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D. Seligson
(Ph.D. Thesis)

May 1983

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PHONON-INDUCED ENHANCEMENTS
OF THE ENERGY GAP AND CRITICAL CURRENT
IN SUPERCONDUCTING ALUMINUM

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(Ph.D. Thesis)

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May 1983

This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.
ERRATA

p.12 The two equations on this page should read,

\[
\begin{align*}
(\dot{n})_D &= \frac{N_R}{2N(0)\nu} \left\{ \left[ N_1(E-\nu)(1 \pm \frac{\Delta^2}{E(E-\nu)}) (n(E-\nu)-n(E)) \right] \\
& \quad + \left[ \nu \to -\nu \right] \\
& \quad + N_1(\nu-E)(1 \pm \frac{\Delta^2}{E(E-\nu)}) (1-n(E)-n(\nu-E)) \right\},
\end{align*}
\]

and

\[
N(0) \int_{\text{all energies}} dE \left( \dot{n} \right)_D \big|_{\Delta=0} = N_R.
\]

p.15 The equation on this page should read

\[
n_o = n - n_T = (\dot{n})_D T_E
\]

\[
= \frac{N_R \tau_E}{8N(0)T_E} \left\{ \left[ N_1(E-\nu)(1 \pm \frac{\Delta^2}{E(E-\nu)}) \right] - \left[ \nu \to -\nu \right] \\
& \quad + N_1(\nu-E)(1 \pm \frac{\Delta^2}{E(E-\nu)}) \right\}.
\]

p.23 Horizontal axes on Fig. 6 should read:

\[
B 2.4 \times 10^{-3}
\]

\[
A 0.1, 0.2, 0.3
\]

In the last line of the caption,

\[
\frac{\nu}{T_c} = 1.14 \to \frac{\nu}{T_c} = 0.88.
\]
\[ \frac{j_s^2}{\Delta^4} \rightarrow \frac{j_s^2}{\Delta^4} . \]

p.27

In the fourth line of the caption,

\[ B(0 \text{ dB}) = 1.37 \times 10^{-3} \rightarrow B(0 \text{ dB}) = 1.37 \times 10^{-2} . \]

p.97

The second and third centered equations should read,

\[ \alpha = A \left( \frac{T_E}{T_0} \right) \left( \frac{\Delta_0}{k_B T_c} \right)^3 = 6.28 \times 10^{-1} A, \text{and} \]

\[ B = 0.162 A \left( \frac{V}{T_c} \right). \]

p.138

The units for \( R_X \) are incorrectly labelled,

\[ \text{cm}^2 \text{ K} \omega^{-1} \rightarrow \text{cm}^2 \text{ K} W^{-1} . \]

p.142

The units for \( R_X \) are incorrectly labelled,

\[ \text{cm}^2 \text{ K} \omega^{-1} \rightarrow \text{cm}^2 \text{ K} W^{-1} . \]

p.143

Same as for p.142.

p.145

In row 11, under the column heading "MAXIMUM \( T_c \) ENHANCEMENT",

\[ -5 \rightarrow -1.5 . \]

In the last line of that page, denoted by (a),

\[ \delta_{300} \rightarrow \rho_{300} . \]

p.151

The reference for ESSS is missing. That reference is


p.153

The entry for Lax, E. is incomplete. It should read,

Phonon-Induced Enhancements of the Energy Gap and Critical Current in Superconducting Aluminum

by Daniel Seligson

Abstract

We investigated the enhancement of the energy gap, $\Delta$, and critical current, $i_c$, in superconducting aluminum thin films under the influence of 8 to 10 GHz phonons. The phonons were generated by piezoelectric transduction of a 1 kW microwave pulse of about 1 $\mu$sec duration. By means of a quartz delay line, the phonons were allowed to enter the aluminum only after the microwaves had long since disappeared. The critical current was measured in long narrow Al strips, in which the current flow is 1-dimensional and well described by Ginsburg-Landau theory. To measure $\Delta$ we used the Al film as one electrode in a superconductor-insulator-superconductor tunnel junction whose current-voltage characteristic gave $\Delta$ directly.

For the measurements of $i_c$, we measured the total critical current in the presence of the phonon perturbation. For the measurements of $\Delta$ we measured the change of $\Delta$ away from its equilibrium value. In both cases we report the first measurements of enhancement of these macroscopic variables under phonon irradiation. The enhancements are given in units of the effective change in reduced temperature, $[\delta T/T_c]$. The maximum enhancements we detected were $[\delta T/T_c]=-0.07$ for $i_c$ and $[\delta T/T_c]=-0.03$ for $\Delta$. We examined the power- and temperature-dependence $(0.82\leq T/T_c\leq0.994)$ of the enhancements and compared them with the prediction of a theory given by Eliashberg and coworkers. We found the gap-enhancement to be in good agreement with the theory, but only for relatively and surprisingly low input power. The critical current measurements are predicted to be in rough agreement with the $\Delta$ measure-
ments but this was not observed. The magnitude of the critical current enhancements was typically more than twice the observed gap enhancements. The measured critical current enhancement was relatively independent of temperature whereas the gap enhancement decreased rapidly as the temperature was lowered. There is at present no theoretical resolution of this matter.
DEDICATION

I dedicate this thesis to my family, and especially, to my father, David, and to my youngest niece, Hanna Lela.
Although I did all of the work for this thesis, I have had a lot of help. John Clarke, my advisor, has given me the impetus to make the kind of samples and measurements that will either stimulate other physicists' interest or answer our own questions. Ever since the post-docs left, my education has been administered by John. The post-docs are C.C. Chi, Dale van Harlingen, Tom Lemberger, and Wolf Goubau, all of whom contributed in innumerable ways to assisting me when I hardly knew electrons from holes. Their general humor, good or otherwise, and wide range of expertise made LeConte memorable and pleasurable. Of a similar vintage is Gerd Schön, who has patiently explained theoretical matters to me while we ran along the roads south of the Acquafreda di Maratea and the fire trails of Berkeley, and while we sat two floors below ground in the basement of Birge. I want to acknowledge the help and amusement I've gotten from all the members, large and small, of the Clarke group over the years.

There is no way to properly acknowledge the influence of one's peers, but I want to thank my fellow graduate students who have spent these years with me. Although there is a necessary tendency towards specialization, in the latter years of graduate school, exchanging ideas with my friends, postponed, at least, the transformation into the caricature of the geographer, in "The Little Prince," who knew more and more about less and less until he knew everything about nothing.

Gloria Pelatowski drew all the figures for this thesis, redrew several of them, and accepted unreasonable requests on unreasonable schedules with no
ill humor, and I thank her. Zack Morowitz did a heroic job of editing this thesis to get it up to department spec, and I take responsibility for any grammatical errors and stylistic anomalies that remain. Rita Jones helped in preparation of the appendices. Charlotte van Andel redrew Fig. 33 after someone lost it. Finally I want to thank Judy Epstein for making the beautiful and ever-so-reliable microwave cavity for me, without which none of this could have been possible.
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CHAPTER 1

INTRODUCTION

The field of enhanced superconductivity began with the investigations of Dayem and Wiegand (1967) and Wyatt et al. (1966) into the effects of microwaves on superconducting tin microbridges and weak links. Independently they found an enhancement of the critical current of these bridges (see Fig. 1). This effect contradicted the results at lower frequencies, where radiation had reduced the critical current. Such a reduction may be understood by thinking of the applied radiation or impressed ac current as an electromagnetic field \( A_w \) which like a static magnetic field \( A_o \), suppresses the dc critical current of a superconductor. This is sometimes called classical rectification. Dayem observed that the enhancement was present only in short, narrowly constricted microbridges and not in longer more uniform ones. Thus it was conjectured that Josephson tunneling was somehow associated with enhancement. Dayem and Wyatt's experimental findings also contradict the Josephson theory prediction concerning the interaction of microwaves and Josephson junctions. In that theory, which has been verified extensively (Shapiro, 1963; Anderson and Dayem, 1964) current steps in the I-V characteristics are generated at voltages \( V = n \hbar \nu / 2e \) for integer \( n \), and the amplitude of the step at zero voltage (the dc supercurrent) diminishes under first application of RF power. At the time of that work, no explanation of this phenomenon was forthcoming.

Three mechanisms have been proposed. Although two of these are no longer in favor, I will describe them briefly. The first (Hunt and Mercereau, 1967) is based on the notion that the applied RF field somehow binds the two
Figure 1. Dependence of critical current of microbridge on RF power. (a) Theoretical expectation, (b) Experimental observation of Wyatt et al. (1966).
large banks of a superconducting bridge and thereby lowers the system energy. In an energy balance equation that includes the kinetic energy of the supercurrent, the condensation energy of the superconductor, and the thermal energy, the binding energy competes with the temperature and increases the maximum supercurrent. Since temperature introduces thermal fluctuations that ultimately destroy the superconductivity, this mechanism has been called the RF suppression of fluctuations. The second mechanism is due to Lindelhof (1976). Given that the critical current and \( T_c \) of a weak link is smaller than both the same quantities in its banks and than equilibrium theory would predict based on material parameters (dimensions, coherence length, etc.), the presumption is that the effect of microwaves is to sweep the Cooper pairs of the banks into the constriction of the bridge, thereby increasing the bridge's capacity for supercurrent. Numerical modeling of each theory results in a predicted increase in \( i_c \) under irradiation. Neither theory can explain either the observed frequency dependence (enhancement is found only within a certain frequency range) or the rapid fall of the critical current after the incident power exceeds some critical value. Dayem also observed apparent enhancements of the critical temperature \( T_c \). The RF suppression of fluctuations model predicts some \( T_c \) enhancement, but Lindelhof's theory does not.

The widely accepted explanation is due to Eliashberg (Ivlev, Lisitsyn, and Eliashberg, 1973 (ILE)) and is described in detail in Chapter 2. It is often called the microscopic theory, whereas the aforementioned ones are usually said to be phenomenological. The key points of Eliashberg's theory are: (1) the increase in supercurrent is the manifestation of an increase in the superconducting energy gap, \( \Delta \) which occurs because the microwaves, as quanta of energy \( E = \hbar \nu \), are absorbed by quasiparticles near the gap edge and are removed from the gap edge to higher energies, (2) the effect does not depend on the
weak link nature or Josephson properties of the structures that had been examined, and it may occur just as well in a homogeneous superconductor. (3) a lower frequency limit is determined by \( \nu_{\text{min}} = \tau^{-1}_E \), where \( \tau_E \) is the inelastic relaxation time of a quasiparticle at the gap edge at \( T_c \), (4) an upper frequency limit is set by competition between the enhancement (proportional to the number of absorbed quanta, \( N_R \)) and the inexorable heating (proportional to \( \nu N_R \)), and (5) the model is extendable to other processes that change the steady-state quasiparticle distribution.

The topic of this thesis is the extension of the Eliashberg mechanism to enhancement by ultrasound or microwave phonons. The next experiments in the field pioneered this method (Tredwell and Jacobsen, 1975 and 1976). Theoretically it is only a modest leap to modify ILE to accommodate the phonons. Tredwell and Jacobsen made aluminum samples of point contacts and microbridges, and measured the temperature dependence of the critical current of the weak link versus temperature. The theoretical equilibrium relation between \( \Delta \) and \( T \) was used to find \( i_e \) vs. \( \Delta \). Then, under 10 GHz phonon irradiation, the enhanced critical current, \( i_e^+ \), was measured and the enhanced gap \( \Delta^+ \) was inferred. The results could be compared quantitatively with ILE. This is difficult in the microwave case because the number of absorbed quanta is difficult to ascertain. The incident fields couple to and are reflected by the films themselves, making a determination of the absorbed power a complicated self-consistency problem in electromagnetism and superconductivity theory. In the phonon case the chosen irradiation scheme is rather like shining a flashlight on the sample: you know quite well that the incident intensity doesn't change when you shine it on something different. The results of the comparison to ILE were convincing evidence that ILE was correct and had predictive power.
There followed on this, around 1976, a deluge of enhancement work. Kommers and Clarke (1977) directly measured gap enhancement by 3 and 10 GHz microwave irradiation of a tunnel junction. Klapwijk, van den Bergh, and Mooij (1977) reported on extensive work measuring the critical current enhancements in long, narrow, superconducting strips, whose properties were calculable from Ginsburg-Landau (GL) theory (Tinkham, 1975). The strips were ideal in the sense that according to GL, $i_c = \Gamma (1-T/T_c)^{3/2}$ and $\Gamma$ depends only on material properties that may be measured. The experimentally derived $\Gamma$ may be compared to the theoretically derived $\Gamma$ and good agreement is found. Critical current enhancements were then interpreted in terms of gap enhancements, and these were compared with ILE. Pals and Dobben (1979) also reported on similar measurements, a notable difference being that they covered an enormous temperature range, from 40 mK to $T_c \approx 1.3$ K. Both groups found $T_c$ enhancements and tried to elucidate an apparent first order transition to the normal state at some critical radiation power.

This was an exciting time in nonequilibrium superconductivity; it had only just acquired its own name (Langenberg, 1975). Measurements and theories abounded on subjects like relaxation times of disequilibria, coefficients of heat coupling between samples, their substrates, and heat sinks, injection and extraction of quasiparticles, charge imbalance, and stability of dissipative states (see conference proceedings edited by Gray, 1981). All of these subjects bear in no small way on our present understanding of gap enhancement. Schmid (1977) and co-workers (Eckern et al., 1979 (ESSS)) made Eliashberg's theory comprehensible and modular, providing us with phonon, photon, and quasiparticle injection plug-ins. Measurements of inelastic relaxation times, in particular by charge imbalance measurements (Chi and Clarke, 1979a), allowed for a more profound comparison of enhancement data and theory because the
only parameter in ILE is proportional to $\tau_E$. The thermal coupling of a sample to its constant temperature environment has a great influence on measured enhancements and measured $\tau_E$'s. Problems of this kind were of general interest at the time, and we now understand better how to include so-called heating and phonon-trapping effects within the framework of ILE.

In spite of these successes, certain difficulties began to emerge. To begin with, Chang and Scalapino (1977, (CS)) made some numerical calculations which probed an ILE assumption that the phonons in the sample are in thermal equilibrium. Their findings indicated that: (1) this isn’t true, (2) the number of quasiparticles is actually reduced below its thermal value and this effect is as significant to the gap enhancement as is the removal of the pair-blocking low-lying quasiparticles, and (3) phonon trapping is a dramatically important phenomenon in these experiments because it alters the magnitude, maximum temperature dependence and even the sign of the enhancement. The importance of these findings was to point out that quantitative comparisons between experiments and ILE must be interpreted more figuratively than literally.

Several research groups tried simultaneously to make more detailed studies of gap enhancement using tunnel junctions. The UCLA group (Dahlberg, Orbach and Schuller, 1979) was unable to see that effect, yet they were able to see critical current enhancement in the same films. An NBS group (Hall, Holdeman, and Soulen, 1980) also experienced difficulty observing enhancement, although they were able to do so a few years later (Holdeman et al., 1981). Similarly, second-generation efforts at Berkeley (Lee and Clarke, private communication) were unable to observe gap enhancement for a few years, but were able to do so, again, in the early 1980’s. The matter is unresolved, but the consensus of opinion is this: (1) gap enhancement by microwaves using tunnel
junctions is confounded by photon-assisted tunneling (Tien and Gordon, 1963; or see Chapter 4), (2) seemingly small differences in sample geometry may make the big difference between success and failure, (3) critical current enhancements are easy to see, and the contrast between $i_c$ and $\Delta$ in this regard is irreconcilable, and (4) the role of heat transfer to the LHe bath is crucial.

In addition to the successes and failures of gap enhancement by radiation, Chi and Clarke (1979b) and Gray (1978) detected gap enhancements in films whose quasiparticle distribution functions were altered by injection or extraction of quasiparticles. Enhancements so generated are small, but this perturbation scheme has the advantage that the change in the distribution function is easy to calculate and control, making comparisons with theory more straightforward than in the microwave case.

The ILE theory predicts a change in the distribution function $n(E)$ of the quasiparticles based on a solution to the Boltzmann equation that describes the interaction of radiation and the quasiparticles. The change in $n(E)$ which is reflected in changes in macroscopic quantities $i_c$ and $\Delta$, and the true nature of the disequilibrium $\delta n(E)$, had not yet been measured. A major effort in the field of nonequilibrium superconductivity was to calculate these $\delta n(E)$ in various circumstances (summarized in Gray, 1981). As the 1970's came to a close, experimenters developed methods to extract the distribution functions from measured I-V characteristics in a wide variety of situations and to compare them with ILE or CS or other theories. Willemson and Gray (1978) examined $n(E)$ in a tunneling injection experiment. Smith, Skocpol and Tinkham (1980) shined a laser on superconducting samples and found $\delta n(E)$ by tunneling methods. Later, Horstman, Wolter and Bartels (1981) used these techniques to verify that changes in $n(E)$ predicted by ILE, or more specifically by CS, were indeed mani-
fest in the I-V's of microwave irradiated tunnel junctions. Of course, since \( \Delta \) increases, the cusp (at the difference of the gaps) and current step (at the sum of the gaps) are displaced, but another consequence is that the depletion at \( n(\Delta) \) causes the tunneling current just below the cusp to be reduced. This experiment is considered to be a very significant confirmation of the validity of the theory.

In research that followed their critical current enhancement studies, Pals and Dobben (1980) measured enhancements of the order parameter in a superconducting cylinder under 9 GHz microwave irradiation. The order parameter \( \psi \) is proportional to \( \Delta \) in the the Ginzburg-Landau theory. They employed the condition of a superconductor's quantization of magnetic flux to relate changes in flux through a SQUID coupled to the cylinder to changes in \( \Delta \) of the film. Further development of this work enabled van Attekum and Ramekers (1982) to make simultaneous measurements of critical current and order parameter enhancements. They found \( \psi \) enhancement to be about one-half of what one would expect it to be based on the observed \( i_c \) enhancements.

This discrepancy has been controversial since the early days of the enhancement field. Theoretical predictions of the difference came from Weiss (1981), Entin-Wohlman (1981), and the Karlsruhe group under Schmid (private communication). These predictions (described in Chapter 2) can account for differences of only 10-20\%, much less than the measured and perceived differences. The only two explicit measurements of this kind are van Attekum and Ramekers and the work of this thesis. The null results of Dahlberg et al. (1979) are an extreme example of the disagreement. Work in progress by Lee and Clarke also points to some large disparity, but this study is not yet complete. An as yet unpublished paper by Mooij and Klapwijk (1982) also tries to
get at this problem. The gist of their analysis is that under identical conditions, the absorbed microwave power may vary by a factor of 2 between $i_c$ and $\Delta$ measurements. Critical current enhancements are measured with a large supercurrent passing through the sample. Gap enhancements are measured with small supercurrents (vanishingly small in principle). For a narrow sample near $T_c$, $\Delta$ falls to $\sqrt{2/3}$ of its equilibrium value when a critical supercurrent passes in it. At low temperatures, when the absorbed power is proportional to $\Delta^{-4}$, the absorbed power during critical current measurements is $9/4$ of the absorbed power during gap measurements. This factor of $9/4$ seems to be in accord with the factor of 2 in the work of van Attekum and Ramekers. This mechanism has no validity whatsoever with regard to the experiments of this thesis.

The experimental study to be described is on the subject of phonon-induced enhancement of superconductivity. Simultaneous measurements of the critical current and the energy gap are made when an aluminum sample is subjected to a beam of monochromatic longitudinal phonons in the range 8-11 GHz. The phonon source is intrinsically decoupled from the sample films so that the interplay of superconductivity and electrodynamics, which complicates the interpretation of microwave measurements, is not relevant to this experiment.

Most of the theoretical considerations are discussed in Chapter 2. In order to facilitate reading other work on this subject, Appendix A relates the notation of this thesis (predominantly ESSS) to that of the other major contributors. Experimental details are described in Chapter 3. All relevant material parameters for aluminum are tabulated (with references) in Appendix B. Chapter 4 is a detailed discussion and analysis of the experimental findings. Appendix C is a
compilation of the sample parameters and gap enhancement data for several of the tunnel junctions I measured. Appendix D is a compilation of the sample parameters and critical current data for several of the strips I measured. Conclusions are summarized in Chapter 5. All footnotes are collected before the references at the end of the text.
CHAPTER 2

THEORY

1. THE STANDARD VERSION

Eliashberg and co-workers (Ivlev, Lisitsyn and Eliashberg, 1973 (ILE)) realized that for a superconductor near \( T_c \), quasiparticles of low energy may be removed from near the gap edge by applying some radiation of frequency \( \nu \). These low-lying quasiparticles block the phase space (whose phase space determines the magnitude of the superconducting energy gap) available to Cooper pairs, and their removal might have the effect of increasing the gap \( \Delta \). This is expressed most eloquently in the BCS integral equation,

\[
1 = \lambda \int_{\Delta} \frac{\hbar \omega_p}{E^2 - \Delta^2} \left( 1 - 2n(E) \right) \, dE,
\]

where \( \lambda \) is a constant that depends on the superconductor in question, \( n(E) \) is the quasiparticle occupation number, and \( \omega_p \) is the Debye frequency. Suppose a solution \( \Delta \) has been found to the integral equation for a thermal distribution \( n(E) = n_\tau(E) = (1 + e^{E/\tau})^{-1} \). The denominator weights the integral in such a way that when \( n(E) \) is either reduced at low energies or increased at higher energies, \( \Delta \) must rise above its equilibrium value to keep the equation satisfied. The ILE theory is thus concerned with calculating the deviation of \( n(E) \) from \( n_\tau(E) \) under the influence of microwave radiation \( \nu \). Tredwell and Jacobsen (1975 and 1976) modified the theory to treat incident ultrasound or phonons, which were also of microwave frequencies. In this chapter I will consider both forms of radiation, but I will emphasize the results for the phonons.
The perturbing radiation $\nu$ may change $n(E)$ by three processes: a state of energy $E - \nu$ may absorb the quantum, a state $E + \nu$ may be stimulated to emit another quantum, and for sufficiently large $\nu$ the quantum may destroy a Cooper pair creating new excitations at $E$ and $\nu - E$. These processes are displayed graphically in Fig. 2. The total scattering into $E$ by the driving perturbation is easily calculated using the Golden Rule,

$$
(\tilde{n})_D = \frac{N_R}{2N(0)\nu} \left\{ N_1(E) \left[ N_1(E - \nu)(1 \pm \frac{\Delta^2}{E(E - \nu)}) (n(E - \nu) - n(E)) \right] 
+ \left[ \nu - \nu \right] \right\} 
+N_1(E)N_1(\nu - E)(1 \pm \frac{\Delta^2}{E(\nu - E)}) (1 - n(E) - n(\nu - E))
$$

Here, $N(0)$ is the single spin density of states at the Fermi level, $N_1(E) = E/\sqrt{E^2 - \Delta^2} \Theta(E - \Delta)$ is the BCS density of states, the terms $(1 \pm \frac{\Delta^2}{E(\nu - E)})$ are coherence factors that determine the energy dependence of the absorption for microwaves (upper sign) and phonons (lower sign), and $N_R = aW/\hbar\nu$, where $a$ is the normal state attenuation coefficient (with dimensions of $1/\text{length}$) and $W$ is the incident radiation flux ($\text{energy sec}^{-1} \text{cm}^{-2}$).

