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Real-Time Optical Tweezing

THESIS

submitted in partial satisfaction of the requirements
for the degree of

MASTER OF SCIENCE
in Electrical and Computer Engineering

by

Shah Mohammed Tamzidur Rahman

Thesis Committee:
Associate Professor Ozdal Boyraz, Chair
Associate Professor Ahmed Eltawil
Professor Nader Bagherzadeh

2016
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ABSTRACT OF THE THESIS

Real-Time Optical Tweezing

By

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Master of Science in Electrical Engineering

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Professor Ozdal Boyraz, Chair

In this thesis a new approach called ‘space-time-wavelength mapping’ has been developed for real-time electronic control of optical tweezers. The proposed technique enables precise control of optical signals in space, time, and frequency through time-domain dispersion and diffractive optics, which in turn enables generation of controlled radiation forces acting on small particles. In this study we show that 150 fs ultrafast optical pulses can be dispersed in time and space to achieve a ~20 μm ×~2 μm focused elliptical beam. The force field at the focal plane of the beam is dependent on local intensity gradients along the plane. The spatial intensity profile can be electronically controlled by assigning local power levels to each wavelength using time-domain RF modulation of dispersed pulses, and sending each wavelength, and hence the assigned power level, to a specific location in space through diffractive optics. We show that by choosing the appropriate RF waveform, one is able to create force fields for cell stretching and compression as well as multiple force hot-spots (of >200 pN force per pulse) for attractive and repulsive
forces. A detailed theoretical model and simulation results from a proposed experimental setup are presented. This approach is significantly more advantageous in terms of flexibility and control, compared to conventional optical tweezers that require mechanical steering or holographic optical tweezers that produce undesired ‘ghost traps’. In addition, it is shown how the technique can also be extended to create tunable 2D force field distributions using a virtually-imaged phased-array (VIPA).
CHAPTER 1

INTRODUCTION

This chapter aims to provide basic information on the concept of optical tweezing and on the history and applications of optical tweezers. In particular, we discuss the developments in electronic control of optical tweezers since that is the heart of this project. This short but succinct literature review and conceptual background should assist greatly in understanding the work presented in the following chapters [1]–[3].

1.1 Optical Tweezers

Optical tweezers use forces arising from intensity gradients in a strongly focused beam of light to trap and move micro- and nano-scale objects. Optical tweezing utilizes photon momentum to accelerate and trap particles using optical forces. It was first demonstrated by Ashkin et al. [4], and has since found a wide range of applications, especially in the biosciences [5]–[8]. Although earlier theoretical work predicted the existence of optical forces [9], until sophisticated laboratory measurements were made possible in the beginning of the twentieth century, their existence was questioned by skeptics [10] because such forces
were deemed to be extremely small for reasonable intensities of light (a maximum of 10 nN/W in air) [11].

One major hurdle in overcoming gravity using optical forces was the need for extremely high irradiances to be focused onto microscopic particles (~100 μm for typical densities), requiring an equally high irradiance source (a 6000 K blackbody for 0.1 mW/μm²) [11], which was not practical before the advent of lasers. This problem was overcome with the invention of lasers. Ashkin’s principal breakthrough was in realizing that they were capable of producing higher irradiances, thus giving rise to significant optical forces for levitation of small particles against gravity [4], [11]. This concept was applied in building the basis for modern optical trapping technology.

An optical trap is formed by tightly focusing a laser beam with an objective lens of high numerical aperture (NA). A dielectric particle near the focus will experience a force due to the transfer of momentum from the scattering of incident photons. The resulting optical force has traditionally been grouped into two components: (a) a scattering force, in the direction of light propagation and (b) a gradient force, in the direction of the spatial light gradient [12]. Ashkin’s seminal work showed that large enough gradient forces could be achieved using a single laser beam, which could overcome the scattering force, and accelerate particles along an intensity gradient [13]. Apart from single-beam experiments, there has been growing literature on double beam optical trapping following Ashkin’s observations. One example is the early work by Roosen and Imbert, demonstrating optical levitation by trapping particles using two counter propagating beams, each contributing an upward force (~70 pN) to balance the particle’s weight [14].
Following the levitation experiments, Barton et al have laid down some theoretical framework for forces and torques acting on an optically levitated spherical particle [15], [16]. Guilloteau, Gouesbet, and co-workers published several studies investigating the scattering of light by optically levitated particles [17], [18]. They proposed and revised a Generalized Lorentz-Mie Theory (GLMT) that accurately represented the experimental data for scattering patterns [17]. It appears that it is not feasible to achieve precise control and tunability over the scattering force using currently existing methods and techniques. The gradient force is a better candidate for this purpose since there are various ways of manipulating the intensity gradient at the focal plane. In the Rayleigh scattering regime in particular, the gradient force is more significant and has been utilized to trap sub-wavelength particles. This opened up possibilities of modulating the intensity gradient of a laser beam to manipulate particles, leading to applications in microfluidic systems, cell sorting, and characterization of microorganisms. Although traditional optical tweezer setups only allowed for pushing forces on objects, a new concept of optical pulling force involving tractor beams has been presented recently [19], [20].

Intensity modulation is traditionally done by mechanically steering multiple parallel beams focused onto a narrow region (~50 μm × ~5 μm) to create desired intensity profiles. This allows for direct control of gradient force across the region, owing to its dependence on the intensity gradient. One example of this is the recent work by Stilgoe and co-workers that demonstrated multiple stable optical traps by steer-controlling the separation between two laser beams [21]. However, an entirely electronic setup that does not involve any moving parts is more desirable for optical micromanipulation.
1.2 Electronic Control and Holographic Optical Tweezers

In contrast to mechanical beam steering, Holographic Optical Tweezers (HOTs) [22]–[26] emerged as an established approach toward electronic control of optical trapping. In an HOT setup, a computer controlled diffractive optical element (DOE) is used to modulate the phase front of a laser beam before feeding it into a conventional tweezer. Such techniques typically use liquid crystal spatial light modulators (SLM) to create a pixel array of intended phase shifts which are then imposed on the beam at the corresponding pixels. By using different configurations of phase shift arrays, one is able to control the 3D location of optical traps at the focal volume. Preece et al. have taken this further by achieving polarization control using the phase shift configurations imposed by the SLM, thereby controlling the spin angular momentum of the traps in addition to their spatial location [27].

