Title
ASYMPTOTIC BEHAVIORS OF ELECTROPRODUCTION AMPLITUDES OF ONE HADRON

Permalink
https://escholarship.org/uc/item/8k54p2d6

Authors
Matsuda, Satoshi
Suzuki, Mahiko.

Publication Date
1969-11-01
ASYMPTOTIC BEHAVIORS OF ELECTROPRODUCTION AMPLITUDES OF ONE HADRON

Satoshi Matsuda and Mahiko Suzuki

November 1969

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY
This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
Asymptotic Behaviors of Electroproduction Amplitudes of One Hadron

SATOSHI MATSUDA
California Institute of Technology, Pasadena, California

and

MAHIKO SUZUKI**
Physics Department and Lawrence Radiation Laboratory
University of California, Berkeley, California

(Received November, 1969)

ABSTRACT

Asymptotic behaviors for electroproduction of one hadron in the limit of $q^2 \to \infty$ (spacelike), with $(\text{laboratory energy})/q^2$ fixed large are derived by means of the Bethe-Salpeter equation which takes account of the vector property of the photon.


** On leave from the University of Tokyo, Japan. Present address: Physics Department, Columbia University, New York, New York 10027.
I. INTRODUCTION

The structure functions of inelastic electron-nucleon scattering have become of considerable experimental and theoretical interest. Bjorken\(^1\) has put forth a conjecture called a scaling rule on the asymptotic behaviors of these functions in accord with the experimental measurement.\(^2\) Combining his conjecture with Regge asymptotic behaviors, Abarbanel, Goldberger, and Treiman\(^3\) have argued that the Pomeranchon in the Regge asymptotics also dominates in the Bjorken limit\(^4\) with \(2mv/q^2\) fixed large where \(q^2\) and \(v\) are the squared four-momentum and the laboratory energy of the photon, respectively. This line was subsequently pursued by Drell, Levy, and Yan,\(^5\) and also by Altarelli and Rubinstein\(^6\) in the ladder approximation to the generation of Regge particles. It was thus argued that the scaling rule is derived from the Pomeranchon contribution. It is, however, believed according to recent developments in hadron physics that the Pomeranchon is generated by a mechanism entirely different from that for the other (ordinary) Regge trajectories. The ladder approximation would be poor for the Pomeranchon so that some doubt may be cast on the \(q^2\) dependence of the structure functions at high energies, while it will be a fair approximation to generating the \(\rho\)-, the \(f\)-, and other trajectories. On the experimental side, people are now planning to measure individual hadrons produced in electroproduction processes.

We are thus motivated to look at the two-body electroproduction processes (like \(e + N \rightarrow e + N + \pi\)) at high energies with a large "photon mass" in the ladder approximation to the ordinary Regge trajectories. We shall use the parametric representation of the solutions to the Bethe-Salpeter equation with a scalar particle exchange which takes proper
account of the vector property of the photon. In contrast to the virtual Compton scattering with highly virtual photons, we have no chance to make use of current algebra techniques in processes where only the incident is highly virtual. Our approach using the Bethe-Salpeter equation would therefore make much sense. We shall discuss the validity of the present model and make a few comments in a final section.

II. SOLUTION TO BETHE-SALPETER EQUATION

We begin with the Bethe-Salpeter equation of scalar rungs as shown in Fig. 1a, 7)

\[
(m^2+k_1^2)(m^2+k_2^2) f(k_1^2, k_2^2, s, t) = \frac{1}{\mu^2 - s - i\epsilon}
\]

\[
+ \frac{\lambda}{\pi^2} \int d^4k_1' \frac{f(k_1'^2, k_2'^2, (p_1 + k_1')^2, t)}{\mu^2 + (k_1 - k_1')^2 - i\epsilon},
\]

where

\[
p_1^2 = p_2^2 = -m^2, \quad s = -(p_1 + k_1)^2, \quad t = -(k_1 - k_2)^2.
\]

The solution is not obtained explicitly, but it is known to satisfy the following integral representation 8)

\[
f(k_1^2, k_2^2, s, t) = \frac{1}{\epsilon} \int dy \int dz \int_0^\infty dy \phi(y, z, \gamma, t)
\]

\[
\left\{y + (1-y) \left[\frac{1}{2} (1+z)(k_1^2+m^2) + \frac{1}{2} (1-z)(k_2^2+m^2) \right] - y(s-m^2) - i\epsilon \right\}^{-3},
\]

where the spectral function \(\phi(y, z, \gamma, t)\) does not depend on any of \(s, k_1^2\) and \(k_2^2\), but on \(t\). The function \(\phi(y, z, \gamma, t)\) satisfies an integral equation
\( \phi(y,z,r,t) = (1-y) \delta(y) + \lambda \int dy' \int dz' \int dy' K(y,z,r; y',z',r'; t) \)