Equivalently, $N_R$ is the number of quanta per unit time per unit volume that the superconductor would have absorbed were it in its normal state with the same radiation intensity present. It is true therefore that

$$
N(0) \int_{\text{all energies}} dE \; N_1(E)(\tilde{n})_D \big|_{\Delta = 0} = N_R.
$$

The most direct path to a solution for the nonequilibrium distribution function is to employ the relaxation time approximation (RTA).
Figure 2. Graphic representation of the three processes that change $n(E)$ by interaction with a phonon.
Precisely, $\tau_E$ is the mean lifetime of a quasiparticle at the gap edge at $T = T_c$ (Kaplan et al., 1976). More generally, $\tau_E$, the inelastic scattering time, is the time scale on which the energy of a nonequilibrium quasiparticle distribution relaxes back to equilibrium. In an Al film of $T_c \approx 1.25$ K, $\tau_E = 12$ nsec. Certain restrictions must be imposed to ensure the validity of the RTA. First, the change in occupation number of a state must be much less than unity,

$$\frac{N_R \tau_S}{2N(0)T_c} \ll 1.$$ 

This has the effect of keeping $n \approx n_T$, and we can make that replacement on the right hand side of Eq.(II). Second, since $\tau_E$ has a temperature dependence, either $\tau_E \propto 1/T^3$ or $1/E^3$ (whichever is shorter), we require that

$$\Delta \nu \ll T.$$ 

This imposes the condition that changes induced by the perturbation occur over an energy range well localized on a scale of $T$, so that $\tau_E$ is relatively constant. Thus we are confined to temperatures close to $T_c$ and energies both close to the gap edge and much smaller than $T_c$. In our expression for $\langle \dot{n} \rangle_D$ we can make the replacements

$$n(E \pm \nu) - n(E) = -\left( \pm \frac{\nu}{4T_c} \right) \text{ and}$$

$$1 - n(\nu - E) - n(E) = \frac{\nu}{4T_c}.$$ 

Putting this into Eq.(I) we get

$$n_0 = n - n_T = \langle \dot{n} \rangle_D \tau_E$$ 

(III)
The subscript "D" is meant to indicate that, due to the perturbing radiation, this time rate of change of \( n(E) \) is the result of a "Drive" term in the governing Boltzmann equation. For \( E < \Delta + \nu \), the quantity in brackets is of order 1, so the leading coefficient may be regarded as the deviation at low energies of the occupation number from its thermal value. It is called \( B \) by Eckern, Schmid, Schmutz, and Schon (ESSS, 1979). The RTA is valid only for \( B \ll 1 \).

The new distribution function for phonon irradiation is shown in Fig. 3. In the case shown \( \nu < 2\Delta \), consequently, no new quasiparticles are generated, states near the gap edge are depleted, and states above \( E = \Delta + \nu \) are enriched. In the figure there is a singularity at \( E = \Delta + \nu \), which is an image of the singular BCS density of states. In reality, this either gets smeared out over a width \( 1/\tau_E \) or by any inherent smearing of the BCS singularity, whichever is larger (Chang and Scalapino, 1977). The smearing over a width \( 1/\tau_E \) indicates that unless \( \nu > 1/\tau_E \), \( n(E) \) will be indistinguishable from a thermal distribution and, in this case, we should expect no enhancement for \( \nu \leq 1/\tau_E \).

By inserting this new value for \( n = n_o + n_T \) into the BCS integral, we may find the nonequilibrium value of \( \Delta \). Since we are interested only in the temperature range \( \Delta / T \ll 1 \), the BCS integral may be reexpressed as the Ginzburg-Landau (GL) equation

\[
\Delta ( T / T_c - 1 + \beta \frac{\Delta^2}{T_c^2} + [ \Delta T / T_c ] ) = 0
\]  

(IV)

where \( \beta = 7\zeta(3) / 8 \pi^2 \approx 0.1066 \), and
Figure 3. The quasiparticle distribution function in equilibrium, and with microwave and phonon radiation ($B=0.03$, $T/T_c = 0.98$, $\nu/T_c = 0.30$, $\nu/\Delta = 0.70$).
\[ [\delta T/T_c] = + \int dE \frac{n-n_T}{\sqrt{E^2-\Delta^2}}. \] (V)

Of course in equilibrium \( n = n_T \) so \([\delta T/T_c]=0\) and the GL equation in its usual form is obtained (Tinkham, 1975). When the nonequilibrium result for \((n-n_T)\) is plugged into Eq.(V), ESSS find that \([\delta T/T_c]\) is a sum of elliptic integrals, proportional to \(B\), and functionally dependent only on the ratio \(\Delta/\nu = u\). Rewriting, the GL equation becomes

\[ u \left( \frac{T}{T_c} - 1 + \beta \left( \frac{\nu}{T_c} \right)^2 u^2 - BG(u) \right) = 0. \] (VI)

where \(G(u)\) is displayed in Fig. 4. Roots of the GL equation may be found either graphically or numerically\[2\]. It is very easy to see with the graphical method that more than one root exists for certain values of the parameters (Fig. 5). One must then discern what these solutions are and which of them is stable. It is the principle purpose of ESSS to examine these questions. It will suffice here to say that in the experimentally accessible domains of Fig. 5 (temperatures sufficiently below \(T_c\) that \(\nu < 2\Delta\) or equivalently \(u > \frac{\Delta}{2}\)), only one solution is found and that is the enhanced one.

What I have presented thus far is the ILE theory of gap enhancement. In the remainder of this chapter I will look at some corrections to ILE, its inherent limitations, and its extension to measurements of critical currents.

2. HEATING CORRECTIONS

As a first amendment to ILE, ESSS assert that in addition to the effect that the redistribution of quasiparticles has on \(\Delta\), the absorbed phonons also have the effect of changing the electronic temperature. Expressed as an additive constant to the quantity \([\delta T/T_c]\), this new term is
Figure 4. The functions $G^{\pm}(u)$ for the microwave (+) and phonon (-) cases. For their analytic representation see ESSS.
Figure 5. Graphical method of finding a solution to Eq. (VI), $t = T / T_c$. The intersection of the straight line, $y_1$, and the curve, $G(u)$, is the root (note horizontal scale is $u^2$). Other notable features of this presentation are: $y_1$ is zero at the equilibrium value of $u^2$, and when $u^2 = 0$, $y_1 = (t - 1)/B$. 

\[ I/B \left[ t - 1 + \beta \left( \frac{\nu}{T_c} \right)^2 u^2 \right] \]
\[ \left[ \frac{\delta T}{T_c} \right]_0 = +0.88 \frac{\nu}{T_c} B. \]

The term has its origin in the fact that in the earlier calculation of \( \langle n \rangle_D \) we assumed that the distribution functions on the right hand side of Eq. (II) were the thermal ones, \( n_T \). By the RTA, \( \dot{n}_D \tau_E \) is the change in \( n(E) \) produced by that assumption, and for self-consistency, the total \( n = n_T + \dot{n}_D \tau_E \) should be used. Carrying out the calculation in a perturbative manner to one higher order, ESSS calculate a correction \( n_a \), which when inserted into Eq. (V) yields the term \( \left[ \frac{\delta T}{T_c} \right]_0 \) \[^3\]. Since \( \left[ \frac{\delta T}{T_c} \right]_0 \) appears in the GL equation on equal footing with temperature \( T/T_c \), and since \( \left[ \frac{\delta T}{T_c} \right]_0 \) does not depend on \( \Delta \), unlike the terms \( \beta \Delta^2/T_c^2 \) and \( BG(\Delta/\nu) \), the implicit meaning of \( \left[ \frac{\delta T}{T_c} \right]_0 \) is that it is a correction to the temperature of the superconductor, resulting from the input of phonon energy, and of such a sign as to increase the temperature.

From a slightly different viewpoint one might argue that because of a finite thermal boundary resistance, \( R_K \), between superconductor and heat reservoir, the superconductor's temperature will be elevated above its surroundings. The usual means for calculating such a temperature rise in a steady state situation is

\[
\text{change in } T = \frac{R_K \times \text{heat input}}{\text{surface area}}.
\]

The heat input is the number of phonons absorbed per unit volume per unit time, \( N_R \), times the sample volume, times the phonon energy \( \nu \). Normalizing the change in \( T \) to \( T_c \) we obtain another correction to the sample temperature

\[ + N_R \frac{\nu}{T_c} R_K l. \]

where \( l \) is the sample thickness. If there is more than one channel for heat exchange, as is the case in the experiments of this thesis, then \( R_K^{-1} = \sum_i R_{K_i}^{-1} \).
which is only a statement that conductances add. If we choose to write $N_R$ in terms of $B$, then this term may be added to the previous one and a single coefficient $\lambda$ used to represent all of the heating effects,

$$[\frac{\delta T}{T_c}]_h = B \frac{\nu}{T_c} 0.88 + B \frac{\nu}{T_c} \frac{R_K l 8N(0)T_c}{\tau_E} = B \frac{\nu}{T_c} \lambda$$

The heating $\lambda$ depends on various material parameters, and in the experiments of this thesis $\lambda \approx 1.5$. The latter term is sometimes called a phenomenological heating term and was first introduced in print in this context by Mooij (1981).

There is good reason to wonder whether the two parts of $[\frac{\delta T}{T_c}]_h$ aren't really the same thing and we're just counting it twice. The purported difference between them, namely $[\frac{\delta T}{T_c}]_0$, refers to the metal's quasiparticle or electronic temperature relative to its phonon temperature, and the phenomenological heating term refers to the difference between the bath temperature and the phonon temperature. By analogy one might say that the source of $[\frac{\delta T}{T_c}]_0$ is the thermal resistance between quasiparticles and phonons.

3. CHANG AND SCALAPINO

The theory discussed up to this point assumes that the phonons within the material (not to be confused with the incident phonon radiation that is absorbed) are in equilibrium with some external heat reservoir at temperature $T$, and consequently, that the energy relaxation of the quasiparticles proceeds as if the entire system were in equilibrium. Chang and Scalapino (1977 (CS)) have examined this assumption and found it to be a serious oversimplification. They looked at the coupled Boltzmann equations for both the nonequilibrium quasiparticles and the nonequilibrium phonons. The extra or nonequilibrium phonons arise from two sources: (i) the relaxation of excited quasiparticles produces phonons, and (ii) recombination of excited quasiparticles to form pairs
produces phonons of large frequency $\Omega$, typically $\Omega \approx 3\Delta$. These extra phonons would not present a problem if they escaped to the bath from the superconducting film without having any other interaction with the quasiparticles. Realistically, though, the phonon escape time, $\tau_{es}$, is a considerable fraction of, or larger than, the pair breaking time, $\tau_B$ (Kaplan et al., 1976). Another point related to the recombination of excited quasiparticles is that on the average, the rate of said recombination is greater than the recombination rate of equilibrium quasiparticles. A typical excited recombination involves quasiparticles of energy $\Delta$ and $(\Delta + \nu) \approx 2\Delta$, yielding a $3\Delta$-phonon. A typical equilibrium recombination involves 2 quasiparticles of energy $\Delta$ and a $2\Delta$-phonon. Since the rate goes like the density of final states and the density of phonon states can be expected to behave like $\Omega^2$, quasiparticle recombination is about twice as fast with pumping as it is without it. For sufficiently rapid escape, the overall quasiparticle density is reduced and this leads to an increase of $\Delta$. And in some cases, about one-half of the enhancement may be attributed to this reduction. See Fig. 6.

The results of Chang and Scalapino are very instructive, but only qualitative. They tell us that the phonon escape time, $\tau_{es}/\tau_B$, is a significant parameter in the problem. The enhancement varies inversely with $\tau_{es}/\tau_B$. In this way it is like the previously discussed heating parameter $[\delta T/T_c]_h$. This is not surprising because $[\delta T/T_c]_h$ is proportional to $R_K$, and $R_K$ can be thought of as arising from the difficulty which phonons encounter upon trying to go from the heated material to the bath. This so-called acoustic mismatch theory of $R_K$ (Little, 1959; Anderson, 1981) also has a place in theoretical calculations of $\tau_{es}$ (Kaplan, 1979). Probably, $\tau_{es} \approx 4l/c_s\eta_{es}$, where $l$ is the thickness of the sample, $c_s$ is the speed of sound, and
Figure 6. The effect of the reduction of quasiparticle density on gap enhancement (from CS). Curves a and b are calculated using the heating parameter $\tau_{se}/\tau_B = 0.2$ and 2.0 respectively. The primed curves are obtained by calculating the total quasiparticle density of the unprimed curved, finding the temperature which in equilibrium has the same quasiparticle density, and then computing the equilibrium gap for that temperature. Symbolically, $\Delta$ for the primed curve is

$$\int dE \frac{n'(E,\Delta')}{\sqrt{E^2 - \Delta^2}} = \int dE \frac{n^o(E,\Delta)}{\sqrt{E^2 - \Delta^2}}.$$

Roughly half of the total enhancement can be seen to result from the reduction in quasiparticle density. The horizontal scale is in CS units, $A$, for conversion to $B$, see Appendix A. ($T/T_c = 0.90$, $\nu/T_c = 1.14$)
\[
\eta_{es} = \frac{\text{number of escapes across the interface}}{\text{number of assaults on the interface}}
\]

One may try with some success to calculate \( \eta_{es} \) from the acoustic theory. What is unique about the CS analysis, and what is not predicted by the embellished ILE theory, is that the enhancement has a maximum as a function of power. To put that into ILE would require an extra heating term proportional to the power squared (\( [\delta T/T_c]h_0 = ((\nu/T_c)B)^2 h_0 \)), so that

\[
[\delta T/T_c]_T = -BG(u) + B \frac{\nu}{T_c} h + [\delta T/T_c]h_0
\]

may have a maximum, forcing the enhancement to roll over. The other result of CS, that the reduced quasiparticle density may be as important as the redistribution, is also difficult to model in the ILE framework. Thus CS suggest using caution when comparing any experimental results with ILE, and they provide two plausible mechanisms that may be responsible for certain discrepancies. It may also be noted that because CS actually calculate the self-consistent solution to the kinetic equations, their method may be used at temperatures far below that for which ILE is valid.

4. TEMPERATURE DEPENDENT ILE?

Although using the ILE theory does not give one a feeling of great security, it is the only analytic theory available. We might therefore try to buttress it up a bit by putting some reasonable temperature dependences into it. Beginning with the enhancement term \(-BG(u)\), the temperature dependence is hidden in \(\tau_g\), which may be replaced by \(\tau_g(T_c) 1(T/T_c)^{-3}\), and in \(1/T_c\), which really came from

\[
[n(E-\nu)-n(E)] + [\nu - \nu] \approx -2n' \nu \approx -\nu/2T_c
\]

for \(E \approx \Delta \ll T\). Evaluating \(n'\) at \(\Delta\), we get
The heating term

$$B \rightarrow B \frac{4e^{\Delta T}}{(1+e^{\Delta T})^2} \frac{1}{(T/T_c)^4} \quad (VII)$$

is more difficult to interpret. The second piece comes from the thermal boundary resistance between the Al film and the LHe bath. It is proportional to the total number of absorbed quanta, which at $T_c$ is $N_R$, but which drops to $N_R 2(1 + e^{\Delta T})^{-1}$ (Tinkham, 1975) as the temperature falls. The Kapitza resistance $R_K$ has a $1/T^3$ dependence in the acoustic mismatch model and is known experimentally to have this behavior in many systems (Lounasmaa, 1974). It also makes sense to replace $\nu/T_c$ by $\nu/T$. Overall the second term undergoes the transformation

$$N_R \frac{\nu}{T_c} R_K l \rightarrow N_R \frac{\nu}{T_c} R_K l \frac{2}{1 + e^{\Delta T}} \frac{1}{(T/T_c)^4},$$

where $R_K$ on the right hand side is now taken to be the Kapitza resistance evaluated at $T = T_c$.

The instructions of ESSS with regard to the first term are that it should be interpreted as an internal thermal resistance between the quasiparticles and the lattice of the Al film. As such it is also proportional to the total number of absorbed phonons, and to elucidate its temperature dependence it is best to use the definition of $B$,

$$B = \frac{N_R T_E}{8N(0)T_c}.$$

Then, in the spirit of the previous replacements we obtain
\[
0.68 \, \frac{\nu}{T_c} \, B \rightarrow 0.68 \, \frac{\nu}{T_c} \, B \, \frac{2}{1 + e^{\frac{\Delta T}{T_c}}} \, \frac{1}{(T/T_c)^4},
\]

where all the temperature dependence on the right hand side is expressed explicitly. To within a factor \((T/T_c)^{-1}\), the two parts of \([\delta T/T_c]_h\) have the same temperature dependence. We write

\[
[\delta T/T_c]_h = B \, \frac{\nu}{T_c} \, \frac{2}{1 + e^{\frac{\Delta T}{T_c}}} \, \frac{1}{(T/T_c)^4} \, \left( \frac{0.68}{T/T_c} + H \right),
\]

where \(H\) is calculable from material properties as discussed earlier. Later we will describe efforts to compare theory with experiment. We choose here to drop the extra factor of \(T/T_c^{-1}\) for two reasons: (1) over the range of \(T/T_c = 0.95 - 0.995\), \(T/T_c^{-1} = 1.03 \pm 2\%\), and to within the accuracy of this experiment, it may be taken as constant, and (2) it is stretching our faith in this analysis to say that we can distinguish the difference in the temperature dependence of these two terms. Since they are close, it is reasonable to say that they are equal. With this final approximation the heating term take the form

\[
[\delta T/T_c]_h = \frac{\nu}{T_c} \, B \, h \, \frac{2}{1 + e^{\frac{\Delta T}{T_c}}} \, \frac{1}{(T/T_c)^4}.
\]

and we seek the fitting parameter \(h\), which has its basis in the microscopic theory (ESSS) and phenomenological heating.

The temperature dependent coefficients are listed in Table I for a range of temperatures relevant to the ILE theory (\(\Delta/T << 1\)) and also for the lowest temperatures at which I made measurements. The trend of these coefficients is such that heating becomes relatively less important at lower temperatures, and the enhancement term more important. The variations are small however and will not permit us to say one or the other of the T-dependent or T-independent theories is correct.
Table I: Temperature Dependence of the Coefficients of ILE

<table>
<thead>
<tr>
<th>Reduced Temperature $T / T_c$</th>
<th>Enhancement term $\Delta / T$</th>
<th>Heating term $T / T_c^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.996</td>
<td>0.194</td>
<td>1.007</td>
</tr>
<tr>
<td>0.993</td>
<td>0.258</td>
<td>1.012</td>
</tr>
<tr>
<td>0.990</td>
<td>0.307</td>
<td>1.017</td>
</tr>
<tr>
<td>0.985</td>
<td>0.378</td>
<td>1.025</td>
</tr>
<tr>
<td>0.980</td>
<td>0.438</td>
<td>1.033</td>
</tr>
<tr>
<td>0.970</td>
<td>0.533</td>
<td>1.049</td>
</tr>
<tr>
<td>0.960</td>
<td>0.628</td>
<td>1.065</td>
</tr>
<tr>
<td>0.950</td>
<td>0.703</td>
<td>1.081</td>
</tr>
<tr>
<td>0.925</td>
<td>0.878</td>
<td>1.138</td>
</tr>
<tr>
<td>0.900</td>
<td>1.031</td>
<td>1.181</td>
</tr>
<tr>
<td>0.820</td>
<td>1.485</td>
<td>1.349</td>
</tr>
</tbody>
</table>

5. CRITICAL CURRENTS

While the ILE theory as presented predicts a gap enhancement during microwave or phonon irradiation, most of the experiments have measured not $\Delta$, but some quantity which depends on $\Delta$. The most important such quantity is the critical current of a long, narrow, superconducting strip, as in the experiments of Klapwijk, van den Bergh and Mooij (1977), and of this thesis. If the cross sections of these strips are sufficiently small,

$$\text{thickness} \times \text{width} < (\text{London length})^2,$$

then current flow is one-dimensional. In equilibrium, GL-theory is often used to treat this situation where an extra term, $-\frac{j_s^2}{\Delta^4}$, is added to the usual self-consistency equation to account for the energy of the supercurrent (Tinkham, 1975; Skocpol, 1976). The maximum value of the supercurrent, $j_s$, and the value of $\Delta$ at $j_s = j_c$ may be calculated readily. Until 1980, all attempts at com-
paring enhanced critical current measurements to ILE predictions were made by inferring changes in $\Delta$ from changes in $j_c$ via the equilibrium relationship, $j_c \propto \Delta^3$ for $\Delta/ T \ll 1$. Weiss (1981), Entin-Wohlman (1981), and Holzman (unpublished) pointed out that when the supercurrent energy term is included in the nonequilibrium GL equation, the critical current enhancement may be about 10% larger than would be otherwise anticipated. One way to understand this is to consider the gap enhancement of a superconductor at two different temperatures, with other conditions being left constant. The enhancement at the higher temperature (smaller gap) is greater (see Fig. 7). When measuring gap enhancement in a superconductor both with and without a supercurrent in it, the gap is smaller (by 2/3) when there is a critical supercurrent. Consequently, on the basis of the last observation we may expect the enhancement to be larger in critical current measurements than in current-free gap measurements.

Entin-Wohlman also pointed out that since the critical current depends on the distribution function at the gap edge,

$$j_c \propto \Delta^2 (1-2n(\Delta)),$$

there is a correction to the equilibrium relationship from this too, and that this correction may also be about 10%.

6. QUANTIFYING THE ENHANCEMENT

It is now appropriate to comment about how these enhancements are presented. A logical first choice might be something like

$$\text{enhancement} = \frac{\text{Change in } \Delta}{\Delta \text{ in equilibrium}},$$

or the analogous quantity for the critical current. An obvious problem arises when considering enhancements above $T_c$ (Klapwijk, 1977; Kommers and Clarke,
Figure 7. This figure is intended to illustrate why $i_c$ enhancements are predicted to be larger than gap enhancements. The line (a) finds the enhanced $u^2 = (\Delta / \nu)^2$ when the bath temperature is such that $u^0 = 1$ (the y-intercept of (a) is $t_a - 1$. The line (b) finds the enhanced $u^2$ at a higher temperature $u^0 = \sqrt{2} / 3$. Because $G(u)$ is essentially monotonic decreasing, $BG(u)|_{u^0 = \sqrt{2} / 3} > BG(u)|_{u^0 = 1}$. By a slight stretch of the imagination, the situation $u^0 = \sqrt{2} / 3$ may be understood to correspond to the case $u^0 = 1, i = i_c$. 
1977), for which the denominator is zero. Once one has become familiar with ILE or ESSS, they realize that since the dimensions of the governing equation are reduced temperature, the enhancement should be expressed as such. That is, \([ \delta T / T_c ]_T\) represents the change in reduced temperature that, in equilibrium, would have resulted in the same change in \(\Delta\) or \(i_e\), that the radiation produced (Fig. 8). This quantity \([ \delta T / T_c ]_T\) which is negative if there is any enhancement is variously called the cooling or the gap control (see Appendix A). This convention makes comparison between \(i_e\) and \(\Delta\) measurements very simple, since the enhancements are expressed in the same dimensionless units and should be about equal. Typical values are \([ \delta T / T_c ] \approx -0.010\). Another advantage is that for \(T > T_c\), \([ \delta T / T_c ]\) remains well defined. Furthermore, the cooling drops out of the theory quite naturally and is convenient to extract from the measured data. Finally, the cooling is predicted to have a much less pronounced temperature dependence then \(\delta \Delta / \Delta\), a fact which is also a virtue when it comes to presenting data.
Figure 8. The cooling, what it is.
CHAPTER 3

EXPERIMENTAL METHOD

1. MICROWAVE APPARATUS FOR GENERATION OF PHONONS

Phonons for this experiment were produced by piezoelectric transduction of a high power microwave pulse. The path from 120 V, 60 Hz to 9 GHz ultrasound is shown schematically in Fig. 9. The klystron was a Varian Type X-13 which put out about 100 mW of continuous microwave power, tunable between 8 and 12 GHz. This power was transmitted via X-band waveguide to the inputs of a Raytheon Type QKW 1458 traveling wave tube (TWT). A 350 V positive pulse on the grid of the TWT yielded 1 kW pulses at the output, for 40 dB gain\(^4\). Schematics for the TWT electronics are shown in Fig. 10. The typical operating frequency and pulse duration were 9 GHz and 1 \(\mu\)sec respectively, with a repetition rate of 50 Hz. I monitored the TWT output power by using a directional coupler to tap off 1% of the forward power, attenuating the tapped-off part another 50 dB or so and then directing it to a diode detector. The diode was shunted with 50 Ω and the resultant signal was viewed on an oscilloscope (Tektronix RM647 with Type A1 preamplifier). The rectified signal at the oscilloscope input was about 8 mV at full power. While an averaging power meter might have done as well, the low repetition rate and high pulse power made it impossible to make reliable measurements this way, thus necessitating the pulse height monitoring just described. Located between the pulse height monitor and the TWT output were two 20 dB calibrated variable attenuators (HP X375A). Microwave power and consequently phonon power were varied by means of these attenuators. The other obvious means of varying the power was to adjust the TWT...
Figure 9. Schematic drawing of the path from ac line current to 9 GHz phonons. Components within the dashed line are immersed in LHe.
Figure 10. Schematics of electronics for TWT amplifier. This equipment was designed and built by Robert Wilson and described in his Ph.D. thesis (Berkeley, 1980). There were some errors in the original drawings and some changes have been made to the circuit, so I include them with corrections here. (a) Low Voltage power supply. (b) Pulse generator, trigger for TWT.
power supply voltage, but this proved to be slow, unreliable, and exasperating.

