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PART III: APPLICATIONS

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Author
Narasimhan, T.N.

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T. N. Narasimhan and P. A. Witherspoon

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NUMERICAL MODEL FOR SATURATED-UNSATURATED FLOW IN DEFORMABLE POROUS MEDIA
PART III: APPLICATIONS

T. N. Narasimhan and P. A. Witherspoon
Earth Sciences Division, Lawrence Berkeley Laboratory
and
Department of Materials Sciences and Mineral Engineering,
University of California, Berkeley, California 94720

ABSTRACT

This is the third and conclusive part of a three-paper series and describes the application of a numerical model for saturated-unsaturated flow in deformable porous media. In all, ten illustrative examples are presented not only to demonstrate the validity of the method but also to highlight the fundamental unity that exists in the basic principles of the fields of hydrogeology, soil mechanics, and soil physics. The chosen examples involve such diverse phenomena as soil consolidation, infiltration and drainage, and generation of fluid pressures due to cyclic loading such as earthquakes.
INTRODUCTION

A theory, as well as a numerical algorithm were presented in Parts I and II, respectively, of this series of papers for numerical analysis of fluid flow in saturated-unsaturated, deformable porous media (Narasimhan and Witherspoon, 1977; Narasimhan, et al., 1978). In this concluding part we present examples illustrating the validity and applicability of the model to a variety of problems related to the flow of water in subsurface systems. The examples have been chosen from the fields of soil mechanics, soil physics and hydrogeology and have been selected partly to illustrate the power of the numerical model and partly to demonstrate the conceptual unity that underlies these fields. The sample problems relate to purely saturated flow with soil deformation; purely unsaturated flow; saturated-unsaturated flow, and finally, the generation of pore pressure in soil masses due to earthquake vibrations. A list of the examples is given below:

I. Saturated Flow with Consolidation:
   a. Consolidation in a heterogeneous, doubly-draining clay column
   b. Simulation of field consolidation observed at Pixley California due to periodic water level changes
   c. Consolidation of a column of clay slurry

II. Unsaturated Flow
   a. Infiltration into a moderately saturated soil
   b. Infiltration into a column of extremely dry soil
   c. Axisymmetric flow to a soil-water sampler

III. Saturated- Unsaturated Flow
   a. Drainage from a one-dimensional soil column
   b. Drainage from a sand box
   c. Consolidation around an excavation in soft clay

IV. Pore Pressure Generation and Dissipation
   a. Pore pressure generation and dissipation in a three-dimensional system.

All the aforesaid examples were solved using the computer program TRUST described earlier (Narasimhan, et al., 1978).
SATURATED FLOW WITH CONSOLIDATION

Consolidation in a Heterogeneous Doubly-Draining Clay Column

The problem of land settlement associated with clayey soils is of considerable interest in the field of soil mechanics. To study such problems, the one-dimensional consolidation theory of Terzaghi (1925) has been used to great practical advantage by many soil engineers with the assumption that horizontal strains are negligible and that all the deformation takes place in the vertical direction. Although the Terzaghi theory provided reasonable results in many field cases, it is known that in many other field situations, the theory predicted smaller settlements or consolidation than was actually observed. While there may be several plausible explanations for the discrepancy between theory and field observations, Schiffman and Gibson (1964) reasoned that the variation of permeability with depth in clayey materials may be an important factor. To verify their hypothesis, they considered a doubly-draining, 30.48 meter column of London Blue Clay (Figure 1) with depth-dependent permeability and compressibility coefficients (Figure 2). Treating $K$ and $m_v$ as continuous functions of depth, and using a finite difference scheme, Schiffman and Gibson computed compaction as a function of time. The same problem was also solved by our numerical model, dividing the column into ten different materials with step-wise variation of material properties (Figure 2) and using 100 volume elements. A comparison of our numerical results and those of Schiffman and Gibson (1964) is presented in Figure 3. As shown in Figure 3, the agreement between the two results is reasonable and the small differences observed are probably to be attributed to the different techniques employed for handling the spatial variation of material properties. In addition to computing the gross consolidation of the clay column, the solution procedure also yielded data on the propagation of the strain wave within the column (Figure 4). As one would expect, at early times, the regions close to the top and the bottom of the doubly-draining system experience maximum deformation rates. With time, however, the strain-field gradually migrates inward as the interior portions of the system, as seen in Figure 4.
Simulation of Field Consolidation Observed at Pixley, California

The subsidence of land surface in many young alluvial basins due to the withdrawal of groundwater and the consequent decline in pore-fluid pressures is widely known (Poland and Davis, 1969). An extremely well documented field example of this occurs near Pixley in the San Joaquin Valley of California. The subsidence of land surface as well as the associated piezometric levels near Pixley have been measured carefully over several years by the U.S. Geological Survey (Lofgren and Klausing, 1969), and are presented in Figure 5. Recently, Helm (1975) successfully modeled observed compaction at Pixley over a twelve-year period from 1958, treating the observed piezometric fluctuations as the mechanism causing subsidence.

Within the alluvial sediments at Pixley, there exist 21 compacting beds of various thicknesses, aggregating a total of 85 m and separated by hydraulically continuous, highly permeable sand beds. In his mathematical simulation of Pixley subsidence, Helm (1975) assumed that practically all the observed consolidation could be accounted for in the clay beds, and consolidation in the sand beds was negligible. Furthermore, he found that for purposes of mathematical simulation, the actual system can be treated as equivalent to seventeen uniform beds, each 4.877 m thick and having $K' = 2.9 \times 10^{-11}$ cm/sec, $S_{s,\text{virgin}} = 7.54 \times 10^{-4}$ m$^{-1}$ and $S_{s,\text{elastic}} = 1.51 \times 10^{-4}$ m$^{-1}$. Here, $S_{s,\text{virgin}}$ denotes the specific storage coefficient under normal loading conditions while $S_{s,\text{elastic}}$ denotes the same coefficient under conditions of rebound or recompression.

To validate our numerical model with an example from the field, the Pixley consolidation was simulated as a one-dimensional consolidation problem, modeling one of the seventeen doubly-draining clay layers. On symmetry considerations, only one half of the clay column need be modeled.