\[ \phi(y',z',r',t) = \frac{1}{2} y(1-y) \Theta(y'-y) \Theta(R(z,z')) - \frac{y(1-y')}{y'(1-y)} \]

\[ \int_0^1 (1-x)^2 dx \delta'(x(1-x)y', y - y \{ (1-x) \gamma' + [x+(1-x)^2 y'] \mu^2 \]

\[ + (1-x)^2 (1-y')^2 \rho(z) \}) \]

where \( R(z,z') = (1+z)/(1+z') \) for \( z > z' \)

and \( \rho(z) = m^2 - \frac{1}{4} (1-z^2) t \)

We find from the analytic property of the kernel that:

(i) \( \phi(y,z,r,t) \) is singular at \( y = 0 \) like \( [(1-z^2)/y]^{\alpha(t)+1} \), if a Regge pole exists at \( t = \alpha(t) \);

(ii) it is regular elsewhere in \( y \) and \( z \) for \( y \in [0,1] \) and \( z \in [-1,1] \);

(iii) it falls off rapidly like \( \gamma^{-1} \) as \( \gamma \to \infty \).

To construct the electropionproduction amplitude out of \( f(k_1^2,k_2^2,s,t) \), we add one more rung, affix a photon line and a pion line to the extreme ends (see Fig. 1b), and supplement the Born term and a possible seagull term.

III. ELECTROPIONPRODUCTION AMPLITUDE

The two invariant matrix elements are defined as

\[ M_\mu = M_1(q_1^2,\nu, t) \left( q_2 \mu - \frac{(q_1 \cdot q_2)}{q_1^2} q_{1\mu} \right) + M_2(q_1^2, \nu, t) \left( p_\mu - \frac{(q_1 \cdot p)}{q_1^2} q_{1\mu} \right) \]
where \( t = -(q_1^2 - q_2^2) \), \( \nu = -(p_1 \cdot q_1)/m \), and \( q_2^2 = -m_\pi^2 \). The ladder diagram for the electropion production leads us to

\[
M_1(q_1^2, \nu, t) = -\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_{-1}^0 d\gamma \left( \frac{1}{2} \right) (1-x)^3 (1-z)(1-y) \phi(y, z, \gamma, t) (\nu - i\epsilon)^2 ,
\]

\[
M_2(q_2^2, \nu, t) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_{-1}^0 d\gamma (1-x)^3 \phi(y, z, \gamma, t) (\nu - i\epsilon)^2 ,
\]

\[
Q = (1-x) \gamma + x m^2 + (1-x)^2 y \mu^2 + (1-x)^2 (1-y)^2 \left[ m^2 - \frac{1}{4} (1-z^2) t \right] - x(1-x) \left[ \frac{1}{2} (1-y)(1-z)(q_1^2 - m^2) + \frac{1}{2} (1-y)(1-z)(m_\pi^2 - m^2) + y (s - \mu^2) \right]
\]

where \( x \) is a new parameter introduced through Feynman parametrization.

The crossed ladder diagram is obtained simply by interchanging \( p_1 \) and \( p_2 \), flipping the signs of \( q_1 \) and \( q_2 \), and replacing \( i\epsilon \) by \(-i\epsilon\).

In accordance with the remark in Ref. 7, these amplitudes may be thought of as electroproduction amplitudes of the nucleon target averaged over spin. Another remark is that because of the approximation we choose, the amplitudes, as they stand, do not satisfy the gauge invariance in general. We have therefore picked up the gauge invariant terms alone.

In the high-energy limit of \( \nu \to \infty \) with \( q_1^2 \) fixed, \( M_1 \) and \( M_2 \) exhibit the usual Regge asymptotic behaviors of \( \nu \gamma(t) \) and \( \nu \gamma(t) - 1 \), respectively. In fact, by replacing \( s y/(1-z) \) by a new variable, we find that the Regge behaviors are factored out of \( M_1 \) and \( M_2 \). Now we go to the limit of \( q_2^2 \to \infty \) with \( 2m \nu/q_1^2 \) fixed large. Watching carefully the function \( Q \) and keeping in mind the analytic properties of \( \phi(y, z, \gamma, t) \).
listed in the previous section, we replace the variables \((s/q_1^2)y/(1-z^2)\) and \(xq_1^2\) by new variables \(\eta\) and \(\xi\), respectively. We then find the Regge behaviors in the variable \(s/q_1^2\) or \(2m\sqrt{q}/q_1^2\). Their leading terms are given as

\[
M_1 - \beta_1(t) \frac{1}{q_1^2} \left( - \frac{2m\sqrt{q}}{q_1^2} \right)^{\alpha(t)}
\]

