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A NEW RATIONALE FOR MARKETS IN BASKETS OF SECURITIES

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Abstract

This paper provides a new rationale for markets in baskets of securities (e.g. the stock index futures markets) by demonstrating that they provide a convenient trading medium for liquidity traders. The reason advanced is that the transaction costs suffered by these liquidity traders due to adverse trades with informed traders will typically be lower in markets for baskets than in markets for individual securities. The paper implies that large financial institutions may be expected to trade heavily in baskets to satisfy the liquidity needs of their clients. Thus, the paper provides a plausible explanation for the remarkable growth in the popularity of the stock index futures market over the past few years.
1 Introduction

Recent years have witnessed a tremendous increase in the popularity of markets in baskets of securities. In a frictionless world, these baskets would be completely redundant. However, ever since the most popular basket security, the S&P 500 futures contract was introduced in 1982, there has been a spectacular growth in its trading volume, so much so that the average daily dollar trade in S&P 500 futures in 1988 was approximately six billion dollars, which was only slightly less than the average daily New York Stock Exchange (NYSE) dollar trade.¹ A basket called the Cash Index Participation (CIP) is actively traded on the Philadelphia Stock Exchange in which long holders have a right to cash-out their positions at an amount equal to the value of the underlying index portfolio. A proposal for introducing a basket wherein all the stocks in the S&P 500 could be traded in a single transaction is under active consideration of the NYSE (see Domijan (1989)). Several other instruments for trading in broad market portfolios are in existence and are exhaustively described in Harris (1988). The aim of this paper is to develop an explanation for the growth in popularity of markets in these baskets of securities.

Two broadly defined motives for trade in financial markets are information and liquidity. Informed traders trade on the basis of their private information. Liquidity traders, however, are typically assumed to trade for reasons not directly related to future payoffs of financial assets, thus their reasons for trade are determined outside the financial market.² The essence of this paper is the demonstration that the introduction of a new market in a basket of securities will typically cause liquidity traders to concentrate their trading in this market, because losses suffered by liquidity traders due to adverse trades with informed traders will typically be lower in the basket than in the individual securities.

Liquidity trades can be thought of as typically being executed by large financial intermediaries whose motives reflect the liquidity needs of their customers. Examples of such

¹See Harris (1988).
²Grossman and Miller (1987) and Holden (1988) are examples of papers in which liquidity traders are assumed to receive exogenous demand shocks and endogenously decide on their optimal trades by maximizing expected utility.
motives are the desire for immediate consumption, idiosyncratic wealth shocks and tax planning. In much of the recent literature on speculation in financial markets, liquidity traders are assumed to trade due to their need for immediacy, in spite of suffering losses due to the presence of informed traders. Notable examples of papers which make this assumption are Kyle (1984,1985), Glosten and Milgrom (1985), Diamond and Verrecchia (1987), and Admati and Pfleiderer (1988). This paper adopts a similar approach and characterizes the strategic trading decision of liquidity traders who wish to trade in several securities. They are modeled as facing a choice of realizing their trade either in the individual securities or in a basket of these securities.

The trading model we use is in the spirit of Kyle (1985) and especially of Admati and Pfleiderer (1988). These papers analyze a speculative market in a single security. We extend their models to multiple security markets and markets for baskets. We assume that in each market risk neutral informed traders and liquidity traders submit their orders to risk neutral competitive market makers who set prices expecting zero profits conditional on all information available to them. Private information possessed by information traders is assumed to be useful for only one period. There are two types of liquidity traders: discretionary and non-discretionary. Discretionary liquidity traders wish to realize their trades in several securities simultaneously, and choose to trade either in the individual securities or in a basket of these securities, depending on where their losses to informed traders are minimized.\(^3\) Non-discretionary liquidity traders are exogeneously constrained to trade a given order size either in a particular security or in the basket. Informed traders are assumed to strategically choose their trades, rationally taking into account the effect their trades will have on prices.\(^4\)

Our main result is that the independent orders submitted by security-specific informed traders in the basket tend to offset each other considerably when the number of securities is large. Due to this 'diversification' benefit, which is not present in the individual securities,

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\(^3\)In contrast, in Admati and Pfleiderer (1988), discretionary liquidity traders optimally choose the timing of their trades. In Kyle (1985), all liquidity trading is non-discretionary.

\(^4\)This assumption is in contrast to the competitive markets model of Grossman and Stiglitz (1980), in which informed agents are assumed to be price takers.
the losses of the discretionary liquidity traders reduce considerably when they trade in the
basket if only security-specific informed trading takes place in the individual securities and
the basket. With the addition of informed traders possessing information about systematic
factors, the basket is preferred if the non-discretionary trades in all markets are roughly
equally dispersed or if the variance of non-discretionary trades is higher in the basket than
in the individual securities. Otherwise, the advantage of the basket rises with precision
of security-specific information and falls with the precision of systematic information. We
argue that stock markets are typically characterized by imprecisely informed systematic
speculators and insiders possessing relatively precise stock-specific information, and there-
fore that liquidity traders desirous of trading in several stocks would tend to prefer to trade
in baskets.

The paper is organized as follows. In section 2, we begin by describing our basic model.
In section 3, we analyze a simple scenario with three markets: two for securities and one for
a basket comprised of the two securities to illustrate the basic ideas underlying our results.
In section 4, we extend the model to an arbitrarily finite number of securities. Section 5
examines the impact of the introduction of ‘systematic’ informed trading into the model.
Section 6 concludes the paper.

2 The Model

Consider a security traded over a single period. Trading takes place at time 0 and at time
1 the security is liquidated. The value of the security at time 0 is denoted by \( \bar{S} \) and its
liquidation value at time 1 (denoted by \( \tilde{S} \)) is given by

\[
\tilde{S} = \bar{S} + \tilde{\gamma} + \tilde{\epsilon}
\]

where \( \tilde{\epsilon} \) and \( \tilde{\gamma} \) are mutually independent, normally distributed random variables each with
a mean of zero. Hereafter, we suppress all tildes unless essential for clarity. We give \( \epsilon

\text{5} The model can easily be extended to many periods if all trades and security value innovations
are assumed to be serially uncorrelated. See Admati and Pfleiderer (1988) for a multi-period version
of this model.
and \( \gamma \) the interpretation of the ‘idiosyncratic’ and ‘systematic’ components of the security price innovation respectively. The specification (1) is consistent with a ‘factor model’ representation of security returns and with the Capital Asset Pricing Model.

All traders in the model are assumed to be risk neutral.\(^6\) \( \hat{S} \) is assumed to be public knowledge and informed traders are assumed to observe the realization of \( \epsilon \) or \( \gamma \) one period ahead and take positions in the security on the basis of this private information.\(^7\) For the purposes of this section, we assume there is one monopolistic speculator in the market\(^8\) who observes \( \epsilon \) (but not \( \gamma \)) with perfect precision at time 0 and submits an order \( z \).