While HOTs achieve electronic control over a grid of optical traps to some extent, the technique suffers from some serious limitations and drawbacks. First of all, a fundamental limitation of HOTs is the confinement of optical traps to discrete spots as opposed to the ability to manipulate an extended continuous intensity landscape [23]. In addition, stemming from the lack of direct control over the intensity landscape, HOTs produce undesired intensity peaks (dictated by symmetry conditions) called ‘ghost traps’ which are strong enough to trap particles [24], [25], [28]. To suppress ghost traps, symmetry of the trapping pattern can be reduced by changing the positions of a few or all the traps as has been shown in [24]. But this approach cannot be applied universally.
as changing the trapping pattern often leads to loss of functionality. Another technique proposed by Polin and co-workers involves defocusing and blocking the zeroth order of diffraction and corresponding ghost traps, while using a direct search algorithm to find appropriate holograms for optical trapping [25]. However, especially when more than eight phase levels are to be applied in order to exploit more degrees of freedom made available by modern SLMs, direct search algorithms are computationally expensive compared to the classical Gerchberg Saxton algorithm which is an example of iterative Fourier transform algorithms (IFTAs) [23]. Another technique that has helped suppress ghost traps is the random mask encoding method presented in [26], [29]. However, this approach significantly reduces diffraction efficiency and cannot be used for more than a small number of traps with reasonable power [23], [26]. Another limitation of HOTs is the inhomogeneity among individual optical traps that arises from the lack of direct control. Intensity variation among generated traps has been reported by Curtis and co-workers to be higher than 25%, and the figure is greater for higher numbers of traps. This is highly undesirable if a uniform system of traps is required. Curtis et al propose several methods to reduce the inhomogeneity but it comes at the cost of reduced efficiency [24].

1.3 Space-Time-Wavelength Mapping: A New Technique

We propose electronic control of optical tweezing by using space-time-wavelength mapping (STWM) technology [1]–[3]. We show that STWM is a powerful approach to create desired intensity gradients in a 2D focal plane and to provide direct control of force fields acting on particles. Previously this technique has been utilized for real-time
imaging and detection of micro particles [30], real-time wide field microscopy [31], [32], and arbitrary waveform generation [33]. It has also been used in photonic time-stretched analog-to-digital converters [34], [35], [36], in waveform digitizers [37], [38], and in investigation of optical rogue waves [39].

In the proposed technique (STWM), time-domain modulation of power spectral density is used to manipulate the intensity gradient at the focal plane of a laser beam, which in turn controls the 2D force profile at the focal plane of the beam. Time-domain modulation is then translated onto space domain using a diffraction grating. Using this method, we demonstrate that commercial femtosecond pulses can generate >200 pN optical force hot-spots, whose polarity and location along the focal plane can be controlled to achieve numerous applications such as cell stretching or particle sorting. The amount of force can be controlled by chirped pulse amplification without any nonlinear distortion. Electro-optic modulation of power spectral density allows tuning of local intensity at the focal plane for greater control and to generate attractive and repulsive forces between particles at specific spots as depicted in Fig. 1.1.

FIG. 1.1 Particles experiencing gradient force at various spots on the focal plane of the beam (particle size has been scaled up for illustration).
Through the ability to directly manipulate a continuous landscape, this technique overcomes the limitations of HOTs discussed above, while allowing a greater and direct electronic control of optical traps. Space-time-wavelength mapping also eliminates the issue of unwanted inhomogeneity among produced traps, while not compromising efficiency. Although our theoretical model presented here considers only one dimension, the analysis can be extended to two dimensions using a virtual imaged phased array (VIPA) [40].
CHAPTER 2

SPACE–TIME–WAVELENGTH MAPPING: THEORY AND MODELING

In this chapter, we discuss the theoretical framework for the proposed STWM technique. The theory is based on an experimental model involving various optical devices and components. Each of these elements have been analytically modeled with certain reasonable assumptions and approximations. The theory and modeling laid out in this chapter serves as the basis for numerical simulations and results presented further along in this work.

2.1 Modeling Setup

To demonstrate the feasibility of optical tweezing by using space-time-wavelength mapping (STWM), we developed an analysis based on the following realistic experimental setup as illustrated in Fig 2.1. The optical tweezer is driven by a fiber laser at 1550 nm that produces 150 fs pulses with ~20 kW peak power. These values correspond to a laser with ~100 mW average power at 50 MHz repetition rate. The laser beam then propagates through a dispersive medium
with ~100 ps/nm chromatic dispersion to broaden the optical pulse width to ~2 ns at the full width half max (FWHM) point. Hence, the dispersive propagation creates a time-wavelength mapping where 1 nm wavelength separation corresponds to 100 ps separation in time domain. Dispersed pulses are then passed through an electro-optic modulator to manipulate the power spectral density of the laser as desired, by using RF waveforms. Here we assume no losses in components and that nonlinear distortion is absent in dispersive fiber. In practice, it is possible to start with a lower peak power and add a chirped pulse amplification to compensate losses and eliminate distortion. The time-wavelength mapped pulses are then coupled to a diffraction grating with 600 lines/mm groove density and a cylindrical lens to create space-wavelength mapping. Finally, a 60x microscope objective is used to generate a ~20 μm long and ~2 μm wide elliptical focal spot. After time-wavelength mapping, the diffraction grating provides a space-wavelength mapping of 1 μm/nm at the focal plane.

FIG. 2.1 Proposed experimental setup. A 150 fs laser with 50 MHz repetition rate undergoes dispersion in time and is then modulated in time domain, and simultaneously in wavelength domain, using arbitrary RF waveforms fed to an electro-optic modulator. The modulated power spectral density is then mapped onto space domain using a diffraction grating. A cylindrical lens and a microscope objective then focus the diffracted beam onto a tight focal spot (20 x 2 μm) for optical manipulation of particles.
2.2 Space-Time-Wavelength Mapping

Space-time-wavelength mapping is the result of time-wavelength mapping followed by space-wavelength mapping. A beam comprised of a spectrum of wavelengths is stretched in time by propagating it through time dispersive media. This gives rise to a time-wavelength mapping $\lambda(t)$. Similarly, the beam is made to undergo spatial dispersion causing the wavelengths in the spectrum of the beam to disperse in space, creating a space-wavelength mapping $\lambda(x)$. Through time-domain modulation of the time-stretched spectrum, one can obtain a desired wavelength-dependent intensity profile $I(\lambda(t)) = I(\lambda)$ for the beam which gets translated into a space-dependent intensity profile $I(\lambda(x)) = I(x)$ through space-wavelength mapping. Since the gradient force exerted by a beam on a particle is proportional to the intensity gradient [11], STWM thus creates a force profile tied to wavelength and space. The analysis presented here involves only the lateral force profile at the focal plane ($z = 0$) and thus any variation of force in the axial direction has been ignored. For further simplicity, only the force along the $x$-axis ($y = 0$) on the focal plane has been considered in this section.
2.3 Modulating the Intensity of a Gaussian Beam