The column was discretized into 20 elements, each 0.122 m thick and subjected to a time-dependent boundary condition, consisting of hydrographs of day by day variations in the piezometric heads in the aquifer. The initial distribution of pore pressure was assumed to be hydrostatic, while that preconsolidation stress was assumed to vary parabolically from the bottom to the center of the bed, as was done by Helm (1975). The initial and boundary conditions used are
shown in Figure 6. The simulation was carried for 4000 days from October 21, 1958. A comparison of the numerical results obtained during the present study with the observed field data is presented in Figure 7, which shows reasonable agreement between the two, with a maximum deviation of about 7 percent in early 1964. A similar accuracy was also obtained by Helm (1975). For the period, 1966-68, Helm obtained slightly better agreement than this study. In this regard, it may be pointed out that Helm plotted the cumulated compaction of 21 beds of varying thicknesses, while Figure 7 relates to 17 beds of the same thickness.

Consolidation of a Column of Clay Slurry

An extreme case of the soil consolidation phenomenon is that of shrinkage of an active clay such as Bentonite. Such materials can undergo very large changes in volume over relatively short intervals of time. The movement of water in such swelling soils may sometimes be of interest e.g., self-weight filtration (Smiles, 1974). An important task here is to establish the hydraulic conductivity of such materials as a function of void ratio.

Smiles and Rosenthal (1968) developed a theory for the one-dimensional movement of water in clay-water systems that are free to swell in the direction of water movement. This theory was developed from the fact that Darcy's law describes fluid flow with reference to the movement of soil particles, and is based on a scale of length defined with reference to the distribution of the mass of clay. In the new material scale, the diffusion equation is an appropriate expression to describe fluid movement.

Smiles and Rosenthal considered the desorption (drainage) of electrolyte from a semi-infinite column of saturated clay that is restrained within smooth cylindrical walls, but is free to contract in the direction of the cylindrical axis. For this system, they defined a new material scale,

\[ m = \int_{0}^{x} \frac{dx}{1 + e} \]
In this scale, the continuity equation for an elemental volume \( dV \) can be shown to be

\[
\frac{\partial}{\partial t} \left( D_m \frac{\partial e}{\partial m} \right) = \frac{\partial e}{\partial t}
\]  

(2)

in which

\[
D_m = \frac{K \frac{\partial x}{\partial e}}{1 + e \frac{\partial e}{\partial m}}
\]  

(3)

The semi-infinite column is subject to the following initial and boundary conditions

\[
e = e_n, \ m > 0, \ t = 0
\]

\[
e = e_0, \ m = 0, \ t > 0
\]  

(4)

Using Boltzmann's transformation, \( \lambda = mt^{-1/2} \), as suggested by Bruce and Klute (1956), the quantity \( D_m \) can be experimentally determined by,

\[
D_m(e) = -1/2 \frac{d\lambda}{de} \int_{e_n}^{e} \lambda de
\]  

(5)

If valid, this theory must lead to the following consequences:

1. The moisture content, or, equivalently, the void ratio, must be uniquely related to \( \lambda \);
2. The cumulative outflow from the column, or equivalently, the change of length of the column must plot as a straight line with the square root of time;
3. The slope of the straight line in 2 should be numerically equal to the integral

\[
\frac{dL}{dt^{1/2}} = \int_{e_n}^{e} \lambda de
\]

(6)

In order to verify their theory, Smiles and Rosenthal conducted a set of experiments with a 13.5 cm tall perspex column filled with a slurry of Wyoming Bentonite at an initial void ratio of \( e_n = 44.76 \) (porosity = 97.81 percent). A constant air-pressure of 1.66 m of water was applied at the top while at the bottom, \( m = 0 \), water was allowed to drain freely through a milli-pore filter, as shown schematically in Figure 8. The void ratio \( e_0(m = 0, t > 0) \) was maintained at 15.91 (94.08 percent). The non-linear relationship between \( e \) and effective stress for Wyoming Bentonite was independently determined experimentally and this relation is plotted in the form of a semi-log plot in Figure 9a. The slope of the straight line in Figure 9a is the compression index \( C_c \), for Wyoming Bentonite.

The desorption experiments were carried out for three weeks, during which the shrinkage of the column at various times and the void ratios within the column at various times were measured. Graphs of shrinkage versus (time)\(^{1/2}\) and \( e \) versus \( \lambda \) were plotted. As predicted by theory, shrinkage versus time\(^{1/2}\) was indeed found to plot as a straight line with a slope of \( dL/dt^{1/2} = 3.8 \times 10^{-3} \text{ cm/sec}^{1/2} \). Also according to theory, there was a fairly well defined relation between \( \lambda = m^{1/2} \) and void ratio, and on graphical evaluation, the integral,

\[
\int_{e_n}^{e_0} \lambda de
\]

yielded a value of \( 3.2 \times 10^{-3} \text{ cm/sec}^{1/2} \). The difference between this value and the value of \( 3.8 \times 10^{-3} \text{ cm/sec}^{1/2} \) (see equation 6) was attributable to experimental errors.
Using the \( \lambda \) versus \( e \) relation, Smiles and Rosenthal proceeded to compute \( D_m(e) \) using (5). The value of \( D_m(e) \), so computed, considered in conjunction with the \( e \) versus effective stress relation shown in Figure 9a, directly leads to the evaluation of hydraulic conductivity \( K \) of the Wyoming Bentonite as a function of void ratio (Figure 9b).

The numerical model developed by us was applied to the Smiles-Rosenthal problem for the reverse purpose of predicting shrinkage, assuming the known moisture characteristic relation 9a and the \( K \) versus \( e \) relation deduced from Smiles-Rosenthal experiments.

It should be pointed out here that the Boltzmann transformation is strictly applicable to a horizontal column. For convenience, however, the experiment had to be conducted with the column in a vertical position. Nevertheless, the potential gradient component due to gravity was less than 8 percent and, hence, could be neglected.

The numerical simulation was carried out to investigate:

1. whether the shrinkage computed with the numerical model agreed reasonably with the actual observations.
2. whether the role of gravity was important.
3. the extent to which consideration needs to be given to a material coordinate system in studying the highly compressible system under consideration.

Lagrangian formulation. Under saturated flow conditions, the discretized equations for an element \( n \) may be written as

\[
\sum_{m} K_{m,n} \frac{(z_m - z_n) + (\psi_m - \psi_n)}{(d_{n,m} + d_{m,n})} \Gamma_{m,n} = (M_c)_n \frac{\Delta \psi_n}{\Delta t}
\]  

(7)

Note that (7) is actually related to a material element fixed in the soil particles (Narasimhan and Witherspoon, 1977). In other words each element is defined to have a constant volume of solids.

Since the bulk volume of the element \( n \) changes with time, it follows that the geometrical quantities \( \Gamma_{m,n} \), surface area between elements \( m \) and \( n \), \( d_{m,n} + d_{n,m} \), the distance between nodal points \( m \)
and n and \( z_m, z_n \), the elevations of nodal points, should all change with time. In evaluating the left hand side of (7), therefore, these time-dependent changes must be given due consideration. For slightly deforming systems, it is customary to ignore these time dependent changes. However, can they be ignored without loss of accuracy in the case of a highly consolidating column of clay slurry? To investigate this we pose the problem in Lagrangian coordinates, as described below.

