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ABSTRACT

A particular case of a Mueller formula describing the absence of long-range two-particle correlations is used to predict double-pomeron cross sections from available data on single-diffraction dissociation. Although success of the formula in this application would immediately verify the double-pomeron hypothesis, the converse is not true because the formula may be invalidated by prominent low-energy resonances (such as the \( f_0 \)) in the pomeron-pomeron total cross section. Low-statistics experiments at \( s = 200 \) and \( 400 \text{ GeV}^2 \) are in order of magnitude agreement with the formula.

There now exists extensive experimental information on single-particle inclusive cross sections of the type \( A + B \rightarrow A + X_A \) illustrated by fig. 1a, in the region \( M_{X_A}^2/s \leq 0.1 \), \( s \) being the square of the total center of mass energy (1-3). Events in this region are often characterized as singly diffractive ("diffractive dissociation" of particle \( B \)) and many of their properties have been elucidated through the concept of pomeron exchange (the squared mass of the pomeron being

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It will soon be possible to study two-particle inclusive cross sections of the type $A + B \rightarrow A + B + X$ illustrated by fig.1b, in the region where both $\frac{M_{X_A}^2}{s}$ and $\frac{M_{X_B}^2}{s}$ are small.* Such events may tentatively be characterized as "double-pomeron exchange", but it remains uncertain whether their distribution and energy dependence will follow the detailed rules of double-pomeron factorization (4). This paper examines the consequences of a simple Mueller formula giving the two-particle inclusive cross section of fig.1b as a product of single-particle inclusive cross sections. This formula does not depend on (double) factorization of the two pomeron links exhibited in fig.1b but rather on (single) factorization at the central vertex. Nevertheless, to the extent that the Mueller formula turns out to be valid in the region where both $\frac{M_{X_A}^2}{s}$ and $\frac{M_{X_B}^2}{s}$ are small, one verifies double-pomeron factorization--because of the recently established experimental fact that within each of the two single-particle inclusive cross sections in the Mueller formula a factorizable pomeron plays a major role (1-3).

Even if valid only in order of magnitude and not in detail, this formula provides a concrete basis for the design of experiments seeking evidence concerning double-pomeron exchange. We shall argue that the formula has useful predictive power for the exclusive measurement

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* In this region, which for convenience we refer to as the "double-pomeron region", $\frac{M_{X_{A,B}}^2}{s} \approx 1 - |X_{A,B}|$, where $X_{A,B}$ are the Feynman longitudinal-momentum ratios. The two-particle inclusive experiment measures a fourfold differential cross section with respect to the variables $t_A, t_B, M_{X_A}, M_{X_B}$.
A + B → A + B + 2π in the double-pomeron region, the experimental study of which has recently been reviewed in Ref. (5).

We begin by recalling the variables introduced in Ref. (5),

\[ Z_{A,B} \equiv \ln \frac{s}{M^2_{X_{A,B}}} \tag{1} \]

and by identifying the two-dimensional single-particle inclusive differential cross section corresponding to fig. 1a:

\[ \frac{d\sigma^{A}}{dt_A dZ_A} \]

The notation for the corresponding cross section when B is the observed particle is obvious. The formula we propose to use for the two-particle inclusive cross section corresponding to fig. 1b is then

\[ \frac{d\sigma^{AB}}{dt_A dt_B dZ_A dZ_B} \approx \frac{1}{\sigma_{tot}^{AB}} \frac{d\sigma^{A}}{dt_A dZ_A} \frac{d\sigma^{B}}{dt_B dZ_B} \tag{2} \]

This formula is a special case of one proposed by Mueller (6), which describes an absence of two-particle correlations and which is usually supposed to become valid when the rapidity gap between the two observed particles is large. In the double-pomeron region (\( Z_A \) and \( Z_B \) both "large"), however, justification of formula (2) requires a more delicate consideration of the dependence on the central vertex in fig. 1b.

This central inclusive vertex, which may be described as proportional to a "pomeron-pomeron total cross section" (4) at total P-P center of mass energy \( M_x^2 \), is presumed to have a Regge expansion for large \( M_x^2 \) with a leading factorizable term proportional to
Formula (2) can be shown to follow to the extent that such a factorizable term, taken alone, is a reasonable approximation (7). Now, as will be shown below, for all double pomeron experiments in the foreseeable future the mean value of $M_x^2$ will be $< 1$ GeV$^2$, at first sight not large enough to justify pomeron dominance of the inclusive central vertex.