2.3.1 Dispersion and Time-Wavelength Mapping

Taking the Gaussian beam as an example, one may develop formulations for modulating the intensity (and hence the force profile) as follows. Assuming the amplitude $A$ of the incident field $E$ of the femtosecond laser source in Fig. 2.1 is given by

$$A(t) = A_0 \exp \left[-\frac{1}{2} \left(\frac{t}{T_0}\right)^2 \right]. \quad (2.1)$$

Taking a Fourier transform to obtain $E(\omega)$, and applying dispersion, the field $E_{\text{disp}}(\omega)$ in the frequency domain is given by

$$E_{\text{disp}}(\omega) = E(\omega) \exp \left[-j \beta_2 L \omega^2 \right]. \quad (2.2)$$

This was followed by an inverse Fourier transform to obtain the dispersed field $E_{\text{disp}}(t)$ and its amplitude $A_{\text{disp}}(t)$ back in time domain as shown in Fig. 2.2.

Applying chirp and RF modulation $f_{\text{RF}}(t)$, the field can be described as

$$E_{\text{RF}}(t) = A_{\text{disp}}(t) e^{j\omega t} e^{j\phi(t)} f_{\text{RF}}(t). \quad (2.3)$$
2.3.2 Electro-Optic Modulation

Based on the Mach-Zehnder Modulator model shown in Fig. 2.3, one can rewrite Eq. (2.3) in terms of a power transfer function instead of field as:

\[ P_{\text{out}}(t) = P_{\text{in}}(t)f_{\text{RF}}^2(t) = P_{\text{in}}(t)\left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{V_{\pi}}(V_{\text{bias}} + v_{\text{RF}}(t))\right)\right], \quad (2.4) \]

where the input power \( P_{\text{in}}(t) = (A_{\text{disp}}(t))^2 \), \( V_{\pi} \) is the half-wave voltage, \( V_{\text{bias}} \) is the bias voltage, and \( v_{\text{RF}}(t) \) is the applied RF waveform. By manipulating \( v_{\text{RF}}(t) \) using an arbitrary waveform generator, one is able to control the power spectral density of the laser emerging from the electro-optic modulator. An example waveform shown in Fig. 2.4 is a super-Gaussian \( v_{\text{RF}}(t) = \pi e^{-st^2n} \) where the integer \( n \) and the constant \( s \) can be used to manipulate the shape and width of the RF waveform, and hence the generated force profile.
FIG. 2.3 A diagram of the electro-optic modulator used.

FIG. 2.4 RF waveform (left, green), corresponding power profile (middle), generating desired force profile (right).

2.3.3 Space-Wavelength Mapping

For a Gaussian beam with beam size $w$, i.e., the spatial full-width at half maximum, the aggregate intensity profile of all wavelengths $I(x, \lambda)$ after the space-wavelength mapping, i.e., after a diffraction grating and a Fourier lens with focal length $f$, is governed by Eq. (2.5) [33]. A derivation of this is given in Appendix A2.
\[ I(x, \lambda) = \sum \frac{P_i}{(f\lambda_i)^2} \exp \left[ - \left( \frac{2\pi w(x - x_i)}{f\lambda_i} \right)^2 \right] \]

Here, \( P_i \) represents the peak power carried by wavelength \( \lambda_i \) in the spectrum, \( x_i \approx G_\beta f (\lambda_i - \lambda_c) \) is the relative position of the first order diffraction peak for the wavelength \( \lambda_i \) with respect to the central wavelength \( \lambda_c \), \( G_\beta = \frac{g}{\cos(\beta)} \) is the effective groove density of the diffraction grating shown in Fig. 2.1, that is defined as a function of groove density \( g \) and the first order diffraction angle \( \beta \). For a beam with wavelength \( \lambda \), and incident angle \( \alpha \), the first order diffraction angle \( \beta \) is given by the grating equation \( \sin(\alpha) + \sin(\beta) = \lambda G \) which reduces to \( 2\sin(\beta) = \lambda G \) when \( \alpha = \beta \) in a Littrow configuration [33].

The different wavelengths along the spectrum are spatially dispersed and hence the force arising from each wavelength is tied to a specific point in space, as shown in Fig. 2.5. Therefore, by tuning the amplitude of individual wavelengths using RF modulation, one is able to generate intensity profiles for desired spatial force profiles.


FIG. 2.5 Force arising from different wavelengths in the spectrum.

2.4 Modulating the Force Profile

Assuming operation under the Rayleigh scattering limit and that the particles are polarizable, such as dielectric particles, one may develop a formulation for the optical force at the focal plane as follows. The trapping gradient force from a laser beam on a particle is proportional to the light intensity gradient. The gradient force described in Eqs. (2.6) and (2.7) can be derived from the basic formulation of Lorentz force [41]. A derivation of this is given in Appendix B.

\[ F = \frac{1}{2} \alpha \nabla E^2 \]  

(2.6)
Incorporating proportionality constants one obtains,

\[ F(r) = \frac{2\pi n_0 b^3}{c} \left( \frac{m^2 - 1}{m^2 + 2} \right) \nabla I(r) \]  

(2.7)

where \( m = n_1/n_0 \) is the relative refractive index of the particle (where \( n_1 \) is the refractive index of the particle and \( n_0 \) is the refractive index of the surrounding medium), \( b \) is the radius of the particle, \( c \) is the speed of light, and \( r \) is the radial distance from the beam axis.

Taking the gradient of intensity in (2.5) with respect to the radial distance, one obtains (2.8) which gives the intensity gradient of the \( i \)th wavelength (\( \lambda_i \)) in the spectrum:

\[ \nabla I_i(x) = \frac{\partial I_i(x)}{\partial x} = -\frac{8\pi^2 w^2 (x - x_i)}{(f\lambda_i)^4} P_i \exp \left[ -\left( \frac{2\pi w (x - x_i)}{f\lambda_i} \right)^2 \right] \]  

(2.8)

Here it has been assumed that operation is at the focal plane and that the beam width is the spot size at the focus, \( i.e., w = w_0 \) for \( z = 0 \), \( z \) being the direction of propagation of the laser. Thus a derivative with respect to \( z \) has not been considered in Eq. (2.8).