Note, that because the problem under consideration is one-dimensional, the surface areas, \( \Gamma_m, n \) in (7), are invariant with time. In order to take into account the temporal variations of the z's and d's in (7), the coordinates of nodal points were recalculated at the end of each time step in the light of past deformation history. In addition, the time rates of change of z and d were also estimated at the end of each time step. Using these data, the mean values of \( \bar{z}_m, \bar{z}_n, \bar{d}_m, n, \) and \( \bar{d}_n, m \) were computed in the same manner as estimating other non-linear coefficients (Narasimhan et al., 1978). These mean values were then used to solve (7).

Horizontal versus vertical column. The first phase of the study consisted in simulating consolidation with: a) the column being vertical and b) the column being horizontal. The boundary and initial conditions are shown in Figure 8. In the vertical column, the initial void ratio \( (e_0) \), was uniformly 44.43 (porosity = 97.74 percent), while at \( m = 0 \), for all \( t \), \( e_0 \) was equal to 15.54 (porosity = 93.58 percent). The condition of uniform void ratio and hence, uniform effective stress, caused a decrease in hydraulic potential from 1.779 m of \( H_2O \) at bottom to 1.774 m at the top. As a result, at \( t = 0 \), there was a small vertical movement of water toward the top. In the horizontal column, the initial void ratio was uniformly 44.76 (porosity = 97.81 percent) while the constant value ratio \( e_0 \) at \( m = 0 \) was 15.05 (porosity = 93.78 percent).

The computed values of consolidation for the horizontal and vertical columns are presented in Figure 10. It is readily seen from the figure that both horizontal and vertical columns agree very well in predicting total consolidation. However, detailed examination of the computed results showed that the pattern of consolidation of similarly located volume elements within the two columns was considerably different. For example, in the vertical column the upward movement of water persisted.
for over 10 days during which the volume elements in the upper part
underwent swelling. On the other hand, none of the elements in the
horizontal column experienced any swelling.

The pattern of deformation occurring within the column is illustrated
in Figure 11. It is seen that for each element the strain-rate
initially increases with time, attains a peak and finally declines
with time. Moreover, nodes located away from the draining end attain
peak strain rates at progressively later times showing the migration
of the consolidation transient.

Departure from experimental results. Referring back to Figure 10,
it is immediately seen that the numerical results show smaller consolidation
than the experimental observations. To investigate this discrepancy,
the numerical results were first checked for internal consistency.
In this regard we see from Figure 10 that the shrinkage versus $t^{1/2}$
plot is indeed a straight line, passing through origin, which is in
accordance with theory. Secondly, for the vertical column, a plot
of $\lambda$ versus $e$ is shown in Figure 12. Also according to theory, this
plot reveals a fairly well defined, unique dependence of $e$ on $\lambda$.
Furthermore, the theory predicts that the slope of the shrinkage versus
$t^{1/2}$ line in Figure 10 should be equal to the integral,

$$\int_{e_n}^{e_0} \lambda \, de$$

Graphical evaluation of this integral from Figure 12 was found to be
$3.1 \times 10^{-3}$ cm/sec$^{1/2}$ while the slope of the straight line in Figure 10
worked out to be $3.06 \times 10^{-3}$ cm/sec$^{1/2}$.

Having examined the consistency of the numerical model, it was
reasonable to question whether the discrepancy was due to the input
values of permeability being smaller than they ought to be. Trial
computations showed that good agreement could be obtained between
the numerical and observed results if the input data on permeability
are multiplied by an arbitrary factor of 2.
A reexamination of the data (Smiles, personal communication) suggested that the experimental results were subject to errors due to difficulties involving measuring shrinkage and computing void ratios. Considering the experimental difficulties, the discrepancy between numerical and experimental results appears reasonable.

Accuracy and adjustments of nodal distances. Note that the Lagrangian treatment requires periodic adjustments of the geometric parameters, nodal distances, elevations, surface areas and volumes. Computationally, while it is fairly easy to make adjustments to nodal volumes occurring in the term $M_c$ on the right hand side of (7), it is far more cumbersome to make such adjustments to nodal elevations, distances, and surface areas.

Since volume involves third power of distances, the geometric parameter likely to undergo maximum change is volume. Also, it is seen from the left hand side of (7) that surface areas occur on the numerator and nodal distances on the denominator, which suggest that the changes in these quantities would tend to compensate for each other. It seems reasonable to infer that, except in extreme cases, neglecting to adjust nodal distances, elevations, and surface areas on the left hand side of (7) may not adversely affect accuracy.

In the one-dimensional problem under study, it is recognized that, $\Gamma_{n,m}$ being constant, volume change is of the same order of magnitude as change in nodal distance or elevation. This problem, therefore, poses an extreme case in which errors due to neglecting changes in nodal distances are likely to be maximum and, hence, merits further attention.

Figure 13 (A and B) show the plots of shrinkage versus time$^{1/2}$ computed with a mesh in which the quantities $z_m, z_n, d_{m,n}$ were held constant on the left hand side of (7) while the variation of nodal volume (incorporated in the $M_c$ term on the right hand side) was allowed to vary in accordance with deformation. For convenience we will call this a "rigid" mesh. Now, since the column is consolidating, the nodal points must move towards each other with time and, hence, the nodal distances must continuously decrease with time. Since this decrease in nodal distances is ignored in the rigid mesh, the gradients are underestimated and the fluxes calculated on the left hand side
of (7) tend to be smaller than they would otherwise be. As a consequence, the rigid mesh solution yields a much slower consolidation history than the rigorous solution, Figure 13A.

Looking at the left hand side of (7) it can be reasoned that instead of continuously adjusting to the d's and z's, one could, from a purely computational point, keep the mesh rigid and apply an equivalent correction to the permeability term, $K_{m,n}$. With this reasoning in mind, the permeability values in the rigid mesh problem were uniformly multiplied by 1.75 (by shifting the $e$ versus log $k$ plot appropriately to the right in Figure 9B), and the calculations repeated. The results are presented in Figure 13B. It is readily seen that the rigid mesh solution with adjusted permeability does indeed agree with the rigorous solution.

These results lead to some inferences of practical interest.

1. A single, simple correction to permeability seems capable of adequately compensating for the far more complex task of deforming the mesh with time.