We assume that liquidity traders in the security have no discretion over their order size, i.e. that they must trade a given number of shares in the security.\(^9\) These traders can be thought of as trading for idiosyncratic life-cycle reasons. Liquidity trading is thus modeled as containing no information about the fundamental forces affecting the price of the security. Let \( z \) be the total random liquidity trade. We assume that the random variables \( z, \epsilon, \gamma \) are mutually independent and multivariate normally distributed, each with a mean of zero.

The informed trader and the liquidity traders submit their orders to a market maker, not knowing the market clearing price when they do so. Competition is assumed to force the expected profits of the market maker to zero. The market maker thus absorbs the net trade in the security and then sets a price which yields zero profits to him. The market maker is assumed to observe only the total order flow, which is denoted by \( \omega \). The price set by the market maker in time 0 is therefore

\[
\hat{P} = E(\hat{S}|\omega) \quad .
\]

\(^6\) The assumption of risk neutrality is made for reasons of tractability and is typical in much of the recent literature on speculation. Examples of important papers in which this assumption is made are Gloslen and Milgrom (1985), Kyle (1985), and Admati and Pfeiderer (1988).

\(^7\) Admati and Pfeiderer (1988) make the same assumption. In Kyle (1985), the informed trader is assumed to observe the liquidation value of the asset several periods ahead and to optimally choose his trading pattern over time.

\(^8\) Throughout this paper, the phrases ‘informed trader’ and ‘speculator’ are used synonymously.

\(^9\) Since traders in this model do not know the market clearing price when they submit their orders (this assumption is stated later in the text), it is difficult to model their trading decision in terms of dollar amounts. The assumption that liquidity traders must trade a given number of shares is also made by Kyle (1985) and Admati and Pfeiderer (1988).
Following Kyle (1985), the market maker is assumed to employ a linear pricing rule to set prices in each period. The pricing rule for the market maker thus takes the form

\[ \hat{P} = \hat{S} + \lambda \hat{\omega} . \]  

(3)

The quantity \( \lambda \) converts order flow into price movements. It can be taken to be a measure of the ‘liquidity’ of the market. A low \( \lambda \) means a more liquid market in the sense that the cost of a given trade is low. This parameter plays a significant role in our analysis.

We now examine the nature of the equilibrium which obtains in this market. Since the informed trader observes only the idiosyncratic innovation, from (2) and (3) and normality, \( \lambda \) is the regression coefficient in the forecast of \( \epsilon \) on the total random order flow \( \omega \). We thus have

\[ \lambda = \frac{\text{cov}(\epsilon, \omega)}{\text{var}(\omega)} . \]  

(4)

The informed trader maximizes his expected profits which are given by the expected value (conditional on his information) of the difference between the price and the liquidation value of the security times his order size. His profits can thus be written as

\[ E(x(S - P(\omega))|\epsilon) . \]

Given the form of the market maker's pricing function, this is equivalent to

\[ E(x(\epsilon - \lambda \omega)|\epsilon) . \]

Substituting \( \omega = x + z \), noting that the liquidity trade \( z \) is independent of the innovation \( \epsilon \) and differentiating with respect to \( x \), we find that profits are maximized when

\[ x = \epsilon/2\lambda . \]  

(5)

Using \( \omega = x + z \) and substituting for \( x \) from above into (4), we get a quadratic equation in \( \lambda \). The unique positive root of this equation is

\[ \lambda = \frac{1}{2} \sqrt{\text{var}(\epsilon)/\text{var}(z)} . \]  

(6)
Equation (6) gives the equilibrium value of $\lambda$ under the above assumptions. Thus $1/\lambda$, the measure of market liquidity is proportional to ratio of the amount of liquidity trading to the amount of private information possessed by the informed trader. The higher the amount of private information, the higher the $\lambda$ and the less liquid the market. This captures implicitly the intuition in Glosten and Milgrom (1985), that market makers set bid-ask spreads (in this context pricing functions), thereby reducing the liquidity of the market to compensate themselves for losses due to trading with agents with superior information. Of course, an increase in the amount of liquidity trading makes markets more liquid; the additional liquidity being provided by competitive market makers who now receive increased compensation from liquidity traders for losses due to adverse trades by informed traders.

The unconditional total losses to the liquidity traders are given by the expected value of the difference between the price of the security and the security’s value times the liquidity traders’ demands. From (1), (3) and (6), these losses are

$$E((P - S)z) = \lambda \text{var}(z) = \frac{1}{2} \sqrt{\text{var}(z) \text{var}(\epsilon)}.$$  

(7)

In keeping with intuition, the unconditional expected losses of the liquidity traders (which equal the unconditional expected profits of the informed traders) are increasing both in the amount of private information $\text{var}(\epsilon)$ and the amount of the liquidity trades $\text{var}(z)$. A higher $\text{var}(z)$ implies a more liquid market, implying greater participation by the speculator, leading to an increase in his profits, corresponding to increased losses incurred by the liquidity traders. Note that from (5) and (6), an increase in the amount of liquidity trading increases the amount of informed trading (through an increase in market liquidity). Thus, here liquidity trading does not decrease the informativeness of prices, unlike Grossman and Stiglitz’s (1980) model. In that model, competitive, risk averse informed traders demand higher risk premia when the amount of the random liquidity trade increases, and the increased premia lead to less informative prices.\(^{10}\)

Thus, in this model, informed traders profit due to their monopolistic access to information and due to the fact that they are able to conceal their trades from the market maker

\(^{10}\)In Grossman and Stiglitz (1980), the random liquidity trade is interpreted as a noisy supply of the risky asset.
by trading along with the liquidity traders. In later sections, we extend this basic model to multiple securities and markets in baskets of securities and to many informed traders.