Substituting (2.8) in (2.7), one obtains an expression for the optical force dependent on power and radial distance along \( x \) of the \( i \)th wavelength in the spectrum, as shown in (2.9):

\[ F_i(P_i, \lambda_i, x) = -\frac{16n_0 w^2 b^3 \pi^3}{c} \left( \frac{m^2 - 1}{m^2 + 2} \right) \frac{(x - x_i)}{(m^2 + 2)} P_i \exp \left[ -\left( \frac{2\pi w (x - x_i)}{f\lambda_i} \right)^2 \right] \]  

(2.9)
Applying a linear superposition on Eq. (2.9) to get the total force $F(x)$ exerted by the entire spectrum, one obtains,

$$F(x) = \sum_i F_i(P_i, \lambda_i, x)$$

$$= - \frac{16n_0 w^2 \pi^3 b^3}{c} \left( \frac{m^2 - 1}{m^2 + 2} \right) \sum_i \frac{(x - x_i)}{(f \lambda_i)^4} P_i \exp \left[ - \left( \frac{2 \pi w (x - x_i)}{f \lambda_i} \right)^2 \right]. \quad (2.10)$$

Note that in equations (2.9) and (2.10), the positive $x$ direction has been defined to be the direction of positive force. Also, equation (2.10) represents a direct relationship between the applied RF modulation and the force profile at the focal plane. RF modulation controls the power spectral density $P_i$ in (2.10), thus modulating the beam intensity profile, and allowing direct control of the force profile $F(x)$. 
CHAPTER 3

ONE DIMENSIONAL ELECTRONIC CONTROL

Several numerical simulations were conducted based on the model described in Chapter 2. Results from these simulations are presented in this chapter. Each simulation explores a different aspect of real-time electronic control of optical tweezers and the results provide greater insight into the possibilities and potential applications of STWM technology. They also allow deeper and further analysis into the capabilities and drawbacks of the proposed model. This chapter is limited to results and applications in 1D. Further extended applications in 2D will be discussed in the next chapter.

3.1 Controlling the Location and Size of Force Hot-Spots

It is important to study and further analyze the relationship between RF Modulation and the intensity and force profiles as predicted by the theory outlined in Chapter 2, in order to explore and demonstrate real-time, electronic location control of optical force hot-spots and how it is related to manipulating the size/length of hot-spots along the $x$ dimension.
3.1.1 Controlling the Location

One of the clear advantages of the proposed method is the high precision in location control of force fields in real-time without altering the rest of the continuous force landscape. The center of the force field is determined by the location of the intensity spikes. Hence, one can change the pulse width and RF voltage to determine the exact location of trap centers. To demonstrate this concept, two different pulse widths were used to maneuver two optical force spikes along the $x$ direction. For instance, for the RF signal shown in Fig. 2.4, a pulse width $\tau_1 = 2$ ns corresponds to a distance $d_1 = 6 \, \mu m$ between force hot-spots, and a pulse width $\tau_2 = 4$ ns corresponds to a distance $d_2 = 12 \, \mu m$ as illustrated in Fig. 3.1, by the red and blue curves respectively. The precision of this technique depends on the RF bandwidth. For a bandwidth of 10 GHz, the narrowest RF pulse possible is $<0.1$ ns, which results in a minimum achievable separation distance of $<0.5 \, \mu m$. This means that two particles separated by, for example, $5 \, \mu m$, can be brought to within $0.5 \, \mu m$ of each other using this approach.

![Graphs](image)

**FIG. 3.1** Illustrating location control of hot-spots. (a) Modulated power distribution, (b) Generated force profile along $x$. 

3.1.2 Controlling the Size and Position

The concept of using different RF pulse widths to control the location of force spikes can be extended to achieve force hot-spots of desired sizes/lengths along the \( x \) dimension. To demonstrate this, three different RF waveforms were applied to a super-Gaussian power profile. Fig. 3.2 illustrates the intensity profiles and optical force contours acting on particles for these three different RF modulation signals. The modulator is biased at the quadrature point and supplied with RF pulses of peak-to-peak voltage \( v_{RF,PP} = \gamma \cdot V_r \), where the amplitude factor \( \gamma \) is controlled by the arbitrary waveform generator.

In the first row of Fig. 3.2, the case when there is no RF signal (Fig. 3.2a) is presented. As expected, a pure elliptical beam that produces a force contour that traps the particle along the central axis of the beam is generated. Effectively, for a beam of size 20 x 2 \( \mu \)m, the particle is trapped along a line-shaped well with dimensions 15 x 1.5 \( \mu \)m as shown in Fig. 3.2b and 3.2c.

In the second row, a 2 ns wide pulsed RF signal is applied, where peak-to-peak voltage \( v_{RF,PP} = 1V_r \) (Fig. 3.2d). In the second row, the RF pulse wave in Fig. 3.2d creates two 3 x 2 \( \mu \)m intensity spikes separated by a 4 \( \mu \)m null region as depicted in Fig. 3.2e. Each intensity spike creates a 4 x 1 \( \mu \)m trapping region as illustrated in Fig 3.2f. Similarly, Figures 3.2g, 3.2h and 3.2i in the third row illustrate the RF waveform and generation of three intensity spikes and three force hot-spots. Here, the RF waveform has two pulses, each of width 1 ns, separated by 2 ns (Fig. 3.2g). It cuts through the beam at two places, leaving three intensity spikes, each measuring 2 x 2 \( \mu \)m (Fig. 3.2h). These three intensity spikes in turn generate three force hot-spots, each of which can trap particles within a 4 x 1 \( \mu \)m region. Beams with wider spectra can be used to produce higher number of hot-spots along the ellipse of the focus. The maximum force generated per pulse at the hot-spots is ~200 pN for 100 nm
particle radius. By using different waveforms one can extend the number of hot-spots, and control the relative strength and position of optical forces acting on polarizable particles. In these calculations, the rise time of RF pulses is assumed to be >200 ps, which allows the RF bandwidth of the arbitrary waveform generator to be well below 10 GHz.