2. In the particular problem studied, the equivalent correction turned out to be multiplication by a factor of 1.75 (increasing permeability by 75 percent). However, the precision of measuring the permeability function experimentally is such that an error of 100 percent may often be tolerable. Thus, the error accrued, due to the use of the rigid mesh, appears to be of the same order of magnitude as that introduced due to errors in the input values of permeability.

3. In the present problem, the total shrinkage at the end of 24 days was 32 percent of the original column length. During this period, the maximum strain attained by the bottom most element was 64 percent. Despite the large deformations involved, the required permeability correction was only 75 percent. In actual field problems, far smaller deformations are likely to be encountered, and the required permeability corrections are likely to be so small that they should be well within the precision of experimentally measuring permeability.
4. For most field problems, therefore, one could use a rigid mesh without any permeability corrections and still obtain reasonably accurate results. Nevertheless, if it is still desired to minimize errors one could use appropriate permeability corrections. A possible method for determining the correction factors will be to carry out parametric studies. The correction factors determined from such parametric studies can be expressed in terms of \( C_c \) or total deformation.

**Deforming and non-deforming coordinates.** In discussing the concept of specific storage, DeWiest (1969) drew attention to the fact that Jacob's (1950) derivation of the governing equation of groundwater flow considers a deformable volume element for the time-derivative part, while, on the spatial derivative part it considers a non-deformable element. To get over this inconsistency, DeWiest then proceeded to redefine specific storage in a fixed coordinate system.

If we consider a saturated flow region and neglect water compressibility, it is obvious that the volume of the flow region must decrease by an amount equal to the volume of water released from storage. Therefore, it is not physically realistic to assume that the overall dimensions of the flow region do not change with time. The formulation based on a deforming volume element is, therefore, more logical. In the light of our study of the results from a rigid mesh, we can interpret Jacob's derivation as pertaining to a deforming volume element, but ignoring, as a reasonable approximation, the effects of deformation on surface areas and nodal distances in evaluating fluxes. Viewed in this fashion, Jacob's derivation if of practical utility.

**INfiltration into a Moderately Saturated Soil**

The next example deals with the early absorption stage of infiltration into a vertical unsaturated soil column. Initially, the soil is at an uniform moisture content of \( \Theta = 0.238 \). At time \( t = 0 \), the top of the column is brought to a state of saturation at \( \Theta_{sat} = 0.495 \) and this boundary condition is maintained indefinitely. If one is interested only in the early absorption history or if the soil column is horizontal, then, gravity is unimportant. Then, the governing equation can be written in the form of the diffusivity equation,
\frac{\partial}{\partial z} \left( D(\theta) \frac{\partial \theta}{\partial t} \right) = \frac{\partial \theta}{\partial t} \tag{8}

in which \( z \) is the depth below soil surface and \( D(\theta) \) is the diffusivity coefficient. For the particular soil considered, \( D \) is a highly non-linear function of \( \theta \) as given in Table 1.

This problem was solved using the numerical technique of this study, by discretizing the flow region into 45 volume elements, each 2 cm long. The resulting moisture content profile at \( t = 10^6 \) is presented in Figure 14 and is compared with the analytical solution of Philip (1969) and the numerical results of Van der Ploeg and Benecke (1974). The agreement is obvious.

In connection with the solution of this problem it must be pointed out that an arithmetic mean was used to evaluate the interface permeabilities (or diffusivities) rather than the harmonic mean permeability. The reason for this is explained in the next section dealing with an extremely dry soil column.
TABLE 1. $D$ versus $\Theta$ Relation for the Absorption Problem.

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$D$, cm$^2$/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2376</td>
<td>0</td>
</tr>
<tr>
<td>0.2440</td>
<td>$1.59 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.2569</td>
<td>$2.13 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.2698</td>
<td>$2.82 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.2826</td>
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</tr>
<tr>
<td>0.2955</td>
<td>$3.73 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.3084</td>
<td>$4.51 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.3213</td>
<td>$6.31 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.3341</td>
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</tr>
<tr>
<td>0.3470</td>
<td>$7.96 \times 10^{-4}$</td>
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<tr>
<td>0.3599</td>
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<tr>
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</tr>
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<tr>
<td>0.4886</td>
<td>$1.04 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Infiltration into a Column of Extremely Dry Soil

The problem of infiltration into extremely dry soils is of considerable interest in the field of agriculture. In considering soil-water relations during rain infiltration, Rubin and Steinhardt (1963) showed that ponding of water on the soil surface can occur only if rainfall intensity exceeds the saturated soil permeability. If not, the soil would tend to attain that uniform saturation at which rainfall intensity exactly matches hydraulic conductivity. To demonstrate this, they considered a vertical, semi-infinite column of Rehovot sand (porosity 39.7 percent) on whose surface rain was assumed to fall at a constant rate (Figure 15). The sand itself was initially very dry (saturation = 12.6 percent; moisture content = 0.05 percent). The hydraulic conductivity $K$ and moisture content $\theta$ of Rehovot sand are highly non-linear functions of moisture content (Figure 16). Empirically, these functions were fitted to the following equations by Rubin et al., (1964):

$$ -\psi = 11.3 + \frac{3.19}{\theta} - 0.05\theta^{15} + e^{-575\theta} + 16.3 $$  \hspace{1cm} (9)

in which $\psi$ is in millibars,

$$ K = \frac{8400}{(-\psi^5 + 14.45^5)} $$  \hspace{1cm} (10)

in which $K$ is in cm/sec. Because of the extremely dry nature of the soil, the initial pressure head within the column was very low and was computed from (8) to be $-6.69 \times 10^3$ meters of water. Computations based on (9) and (10) showed that at a moisture content of $\approx 0.168$ and a pressure head of $\approx -0.3$ m of water, the hydraulic conductivity of the soil will equal the assumed rainfall intensity of $3.528 \times 10^{-4}$ cm/sec.

Rubin and Steinhardt (1963) posed the infiltration problem outlined above in the form of the diffusivity equation and solved it with the help of a finite difference scheme. Their results are shown by the solid line in Figure 17.
Although the diffusivity formulation is useful in solving the aforementioned highly non-linear problem, it has been rightly pointed out (Bruce and Klute, 1956) that the diffusivity equation cannot be used when the porous medium is heterogenous. This is because of the jump discontinuity of the $\phi$ distribution at material interface. It is therefore desirable to be able to solve the infiltration problem with $\psi$ as the dependent variable. However, due to the strong non-linearity of the coefficients in relation to $\psi$, the solution of the $\psi$-dependent equation is considered difficult. For example, Neuman (1972) who attempted to solve the above problem using the Galerkin finite element method found that the infiltration process could not even be initiated with the $\psi$-dependent formulation, and concluded that the diffusivity formulation was the best way to solve the problem.

