Submitted to Journal of Applied Physics

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January 1986
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*This was supported by the Office of Energy Research, Office of Basic Energy Sciences, U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Beam end erosion

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ABSTRACT

The time evolution of the line charge density and energy at the ends of an initially rectangular ion bunch due to the space charge of the bunch are measured and found to be in good agreement with a one-dimensional dynamical theory. The ends erode at the space charge wave speed into the bunch, and the end particles move at twice that speed away from the bunch.

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The longitudinal space charge fields of a bunched beam inside a conducting pipe cause the ends of the bunch to eventually move outwards and erode the bunch in the absence of restoring forces. While there is an infinite number of possible bunch-shape longitudinal-energy-distribution combinations and their subsequent evolutions, a particularly interesting and relevant case is the evolution of a bunch which starts with a monoenergetic distribution, constant line density and abrupt ends. This particular case has an analytic solution, as detailed below, for a one-dimensional model, which is a good approximation for the real 3-dimensional bunch emerging from the SBTE machine. Because the equations of motion are time reversible, this case also is relevant for obtaining a rectangular power pulse onto a heavy ion driven (HIF) inertial confinement fusion target. The SBTE machine consists of a Cs gun, a matching section, and an electrostatic quadrupole focused transport line which is used to study transverse focusing in the space charge dominated limit. The transverse focusing experiments have been performed on the constant energy and constant intensity undisturbed interior portions of a bunch, away from the ends, and in the design stages of the experiment calculations and computations were made to estimate the bunch end erosion in order to determine the location of the undisturbed beam. While the distention of current pulses in the experiment has long been evident, the predicted accompanying energy changes at the bunch ends could only be measured recently, after the installation of an electrostatic energy analyzer at the exit of the transport line.
The most commonly used approximation in describing a drifting bunch for HIF is that it is monoenergetic at any particular longitudinal position and time, and that the space charge fields are well approximated by either the longitudinal electric field at \( r = 0 \) or the somewhat lower average electric field acting over the beam cross section. The real bunch, of course, has a variation from \( r = 0 \) to the "edge" depending on its transverse distribution, and, in the present case of transport in an electrostatic quadrupole lattice, it has an additional variation corresponding to the electrostatic potentials within the lattice. These bunches differ from bunches in r.f. accelerators in being colder or more monoenergetic at any location within the bunch because of the absence of synchrotron oscillations, leaving space charge forces dominant for longitudinal dynamics. In the present experiment with a Cs\(^+\) beam, the potential due to the space charge of the beam is about 600 volts, and the quadrupole potential is 7250 volts, which reduces quadratically to a maximum of about ±3000 volts at the edge of the beam. The nominal beam energy is 120 keV and the beam current is 12 mA. Ordinarily, when the voltage in a gun is turned on and off, one would expect transients in energy and current. These transients are the subjects of other studies,\(^{11,12}\) are different at the two ends of the pulse, and are significant for the HIF program. The charge contained within the gun in the steady state amounts to the current times the steady state transit time of about 600 ns, and thus is sufficiently large to generate a tail of some microseconds duration with large energy and current modulations. In the SBTE setup, there is no convenient way of measuring these initial conditions; however, the matching section between the gun and the lattice serves as an effective energy filter with only about a 5\% bandwidth in energy, thereby clipping off much of the bunch end transients. This explanation is consistent with the observed time evolution of a low current (attenuated after the gun) pulse, as shown in Fig.1(a), where the current is 70 \( \mu \)A. If it were possible to measure very low current beams, then the initial conditions could be inferred from the beam at the exit of the transport line; however, the smallest
experimentally available and measurable current still has sufficiently large space charge fields to elongate the bunch. The fall time of the low current bunch of 600 ns is near the calculated values for the evolution of a rectangular pulse. That is, where the full intensity beam tail elongates to about 5 μs, as seen in Fig. 1(b), the low intensity beam tail elongation, proportional to $\sqrt{g\lambda}$ as given below, would be about 600 ns when account is taken of the smaller diameter of the low current beam. An initial energy modulation of more than 4% would lead to a noticeably longer tail; this, together with the much larger measured energy modulations on the full intensity (12 mA) beam allow us to approximate the starting conditions as constant energy and line density with abrupt ends.