**FIG. 3.2** Different RF waveforms create different intensity profiles, and hence can be used to produce different 2D force profiles. Figures on the left column (a, d, and g) show RF waveforms used to modulate the intensity. The middle column (b, e, and h) shows intensity profiles corresponding to each RF waveform, and the right column (c, f, and i) shows corresponding force contours arising from each intensity profile. Hot-spots or points of maximum force can be maneuvered over the focal plane by selecting an appropriate RF waveform.
3.2 Polarity Control of Optical Forces

In addition to location control of hot-spots, Fig. 3.3 illustrates how one may also alter the direction of the force at the hot-spots by generating suitable intensity profiles through RF modulation. To achieve this, one needs the ability to change the sign of the intensity gradient. A default super-Gaussian beam profile (Fig. 3.3a) results in a trap located at the beam center (Fig. 3.3c) due to a positive intensity gradient, followed by a negative intensity gradient (Fig. 3.3b). As depicted in Fig 3.3b, a positive gradient (~ +1.5 W/nm) produces a positive force spike (~250 pN), followed by a zero-force region spanning ~5 μm. Then a negative gradient (~ -1.5 W/nm) creates a negative force spike (~ -250 pN). A particle positioned at each of these spikes will experience a force pulling them toward the center, creating an attractive force between the particles. In general, the precise control of DC bias and RF waveform facilitates the generation of arbitrary force polarity and magnitude. Such pulse profiles are particularly useful for bio applications that compress cells. Although a wavelength of 1550 nm presents challenges for bio applications due to absorption in water, the method presented in this paper can be tailored for Ti-sapphire femtosecond pulse lasers and Ytterbium and Erbium doped fiber lasers, along with frequency doubling mechanisms, to operate for all wavelengths from 500 nm up to 1 μm.
FIG. 3.3 Demonstrating polarity control of hot-spots. The green curves on the left column (a, d, and g) show the normalized RF waveform; the middle column (b, e, and h) shows the corresponding modulated power profile for each RF waveform, and the right column (c, f, and i) shows the desired force profiles generated using the applied modulation. Different sequences of positive and negative power gradients result in different arrangements of polarities (i.e. push or pull) of the generated force spikes, creating attractive (top row: a, b, and c), repulsive (middle row: d, e, and f), and other arbitrary (bottom row: g, h, and i) force configurations.
To reverse the force direction, one can modulate the intensity at the beam center and create negative gradients. For instance, a pulsed RF waveform such as the one shown in Fig. 3.3d., can be used to create a power dip in the middle (Fig. 3.3e), essentially producing a negative gradient followed by a positive gradient at the beam center. This results in an inverted force profile (Fig. 3.3f compared to 3.3c), where a negative force spike is followed by a positive one, with a zero-force region in between. In this particular example, we used a 2 ns wide pulsed RF waveform with a peak-to-peak amplitude of 0.5\( V_r \) to create a negative gradient of \( \sim 0.5 \) W/nm followed by a zero gradient and then a positive gradient of \( \sim +0.5 \) W/nm. Fig. 3.3f shows the resulting force profile, which is composed of a negative force spike (\( \sim -190 \) pN) and a positive force spike (\( \sim +190 \) pN), separated by a zero-force region spanning \( \sim 4 \) \( \mu \)m. The configuration in Fig. 3.3f can be used to repel particles away from the center of the beam. Similarly, one can use more complicated RF waveforms to generate compressive and repelling forces or control multiple particles at the same time as shown in Fig. 3.3g, 3.3h and 3.3i. Here, we use the RF waveform shown in Fig. 3.3g to modulate the power spectral density such that the power rises in two discreet steps, creating two successive positive gradients. First, the peak power is increased from 0 W at \( \sim 1541 \) nm to a steady value of \( \sim 0.7 \) W at \( \sim 1543 \) nm, which is maintained until \( \sim 1549 \) nm as depicted in Fig 3.3h. The power profile in Fig. 3.3h starts with a positive gradient of \( \sim +0.35 \) W/nm, which produces a positive force peak of \( \sim +140 \) pN, followed by a zero-force region spanning \( \sim 4 \) \( \mu \)m as shown in Fig. 3.3i. Then, instead of a power fall off as characteristic of regular pulses, the peak power undergoes a second increase from \( \sim 0.7 \) W at \( \sim 1549 \) nm to \( \sim 1.4 \) W at
~1550 nm, with a gradient of ~+0.5 W/nm, followed by a steady region where it remains at 1.4 W until ~1556 nm, before sliding down to zero with a negative gradient of ~-0.47 W/nm as illustrated in Fig. 3.3h. Thus, one obtains an additional positive force peak of ~+150 pN due to the second rise in peak power, followed by a zero-force region of width ~6 μm and a negative spike of ~-150 pN as depicted in Fig. 3.3i.
CHAPTER 4

TWO DIMENSIONAL ELECTRONIC CONTROL

Thus far only one dimensional control of force hot-spots has been proposed and analytically demonstrated. Given the vast range of possible applications, particularly in cell manipulation ad transportation, it is only appropriate to try to extend the STWM technique to a second dimension. It is perhaps not surprising that this can be done with minor modifications to the original setup. In fact, to achieve 2D control, one needs only the additional ability to electronically modulate the beam intensity profile over a 2D plane, i.e. in both $x$ and $y$ directions. This chapter discusses details of one such technique.

4.1 Two Dimensional Spatial Dispersion using a VIPA

The goal of this endeavor is to be able to maneuver force hot-spots over a two dimensional plane – to create and manipulate, for instance, a 2D grid of traps (Fig. 4.5) or any other arbitrary 2D pattern (Fig. 4.6), by gaining control over the 2D force field. One way to create and control a 2D force field is to use a 2D spatial disperser to generate a 2D spectral shower using two
orthogonally oriented 1D spatial dispersers: a diffraction grating and a virtually-imaged phased array (VIPA), as prescribed in literature [40].

The angular dispersion by a VIPA is 10–20 times larger than those of common diffraction gratings, which have a Blaze angle of ~30° [42]. The VIPA employs a thin plate of glass and a semi-cylindrical lens (C lens); the input light is line focused with the semi-cylindrical lens into the glass plate. The collimated light then emerges on the other side of the plate, where the angle of propagation is dependent on the wavelength [42].