Recent application of the TRUST computer program has shown that it is indeed possible to solve the infiltration problem using the $\psi$-dependent governing equation. The key to the understanding of the problem here is that of determining the appropriate mean permeability at the interface between two volume elements. Recall from equation 3a of Part II of this series of papers (Narasimhan et al., 1978), that in the present work, the mean interface permeability was defined to be the harmonic mean permeability,

$$
\bar{K}_{\phi,m} = \frac{K_{\phi}K_{m} (d_{\phi,m} + d_{m,\phi})}{K_{m}d_{\phi,m} + K_{\phi}d_{m,\phi}}
$$

This mean assures continuity of fluxes at the interface, and is found to be a good estimate in linear and moderately non-linear problems. However, due to the occurrence of the product $K_{\phi}K_{m}$ on the numerator of (11), $\bar{K}_{\phi,m}$ tends to be strongly weighted towards the lower of the two permeabilities. As a result, in extremely dry soils, permeability of the interface just ahead of the front becomes too low to effectively drain the fluid on the downstream side. This in turn causes abnormal buildup of potential in the volume element.

It therefore appears that a mean value based on a higher weight for the upstream permeability is necessary for handling this problem.
Accordingly, it was first decided to use an arbitrary definition of $K_{\ell,m}$ according to

$$
K_{\ell,m} = 0.8K_{\text{upstream}} + 0.2K_{\text{downstream}}
$$

(12)

Moreover, two further precautions are needed before attempting to solve the problem. First is the abruptness of the boundary condition at $t = 0$. To start the problem smoothly therefore, it is necessary to keep the time steps relatively small during the initial stages of calculations. Secondly, noting that the present algorithm uses a quasi-linearization technique in which a mean value of permeability over a time step is estimated and used in the calculations, it becomes necessary to limit the time steps to small magnitudes so that the permeability and capacity coefficients do not vary widely from one time step to the next.

The flow region was discretized into volume elements 0.01 m wide. The dependence of $S$ on $\psi$ and $K$ on $\psi$ were approximated by means of tables with over 90 data points with linear interpolation. The size of $\Delta t$ was gradually increased from a 0.1 second to a maximum of 8 seconds. Since the stable time step was never below 10 seconds, it was possible to carry out the calculations explicitly. The results obtained are shown compared with Rubin and Steinhardt results in Figure 17. It is seen that the agreement is generally good. Check on mass balance showed mass balance errors to be less than 0.1% at the end of 22,500 seconds.

The importance of the weighting factor. As seen in (12), a factor of 0.8 is used for upstream weighting. This choice is arbitrary and the arbitrariness does indeed question the credibility of the solution process. To further study this question, different weighting schemes were studied for $t > 5,625$ seconds. The results are presented in Figure 18. It is surprising to note from Figure 18 that the solution is considerably insensitive to the choice of the weighting function and indeed one does not incur any significant error if one simply chooses to arbitrarily use the arithmetic mean. However, it must be mentioned here that in the early stages of the solution process it is, indeed, necessary to use a high upstream weight of 0.8.
An attempt was made to solve this problem implicitly by increasing \( \Delta t \) to 20 seconds. This gave poor results due to large variations in permeability and specific moisture capacity from one time to another.

**Axisymmetric Flow to a Soil-Water Sampler**

A practical problem of the soil scientist is to obtain a sample of soil solution from a small region surrounding the sampling device. In order to justify confidence in the sampling procedure, it is essential to understand, in detail, the flow field around the sample, or especially when the solution is rapidly moving past the probe. For this purpose, the present numerical model was used to simulate radial flow to a soil probe in partially saturated soils.

The physical model (Figure 19) consists of a horizontal layer of wet Geary silt loam in an impermeable cylindrical shell with a porous probe at its center. It is assumed that the model is suitably protected from evaporative losses. It was also assumed, that through an elaborate experimental set up, the effluent could be continuously withdrawn for volume measurement while simultaneously applying a constant suction at the probe. The model is subjected to several cycles of different suction, as follows. The soil is first saturated with water and brought to a constant suction and moisture content by applying a controlled probe suction until all outflow has ceased. An additional known suction is then applied, and the outflow rate observed. Similar routines are repeated at other suctions and the results compared.

In the numerical problem, the probe was assumed to be 3.7 cm tall, 1.1 cm in diameter in a cylindrical sample, 51.1 cm in diameter. The physical properties of the silt loam (Figure 20) was obtained from Hanks and Bowers (1962). The flow region was discretized into 30 cylindrical volume elements, the inner most ten being 0.5 cm wide and the rest 1 cm wide. The soil was assumed to be rigid and non-swelling. Due to the small height of the sample, gravitational effects were neglected.

Numerical solutions were generated for various simulated sampling routines to evaluate the spatial distribution of pressure head and moisture content as well as the rate of flow into the soil probe. A few of the results are summarized below.
Figure 21 shows the spatial variation of pressure head produced by a suction of 0.2 m at the probe corresponding to an initial soil suction of -.0005 m. For the same conditions, Figure 22 shows the spatial distribution of soil moisture content $\theta$, while Figure 23 depicts cumulative outflow at the probe as a function of time.

The quantity of solution released from storage by different parts of the flow region to make up the effluent withdrawn at the probe is illustrated in Figure 24. It is seen that at early times a bulk of the cumulative drainage takes place in the immediate vicinity of the probe. As time progresses, more of the depletion from storage occurs in the regions away from the probe. It may be pointed out here that due to the cylindrical symmetry of the flow region, the outer shells have far greater volumes than the inner ones and hence, despite the relatively small changes in moisture content, the outer shells can dominate the contribution to total cumulative depletion from storage.

**SATURATED-UNSATURATED FLOW**

**Drainage from a One-Dimensional Soil Column**

Liakopoulus (1965), conducted several experiments on the drainage of water from vertical columns of Del Monte sand. One of these experiments was chosen for simulation by the present model.

A one-meter high perspex column (Figure 25) was packed with Del Monte sand, and was instrumented with a sufficient number of tensiometers to continuously measure moisture tension at various points within the column. Prior to the start of the experiment ($t < 0$), water was continuously added from the top of the column and allowed to drain freely at the bottom through a micro-pore filter. The flow was carefully regulated until all tensiometers read zero pore-water-pressure. This meant that the vertical gradient in potential equals unity ($\frac{\partial \phi}{\partial z} = 1$). On obtaining this initial condition, the addition of water from the top was discontinued and the upper boundary rendered impermeable. From then on ($t > 0$), the tensiometer readings were recorded while periodically measuring the flow rates at the draining end.

