Asset Prices and the Fundamentals: A $Q$ Test

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Abstract

This paper presents a test of the relationship between financial asset values and the fundamentals. The test is an indirect test based on Tobin's (1969) $q$ model of investment. The advantage of this test is that it avoids conditioning on a specific model of equilibrium returns by substituting an observable proxy, capital, for the unobservable fundamental value of the firm. Section 1 of this paper shows that under mild restrictions the value of capital and the financial value of the firm should move together even if marginal or average $q$ does not accurately describe the actual investment decision rule. Average $q$ is a mean reverting process. Section 2 of the paper presents the results of tests for violations of the mean reversion restriction. The tests cannot reject the unit root null hypothesis for a long annual sample from 1926 through 1988 or a quarterly post WWII sample. Section 3 examines the power of the tests with Monte Carlo simulations. The simulations show that the tests have good power until the serial correlation in the series gets quite high. The tests cannot distinguish between no mean reversion and very slow mean reversion. The empirical evidence presented in this paper provides fairly strong evidence against the hypothesis that the financial and real values are closely linked in the short run or even in the medium run.

Key words: investment, fundamentals, Tobin's $q$

JEL Classification: E44, C15
firms should move together even if marginal or average q do not accurately describe the actual investment decision rule. Average q is a mean reverting process.

Section 2 of the paper presents the results of tests for violations of the mean reversion restriction. The tests cannot reject the unit root null hypothesis for a long annual sample from 1926 through 1988 or a quarterly post WWII sample. The mean reversion restriction is robust to specification error since it imposes very little structure on the model, but the test could have low power because so little structure is imposed. Section 3 examines the power of the tests with Monte Carlo simulations. The simulations show that the tests have good power until the serial correlation in the series gets quite high. The tests cannot distinguish between no mean reversion and very slow mean reversion.

The unit root tests and the Monte Carlo simulations indicate that the low frequency movements in the value of capital and the value of financial assets are not highly correlated. Financial and real value do not move closely together.

Section 1: The Restriction

Tobin's q links the firm's production process to the payoffs. Direct estimation of a q model of investment requires a very detailed specification which includes the firm's production process and a model of the discount factor. But the implication that q is a mean reverting process can be tested without committing to a particular specification.
Definition

Let $V_t$ denote the current price of an asset. Define the fundamental value of the asset as,

$$V^*_t = E_t \left[ \sum_{j=1}^{\infty} D_{t+j} d_{t+j} \right]$$  \hspace{1cm} (1)

the conditional expectation of the discounted asset payoffs. Here $D$ denotes the (stochastic) discount factor and $d$ the asset's payoff.

Direct Tests

The relationship between stock prices and their fundamental value has been extensively tested. A crucial step in calculating the fundamental value is the specification of the unobservable discount factor. The payoff sequence for stocks, dividends, is observable. Specifying the discount factor is tantamount to specifying an equilibrium model of returns. The consumption-capital asset pricing model makes the discount factor a function of consumption growth, the capital asset pricing model makes the discount factor depend on the covariance between a security's return and the market return, and the random walk model assumes a constant discount factor. Most applications of the popular volatility tests approximate the discount factor with a constant, eg, see the surveys by Cochrane (1991b), LeRoy (1991), and West (1988). The test results are very sensitive to the specification of the discount factor. For example, see Cochrane (1991a) and Craine (1993).
Tobin's q

The financial value of the firm is the expected discounted value of the payoff sequence, as in equation 1. q models of investment link the payoff sequence, \(<d>\), to the firm's production process. The value of capital is the expected discounted value of capital's contribution to the payoff sequence.