After the pulse height monitor, the waveguide went through the wall of the screened room in which the experiments were performed. The #30 mesh copper shielding was necessary to eliminate noise rounding of the sharp nonlinearities of the superconducting devices being investigated. A major source of the electromagnetic interference (EMI) was the switching transient of the pulsed TWT. Next there were more attenuators for convenient adjustment of the power while the operator was inside the screened room, and then another directional coupler to tap off -20 dB of the power reflected from the end of the apparatus. Before entering the top plate of the experimental insert, the waveguide was vacuum sealed with an infrared filter. The filter prevented the waveguide from acting as a light pipe for room temperature radiation, and the seal was essential for pumping the LHe bath down to temperatures in the range 1.0–1.5 K. See Fig. 11.

Copper waveguide extending from the room temperature top plate down into the LHe would have presented an enormous heat leak that could not have been tolerated. Most of the center length of the waveguide (≈ 35 cm) was made of thin wall stainless steel and this reduced the heat leak to acceptable levels. The joints between each pair of waveguide sections on the insert were made with indium O-rings and were superfluid leak tight. They showed excellent stability upon thermal cycling. The need for sealing off the waveguide from the bath arose with the large microwave powers required in this experiment. The vapor pressure of helium is around 1 Torr at typical temperatures, and the strong electric fields of the microwave pulse can produce some kind of breakdown, or arcing, in the dilute vapor. When it arcs, one hears mechanical vibrations of the low temperature apparatus, presumably at the frequency of the
Figure 11. Reflected power monitor, screened room, and top of the low temperature insert.  

- **a**: microwave diode detector,
- **b**: -20dB coupler, to direct power reflected from the cavity to the detector,
- **c**: room temperature waveguide seal and IR filter,
- **d**: cryostat top plate,
- **e**: one of several superfluid seals.
pulse repetition rate, and the envelope of the reflected power, as viewed on the oscilloscope, is not square as it should be. By sealing off the room temperature end of the waveguide, the low temperature end just above the cavity and all the joints exposed to the vapor or the superfluid, the waveguide may be evacuated, and no arcing is then observed. The cavity itself is not sealed against the superfluid, but is completely immersed in and therefore full of the liquid, and dilute vapor should not be present. Indeed there is no arcing when the seals work.

The final or cold waveguide seal must be microwave transparent, superleak tight, recyclable, and able to withstand a pressure difference of 1 atmosphere. A seal as shown in Fig. 12 will work. A blank X-band flange already soldered to the waveguide was faced-off on a lathe, and then a sharp cutting tool was used to produce a small groove between the waveguide cutout and the flange mounting holes. The groove was \( \approx 30 \) mils deep but that is not critical. It was necessary to remove all burrs, rough spots, and sharp corners from the waveguide, using emery paper, steel wool, or a small file. This was most important at the cut-out since one of the principal failure modes was tearing of the mylar at the lip of the waveguide cut-out. A piece of mylar 2 or 3 mils thick was sandblasted lightly on one face to roughen it and increase its adhesion to the epoxy. Probably many different epoxies would have worked, but the only one I tried is Miller-Stephenson Chemical Co., Inc. Epoxy 907 adhesive. A small amount was applied, making a thin closed ring between the cut-out and the groove. The ring must be continuous in order to get a superfluid seal. The mylar was then placed in final position against the epoxy and flange, and the whole assembly was clamped together and baked according to the instructions. If the groove was deep enough to accommodate the epoxy as it squeezed out it was possible to separate the clamp from the flange. The seal was then examined, and if any
Figure 12. Exploded view of superleak tight waveguide seal.
fissures could be seen in the glue I then found it worthwhile to tear it apart, clean all the surfaces, and start over. This type of seal worked quite well, and had the single disadvantage that since the mylar is porous to He gas at 300 K it may not be leak tight while warm. Of course it loses its porosity at LN\textsubscript{2} temperature.

Continuing down the waveguide, we arrive finally at the end of the insert, which is a re-entrant microwave cavity with a cylindrical, single crystal, piezoelectric, quartz rod inserted into it. Shown in Fig. 13, the cavity may be tuned from 8-11 GHz by moving the tuning plunger in and out. The cavity may be thought of as an LC resonator, the capacitor being formed by the center plunger and the opposite wall, the L being a toroidal inductor. Moving the foil wall changes the effective capacitance and consequently the resonant frequency. The quartz crystal rests in a region of strong RF electric fields, essentially perpendicular to its face. The quartz is an X-cut single crystal\textsuperscript{[5]}, 1.00" long and 4 mm in diameter. The end faces are polished flat to $\approx 50$ nm (1/10 wavelength of sodium light) and are cut parallel to 4" of arc, or so the manufacturer claims.

The piezoelectricity of an X-cut crystal is such that compressional ultrasound or longitudinal phonons are produced when an RF electric field is normal to its surface. The acoustic strain is proportional to the gradient of the acoustic stress, $E \times d_{11}$, where $E$ is the electric field and $d_{11} = 2.3 \times 10^{-12} \text{ m V}^{-1}$ is the appropriate piezoelectric constant (Landolt-Bornstein, 1969). As made clear by Jacobsen (1960), this gradient is greatest at the end face of the crystal. Ultrasound generated there will propagate along the rod with the speed of sound, $c_1 = 6.0 \times 10^3 \text{ cm/sec}$,\textsuperscript{[6]} with very little attenuation at low temperatures. Phonons will arrive at the other end of the crystal (on which end the sample is
Figure 13. Cutaway view of reentrant cavity and coupling hole, (a), which may be thought of as an impedance transformer. The foil, (c), and attached pin, (b), may be moved along the crystal axis by a cam and spring. This varies the effective capacitance and, therefore, the cavity resonance. The sample is grown and deposited on the end of the crystal which is not within the cavity. The coupling hole is actually part of a separate diaphragm, (d), which lay on top of the cavity.
grown) 4.2 \mu\text{sec} after initiation of the microwave pulse. The pulse itself being \approx 1 \mu\text{sec}, the phonons arrive long after the microwaves have come and gone. The approximate transduction efficiency is

$$\eta = \frac{4\pi c_{11} d_{11}^2}{\kappa^2 \varepsilon_0} \frac{Ac_i Q}{2\pi v V'}$$

where $c_{11} = 8.7 \times 10^{10} \text{ N m}^{-2}$ is the elastic constant (Landolt-Bornstein, 1969), $\kappa = 4.5$ is the dielectric constant of quartz, and $V$ is some effective volume approximately equal to $A$ times the gap (0.3-0.5 mm) between the tuning stub and the opposite wall of the cavity (Jacobsen, 1960). For a $Q \approx 800$, $\nu = 9$ GHz and $\eta = 6.9 \times 10^{-3}$. According to Jacobsen (private communication), this $\eta$ is typically 3 to 4 times greater than the measured transduction efficiency. The $Q$ of the cavity ranged from 300-1200, depending on the frequency, the protrusion of the crystal into the cavity, the number of days the entire assembly had been cold, how tight the screws were holding the cavity together, and other factors. Somewhat better than average performance was achieved by painting the crystal entrance hole with silver paint after the crystal was mounted. A $Q$ of 1000 was a practical upper limit for these experiments because at full microwave power and at the highest $Q$, the cavity seemed to arc and thus was not useful in this range. The $Q$ was measured by replacing the TWT with a sweep oscillator (HP 694A) and using a wavemeter to measure the dip in reflected power as the frequency swept over the cavity resonance. The depth of that dip indicated the fraction of incident power actually absorbed in the cavity, $f_i$. The coupling hole (Fig. 13) was painstakingly filed by hand to strike the best compromise between high $Q$ (small hole, low losses) and high $f_i$ (large hole, ready entrance to the cavity). Typically, $f_i$ varied from 0.9-1.0, usually improving with time and application of high power pulses. The predominant loss mechanisms were in the cavity walls, through the coupling hole, and at the joints between the cavity
parts. That it was not in the crystal entrance hole or by phonon generation is demonstrated by the fact that replacing the crystal with a copper rod did not change the $Q$ noticeably. The maximum $Q \approx 1000$ places an upper limit on the conversion efficiency, $\eta < 0.001$, which is less than one might anticipate from the previous calculation of $\eta$. We will use $\eta = 5 \times 10^{-4}$ as the standard value.

Alternatives to the chosen method were available. A number of materials would have yielded higher conversion efficiencies because of their higher piezoelectric constants. CdS and ZnO ($d_{15} = 14 \times 10^{-12} \text{ m} \cdot \text{V}^{-1}$) may be deposited as thin films. LiNb ($d_{15} = 71 \pm 3 \times 10^{-12} \text{ m} \cdot \text{V}^{-1}$) and tourmaline ($d_{15} = 3.7 \times 10^{-12} \text{ m} \cdot \text{V}^{-1}$) are crystals that may be obtained either at great expense as rods of the dimensions I used, or in smaller pieces which may be bonded to a Z-cut quartz delay line. I chose to avoid the technical challenges associated with these materials. Deposition of the thin film piezoelectrics requires a dedicated fabrication facility, which was available at the Electronics Research Laboratory (ERL) in EECS at Berkeley. But the potential yield of a procedure requiring complex depositions on each end of a quartz rod was depressing to consider. The bonding procedure is tricky and unreliable, and although it is standard procedure in some types of experiments (Coleman, 1974), I was not able to get it to work in about two weeks of effort. Relatively speaking, the X-cut quartz is cheap, easy and effective.
2. THE SAMPLES

2.1. Materials

There was really no choice at all about materials. High $T_c$ superconductors, like Nb, would show little response because of their very short relaxation time, $\tau_B=18$ ps. A low $T_c$ superconductor like Zn ($T_c = 0.88 K, \tau_B=93$ ns) would be inconvenient because of the difficulty of maintaining the sample or bath below $T_c$ with a heat load of 50 mW. Aluminum is therefore the only sensible choice, since its $T_c = 1.25 K$ for typical evaporated films makes the cryogenics trivial, and its long $\tau_B$ (13 ns) ensures a large nonequilibrium effect. To see how large, we must estimate $B=NR\tau_B/8N(0)T_c$, which parametrizes the disequilibrium. Here, $N_R$ is the number of phonons absorbed per unit volume per unit time for the metal in the normal state. Consequently, what we need is the incident phonon flux and the normal state ultrasonic attenuation coefficient for longitudinal phonons, $\alpha_i$. The incident phonon flux is the available microwave power, $P_{\text{microwave}}$, times the conversion efficiency $\eta \approx 10^{-4}$, divided by both the cross-sectional area of the crystal, $\pi (0.2 \text{ cm})^2$, and the energy per phonon. In the limit that the product, $qL_e$, of phonon wavevector and electron mean free path is small compared to 1, Pippard (1955) tells us that

$$\alpha_i = \frac{4\pi m_e v_F q^2 l_e}{15 \rho c_i} \quad \text{(IX)}$$

where $N$ is the free electron density, $m_e$ is the electron mass and $\rho$ is the metallic density. At 9 GHz and using values from the standard table, we find $qL_e=0.66$ and $\alpha_i=2.85\times10^2 \text{ cm}^{-1}$. It is known (Lax, 1959 and Wang and McCarthy, 1969) that in pure Al samples, the measured ultrasonic attenuation is about 25% larger than Eq.(IX) would predict, when $l_e$ is obtained from conductivity data. Therefore,
\[ N_R = \text{phonon flux} \times \alpha_i \]

\[ \approx 1.9 \times 10^{26} \text{ phonons sec}^{-1} \text{ cm}^{-3} \text{ k}W^{-1}, \text{ and} \]

\[ B \approx 0.115 \text{ k}W^{-1} \]

Measurements of \( I_c \) enhancements require no more than a thin strip of the material described above, but measurements of gap enhancements require another film. The single particle tunneling characteristic of a superconductor-insulator-superconductor (SIS) tunnel junction allows us to measure the gap of either junction film quite directly. If I made both films of identical Al, then changes in the I-V induced by the phonons would be the net result of changes in each film. From the standpoint of analysis it is preferable to examine only one change at a time. Thus I sought a second film, or counterelectrode, which would be relatively inert with respect to the phonons.

I found a solution to this in \( O_2 \)-doped Al. The oxygen-doping allows the \( T_c \) to be raised above the \( T_c \) of clean Al. Typically we added \( O_2 \) to the extent that the transition temperature was raised to \( T_c \approx 1.8 \text{ K} \). It is common to refer to this doped Al as dirty, or granular Al. The impurity scattering caused by the oxygen shortens the mean free path, \( l_\phi \), by almost two orders of magnitude. This was determined via a resistivity measurement and the known constancy of \( \rho l_\phi \). This reduces \( \alpha_i \) by the ratio of the mean free paths and similarly reduces \( N_R \) (at constant phonon flux). Chi and Clarke (1979a) measured \( \tau_E \) for similar dirty films and found it reduced by 10 from that of the clean films, 3 times more reduction than one would expect from the \( 1/T^g \) dependence of \( \tau_E \). Although Chi and Clarke did not measured \( \tau_{E,\text{dirty}} \) at the temperature of interest, we can be confident that it is less than \( \tau_{E,\text{clean}} \) at these temperatures. The significance of this is simply that through B's linear dependence on \( \tau_E \), the effect of the pho-
nons on the dirty film is less than on the clean one. Furthermore, since the function \( G(u=\Delta/\nu) \) decreases with increasing \( u \), at 1.2 K \( G(u_{\text{dirty}}) \approx 0.2 \) and \( G(u_{\text{clean}}) \approx 2.0 \). Another way of saying the same thing is that the counterelectrode will be at much lower reduced temperature than the test film and therefore will have fewer quasiparticles available to absorb phonons. Overall we may conclude that \([\delta T/ T_c]_{\text{dirty}} < 0.01 [\delta T/ T_c]_{\text{clean}}\). Worth more than all of the above reasoning is the fact that on the one dirty sample on which I made critical current enhancement measurements, the measured enhancement was \( \approx 1.5 \times 10^{-3} \) at \( T/ T_c = 0.98 \), which is 30 times less than on clean strips at the same reduced temperature and comparable input power.

The high \( T_c \) of the counterelectrode has another valuable consequence. At 1.2 K its reduced temperature is \( T/ T_c \approx 0.87 \). In any real film, granularity, anisotropy and inhomogeneity of the material imply that a distribution of \( T_c \)'s must replace a single-valued \( T_c \). Via the temperature dependence of the gap, this distribution of \( T_c \)'s rounds off the singularity in the BCS density of states. This is known as gap smearing. Since it is the BCS singularity that gives rise to the sharp features in the I-V of an SIS junction, at low \( T/ T_c \), where the temperature dependence of the gap is small, the smearing will be small and the I-V's are wonderfully sharp (witness all the equilibrium I-V's in this thesis), allowing precise location of the equilibrium gap and precise measurement of deviations from it.

2.2. Layout of the Sample

The samples for this experiment were generally composite structures consisting of an SIS tunnel junction and a narrow superconducting strip etched in the test film of the junction. Both of these were on the 4 mm diameter end face of a quartz rod 1” long. With the samples having this in common, we made two
varieties, shown in Fig. 14.

The distinguishing feature of the wide samples was that the strip was outside the junction area, but it was etched in the same test film, and the junction was wide. Wide samples were made of both possible configurations: the test film being the lower film, and vice versa. This precaution was taken against the possibility that observed differences in enhancement between strip and junction were the result of either different acoustic couplings or heating conditions. These might have arisen from the fact that the strip can be against both the crystal and the superfluid bath, whereas the test film of the junction cannot be in contact with both of these. I observed no differences between these two configurations and will not make any further distinction between them.

In narrow samples, the strip and test film of the junction were the same (see Fig. 14b). The junction is 10 \( \mu m \) wide and 300 \( \mu m \) long. Since it was so narrow, \( I_c \) measurements could be made in the test film of the junction. Be reminded that \( I_c \) refers to the critical pair-breaking current of the strip and not to a Josephson-like current in the junction. This kind of sample is truly ideal, because whatever parameters govern the enhancement (\( \tau_E, N_R \), heating, etc.), they must be the same for both measurements and thus a very direct comparison between the two is permitted. All narrow samples were made with the counterelectrode in contact with the crystal.

2.3. Substrate Preparation

Preliminary to making anything on the quartz rods, I made many junctions and strips on glass slides in order to test and become familiar with the fabrication procedure while avoiding the extra care needed for handling the crystals. Both the slides and the crystals were prepared in the same way. Using rubber gloves at all times, I washed the substrate in Labtone and water, patiently and
Figure 14. Two types of composite samples. a: Wide junction with nearby strip. b: Narrow junction in which clean film is also the strip.
gently rubbing the crystal surface with a Q-tip. This step removed any grease that might have prevented adhesion of the evaporated films. Next I rinsed the soapy water off my gloves and then began to rinse it off the substrate, again using a clean wet Q-tip to brush the surface. I repeated this and then squirted a large amount of distilled water over the clean substrate. The distilled water washed away any particulates left by the industrial water of LeConte Hall. Then I used bottled \( N_2 \) to blow dry the sample, taking care not to blow water from the wet gloves all over the newly clean substrate. I cleaned the substrates in the final step prior to beginning the evaporation. Dust is omnipresent in LeConte and it will certainly short out any junctions if given the chance. This is also the principal reason for the final rinse in distilled water. One can be certain that one hasn't cleaned well enough when films peel off and particulates puncture the junction. A bright light at an oblique angle will make both water spots and particulates visible. Visual inspections of this kind saved a lot of time.

The only subtle point about preparation concerned the photolithography. In order to get fine line widths and sharp edges, it is essential that the substrate be in intimate contact with the photographic plate that is the mask defining the line. Alignment of a pattern already on the substrate and the mask is accomplished when the two are just out of intimate contact and may be moved with respect to one another. The quartz rods have microscopic burrs on their edges which are big enough to scrape the image off the mask, even when crystal and mask are not in contact. Removing these burrs with a light chamferring made the difference between a high rate of success and failure.

2.4. Evaporations and Oxidations

The films were deposited using standard evaporation techniques and aperture masks. Usually the first film was the dirty Al counterelectrode. After
pumping the evaporator down below $10^{-5}$ Torr, a micrometer needle valve connecting an oxygen bottle to the vacuum space was opened, and sufficient flow was established to reach $10^{-4}$ Torr. The evaporation was done at about 20-25 Angstroms/sec. Consistent dirtiness may be characterized by the resistivity, the $T_c$, and the reflectivity of the film as seen by the naked eye. As the film got more doped up, it looked increasingly less metallic and more black, rough, and non-reflective. If sufficient oxygen is added, the resistivity may be increased to several hundred $\Omega$-cm and the transition temperature may be pushed above 2 K (Deutscher et al., 1973, Chi and Clarke, 1979a). Another more subjective test of $O_2$-doping is how easily an oxide is grown on it. On clean Al, an oxide suitable for making high quality tunnel junctions may be grown in air in a few minutes. On dirty Al of this kind, an oxide is more difficult to obtain. The oxide must provide a junction resistance small compared to 50 $\Omega$ and yet large enough not to be prone to superconducting shorts. In order to operate within these limits one must attain reproducibility in the oxidation procedure, which as we have seen is linked to the doping of the first film.

Consistent doping may be achieved if the following precautions are taken. First, one must always use the same $O_2$ pressure in the chamber. Since the vacuum gauges are notoriously variable over a period of months, this constancy of pressure is most easily achieved by regulating the $O_2$ pressure on the high side of the valve (at say 10 lbs), and by setting the micrometer needle valve to the same point every time. Second, one must be able to see the aluminum melt and evaporate in order to provide human feedback to the current through the evaporation boat. Third, in order to get the Al to wet the boat and to produce an even evaporation rate, one must apply much more heat than one would think was necessary. Practically, this means using 60-100 A until the Al gets soft and sags under its own weight into the boat. Then, applying a little more current
will produce rapid melting, wetting of the surface, swirling in the boat as a result of eddy currents, and evaporation at more than 10 nm/sec. Finally, turn the heat down, monitoring the decrease in rate until it is about 2.0-2.5 nm/sec.

It is typical that the pressure will drop in the bell jar from $10^{-4}$ to $8 \times 10^{-5}$ Torr during the evaporation. This recipe will yield $O_2$-doped Al of $T_c \approx 1.8$ K, with resistivities of 30-100 Ω-cm; it will be reflecting but somewhat brown and not as shiny as the clean material. Using a new, tungsten filament, spiral boat every time is essential, as is using enough Al, so that the rate doesn't drop before enough material has been deposited. In my experiments I used a 100 nm counterelectrode.

The four masks used are shown in Fig. 15. Mask I was used for the counterelectrode: it is just a 1/32" strip running across the center of the crystal face. The masks were designed so that up to 4 crystals could be prepared simultaneously, or so that 4 junctions could be made on a single glass slide, as in Fig. 16. Masks II and III allowed the mutually perpendicular sides of an insulating SiO window to be deposited over the first strip. After oxidation, Mask IV made the cross strip centered over the window, thereby completing the junction.

The two SiO evaporations were fairly trivial; 65nm of SiO, deposited at 1 nm/sec gave consistently insulating windows. The only problem was with SiO molecules bouncing into the junction region, reducing its effective area and producing much too high a resistance. This occurred when the mask and substrate were not in good contact, as in Fig. 17. It is vitally important to make sure that this doesn't happen.