3 The Analysis with Two Securities

In this section, we examine a simple model with three markets: two for securities and one for a basket and two monopolistic security-specific speculators, and examine extensions in following sections. We now introduce multiple classes of liquidity traders. These classes are denoted by $L$, $M_1$, $M_2$ and $R$. Each trader belonging to the class $L$ wishes to trade in both securities. The class $L$ can be interpreted as mutual fund or index fund portfolio managers satisfying the liquidity needs of their clients.\footnote{This interpretation becomes more realistic in later sections, where we consider the case of many securities.} Traders belonging to the classes denoted by $M_1$, $M_2$ and $R$ are constrained to trade a given order size in securities 1 and 2 and the basket respectively. We refer to the traders denoted by $L$ as discretionary liquidity traders, in that they are assumed to strategically choose the market(s) in which their losses to informed traders are minimized. On the other hand traders $M_1$, $M_2$ and $R$ are referred to as non-discretionary traders, in the sense that they are constrained to trade a fixed order size in a particular market. The classes $M_1$ and $M_2$ can be interpreted as either (a) traders for whom the other security is not part of their optimal portfolios, and they do not wish to trade in it, or (b) they are informationless or irrational speculators in the Black (1986) sense. For the class $R$ only the interpretation (b) is justified. This class is not crucial for the model; it prevents the market for the basket from breaking down in the event that the $L$ traders prefer to trade in the individual securities, since market makers do not wish to trade in the absence of liquidity trading.\footnote{An additional interpretation can be given based on the analysis of Holden (1988). His model creates clienteles for different markets based on different investment horizons for different investors and securities with differing maturities. In our context, the basket can be interpreted as an index futures contract which matures more frequently than the securities and the classes $R$, $M_1$ and $M_2$ can be interpreted as investors with differing investment horizons.}
3.1 Two Securities and No Basket

Let there be two securities, each simultaneously traded over one period, with liquidation values given by

\[ \tilde{S}_i = \tilde{S}_i + \tilde{\gamma} + \tilde{\epsilon}_i , \quad i = 1, 2 . \tag{8} \]

Note that the systematic innovation is assumed to be the same for both securities, while the idiosyncratic components differ across securities.\(^{13}\)

There are two monopolistic speculators, one in each market (indexed by \(i = 1, 2\)), who submit trades in the respective markets based on their private knowledge of the realization of \(\epsilon_i\). The idiosyncratic innovations \(\epsilon_1\) and \(\epsilon_2\) are assumed to be independent. This implies that these traders do not submit orders in the market for the security for which they do not possess information.

We assume for simplicity that the discretionary traders \(L\) submit equal (random) orders \(l\) in either security.\(^{14}\) Let \(m_1\) and \(m_2\) be the (random) demands of the non-discretionary traders \(M_1\) and \(M_2\). Further, assume \(l, m_1, m_2, \epsilon_1, \epsilon_2, \gamma\) are mutually independent and multivariate normally distributed, each with a mean of zero.\(^ {15}\)

The market makers' pricing functions in each market are given by

\[ P_i = \tilde{S}_i + \lambda_i \omega_i , \quad i = 1, 2 , \tag{9} \]

where \(\omega_i\) is the total random order flow in security \(i\). From the analysis in the previous section (see (6)), we have

\[ \lambda_i = \frac{1}{2} \sqrt{\text{var}(\epsilon_i)/\text{var}(l + m_i)} \tag{10} . \]

The preceding result shows that, as in Section 2, the pricing parameter \(\lambda_i\) depends on the private information possessed by the informed traders (an increase in which reduces the

\(^{13}\)Equation (8) can be interpreted as a linear factor model with identical systematic factor weights for all securities. The assumption of identical factor weights is not crucial to the analysis, and is made for ease of notation.

\(^{14}\)The weaker assumption that the class \(L\) submits orders of equal variance in either security and the basket is sufficient for our analysis to hold - the stronger assumption is made for notational convenience.

\(^{15}\)Our analysis remains unchanged even if the demands of the traders \(M_1\) and \(M_2\) are assumed to be correlated.
liquidity of the market - alternatively, increases the adverse selection faced by the market maker) and the total amount of liquidity trading (an increase in which has the opposite effect).

Losses to the class \( L \) of liquidity traders from trading in security \( i \) (denoted by \( C_{Li} \)) are given by

\[
C_{Li} = E((P_i - S_i)l) = \lambda_i \text{var}(l) = \frac{1}{2} \sqrt{\frac{\text{var}(\epsilon_i)}{\text{var}(l + m_i) \cdot \text{var}(l)}} .
\]

(11)

As before, an increase in \( \text{var}(l) \) implies greater participation by the speculator and increases the losses of the class \( L \).

3.2 Two Securities and a Basket

We now examine the consequences of introducing a third security which has payoffs identical to the sum of the payoffs on securities 1 and 2. We refer to this security as the ‘basket’.\(^{16}\) Let the order submitted by the non-discretionary traders in the basket (the class \( R \)) be denoted by \( r \). We let \( r \) be normally distributed with zero mean and independent of all the other liquidity trades and the security innovations.

To simplify the analysis, we assume discretionary liquidity traders must realize their entire trades either in the individual securities or in the basket. Though such an assumption can be justified on the basis of per trade transaction costs, in Appendix 2 we show that our basic results continue to obtain in the limit as the number of securities becomes large even if discretionary liquidity traders are allowed to allocate their trades between the basket and the individual securities.

The discretionary liquidity traders choose to trade either in the individual security markets or in the basket, depending on where they suffer the least transaction costs due to trading with informed traders. We assume that the market maker in a particular market does not observe the order flow in the other markets.\(^{17}\) Risk neutral speculators will also

\(^{16}\)Note that, as a matter of convention, the basket is defined as a simple sum and not as a weighted average.

\(^{17}\)The basic nature of our results will not change if the market maker were assumed to condition on all order flows.
submit orders in the basket, so long as they can profit from doing so. The following Lemma describes the optimal trades and profits of the informed traders in the basket given that discretionary liquidity traders choose to trade only in the basket. (All proofs are in Appendix 1 unless otherwise stated.)

**Lemma 1** Assuming that discretionary liquidity traders trade only in the basket, each speculator $i$ chooses a quantity

$$x_{im} = \frac{\epsilon_i}{2\lambda_m}, \quad i = 1, 2,$$

(12)

to trade in the basket, where

$$\lambda_m = \frac{1}{2} \sqrt{\frac{\text{var}(\epsilon_1 + \epsilon_2)}{\text{var}(l + r)}}$$

(13)

is the equilibrium pricing parameter in the market for the basket.

The pricing parameter in the market for the basket $\lambda_m$ is increasing in the total amount of information possessed by both information traders, as seems reasonable. Note that the functional form of the informed traders' orders is unchanged from that in the single security case (see (5)). This is because they trade on independent information in the market and therefore do not compete strategically against each other.\(^{18}\) However, the equilibrium orders placed by these traders are smaller in the basket than in the individual securities because they face a larger pricing parameter in the basket.

