Runaway-electron model for x-ray emission in pinched discharges

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We consider a simple model in which a small fraction of runaway electrons produce the often-observed x-ray emission in pinches. We show the turbulent collision frequency scales as \( \nu^{-1} \), from fairly general arguments. Simple scalings result, linking the number of runaways with their energy. Fitting to a recent experiment gives a pinch field exceeding the applied field by \( \sim 400 \). Future experiments can check the scaling without knowing the pinch electric field.

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Hard x rays are often observed near or in pinched discharges. Bursts of x rays appear when the pinched current peaks. Discharge voltages of 10–20 kV yield x rays of several hundred keV. Very small pinches (\( \leq \) mm) are seen, in which typical plasma parameters are electron densities \( n \leq 10^{21} \) cm\(^{-3} \), temperature \( T_e \sim 10 \) keV, and with azimuthal magnetic fields \( B \sim 10^7 \) G.\(^{-1} \). Acceleration of electrons to produce these x rays may occur when the conductivity of the pinched plasma drops abruptly in a few nsec, giving a quasistationary turbulent electric field. Estimates of the pinch phenomena often assume that the entire electron distribution is accelerated significantly.

This note explores briefly the notion that only a small portion of the dense plasma is accelerated, as is typical in a “runaway” distribution. The electric fields generated by the micropinches are difficult to calculate, so whether they are powerful enough to accelerate most or all of the electrons remains to be seen after a detailed (probably numerical) analysis. Here, we take the view that it is not necessary to assume most of the plasma is accelerated by these huge fields to explain the observed number and energies of the x rays or the presence of \( \sim 200 \)-keV ions transported oppositely to the x-ray-producing electrons. A simple scaling of number and energy results, which agrees with a recent experiment and may be used to scale to future observations.

In fitting this experiment, pinch electric fields exceeding the applied field by \( \sim 400 \) are required to give enough acceleration to produce the observed energy and number of x rays.

Consider a Maxwellian electron distribution of thermal velocity \( v_e \), drifting along z at velocity \( v_z \) in an electric field \( E \). For slow drift \( v_z < v_e \). Each electron moves in one-dimensional fashion down the throat of the pinch, obeying

\[
\frac{dv}{dt} = \frac{e}{m} E - \nu v(v).
\]

We can find the collision frequency \( \nu(v) \) for scattering from drift-generated turbulence if the electrons are moving faster than the turbulent waves. This is a reasonably typical circumstance. The electrons undergo a deflection to the side \( \delta v \) in a time \( \delta t \sim (kv)^{-1} \). The angular deflection \( \delta \vartheta = \delta v/v \) is related in scaling by

\[
\nu(v) \sim (\delta \vartheta)^2/\delta t \sim v^{-3}.
\]

This gives a scaling of

\[
\nu(v) = v^*(v/v_e)^3, \quad v > v_e,
\]

where \( v^* \) is the turbulent scattering frequency of the bulk of the electrons, \( v < v_e \). Scaling of \( v \) with \( v_e \) is suggested by the arguments of Kaplan and Tsyutovich.\(^4\) This \( v^{-3} \) scaling can be regarded simply as coutomb-like with scaling factors to account for the turbulence. Using Eq. (3) in Eq. (1), the runaway velocity is

\[
v_r = (m v^* v_e^3/eE)^{1/2}
\]

and the steady-state drift velocity of the bulk is

\[
v_s = eE/mv^*.
\]

A sketch of the electron distribution, Fig. 1, shows the runaway regime, \( v > v_r \). Using Eq. (4),

\[
v_r = v_s(v/v_e)^{1/2}.
\]

For a Maxwellian distribution of \( N \) electrons, the fraction of runaways is

\[
N_r/N = \exp(-v_r/v)
\]

The energy acquired in passing through a pinch of length \( L \) in essentially free acceleration for electrons with \( v > v_e \) is

\[
W_f \equiv 5E_{10}L \quad (\text{keV}),
\]

where \( E_{10} \) is the pinch electric field in units of 10 keV and \( L \) in cm. To proceed further and connect Eq. (7) to Eq. (8), we must assume a form for \( v^* \), a subject fraught with controversy. We shall side with tradition and use the Sagdeev form

\[
v^* = \frac{1}{100} \alpha(T_0/T_1)(v_e/v)\nu_{pe},
\]

where \( \nu_{pe} \) is the electron plasma frequency and \( \alpha \) is an adjustable number; \( \alpha = 1 \) gives the usual Sagdeev magnitude. Using Eq. (9) in Eq. (5), we than eliminate \( v^* \) from Eq. (7) and find

\[
W_f \equiv 5E_{10}L \quad (\text{keV}).
\]