Examples

Consider a simple linearly homogeneous production process in capital, \(K\), and labor, \(L\),

\[ f(K_{t-1}, L_t) = f_K K_{t-1} + f_L L_t \]  \hspace{1cm} (2)

Assume labor receives its marginal product in wage payments and investment is \(K_{t-1} - K_t\). Then net profits are,

\[ d_{t+1} = f_K K_t - (K_{t+1} - K_t) \]  \hspace{1cm} (4)

capital's contribution to output minus the value of investment. The value of the firm's capital equals,

\[ K_t = E[D_{t+1} (d_{t+1} + K_{t+1})] = E \sum_{j=1}^{\infty} D_{t+j} d_{t+j} = V_t \]  \hspace{1cm} (3)

the expected discounted value of the payoffs to capital which is the value of the firm.

In this example Tobin's q always equals one.
Hayashi (1982) presented a deterministic q model of investment with a cost to adjusting capital so that capital adjusts slowly and q does not always equal one. Hayashi’s model gives a closed form solution where the investment capital ratio is an increasing function of deviations of q from its long run equilibrium value,

\[
\frac{I}{K} = g(q - 1)
\]  

(5)

Hayashi's investment equation has an intuitive "error correcting" form. If q exceeds its long run equilibrium value, investment increases; if q falls short of its long run equilibrium value, investment decreases. q is a mean reverting process.

A Generalized Mean Reversion Relationship for Average q

Almost any stochastic growth model with an interior solution satisfies the restriction that q is stationary, eg, see Brock (1982) or Prescott and Merha (1980). Essentially the economic assumption of a convex production set restricts the ratio of earnings to capital to a stationary stochastic process. Specification of the earnings-capital relationship gives the link from capital to the financial value of the firm. This paper substitutes statistical restrictions for an explicit model.

Assumptions:

A1. The current financial value of the firm is the expected present value of the earnings,

\[
V_t = E_t[D_{t+1}(d_{t+1} + V_{t+1})] = E_t[\sum_{j=1}^{\infty} D_{t+j}d_{t+j}],
\]
Define:

\[ D_{t+j} = \beta^j c_{t+j}, \quad 0 < \beta < 1 \]

\( c_{t+j} \) as a stochastic element of the discount factor, e.g., the marginal intertemporal rate of substitution for consumption \( U_{c_{t+j}}/U_{c_t} \).

\( g_{t+j} \equiv \frac{K_{t+j+1}}{K_{t+j}} \), as one plus the growth rate of capital. Notice that,

\[ \frac{K_{t+j}}{K_t} = \prod_{i=0}^{j} g_{t+i}, \]

one plus the growth of capital over \( j \) periods equals the compounded growth factor.

\( p_{t+j} = d_{t+j}/K_{t+j} \), as the earnings capital ratio.

A2. \( c, p, \) and \( g \) are a jointly stationary stochastic process.

Now dividing A1 by the value of capital gives \( q_t \),

\[ \frac{V_t}{K_t} = q_t = E_t[D_{t+1} \mid \frac{K_{t+1}}{K_t}, \frac{d_{t+1} + V_{t+1}}{K_{t+1}}]. \]

Substituting the definitions, and solving the forward difference equation,

\[ q_t = \sum_{i=1}^{\infty} \beta^i E_t[\prod_{j=0}^{i} g_{t+j} c_{t+j} p_{t+j}], \]

expresses average \( q \) as the expected present value of the earnings to capital ratio weighted by capital growth.

These statistical restrictions relax the explicit economic assumptions made, e.g., by
Hayashi, so that the environment is stochastic and firms can earn rent (because of a concave technology or earn monopoly profits—so marginal q does not equal average q), and they permit temporary disequilibrium, or time-to-build or delivery lag specifications, or tax wedges, and "noise trading" in financial markets. In addition, the econometrician's data set can contain measurement error. The statistical assumptions rule out specifications that depend on calendar time.

**Theorem (White p42 Theorem 3.35)**

Let \( h \) be an \( F \)-measurable function onto \( R^k \) and define \( Y_t = h(X_t, X_{t+1}, \ldots) \) where \( X_t \) is (an) \( n \times 1 \) (vector). (i) If \( <X_t> \) is stationary, then \( <Y_t> \) is stationary. (ii) If \( <X_t> \) is stationary and ergodic, then \( <Y_t> \) is stationary and ergodic.