A VIPA can be employed to disperse a spectrum in the vertical dimension, while a grating diffracts it in the horizontal dimension, creating a 2D space-wavelength mapping. Through electro-optic modulation in time domain, one is able to manipulate the power spectral density of the spectrum at individual wavelengths, and hence control the 2D intensity landscape – enabling creation of desired intensity gradients that define the 2D force profile. This section presents the analytical framework and sample time-domain waveforms required for generating multiple particle traps, and force configurations for exerting stretching and compressive forces on objects.

4.1.1 Modeling Setup and Theory

The required setup, shown in Fig. 4.1, can be obtained by modifying the original setup proposed in Chapter 2 (Fig. 2.1). In this new setup, the optical tweezer is driven by a modelocked laser with broad bandwidth and high peak power. Working with the same 150 fs pulses with ~20 kW peak power at 50 MHz repetition rate, the laser beam propagates through a dispersive medium with ~200 ps/nm chromatic dispersion to broaden the optical pulse width
to ~4 ns at the full width half max (FWHM) point. Hence, like the original model, the dispersive propagation creates a time-wavelength mapping where 1 nm wavelength separation corresponds to 200 ps separation in time domain. Dispersed pulses are then passed through an electro-optic modulator to manipulate the power spectral density of the laser as desired, by using RF waveforms. The time-wavelength mapped pulses are then coupled to a diffraction grating with 600 lines/mm groove density to create a 1D space-wavelength mapping in the $x$ direction. The diffracted beam is then dispersed in the $y$ direction by an air-spaced VIPA with tilt angle 1.26°, thickness 1.5 mm and surface coating reflectivities 1 and 0.98, to produce a 2D space-wavelength mapping. Finally, an imaging lens and a 60x microscope objective are used to generate a ~25 μm x ~25 μm focal spot.

**FIG 4.1** Proposed experimental setup for 2D electronic control of optical tweezers. A VIPA and a diffraction grating are orthogonally oriented to create a 2D spatial disperser, whose 2D spectral shower can be modulated by arbitrary RF waveforms using the electro-optic modulator, thereby achieving electronic control over the 2D force field.
One may develop an analysis for the setup in Fig. 4.1 by assuming a Gaussian beam approximation. Under this assumption, the intensity distribution after the diffraction grating and the VIPA is given by [43], [44]:

$$I(x, y) = \sum_i \frac{P_i}{(f\lambda_i)^2} \exp \left[ -\left( \frac{2\pi w(x - x_i)}{f\lambda_i} \right)^2 + \left( -\frac{2F^2 y^2}{f^2 w^2} \right) \times \frac{1}{(1 - Rr)^2 + 4(Rr)\sin^2 (k\Delta/2)} \right]. \quad (4.1)$$

In Eq. (4.1), $P_i$ represents the peak power carried by wavelength $\lambda_i$ in the spectrum, $f$ is the focal length of the Fourier lens, $w$ is the spot size of the beam, $x_i \approx G_\beta f (\lambda_i - \lambda_c)$ is the relative position of the first order diffraction peak for the wavelength $\lambda_i$, with respect to the central wavelength $\lambda_c$, $G_\beta = G / \cos (\beta)$ is the effective groove density of the diffraction grating, that is defined as a function of groove density $G$ and the first order diffraction angle $\beta$ [29], $t$ is the thickness of the VIPA, $F$ is the focal length of the cylindrical lens, $\alpha$ is the VIPA tilt angle, $Rr$ is the product of the reflectivities of the two surfaces of the VIPA, $k = 2\pi/\lambda$, and

$$\Delta = 2t\cos(\alpha) - \left( \frac{2\sin(\alpha) y}{f} \right) - \left( \frac{t\cos(\alpha) y^2}{f^2} \right). \quad (4.2)$$

The force depends on the intensity gradient, as mentioned in Chapter 2, and is given by

$$F(r) = \left( \frac{2\pi n_0 b^3}{c} \right) \left( \frac{m^2 - 1}{m^2 + 2} \right) \nabla I(r), \quad (4.3)$$

where $m = n_i/n_0$ is the relative refractive index of the particle (where $n_i$ is the refractive index of the particle and $n_0$ is the refractive index of the surrounding medium), $b$ is the
radius of the particle, $c$ is the speed of light, and $r$ is the radial distance from the beam axis.

### 4.1.2 Modulating the 2D Intensity and Force Profile

The 2D spectral shower produced by the combination of the VIPA and the grating creates a 2D space-wavelength mapping. This mapping, focused onto a $\sim 25 \times 25 \, \mu m^2$ area by the imaging lens and the microscope objective, is shown in Fig. 4.2. By using different RF waveforms, it is possible to manipulate the power profile in accordance with this mapping, to obtain desired intensity gradient and force profiles in 2D space. Any change in the intensity profile in turn creates intensity gradients in the lateral direction. For instance, Fig. 4.3a shows the modulated power profile with 25 pulses of width $\sim 80$ ps each, used to obtain the force profile in Fig. 4.3b, which is a 2D grid of 25 traps, each with $\sim 10$ pN force hot-spots, in a total area of $\sim 25 \times 25 \, \mu m^2$. The numbers 1 through 25 link each power spike in Fig. 4.3a to the corresponding trap in Fig. 4.3b based on the space-wavelength mapping shown in Fig. 4.2. Fig 4.3c shows a close up of each pulse tip in Fig 4.3a, displaying fine details such as rise time and pulse width at the tip. To sustain the depicted rise time of $\sim 26$ ps, an RF bandwidth of $\sim 20$ GHz is needed. Fig. 4.3d illustrates the force contours in each trap shown in Fig. 4.3b.
FIG 4.2 2D Space-Wavelength mapping produced by the 2D spatial disperser in Fig. 4.1 consisting of a VIPA and a diffraction grating.