FIG. 1. Assumed drifting electron distribution, moving at speed \( v_n \), with thermal spread \( v_e \). Electrons with \( v > v_n \) run away. The collision frequency appears as scaled in the upper dashed line.
\[ N_z = \exp \left( -5 a \frac{\left( \frac{n_{1a} T_{z/\gamma}}{E_{10}^{1/2}} \right)^{1/4}}{E_{10}^{1/2}} \right), \]

where we assumed \( T_a = T_{z/\gamma} \) in keV and \( n_{1a} \) is the plasma density in units of \( 10^{18} \text{ cm}^{-3} \). Equations (8) and (10) relate number and energy of the runaways. When they strike a target we should expect \( N_z \), photons of maximum energy \( \sim W_{10} \). Most experiments give some fix upon \( L_z, T_{z/\gamma}, \) and \( n_{1a} \). The pinch field \( E_{10} \) depends on detailed dynamics of the pinch; certainly, it is bounded from below by the applied field. A separate dynamical model giving \( E \) could be folded into Eqs. (8) and (10) to give a verifiable scaling.

Note that Eq. (8) also gives approximately the energy of ions accelerated in the direction opposite the electrons. These have been observed.\(^3\)

In a recent experiment\(^2\), micropinches produced \( \sim 200 \text{ keV} \) oppositely streaming ions. Photons were seen from the pinch itself for shots with the lowest applied electric fields. Higher-energy shots gave radiation from fast electrons escaping the pinch and striking the anode. The x-ray fluence indicates the order of magnitude of \( N_z \sim 10^6 \). In a typical micropinch of length \( L = 1 \text{ mm} \) and radius \( 0.3 \text{ mm} \), there are \( N = 3 \times 10^5 \) electrons if \( n_{1a} = 10^8 \), as observed. Then, from Eq. (10)

\[ \frac{N_z}{N} = 3 \times 10^{-10} = \exp \left( - \frac{28 a T_{z/\gamma}^{1/4}}{E_{10}^{1/2}} \right), \]

and we require, for \( W_{10} = 200 \text{ keV} \), \( E_{10} \gtrsim 400 \). From Eq. (11) we find

\[ E_{10} \gtrsim 6.4 a^{1/2} T_{z/\gamma}^{1/2} \gtrsim 400. \]

Thus, for \( T_{z/\gamma} \gtrsim 4 \) we must have \( a^2 \gtrsim 31 \). This is a high resistivity, reflecting the highly pinched state in which the effective \( E \gtrsim 4 \text{ MV/cm} \). This field gives the ion energies which are also observed.

This treatment relies on several assumptions, of course: a Maxwellian distribution, a simple scaling of the collision frequency, and the existence of anomalous resistivity. It yields the simple scalings of Eqs. (8) and (10), which can be tested in future experiments. Further observables, such as the detailed energy spectrum of the runaway electrons which produce the x rays, can be calculated. The point of this approach is that, given a theoretical model which predicts the pinch field \( E \), we then can check if a simple runaway model works. Indeed, mutual consistency of Eqs. (8) and (10) over a range of parameters can check this assumption without knowing \( E \), since if the inequality in Eq. (8) is taken to be a useful equality, \( E \) can be eliminated from the scaling.

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Laser heating and magnetic compression of plasma in a fast solenoid

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A low-\( B \) plasma column a few mm in diameter by 22 cm in length is heated by an axially directed CO\(_2\) laser to a high-\( B \) state in a fast rising solenoidal magnetic field. Successful heating depends on proper timing between the laser pulse and rising field. Typical conditions attained are a line energy density of \( 6 \text{ J/cm} \), \( T \gtrsim 40 \text{ eV} \), and \( n_{1a} \sim 3 \times 10^{18} \text{ cm}^{-3} \), with conditions quite uniform along the length. The heating suppresses instabilities which appear under certain conditions in the non-laser-heated case.

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We describe an experiment designed to study the interaction of a CO\(_2\) laser pulse with an existing plasma in a fast rising solenoidal magnetic field. For the electron densities and length scales involved, refraction of the beam by the plasma is an important consideration. The purposes of the investigation are thus (a) to determine the conditions under which the beam is refracted towards the plasma tube axis ("beam trapping"), as in a light pipe, and (b) to investigate the plasma conditions which can be attained by a combination of laser-energy absorption and magnetic compression.\(^1\) Earlier work on laser-heated \( \theta \)-pinches has been carried out either at low laser energies\(^4\) or with very short high-\( B \) columns for different purposes.\(^4\) The length (22 cm) and bore (3 cm) of the 16-turn helical solenoid used in the present work permits good diagnostic access but precludes studies of certain phenomena which can only be addressed in a longer magnet such as that described in the accompanying paper.\(^5\)

A schematic of the experiment is shown in Fig. 1. The magnetic field rises to a peak of 100 kG in 4.2