The next step was oxidation of the first metallic layer. Daily and seasonal variations in humidity and temperature do not permit consistent thermal oxida-
Figure 15. The four masks used in sample fabrication.
Figure 16. Four junctions prepared on a glass slide using the masks of Fig. 15 and the procedure in the text.
Figure 17. Origins of SiO creep. A small space between the crystal surface and the aperture mask defining the junction may allow hot SiO molecules to bounce into the junction area, thereby reducing its area. Molecule A sticks on impact. Molecule B bounces off its targeted area and eventually lands where the junction is supposed to be.
tion of Al in the open air of the laboratory. It is easy to grow an oxide on Al in the open air, but difficult, as the weather changes, to grow the desired one without considerable trial and effort. In pursuit of a weather resistant oxidation procedure, I began to oxidize the counterelectrode "without breaking vacuum." Rather than open the bell jar to the atmosphere, I admitted a fixed pressure of bottled $O_2$, and then pumped it out for the final clean Al evaporation. The conditions were 150 Torr for a period of 10 minutes up to a couple of hours, depending on the desired junction specific resistance. I believe that the pressure was not relevant, although I kept it at that value in all of my oxidations, and that even with much less $O_2$, the oxidation would have proceeded at nearly the same rate. The small leaks to the atmosphere that prevent a typical evaporation set-up from achieving pressures below $10^{-7}$ Torr admit the constituents of air to the bell jar at all times. It is for this reason that one can oxidize an Al film in vacuum.

The final step in the evaporation was the deposition of the clean Al test film. Maintaining the chamber pressure below $4 \times 10^{-6}$ Torr and depositing at rates greater than 5 nm/sec produced films of $T_e \approx 1.25 \, K$ and electron mean free path $l_e \approx 70$ nm. As in the procedure for the dirty films, it is imperative that one be able to see the material as it begins its melting. Only 40 A of current to the filament is required for the initial heating, with more once the evaporation begins.

Thus one fabricates an Al-AlOx-Al tunnel junction, consisting of two 1/32" Al strips, masked off at their point of crossing to give a junction area of $(\approx 300 \, \mu m)^2$. Before wasting any time with the photolithography, the junctions and strip resistivities were checked with a low power ohmmeter.
2.5. Photolithographic Steps

The 10 μm wide strip for critical current determination was etched out of the wide, clean strip using photolithographic means. The narrow line width is easy to achieve with the machinery at ERL. In outline, the procedure for making this narrow line is given below:

(1) Use compressed air to blow the sample clean.

(2) Spin on Shipley 1350J photoresist (diluted 1:1) at 5000 rpm for 30 seconds.

(3) Bake the coated substrate at 85 K for 25 minutes.

(4) Blow the sample clean.

(5) Using the Kasper Contact Aligner, orient the photomask and substrate with respect to one another. For wide samples, orient the 10 μm line on the clean strip but away from the Si0 and junction area. For narrow samples, orient the line in the junction area. Special photomasks were made for the narrow samples so that the strip length exactly equalled the junction width. After alignment, expose the photoresist.

(6) Develop in Shipley AZ Developer, diluted 1:1 with water. Exposure should be long enough to give a well defined image in less than 60 seconds, but short enough so that 60 seconds is required for full development. Any other exposure length gives results that are very susceptible to human error. Development is terminated by rinsing in water.

(7) Etch the developed sample in Aluminum Etchant, which is probably mostly phosphoric acid. The process may be inspected with the naked eye while the sample is in the etchant, or it may be checked by rinsing with water, allowing inspection under a microscope. Etching 100 nm of clean Al normally takes 2-5 minutes. The same thickness of dirty Al seems to take
somewhat longer.

(8) Soak the etched samples in clean acetone to remove the photoresist.

(9) Rinse in distilled water and blow dry.

(10) In narrow samples, if the finished junction/strip shows cloudy areas of dirty Al inside the masked off area (Fig. 18), then that is probably evidence of SiO creep and the effective junction area may be correspondingly smaller than it appears to be.

(11) Before wasting any time mounting and/or testing the sample, junction and strip resistances should be checked with a low power ohmmeter.

The real difficulties with this procedure are hidden in step (5). The 1" quartz crystal is longer than the distance between the plane that holds the substrate and the plane of the photomask surface. This is not surprising since the Kasper was designed for thin, silicon wafers. The problem could be circumvented only by raising the plane of the photomask, reversibly, without altering the Kasper. This was accomplished by suspending the mask higher in the Kasper's optical path than it is normally (Fig. 19). The other difficulties were also associated with how to handle a rod-like substrate with equipment designed for wafers. After having reduced those problems to a minimum, the photolithographic steps were quite standard and easy.

2.6. Mounting and Alignment

After satisfying myself that the finished junction and strip were worth investigating further, I attached it to the probe, electrically and mechanically. For wide samples I attached 5 leads, and 6 leads for narrow ones, as in Fig. 20. In this way four terminal I-V measurements could be made on either of the strips or the junction. The leads were #38 or #39 copper wire, tinned with
Figure 18. Narrow junction, (a), showing evidence of SiO creep. The shady region, (b), inside the SiO window results from SiO that bounces into the window, covers the Al underlayer, and then prevents the etchant from attacking the Al. In this figure the effective junction area is reduced 15-20%.
Figure 19. This is an edge-on view of the Kasper Contact Mask Aligner used to expose photoresist patterns on the end of a 1" quartz rod. Also shown are some accessories used to circumvent the limitations of the machine. A special mask, a, is suspended above the usual plane, e, by a mask holder, b, attached with screws, c, to the Kasper mask holder, d. A jig, f, for holding the crystal is held in place against the substrate table by vacuum supplied by the Kasper. The O-ring, g, and mylar seal, h, aid in that sealing process.
Figure 20. Arrangement of 4-terminal measurements of I-V characteristics of junctions (a,c) and strips (b,d) for wide (a,b) and narrow (c,d) samples.
solder and coated with indium just prior to attachment. This attachment to the films was made with indium pads pressed onto the film, followed by similar pressing of the lead onto the pad, and finished with a second indium pad pressed onto the first. This was performed under magnifying lenses using tweezers and a teflon 'pencil' to squeeze the pieces together. I rarely had a lead come off on the first cool-down, but occasionally one would come off on a later cool-down after thermally cycling it. Hence, the experimenter must check these connections assiduously. Once electrically connected, the crystal was inserted into the cavity (sample end out, of course). The probe was attached to the microwave sweep oscillator while still at room temperature. The cavity resonance was monitored and the crystal moved in or out until the desired frequency was obtained. A spring clamp was tightened against the rod and the narrow annulus between crystal and cavity was closed with silver paint, completing the mounting process. The cavity was tunable over nearly 2 GHz, but this preliminary alignment served to ensure that the desired TWT frequency was within that tuning range. Also, at the low end of the range, the resonant frequency was least susceptible to changes resulting from vibrations or other motions or just plain drift. With a Q near 1000, eliminating such susceptibilities was very desirable, so the preliminary alignment served to place the desired TWT operating frequency as near as possible to the low end of the range.

3. OBSERVATION AND DETECTION OF ENHANCEMENT

3.1. Junctions

Wide and narrow samples were measured in the same way. Unless a specific distinction is made, all the details of this section refer to both types.
3.1.1. Determination of Equilibrium Properties of Junctions

The equilibrium value of the gap, $\Delta$, was measured using a 4-terminal measurement of the most standard sort, as shown in Fig. 20. The junctions were always current biased. The current through a series resistor was monitored with one PAR 113, while the voltage across the junction was monitored with another. For quick measurements, I used a low frequency oscillator to sweep the current and I monitored the I-V on an oscilloscope. For more permanent and easy-to-read records, and also for the nonequilibrium measurements, it was necessary to use a battery operated dc source, whose output could be swept either slowly by hand or with a motor driven potentiometer; the I-V was recorded on an X-Y recorder. Using the cusp at the difference of the gaps and the step at the sum of the gaps, one could determine $\Delta$ for each film.

The bath temperature was regulated by means of an ac resistance bridge providing feedback current to a heater immersed in the helium. The bath temperature was measured by using that same bridge to determine the resistance of a carbon resistance thermometer calibrated against a germanium standard. Regulation was better than 0.1 mK, but absolute accuracy was only within a few mK. An early cause celebre of enhancement experiments was the issue of the shielding of the thermometers from the microwave sources. In particular, if the resistance of a carbon thermometer without microwaves at a given temperature was the same as the resistance of the thermometer with microwaves at a lower temperature, then the bridge/heater circuit will cause the bath temperature to drop when microwaves are applied and the measured superconducting energy gap will increase. The naive observer will then deduce incorrectly that the influence of microwaves on a superconductor is to increase its gap, or critical current if that's what one is measuring. Because of the very low duty factor of
the perturbation in this experiment, it is possible to measure the equilibrium I-V even with the perturbation on, and then to compare it with the equilibrium I-V without the perturbation. Upon doing this, one finds that the I-V's are identical and therefore that the bath temperature is not changed by some interaction of the microwaves (or phonons!) with the thermometer.

After having determined \( \Delta \) (for the test film only) and \( T \), one plots \( \Delta^2 \) vs. \( T \), which is linear near \( T_c \), according to Ginsburg-Landau theory,

\[
(T/T_c - 1) + \beta \frac{\Delta^2}{(k_B T_c)^2} = 0.
\]

Without resorting to derivative techniques to resolve the cusp and the step, I could usually measure \( \Delta \)'s as low as 10 \( \mu eV \), which means \( T/T_c \approx 0.999 \) and \( 2\Delta < h\nu \). The extrapolation to \( T_c \) was quite straightforward. After determining this, the slope could be compared with the GL or BCS prediction; agreement to within a couple of percent was always found. Predictions of BCS or extensions of GL down to temperatures as low as \( T/T_c \approx 0.82 \) agreed with measurements of \( \Delta^2 \) vs. \( T \) using \( T_c \) as the only parameter. Also on the subject of \( T_c \)'s, the transition temperature of the counterelectrode was determined roughly by measuring its \( \Delta/T \) for some \( T \) at which both films were superconducting, and finding its \( T/T_c \) from tables.

A feature of SIS junctions that is sometimes used to characterize their idealness is how close the I-V approaches the origin as the current is swept towards zero. Any deviation from zero may be regarded as a superconducting short in parallel. It is easy to misinterpret the effect of such shorts if one is concerned with Josephson phenomena. In this experiment I wasn't looking at Josephson phenomena, but there was no shorting either. In other words, these were extremely high quality SIS junctions, with razor sharp features at all but
the highest temperatures.

3.1.2. Measurement of Nonequilibrium Effects in Junctions

As in the equilibrium measurements, detection of nonequilibrium effects was effected by the junction being current biased. Under these conditions the response produced by the pulsed perturbation was a pulsed change in voltage across the junction. One might conceivably measure the nonequilibrium I-V by using a suitably wideband dc amplifier to measure the voltage during the perturbation, but one loses sensitivity that way. More sensitivity is achieved by using an ac-coupled amplifier and by measuring first the change in voltage, $\Delta V$, due to the phonons. The ac-amplifier I used was a B&H Products AC3020LN, which provided 30 dB power gain from about 10 kHz to 1.3 GHz with a noise figure of 1.2 dB. It had 50 $\Omega$ input and outputs. Needless to say, not all that bandwidth was needed for this experiment; so the output was shunted with a 0.001 $\mu$F capacitor, rolling off the wideband noise at about 3 MHz, which allowed for 100 ns resolution of the leading and trailing edge of the change signal. The leads on the probe were only twisted pairs, yet they possessed plenty of bandwidth for the task. Following the preamplifier was a PAR 115, operated on its 50 $\Omega$ input with a voltage gain of 10. Next came the input of a Tektronix RM647 oscilloscope, using a 10A2 plug-in which has a 10x output, but which is actually about 11 or 12 (see Fig. 21). Calibrated as a string from low noise input to final oscilloscope output, these amplifiers provided an overall gain of 3100±1%.

It will be useful to digress for a moment and describe the change signal so that the purpose of the detection electronics yet to be described can be understood. The oscilloscope was triggered a microsecond or so before the microwave pulse. The output of the amplifiers is zero until that time, which is
Figure 21. Block diagram of the electronics for measurement of $\Delta V$ and reconstruction of nonequilibrium I-V. All electronics except the junction are at room temperature.
essentially the moment the microwaves arrive at the junction. The effect of a 600 W pulse on the Al films is dramatic. Admittedly, the pulse is not well coupled to the films, the energy being absorbed mostly in the cavity, and the films being outside it. Nevertheless, the films see it, they absorb it, and they get hot, or at least their quasiparticle distributions are effected. Consider such an effect on the junction whose I-V is shown by the solid line in Fig. 22. If the pulse is intense enough, then for its 1 μsec duration the films will be normal and its I-V (if one could record it in a microsecond) would be the dashed line. Biased let's say at 500 μA, the amplifier string would see a negative going pulse of about 80 μV, and via its 3100 gain would try to produce a 250 mV output. The phonons, when they arrive 4.2 μsec later will produce a ΔV of the order of 2 μV and consequently about 6 mV at the output of the amplifiers. After amplification, the signals are fed into a PAR 162 Boxcar Integrator with a PAR 165 plug-in whose input range may be varied from ±50 mV full scale to ±5 V. In order not to overload the boxcar with the microwave change signal and yet to retain adequate sensitivity for measuring the phonon change signal, the microwave part had to be clipped or attenuated. The only effective means of doing this was to use an FET switch to short the amplifier output just while the microwaves were on. The schematic for this is shown in Fig. 23, and it worked.

The amplified change signal, with microwave component excised, was fed into the boxcar and averaged there, yielding a dc signal level proportional to the change signal at the time of the boxcar's sampling window. Whatever the boxcar input range (in volts), its full scale output is ±10 V, so the system gain up to this point is

\[ 3100 \times \frac{10 \, V}{\text{Input range}} \]

With the window at some fixed time, in the middle of the phonon change signal
Figure 22. Representative I-V of junction at $T/T_c = 0.96$. Dashed line is meant to represent fairly the normal state resistance of the junction.
Figure 23. Schematic for FET switch used to isolate the amplifiers from $\Delta V$ due to the microwaves.
for instance, one could sweep the bias current slowly enough to give the boxcar time to integrate, and plot I vs. $\Delta V$ overlayed on I vs. $V_{eq}$, as in Fig. 24a. The horizontal scales are made different for purposes of resolution. The point is to display how the changes $\Delta V$ are correlated with the features of the equilibrium I-V.

The nonequilibrium I-V is the sum of the two curves just described. That is, the full nonequilibrium $V$, at bias current $I$, is the sum of the equilibrium $V$ and the change $\Delta V$ induced by the perturbation. Thus the nonequilibrium I-V is obtained by adding $\Delta V$ and the equilibrium $V$, at of course the same bias current. This may be performed in one's head, by hand, or even digitally. I chose to do using a summing amplifier to add the output of the boxcar ($\Delta V$) and the output of the dc amplifier that make the equilibrium voltage measurements (see Fig. 21). It is possible to get the equilibrium $V$ from the dc amplifier even when the perturbation is on because of the low frequencies, relative to the pulse width, at which one may roll off the response of the PAR 113, and also because of the very low duty factor at which the experiment was run. The X-Y recorder was used to plot I vs. "output of the summing amplifier," or what I called the reconstructed I-V. Fig. 24b is an example of a typical reconstructed I-V, superimposed on its equilibrium I-V. By viewing the data this way one gets a very clear understanding of the changes produced by the phonons.

Thus we have seen three methods of recording the change signal $\Delta V$: examining the real time record on the oscilloscope, plotting I vs. $\Delta V$ on the X-Y recorder, and also plotting I vs. $V_{neq}$. In addition I sometimes swept the boxcar window and used the chart recorder to get a hardcopy of $\Delta V$ vs. time. Another method I used extensively was to measure the boxcar output with a battery operated Fluke voltmeter, Model 8020B. This was useful when all I wanted to
Figure 24. (a) Equilibrium I-V of a typical junction superimposed on the change signal $\Delta V$ at high phonon power. The large spike at $\approx 140 \mu A$ is an artifact of the measurement. (b) Reconstructed I-V superimposed on equilibrium one.
record was $\Delta V$ at a given bias current, temperature, and input power. The digital output is much easier to read than the analog meter on the face of the boxcar. Digital circuits have their own problems with sensitive superconductivity measurements. A line-powered DVM, plugged in and turned on but not connected to anything, would introduce enough high frequency noise to make measurements impossible, increasing background noise on the oscilloscope trace to unacceptable levels. Even the handheld DVM had to be kept away from the current and voltage leads.

General debugging and surveying of the experimental situation was aided greatly by being able to see the change signal on the oscilloscope. With the bandwidth limited to 3 MHz, the amplifiers will yield a background of 5 mV RMS, or 1.6 $\mu$V RMS referred to the input. Without averaging, 1 $\mu$V or so represents the limiting change signal one might be able to see on the oscilloscope. Arduous experimenting with cabling was the only route to eliminating additional sources of noise that would have made life in the screened room that much more difficult. Both the current and dc-voltage leads were twisted pairs of 50 $\Omega$ coax. The connection to the single-ended low noise ac-preamplifier was made via another coax, this one covered with an extra braided shield. Ground had to be established at the cryostat with the heaviest grounding strap I could arrange to use. The signal levels across the series resistor in the current source were large enough for direct measurement with the X-Y recorder, but unless that signal was buffered first (with a PAR 113) I couldn't do the reconstruction.

3.2. Strips

The two sample configurations called for slightly different wiring of the strips for $i_c$ measurements, as shown in Fig. 20b,d. In both cases they were 4-terminal measurements that were identical except for the placement of one
3.2.1. Determination of Equilibrium Properties of Strips

Equilibrium characterization of the superconducting strips was made by comparing the magnitude and temperature-dependence of the critical current with predictions of the GL theory. Unfortunately this is a more subjective task than in the Δ-case. GL theory for strips (Tinkham, 1975; Skocpol, 1976; Klapwijk, 1977) describes the critical current in terms of material parameters like the resistivity in the normal state, thermodynamic critical field $H_c(0)$, and coherence length $\xi_0$, and the London penetration depth $\lambda_L$:

$$j_c = 1.55 \frac{H_c(0)}{\lambda_L(0)} \left( \frac{l}{\xi_0} \right)^{1/2} \left( 1 - T/T_c \right)^{3/2}.$$

Thus, one has to rely on previously measured values of the last three, and on geometrical measurements of the sample in question, to determine the first (to get the mean free path, $l_*$). To further complicate matters, the validity of the one-dimensional approximation will certainly fail at lower reduced temperatures, and the current distribution will cease to be uniform; then, one of the few things one can be sure will happen is that the critical current will fall below the GL prediction. This contrasts with measurements of the gap, where the only parameter one needs to know is $T_c$, and then the junction acts like a thermometer at all temperatures, at least for weak coupling superconductors like aluminum.

One needs to know how close to ideal is the material in the superconducting strip, in order that there be some basis for comparison of nonequilibrium measurements to nonequilibrium theory, which uses as a starting point an ideal superconductor in equilibrium. Not being able to make this comparison as well as one would hope, one does the best one can. That is, given the uncertainties
of the relevant quantities, can one be confident that the strip is behaving nearly ideally?

The quantities one has to measure are the strip $T_c$, resistivity, length, and cross-sectional area. Near $T_c$, $i_e^{2/3} = \Gamma(1-T/T_c)$, where $\Gamma$ is a constant that comes from the theory. The usual procedure is to plot $i_e^{2/3}$ vs. $T$ and extrapolate the line to zero current. The $T$ intercept is $T_c$, and the slope tells one what fraction of the ideal current one has. As in equilibrium junction measurements, a low frequency oscillator was used to sweep the current and the complete I-V was viewed on an oscilloscope. One was always able to measure a critical current up to temperatures such that $i_e \approx 20 \mu A$, and often up to $i_e \approx 1 \mu A$. Thus the extrapolation to $T_c$ was only a couple of millikelvins. The oft-mentioned tails of $i_e^{2/3}$ vs. $T$ were neither observed nor of concern (Falco, Werner and Schuller, 1980).

The resistance measurements at room temperature and just above $T_c$ were easy enough to determine within a few percent, but extracting the resistivity required knowing the length and cross-sectional area. The length of several hundred micrometers was always determined visually using a calibrated grating under a microscope. The grating divisions were 12.5 $\mu m$ apart so realistically one could say that the length was known to 1-2%. Thickness was usually determined by a crystal film thickness monitor used during the evaporation procedure. This had been calibrated against a Dektak film thickness measuring instrument. Occasionally, consistency checks were made and the results were positive, <10% deviation. Thickness thus measured does not concern itself with electrical properties, only mechanical ones; thus it leaves room for errors in the resistivity calculations because it is possible that portions of the film are not electrically active, e.g. the upper 5 nm of the film may become oxidized and
non-conductive. The film width was predetermined by a photomask whose dimensions were set macroscopically and then reduced precisely. Nevertheless, variations in photolithographic procedure produced width variations which could easily have been 20%. The film width was measured visually, using a graticule under a light microscope. Given the 12.5 \textmu m grating spacing, the measurement of lines 3-10 \textmu m wide consisted of nothing more than an acknowledgement that they were close to what they should have been. To the accuracy of the measurement the lines were sharp, but their electrical dimensions may have been somewhat less than they appeared to be. The resistivity extracted from the resistance using these geometrical dimensions yielded a \( \Gamma \) that was consistently about 30% larger than the measured value. For the very narrowest lines (3 \textmu m) the discrepancy was as much as 60%. Considering the accuracy of the measurements and their implicit limitations, this is very good agreement. One worries that the strip is not in fact an homogeneous superconductor, but instead some kind of weak link. In that case the \( i_c \)-enhancement is not necessarily due to \( \Delta \)-enhancement (Hunt and Mercereau, 1967; Lindelof, 1976) and the ILE theory is not directly applicable. On the basis of the consistency of the ratio of predicted to measured \( \Gamma \) values, I am confident that \( i_c \) was determined by the behavior of the film overall and not by some constriction in it. Another approach to calculating the film resistivity and \( i_c \) is to assume that the room temperature resistivity of the film is the phonon-limited handbook value, 2.72 \( \mu \Omega \)-cm (Fickett, 1971). Then, using the measured length, extract the effective film area, and from \( R_T \approx T_c \), get \( \rho \) and then \( \Gamma \). With this method, the measured \( \Gamma \)'s are only a few percent below the the calculated ones. Using either method, I believe there is satisfactory agreement to convince ourselves that sufficiently close to \( T_c \) the superconducting strip is behaving ideally.
3.2.2. Nonequilibrium Measurements of Strips

Unlike equilibrium measurements, it is impossible to observe $i_e$ enhancement using an audio oscillator as a swept current source. The phonon experiments of Tredwell and Jacobsen (1975 and 1976) employed a method whereby the weak-link was dc-biased at some current greater than the critical current in equilibrium, $i > i_e$. Fig. 25. Upon arrival of the phonons, the weak-link switched to a zero-voltage state and produced a large negative-going voltage pulse. The largest current at which this switching occurred was identified as the enhanced critical current, $i_e^*$. This method was also of no use in my experiments. They both fail because the primary pulse drives the strip normal, and hysteresis won't permit it to return to the zero-voltage state until the current is returned to some very low value, for example, zero (Pals and Dobben, 1979).

To avoid the hysteresis problem, I kept the current turned off until the phonons were on the sample. Then, once an enhanced state had been reached, I injected a short, square current pulse, increasing its magnitude with each successive firing of the TWT until I detected a voltage across the strip. I identified that threshold current as $i_e^*$. I used this same technique to examine the equilibrium critical current and found that it agreed with the dc measurements.

