input unambiguously causes the related industry’s input price to increase while it causes the industry’s own input price to increase (decrease) if inputs are substitutes (complements). Demand for the concentrated industry increases if (i) outputs are complements or (ii) outputs are perfect substitutes and the marginal cost of producing the competitive industry’s input is increasing.

To verify the latter claim of Proposition 16, note that the latter numerator term of (12\*') vanishes under perfect substitutes \( u_{zz} = u_{yz} = u^* \), but is positive (excluding the minus sign that is offset by negativity of the denominator) if \( u_{zz} - u_{yz} < 0 \). While this may appear to include all possible output relationships, some cases of near-perfect substitutes can have \( u_{zz} - u_{yz} > 0 \) without violating concavity conditions if \( u_{yy} \) is large relative to \( u_{zz} \). On the other hand, the former numerator term is negative (vanishes) when the marginal cost of producing the competitive industry’s input is increasing (constant), which together with the denominator contributes to non-negativity of \( dp_j/dx_0 \).

The firm with market power evaluating naked overbuying solves the profit maximization problem given by

(8\*') \[
\max_{x_j, x_k \geq 0} \pi_j = p_j y - w_j x_j - w_k x_k
\]
using (12\textsuperscript{m}), (13\textsuperscript{m}), (12\textsuperscript{*}), (13\textsuperscript{*}), (29), and (30). The first-order condition for $x_y$ again leads to (9\textsuperscript{m}) where (23) applies if $x_o = 0$, while the first-order condition for $x_0$ yields

$$
\frac{dp_y}{dx_0} - \frac{dw_y}{dx_0} x_y - \frac{dw_z}{dx_0} x_0 - w_z
$$

(31)

$$
= \frac{c_{zz} u_{zz} y' - (u_{zz} z' + u_{zz} z'') (u_{zz} - u_{yz}) y z'}{\pi_{zz}} - \frac{(u_{zz} z' + u_{zz} z'') (c_{yz} x_y + c_{zz} x_0)}{\pi_{zz}} - w_z.
$$

Because the signs of terms in (12\textsuperscript{m}) and (13\textsuperscript{m}) are unaffected by the addition of $x_0$ to the problem at $x_0 = 0$, the firm with market power is better off with naked overbuying if and only if the first-order condition for $x_0$ is positive when evaluated at $x_0 = 0$ where $x_y$ solves the profit maximization problem at $x_0 = 0$ (assuming second-order conditions hold). If this first-order condition is negative at this point, then the results without $x_0$ in the problem apply because the firm would choose $x_0 = 0$ at the boundary condition.

The result in (31) is qualitatively ambiguous. The first right-hand term is clearly positive if outputs are complements and the second right-hand term evaluated at $x_0 = 0$ is clearly positive if inputs are complements. Further, the left-hand term is also positive when outputs are perfect substitutes. The second right-hand term can be positive or negative but, evaluated at $x_0 = 0$, is negative (positive) if inputs are substitutes (complements). Of course, the third right-hand term is negative and can dominate if the related industry’s input price is sufficiently high.

**Proposition 17.** With the market structure in (1)-(5), (6\textsuperscript{*}), and (7)-(8) where the concentrated industry has market power in both its input and output market, naked overbuying of the related industry’s input is profitably sustainable if inputs are complements, outputs are complements or perfect substitutes, and the related input industry’s input price is sufficiently low.

The case where both inputs and outputs are complements is the case where the concentrated industry overbuys the input because the beneficial effects on its output market dominate the increased cost of input purchases. The intuition of the major case of Proposition 17
is similar but the concentrated industry is better off because it does not have to use the increased purchase of inputs to relax the monopoly-restricted size of its output market. On the other hand, if inputs are complements and outputs are substitutes then buying the competitive sector’s input and discarding it both increases the supply of the concentrated industry’s input and, because of indirect effects though discouraging industry activity, increases the concentrated industry’s demand. These effects tend to improve the concentrated industry’s ability to exploit both its input and output markets. By comparison, if inputs are substitutes then buying the competing sector’s input and discarding it not only raises the input price of the competing sector but also the input price of the concentrated sector. In this case, the output market effect of causing a contraction in industry activity must be greater to make such action profitable.

**Conclusion**

This paper has developed a framework to evaluate static explanations for predatory overbuying in input markets and predatory overselling in output markets. The intent is to fully understand predatory behavior that is profitably sustainable. Much can be learned from the comparative static analysis before developing the two-stage predatory formulation where optimality depends on a second-stage recoupment period (at least in the case with related industries).¹⁹

While the literature on predatory behavior has drawn a distinction between raising rivals’ costs and predatory overbuying that causes contraction of a related industry, our results show