(12)

\[
M_2 - \beta_2(t) \frac{1}{q_1^2} \left( - \frac{2m\sqrt{q}}{q_1^2} \right)^{\alpha(t)-1}
\]

(13)

\[
\beta_1(t) = - \int_0^\infty d\xi \int_0^\infty d\eta \int d\gamma \int d\gamma \, (\frac{1}{2}) (1-z(1-y)) X(z,\gamma, t) (R - i\epsilon)^{-2}
\]

(14)

\[
\beta_2(t) = \int_0^\infty d\xi \int_0^\infty d\eta \int d\gamma \int d\gamma X(z,\gamma, t) (R - i\epsilon)^{-2}
\]

(15)

\[
R = \gamma + (m^2 - \frac{1}{4} (1-z^2) t) + i \left[ \frac{1}{2} (1+z) - (1-z) \eta \right]
\]

(16)

where \(X(z,\gamma, t)\) is defined by the relation

\[
\varphi(y, z, \gamma, t) = \left[ (1-z^2)/y \right]^{\alpha(t)+1} X(z, \gamma, t)
\]

(17)

Adding the crossed ladder gives the correct signatures to \(M_1\) and \(M_2\). Non-leading Regge terms are also obtained in the same way.

The asymptotic behaviors \(M_1\) and \(M_2\) given in Eqs. (12) and (13) are rewritten in terms of the transverse and longitudinal cross sections defined by Hand\(^9\) as

\[
\frac{d}{dt} \sigma_T(q_1^2, \nu, t) \rightarrow \left( \frac{1}{q_1^2} \right)^4 \left( \frac{2m\sqrt{q}}{q_1^2} \right)^{2\alpha(t)-2}
\]

(18)

\[
\frac{d}{dt} \sigma_L(q_1^2, \nu, t) \rightarrow \left( \frac{1}{q_1^2} \right)^3 \left( \frac{2m\sqrt{q}}{q_1^2} \right)^{2\alpha(t)-2}
\]

(19)
as $q_1^2 \to \infty$ with $2m\nu/q_1^2$ fixed large. The trajectory $\alpha(t)$ may be any ordinary trajectory, but not the Pomeranchuk trajectory, for which the whole calculation would be doubtful.

If one connects our amplitudes by unitarity, one would obtain the lower bound of the two structure functions $W_1$ and $W_2$ in the inelastic electroproduction. With the usual definitions of $W_1$ and $W_2$, we find

$$W_1(q^2, \nu) \geq \left( \frac{1}{q_1^2} \right)^3 \left( \frac{2m\nu}{q_1^2} \right)^{2\alpha(0) - 1} \left[ \log \frac{2m\nu}{q_1^2} \right]^{-1}, \quad (20)$$

$$W_2(q^2, \nu) \geq \left( \frac{1}{q_1^2} \right)^3 \left( \frac{2m\nu}{q_1^2} \right)^{2\alpha(0) - 3} \left[ \log \frac{2m\nu}{q_1^2} \right]^{-1}, \quad (21)$$

where the right-hand sides are the lower bounds obtained by unitarizing the electropionproduction amplitudes. Experimentally, the scaling rule seems to hold for $W_1$ and $W_2$ so that our $M_1$ and $M_2$ do not contradict with them.

IV. COMMENTS

In the present model we have not taken account of momentum dependences of the vertex function. It may change our conclusion on the $q_1^2$ dependence by the amount determined by the momentum dependence of the vertex functions. There may be some other reasons to choose constant vertex functions, but we shall present here a new argument in favor of a constant vertex in the case of scalar hadrons. The three-body scalar vertex is defined by a vacuum expectation value as

$$\int d^4x \int d^4y \ e^{iP_a x + iP_b y} \langle 0 | T(\phi_a(x) \phi_b(y) \phi_c(0)) | 0 \rangle$$

$$= \Delta_a(p^2) \Delta_b(p^2) \Delta_c(p^2) \Gamma(p_a^2, p_b^2, p_c^2) \ , \quad (22)$$
where \( p_a + p_b + p_c = 0 \), and \( \Delta(p^2) \) is a scalar-meson propagator. To avoid complications, let us assume that \( \Gamma(p_a^2, p_b^2, p_c^2) \) is totally symmetric in \( p_a^2, p_b^2, \) and \( p_c^2 \). The argument goes through in the asymmetric cases too. In the limit of \( (p_c)_4 = - (p_a + p_b)_4 \to \infty \) with \( p_c^2 \) fixed, we can use Bjorken's technique. The left-hand side of Eq. (22) turns out to be of the fourth power of \( (p_c^4)^{1} \). In fact, the coefficient of \( (p_c^4)^{-1} \) vanishes because of canonical commutation relations, the coefficient of \( (p_c^4)^{-2} \) vanishes on the assumption that \( \langle 0 | \phi(x) | 0 \rangle = 0 \) for any field describing a physical particle, and the coefficient of \( (p_c^4)^{-3} \) drops also as a consequence of equations of motion in which the source functions do not involve a derivative of \( \phi \). Therefore, comparing both sides of Eq. (22), we deduce