Fig. 26 is a pulse timing diagram, indicating the strip's response for a variety of pulse heights, arrival times, and durations. It was necessary to keep the pulse short so that it was off before the phonons turned off; otherwise, for currents $i_e^* < i < i_e^*$, I detected a pulse that was zero while the phonons were on, but finite for that part of the current pulse following the end of the phonons. In the phonon intensity diagram, the second and third pulses signify echoes of the first phonon pulse. Most of that pulse is reflected at the quartz-Al-He interface, travels back down along the crystal, is reflected at its other
Figure 25. Representation of the method used by Tredwell and Jacobsen (1975 and 1976) to observe Josephson-like critical current enhancement.
Figure 26. Pulse timing diagram for nonequilibrium critical current measurements. The current pulse, $a$, is less than the equilibrium critical current, $i_c^0$, but since $a$ occurs when the microwaves are on and $i_c$ is suppressed, there is a voltage across the strip. The pulses, $b - e$, all satisfy the inequality $i_c^0 < i < i_c^*$. The pulse, $a$, is entirely within the duration of the phonon pulse so $V_s = 0$, except for some switching transients. The pulse, $c$, is identical to $a$, except that the phonons are off during $c$, so $V_s > 0$. The pulse, $d$, begins once the phonons are on, $V_s = 0$, and continues until after they are off, $V_s > 0$. The last pulse shown, $e$, begins before the phonons turn on, so due to hysteresis, $V_s > 0$ for the duration of $e$. 
end, and returns back towards the Al, repeating this sequence until all the energy is absorbed, transmitted, or scattered away. No systematic study of the intensity of the echoes was undertaken. For the purpose of illustration in the figure they are shown to be nearly equal. One could also see the echoes producing a $\Delta V$ in the junction measurements. Under certain conditions the train of $\Delta V$s could be quite long, showing as many as 17 echoes.

The electronics set-up for pulsed $i_c$ measurements is shown in Fig. 27. The preamplifier (the same one used for $\Delta$ measurements) is single-ended and its use as a threshold voltage detector necessitated using a floating pulsed current source. For this I used a Chronetics Type PG-10 pulse generator, floated with an isolation transformer between it and the power line. A Wavetek 5080 1 dB/step attenuator was used to set the current range and obtain maximum sensitivity with the output potentiometer on the pulse generator. The pulse was rounded and current limited with the low pass filter, and the current was measured by looking at the voltage across R2. The only available amplifiers at these frequencies were single-ended ones so a pulse transformer (Pulse Engineering, PE 2273 132-AW 2F) was necessary to eliminate grounding problems. To a first approximation, the boxcar was looking at the voltage across 50 $\Omega$. The boxcar window was set by examining the traces of its timing pulse and $V_{R2}$, and adjusting the timing window accordingly. The window was usually set to be about 400 ns, or one-half the width of the current pulse. The output of the boxcar was read with the Fluke DVM. At the bottom of the cryostat, a 25 $\Omega$ resistor was inserted in each current lead. This had the effect of roughly impedance matching the current source to the strip. This greatly reduced the turn-on and turn-off transients seen at the strip voltage preamplifier. That preamplifier was terminated with 50 $\Omega$ and 1000 pf to reduce the effective bandwidth and improve the resolution of threshold detection.
Figure 27. Instrument schematic for pulsed detection and measurement of critical currents. Connection of the strip to the outside world is via twisted pairs, TP.
After timing the arrival of the current pulse with the arrival of the phonons, and after tuning the cavity to the input frequency by minimizing the reflected power, the threshold was found by turning the potentiometer on the pulse generator until the first hint of a transition to a finite voltage was observed. This involved art, craft, and sometimes science, but it became possible to obtain consistent results. After the output of the DVM was recorded the current polarity was changed and the threshold again detected and recorded. These two were averaged before comparing with dc-equilibrium results, which were found to be in agreement. Measurements of $i_c^2$ were made regularly during a run, both as a consistency check and as a guard against drifts or other ill-understood variables.

3.3. Timing

The timing pulse that triggered the oscilloscopes, the boxcar, and the TWT was derived originally from the Chronetics pulse generator, which was not grounded. Using an opto-electronic isolator, that timing pulse was given a reference to ground. Because such devices are slow and can't provide much current, the first oscilloscope was triggered off it, and the boxcar and second oscilloscope were triggered off the gate output of the first. The gate output of the second was used to trigger the TWT, by means of a Systron-Donner 100A pulse generator triggering an E-H Research Laboratories 132A HV pulser, which then triggered the TWT controller. The E-H 132A set the duration of the microwave pulse. It was convenient and maybe even necessary to make the connections this way. By using the second oscilloscope as I did, I could turn the TWT on or off from inside the screened room without disturbing any of the carefully timed pulses and windows. The gate output of a single oscilloscope did not have enough fan-out to permit connecting or disconnecting anything without
changing the relative timing of everything. No doubt there exists a different solution to all these problems; but the apparatus evolved, it was not born of a master-plan. In fact, little of what was ultimately necessary was even conceived of at first.
CHAPTER 4

ANALYSIS

1. GAP ENHANCEMENT

A fundamental premise of the analysis of the gap enhancement data, supported by the arguments of Chapter 3 Section 2.1, is that only the test film of the junction is perturbed by the phonons. Examining a typical reconstructed I-V (Fig. 24b), we can immediately identify the new current-step-like feature as the step at the sum of the nonequilibrium gaps. If we call the gap of the higher \( T_c \) counterelectrode \( \Delta_\text{h} \), and that of the lower \( T_c \) film \( \Delta \), this step occurs at a voltage \( eV = \Delta_\text{h} + \Delta \). By our premise, the displacement \( \Delta V \) of the step from its equilibrium value, \( \Delta_\text{h} + \Delta^0 \), is equal to the change in the gap of the test film, as

\[
\Delta^* = \Delta^0 + \Delta V
\]

The output of the boxcar is \( \Delta V \), so at the appropriate bias current, one may measure \( \delta \Delta \), the change in \( \Delta \), directly. The reconstruction is needed only to give us insight into the meaning of \( \Delta V \), and to allow us to set the bias current for measurement of \( \delta \Delta \).

The value of \( \Delta_\text{h} \) is found in equilibrium by locating the midpoint of the cusp and the step

\[
\Delta_\text{h} = \frac{(\Delta_\text{h} - \Delta) + (\Delta_\text{h} - \Delta)}{2}
\]

One might argue that the invariance of \( \Delta_\text{h} \) with respect to the phonons should be ascertained by considering the above sum, both in and out of equilibrium \(( \Delta \rightarrow \Delta^0, \Delta^* \) respectively). In practice this is impossible because the true cusp
is not observed in the nonequilibrium situation. The cusp is actually an unstable point because of the negative resistance region between cusp and step. As soon as the cusp voltage is reached, the voltage switches across to the step. Worse yet, the voltage region just below the cusp is stable only to the extent that the system noise isn't large enough to produce that switching. Thus in a noisy system, one won't be able to bias the junction as high up onto the cusp as one would in a quiet system, and the apparent cusp will appear at lower voltages than the true one. I found that turning on the TWT (but having the cavity kept off resonance so that no phonons were generated) had the effect of producing this premature switching. Let me emphasize that the equilibrium I-V's were otherwise identical, but the one with microwaves-on showed an earlier switching to the rise at the sum of the gaps. From the I-V's with microwaves off we measured $\Delta^0$ and $(\Delta^0)^2$ vs. $T$, and since that agreed with BCS, we know the cusp on those I-V's must be in the right place, and consequently, the cusp on the I-V's with microwaves off is in the wrong place. So, while it would have been persuasive to make the point about $\Delta_0$'s inertness in the way described, the necessary information was not available. I also could have had beautifully symmetric nonequilibrium I-V's, with the step displaced an amount $\delta\Delta$ and the cusp displaced the same. But alas it was not to be, and the nonequilibrium I-V's of different junctions were quite different, some showing a lot of this symmetry and others showing not much at all. All measurements of gap enhancement were made by measuring $\Delta V$ at bias currents in the step and making the assignment $\delta\Delta = \Delta V$.

From $\Delta V$ I computed the cooling $[\delta T/ T_c]$ and plotted $[\delta T/ T_c]$ vs. either $T$ or $T/ T_c$. To make a comparison with the theory we needed to know or guess the power parameter $B$ and the heating parameter $h$. The following subsections are devoted to that cause.
Before getting to that analysis, it is interesting to point out another aspect of Fig. 24b. As described earlier, the ILE theory predicts a depletion of the quasiparticle distribution function at low energies and an increase at higher energies. In addition to the induced change in the energy gap, this redistribution has consequences that may be seen in the tunneling characteristics of a junction one of whose films is so perturbed. At low voltages \( V \), the current is determined largely by the quasiparticle density \( n(E) \) in the test film (because the counterelectrode is far below its \( T_c \)) at an energy \( E = E_{\text{cusp}} - eV \). Very close to \( eV = 0 \), \( E_{\text{cusp}} - eV \) is large, and the equilibrium current and the excess quasiparticle density are very small. Thus, the equilibrium and nonequilibrium tunneling current are small and about equal. At somewhat higher voltages, the effect of the perturbation becomes larger, the quasiparticle excess becomes significant, and the nonequilibrium tunneling current becomes noticeably larger than that in equilibrium. Finally, when the tunneling current is predominately via quasiparticles of \( E < \Delta + h\nu \), that is \( E_{\text{cusp}} > eV > E_{\text{cusp}} - h\nu \), the current should drop, since the quasiparticle density is reduced by the perturbation at those energies. As discussed in the text that relates to Fig. 3, the perturbation induces an image of the singular BCS density of states at an energy \( E = \Delta + h\nu \). This image shows up in the tunneling current for \( eV < E_{\text{cusp}} \) as an additional cusp at \( eV = \Delta > \Delta - h\nu \). All of these features are clearly visible in Fig. 24b.

1.1. Heating

The principal heating mechanism in this experiment is the thermalization of the energy deposited in the quasiparticle system by the incident phonons. The magnitude of this effect can be approximately 50 mK, or in units of the reduced temperature, \( \approx 4 \times 10^{-2} \). Of considerably less importance is the self-
heating of the junction arising from dissipation of the tunneling current. This heating is obviously bias dependent, but for currents relevant to the gap measurement, I can take $V_{\text{bias}} \approx 300 \, \mu\text{V}$. The change in temperature, $\Delta T / T_c$, is estimated (in the worst case) to be

$$
\Delta T / T_c = \frac{R_K}{T_c} \frac{V^2}{R} \frac{1}{A} \approx 5.0 \times 10^{-3}.
$$

for $R=4 \, \Omega$, $A=3.6 \times 10^{-3} \, \text{cm}^2$, and $R_K = 10 \, \text{cm}^2 \, \text{K} \, \text{W}^{-1}$ (see next section). I can also conclude that the bias heating is relatively insignificant because the measured equilibrium gap agrees with the BCS prediction. Throughout the remainder of the text, I will be concerned with the principal mechanism.

Above $T_c$, the phonon absorption mechanism is well understood: phonons will be taken up by the electrons and the energy then given to the thermal phonons, the temperature of the metal rising in accordance with the thermal link to the LHe. For temperatures between $T_c$ and $T_{c>}$ (the transition temperature of the higher $T_c$ counterelectrode), the films comprise an S-I-N junction whose equilibrium voltage at constant current is temperature-dependent. It may be used therefore as a bolometer. With the phonons on, at a temperature just greater than $T_c$, and at some appropriate bias current, I observed a change in the junction voltage $\Delta V$ (or equivalently its resistance) which I interpreted as a change in temperature (Fig. 28). The temperature dependence of the resistance of this S-I-N junction is due to the temperature dependence of the gap in the superconducting film. Thus the detected $\Delta V$ arises from a decrease in $\Delta$ in the counterelectrode. The quasiparticles in that film are warmed as they approach equilibrium with phonons in it, and those phonons are in equilibrium with the phonons in the test film. The equilibrium of the phonons is established because both films are aluminum and they are separated by only a few nanometers of oxide. The phonons are heated above $T_{\text{bath}}$ because they are
Figure 28. The effect of injected phonons on the I-V characteristic of an S-I-N tunnel junction is to shift that I-V into near coincidence with an equilibrium I-V of the same junction at a higher temperature. a) I-V characteristic of sample N1 with and without injected phonons. b) Power dependence of the measured change in junction temperature above $T_c$ in sample W7.
Figure 28b.
coupled by the electron-phonon interaction to the normal electrons in the test film. The normal electrons are heated by absorption of the incident intense monochromatic ultrasound. The electrons are actually hotter than the phonons because of a thermal resistance between the two systems (as described in Chapter 2, Section 2.). In terms of the heating corrections introduced in that section, the electrons are at a temperature

\[ T_e = T_{\text{bath}} + B \nu h = T_{\text{bath}} + \delta T_{e-p} + \delta T_{p-bath} \]

where \( \delta T_{e-p} = 0.68 \times B \nu \). The phonon temperature is elevated by \( \delta T_{p-bath} \), which is given either by the Kapitza resistance or by the phenomenological heating term. Thus I may conclude that the detected change in temperature is equivalent to the change in the phonon temperature of films comprising the junction, and this that is identically the phenomenological heating term.

\[(\delta T / T_e)_{\text{phonons}} = B (h - 0.68) \nu / T_e \]

In order to convert the measured change in voltage to a change in temperature, I measured the equilibrium temperature dependence of that voltage. This proved to be linear over more than 100 mK, with a sample dependent slope of \( \approx 6 - 9 \mu V / mK \) at bias currents such that \( V(I_B) \approx \Delta / e \). With this and my interpretation, I deduced the phonon heating in the test film, which was as much as 100 mK at full radiation power.

As a check on the validity of my scheme, I carefully measured the power dependence of the heating. For small changes in temperature this should be linear, but it should roll off as the change \( \Delta T / T \) becomes sizeable compared to unity. The reason for this is that the Kapitza resistance is the coefficient of the linearized analysis of the heat flux between two reservoirs at temperatures \( T \) and \( T + \Delta T \).
As $\Delta T$ increases, the validity of that approximation fails and $R_K^{-1} \rightarrow R_K^{-1}(1 + 1.5\Delta T/T)^{-1}$. At the highest phonon power $\Delta T/T \approx 0.1$, which means that the correction is significant. We found $\Delta T(1 + 1.5\Delta T/T)$ to be a linear function (see Fig. 28b) of power except at the highest power, where the measured change in temperature was $\approx 15-20\%$ less than predicted. Nonetheless it seems fair to say that the observed power dependence is consistent with our interpretation of the effect as heating. This concept of a finite temperature difference correction to the heat flux is called the second order Kapitza resistance, $R^{(2)}_K$.

I also repeated the measurement at another bias current. The results of that determination agreed to within 10\%. I should point out that both bias currents had to be in the vicinity of $I_B$ such that $V(I_B) \approx \Delta \theta$, because at much lower currents the temperature dependence of the resistance was too small and at much higher currents the incident phonon pulses produced very little effect.

In the above analysis we claimed to have measured the heating as a function of phonon power. In fact, our means for determining the phonon power consist only of reading the dial on a calibrated attenuator which controls the microwave power going into the cavity. The outgoing phonon power is proportional to the incoming microwave power only to the extent that the transduction efficiency is constant and independent of microwave power. The measured linearity of the heating is our best indication that this is so. Measurements of the nonlinear piezoelectric behavior of quartz (Gagnepain and Berson, 1975), in conjunction with worst case estimates of the electric fields in the cavity ($\approx 2 \times 10^7 \, V \, m^{-1}$), support this observation.

$$Q = \frac{\text{Area}}{4} R_K^{-1} [(T + \Delta T)^4 - T^4] \approx \text{Area} \, T^3 \, R_K^{-1} \, \Delta T.$$
In summary, I would like to draw two important conclusions from the heating measurements. First, they represent a determination of the change in the phonon temperature of the test film and therefore

\[ \frac{\Delta T}{T_c} = B\left(\nu / T_c\right)(h - 0.68). \]

Since the measurements are made just nominally above \( T_c \), all the temperature dependences (which are normalized to unity at \( T_c \)) may be neglected. The second conclusion is that the phonon power is proportional to the input microwave power, high power roll off and uncertainties in attenuator calibration notwithstanding. Finally, it is worth pointing out that the virtue in doing this measurement and analysis above \( T_c \) is that it is then possible to identify the change signal \( \Delta V \) with the Kaptiza heating term, which is not possible below \( T_c \) because then \( \Delta V \) is the sum of \[ \left[ \frac{\delta T}{T_c} \right]_h \] and the enhancement term \( BG(\nu) \).

1.2. Large Area Junctions

From the night of July 17, 1979, when I first saw gap enhancement change signals on the oscilloscope, to March 1983, when I completed the experiment, the magnitude of the cooling I observed on all large area junctions was \( 5-20 \times 10^{-3} \), measured close to \( T_c \) and at full TWT power. During that time I looked at dozens of junctions made on at least 8 different quartz crystals. The junctions were distributed at random near the center of each crystal. The great level of consistency I found indicates that this effect is largely independent of crystal-to-crystal and sample-to-sample variations, and that the phonon beam is essentially a plane wave perpendicular to the crystal axis; otherwise different results would obtain when a junction was dead center on the crystal face and when it was a millimeter off center.
Another universal feature was the disappearance of a change signal as the
temperature was lowered. Generally, $\Delta V$ was negligible, or below the noise at
$T/T_c \approx 0.90$. The theory predicts that the cooling falls as the temperature is
lowered, but the change signal falls off even faster. We have

$$
\frac{T}{T_c} - 1 + \beta \frac{\Delta^2}{T_c^2} + \left[ \frac{\delta T}{T_c} \right] = 0,
$$

$$
\Delta^* = \Delta^0 + \delta \Delta, \text{ where } \delta \Delta = \Delta V \text{ when the junction is biased in the step. By expand-
ing } \Delta^*, \text{ we can write}
$$

$$
\frac{T}{T_c} - 1 + \beta \frac{\Delta^0^2}{T_c^2} + \frac{2 \beta \Delta^0 \delta \Delta}{T_c^2} + \frac{\beta (\delta \Delta)^2}{T_c^2} + \left[ \frac{\delta T}{T_c} \right] = 0.
$$

The first three terms are the equilibrium GL equation and their sum is identi-
cally zero. Ignoring $(\delta \Delta)^2$, we now have

$$
-2 \beta \frac{\Delta^0 \delta \Delta}{T_c^2} = \left[ \frac{\delta T}{T_c} \right]_T, \text{ or}
$$

$$
-\delta \Delta = \frac{\left[ \frac{\delta T}{T_c} \right]_T T_c^2}{2 \beta \Delta^0}.
$$

This implies that as the change in effective temperature $\left[ \frac{\delta T}{T_c} \right]_T$ falls off at
lower temperatures, the change signal $\Delta V = \delta \Delta/e$ gets even smaller by the fac-
tor $\Delta^0$. This is a major limitation in measuring the cooling at lower tempera-
tures. Since the theory is valid only at higher temperatures, this limitation
does not affect the measurements that may be compared with the theory.

Figure 29 displays enhancement data of a typical large area sample (sam-
pie W6), presented as $\left[ \frac{\delta T}{T_c} \right]_T$ vs. $T/T_c$. Before extracting the parameters $B$
and $h$ from the data one has to choose a theory, and the choices are between
one with the conjectured temperature dependences of the parameters and one
without. Fitting both versions to the data at two points, $T/T_c \approx 0.95$ and 0.39, i
Figure 29. Data from a large area junction (sample W6) at large phonon power ($\nu = 8.91$ GHz). The solid line is a fit to the data with the parameters $B=0.0174$ and $h=3.15$ in the temperature dependent theory or $B=0.0136$ and $h=2.23$ in the temperature independent theory. The dashed line is meant to guide the eye to the highest temperature point, which is not spuriously low, and which is not accounted for by either version.
obtained the values

\[ B = 0.0174, h = 3.15 \text{ (with temperature dependences), and} \]
\[ B = 0.0136, h = 2.23 \text{ (without temperature dependences).} \]

Recalling the results of our earlier a priori estimate of \( B \) (Chapter 3, Section 2.1), at full microwave power (about 600 Watts)\(^7\) \( B \approx 0.069 \), my best attempts to fit the data yield values of \( B \) that are not far off, especially considering the ad hoc nature of that estimate. As a general rule, the effect of removing the temperature dependences was to reduce \( h \) by \( \approx 1 \) and \( B \) by 25-50\%. To within instrumental resolution, both theories yield curves which are overlapping over the intermediate temperatures. The solid line in the figure is the numerical prediction of either theory with the appropriate parameters. Examining Appendix C, (the column labeled \( B(0 \text{ dB)} \)), we see that this sample is in rather poorer agreement than most of the other samples tested. It is plain that there is some scatter, but this is due presumably to crystal variations, subtleties in cavity performance, sample-to-substrate adhesion problems, different mean free paths in different films, and possibly other mechanisms. I can also conclude at this point that \( B \) is small enough that we can believe that the theory (which is essentially a perturbation theory in \( B \)) has some validity.

The first issue to be dealt with is the introduction of the temperature dependence of the parameters. The ILE theory employs a relaxation time approximation (RTA) to the Boltzmann equation governing the distribution of excitations (quasiparticles and phonons) in the irradiated superconductor. It is strictly correct only at \( T = T_c \) and for no irradiation. As the temperature is lowered two problems arise. First, the parameters in the ILE equation change with temperature. Specifically, the inelastic relaxation time, \( \tau_E \), the Kapitza resistance and the absorbed phonon power have known temperature dependences which may be included properly in ILE. The second and more significant
problem is that as $T$ is lowered and $\Delta T$ grows, the RTA itself breaks down (see Chapter 2, Section 1.). There is no obvious way to incorporate the energy dependence of $\tau_E$ into the embellished Ginzburg-Landau equation given by ILE. Since we can only make an incomplete correction to the theory, it is still faulty. For this reason I choose to emphasize the results obtained by using the barest form of ILE, one without any temperature dependence of the parameters included. Occasionally I will make reference to the temperature dependent theory, but unless I do so, it is to be understood that the temperature independent version is being used. Finally, I reiterate the results of Chang and Scalapino, who point out the failure of ILE's assumption that the phonons are in equilibrium.

We may try to calculate $h$ in terms of known quantities,

$$h = 0.68 + \frac{8 (R_K k_B) l (N(0) k_B T_c)}{\tau_E} = 0.68 + 0.23 R_K \frac{l}{100 \text{ nm}} \frac{T_c}{1.25 \text{ K}}$$

where $l$ is the film thickness and $R_K$ is in cm$^2$ K W$^{-1}$. The Kapitza resistance is the result of the heat exchange through both of the sample's surfaces, one in direct contact with the helium bath and the other in contact with the single crystal quartz substrate. From the work of Follinsbee and Anderson (see Wyatt, 1981), we know that the Kapitza resistance between copper and liquid helium at $T=1.25$ K is $R_K = 10$ cm$^2$ K W$^{-1}$. This quantity has not been measured for aluminium, but according to Lounasmaa (1974), $R_K$ at a liquid helium/metal interface is remarkably insensitive to the choice of metal. It is also true that the acoustic mismatch model of $R_K$ works very well for solid-solid interfaces, but when applied to LiHe/solid interfaces near $T=1$ K it predicts that $R_K$ will be two orders of magnitude larger than it is found to be. In addition to the magnitude being wrong, the dependence of the resistance on the acoustic impedance, $\rho v_s$, is incorrect, there being less observed dependence on $\rho v_s$ than theory predicts.
It is also interesting to note that the temperature dependence of $R_K$ is not the traditional $T^{-3}$, but something more like $T^{-5.1}$ for $0.65 \, K < T < 1.25 \, K$. For lack of a better number, I will use the Folinsbee and Anderson value of $R_K(T=1.25 \, K)=10 \, \text{cm}^2 \, \text{K W}^{-1}$. Heat conduction through the other surface is probably better understood, since it involves two solids, one of which is an insulator. From the work of Cheeke et al. (1976), we can find the Kapitza resistance between virtually any pair of solids as long as we know their densities and various speeds of sound. If we follow their instructions we get $R_K(T^3)=7.13 \, \text{cm}^2 \, \text{K W}^{-1}$, and at $T=1.25 \, K$ this amounts to 3.65. This is considerably less than the resistance of the LHe channel, so the quartz provides the more important heat loss channel, although without both channels there would probably be no enhancement. The combined resistance is

$$R_K = \left( 10^{-1} + 3.65^{-1} \right)^{-1} = 2.67 .$$

It is also true that $R_K$ is sensitive to contaminating layers between the sample and reservoir, to surface preparation, and also to vagaries of the contact between sample and substrate (Lounasmaa, 1974, and Holt, 1966). It has been my experience that unless sufficient preventive care is taken, a layer of ice (frozen air or nitrogen or water) will form over the sample, insulating it to some degree from the cooling of the helium bath. In that extreme case, the balance of enhancement and heating shifts, and instead of enhancement, all that was observed was an effect of the opposite sign, heating. In all of the data to be discussed, the samples were free of this 'ice,' and consequently, it is appropriate to use the value $R_K(1.25 \, K)=2.67 \, \text{cm}^2 \, \text{K W}^{-1}$. Then, with $l = 100 \, \text{nm}$ and $T_c = 1.25 \, K$.

$$h \approx 1.29 .$$

In the extreme ice case, $R_K=3.65, h \approx 1.52$. 
The heating parameter, \( h \), thus extracted is equivalent to a determination of the Kapitza resistance, \( R_K = 6.7 \text{ cm}^2 \text{ K W}^{-1} \). This value is intermediate between the single channel resistance for the quartz/Al and Al/He interfaces. Given the uncertainties in the analysis (both theoretical and experimental), I accept this value as being in satisfactory agreement with theoretical predictions. Appendix C is a compilation of useful data for several junctions. From the column labelled \( h_{fu} \) one can see that the heating term for wide samples is typically 1.7, with some examples near 1.2 and others >2.0. This corresponds to a mean Kapitza resistance of \( \approx 3.6 \pm 2 \text{ cm}^2 \text{ K W}^{-1} \), as compared with the theoretical value of 2.7 cm\(^2\) K W\(^{-1}\).