Given these results, the losses incurred by the discretionary liquidity traders from trading in the basket (denoted by $C_{Lm}$) are

$$C_{Lm} = \lambda_m \text{var}(l) = \frac{1}{2} \sqrt{\frac{\text{var}(\epsilon_1 + \epsilon_2)}{\text{var}(l + r)}} \text{var}(l).$$

(14)

The discretionary liquidity traders will choose to trade either in the individual securities or in the basket, depending on where they suffer the least losses to informed traders. They will prefer to trade only in the basket if and only if

$$C_{Lm} < C_{L1} + C_{L2},$$

(15)

\(^{18}\)If the signals of the two traders were correlated, this result would no longer obtain.
or equivalently when

\[ \lambda_m < \lambda_1 + \lambda_2 \]  \hspace{1cm} (16)

Substituting for \( \lambda_1, \lambda_2 \) and \( \lambda_m \) from (10) and (13) we have

**Proposition 1** Under the assumptions of this section, discretionary liquidity traders prefer to trade in the basket over the individual securities if and only if

\[ \sqrt{\frac{\text{var}(\epsilon_1 + \epsilon_2)}{\text{var}(l + r)}} < \sqrt{\frac{\text{var}(\epsilon_1)}{\text{var}(l + m_1)}} + \sqrt{\frac{\text{var}(\epsilon_2)}{\text{var}(l + m_2)}} \]  \hspace{1cm} (17)

Thus discretionary liquidity traders prefer to trade in the basket if the variance of non-discretionary trades is high in the basket and low in the securities. The intuition is that low variance of non-discretionary security trades causes the security markets to become less liquid, alternatively the adverse selection faced by the market maker becomes more severe (i.e. \( \lambda_i \) is set higher), which causes the advantage of the basket to increase. The opposite intuition holds for low variance of non-discretionary basket trades.

Let \( \text{var}(m_1) = \text{var}(m_2) = \text{var}(r) \). Then the above inequality would hold, since the sum of the standard deviations of independent non-degenerate random variables (in this context \( \epsilon_1 \) and \( \epsilon_2 \)) is greater than the standard deviation of their sum. This leads us to the following Corollary.

**Corollary to Proposition 1** If the variances of non-discretionary trades in all markets are equal, discretionary liquidity traders prefer to trade only in the basket.

The above result can be explained as follows. In equilibrium, the pricing parameter depends on the ratio of the variability of the total information possessed by the information traders in the market to the total variability of liquidity trades. The variability of liquidity trades in all markets is the same. (The discretionary liquidity traders will submit the same order \( l \) in the basket as in the securities.) However, the variability (i.e. the standard deviation) of information in the basket is lower than the sum of the variabilities of information in the individual securities, because of the ‘diversification’ effect of the independent trades of the two informed traders in the basket. This effect arises because these informed traders
trade on independent information in the basket. Due to the diversification effect, the market maker in the basket sets \( \lambda_m \) to be less than \( \lambda_1 + \lambda_2 \), even though the variability of the total liquidity trades is equal in all markets.\(^{19}\) The total effect of informed trading is thus less damaging to the discretionary liquidity traders in the basket than in the individual securities.

For further illustration of this point assume \( \tilde{\epsilon}_1 = \tilde{\epsilon}_2 \). Then assuming the functional form of the optimal trades of the informed traders remains unchanged\(^{20}\) \( \lambda_m \) would be exactly equal to \( \lambda_1 + \lambda_2 \) (see (13) and (10)), making the discretionary liquidity traders indifferent between the basket and individual securities. As the correlation between \( \epsilon_1 \) and \( \epsilon_2 \) declines, the diversification effect increases. If \( \tilde{\epsilon}_1 = -\tilde{\epsilon}_2 \) then \( \lambda_m \) is zero since only liquidity traders submit orders in the basket.

Intuitively, one would expect the above effect to markedly intensify as the number of securities comprising the basket became large. The next section formalizes this intuition.

4 The Case of Many Securities and a Basket

Assume there are \( N \) securities, each with payoff given by (8), with \( i = 1, \ldots, N \). There are \( N \) monopolistic speculators, one in each security (indexed by \( i = 1, \ldots, N \)) each of whom observes the realization of \( \epsilon_i \) for a particular security \( i \). The idiosyncratic innovations \( \epsilon_i \), \( i = 1, \ldots, N \) are assumed to be mutually independent. There are \( N + 1 \) classes of liquidity traders: discretionary traders \( L \) and non-discretionary traders \( M_1, \ldots, M_N \). Analogous to the two securities case, each member of the class \( L \) of discretionary liquidity traders desires to submit equal orders in all \( N \) securities. Non-discretionary traders \( M_1 \) through \( M_N \) are constrained to trade in securities 1 through \( N \) respectively. Denote the demands of the

\(^{19}\) Note that \( \lambda_m \) depends on the standard deviation of the information possessed by informed traders and not the variance. Since the pricing parameter converts order flows to price movements it must be in units of dollars per share. The ratio of the variance of information to the variance of liquidity trades is measured in units of dollars squared per share squared. The market maker is concerned only with the square root of this ratio, i.e. the ratio of standard deviations.

\(^{20}\) We assume this for the purposes of illustration. In the actual equilibrium, the functional form of the optimal trades would change, since the traders would now compete, their information now being perfectly correlated.
liquidity traders as \( l, m_1, \ldots, m_N \) respectively. We assume the random variables \( l, m_i, \gamma \) and \( \epsilon_i, i = 1, \ldots, N, \) are mutually independent and multivariate normally distributed, each with a mean of zero. Further, for simplicity assume \( \text{var}(m_i) = \text{var}(m) \) for all \( i = 1, \ldots, N. \)

The functional forms of the equilibrium pricing parameter in each of the security markets and the losses incurred by the discretionary liquidity traders from trading in the security \( i \) (where now \( i = 1, \ldots, N \) remain unchanged from (10) and (11). Of more interest is the scenario when we introduce a ‘basket’ having identical payoffs as \( \sum_{i=1}^{N} S_i. \) Assume the existence, as before, of an independent class \( R \) of non-discretionary traders (with their demands being denoted by \( r \)) in the basket. The next result describes the resulting equilibrium in the market for this ‘basket’ given that discretionary liquidity traders switch to trading in the basket.

**Lemma 2** Assuming discretionary liquidity traders trade only in the basket, the information trader possessing information about security \( i \) submits an order

\[
x_{im} = \frac{\epsilon_i}{2\lambda_m}
\]

in the market for the basket, where

\[
\lambda_m = \frac{1}{2} \sqrt{\frac{\text{var}(\sum_{i=1}^{N} \epsilon_i)}{\text{var}(l + r)}}
\]

is the pricing parameter set by the market maker in the basket in equilibrium. The profits to information traders from trading in the basket are given by

\[
\pi_{im} = \frac{\text{var}(\epsilon_i)}{2} \sqrt{\frac{\text{var}(l + r)}{\text{var}(\sum_{i=1}^{N} \epsilon_i)}}.
\]

Note that the pricing parameter in the basket \( \lambda_m \), as in Section 3.2, depends on the total information of all the informed traders. Also, the profits earned by a particular informed trader in the basket are related to the relative importance of the trader’s information as compared to that of all other traders. The profits drop to zero as \( N \) becomes large, because each informed trader’s information becomes unimportant relative to the total information. Finally, as \( N \) becomes large, the quantity traded by each informed trader in the basket
also goes to zero, because each individual informed trader faces a large $\lambda_m$ due to the large number of informed traders trading on independent information.