\[
\Gamma(p_a^2, p_b^2, p_c^2) \to \text{const.} \quad (23)
\]

as \( p_c^2 \to \infty \) with \( p_a^2 \) fixed. Repeating the same argument in different limits, we find that \( \Gamma(p_a^2, p_b^2, p_c^2) \) behaves like a constant in all limits. This implies that a constant vertex is a good approximation as far as the asymptotic behaviors are concerned. The \( q^2 \) dependence in the low \( q^2 \) region will be naturally unreliable. The argument given above indicates that multiplying damping form factors at each vertex may be quite misleading. The electromagnetic form factor would not fall off so rapidly as \( (q^2)^{-2} \) when one or two of the hadron lines attaching the photon line are off the mass shell. In more general cases where hadrons have spins and derivatives of boson fields enter in the interaction Lagrangian, the argument may not go through exactly in the same way. We have presented this argument to make less unreasonable the assumption of constant vertex functions.
The other comment is in connection with finite energy sum rules. We have found that the contributions from ordinary trajectories do not fall off so rapidly as the electromagnetic form factor as \( q_1^2 \to \infty \). It was argued by Harari\(^{11}\) in the case of the virtual Compton scattering that the contributions from ordinary trajectories must fall off rapidly to be consistent with finite energy sum rules. The idea was as follows: The finite energy sum rules connect the low-lying resonance contribution in the s- (and u-) channel to the Regge contributions in the t-channel. Since the electromagnetic form factors of transitions between low-lying resonances are expected to fall off rapidly, the Regge residue in the t-channel also must fall off in conformity with them. However, we are able to avoid this difficulty in the following way. When we write a finite energy sum rule with the upper bound of the integral being \( \nu_{\text{cut}} \), we choose \( 2m v_{\text{cut}}/q_1^2 \) to be a large finite number, say \( L \), to assure the dominance of a few leading Regge trajectories. For a small value of \( q_1^2 \), \( \nu_{\text{cut}} = q_1^2 L/2m \) is not very large. But, as \( q_1^2 \) increases, we must choose larger and larger values of \( \nu_{\text{cut}} \) in order to maintain the same accuracy for the finite energy sum rule. Since \( s \sim 2mv \), higher resonances contribute to the sum rules more and more as \( \nu_{\text{cut}} \) increases.

The transition form factor from the nucleon to higher resonances may be well expected to damp mildly enough to be consistent with our predictions. In the other way around, if one increases \( q_2^2 \) alone with \( \nu_{\text{cut}} \) fixed in the sum rules, exchanges of lower Regge trajectories become more and more important so that the \( q_2^2 \) dependence of a single Regge trajectory does not describe that of the whole amplitude.
In conclusion, we feel that our results of the asymptotic behaviors of electroproduction amplitudes may hold in more realistic cases although they are derived rather in a simplified model. They will be tested by experiment quite soon.

ACKNOWLEDGMENTS

We would like to thank Professor G. F. Chew for the kind hospitality of the Lawrence Radiation Laboratory. One of us (M.S.) is grateful to Professor S. Mandelstam for the hospitality at the Department of Physics, University of California at Berkeley, and his colleagues for helpful conversations at the Department of Physics, University of Tokyo.
REFERENCES

7. The target particle is treated as a scalar particle while the photon as a vector particle. Strictly speaking, therefore, our conclusions apply to the electroproduction with, for instance, a pion target. But when we are interested in amplitudes averaged over the target spin, we expect that this simplification will not affect our main conclusions.
9. L. N. Hand, Phys. Rev. 129, 1834 (1963). Our $\sigma_T(q_1^2,\nu,t)$ and $\sigma_L(q^2,\nu,t)$ correspond to $\sigma_{\text{transverse}}(q_1^2,K)$ and $\sigma_{\text{scalar}}(q^2,K)$, respectively.
10. In the foregoing Refs. 3, 5, and 6, the derivation of the scaling rule is always based on a constant vertex function.
FIGURE CAPTIONS

Fig. 1a: Ladder diagram of Feynman amplitude for hadron-hadron scattering.

Fig. 1b: Electropion production process.
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.