A problem arises when we ask questions about the power dependence of the enhancement. For instance, having found an \( h \) and a \( B \) at the highest power (0 dB) how do the data at low power agree with the theory given \( h \) and a suitably reduced \( B \)? They don't agree well, and the next question is why? The route to a solution is to look at the power dependence of the enhancement at some high temperature where one expects the theory to be valid and where the signals are sufficiently large that so the error bars are relatively small, as in Fig. 30 (sample W4). We see that at low relative power the enhancement is linear (as it should be), but at higher power the enhancement rolls off. The theory is intrinsically nonlinear, but at what level do the nonlinearities dominate? If we just tried to fit the data at constant \( T/T_c \) and demanded a good fit at the high power points, we’d get values like \( B(0 \text{ dB})=0.042 \) and \( h=4.05 \), as compared with \( B(0 \text{ dB})=0.005 \) and \( h=1.97 \) obtained from a temperature independent fit at \( B =0 \text{ dB} \). Naturally, if we use the power-fit parameters we get terrible agreement at all other temperatures, and if we use the temperature-fit parameters we get terrible agreement at all other powers. The linearity of the heating measurements eliminates the possibility that the transducer is
Figure 30. Power dependence of the gap enhancement for sample W4. $T/T_c = 0.99$, $v/T_c = 0.33$. The horizontal scale is the power referred to the maximum available (1.0 = 0 dB). The curve, A, is the prediction of the temperature independent ILE theory for the parameters $B(0 \text{ dB}) = 1.37 \times 10^{-3}$ and $\delta = 1.60$, which are obtained by fitting the data at all temperatures for $B \leq -10 \text{ dB}$. The effect of introducing the 2nd order Kapitza resistance $R^{(2)}_K$ (Chapter 4, Section 1.1) is negligible. This may be understood from the magnitude of the maximum correction introduced by that analysis, $1.5Bh_{\text{c}}(v/T_c) = 0.011$. Curve B, $(B(0 \text{ dB}) = 4.2 \times 10^{-2}$, $\delta = 3.81$, $1.5Bh_{\text{c}}(v/T_c) = 8.0 \times 10^{-2}$), is the best fit, with $R^{(2)}_K$ included, of the power dependent data at $T/T_c = 0.99$. Curve C, $(B(0 \text{ dB}) = 3.70 \times 10^{-2}$, $\delta = 4.05$, $1.5Bh_{\text{c}}(v/T_c) = 7.5 \times 10^{-2}$), is the best fit without $R^{(2)}_K$ included. For both B and C the correction is significant and affects the shape as well as the magnitude of the theoretical prediction.
somehow nonlinear. For some narrow samples, the parameters were $B(0 \, dB) \approx 0.45$ and $h = 2.5$. Such very high values of $B$ are absurd. As mentioned in Chapter 2, $B$ is essentially the deviation of the distribution function from its equilibrium value at the gap edge. If $B=0.5$, then those states near the gap edge would be completely empty and surely the theory is not valid so far from equilibrium. Furthermore such a high $h$ and $B$ would predict that the measured beating above $T_c$ is

$$\Delta T = \nu B (h - 0.68) \approx 340 \, \text{mK},$$

which is totally out of line with what’s observed. As a last comment on the inappropriateness of $B(0 \, dB)=0.45$, our earlier a priori calculation predicts $B \approx 0.069$. The uncertainties in that calculation are of such a sign and magnitude that $B = 0.5$ is untenable.$^8$

Finally it occurred to us to try to fit just the low power data. For the same sample (N4) whose complete data set is shown in Fig. 31, I used the enhancements at -10 dB down and below. If we try to do a power fit, we find that within reasonable limits, for any $B$ we can find an $h$ that fits the data. This is a consequence of both the data and the theory being linear for low power. Thus power-fitting doesn’t give us a unique set of parameters. If on the other hand we try a temperature-fit -10 dB down, we can get a good and sensible fit,

$$B(-10 \, dB) = 1.37 \times 10^{-3}, \ h = 1.60,$$

and by virtue of our experience with power-fits, we know it will fit all the low power data. In fact that happens, and Fig. 32 displays that prettily. If we use those parameters and compare the theory with -7 dB down data we find strong deviations, which become even more pronounced at higher power. Thus $B(0 \, dB)=1.37 \times 10^{-2}$; and while our intuition tells us that the theory should still be valid for such a small value of the expansion parameter, it doesn’t seem to be
Figure 31. Data from a large area junction (sample W4) for a range of input power (measured relative to full power) and over a substantial temperature range. ($\nu = 9.26$ GHz, $T_c = 1.33$ K)
Figure 32. The data of sample W4 and curves generated from a fit at -10 dB down ($B(-10\,dB)=1.37\times10^{-3}, a=1.60$). A: -10 dB, B: -12 dB, C: -15 dB, D: -17 dB, E: -20 dB, F: -7 dB. The full data for this sample is shown in Fig.31.
valid. The Kapitza resistance corresponding to $h = 1.6$ is $R_K = 3.9$ cm$^2$ K W$^{-1}$.

The conclusion to be drawn must be the one motivated by Chang and Scalapino. The effect of the nonequilibrium phonons and the energy dependence of $\tau_F$ must become important at a fairly low value of $B$, at the order of $5 \times 10^{-3}$. From Fig. 33, which shows the power dependence of the enhancement for several values of $h$, one would not expect marked deviations from linearity for $B < 5 \times 10^{-2}$. The calculations of Chang and Scalapino (which were performed for the microwave case at $T/T_c = 0.90$ and $\nu/T_c = 0.98$, or $\nu \approx \Delta$) indicate that the enhancement rolls off noticeably for $B > 5 \times 10^{-3}$ (see Fig. 6). Thus our data seems to confirm their prediction qualitatively, although a more detailed comparison awaits a repeat of their numerical calculation with the coherence factors appropriate to the phonons and with suitable temperature and frequency parameters.

There is another problem that is obvious from Figs. 29, 31, and 32. At the highest temperatures, the measured enhancement drops off while the theory predicts it will continue to increase right up to pair-breaking. This is an effect I have observed many times at $T/T_c \approx 0.99$ but less than pair-breaking ($T/T_c \approx 0.997$). At temperatures just greater than the highest at which I have presented measurements, the change signal in the step changes sign, indicating a change from gap enhancement to gap reduction. I've never quantified that reduction. This transition occurs at temperatures a few millikelvins below the pair-breaking temperature. A possible mechanism within the confines of the ILE model presents itself if at higher temperatures the relative weight of the enhancement, $BG(\Delta/\nu)$, and heating terms, $B h \nu/T_c$, is slightly different than in the ILE theory. Consider the $\Delta$ dependence of the heating term. In ILE there is none, but in reality we know that we must at least include a prefactor.
Figure 33. The gap enhancement \( \frac{\delta T}{T_c} \) vs. power \( B \) as predicted by the temperature independent ILE theory, for several values of the heating parameter, \( h \) (\( T'/T_c=0.98, \nu/T_c=0.33 \)). The 2\textsuperscript{nd} order Kapitza resistance \( R_{K}^{(2)} \) is included, but its effect is negligible at power levels, \( B \leq 5 \times 10^{-3} \) where we find the ILE theory to be applicable.
\( \frac{2}{1 + e^{\Delta/T}} = 2n(\Delta) \) to account for the \( \Delta \)-dependence of the ultrasonic absorption. It is assumed in this that \( \nu \ll \Delta \), and near pair-breaking (which is the temperature region we're concerned with at this juncture) this assumption breaks down. Near pair-breaking the relative absorption is greater than \( 2n(\Delta) \). Thus the heating term will be larger, and its importance relative to the enhancement term will grow. It is possible that proper inclusion of this effect may cause the measured enhancement to go through a maximum below pair-breaking. I have studied the ILE equation with the temperature dependent coefficients, in which I included \( 2n(\Delta) \) in the heating term (see Theory, Section 2.4), and no maximum is predicted, so at the very least some deviation from the usual \( 2n(\Delta) \) behavior is required.

In my interpretation of the general picture everything agrees, but one should not take any of the results too literally. ILE is only good for \( \Delta/T \ll 1 \). At \( T/T_c = 0.95 \) and \( \Delta/T = 0.72 \), so we are at the limits of validity of the theory while trying to fit down there. The ILE theory assumes explicitly that the phonons are in some kind of thermal equilibrium, which Chang and Scalapino have proven explicitly is not the case. For reasons not obvious to me, CS performed their calculation at \( T/T_c \approx 0.90 \), which is outside the range of ILE and therefore does not invite comparison between the two. Similarly, recent CS-like calculations have been made for the phonon case (Cirillo et al., 1982), and they too are concerned primarily with that temperature. They do seem to indicate that at higher temperatures the deviations from ILE are not very large. It is reassuring to find that the fitting parameters \( B \) and \( h \) are close to what one might expect.

The sample (W6) of Fig. 29 was also examined at two other frequencies. The complete data at full microwave power is shown in Fig. 34. I would have thought that \( h \) would remain relatively constant[9] at each of the three frequencies
Figure 34. The same junction as in Fig. 29 (sample W6), but at three different frequencies. The lines are meant to guide the eye only.
(ν = 9.63, 8.91 and 8.35 GHz), and that B would vary depending on the frequency dependence of the overall transduction process. The magnitudes of the enhancement might vary, but the temperature dependences should be approximately the same except for the fact that \[ \delta T / T_c \] is proportional to \( \nu / T_c \). The observed temperature dependences are much more \( \nu \)-dependent. The maximum enhancement varies by almost 2, but it is easy to accept that fact, since the klystron, TWT, cavity, crystal, and film all contribute in some complicated and immeasurable way to B. The heating parameter \( h \) in the temperature independent theory is found to vary from 1.6 to 2.5. These numbers are not outside the plausible range of \( h \), but since \( h \) is essentially a property of the Al/quartz and Al/LHe interface, it is peculiar that it is not more constant. If \( h \) is truly constant, this observed variation in it implies that the errors in the enhancement measurement are much greater than we believe them to be.

Something is missing from the model. The following does not constitute an explanation, but it is an idea. Recall that \( R_K \) is the coefficient of the thermal boundary resistance, which itself behaves like \( 1 / T^3 \), according to the acoustic mismatch theory (Little, 1975; Anderson, 1981). The \( 1 / T^3 \) comes from an integral for the heat flux that extends over all frequencies, but which is actually cut off by a Bose factor \( (e^{\nu / T} - 1)^{-1} \). The meaning of the \( 1 / T^3 \) is that the principal mode of heat removal from the sample is by phonons of frequency \( \nu = T \), and that they are present in numbers proportional to \( \nu^2 \) if the density of phonon states is Debye. If in the process of injecting phonons of frequency \( \nu \) the principal heat escape mode is via those phonons, and if \( T^3 \rightarrow \nu^2 \), then at constant T, \( \nu^2 h \) should be constant. For convenience I normalize \( \nu \) to \( T_c \). The table below shows that \((\nu / T_c)^3 h\) is strikingly constant. A few other junctions were tested at different frequencies and they did not reveal this same apparent
sensitivity to frequency, so it is possible that the effect is spurious. The accord is probably coincidental, but may be noteworthy.

<table>
<thead>
<tr>
<th>TABLE II</th>
</tr>
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<tbody>
<tr>
<td>$T_c/\nu$</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>3.10</td>
</tr>
<tr>
<td>2.91</td>
</tr>
<tr>
<td>2.69</td>
</tr>
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</table>

1.3. Small Area Junctions

Categorically speaking, enhancements of the small area or narrow tunnel junctions were larger than those of the large area ones. Of equal importance, the temperature dependence they displayed was much flatter, as in Fig. 35 (sample N1). Interpreted in the context of $B$'s and $h$'s, the described behavior means $B$ is larger than before, or $h$ is smaller than before, or both. The fitting parameters for all the narrow junctions I tested are listed in Appendix C. The parameters were chosen to give a good fit above $T/T_c \approx 0.92$. At lower temperatures the measurements consistently lay above the theoretical curves with the chosen parameters. Another way of saying this is that the enhancement for narrow junctions had a flatter temperature dependence than may be accounted for by this type of theory. If we choose to fit the low $T/T_c$ data (as well as one point at $T/T_c \approx 0.98$), then $h$ increases by $\approx 1.0$ and $B$ by $\approx 50\%$. As in the last section I am only quoting results of the temperature independent analysis. At present we have no explanation of why either one of those two possibilities should obtain. The acoustic power $B$ varies from sample to sample, but these variations are not adequate to explain its being systematically larger in the
Figure 35. Gap enhancement in a narrow junction (sample N1).
narrow samples. A similar statement may be made for $h$. The ESSS heating term $[\delta T/T_c]_0 = 0.68 B \nu/T_c$ makes no distinction about sample geometry. The phenomenological heating contains a term $\text{Volume} / \text{Area}$, which is essentially the sample thickness, but which also contains a correction from the sample sides. This serves to reduce the term by $\text{thickness} / \text{width} \approx 1\%$ in the narrow case and $0.3\%$ in the wide case, and is therefore negligible.

As in the case of the wide junctions, it was necessary to fit the data at low power, when that data was available. The $h$'s so extracted are in quite good agreement with the predictions of the theory. We find $h_{fu} = 0.8-2.0$ ($R_K = 0.52-5.7 \, \text{cm}^2 \, \text{K} \, \text{W}^{-1}$) with a mean of $h_{fu} = 1.3$ ($R_K = 2.7 \, \text{cm}^2 \, \text{K} \, \text{W}^{-1}$), and $h_{\text{theory}} \approx 1.3$ ($R_K = 2.7 \, \text{cm}^2 \, \text{K} \, \text{W}^{-1}$). For comparison we remind the reader that the wide sample of Fig. 31 and 32 (sample W4) gave $R_K \approx 3.9 \, \text{cm}^2 \, \text{K} \, \text{W}^{-1}$, and $h_{fu} \approx 1.7$, $h_{\text{theory}} \approx 1.3$.

The narrow samples also differed from the wide ones in another marked way. Figure 36 (sample N2) is typical of reconstructed I-V characteristics of the narrow tunnel junctions. The stair-like structure reminds us of a well-known phenomenon, photon-assisted tunneling (Tien and Gordon, 1963; Solymar, 1972), but in this case it is phonon-assisted tunneling (abbreviated PAT, see Solymar, 1972 and references therein). In both cases the underlying mechanism is movement of a quasiparticle across the tunneling barrier by absorption or emission of a quantum $\hbar \nu$. To see PAT, the RF must couple both films together. The structures in the I-V are images of the structures in the I-V without radiation, displaced by $\pm n\hbar \nu$. PAT is the most serious obstacle for all microwave gap enhancement experiments, because while irradiating the junction with RF it is almost impossible not to have that RF coupling the films together. With sufficient radiation to observe gap enhancement, there is usu-
Figure 36. The combined effects of phonon-assisted tunneling and phonon-induced gap enhancement on a narrow tunnel junction (sample N2), superimposed on its equilibrium I-V.
ally enough RF coupling the films to wash out completely the nonlinearities whose positions locate the equilibrium and nonequilibrium gaps. One must be either clever or lucky to avoid this (Horstman and Wolter, 1981). For phonons, on the other hand, the effect is difficult to observe and has, before this experiment, been seen only in 2nd-derivative measurements of the I-V. In the large area junctions, for instance, we saw indications of PAT in the form of a current depression of width \( V = \hbar \nu / e \), at a voltage just above the step. In the narrow junctions the PAT was often strikingly noticeable, and was even more so at lower temperatures. The spacing of the steps can serve as a check of the calibration of the amplifiers and recording system, since the frequency may be measured to higher accuracy by conventional microwave methods. The problem is that the measured steps were neither equally spaced nor exactly \( \hbar \nu / e \), and yet the system is calibrated to within a few percent by alternate means. The steps are almost always within 15\% of the correct value, and usually within 5\%. It is difficult without derivative techniques and a special interest in it to locate the step positions to high accuracy, but the systematic behavior is quite clear: features at \( \Delta_\nu + \Delta + n\hbar \nu \) appear at a greater apparent \( \nu \) than features at \( \Delta_\nu + \Delta - n\hbar \nu \) and the spacing increases for larger \( -n \).

We might ask why the narrow samples are more susceptible to PAT than wide ones. The answer may be that the phase of the phonon field across the junction is more constant in the narrow than in the wide samples. This is purely a geometrical effect. The effect of a wedge in the crystal (non-parallelism of its endfaces) is to cause the phonons to arrive with a different phase at different points along the sample. The manufacturers claim \( \alpha_{\text{wedge}} <4^\circ \), which is astonishingly small. Such a wedge would introduce a phase difference of
for a 300 μm sample and Δθ = 0.002 for a 10 μm sample. It’s possible that the wedge is actually quite a bit larger, although still small by optical standards, in which case it might be large enough to effect the presence or absence of PAT steps. In any case a wedge would not effect enhancement.

When measuring gap enhancement in the presence of PAT one measures the displacement of the zeroth order step from the equilibrium value of the gap; so it may not matter that the π≠0 steps appear a little out of place, but one likes to understand the observations. The PAT does complicate matters because now instead of there being a single current step at the sum of the gaps there are several, and we have to pick the correct one out of a multitude. This is easy, but the results I’ve described for narrow junction gap enhancement are strange, particularly at low temperatures, so the question arises: is the zeroth order step displaced from its equilibrium value because of gap enhancement?

At the lowest temperatures (T=1.04 K, T/Tc ≈ 0.82), large area junctions were inert from the point of view of enhancement. This is without exception. At these temperatures, most of the narrow junctions showed a very distinct change signal in the step, even those narrow junctions that did not show a lot of PAT, like the one in Fig. 37 (sample N4).

1.4. Heating Measurements Above the Critical Temperature

As mentioned earlier, I made some heating measurements above Tc. These measurements determine \[ \frac{\delta T}{T_c} \mid_{T>T_c} = B(h - 0.68)(\nu / T_c). \] We have therefore, in principle reduced the problem from finding the two parameters B and h to finding a single parameter, since the heating measurements provide another
Figure 37. Reconstructed I-V of a narrow tunnel junction (sample N3), showing gap enhancement and relatively little PAT. ($T/T_c=0.908, \nu=8.81$ GHz)
constraint. Instead of performing the analysis that way we choose to compare side by side the measured values of this heating, \( \delta T/ T_c \) \text{meas}, and the value of the same quantity calculated with the \( B \)'s and \( h \)'s obtained by the fitting procedure, \( \delta T/ T_c \) \text{fit}. This comparison may be found in Appendix C, but I repeat the essential elements of it below for clarity.

Clearly, some of the samples are in excellent agreement and others are not. Most of the samples on which I obtained heating measurements were also those samples on which I did not take any low power enhancement data. We know from our discussion of the power dependence of the enhancement that \( B \)'s obtained from fits at 0 dB are characteristically low. This is because the enhancement rolls off faster than predicted by the theory used in the fitting

<table>
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<th>Sample</th>
<th>Power of Fit</th>
<th>( \nu/ T_c )</th>
<th>( h_{\text{theory}} )</th>
<th>( h_{\text{fit}} )</th>
<th>Heating ( B(h-0.68)(\nu/ T_c) ) Measured ( (\times 10^{-3}) )</th>
<th>Heating ( (\times 10^{-3}) )</th>
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</tbody>
</table>
procedure, and so the $B$'s found by fitting at high power are lower than they are in reality. If such a $B$ is used to estimate the heating $B(h-0.68)(\nu/T_c)$, then the estimate will be low by some amount, depending on the saturation. We can anticipate therefore that those samples on which I had to fit at 0 dB will show poor agreement between measured and fit values of the heating, and the fit values will be low. This is born out in the table above.

Another point to understand is that since the heating is proportional to $(h-0.68)$, if $h \approx 0.68$, then the fit heating will be very sensitive to the exact value chosen for $h$. In the case of sample N4, the observed temperature dependence was quite flat (like sample N2, see Appendix C) and the $h$ obtained was quite small. This is more probably a manifestation of some other physics at play than it is of some unusually low Kapitza resistance. With such a low $h$, we anticipate less than perfect agreement between measurement and fitting results. Indeed the fit heating is 3 or 4 times less than the measured heating for this sample. If the fitting procedure had been optimized to emphasize those data points nearest to $T_c$ instead of looking at a broader temperature range ($T/T_c = 0.95 - 0.99$), then both $B$ and $h$ would have been found to be larger, and the fit heating would have been in a finer state of accord with the measured heating. To present those results would have been a bit of a misrepresentation, because they would not have indicated the observed very flat lower temperature behavior.

Our criteria therefore eliminate all but two samples: W7 at 8.43 GHz and N1 at 8.6 GHz. The measurements of $[\delta T/T_c]_{T>T_c} = B(h-0.68)(\nu/T_c)$ agree very well with the fit value and lend considerable credibility to our analysis. We should point out here that the agreement relies on an analysis that is not above suspicion. The key to this analysis is the assumption that the quasiparticles in
the counterelectrode (the film that remains superconducting in the heating measurement) are in a quasithermal distribution in equilibrium with the phonons in that film. One can argue that because the quasiparticles cannot escape the film and because there is electron-phonon coupling, the quasiparticles must have the same temperature as the phonons. This is similar to the result obtained in thermodynamics where two closed couple systems in equilibrium are found to have the same temperature. Nevertheless some detailed, kinetic equation analysis would be of interest here.