The losses incurred by the discretionary liquidity traders from trading in the basket are

$$C_{Lm} = \lambda_m \text{var}(l) = \frac{1}{2} \sqrt{\frac{\text{var}(\sum_{i=1}^{N} \epsilon_i)}{\text{var}(l + r)}} \text{var}(l).$$  \hspace{1cm} (21)

For simplicity of exposition, we now assume $\text{var}(\epsilon_i) = \text{var}(\epsilon)$ for all $i = 1, \ldots, N$. Comparing the losses to the discretionary liquidity traders from trading in the basket and trading in the individual securities, we have

**Proposition 2** Under the assumptions of this section, discretionary liquidity traders prefer to trade in the basket if and only if

$$\sqrt{\frac{1}{\text{var}(l + r)}} < \sqrt{\frac{N}{\text{var}(l + m)}}.$$  \hspace{1cm} (22)

For $N$ large, we see that the condition described above would most likely hold. The source of this result is again the diversification effect of the informed traders' independent trades in the basket, which benefits the discretionary liquidity traders. The effect is much larger than in the two securities case, in keeping with intuition.

Equation (22) holds for large $N$ across a wide range of parameter values. Assuming its validity, in our final equilibrium we have informed traders trading extensively in individual security markets, to a small extent in the basket, and the discretionary liquidity traders trading in the basket. The speculation in individual securities is supported by the non-discretionary traders $M_1, \ldots, M_N$.\(^{21}\)

\(^{21}\)The reader may well wonder whether the model is consistent with arbitrage by traders between the individual securities and the basket to remove price differences. It turns out, however, that there are no arbitrage opportunities in our model. To see this note that the price of the basket incorporates all the individual security initial values $\bar{S}_1, \ldots, \bar{S}_N$. Thus, the **unconditional expected** difference between the sum of the prices of the securities and the price of the basket is zero. Traders do not know the market clearing price when they submit their orders and no one group of traders can predict the total order flow in any market with precision. It follows that consistently profitable arbitrage between the markets is not possible. Note also that the link between the markets is maintained by the informed traders, who trade in both the individual securities and the basket in equilibrium. The actual sizes of their trades are determined by the zero profit condition imposed on the market makers.
5 The Effect of Systematic Informed Trading

In this section we examine the impact of introducing information traders who are informed only about the systematic component of the security values to the scenario of the last section. We refer to these speculators as 'systematic' informed traders, as opposed to the 'security-specific' informed traders described earlier. In Subsection 5.1 we analyze the model of the previous section, with $N$ securities, $N$ monopolistic security-specific speculators and $N+1$ classes of liquidity traders, now assuming the presence of a monopolistic systematic speculator. In Subsection 5.2 we consider the case of many security-specific informed traders per security and many systematic informed traders.

In order to focus our attention on the impact of different precisions of information on the liquidity traders' choice of markets, we now assume that all informed traders are imperfectly informed. Specifically, the security-specific trader in security $i$ ($i = 1, \ldots, N$) is assumed to observe $\tilde{\epsilon}_i + \tilde{u}_i$, where $\text{var}(u_i) = \theta$ for all $i = 1, \ldots, N$. The systematic trader is assumed to observe $\tilde{\gamma} + \tilde{v}$, where $\text{var}(v) = \kappa$. We further assume that $u_1, \ldots, u_N, v$ are mutually independent and independent of $\epsilon_1, \ldots, \epsilon_N, \gamma$ and all the liquidity trades, and are multivariate normally distributed with zero mean. To avoid notational clutter, we set $\text{var}(\epsilon) = \text{var}(\gamma) = 1$ for the purposes of this section.

5.1 Monopolistic Systematic Information Traders

Recall that each security has payoffs given by (1). Therefore, the value of the basket is given by

$$\tilde{S}_m = \tilde{S}_m + \gamma_m + \sum_{i=1}^{N} \epsilon_i \quad \text{where} \quad \gamma_m \equiv N\gamma \quad \text{and} \quad \tilde{S}_m \equiv \sum_{i=1}^{N} \tilde{S}_i \quad .$$

We assume there is a monopolistic speculator, who observes the realization of $\gamma + v$ and takes a positions in each of the securities and the basket based on this information. It is then possible to show the following.

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22 Admati and Pfeiderer (1988) make a similar assumption. However, their model does not distinguish between idiosyncratic and systematic components of the security value innovation.
Lemma 3 Under the above assumptions, assuming discretionary liquidity traders trade only in the basket, the equilibrium pricing parameter in the basket is given by

\[
\lambda_m = \frac{1}{2} \sqrt{\frac{N}{1+\bar{s}}} + \frac{N^2}{1+\kappa \var(l+\eta)}. \tag{24}
\]

The equilibrium pricing parameter in security i given that discretionary liquidity traders trade in securities is

\[
\lambda_i = \frac{1}{2} \sqrt{\frac{1}{1+\bar{s}}} + \frac{1}{1+\kappa \var(l+m)}. \tag{25}
\]

Two aspects of the above result are worth mentioning. First, \(\lambda_m\) and \(\lambda_i\) now reflect the additional component of adverse selection faced by the market makers due to their trades with the systematic informed trader. Second, all pricing parameters are increasing in the precision of the informed traders' information (i.e. they are decreasing in the variance of the noise in their signals). More precise signals imply greater participation by the speculators, leading to an increase in the adverse selection faced by the market maker.

The losses suffered by liquidity traders from trading in the security \(i \ (i = 1, \ldots, N)\) and the basket are respectively \(C_{Li} = \lambda_i \var(l)\) and \(C_{Lm} = \lambda_m \var(l)\). Define \(P_s \equiv 1/(1+\theta)\) and \(P_m \equiv 1/(1+\kappa)\). \(P_s\) and \(P_m\) then reflect the precision of the information possessed by security-specific and systematic informed traders. As in earlier Sections, comparing the losses to the discretionary liquidity traders from trading in the basket and trading in the individual securities, we have

Proposition 3 Under the above assumptions, when there exists a monopolistic informed trader who possesses private information about the systematic component of security returns, discretionary liquidity traders prefer to trade in the basket if and only if

\[
\frac{P_s + P_m}{\var(l+\eta)} < \frac{P_s + P_m}{\var(l+m)}. \tag{26}
\]