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¹⁹ The conceptual results of this paper apply for various time horizons. Any substantive difference in a two-stage model will depend on having costs of expansion and contraction that differ from one another or that differ between industries. If the costs of expansion and contraction follow standard cost curves over longer time periods and are reversible as in classical theory of short-, intermediate-, and long-run cost curves, then the model of this paper is applicable and two-stage issues are inapplicable. So understanding of how two-stage results differ from classical theory depends on understanding how marginal costs of expansion differ from marginal costs of contraction.
that optimal behavior can involve a combination of the two.\textsuperscript{20} In the case of substitutes in a static model, raising rivals’ costs is the means by which contraction of the related industry is achieved. Given the existence of a related competitive industry, a firm with market power in both its input and output markets can be attracted either to overbuy its input as a means of raising rivals’ costs so as to take advantage of opportunities to exploit monopoly power in an expanded output market. Interestingly, this can be attractive even though a similar explanation over overselling is not applicable. That is, overbuying can be profitable sustainable whereas overselling appears to require a two-stage explanation with irreversibility. In contrast to the Supreme Court ruling in \textit{Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.,} 509 US 209 (1993) and \textit{Weyerhaeuser Co. v. Ross-Simmons Hard Wood Lumber Co., Inc.,} 549 US ___ (2007), these results show that (i) predatory buying in input markets will not necessarily lead to short-run costs above prices because the output market is exploited to increase output prices relatively more, and that (ii) a second-stage recoupment period after driving competitors from the market is not necessary to make this behavior profitable.

Moreover, such action may result in raising prices to consumers, which not only causes loss in overall economic efficiency, but also loss in consumer welfare in particular (thus satisfying the narrower legal definition of efficiency emphasized by Salop 2005). But this loss in consumer welfare may occur either through higher prices for the primary consumer good (in cases of overbuying where $dp_{y}/dx_{y} > 0$), or by causing a relatively higher price for a related consumer good (in cases of overbuying where $dp_{z}/dx_{y} > 0$).

A further set of results in this paper apply to the case of complements. While apparently not considered in the legal literature defining predatory behavior, overbuying can reduce costs to a related industry in the case of complements, and thus increase the ability to exploit an output

\textsuperscript{20} Of course, we recognize that much of the literature on predatory overbuying is based on the presumption that overbuying causes firms to exit, as in a two-stage case of recoupment.
market if the related output is also a complement. The general equilibrium model reveals that the case where both inputs and outputs are complements is virtually identical in effect as the case where both are substitutes. While the case of complements is less common in reality, it seems that any legal standard should treat the cases symmetrically. In any case, the generality of the results here is broad enough to consider the fundamental case that is the core of the debate between Salop (2005) and Zerbe (2005).

With the analytical understanding provided by the framework of this paper, the four-step rule proposed by Salop (2005) is shown to relate to a special case. That is, overbuying can be associated with Salop's first step of artificially inflated input purchasing. However, in the case of complements, this will not lead to injury to competitors according to Salop's second step. Yet, market power may be achieved in the output market (Salop's third step), which may cause consumer harm in the output market if outputs are also complements (Salop's fourth step).

Our results also show that issues in "buy-side" monopsony cases are not simply a mirror image of issues in "sell-side" monopoly cases when related industries are considered, especially when proprietary restrictions on data availability cause partial equilibrium analysis of monopsony to be conditioned on quantities rather than prices in related markets. Further, a sustainable form of overbuying in the input market is possible in absence of the typical two-stage predation-recoulement approach, which dominates the overselling literature, and which cannot be detected by a period when marginal costs exceed prices. These issues have previously been understood as mirror images of one another in the conventional partial equilibrium framework. However, once the equilibrium effects of market power and typical data availability are considered, partial equilibrium analysis of monopoly turns out to understate the true distortionary effects but partial equilibrium analysis of monopsony overstates the true distortionary effects.

The framework of our analysis allows standard estimates of supply, demand, and production technologies to be used to determine the resulting behavior and its deviation from
competitive standards. The general equilibrium model is the basis for determining, by standard measures of welfare economics, whether overbuying leads to consumer harm and thus violates the rule of reason under the Sherman Act. In particular, results show in a static model of perpetual predatory overbuying that the purpose of overbuying and consequent raising of rivals’ costs is to more heavily exploit the output market, which necessarily harms consumers. This can happen even if the market output of the subject good does not contract from competitive levels because greater market demand for the subject good is achieved by influencing the related output market through predatory buying of its input.

References


Figure 1. Equilibrium Measurement of the Welfare Effects of Monopolization.

$p_n$ vs $q_n$

$p_n^d(\delta_0) = p_n^*(0)$

$q_n^d(p_n^d(\delta_0)) = q_n^*(p_n^*(\delta_0))$

$q_n^d(p_n^d(0)) = q_n^*(p_n^*(0))$
Figure 2. Use of Market Power by One Industry with Parallel Vertical Structures.

(a) Graph showing the relationship between $p$, $w$, $p(y+z)$, and $w(x_y + x_z)$.

(b) Graph showing the relationship between $x_z$, $p/w$, and $x^0$.

(c) Graph showing the relationship between $w$, $p^0/y^r$, $w^0$, $ED^*$, $MO$, $x_y$, and $D^*$, $MR^*$.
Figure 3. Equilibrium Welfare Effects of Monopoly with Vertically Parallel Structure.
Figure 4. Equilibrium Welfare Effects of Monopsony with Vertically Parallel Structure.