2. CRITICAL CURRENT ENHANCEMENTS

2.1. Generalia

The labels given to the two types of samples, narrow and wide, make reference to the SIS junctions on each type. The other component of each sample, the strip, was 3–10 µm wide, 100-200 nm thick, and 100–300 µm long. The major distinction between narrow and wide was whether the strip lay in intimate contact with the crystal or whether there was a counterelectrode of dirty Al between it and the crystal. I observed no systematic differences between enhancements in strips of either type. Also, as one might expect, measurements made on samples consisting of only a single strip were consistent with those made on composite samples. How, for instance, could a strip be affected by the junction that is lying next to it? As in the gap case, the strips were placed largely at random near the centers of several different crystals, with no striking difference coming from placement or preparation variations. The picture which emerges is one intrinsic to these narrow strips and is not the result of less interesting complications like variations in Kapitza resistance, ground planing, contact with the crystal, proximity to nearby structures, LHe level, or others. Except for explicit comparisons to gap data taken for the same sample,
we will make no distinctions between strips from wide and narrow samples.

The $i_c$ data was not as consistent as the gap data. The strip parameters and relevant enhancement data are tabulated in Appendix D. By comparing the columns labelled Maximum $i_c$ Enhancement and Maximum Gap Enhancement, it is clear that there is a considerable discrepancy of magnitudes. The critical current enhancement is consistently larger; and we can say with certainty, for temperatures above 0.95, that the $i_c$ enhancement was larger (typically by 2 to 5) than the gap enhancement under the same conditions, and that the $i_c$ enhancement fell off much more slowly with decreasing temperature than the gap enhancement. This latter effect made the ratio of enhancements greater at low temperature. Furthermore, the flat temperature dependence of the critical current enhancement made it impossible to fit the data to the ILE model.

Enhancements of both $i_c$ and $\Delta$, always expressed as $[\delta T/ T_c]$, are predicted by the so called theory to be close to one another, under the same operating conditions. To digress momentarily, if the mechanism for gap enhancement was exactly the same as for $i_c$, and $i_c \propto \Delta^3$ as in equilibrium, then changes in one or the other expressed as $\delta \Delta/ \Delta$, or $\delta i_c/ i_c$ would yield the relationship

$$\frac{\delta i_c}{i_c} = 3 \frac{\delta \Delta}{\Delta}$$

If, on the other hand, the changes are expressed as effective temperatures, $\Delta^2 \propto (1 - T/ T_c)$ and $i_c^{2/3} \propto (1 - T/ T_c)$, then

$$\frac{\delta \Delta^2}{\Delta^2} = \frac{\delta i_c^{2/3}}{i_c^{2/3}}$$

and the enhancements are the same. In Chapter 2 I discussed how this description is incomplete, but I include these words here to remind ourselves that up to a correction of perhaps 20%, $i_c$ and $\Delta$ enhancements are expected to be
equal. I would like to point out here that one similarity between strip and junction behavior was the saturation of the enhancement at high power, both showing marked roll-off.

Figure 38 shows data from the first sample on which I made simultaneous gap and $i_c$ measurements. It does not extend to low temperatures because the importance of probing that region was not understood at the time; but it is clear that the measured $[\delta T/T_c]$s are not the same, or even close. This is a wide junction and the strip lies off to one side. This junction is somewhat unusual in that it is fabricated with the test film under the dirty film, while all other samples were made in the reverse order, test film up. The ultrasound strain wave probably has a node at the interface of aluminum and liquid helium (because of their different acoustic impedances, $\rho c$ \[^{10}\]) and therefore should be of greater magnitude in the test film down variety. With more ultrasonic energy, there should be more absorption and more enhancement than in the strip, where the test film is right up against the LHe. Whether this understanding is correct or not, the $i_c$ enhancement is about 5 times the gap enhancement for this sample. Shown in Fig. 38b is the raw critical current data, $i_c \ast$ vs. $T/T_c$. From the equilibrium curve, it is plain that $i_c \ast$ does not follow the usual $(1-T/T_c)^{3/2}$ dependence very far below $T_c$. Some strips behaved better than others in this regard, obeying the GL theory for $T/T_c > 0.93^{[11]}$. In all cases $[\delta T/T_c]$ was determined by comparing $i_c \ast$ with $i_c \circ$ and by defining $[\delta T/T_c] = (T^* - T)/T_c$; where $T^*$ is defined implicitly, $i_c \ast(T/T_c) = i_c \circ(T^*/T_c)$.

Figure 39 shows some results of $\Delta$ and $i_c$ enhancements of a narrow sample at two different frequencies. There is no intended correspondence between relative power levels at the different frequencies, but at a given $\nu$, $N \mathrm{dB}$ is the same input power for both $i_c$ and $\Delta$. It is not possible to say whether more
Figure 3B. Gap and critical current enhancements in a wide sample (sample W2). Note the different scales for the cooling of $\Delta$ and $i_c$. a: 0 dB, b: -2 dB, c: -3 dB, d: -5 dB, e: -7 dB, f: -10 dB, g: -13 dB, h: -17 dB, i: -20 dB.
Figure 39. Gap and critical current enhancements in a narrow sample. The left hand figures are gap enhancements and the right hand figures are critical current enhancements. This is the same sample as in Fig. 35 (sample N1).
power was available at the lower frequency. The ratio of maximum enhancements is approximately 2 at \( v = 8.6 \) GHz and 1.3 at 9.7 GHz.

2.2. Unexpected Phenomena

In general, the critical current enhancement data raises more questions than it resolves. Curves of measured cooling as a function of \( T / T_c \) do not avail themselves of interpretation in terms of \( B \) and \( h \). In many cases, for instance, enhancement increased gradually below \( T / T_c = 1 \) before decreasing. In other cases, I found \( i_c' < i_c^0 \) within an isolated range of temperatures and input power levels. In the sample of Fig. 38, \( i_c' < i_c^0 \) for \( T / T_c \approx 0.86 \) and \( v=9.66 \) GHz; that is why the plotted data does not extend to lower temperatures. In fact at 8.6 GHz, \( i_c' < i_c^0 \) for \( T / T_c \approx 0.87 \), but only when the input was attenuated 10 dB or more. This kind of behavior was never observed in \( \Delta \).

Equally bizarre and unexplained is some data from a narrow strip whose enhancement I examined during irradiation by the first and second phonon echoes, as well as by the primary phonon pulse\(^{[12]}\), Fig. 40. These measurements are performed for each pulse by moving the current pulse and boxcar window to the time when that pulse is incident on the films. The phonons generated by the X-cut quartz are longitudinal, that is, they are compressional waves in the solid. We label the three pulses \( l_1, l_2, \) and \( l_3 \), and they arrive 4.6, 13.8, and 23.0 \( \mu \)sec respectively, after the microwaves turn on. In each successive pulse there is less ultrasonic energy, so we expect each successive pulse will produce less enhancement and less \( i_c' \). This is the case near \( T_c \), but as the temperature is lowered to \( T / T_c \approx 0.96 \), \( i_c' < i_c'_{l_1} \) and when \( T / T_c \approx 0.89 \), \( i_c'_{l_1} < i_c'_{l_2} \). The expected relationship between \( i_c'_{l_2} \) and \( i_c'_{l_3} \) is maintained at all temperatures. Another odd feature of these measurements is the enhancement by a given pulse at two different power levels, 0 dB and one-half of that.
Figure 40. Enhanced critical current versus temperature for each of three successive phonon pulses $t_1$, $t_2$, and $t_3$ at full power (A,B,C) and 3 dB down (a,b,c). The solid line is a smooth curve through the equilibrium data which agrees with GL theory close to $T_c$. 

$T_c = 1.220$ K
$
u = 8.57$ GHz
For the primary pulse \( l_1 \), \( i_c^{*0dB} > i_c^{*-3dB} \) for \( T/ T_c > 0.88 \), at which temperatures they either merge or cross. Also, \( [ \delta T/ T_c ] \) should be nearly proportional to power, and it most certainly is not. For \( l_2 \) the situation is somewhat better, but for \( T/ T_c = 0.87 \), \( i_c^{*0dB} = i_c^{*-3dB} \). The second echo \( l_3 \) behaves best. The curves of \( i_c^* \) vs. \( T/ T_c \) at both powers are similar or congruent. That fact does not prove anything, but the laws of corresponding states, that appear throughout the theory of superconductivity, are laws about congruences, so my intuition has been instructed to recognize further congruences.

I did not make measurements of this kind on enough samples to know if this behavior is spurious or generally valid, though the \( l_1 \) data itself is entirely consistent with the \( l_1 \) data from other samples. Since each successive \( l_n \) arrives 9.2 \( \mu \)sec after the previous one and \( 4.6 + (n-1)\times9.2 \mu \) sec after the microwave pulse, it is interesting to think of the measurements on successive \( l_n \) as a probe of the time dependence of the enhancement, where time refers to time following the microwave pulse. The inelastic relaxation time is only 12 nsec so it is likely that within a few hundred nanoseconds of the microwave turn-off the superconductor will have reached thermal equilibrium. Some of the evidence of this being the case are the square-topped gap change signals, which indicate a steady state has been reached within 100 nsec of the microwave turn-on, and also the 100 nsec trailing edge of the change signal pulse, which indicates the return to equilibrium is fast on the heels of the microwave turn-off.

In contrast to this comprehensible evidence from \( \Delta \)-measurements are some measurements of \( i_c^* \) vs. time which I made on one strip, shown in Fig. 41. These were motivated by the observation that following the initial pulse, \( i_c^* \) was depressed for a time before it returned to its equilibrium value, \( i_c^0 \). The exact
Figure 41. The time dependence of the critical current following a microwave pulse at time=0. Measurements were made only during the periods A,B,C,D.
duration of that depression varied from approximately 1 \( \mu \text{sec} \) to many 10's of microseconds. Any sample for which the suppression time was much greater than 4.6 \( \mu \text{sec} \) (the time for longitudinal phonons to traverse the crystal once) was fairly useless. The very long times were attributable to the presence of a thermally insulating ice-layer between sample and bath which drastically increased the thermal boundary resistance. By warming the sample and evaporating the ice-layer, the depression time was reduced to acceptable values. Only on one or two occasions was this a problem and no extensive study was made. The data shown are more or less typical. For a few microseconds following the microwave pulse the critical current is suppressed altogether, and then it begins to return towards \( i_c^0 \). At about the same moment when \( i_c^* = i_c^0 \) for the first time, the phonons arrive and produce a dramatic increase in \( i_c^* \), which returns to equilibrium on a much slower time scale than \( \tau_E = 12 \, \text{nsec} \). As \( i_c^* \) falls, another phonon pulse hits the film and \( i_c \) is enhanced again, after which it continues to fall slowly. When \( t=20 \, \mu \text{sec} \), we find \( i_c^* = i_c^0 \).

We might speculate that the energy deposited by the microwave radiation is not completely out of the film when the phonons arrive. By reasoning this way one could argue for a time dependent heating term, e.g. \( Be^{-t/\tau} \), to help explain Fig. 41. Phenomenologically, one might find a time scale \( \tau \) of the order of microseconds to fit the data, but the known mechanisms are two orders of magnitude less than this. We might also speculate that within the context of ILE there are non-linear, e.g. \( B^2 \), enhancement or heating terms which alter the expected dependence of enhancement on power or temperature. A third possibility is that the failure of the ILE theory to calculate correctly the nonequilibrium distribution function results in an underestimate of the importance of distribution function corrections to the critical current, as calculated by Entin-Wohlman (1981). The kinetic equation calculations of Chang and
Scalapino are needed to resolve this question. This assumes, though, that the operating principle of $i_c$ enhancement is the same one we've been discussing all along. A final possibility is that another mechanism is needed to explain $i_c$ enhancement. Earlier measurements of critical current enhancements by microwave irradiation were dc measurements (see Chapter 1 for references) and would not have unearthed the aforementioned time-dependences. Also, these earlier studies did not seek to fit ILE to their data at more than one point, and therefore they do not comment on the validity of the temperature dependent ILE. It is my feeling that these $i_c$ enhancement measurements, which are purported to be confirmation of the Eliashberg mechanism of gap enhancement, do not in fact confirm it.

3. MICROWAVE ENHANCEMENT

As described earlier, the phonon generation apparatus for this experiment required a pulsed microwave source and also a low power microwave sweep oscillator for diagnostic purposes. Microwaves injected into the waveguide were confined primarily to the cavity, but some leaked out and influenced the superconducting films. It was not the purpose of this thesis to study the microwave effects of gap or critical current enhancement, but I did look for them just to see what was happening.

The high power microwave pulses produced a large change signal that I could interpret only as heating. The enhancement signature is a positive change signal at bias currents such that $eV = \Delta_e + \Delta$. At normal operating power, the change signal was always negative, and the reconstructed I-V had no structure remaining in it (see Fig. 42a). For an input power 50 dB below the lowest power at which any phonon-induced gap enhancement effects were seen, the reconstructed I-V, shown in Fig. 42b, is strongly reminiscent of photon-
Figure 42. (a) The equilibrium and reconstructed I-V of a junction under saturating microwave power. (b) Reconstructed I-V of another junction that has been exposed to less than -50 dB of full microwave power.
Figure 42b.
assisted tunneling, and it shows no evidence of any gap enhancement.

I used the microwave sweep oscillator as a low power, continuous source and was thus able to make dc measurements of the microwave effects on these samples. I monitored the I-V continuously on an oscilloscope, and by switching the microwaves on and off at a low rate, I could view the equilibrium and nonequilibrium I-V's superimposed on one another. All I saw was PAT with the zeroth order step seemingly locked on the rise at the sum of the gaps, that is, no enhancement, (see Fig. 43). No difference was detected either over the 6-11 GHz range of the cavity or over the range of available input power. Sufficient power was available to wash out the PAT steps. It is possible that derivative techniques would detect some enhancement in these apparently flat I-V's. Generally speaking, microwave effects in this configuration are dominated by PAT. No difference was observed between the two types of samples.

This apparent lack of gap enhancement induced by microwaves is made more striking by the simultaneous critical current enhancement measurements. Near $T_c$, the output of the sweep oscillator was sufficient to produce large $i_c$ enhancements, and at certain frequencies was great enough to drive the strip normal. When an audio oscillator was used to sweep the current through $=i_c$ in the presence of some cw microwave source, increasing the RF power marginally caused the supercurrent to vanish. Pals and Dobben (1979) have shown that this sense of 'normal' is artificial,[14] and higher critical currents may be achieved if the experiment is more artful. At no time did this experimenter investigate $i_c$ as directed by Pals and Dobben. Near $T_c$, the maximum critical current enhancements so obtained were smaller, but of the same order of magnitude as in the phonon experiments. At lower temperatures the enhancement was less, but that may be an indication that I had insufficient microwave power
Figure 43. Microwave photon-assisted tunneling, no gap enhancement. a. Large microwave power. b. Less microwave power.
and not of some intrinsic differences between microwave and phonon experiments.

The most pronounced difference was the enhancement very close to $T_c$, above pair-breaking, and above $T_c$. At temperatures where the incident phonons destroyed the superconductivity, $T_{h\nu=2\Delta} < T < T_c$, or were unable to create superconductivity from an otherwise normal state, $T_c < T$, the microwaves produced enhancements. $[\delta T/\Delta] = \text{several } \times 10^{-3}$. For 4 mK above $T_c$, the microwaves yielded a supercurrent. In this case, $T_c$ means the extrapolation to zero current of the curve $i_c^{2/3}$ vs. $T$, and it involved an extrapolation of only about 1 mK. Above $T_c + 4 \text{ mK}$, there was no supercurrent within this experimental scheme. Critical temperature enhancement is expected (and documented) to be achievable with microwave irradiation than with the phonons. This is because of the coherence factor for absorption by pairs when $\nu$ exceeds $2\Delta$. The coherence factor

$$1 \pm \frac{\Delta^2}{E(E - \nu)}$$

is approximately zero for the microwaves (+) when $E \approx \Delta$ and $\nu = 2\Delta$, while it is approximately 2 in the phonon case (-). Once the pair-breaking threshold is reached, the pair absorption channel turns on strongly for phonons, destroying the enhancement. It turns on only weakly for the microwaves, allowing superconductivity to remain at slightly higher temperatures. No systematic study of these effects was pursued. Let me just mention that at the high RF powers used to make phonons, the leaking RF suppressed any propensity for superconductivity. It was possible to see some $i_c$ enhancement by RF only when the input power was attenuated severely. This method would have allowed the true maximum critical current to be measured. Again, no studies like this were made.
Given the null results of gap enhancement under irradiation with the low power source, it is surprising that the $i_c$ measurements were positive at the same power. If ILE is the mechanism for critical current enhancement, then phonons are being absorbed by the quasiparticles. Why no gap enhancement ensues is a mystery. One might argue from the other point of view that the null gap enhancement in the presence of PAT means that RF couples the films together but is not necessarily absorbed in the test film. Why there is substantial $i_c$ enhancement is a mystery. The absence of microwave gap enhancement in a narrow, Ginsburg-Landau superconductor should be sufficient evidence that inhomogeneous current flows are not responsible for the lack of gap enhancement in so many other experiments.
I have described in detail some recently completed experiments which measure the enhancing influence of microwave frequency ultrasound on superconducting aluminum. The measurements have been mostly of two types: critical current enhancements in long narrow microbridges and gap enhancement as detected by SIS quasiparticle tunneling. Near the transition temperature and for sufficiently narrow strips, the equilibrium relationship between $i_c$ and $\Delta$ is well known theoretically and is reconfirmed here. In this Ginsburg-Landau regime, $i_c$ and $\Delta$ are both parametrized by the order parameter, $\Psi$; and when we say that measurements of either $i_c$ vs. $T/T_c$ or $\Delta$ vs. $T/T_c$ agree with theory, what we really mean is that our understanding of the connection either between $i_c$ and $\Psi$ or $\Delta$ and $\Psi$ is confirmed. Thus the determination of $i_c$ and $\Delta$ represent two different views of the order parameter $\Psi$. In the work described, I have examined $i_c$ and $\Delta$ out of thermal equilibrium and tried to compare the results with predictions of a GL-type theory that has been extended to allow for the nonequilibrium distribution function induced by the perturbing phonons. As they do in equilibrium, these comparisons test our understanding of the connections between either $\Psi$ and $\Delta$, or $i_c$, two manifestations of $\Psi$.

I described a theory (and its limitations) of two parameters, a power parameter $B$ and a heating parameter $h$. Both parameters may be estimated from known material properties or auxiliary measurements. I plotted the enhancements $[\delta T/T_c]$ vs. $T/T_c$; and finding the shapes and magnitudes of those curves in accord with predictions of the theory, I tried to fit the data to
the theory, thereby extracting values of $B$ and $h$ that could be compared with the a priori estimates.

It was discovered that in order to get a satisfactory fit of the gap enhancement data over a wide range of temperatures and applied power, the high power data must be discarded. The high power dependence of the measured enhancement could not be fit with realistic values of $B$ and $h$. From our a priori estimates, $B(0 \text{ dB}) \approx 0.07$ and $h = 1.3$ for a 100 nm film. The high power data required values of $B$ as large as $B = 0.45$ and $h = 3.6$, whereas if only the low power data were considered, we found typical values $0.01 \leq B(0 \text{ dB}) \leq 0.15$ and $1.2 \leq h \leq 2.0$. Because of the uncertainties in the calculation of the ultrasonic transduction efficiency, this range of $B(0 \text{ dB})$ is entirely plausible. Similarly, the observed range of $h$ is comprehensible, given the model used to calculate it.

An equivalent way of expressing the failure of ILE to fit the high power data is to say that the measured gap enhancement rolled off, or saturated, at lower power than predicted by ILE. If one looks at the predictions of gap enhancement made by Chang and Scalapino, one finds that their predictions too roll off for $B \approx 0.005$, 10 times lower power than do ILE's for a similar choice of parameters. From my own measurements, supported by the results of Chang and Scalapino, I take the point of view that agreement with the ILE theory may only be obtained at low power. In the tables of Appendix C, the power (referred to maximum) at which the fit was made is listed for each sample. Results obtained at 0 dB are suspect therefore, particularly with regard to the value of $B$. The $h$'s found at 0 dB are roughly the same as those found at lower power, which is the same as saying that lines of $[\delta T/T_c]$ vs. $T/T_c$ at constant $B$ are roughly the same shape, independent of $B$. 
Gap enhancement measurements were made on two different types of samples, wide ones, \((200-350 \, \mu m)^2\), and narrow ones, \((10 \times 350 \, \mu m)^2\). The narrow ones revealed more gap enhancement, \(\left[ \delta T / T_c \right] = -(0.02-0.03)\) at 0 dB and \(T_c = 0.985-0.993\), than the wide ones, \(\left[ \delta T / T_c \right] = -(0.007-0.020)\), maximum. Also, narrow junctions seemed to have a consistently flatter temperature dependence than the wide ones. This latter effect may be attributed to a lower Kapitza resistance or \(h\) in the narrow samples. We have no understanding of how that might obtain, but I found an average Kapitza resistance of 2.7 cm\(^2\) K W\(^{-1}\) for narrow samples and 3.6 cm\(^2\) K W\(^{-1}\) for wide ones. It is also true that for constant input power, a lower \(R_K\) or \(h\) is consistent with larger enhancements.

In a few instances we made measurements of the heating of the junction at \(T > T_c\). We may extract from these measurements the quantity \(B(h-0.69)\nu / T_c\) and compare that with \(B_{fit}(h_{fit} -0.68)\nu / T_c\). In order that \(B_{fit}\) accurately represents the input power, \(B_{fit}(0 \, \text{dB})\) must be obtained by scaling upwards the \(B\) obtained by fitting at low power. If this is not done then for the reasons mentioned earlier \(B_{fit}(0 \, \text{dB})\) will be low. The results of this comparison are given in Table III and Appendix C. There were only two junctions, one wide and one narrow, whose fit and measured heating above \(T_c\) could be compared sensibly. Sample W7, I fit at -10 dB and found \(h=1.98\) and \(B(0 \, \text{dB})=0.164\). From the acoustic mismatch model for the Kapitza resistance and other measurements of the Kapitza resistance, we guess \(h_{theory}=1.78\), and our \(a \, priori\) estimate of the acoustic power is \(B(0 \, \text{dB})=0.07\). The measurement of heating gave \(\left[ \delta T / T_c \right]_{T>T_c}=0.070\) and our fit expression for the heating is \(B_{fit}(0 \, \text{dB})(h_{fit} -0.68)\nu / T_c = 0.070\). From just the theoretical guess we get \(B_{guess}(h_{theory} -0.68)\nu / T_c = 0.028\). The other sample N1 we fit at -10 dB and found \(h=1.66\) and \(B(0 \, \text{dB})=0.22\). Because this film is thinner than W7, we get \(h_{theory}=1.29\), but \(B_{guess}=0.070\) as before. The measured heating was
\[
\left[ \frac{\delta T}{T_c} \right]_{T>T_c} = 0.051 \pm 0.010 \text{ and the fit heating is } 0.069. \text{ Just from the guess values of } B \text{ and } h \text{ we get } 0.020. \text{ Given the uncertainties involved, the accord of theoretical estimates and two independent experimental measurements (heating and enhancement) is quite good.}
\]

Critical current measurements were made on narrow strips etched from the Al films and exposed to the same phonon flux and heating conditions. They showed a markedly different magnitude (several times larger, \[ \left[ \frac{\delta T}{T_c} \right] = -(0.04-0.08) \]) and temperature dependence (much flatter) of the gap enhancement or cooling. The theory does not predict that such differences should occur, although it does predict some differences. This points to the conclusion that we fail to grasp the connection between \( i_c \) and \( \Psi \) in this nonequilibrium situation, even though it seems we can make the connection between \( \Delta \) and \( \Psi \) in this situation and even though in equilibrium we understand all the connections between \( i_c, \Delta, \) and \( \Psi \). It is a rare circumstance that we can make nonequilibrium measurements of functions of the order parameter. Critical current measurements were essentially identical in wide and narrow samples (as they should have been). Since the gap cooling in the narrow case is larger and flatter than in the wide case, gap and critical current cooling are here in closer agreement than in the wide case. Nevertheless they are not in agreement. It is arguable whether the better agreement between theory and experiment is found for wide or narrow junctions.