Two considerations may nevertheless be invoked to support the conjecture that formula (2) will be useful throughout a double-pomeron region defined as $Z_A Z_B > \ln 10$ without regard for the magnitude of $M_x^2$. (1) Dash and collaborators have discovered that a single factorizable effective (or "bare") pomeron with intercept $\alpha_p(0) \approx 0.85$ is capable of representing a large variety of diffractive data, both inclusive and exclusive, down to remarkably low energies (8).

(2) Dual-resonance models suggest that the (inclusive) pomeron controlling the large-$M_x$ region may be dual to resonances in external-pomeron cross sections and thus may give a reasonable average approximation at low $M_x$ (9).

It can be shown (7), that in a rough statistical sense

$$M_x^2 \approx s e^{-(Z_A Z_B)}$$

and henceforth in this paper the symbol $M_x$ is to be understood as meaning the quantity given by eq. (3).* If one wishes to improve formula (2) for low values of $M_x$ so as to include the effect of resonances such as the $f_0$, one may add to the right-hand side of this

* The actual cluster mass, as illustrated in fig. 11 of Ref. (5), lies somewhat lower.
formula an appropriate $M_x$-dependent factor. At the present time, however, it is unclear how much alteration is required of the low-$M_x$ extrapolation of the presumed smooth and factorizable behavior at large $M_x$—an extrapolation implicit in formula (2).

To simplify analysis of the implications of formula (2), let us fix the two variables $t_A$ and $t_B$, the limits on which in the double-pomeron region are almost independent of $s$, $Z_A$, and $Z_B$, and concentrate on the latter three variables. As shown in Ref. (5) the allowed region of $Z_A$ and $Z_B$ at fixed $s$ is approximately the triangular domain of Fig. 2, where the parameter $s_0$ has been estimated from the mean transverse momentum of produced particles to be $s_0 \approx 0.14$ GeV$^2$.** The double-pomeron region, by our definition, is the inner triangle where both $Z_A$ and $Z_B$ are larger than $\approx 10$.

Formula (2) makes detailed predictions about the distribution of events within the inner triangle which it will be important to verify. We confine ourselves here, however, to discussing the integral over this triangle.

It has been experimentally established that within the double-pomeron region the single-particle inclusive cross sections on the right-hand side of eq. (2) are slowly-varying functions of $Z_A$, $Z_B$, and $s$. To estimate the integrated magnitude of the double-pomeron

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* As emphasized in Ref. (7), an immediate indication of the breakdown of formula (2) would be given by the existence of a correlation between the azimuthal angles of the leading final particles $A$ and $B$.

** Formula (3) implies that $s_0$ is the minimum value of $M_x^2$. 
cross section we therefore take the observed values at the centroid of the double-pomeron triangle,

$$Z_A = Z_B = \overline{Z}(s) = \frac{1}{3} \ln \frac{10s}{s_0}$$  \hspace{1cm} (4)

and multiply by the area of this triangle

$$A_{D.P.}(s) = \frac{1}{2} \left( \ln \frac{s}{100s_0} \right)^2.$$  \hspace{1cm} (5)

We thus have

$$\frac{d\sigma_{AB}(s)}{dt_A dt_B} \cong \frac{A_{D.P.}(s)}{c_{tot}(s)} G_{AB}(s,t_A) G_{AB}(s,t_B),$$  \hspace{1cm} (6)

with

$$G_{AB}(s,t_A) = \left( \frac{d\sigma_{AB}}{dt_A dZ_A} \right)_{Z_A=\overline{Z}(s)}.$$  \hspace{1cm} (7)

If we roughly parametrize experimental information on the single-particle inclusive cross section by the (large $Z_A$, small $t_A$) form

$$\frac{d\sigma_{AB}}{dt_A dZ_A} \cong C_{AB} e^{-\epsilon Z_A} b_{AB}^{t_A},$$  \hspace{1cm} (8)

adequate for $pp$ collisions in the range $100 \text{ GeV}^2 \leq s \leq 1000 \text{ GeV}^2$, $2 \leq Z_A \leq 5$, $0 < t_A < 0.5 \text{ GeV}^2$, with $\epsilon \approx 0.5$, $b_{pp} \approx 6 \text{ GeV}^{-2}$, $C_{pp} \approx 23 \text{ mb} \text{ GeV}^{-2}$, then

$$G_{AB}(s,t_A) \cong C_{AB} \left( \frac{s_0}{10s} \right)^{\epsilon/3} e^{b_{AB}^{t_A}}.$$  \hspace{1cm} (9)

Integrating the form (8) over the triangle, without using the centroid-average approximation, gives in place of (6) the result
With a factorizable t-dependence as in the form (8), the integral over \( t_A \) and \( t_B \) is a simple multiple of the value at \( t_A = t_B = 0 \), the energy dependence remaining the same. Figure 1 shows the predicted total double-pomeron cross section on the basis of formulas (6) and (10).