The physical properties of Del Monte sand were measured by Liakopoulous by an independent set of experiments. The sand had a porosity of 29.75 percent. The dependencies of $\theta$, $S$ and $k$, as deduced
from the experimental data (Liakopoulos, 1965) are given in Figure 26. It can be seen from the figure that both $\theta$ and $S$ remain unchanged in the range $-0.2 \, \text{m} \leq \psi < 0$. In other words, moisture suction has to exceed the "air entry" value of $-0.2 \, \text{m}$ before the pores can be desaturated; that is, the soil remains saturated, because $\psi$ remains negative, but greater than $-0.2 \, \text{m}$.

For purposes of numerical simulation the column was divided into 10 volume elements of equal size. As a first step in the simulation the sand was assumed non-deformable and rigid, as is customarily done in dealing with unsaturated flow. The profiles of pore pressure distribution, computed in this fashion, are shown in Figure 27A. It is seen that although the computed values agree reasonably with the experimentally observed values after about ten minutes of starting the experiment, the computed suctions are markedly greater than the observed suctions at the end of five minutes. Note from the figures, that at the end of five minutes the observed pressure is everywhere greater than $-0.2 \, \text{m}$. In light of the saturation and moisture content curves of Figure 26, this means that no drainage could have occurred during the first five minutes, since the soil is rigid, and the expansivity of water is extremely small. Yet experimental observations (Figure 28) indicate the drainage had indeed occurred during the first five minutes. The most reasonable conclusion that one can draw from this is that during the first five minutes the drained water was released from storage by a slight deformation of the soil skeleton and a consequent decreased in-void-volume.

The next question to be considered, then, is whether better simulation of experimental results could be achieved by assuming the sand to be deformable.

**Moisture suction and effective stress.** To assess the importance of deformation, the sand was assigned a $C_c$ value of 0.017 which was assumed to be reasonable for an unconsolidated sand. At $t = 0$, the effective stress at a point 0.55 m from the top was computed to be 0.825 m of water or 8.1 newtons/m$^2$. At this effective stress a $C_c$ of 0.017 amounts to an $a_v$ of approximately $2.1 \times 10^{-6} \, \text{m}^2/\text{newton}$ ($m_v \approx 1.7 \times 10^{-6} \, \text{m}^2/\text{newton}$).
Using the assumed value, a second simulation was carried out, assuming that all the moisture suction developed was fully converted to effective stress according to the relation, \( \sigma' = \sigma - \rho_w g \psi \). This is equivalent to setting \( \chi = 1 \) in Equation 13 of Part I, (Narasimhan and Witherspoon, 1977). The results of this simulation are given in Figure 27B. It can now be seen that there is a closer agreement between the results at the end of five minutes but, for longer times, the computed pore pressures are consistently larger than the observed pressures, suggesting that the fluid mass capacities of the volume elements are larger than they should be. Such, indeed, will be the case if the elements are deforming in addition to desaturating.

From figures 27A and 27B, one could infer that while deformation is important at small values of suction (and hence, small values of time) it must drastically decrease, or even cease to exist at higher magnitudes of suction. If this were so, \( \chi \) cannot be unconditionally set to 1 in the unsaturated zone, but should be a function of \( \psi \). In the absence of any data on the \( \chi \) versus \( \psi \) relationship for Del Monte sand, the following arbitrary assumption was made,

\[
\chi = \begin{cases} 
1, & \text{if } \chi \geq -0.2 \text{ m} \\
0, & \text{if } \chi \leq -0.2 \text{ m} 
\end{cases}
\]  

(13)

implying that the flow is purely saturated when \( \psi \) is greater than air-entry value, and purely unsaturated flow otherwise.

The results obtained on the basis of (13) in conjunction with \( C_c = 0.017 \) are given in Figure 27C. It can now be seen that there is reasonable agreement between experimental and observed results at early as well as late times.

The results presented in Figure 28 show a good agreement between the experimental and computed drainage rates. The simulations described above strongly suggest that in certain types of problems the importance of deformation cannot be ignored when the soil is under moisture suction.
Drainage from a Sand Box

The next problem concerns the simulation of saturated-unsaturated flow in a two-dimensional sand box. Vachaud et al. (1971) conducted very carefully controlled drainage and infiltration experiments on a sand box 3 m long, 2 m high and 0.05 m thick. The particular experiment chosen for numerical simulation is schematically described in Figure 29. In this experiment the sand box was first allowed to attain hydrostatic equilibrium with the water levels on the left and the right hand sides of the sand box standing at an elevation of 1.43 m above the base for a long period of time. Then at \( t = 0 \) the water level on the left hand side was allowed to drop, by .63 m, to .8 m above base, and was maintained at that elevation for the duration of the experiment. During this experiment, lasting for over 50 hours, pressure heads were monitored in 20 tensiometers distributed over the sand box, and the distribution of moisture content was also periodically monitored. These data enabled the determination of the distribution of \( \psi, \theta \), and \( \phi \) over the sand box, and the change in the quantity of stored water in the system.

The aim of the simulation was to use the known material properties and boundary conditions, and to determine temporal and spatial variations of \( \psi, \theta \) and change in stored water.

The experimentally determined relationships between \( \psi, \theta \) and hydraulic conductivity are shown in Figure 30. It can be seen from the figure that the experimental data points show considerable scatter. But, for purposes of simulation, the sand was assumed to be homogenous with mean properties corresponding to the solid lines shown in the figure. The mean values used are shown in Figure 31.

According to the authors of the experiment, there was hydrostatic equilibrium at \( t = 0 \). Accordingly, for purposes of simulation, hydrostatic initial conditions were used as shown in Figure 32.

In the simulations, three different values of deformation parameters were considered: \( C_C = 1 \times 10^{-6} \) (rigid sand), \( C_C = 5 \times 10^{-4} \), and \( C_C = 1 \times 10^{-2} \) (most deformable case). In addition, \( x \) was set to 1 for \( \psi \geq -0.2 \), and to zero otherwise. Comparison of the results showed that since the problem is dominated by the phenomenon of desaturation,
there was practically no difference in the results obtained with the three different values of soil deformation.

As a first step in the comparison of experimental and numerical results, the computed position of the surface, $\psi = 0$ is compared with the experimental data in Figure 33. As can be seen, but for slight departures at early times, the agreement is reasonable, considering the difficulties associated with the experiment.