Some brief measurements of the microwave induced enhancement of \( i_c \) and \( \Delta \) corroborate the results of the phonon experiments, experimental difficulties of photon-assisted tunneling and theroretical considerations (Mooij and Klapwijk, 1982) notwithstanding. I was always able to detect considerable \( i_c \) enhancement at microwave powers which produced essentially no gap enhance-
ment in the same sample. This result is not new, but one may always speculate that the absence of an effect (particularly one proved to be intermittently detectable) is the result of some limitation of the experimenter or the sample. The samples on which I made these microwave measurements were the same ones on the same cool-down in the same cavity etc. as the samples on which I made successful determinations of gap enhancement.

Gap enhancement and critical current enhancement by phonon irradiation have been seen for the first time\[18\]. Some agreement is found with the theory of Eliashberg (IIE). The discrepancies suggest that some physics is missing from our understanding of critical current enhancements, in both the microwave and the phonon case.
APPENDIX A

NOTATIONS COMPARED

There is no standard notation among the papers written in this field. The most important quantities are what I've called $B$, the power parameter, and $[\delta T/T_c]$, which I've called the cooling, the gap enhancement, or even the heating if it is positive. The cooling is the best quantity with which to identify the enhancement, as it falls naturally out of the theory and it is not divergent.

Eckern, Schmid, Schmutz, and Schon (1979) call the negative of the cooling "the gap control" and give it the symbol $\chi$. They call it the gap control because it amounts to an inhomogeneous term in the GL equation, and such a term controls the deviation of the gap from its equilibrium value. Mooij (1981) calls it the cooling and uses the symbol $\delta_{\text{exp}}$, which is a perfectly good symbol. For the purpose of suggesting the fact that the cooling represents a change in the reduced temperature, I chose $[\delta T/T_c]$, enclosing it in brackets to indicate that it is a single symbol. Positive $[\delta T/T_c]$ means heating, which is intuitive, and positive $\chi$ means enhancement.

The power parameter also is a problem when comparing results of different discussions (mostly theoretical). ESSS use $B$ for reasons I discussed in the text. Mooij (1981), following ILE, uses $\alpha/\gamma$ where $\alpha = A_w^2 2e^2 D/\hbar c^2$ is related to the rate of energy uptake from the microwave field $A_w$, $D = v_F l/3$, and $\gamma = \hbar/\tau_F$. It is awkward to relate $\alpha/\gamma$ to $B$ by comparing terms, but if one goes back to the derivations of the nonequilibrium distribution functions, it is trivial to show that
\[
B = \frac{1}{4} \left( \frac{\alpha}{\gamma} \right) \left( \frac{\nu}{T_c} \right).
\]

The same problem arises for the Chang and Scalapino (1977) power parameter \( A \), but by the same means one finds

\[
\frac{\alpha}{\gamma} = \frac{A}{2} \left( \frac{T_E}{\tau_o} \right) \left( \frac{\Delta_o}{k_B T_c} \right)^3 = 3.243 \times 10^{-1} A \text{ or}
\]

\[
B = 8.1 \times 10^{-2} A \left( \frac{\nu}{T_c} \right).
\]

Note that this is different from the result given by Mooij (1981), \( \alpha/\gamma = 1.09 \times 10^{-2} A \). Finally in a recent paper by Cirillo et al. (1982) they introduce the new quantities \( Q_o \) and \( Q_1 \) such that

\[
B = \frac{1}{4} Q_o \frac{\nu}{T_c} \frac{T_E}{\tau} = 2.98 \times 10^{-2} \frac{\nu}{T_c} Q_1.
\]

Without doubt \( B \) is the best choice, because it is not specific as to whether the drive is by phonons, microwaves, or injected quasiparticles, because it is essentially the deviation of the distribution function near the gap edge, and because it is easy to understand \( B \) in terms of \( N_R \), the normal state rate of uptake of radiation (see Chapter 2).
APPENDIX B

A TABLE of CONSTANTS

Useful things to know about aluminum

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bulk properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lattice spacing</td>
<td>4.05 Angstroms</td>
<td>Ashcroft and Mermin (1976)</td>
</tr>
<tr>
<td>Atomic weight (FCC)</td>
<td>26.982 m&lt;sub&gt;p&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Atomic number</td>
<td>13</td>
<td>&quot;</td>
</tr>
<tr>
<td>Fermi velocity &lt;em&gt;v&lt;/em&gt;&lt;sub&gt;F&lt;/sub&gt;</td>
<td>2.03×10&lt;sup&gt;6&lt;/sup&gt; cm sec&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Free electron density, &lt;em&gt;N&lt;/em&gt;</td>
<td>18.1×10&lt;sup&gt;22&lt;/sup&gt; cm&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Single spin density of states, &lt;em&gt;N&lt;/em&gt;(0)</td>
<td>2.32×10&lt;sup&gt;22&lt;/sup&gt; eV&lt;sup&gt;-1&lt;/sup&gt; cm&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Metallic density</td>
<td>2.66 gm cm&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>&quot;</td>
</tr>
<tr>
<td>Longitudinal speed of sound &lt;em&gt;c&lt;/em&gt;&lt;sub&gt;L&lt;/sub&gt;</td>
<td>6.65×10&lt;sup&gt;5&lt;/sup&gt; cm sec&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>Simmons and Wang (1971)</td>
</tr>
<tr>
<td>Coherence length &lt;em&gt;ξ&lt;/em&gt;&lt;sub&gt;0&lt;/sub&gt;</td>
<td>1.6×10&lt;sup&gt;-4&lt;/sup&gt; cm</td>
<td>Klapwijk et al. (1977)</td>
</tr>
<tr>
<td>Critical field &lt;em&gt;H&lt;/em&gt;&lt;sub&gt;c(0)&lt;/sub&gt;</td>
<td>8×10&lt;sup&gt;3&lt;/sup&gt; A m&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>&quot;</td>
</tr>
<tr>
<td>London penetration depth, &lt;em&gt;λ&lt;/em&gt;&lt;sub&gt;L(0)&lt;/sub&gt;</td>
<td>1.6×10&lt;sup&gt;-8&lt;/sup&gt; cm</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
Resistivity, $\rho$
(phonon limited, 300 K)
\[ \rho l = \frac{m_3}{N_{e^2}} \]
2.72 $\mu\Omega$ cm
Fickett (1971)
9 $\times 10^{-12}$ $\Omega$ cm$^2$

<table>
<thead>
<tr>
<th>Thin film properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$ (clean evaporation)</td>
</tr>
<tr>
<td>$T_c$ (dirty)</td>
</tr>
<tr>
<td>$\tau_F$ (clean)</td>
</tr>
<tr>
<td>$\tau_F$ (dirty)</td>
</tr>
<tr>
<td>mean free path, $l_*$ (clean)</td>
</tr>
<tr>
<td>mean free path, $l_*$ (dirty)</td>
</tr>
<tr>
<td>$R_{T_c}$</td>
</tr>
</tbody>
</table>

### APPENDIX C MATERIAL PARAMETERS AND ENHANCEMENT DATA FOR JUNCTIONS

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>AREA</th>
<th>THICKNESS</th>
<th>$R_J$</th>
<th>$T_{C&lt;}$</th>
<th>$T_{C&gt;}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1(a)</td>
<td>(200 - 300)$^2$</td>
<td>200</td>
<td>0.33</td>
<td>1.31</td>
<td>1.80</td>
</tr>
<tr>
<td>W2</td>
<td></td>
<td>210</td>
<td>1.26</td>
<td>1.274</td>
<td>2.11</td>
</tr>
<tr>
<td>W3</td>
<td></td>
<td>100</td>
<td>8.5</td>
<td>1.330</td>
<td>1.90</td>
</tr>
<tr>
<td>W4</td>
<td></td>
<td>100</td>
<td>0.95</td>
<td>1.33</td>
<td>1.86</td>
</tr>
<tr>
<td>W5</td>
<td>140 x 150</td>
<td>100</td>
<td>19.1</td>
<td>1.261</td>
<td>1.91</td>
</tr>
<tr>
<td>W6</td>
<td>240 x 250</td>
<td>100</td>
<td>6.1</td>
<td>1.256</td>
<td>1.91</td>
</tr>
<tr>
<td>W7</td>
<td>330 x 360</td>
<td>185</td>
<td>0.27</td>
<td>1.212</td>
<td>1.62</td>
</tr>
<tr>
<td>N1</td>
<td>10 x 360</td>
<td>100</td>
<td>4.3</td>
<td>1.264</td>
<td>1.86</td>
</tr>
<tr>
<td>N2</td>
<td>10 x 360</td>
<td>100</td>
<td>10.0</td>
<td>1.252</td>
<td>1.93</td>
</tr>
<tr>
<td>N3</td>
<td></td>
<td>100</td>
<td>8.6</td>
<td>1.251</td>
<td>1.92</td>
</tr>
<tr>
<td>N4</td>
<td></td>
<td>100</td>
<td>9.6</td>
<td>1.234</td>
<td>2.05</td>
</tr>
</tbody>
</table>

(a) Inverted configuration. Unlike all other junctions, this one had its test (clean) film against the quartz substrate/transducer.
### ENHANCEMENT DATA/WIDE JUNCTIONS

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>$v$ (6Hz)</th>
<th>$(\hbar v/k_B T_C)$</th>
<th>POWER OF FIT (dB)</th>
<th>$B$ ($10^{-3}$)</th>
<th>$h_{\text{fit}}$</th>
<th>$h_{\text{theory}}$</th>
<th>$B(\hbar v/T_C)$ ($10^{-3}$)</th>
<th>$B(0\text{dB})$ ($10^{-3}$)</th>
<th>HEATING (a)</th>
<th>$R_{k,\text{fit}}$ cm$^2$ K$^{-1}$ (10$^{-3}$)</th>
<th>ENHANCEMENT (d)</th>
<th>6T/T$_C$ (10$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>9.01</td>
<td>0.328</td>
<td>-9</td>
<td>5.56</td>
<td>1.21</td>
<td>1.97</td>
<td>2.20</td>
<td>44.2</td>
<td>7.7</td>
<td>1.1</td>
<td>-7.5</td>
<td></td>
</tr>
<tr>
<td>W2</td>
<td>8.87</td>
<td>0.333</td>
<td>-5</td>
<td>3.79</td>
<td>1.60</td>
<td>1.99</td>
<td>2.02</td>
<td>12.0</td>
<td>3.7</td>
<td>1.9</td>
<td>-8.0</td>
<td></td>
</tr>
<tr>
<td>W3(c)</td>
<td>8.83</td>
<td>0.317</td>
<td>-10</td>
<td>1.26</td>
<td>1.37</td>
<td>1.33</td>
<td>0.55</td>
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<td>2.8</td>
<td>2.8</td>
<td>-6.0</td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>9.26</td>
<td>0.333</td>
<td>-10</td>
<td>1.37</td>
<td>1.60</td>
<td>1.33</td>
<td>0.72</td>
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<td>4.2</td>
<td>3.8</td>
<td>-4.2</td>
<td></td>
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<tr>
<td>W5</td>
<td>8.88</td>
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<td>-6</td>
<td>6.2</td>
<td>1.91</td>
<td>1.30</td>
<td>3.99</td>
<td>15.6</td>
<td>6.5</td>
<td>4.2</td>
<td>-7.0</td>
<td></td>
</tr>
<tr>
<td>W6</td>
<td>9.63</td>
<td>0.372</td>
<td>0</td>
<td>6.9</td>
<td>1.57</td>
<td>1.30</td>
<td>4.0</td>
<td>6.9</td>
<td>2.3</td>
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<td>-5.0</td>
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<tr>
<td></td>
<td>8.91</td>
<td>0.344</td>
<td>0</td>
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<td>2.23</td>
<td>1.30</td>
<td>10.4</td>
<td>13.6</td>
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<td>16</td>
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<tr>
<td></td>
<td>8.69</td>
<td>0.331</td>
<td>0</td>
<td>12.6</td>
<td>2.01</td>
<td>1.30</td>
<td>8.3</td>
<td>12.6</td>
<td>5.6</td>
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<td>-7.0</td>
</tr>
<tr>
<td></td>
<td>8.35</td>
<td>0.323</td>
<td>0</td>
<td>23.3</td>
<td>2.53</td>
<td>1.30</td>
<td>19.0</td>
<td>23.3</td>
<td>13.9</td>
<td>28</td>
<td>8.0</td>
<td>-8.0</td>
</tr>
<tr>
<td>W7</td>
<td>9.78</td>
<td>0.386</td>
<td>0</td>
<td>21.7</td>
<td>1.25</td>
<td>1.78</td>
<td>10.5</td>
<td>21.7</td>
<td>4.8</td>
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<td>-15.0</td>
</tr>
<tr>
<td></td>
<td>8.99</td>
<td>0.355</td>
<td>0</td>
<td>31.2</td>
<td>1.52</td>
<td>1.78</td>
<td>16.8</td>
<td>31.2</td>
<td>9.2</td>
<td>47</td>
<td>2.0</td>
<td>-17.0</td>
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<td>8.46</td>
<td>0.334</td>
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<td>1.78</td>
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<td>21.0</td>
<td>4.1</td>
<td>27</td>
<td>1.4</td>
<td>-14.5</td>
</tr>
<tr>
<td></td>
<td>8.43</td>
<td>0.333</td>
<td>-10</td>
<td>16.4</td>
<td>1.96</td>
<td>1.78</td>
<td>10.7</td>
<td>164</td>
<td>70</td>
<td>70</td>
<td>3.1</td>
<td>-17.0</td>
</tr>
</tbody>
</table>

(a) The heating we define to be $B(0\text{dB})(h - 0.68)v/T_C$. It is what is measured above $T_C$.
(b) Our best estimate of $R_K$ from material properties is 2.7 cm$^2$ K$^{-1}$.
(c) The phonons in this measurement were transverse phonons.
(d) $T/T_C \approx 0.98$, power = 0dB.
### ENHANCEMENT DATA/NARROW JUNCTIONS

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>(\nu) (6Hz)</th>
<th>((h\nu/k_BT_C))</th>
<th>POWER OF FIT (dB)</th>
<th>(B) ((10^{-3}))</th>
<th>(h_{\text{fit}})</th>
<th>(h_{\text{theory}})</th>
<th>(B\nu/T_C) ((10^{-3}))</th>
<th>(B(\text{dB})) ((10^{-3}))</th>
<th>FIT MEASURED ((10^{-3}))</th>
<th>(\delta I/I_C) ((10^{-3}))</th>
<th>(R_{k,\text{fit}} , \text{cm}^2 K_w^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>8.6</td>
<td>0.328</td>
<td>-10</td>
<td>21.6</td>
<td>1.66</td>
<td>1.29</td>
<td>11.7</td>
<td>216.</td>
<td>69</td>
<td>51.±10</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>9.66</td>
<td>0.368</td>
<td>-4</td>
<td>11.0</td>
<td>2.00</td>
<td>1.29</td>
<td>8.1</td>
<td>27.6</td>
<td>13.4</td>
<td>5.7</td>
<td>-14</td>
</tr>
<tr>
<td>N2</td>
<td>8.81</td>
<td>0.337</td>
<td>-6</td>
<td>9.8</td>
<td>0.80</td>
<td>1.29</td>
<td>2.6</td>
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<td>1.7</td>
<td>0.5</td>
<td>-15</td>
</tr>
<tr>
<td>N3</td>
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<td>0.339</td>
<td>0</td>
<td>52.9</td>
<td>1.33</td>
<td>1.30</td>
<td>23.8</td>
<td>52.9</td>
<td>11.7</td>
<td>52.0</td>
<td>2.8</td>
</tr>
<tr>
<td>N4</td>
<td>8.81</td>
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<td>10.2</td>
<td>0.92</td>
<td>1.28</td>
<td>3.2</td>
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</tr>
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<td>9.94</td>
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<td>16.6</td>
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<td>6.7</td>
<td>52.5</td>
<td>7.3</td>
<td>20.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>

\(a\) \(T/T_C \approx 0.98\), power = 0dB.
For number sequence only.
### APPENDIX D MATERIAL PARAMETERS AND ENHANCEMENT DATA FOR NARROW STRIPS

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>WIDTH (µm)</th>
<th>THICKNESS (nm)</th>
<th>( R_{300}/R_{T_c} )</th>
<th>( \nu/T_c )</th>
<th>MAXIMUM ( T_c ) ENHANCEMENT ((10^{-3}))</th>
<th>JUNCTION TYPE</th>
<th>MAXIMUM GAP ENHANCEMENT ((10^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>210</td>
<td>3.0</td>
<td>0.33</td>
<td>-40</td>
<td>wide (W2)</td>
<td>-8</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>190</td>
<td>2.5</td>
<td>0.33</td>
<td>-38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>100</td>
<td>2.5</td>
<td>0.34</td>
<td>-40</td>
<td></td>
<td></td>
</tr>
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<td>4</td>
<td>10</td>
<td>50 - 100</td>
<td>2.6</td>
<td>0.35</td>
<td>-19</td>
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</tr>
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<td>5</td>
<td>10</td>
<td>100</td>
<td>2.4</td>
<td>0.34</td>
<td>-35</td>
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<td>6</td>
<td>10</td>
<td>100</td>
<td>2.65</td>
<td>0.33</td>
<td>-65</td>
<td>narrow(N1)</td>
<td>-29</td>
</tr>
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<td></td>
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<td>0.37</td>
<td>-20</td>
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<td>8</td>
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<td>100</td>
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<td>0.34</td>
<td>-37</td>
<td>narrow(N2)</td>
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</tr>
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<td>9</td>
<td>10</td>
<td>100</td>
<td>2.4</td>
<td>0.39</td>
<td>-38</td>
<td>narrow(N4)</td>
<td>-30</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>0.34</td>
<td>-45</td>
<td></td>
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<td>10</td>
<td>100</td>
<td>4.3</td>
<td>0.33</td>
<td>-78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11(a)</td>
<td>10</td>
<td>150</td>
<td>1.25</td>
<td>0.25</td>
<td>-5</td>
<td></td>
<td></td>
</tr>
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<td>12</td>
<td>10</td>
<td>410</td>
<td>3.0</td>
<td>0.32</td>
<td>-15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) This strip was doped with \( O_2 \). \( T_c = 1.7K, \delta_{300} \approx 20\mu\Omega \cdot \text{cm} \).
FOOTNOTES

(1) Phonon trapping labels the problem caused by recombination or scattered phonons that stay in a sample, do not escape to the bath, and may be reabsorbed. It is akin to complications caused by multiple scattering in other systems.

(2) It is a great aid in computations to realize that since \( u \) need never be very large and that \( G(u) \) is smooth for \( u > \frac{1}{2} \), \( G(u) \) may be well approximated by a simple function which is calculated easily. In the phonon case the approximation

\[
G(u) = \exp \left( A_0 + A_1 u + A_2 (u^2) \right)
\]

with \( A_0 = 0.4789, A_1 = -0.7638, A_2 = -0.0749 \) fits the exact \( G(u) \) for \( \frac{1}{2} < 5 \) with an accuracy of better than 1%.

(3) Actually, this calculation yields a \( \left[ \delta \bar{T}/T_c \right]_o \), which is bigger than the one quoted. ESSS cut it down to size by invoking the results of some more exact studies, which differ by a factor, but not in spirit.

(4) The pulse circuitry for this TWT was designed and built by Robert Wilson, and is described in his thesis (1981). Some small changes were necessary and the schematic is shown in Fig. 3.

(5) Obtained from Valtec Inc.

(6) Measured by time of flight in this experiment. According to Simmons and Wang (1972) \( c_t = 6.09 \times 10^5 \) cm/sec. The measure transverse speed of sound is \( c_t = 3.2 \times 10^5 \) cm/sec.
(7) Full microwave power delivered to the cavity is \( \approx 600 \text{ W} \). This is measured by monitoring the current through the heater in the bath. All other factors being equal, to maintain a given bath temperature, the heat input \( Q \) to the bath must be the same whether the microwaves are on or off. Thus

\[
Q_{\text{total}} = Q_{\text{m}} + P_{\mu} \times \text{Duty factor}
\]

The heater power \( (P_R) \) and duty factor are measured easily, and so therefore is \( P_{\mu} \). The TWT probably put out more than \( 1000 \text{ W} \), but a few dB was lost in the transmission through a score of flanges and bends and couplers and connectors.

(8) Another variable enters the fitting problem at the highest powers. One may contrive to include the second order Kapitza resistance in ILE by making a transformation like

\[
B h \frac{\nu}{T_c} \rightarrow B h \frac{\nu}{T_c} (1 - 1.5 B h \frac{\nu}{T_c})
\]

For sufficiently low power, meaning \( B h \nu / T_c \approx [\delta T / T_c]_h < 0.01 \), the presence or absence of the extra term has a negligible effect on the enhancement. As \( [\delta T / T_c]_h \) grows to \( \approx 5\% \), the contribution is significant and because of its sign, the heating becomes less important and the predicted enhancement grows, i.e. it does not saturate as quickly. The disparity is not gross at values of \( [\delta T / T_c]_h \) obtained by temperature-fitting. In order to mimic the observed saturation of the enhancement at high powers, it was necessary to eliminate this extra term. Since we are going to disregard the numerical values obtained by the power-fits, this is not a terribly important point and it does not comment on the validity of that correction. Reiterating the conclusion, at relevant values of the heating \( [\delta T / T_c]_h \), the second order Kapitza term does not have a marked effect on the pred-
ictions of ILE.

(9) It is an intensive parameter in the thermodynamic sense.

(10) For aluminum \( \rho c = 1.9 \times 10^8 \text{ gm cm}^{-2} \text{ sec}^{-1} \), and for liquid helium \( \rho c = 3.4 \times 10^3 \text{ gm cm}^{-2} \text{ sec}^{-1} \) (Wyatt, 1981).

(11) The narrowest strips that I made were 3 \( \mu \text{m} \) wide and were prepared on glass slides. They obeyed GL for \( T/ T_c > 0.83 \). My photolithographic technique improved with time, providing me with sharper lines, which were correlated with improved low temperature behavior.

(12) Recall that most of the incident phonon energy is not absorbed in the Al films but is reflected at the LHe and returns to the other end of the crystal where it is reflected again and then incident on the films, etc.

(13) This is not \( t_2 \). It is \( t_1 \), a pulse of primary transverse phonons which the crystal manufacturers claim should not be present. However, their time of flight is their signature. It is probably cracks in the crystal surface which are responsible for their presence. Different crystals revealed them to different extents. In the gap experiments where \( \Delta V \) vs. time could be monitored on an oscilloscope, their presence was unmistakable. Some gap data was taken for \( t_1 \), with no substantial differences discovered. The theory is written for 1-phonons but since the absorption mechanism is essentially the same (Ginsburg, 1969) the enhancement mechanism will be the same.

(14) "Normal" in this sense only implies the vanishing of the hysteretic return current.

(15) An example of one of the uncertainties, for instance, is \( \tau_E \), which varies by a factor of 4 throughout the literature. We have used the canonical Clarke group value \( \tau_E = 12 \text{ ns} \). The relaxation time enters the calculation for our guess of both \( B \) and \( h \), but it cancels when one takes the product
\[ B(h - 0.68) \nu / T_c. \]

(16) Tredwell and Jacobsen (1975 and 1976) did not measure either of these. Instead they measured the enhancement of a Josephson-like tunneling current.
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