If one of the incident particles is not a proton, one expects from factorization

\[
\frac{d\sigma_{AB}^{\text{el}}}{dt_A dt_B} = \frac{C_{AB} C_{BA}}{C_{\text{tot}}^{AB}} e^{b_{AB} t_A + b_{BA} t_B} \left\{ \left( \frac{1}{10} \right)^{2\epsilon} - \left( \frac{s_0}{s} \right)^\epsilon \right\} 
\]

\[
\left[ 1 + \epsilon \ln \left( \frac{s}{100 s_0} \right) \right] . \quad (10)
\]

With a factorizable t-dependence as in the form (8), the integral over \( t_A \) and \( t_B \) is a simple multiple of the value at \( t_A = t_B = 0 \), the energy dependence remaining the same. Figure 3 shows the predicted total double-pomeron cross section on the basis of formulas (6) and (10),

If one of the incident particles is not a proton, one expects from factorization

\[
\frac{d\sigma_{Ap}^{\text{el}}}{dt} / \frac{d\sigma_{pp}^{\text{el}}}{dt} = \frac{C_{Ap}}{C_{pp}} e^{(b_{Ap} - b_{pp})t} \quad (11)
\]

and

\[
\frac{C_{pA}^{\text{tot}}}{C_{pp}^{\text{tot}}} = \frac{C_{pA}}{C_{pp}} . \quad (12)
\]

For example, if particle A is a pion, formulas (11) and (12) give \( b_{\pi p} = 4 \text{ GeV}^{-2} \), \( C_{\pi p}/C_{pp} \approx 0.36 \) and \( C_{p\pi}/C_{pp} \approx 0.6 \). The corresponding \( \pi p \) total double-pomeron cross section is then predicted to be smaller than that in \( pp \) collisions by a factor \( \approx 0.5 \), but the energy dependence should be the same. The \( Kp \) double-pomeron integrated cross section is predicted by the same reasoning to be roughly half that of \( \pi p \).

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\* With an exponential dependence on \( t \) as in (8), the factor is \( 1/b_{AB} b_{BA} \).
We close with a discussion of the low $M_x^2$ region—which one expects to be dominated by central clusters consisting of two pions $(2/3 \pi^+\pi^- \text{ and } 1/3 \pi^0\pi^0)$. Much of the total double-pomeron cross section should lead to such clusters when $M_x^2 < 1 \text{ GeV}^2$ at the centroid of the double pomeron triangle, i.e., according to formulas (3) and (4), when

$$s e^{-2\Sigma} = s \left( \frac{s_0}{10 s} \right)^{2/3} \leq 1 \text{ GeV}^2,$$

or

$$s \leq 5 \times 10^3 \text{ GeV}^2.$$

For the present and the immediate future, in other words, a large proportion of double-pomeron events will be of the variety $A + B \to A + B + 2\pi$, and the foregoing cross-section estimates, if valid at all, may be applied to this special reaction. The experimental points in fig. 3 represent measured double-pomeron cross sections $(5,10)$ to produce $\pi^+\pi^-$, multiplied by a factor 3/2 to include $\pi^0\pi^0$ production.

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REFERENCES
1. Two recent surveys of the reaction $pp \to pX$ have been given by D. P. Roy and R. G. Roberts, Rutherford Laboratory preprint RL-74-022, T79 (January 1974), and by R. D. Field and G. C. Fox, Cal Tech preprint CALT-68-434 (May 1974).
3. For a review of all data available on diffractive processes of the type $\pi p \rightarrow pX$ and $Kp \rightarrow pX$, see D. W. G. S. Leith, SLAC-PUB-1330 (October 1973) and the supplement to the foregoing (November 1973).


10. The plotted value corresponds to data taken from E. L. Berger et al., CERN/D.Ph.II/Phys 74-26 submitted to the London Conference on High Energy Physics (July 1974), together with the definition of double-pomeron cross section proposed in Ref. [5] and employed consistently throughout the present paper.
FIGURE CAPTIONS

Fig. 1. (a) Diagram defining the variables appropriate to the description of single diffraction dissociation (single pomeron exchange).

(b) The variables needed to describe double pomeron exchange.

Fig. 2. The triangular phase-space plot showing the double-pomeron region.

Fig. 3. The integrated double-pomeron cross section as a function of lab momentum. The dashed curve is the centroid-average approximation for pp, while the solid curves represent exact integrations of the form (10) over the double-pomeron triangle. The experimental points correspond to the measured cross section for a central $\pi^+\pi^-$ cluster, $(5,10)$ multiplied by $3/2$. 
Fig. 1
Fig. 2
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