For \(N\) large, if the non-discretionary trades in the basket and in the individual securities are roughly equally dispersed, condition (26) would hold. Suppose \(\var(r) = \var(m)\). The diversification benefit from the independent trades of the security-specific informed traders...
in the basket is present in this case too. However, the systematic informed trader trades on
the basis of information which is \textit{perfectly} correlated across securities. Therefore, the effect
of systematic informed trading is the \textit{same} in the basket and in the individual securities.
Equivalently, the diversification benefit of the security-specific informed traders described
earlier is \textit{absent} in the case of systematic informed trading. Further illustration of this
point can be obtained by assuming there is no security-specific speculation. Then we get

\[
\lambda_m = \frac{1}{2} \sqrt{\frac{N^2 P_m}{\text{var}(l + r)}}
\]

and

\[
\lambda_i = \frac{1}{2} \sqrt{\frac{P_m}{\text{var}(l + m)}}.
\]

Then, if \(\text{var}(m) = \text{var}(r)\), we have \(\lambda_m = \sum_{i=1}^{N} \lambda_i\) and discretionary liquidity traders are
indifferent between trading in the individual securities and trading in the basket. The
addition of the security-specific informed traders provides a diversification benefit to the
discretionary liquidity traders and the basket is then preferred. If \(\text{var}(r) > \text{var}(m)\), condition (26) holds more strongly. The latter follows because a high \(\text{var}(r)\) relative to \(\text{var}(m)\)
tends to increase the liquidity of the market for the basket relative to individual securities,
increasing the basket’s relative advantage.

We now consider the case of \(\text{var}(r) < \text{var}(m)\). To illustrate this case, rewrite (26) as

\[
\text{var}(l + r) > \frac{\text{var}(l + m)}{N} + \frac{P_m}{P_s} [\text{var}(m) - \text{var}(r)].
\]  \hspace{1cm} (27)

In this case, the validity of the condition (26) depends on the precisions of information
possessed by systematic and security-specific informed traders. From (27) above, we see
that an increase in the precision of systematic information causes the advantage of the
basket to decline. The reason is that when \(\text{var}(r) < \text{var}(m)\), i.e. when the amounts of
non-discretionary trades in the basket are lower than those in the securities, the marginal
impact of an increase in the precision of systematic information is relatively more on the
adverse selection in the basket than on the adverse selection in the individual securities.
This is evident upon examining the expressions (24) and (25) for \(\lambda_m\) and \(\lambda_i\) respectively.
Thus an increase in the precision of systematic information is disliked more by liquidity
traders trading in the basket than by liquidity traders trading in the individual securities. A similar, but reversed intuition can be used to explain the impact of the precision of security-specific information \( P_s \) on the relative advantage of the basket. The above discussion implies that if \( \text{var}(r) < \text{var}(m) \), the advantage of the basket declines as the ratio of the precision of systematic information to the precision of security-specific information increases. If \( \text{var}(r) \geq \text{var}(m) \), the basket is always preferred.

5.2 Non-Monopolistic Informed Trading

Hitherto, we have assumed that markets are characterized by traders who have monopolistic access to a particular type of information. We now relax this assumption and characterize these markets as each being composed of many imperfectly competitive informed traders, as in Admati and Pfleiderer (1988). We assume for simplicity that all security-specific traders in a particular security and all systematic traders observe the same signal, specifically that all the security-specific traders in security \( i \) observe \( u_i + v_i \) (\( i = 1, \ldots, N \)) and the systematic traders observe \( \gamma + v \). Let \( k_i \) be the number of informed traders in the \( i \)th security (\( i = 1, \ldots, N \)) and let \( g \) be the number of 'systematic' informed traders. We assume that traders in each market choose trading strategies taking the strategies of all other traders as given and characterize the resulting Nash equilibrium in each market.

**Lemma 4** Under the above assumptions, the equilibrium pricing parameters in the basket given that the discretionary liquidity traders trade only in the basket is given by

\[
\lambda_m = \sqrt{\frac{\sum_{i=1}^{N} \frac{k_i}{(k_i + 1)^2} P_s + \frac{g}{(g+1)^2} P_m N^2}{\text{var}(l + r)}}. \tag{28}
\]

The equilibrium pricing parameter in security \( i \) given that the discretionary liquidity traders trade in the individual securities is

\[
\lambda_i = \sqrt{\frac{\frac{k_i}{(k_i + 1)^2} P_s + \frac{g}{(g+1)^2} P_m}{\text{var}(l + m)}}. \tag{29}
\]

We see that the equilibrium pricing parameters in each market are decreasing in the number of informed traders. On the face of it, this seems counter-intuitive, since one would think
that an increase in the number of informed traders would make the adverse selection problem faced by the market maker more severe. However, since informed traders who observe the same signal compete with each other, the adverse selection actually reduces with an increase in their number.\(^{23,24}\) Thus, as in Kyle (1984) and in Admati and Pfleiderer (1988), a larger number of informed traders leads to better terms of trade for liquidity traders due to heightened competition among informed traders who all observe the same signal.

To provide a convenient basis for comparison let \(k_i = k\) for all \(i = 1, \ldots, N\).\(^{25}\) Let \(K \equiv \frac{(k+1)^2}{k}\) and \(G \equiv \frac{(q+1)^2}{q}\). Note that \(K\) and \(G\) rise increase with an increase in the numbers of systematic and security-specific informed traders respectively. The following Proposition then follows.

**Proposition 4** Under the above assumptions, discretionary liquidity traders prefer to trade in the basket if and only if

\[
\frac{P_s + P_m K}{\text{var}(l + r)} < \frac{P_s + P_m K}{\text{var}(l + m)}.
\]

Again, the basket is preferred if \(\text{var}(r) \geq \text{var}(m)\). As in the discussion following Proposition 3, we consider the case of \(\text{var}(r) < \text{var}(m)\). The advantage of the basket is still increasing in \(P_s\) and decreasing in \(P_m\). The intuition for this result remains unchanged from the intuition given in Section 5.1. We also see, however, that as the number of systematic traders grows large, the RHS of this equation reduces more relative to the LHS, increasing the parameter range under which the inequality holds. The explanation is found by examining the expressions (28) and (29) for \(\lambda_m\) and \(\lambda_l\) respectively. We see that if \(\text{var}(r) < \text{var}(m)\) the marginal impact of an increase in the number of systematic informed traders on the pricing parameters is more in the basket than in the securities. Equivalently, as the number of systematic informed traders increases, they compete more vigorously with each other,

\(^{23}\)This result could disappear if information traders were assumed to observe weakly correlated signals.

\(^{24}\)An alternative interpretation of this result is that the total order flow as a function of the information is now larger than in the monopolistic speculation case, thus a lower pricing parameter than before suffices to provide the competitive market maker adequate protection against informed trades.

\(^{25}\)Given the other assumptions, this is equivalent to assuming that the cost of acquiring security-specific information for each security is identical.
and the marginal benefit of increased competition is relatively more in the basket than in the individual securities due to the assumed smaller non-discretionary trades in the basket. A similar, but reverse intuition can be given describing the impact of an increase in the number of security specific traders. The above discussion implies that if \( \text{var}(r) < \text{var}(m) \), the advantage of the basket increases as the number of systematic informed traders increases relative to the number of security-specific informed traders. If \( \text{var}(r) \geq \text{var}(m) \), the basket is always preferred.