FIG 4.3 Electro-optic modulation of 2D intensity and force profile using arbitrary RF waveforms, to generate a 2D grid of optical traps (a) Modulated power profile with 25 spikes along the spectrum. (b) Corresponding force profile with a 2D grid of 25 optical traps generated by the power profile in (a) based on the 2D space-wavelength mapping in Fig. 4.2. The marked numbers 1 – 25 link each spike in (a) to the corresponding trap location in (b). (c) Details of each pulse tip in (a). (d) Force contours for each trap in (b).
4.1.3 Maneuvering Traps over a 2D Plane

The technique described above can be taken further to maneuver optical traps over a 2D plane. Based on the correspondence between RF pulses in Fig 4.3a and trap locations in Fig. 4.3b, selecting certain pulses to remain active while turning off others will produce traps in the corresponding locations on the 2D grid. Hence, one may ‘move’ and ‘shift’ the pulses along the spectrum in arbitrary ways to obtain traps in desired locations. This approach can be used, for instance, to move two traps closer to each other or farther away from each other, providing suitable force configurations for compressing, stretching and transporting cells and molecules. This is illustrated in Fig. 4.4. Figs. 4.4a and 4.4b demonstrate how different power profiles can be applied to maneuver a set of optical traps on the focal plane. When pulses a1 and a2 are applied, two optical traps in locations A1 and A2 are produced (Fig. 4.4a). These two traps can be moved to locations B1 and B2 by changing the power pulses to b1 and b2 in Fig. 4.4b. This ability to electronically maneuver optical traps in a 2D plane can be applied in compression, stretching, and transportation of cells and other organic and inorganic particles.
FIG. 4.4 Maneuvering two optical traps from locations A1, A2 to B1, B2. Two pulses a1 and a2 in the power profile in (a) are replaced by b1 and b2 in (b) to move two optical traps in the corresponding force profile closer to each other from A1, A2 in (a) to B1, B2 in (b), respectively.
4.1.4 2D Grid of Traps and Arbitrary Patterns

When it comes to 2D manipulation, holographic optical tweezers are typically limited to a grid of optical traps. While our approach has no such limitation, it is useful to mimic such a grid for existing applications. Using the proposed technique, each trapping region and its associated force spikes take up a minimum area of $7 \times 4 \ \mu m$, meaning a $\sim 1200 \ \mu m^2$ area can accommodate $\sim 45$ traps as shown in Fig. 4.5. In addition to a rigid grid of traps (Fig. 4.5), one can achieve any arbitrary pattern with extended areas of high intensity, leading to bigger force hot-spots, whose location and polarity can be precisely controlled over the entire 2D plane as described above and illustrated in Fig. 4.6. This opens up new possibilities and further prospects stemming from the greater flexibility and control over the entire landscape.

**FIG. 4.5** Using 2D STWM, one can generate the beam profile on the left, which can be used to obtain the 2D grid of optical traps on the right.
FIG. 4.6 An arbitrary 2D force profile (right) may be generated using a corresponding intensity profile (left).
CHAPTER 5

DISCUSSION AND SUMMARY

We have presented a method to achieve electronically controlled force profiles for optical tweezing applications by extending the idea of STWM. The proposed approach can generate a continuous force profile at the focal point where polarity, location and magnitude can be controlled precisely by using time-domain electro-optic modulation. The created hot-spots can be actively tuned to accelerate particles in a particular direction, to maneuver them along the focal plane, and to create more complex scenarios for a wide range of applications, especially in the biosciences.

We also presented an extension of the 1D STWM technique to two dimensions, in order to achieve electronically controlled optical tweezing in two-dimensional space. We have proposed that by using time-domain electro-optic modulation, followed by space-wavelength mapping using a grating and a VIPA, STWM can achieve direct control over the entire continuous 2D intensity landscape, and hence is able to manipulate the location, area, size, and polarity of optical force hot-spots as desired.
One major obstacle that continues to be a challenge in optical manipulation is the diffraction limit when working with nanometer-sized objects. Techniques in plasmon nano-optics exploit surface plasmon resonances supported by metallic nanostructures, to effectively concentrate evanescent fields well beyond the diffraction limit [45–54]. The STWM technique proposed here, however, uses time-stretch to overcome the diffraction limit. Precision of the proposed technique can be enhanced by using a wider RF bandwidth, or by increasing the optical bandwidth of the laser with an appropriate dispersion value. Overall, one needs to consider trade-offs between dispersion, RF bandwidth, spectral width, power requirement, and spatial precision, in order to implement the STWM approach. Future work building up on this study will involve conducting the proposed experiment to corroborate the theory and simulation with experimental results, and further investigation into the capabilities and limitations of the proposed method. In particular, the analysis will be extended by considering scattering forces in the axial direction, which have been ignored in the current model.
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APPENDIX A

DERIVATION OF BEAM INTENSITY PROFILE

A1. Plane Wave

Given a plane wave, beginning with the field profile $E_p$,

$$E_p(x, d) = \text{rect} \left( \frac{x}{d} \right)$$  \hspace{1cm} (A1.1)

where $d$ is the beam size that is defined as the aperture diameter, on passing through a diffraction grating, it acquires the field profile $E_{pi}$:

$$E_{pi}(x, d) = \text{rect} \left( \frac{x - x_i}{d} \right).$$  \hspace{1cm} (A1.2)

After passing through a Fourier lens where the distance between the lens and the grating is equal to the focal length of the lens $f$, the Fourier Transform of the field is taken with the following transformations:

$$x \rightarrow \frac{x}{\lambda f},$$

and

$$\text{rect}(ax) \rightarrow \text{sinc} \left( \frac{x}{a\lambda f} \right).$$
The output field $E_{\text{out}}$ thus becomes:

$$E_{\text{out}} = \text{sinc} \left[ \frac{x - x_i}{\lambda} \left( \frac{d}{f} \right) \right]. \quad (A1.3)$$

Since the intensity $I$ is given by

$$I \left( x, \lambda, \frac{d}{f} \right) = E_{\text{out}}^2,$$  \hspace{1cm} (A1.4)

the intensity profile at the output can be obtained from (A1.3) by using (A1.4) as:

$$I \left( x, \lambda, \frac{d}{f} \right) = \text{sinc}^2 \left[ \frac{x - x_i}{\lambda} \left( \frac{d}{f} \right) \right], \quad (A1.5)$$

where $x_i \approx G_\beta f (\lambda_i - \lambda_c)$ is the relative position of the first order diffraction peak for the wavelength $\lambda_i$ with respect to the central wavelength $\lambda_c$, $G_\beta = \frac{G}{\cos(\beta)}$ is the effective groove density of the diffraction grating, that is defined as a function of groove density $G$ and the first order diffraction angle $\beta$. For a beam with wavelength $\lambda$, and incident angle $\alpha$, the first order diffraction angle $\beta$ is given by the grating equation $\sin(\alpha) + \sin(\beta) = \lambda G$ which reduces to $2\sin(\beta) = \lambda G$ when $\alpha = \beta$ in a Littrow configuration [33].
A2. Gaussian Wave