A special feature of the experiment was the measurement of the spatial and time-dependent variation of moisture content at different points in the sand box. Thanks to this, the experiment also yielded data on the quantity of water removed from storage in the sand box. A comparison of the numerical and experimentally observed, cumulative depletion from storage is shown in Figure 34. It is seen from this figure that, although the two curves approach each other toward steady state ($t > 50$ hours), they depart significantly between one and twenty hours during the transient behavior. This suggests that the mean physical properties used in the simulation are not quite appropriate.

A detailed comparison of the experimentally observed isopotential lines with the computed ones (Narasimhan, 1975) showed that these agreed very well over most of the flow region except at the upper left hand portion. A careful study of the computed results showed that over 55 percent of the total depletion from storage occurred in a part of the unsaturated zone about 1 m long, 0.7 m high, located in the upper left hand part of the sand box. Moreover, this region was also experiencing maximum moisture suctions and, hence, the lowest permeabilities within the flow region.

It was brought to the attention of the authors (Vachaud, personal communication), that Vauclin, et al., (1975) had used much lower permeability values for $\psi < -0.1$ m than those used by Narasimhan (1975) and obtained good agreement with experimental observation. A comparison of the permeability characteristics used by Narasimhan (1975) and Vauclin (1975) is shown in Figure 35.

A subsequent numerical simulation of the same problem based on Vauclin's new physical properties showed that a much better agreement could be obtained between numerical and experimental values of cumulative depletion from storage (Figure 36).
It may be pointed out here that the use of the new physical properties did not considerably improve the agreement in the position of the surface $\psi = 0$, indicating that the position of the free surface is not very sensitive to changes in physical properties. Moreover, careful study of the simulated results showed that the computation of cumulative storage change was sensitive especially to the permeabilities in the ranges of pressure head less than -0.3 m, confirming the previous inference that the upper left hand part of the flow region, releasing most of the water from storage and experiencing lowest pressure heads and lowest permeabilities, dominates the unsaturated flow system behavior.

Consolidation Around an Excavation in Soft Clay

Next is a hypothetical problem which is of interest to the soil and foundation engineer, and involves settlement associated with the development of moisture suction.

A 20-meter bed of soft clay (San Francisco Bay Mud), fully saturated up to ground level is under hydrostatic equilibrium with the fluid potential, $\phi$, everywhere equal to 20 m. The laterally infinite clay bed rests on an impermeable foundation. The upper part of the bed is preconsolidated to 0.35 atm. while the rest is normally consolidated. An excavation 20 m long, 10 m deep and very wide is made in this bed. The problem is to study the new drainage pattern imposed within the clay bed due to the excavation, and the resultant consolidation and swelling phenomena caused by the pore pressure dissipation.

Conceptually, the first step in the solution process is to determine the effect of unloading on the antecedent hydrostatic pore pressure distribution. To do this it is assumed that a) the process of excavation is rapid, and considering the low permeability of the clay, can be considered instantaneous, and b) that the soil responds elastically in an undrained state in such a way that the change in loading is instantaneously borne by equivalent pore-pressure changes, and that the soil skeleton does not immediately experience any change in effective stress. With the gradual dissipating pore pressures, the effects of unloading are gradually transferred to the soil skeleton.

The initial and boundary conditions of the problem are shown in Figure 37. In order to compute the distribution of $\psi$ and $\phi$ at $t = 0$ (Figure 37B, 37C) the following procedure was adopted. Assuming
a Poisson's ratio of \( v = 0.5 \), the new stress distribution was calculated at 15 points in the flow region, using the elastic theory stress distribution charts and tables given in Poulos and Davis (1974). The differences between the old and new stress distributions, evaluated with the help of Mohr's Circle provided the changes in the principal stresses, \( \Delta \sigma_1', \Delta \sigma_2', \) and \( \Delta \sigma_3' = \Delta \sigma_3 \). The changes in stresses cause changes in pore pressure within the clay. According to Skempton (1954), change in pore pressure can be related in effective stresses by the relation

\[
\Delta u = \rho_w g \Delta \psi = \frac{1}{3} (\Delta \sigma_1' + \Delta \sigma_2' + \Delta \sigma_3')
\]  

(14)

The change in pore pressure computed by (14) at the fifteen control points were contoured to arrive at the new distribution of pore water pressures and potentials, which form the initial conditions for drainage-settlement problem solved by the numerical model. The initial and boundary conditions of the problem are presented in Figure 37. Note in Figure 37B that, as a result of the excavation process, a portion of the flow region immediately surrounding the excavation goes into a state of negative pore pressures. We will see later that this has profound influence on the generation of the seepage face and the settlement in the vicinity of the excavation. In the simulation, the floor of the excavation is assumed to be constantly covered by a film of water, and hence, is treated as a prescribed potential boundary with \( \psi = z \). The wall of the excavation is partly impermeable, and partly a seepage face, while all the other boundaries are impermeable.

The physical properties of Bay Mud were experimentally determined by W. N. Houston of the University of California, Berkeley, and are presented in Figure 38. These properties represent the "drained" parameters of the clay. Since the clay is very plastic it was assumed that desaturation could be initiated only at very large suction, and that the clay remains fully saturated throughout the range of negative pore pressures encountered in the present problems. The flow region was discretized into 256 volume elements (Figure 39) of unequal size.

The drainage-consolidation computations were carried out for 360 days from \( t = 0 \), and the distributions of potentials computed at different times are shown in Figure 40.
The evolution of the seepage face in Figure 40 merits special attention. As seen from Figure 37B, the region immediately around the excavation is in a state of moisture suction at t = 0. At t = 0, therefore, no seepage face could exist along the wall, since, by definition, water can only leave the flow region across a seepage face. Hence, at t = 0, the entire wall forms an impermeable boundary. However, with the passage of time, as pore pressure begins to build up behind the wall, an incipient seepage face forms at the base of the wall and gradually begins to grow upward. As can be seen from Figure 37B, the seepage face attains a maximum length after about 40 days, and then begins to contract.

The numerical computations provide, for each volume element in the flow region, the total change in volume as a function of time. If we assume that all the volume change takes place in the vertical direction, then, the volume change divided by the base area of an element yields the vertical deformation. The summation of the vertical deformations along a given column of volume elements gives the settlement or swelling over the column length.

The pattern of swelling and consolidation around the excavation is illustrated in Figure 41. It can be seen that the floor of the excavation progressively heaves upward with time, the maximum swelling being at the axis of the excavation.

In the region to the right of the excavation the ground gradually settles with time. However, it is interesting to note that the maximum settlement does not occur at the excavation wall, but a few meters behind it. The reason for this is as follows: during the initial growth of the seepage face, the elements behind the excavation receive more water from the region to the right than can be drained by the still growing seepage face. As a result, these elements experience a net buildup of pore pressure and swell rather than consolidate. As time progresses, and the seepage face dominates drainage, these volume elements begin and continue to lose pore pressure and consolidate.