It is an empirical matter as to whether one would find more systematic or security-specific informed traders in financial markets. We do, however, claim that the precision of information possessed by stock-specific speculators is typically far higher than the precision of systematic information. Stock-specific information (e.g. hiring and firing of management, impending mergers/acquisitions) is usually possessed by insiders in the relevant company whose information is oftentimes very precise relative to the information of the general. On the other hand, systematic information typically consists of forecasts inferred from public information (e.g. published government statistics about macroeconomic factors) and systematic informed traders are typically characterized as agents who can process this information more efficiently than others. The forecasts of these traders are likely to be far noisier than the security-specific traders' signals.

We have shown earlier that if the non-discretionary trades are at least as high in the basket as in the securities then the basket is always preferred and that otherwise the advantage of the basket rises as the precision of the systematic information falls relative to the precision of security-specific information. Based on the above results and the observations in the preceding paragraph, we conclude by claiming that the tendency of (30) to hold in stock markets would be typically quite strong.

6 Concluding Remarks

We have attempted to define a new theoretical basis for markets in baskets of securities by claiming that these markets allow liquidity traders to realize their trades more efficiently.
The reason advanced is that their losses to informed traders are typically lower in baskets than in the individual securities.

With a large number of securities comprising the basket, the advantage of the basket was found to be substantial when only informed traders with security-specific information were assumed to trade in our economy. The presence of a monopolistic informed trader possessing systematic information reduced this advantage somewhat. The basket, however, was still preferred if the total non-discretionary liquidity trades were at least as high in the basket as in the securities. Otherwise, the advantage of the basket was found to decrease in the ratio of the precisions of systematic to security-specific information. Also, given the existence of at least one systematic informed trader, it was found that the advantage of the basket increased with an increase in the number of systematic informed traders relative to security-specific informed traders. We argue that the advantage of baskets in stock markets is strong because the precision of stock-specific information is typically far higher than that of systematic information.

The paper implies that large financial intermediaries, such as mutual funds, can be expected to trade heavily in baskets of securities. It thus provides a possible explanation for the growth in trading volume of the S&P 500 futures contract in the past few years. Another stylized fact which can at least partially be explained by this paper is the explosive growth of mutual funds. Net assets of mutual funds grew by an average of 1.7% per year in the 1967-81 period and by an average of approximately 75% per year in the period 1982-87. The year in which the period of growth of the mutual funds began coincides exactly with the year in which the S&P 500 futures contract was introduced. The implication of this paper is that the introduction of the futures contract would have provided considerable savings in trading costs for mutual funds and other large financial institutions. The dramatic increase in execution efficiency of trades thus provided by the stock index futures market could have triggered off the phenomenal growth in mutual funds.

Additional empirical implications of the paper are that the adverse selection problem

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(as reflected in percent bid-ask spreads) can be expected to be lower in markets for baskets of securities than in markets for individual securities. Also, upon introduction of a basket, bid-ask spreads can be expected to widen in individual securities comprising the basket due to lowered liquidity trading in the individual securities. The above implications can be tested using stock index futures and individual stock transaction data.

All our results are obtained by analyzing the strategic trading decisions of informed traders and uninformed liquidity traders. Possible further extensions of this work include extending the model to the case of diverse private information and making the information acquisition endogeneous to the model.
Appendix 1

Proof of Lemma 1: In choosing the quantity $x'_{1m}$ to trade in the market for the basket, the trader informed about security 1 maximizes

$$E(x'_{1m}(S_m - P_m)|\epsilon_1) ,$$

where $S_m$ is the true value of the basket and $P_m$ is its price. From the form of the pricing function, this is equivalent to

$$E((x'_{1m}(\epsilon_1 + \epsilon_2) - x'_{m1}\lambda_m(x'_{1m} + x'_{2m} + l + r)|\epsilon_1)) .$$

Let trader 1 conjecture that in equilibrium, trader 2's demand will be independent of $\epsilon_1$ (this conjecture proves to be consistent with the equilibrium). Then the optimal order size for trader 1 in the basket is $x_{1m} = \epsilon_1/2\lambda_m$. Then (12) follows by symmetry. The pricing parameter is given by

$$\lambda_m = \text{cov}(\epsilon_1 + \epsilon_2, x_{1m} + x_{2m} + l + r)/\text{var}(x_{1m} + x_{2m} + l + r) .$$

(31)

Substituting for $x_{1m}$ and $x_{2m}$ in (31), we get (13).

Proof of Lemma 2: The proofs of (18) and (19) are straightforward extensions of the proof of Lemma 1 and are therefore omitted. To obtain (20), we substitute for $\lambda_m$ and for each informed traders’ optimal trade into informed trader i’s given by

$$E((x_{im}(\sum_{i=1}^{N} \epsilon_i) - x_{im}\lambda_m(\sum_{j=1}^{N} x_{jm} + l + r)|\epsilon_i) ,$$

and take an unconditional expectation of the above expression.

Proof of Proposition 2: Discretionary liquidity traders prefer to trade in the basket if and only if

$$C_{Lm} < \sum_{i=1}^{N} C_{Li} ,$$

(32)

i.e. if they suffer lower losses due to trades with speculators in the market for the basket than in the market for individual securities. Equation (32) is equivalent to

$$\lambda_m < \sum_{i=1}^{N} \lambda_i .$$

(33)
Substituting for $\lambda_m$ and $\lambda_1, \ldots, \lambda_N$ from (19) and (10) completes the proof. □

**Proof of Lemma 3:** Let $x'_{sm}$ denote the optimal trade of the systematic informed trader in the basket. Then, he maximizes
\[
E((x'_{sm}(\sum_{i=1}^{N} \varepsilon_i + \gamma_m) - \lambda_m x'_{sm}(\sum_{j=1}^{N} x_{mj} + x'_{sm} + l + r))|\gamma + v).
\]
Let the trader make the consistent conjecture that the demands of all other traders are independent of $\gamma_m$. The solution to this problem then yields $x_{sm} = (\gamma_m + Nu)/2\lambda_m(1 + \kappa)$. Similarly, we find that the quantity traded by the security-specific speculator $j$ in the basket is $x_{jm} = (\varepsilon_j + u_j)/2\lambda_m(1 + \theta)$. Note that
\[
\lambda_m = \text{cov}(\gamma_m + \sum_{i=1}^{N} \varepsilon_i, \sum_{j=1}^{N} x_{jm} + x_{sm} + l + r)/\text{var}(\sum_{j=1}^{N} x_{jm} + x_{sm} + l + r).
\]
Substituting for $x_{sm}$ and $x_{jm}$ completes the proof of (24).