Given a Gaussian wave, beginning with the field profile $E_g$, 

$$E_g(x, w) = \exp \left( -\frac{x^2}{2w^2} \right), \quad (A2.1)$$

where $w = \frac{d}{2\sqrt{\ln 2}}$, $d$ is the beam size that is defined as the spatial full width at half maximum (FWHM), on passing through a diffraction grating, it acquires the field profile $E_{gi}$:

$$E_{gi}(x, w) = \exp \left( -\frac{(x - x_i)^2}{2w^2} \right). \quad (A2.2)$$

After passing through a Fourier lens where the distance between the lens and the grating is equal to the focal length of the lens $f$, the Fourier Transform of the field is taken with the following transformations:

$$x \rightarrow \frac{x}{\lambda f},$$

and

$$\exp(ax^2) \rightarrow \exp \left( -\frac{\pi^2 x^2}{a(\lambda f)^2} \right).$$
The output field $E_{\text{out}}$ thus becomes:

$$E_{\text{out}} = \exp \left[ -\frac{\pi^2 (x - x_i)^2}{2 \ln 2 \ \lambda^2} \left( \frac{d}{f} \right)^2 \right]. \quad (A2.3)$$

Since the intensity $I$ is given by

$$I (x, \lambda, \frac{d}{f}) = E_{\text{out}}^2,$$  \hspace{1cm} (A2.4)

the intensity profile at the output can be obtained from (A2.3) by using (A2.4) as:

$$I (x, \lambda, \frac{d}{f}) = \exp^2 \left[ -\frac{\pi^2 (x - x_i)^2}{2 \ln 2 \ \lambda^2} \left( \frac{d}{f} \right)^2 \right]. \quad (A2.5)$$

where $x_i \approx G_\beta f (\lambda_i - \lambda_c)$ is the relative position of the first order diffraction peak for the wavelength $\lambda_i$ with respect to the central wavelength $\lambda_c$, $G_\beta = \frac{G}{\cos(\beta)}$ is the effective groove density of the diffraction grating, that is defined as a function of groove density $G$ and the first order diffraction angle $\beta$. For a beam with wavelength $\lambda$, and incident angle $\alpha$, the first order diffraction angle $\beta$ is given by the grating equation $\sin(\alpha) + \sin(\beta) = \lambda G$ which reduces to $2\sin(\beta) = \lambda G$ when $\alpha = \beta$ in a Littrow configuration [33].


APPENDIX B

DERIVATION OF GRADIENT FORCE

The force exerted on a particle (for instance, Particle 1) of unit charge in an electromagnetic field is known as the Lorentz Force and is expressed in terms of the Electric and Magnetic fields as:

\[ F_1 = q \left( E_1 + \frac{dx_1}{dt} \times B \right) \]

Here, \( F_1 \) represents the force exerted on Particle 1 of unit charge \( q \), \( E_1 \) is the Electric field, \( dx_1/dt \) is the velocity of the particle's motion, and \( B \) is the Magnetic field.

If the diameter of the trapped particle is significantly smaller than the wavelength of light, the conditions for Rayleigh scattering are satisfied and the particle can be treated as a point dipole, i.e., the dipole approximation. For a point dipole, the distance between its two ends, \( x_1 - x_2 \), with opposite charges, is infinitesimal and hence the Lorentz force \( F \) can be expressed as:

\[ F = q \left( E_1 (x, y, z) - E_2 (x, y, z) + \frac{d(x_1 - x_2)}{dt} \times B \right) \]

\[ = q \left( E_1 (x, y, z) + ((x_1 - x_2) \cdot \nabla) E - E_1 (x, y, z) + \frac{d(x_1 - x_2)}{dt} \times B \right). \]

Here, \( E_1 \) and \( E_2 \) stand for the Electric field at the two ends of the dipole.
\( \mathbf{E}_1 \) cancels out and on multiplying by \( q \), this simplifies to:

\[
\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} + \frac{d\mathbf{p}}{dt} \times \mathbf{B} = \alpha \left[ (\mathbf{E} \cdot \nabla) \mathbf{E} + \frac{d\mathbf{E}}{dt} \times \mathbf{B} \right]
\]

where \( \mathbf{p} = q(x_1 - x_2) = \alpha \mathbf{E} \) is the dipole moment, which is assumed to be linear.

The proportionality constant \( \alpha \) constitutes the following:

\[
\alpha = 4\pi n_0^2 \varepsilon_0 r^3 (m^2 - 1)/(m^2 + 2).
\]

Here, \( r \) is radius of particle, \( n_0 \) is the refractive index of medium, and the relative refractive index \( m = n_{\text{particle}}/n_0 \).

The following two identities are of interest at this point:

1. \((\mathbf{E} \cdot \nabla) \mathbf{E} = \nabla \left( \frac{1}{2} \mathbf{E}^2 \right) - \mathbf{E} \times (\nabla \times \mathbf{E})\]
2. \(\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\)

The first one is (1) Vector Analysis Equality and the second one is (2) One of Maxwell’s Equations.

Using these identities in unison with the expression for force above, one obtains,

\[
\mathbf{F} = \alpha \left[ \frac{1}{2} \nabla \mathbf{E}^2 - \mathbf{E} \times (\nabla \times \mathbf{E}) + \frac{d\mathbf{E}}{dt} \times \mathbf{B} \right] = \alpha \left[ \frac{1}{2} \nabla \mathbf{E}^2 - \mathbf{E} \times \left( -\frac{d\mathbf{B}}{dt} \right) + \frac{d\mathbf{E}}{dt} \times \mathbf{B} \right] = \alpha \left[ \frac{1}{2} \nabla \mathbf{E}^2 + \frac{d}{dt} (\mathbf{E} \times \mathbf{B}) \right].
\]
The second term in the above equation is 0 because $\mathbf{E} \times \mathbf{B}$ is a scale factor of the Poynting Vector, the power per surface area, and we assume that the laser power remains constant.

The square of the magnitude of the electric field is equal to the intensity of the beam as a function of position.

Therefore:

$$F = \frac{1}{2} \alpha \nabla E^2 = \frac{2 \pi n_0 r^3}{c} \left( \frac{m^2 - 1}{m^2 + 2} \right) \nabla I(r),$$

Here $\nabla I(r)$ represents the intensity gradient.

This indicates that the force on the dielectric particle, when treated as a point dipole, is proportional to the gradient along the intensity of the beam. The gradient force described here tends to attract the particle to the region of highest intensity.