Given assumptions A1 and A2 the theorem assures that average \( q \) is a stationary stochastic process for bounded expected present value functions defining \( q \).

2: Empirical Evidence

Data

The test requires data on the financial value of the "firm" and the value of physical capital. Real capital and value of financial assets are harder to measure accurately than the S&P index, but stationary measurement error will not invalidate the tests.

The Commerce Department publishes an annual capital stock series starting in
Figure 1

Capital and the S&P500

- COMNK
- S&P/CP1
1925 and a quarterly investment series for the post WWII period. This study uses the constant cost net nonresidential capital stock series for an annual series and forms a quarterly series for the post WWII period using investment to interpolate the annual data. The S&P500 index is deflated by the CPI to form a real annual financial series, and a quarterly financial value series is constructed for the post WWII period that includes debt.\(^1\)

Figure 1 plots the annual series (normalized by their sample means). The figure shows the renowned volatility of financial markets accentuated by the '29 and '87 crashes and the extended '60's boom. The capital series displays a fairly smooth upward trend after the Great Depression and WWII. Low frequency movements dominate both series, but the low frequency movements are not highly correlated.

Insert Figure 1

Table 1 presents some summary statistics for the "growth rates" \((\log(x_t) - \log(x_{t-1}))\) of the series. The growth rates series pass standard stationary tests. The growth rates of the financial series have very low serial correlation and are extremely volatile. The growth rates of the capital series have high serial correlation and are extremely smooth.

\(^1\)The financial value of the firm is equity plus debt. The annual series only includes equity. See the appendix for a complete data description.
Table 1

Sample Statistic for Univariate Growth Rates

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The data do not appear to contradict the assumptions of the theorem.

Mean Reversion Test Results

This portion presents the results from four popular tests for unit roots: the Dickey Fuller test (DF) and the Augmented Dickey Fuller (ADF) regression suggested by Engle and Granger (1987), Stock and Watson’s (1988) tests for common trends (CIOC and CIOD), and the variance ratio (VR) tests on long interval differences suggested by Cochrane (1988). A unit root violates the mean reversion restriction.

Table 2 reports the significance levels at which the null can be rejected taken from Table 8.5.2 in Fuller, Table II in Granger and Engle, and Table I in Stock and Watson. The variance ratio test has an asymptotic normal distribution. $q_a$ denotes the logarithm of the annual series and $q_Q$ the logarithm of the quarterly series.
Table 2
TEST RESULTS

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<td>&lt;10%</td>
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<td>&lt;15%</td>
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<td>&lt;10%</td>
<td>&lt;15%</td>
<td>&lt;15%</td>
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None of the tests can reject the unit root null hypothesis at moderate significance levels, and these results are robust. One gets similar results substituting (1) nominal series (so spurious serial correlation introduced by the deflators does not seem to be the culprit) or, (2) the value weighted NYSE index (so the results are not sensitive to the particular series), or (3) the level rather than the log of $q_s$ or (4) if a cointegrating regression is estimated for $V$ and $K$.

3 Power

Many suspect that unit root and cointegration tests have low power, so a failure to reject the null hypothesis may not provide much evidence in its favor. The power of the test against a specific stationary ARMA(1,1) alternatives2 was examined using Monte Carlo simulations. Estimates of the ARMA(1,1) specification yield a large AR coefficient, .8 for the annual data and .96 for the quarterly, and a small MA coefficient around .2.