The systematic informed trader's objective function in security $i$ is given by
\[
E((x'_{si}(\varepsilon_i + \gamma) - \lambda_i x'_{si}(x_{si} + x_i + l + m))|\gamma).
\]
Again, if we let the traders conjecture their demands to be independent in equilibrium, maximization of the above yields $x_{si} = (\gamma + v)/2\lambda_i(1 + \kappa)$. Similarly, maximizing the objective function of the security-specific trader yields $x_i = (\varepsilon_i + u_i)/2\lambda_i(1 + \theta)$. Now
\[
\lambda_i = \text{cov}(\varepsilon_i + \gamma, x_i + x_{si} + l + m)/\text{var}(x_i + x_{si} + l + m). \tag{34}
\]
Substituting for $x_{si} + x_i$ yields (25). □

**Proof of Proposition 3:** Discretionary liquidity traders prefer the basket if (33) holds. Substituting for $\lambda_i$ and $\lambda_m$ from (24) and (25) and simplifying yields (26). □

**Proof of Lemma 4:** Let the security-specific informed trader informed about security $i$ make the consistent conjecture that the demands of all other security-specific traders informed about security $i$ in the basket is given by $\beta_i(\varepsilon_i + u_i)$ and that the demands of all other informed traders is independent of $\varepsilon_i + u_i$. Then this informed trader's objective function is
\[
E((x'_{im}(\sum_{i=1}^{N} \varepsilon_i) - x'_{im}\lambda_m(x'_{im} + (k_i - 1)\beta_i(\varepsilon_i + u_i) + \sum_{j \neq i} x_{jm} + nx_{sm}))|\varepsilon_i + u_i).
\]
The optimal trade of this trader is then

\[ x_{im} = \left[ \frac{1}{2\lambda_m(1 + \theta)} - \frac{(k_i - 1)\beta_i}{2} \right] (\epsilon_i + u_i) . \]

Setting this equal to \( \beta_i(\epsilon_i + u_i) \) yields

\[ \beta_i = \frac{1}{(k_i + 1)\lambda_m(1 + \theta)} \]

and so \( x_{im} = (\epsilon_i + u_i)/(k_i + 1)(1 + \theta)\lambda_m, \ i = 1, \ldots, N. \)

Similarly, it can be shown that

\[ x_{sm} = \frac{\gamma_m + N\nu}{(n + 1)(1 + \kappa)\lambda_m} . \]

Noting that \( \lambda_m = \text{cov}(\sum_{i=1}^{N} \epsilon_i + \gamma_m, \omega_m)/\text{var}(\omega_m) \) where \( \omega_m \) is the total order flow in the market for the basket and substituting for \( x_{im} \) and \( x_{sm} \) from above yields (28). Using a method similar to the above for the \( i \)th security market, we get \( x_i = (\epsilon_i + u_i)/(k_i + 1)(1 + \theta)\lambda_i \) and \( x_{si} = (\gamma + \nu)/(n + 1)(1 + \kappa)\lambda_i \). Now \( \lambda_i = \text{cov}(\epsilon_i + \gamma, \omega_i)/\text{var}(\omega_i) \), where \( \omega_i \) is the total order flow in security \( i \). Substituting for \( x_i \) and \( x_{si} \) yields (29). \( \square \)

**Proof of Proposition 4:** Losses to liquidity traders in the basket are \( C_{Lm} = \lambda_m \text{var}(l) \) and in security \( i \) are \( C_{Li} = \lambda_i \text{var}(l) \). Using condition (32) and substituting for \( \lambda_m \) and \( \lambda_i \) from (28) and (19) yields (30). \( \square \)
Appendix 2

This Appendix demonstrates that our basic results continue to obtain in the limit if discretionary liquidity traders are assumed to allocate their demands between the securities and the basket. Let us assume that they trade a fraction $w$ of their total demand in the securities and $1 - w$ in the basket. Thus, they are assumed to trade $wl$ shares in each of the securities and $(1 - w)l$ shares in the basket. The traders minimize their losses, taking the pricing parameters in each of the markets as given. Their objective function is

$$ (w^2 \sum_{i=1}^{N} \lambda_i + (1 - w)^2 \lambda_m) \text{var}(I) \ . $$

This is minimized when

$$ w = \frac{\lambda_m}{\sum_{i=1}^{N} \lambda_i + \lambda_m} \ . \quad (35) $$

Consider first the case of Section 4, where only security-specific speculators trade. Then $\lambda_m = (1/2) \sqrt{N \text{var}(e)/\psi_m}$ where $\psi_m$ is the total (finite) liquidity trade in the basket in equilibrium. Similarly, $\lambda_i = (1/2) \sqrt{\text{var}(e)/\psi}$, where $\psi$ is the total liquidity trade in security $i$. Note that the liquidity trades in all securities are the same, by virtue of the assumption that $\text{var}(m_i) = \text{var}(m)$ for all $i = 1, \ldots, N$. We do not attempt to solve for $\psi_m$ and $\psi$ as this is a complicated algebraic exercise and it is possible to find them in closed form only in certain special cases. Instead, we examine limiting results as $N \to \infty$. Substituting for $\lambda_m$ and $\lambda_i$ in (35), we have

$$ w = \frac{\sqrt{\text{var}(e)/\psi_m}}{\sqrt{N \text{var}(e)/\psi} + \sqrt{\text{var}(e)/\psi_m}} \ . \quad (36) $$

It is apparent that as $N \to \infty$, $w \to 0$, i.e. the trading becomes more and more concentrated in the basket as the number of securities in the basket becomes large. We thus have

**Proposition 2A** As the number of securities becomes large, the fraction of trades realized in the individual securities approaches zero.

Repeating the same exercise as above for the case of Section 5.1, where both systematic and security-specific traders trade, we have

$$ w = \frac{P_e/N + P_m}{\psi_m} \sqrt{\frac{P_e}{\psi} + \sqrt{\frac{P_e/N + P_m}{\psi_m}}} \ . \quad (37) $$
As $N \to \infty$,

$$w \to \frac{\sqrt{P_m/\psi_m}}{\sqrt{(P_s + P_m)/\psi} + \sqrt{P_m/\psi_m}}$$

Thus, with $N = \infty$, $w \to 0$ as $P_m/P_s \to 0$. This leads us to

**Proposition 3A** With a large number of securities, the trades tend to be more and more concentrated in the basket as the precision of systematic information becomes very small relative to the precision of security-specific information.

This result is similar to the one obtained in Section 5. The intuitions for Propositions 2A and 3A are similar to the intuitions provided in the main text for Propositions 2 and 3. We have thus shown that our results continue to hold in the limit if discretionary liquidity traders are allowed to allocate their trades between the securities and the basket.
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