It would appear that the offset of the trough of maximum consolidation, at some distance behind the excavation wall, is sometimes observed in the field Peck (1969) as shown in Figure 42. In this
particular case, however, the settlement pattern has been attributed to the inward movement of the vertical wall of the excavation rather than to seepage.

It has been pointed out (Peck, and O'Rourke, personal communications), that in an extremely soft clay, like the one used in the simulation, and using elastic theory to obtain the initial conditions, the simplified stress calculations may not be completely valid, therefore, other factors should be considered. These assumptions, however, may be valid in less soft materials. It must be emphasized that the purpose of this hypothetical simulation has only been to qualitatively understand the phenomenon of drainage and consolidation around an excavation.

Note that in Figure 41 certain oscillations occur in the settlement profile to the right of the excavation. Whether these are due to the particular initial conditions used, or are due to the variation in mesh size, is not quite clear.

In a consolidation problem of this nature it is of interest to gain an insight into the pattern of deformation in different parts of the flow domain. The time-rates of strain of a few typical points in the flow region are presented in Figure 43.

Figure 43A illustrates the behavior of a node which has been undergoing swelling at a progressively decreasing rate with time (by convention, swelling is negative deformation), while Figure 43B illustrates an element which has been consolidating ever since \( t = 0 \). Figure 43C summarizes the history of a node which swells slightly to start with, reverses trend, consolidates at increasing strain-rate, attains a peak strain-rate, and finally experiences an ever decreasing strain-rate. The abrupt increase in strain rate, as evidenced by the inflection in the left limb of the curve in Figure 43C, indicates the transition of the node from a state of over-consolidation to that of normal consolidation. Figure 43D illustrates the deformation history of a node which passes from an overconsolidated to a normally consolidated state during consolidation. It can be seen this node attains two peaks of stress-rate, one in the domain of recompression, and another in the domain of normal consolidation.
IV. PORE PRESSURE GENERATION DUE TO VIBRATION

Pore Pressure Generation and Dissipation in a Three-Dimensional System

As the last illustrative example, we will consider the problem of soil liquefaction associated with the response of cohesionless, saturated soils to earthquake motion. It is known (Seed and Lee (1966) that cyclic stresses associated with earthquakes and other causes can generate pore pressures in saturated soils. Abnormal build up of such pore pressures can lead to the loss of strength and consequent liquefaction of shallow cohesionless soils. Liquefaction results when pore pressures equals or exceeds existing total stress. In earthquake prone regions, such as coastal California, design of civil engineering structures must take into consideration the liquefaction potential at the site.

Assuming that liquefaction potential does indeed exist at a given site, one possible method of reducing that potential is the dissipation of pore pressures (concomitant with the generation process) by providing relief drainage wells.

In Figure 44 we consider such a system of drainage wells (12-inch dia.) located at 8-foot centers at a site underlain by silty sands intercalated with thin layers of organic material. On consideration of symmetry, the shaded region of Figure 44 was modeled as a pie-shaped three-dimensional region shown in Figure 45.

The system was modeled as a saturated-unsaturated flow region with initial hydrostatic condition corresponding to the surface $\psi = 0$ located at 20 feet below ground surface. Based on design earthquake magnitudes and soil dynamic properties it was calculated using the method of Seed et al., (1976), that the soil would liquefy in a magnitude 7 earthquake lasting for 20 seconds. In the light of this calculation, appropriate pore-pressure generation rates were assigned to each saturated volume element in the flow region. It was assumed, from the nature of construction of the relief wells that pore pressure generation would not occur, either within the well or in the unsaturated elements.

The computed results showed that, under the given conditions, pore pressures can be dissipated sufficiently during the 20-second period of the earthquake so as to minimize the hazard of liquefaction.
In particular, the computations showed that the presence of the thin, poorly permeable organic layers significantly influence the drainage pattern of the system. Figure 46 presents the pore pressure dissipated during the 20-second period expressed as a percent of the total pore pressure generated. It is seen that pore pressures are dissipated effectively from within the sands, and that difficulty of dissipation is restricted mainly to the these organic layers.
<table>
<thead>
<tr>
<th>NOTATION</th>
<th>Definition</th>
<th>Unit(s)</th>
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<tr>
<td>$a_v$</td>
<td>coefficient of compressibility</td>
<td>$(LT^2/M)$</td>
</tr>
<tr>
<td>$C_c$</td>
<td>compression index; slope of $e$ versus log $\sigma'$ straight line in the normal consolidation region</td>
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<tr>
<td>$C_k$</td>
<td>slope of $e$ versus log $k$ straight line</td>
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</tr>
<tr>
<td>$C_s$</td>
<td>swelling index; slope of $e$ versus log $\sigma'$ straight line in the rebound region</td>
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<tr>
<td>$d_{\ell,m}$</td>
<td>distance between nodal points $\ell$ and $m$</td>
<td>(L)</td>
</tr>
<tr>
<td>$D$</td>
<td>soil moisture diffusivity</td>
<td>$(L^2/T)$</td>
</tr>
<tr>
<td>$D_m$</td>
<td>soil moisture diffusivity in a material coordinate system</td>
<td>$(L^2/T)$</td>
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<tr>
<td>$e$</td>
<td>void ratio</td>
<td>(1)</td>
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<tr>
<td>$e_n, e_o$</td>
<td>initial and boundary void ratios, respectively, in the clay slurry problem</td>
<td>(1)</td>
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<tr>
<td>$g$</td>
<td>gravitational constant</td>
<td>$(L/T^2)$</td>
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<td>$k$</td>
<td>absolute permeability</td>
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<td>$K$</td>
<td>hydraulic conductivity</td>
<td>$(L/T)$</td>
</tr>
<tr>
<td>$K_{\ell,m}$</td>
<td>mean hydraulic conductivity at interface between nodes $\ell$ and $m$</td>
<td>$(L/T)$</td>
</tr>
<tr>
<td>$K'$</td>
<td>hydraulic conductivity of aquitard</td>
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<td>length of flow column</td>
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<td>( \psi_L )</td>
<td>pressure head at nodal point</td>
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\[ \phi = \phi(t) \quad t > 0 \]

\[ \phi(z) = \Phi, t=0 \]

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\[ \theta_0 = 0.460 \]
\[ \psi_0 (\text{soil}) = 0.0005 \text{m} \]
\[ \psi (\text{probe}) = -2.0 \text{m} \]
\[ -0.2 \text{ m} \]
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