The one-dimensional model consists of the equation of continuity,

$$\frac{\partial \lambda}{\partial t} + \frac{\partial \lambda v}{\partial x} = 0 ,$$

(1)

the force equation,

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{e\epsilon}{M}$$

(2)

and the "long wavelength" field equation,

$$\epsilon = -g \frac{\partial \lambda}{\partial x} .$$

(3)

Here $\lambda$ is the line charge density, $v$ is velocity in the beam frame, and

$$g = \frac{1}{4\pi\epsilon_0} \left( \frac{1}{2} + 2 \ln \frac{b}{a} \right)$$

(4)
is the long-wavelength geometrical factor. A wave speed for small amplitude disturbances $c = \sqrt{e g \lambda_0 / M}$ is readily determined from these equations, where $\lambda_0$ is the undisturbed line charge density. The solution for the initially square profile in energy and line density is then found to be

$$\frac{\lambda}{\lambda_0} = \begin{cases} 
1 & x < -c_s t \\
\left(\frac{2}{3} - \frac{x}{3c_s t}\right)^2 & -c_s t < x < 2c_s t \\
0 & x > 2c_s t 
\end{cases}$$

(5)

$$\frac{\nu}{c_s} = \begin{cases} 
0 & x < -c_s t \\
\frac{2}{3} \left(1 + \frac{x}{c_s t}\right) & -c_s t < x < 2c_s t 
\end{cases}$$

(6)

Here, $x$ and $v$ are measured backward in the beam frame. The analytical results given above are plotted in Fig. 2.

The experimentally measured results are shown in Fig. 3(a) and 3(b), along with the analytical results, where the conversion from the quantities defined in the beam frame has been made to conform with measurements at one fixed location in the lab frame.

The rather good agreement between the two sets of curves should not be taken to mean that we attach high significance to this circumstance. There are several known limitations to the one-dimensional approximation, such as implicitly assuming that the space-charge modulations are line density modulations rather than envelope modulations, that all particles respond to the average force, and that the force is well represented by $a\lambda/ax$. At the start of the transport line $\lambda$ is close to a step function, and the approximation greatly overestimates the longitudinal fields, which for a real bunch do not exceed the radial fields. Another prevalent approximation is that in highly
space-charge depressed transport the transverse space charge is just balanced by the restoring focusing fields. Obviously both approximations can not be right, because a longitudinal density modulation then would automatically show up as a change in transverse dimension. The ends of the bunch would neck down as they elongate, and the waves on the bunch would have the appearance of arterial pulses instead of sound waves in a pipe. As the ends of the beam elongate, the approximation with a constant $g$ therefore underestimates the forces. Insofar as changes in beam radius affect the electric field only logarithmically, in $g$, and most of the impulses at the bunch ends occur early in time, before the beam radius has changed significantly, the approximation is reasonable. It is fortunate that the simplest approximation provides such good agreement.

ACKNOWLEDGEMENTS

This was supported by the Office of Energy Research, Office of Basic Energy Sciences, U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Fig. 1a. Current versus time at the exit of the 13 meter long transport line for a low intensity beam pulse obtained with an attenuator immediately after the gun. This shows a 600 ns fall time. Vertical scale: 100 μA/div.; horizontal scale: 1 μs/div.

Fig. 1b. Current versus time for a high intensity beam with identical gun conditions but without an attenuator. This shows a 5 μsec fall time at bunch end due to space charge expansion in the absence of longitudinal focusing. A more abrupt bunch end in the presence of longitudinal focusing is displayed for reference. Vertical scale is 2 mA/div. and horizontal scale is 0.5 μs/div.
Fig. 2. Plot of expressions (5) and (6); the dashed lines indicate the initial line charge density ($\lambda/\lambda_0$), and velocity ($v/2c_s$), and the solid lines indicate these quantities at any subsequent time. Distance from the initial bunch end is measured in terms of space charge wave speed ($c_s$) multiplied by time.
Fig. 3a. Comparison of the experimentally measured current fall with the analytic prediction.
Fig. 3b. Comparison of the experimentally measured beam energy in the bunch with the analytic prediction.