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2 A number of recent papers examine the small sample distribution of cointegration and unit root test statistics, eg, see Blaugh (1988), or Schwert (1987, 1989). Schwert argues economic data frequently contain moving average errors.
FIGURE 2

Power of Dickey-Fuller Test (10%)

Probability of Rejection

ar = .7

ar = .8

ar = .9

MA parameter

0 -0.5 -0.3 -0.1 0.1 0.3 0.5 0.7 0.9
FIGURE 3

Power of Augmented Dickey-Fuller (10%)

Probability of Rejection

MA Parameter

ar = .7

ar = .8

ar = .9
The Monte Carlos use data generated from an ARMA(1,1) process to calculate the power of the unit root tests with sample sizes of one hundred. Figure 2 plots the probability of rejection (fraction of rejections in 1000 trials) at the 10% level for the Dickey-Fuller test as a function of the MA parameter. Figure 3 plots the probability of rejection for the Augmented Dickey-Fuller tests.

INSERT FIGURES 2&3

The Monte Carlos confirm suspicions that it's hard to separate a unit root from a large low frequency component, but the tests have surprisingly good power against moderate low frequency movements. When the AR coefficient reaches .9 the tests only reject the unit root null about 25% of the time, but the Dickey-Fuller test rejected 80% of the time for AR coefficients up to .8.³

After the empirical work in this paper was completed Blanchard, et al (1993) published a paper examining the relationship between stock valuation and investment. The appendix to their paper gives an annual q series which is longer than the q series used in this paper. The Blanchard, et al series starts in 1900 and includes the value of debt in the financial value of the firm. Augmented Dickey-Fuller tests reject the unit root null at the 5% level for their series using the full sample, but fail to reject the null at the 10% level using the a truncated sample starting in 1925. The first order serial correlation in their series also is about 0.8.

The Monte Carlo evidence and test results from the Blanchard, et al data confirm the suspicion that unit root tests cannot distinguish between very slow mean

³ I thank Mark Carey for the simulations and other valuable research assistance.
reversion and no mean reversion. In practical terms, however, a stationary low
frequency deviation from the long run equilibrium and unit root (which gives the
implausible result of no long run equilibrium) have similar implications. Financial asset
values are not closely related to the fundamentals.\footnote{The estimated coefficients in the ARMA model imply the half life of a shock is
7 years.}

4. Summary and Conclusion

This paper presents a robust test of the relationship between asset prices and
the fundamentals based on Tobin’s $q$. The test is intuitive and simple. In theory $q$
should be a mean reverting process. The data indicate that mean reversion is very
slow. Low frequency movements in aggregate capital and the aggregate financial value
of firms are not highly correlated. This fact presents difficult puzzle for economists
trying to understand and model the linkages between the real and financial sectors.
REFERENCES


———, and Christian Gilles, 1988, "Econometric Aspects of the Variance-Bounds Tests", reproduced, University of California at Santa Barbara


APPENDIX

DATA

This gives the components of q for the annual series from 1925-1987:

\[ q = TOBQA = \text{SP5ARS}/\text{COMWKS} \]

SP500  =  the S&P's Composite Common Stock Price Index  
1970-1981 CITIBASE (FSPCOM)

CPI    =  Consumer Price Index  
1970-1981 CITIBASE (PUNEW)

SP5ARS = SP500/CPI

COMNKS = Constant Cost Net Stock of Private Total Nonresidential Fixed Capital  
Source:  Survey of Current Business, various issues.

This gives the components of q for the post WWII quarterly series:

\[ q = TOBQO = V/K \]

Definitions:

\[ NV = MVD + MVE \]

\[ MVD = \text{INT/YA}, \text{ the market value of debt} \]

\[ MVE = \text{DIV/YSP}, \text{ the market value of equity} \]

This follows Abel and Blanchard's construction of the financial value of the firm, see their appendix. The data comes from DRI's data bank with the DRI mnemonic in parenthesis.

INT is net interest payments by nonfinancial business corporations (INTBUSCORPNF).

YA is the yield on Moody's A corporate bonds (RMMBCANS).

DIV is dividends paid by nonfinancial business corporations (NFCDIV).

YSP is the quarterly average of the monthly yield on the S&P 500.

\[ V = NV/PUNEW, \text{ financial value of the firm in consumption units} \]

K is the COMNKS interpolated to a quarterly series using gross real private nonresidential investment (GIF CITIBASE) as weights.
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