Modeling and Control of an Active Magnetic Bearing Spindle System

Cavalier, Grant

2015

Peer reviewed|Thesis/dissertation
Abstract of the Thesis

Modeling and Control of an Active Magnetic Bearing Spindle System

by

Grant Cavalier

Master of Science in Mechanical and Aerospace Engineering

University of California, Los Angeles, 2015

Professor Tsu-Chin Tsao, Chair

Magnetically levitated spindles have been considered for high-speed high-precision machining. These systems eliminate contact between the spindle and housing by replacing traditional bearings with rings of electromagnets which levitate the spindle in their center. Levitated spindles may be spun much faster than conventional ones; and with greater precision and longer median time to failure. Furthermore, the close coupling between the spindle dynamics and the cutting process allows for monitoring of cutting force which can be used to infer tool wear and part quality. Magnetic levitation systems are inherently unstable and require some form of feedback control. Challenges for the control system include robust stability, precise positioning of the spindle, and rejection of disturbances caused by rotor imbalance and the cutting process. This work presents the system identification and modeling of an Active Magnetic Bearing Spindle (AMBS) system and the design of a stabilizing controller. An internal model principle type control scheme, the plug-in resonator, is presented for the rejection of imbalance disturbances and reference tracking. The control system is implemented on the AMBS and experimental results are presented.
The thesis of Grant Cavalier is approved.

James S. Gibson

Robert T. M’Closkey

Tsu-Chin Tsao, Committee Chair

University of California, Los Angeles

2015
# Table of Contents

1 Introduction ................................................................. 1

1.1 Active Magnetic Bearing Spindle Systems ......................... 1

1.1.1 Advantages of AMBS ............................................... 2

1.1.2 Challenges of AMBS ............................................... 3

2 System and Modeling ....................................................... 6

2.1 System Overview ....................................................... 6

2.1.1 Axis Orientations ................................................. 6

2.1.2 Hardware ............................................................ 7

2.2 System Identification and Modeling ............................... 9

2.2.1 Experimental Frequency Response ........................... 12

2.2.2 Modeling ............................................................ 17

3 Feedback Control ......................................................... 21

3.1 Stabilizing Control .................................................... 21

3.1.1 Design of Stabilizing Control ................................. 21

3.1.2 Analysis of Stabilizing Control ............................... 24

3.2 Plug-In Resonator ..................................................... 31

3.2.1 Resonator .......................................................... 32

3.2.2 Discrete Approximate Plant Inversion ....................... 34

3.3 Implementation and Experimental Results .................... 36

3.3.1 Spindle Rate of 77 Hz .......................................... 37

3.3.2 Spindle Rate of 200Hz ......................................... 43
4 Conclusion ....................................................... 51

References ....................................................... 53
# List of Figures

1.1 Cross Section of Typical AMBS ............................................. 2
1.2 AMBS Machine Tool System Diagram ..................................... 3

2.1 AMBS Axis Orientations .................................................. 7
2.2 Experimental System Diagram .......................................... 8
2.3 Experimental Setup in the Laboratory .................................. 9
2.4 Block Diagram for System Identification ............................. 10
2.5 Schematic of Signals for System Identification ....................... 10
2.6 Reference to Output Raw Frequency Response Data ................. 13
2.7 Reference to Control Raw Frequency Response Data ............... 14
2.8 Back Calculation of the Controller Frequency Response .......... 16
2.9 Block Diagram for Analyzing Model Closed Loop Stability .... 17
2.10 Plant Model Frequency Response ..................................... 19

3.1 Implementation of the LQGi Controller .............................. 25
3.2 Frequency Response of the Stabilized System - Direct Fit ....... 26
3.3 Frequency Response of the Stabilized System - Modified Fit .... 27
3.4 Step Response of the Closed Loop System Compared with Simulation .... 28
3.5 Step Response Comparison of Controllers .......................... 29
3.6 LQGi Regulation RMS Error .............................................. 30
3.7 Comparison of LQGi Designs - RMS Control Signal ............. 31
3.8 Structure of the Plug-in Peak Resonator .............................. 32
3.9 Structure of the Resonator .............................................. 33
3.10 Equivalent Block Diagram of the Resonator and Closed-Loop Plant ... 34
3.11 Frequency Response of the Peak Filter, 77 Hz Design . . . . . . . . . . . . . 37
3.12 Magnitude of Shifted MIMO Nyquist Contour, 77 Hz Design . . . . . . . . 38
3.13 Magnitude of the Sensitivity Function, 77 Hz Design . . . . . . . . . . . . . 39
3.14 Disturbance Rejection at Spindle Speed of 77 Hz . . . . . . . . . . . . . . . 40
3.15 Error Spectrum During Regulation at Spindle Speed of 77 Hz . . . . . . . . 41
3.16 Steady-State Error for Tracking 154 Hz Sinusoid at Spindle Speed of 77 Hz . 42
3.17 Scatter Plot of Elliptical Trajectory Following - 77 Hz . . . . . . . . . . . . 43
3.18 Frequency Response of the Peak Filter, 200 Hz Design . . . . . . . . . . . . 44
3.19 Magnitude of Shifted MIMO Nyquist Contour, 200 Hz Design . . . . . . . . 45
3.20 Magnitude of the Sensitivity Function, 200 Hz Design . . . . . . . . . . . . 46
3.21 Disturbance Rejection at Spindle Speed of 200 Hz . . . . . . . . . . . . . . 46
3.22 Error Spectrum During Regulation at Spindle Speed of 200 Hz . . . . . . . . 47
3.23 Steady-State Error for Tracking 400 Hz Sinusoid at Spindle Speed of 200 Hz 48
3.24 Saturation of the AMB Control Signal . . . . . . . . . . . . . . . . . . . . . 49
3.25 Scatter Plot of Elliptical Trajectory Following - 200 Hz . . . . . . . . . . . 50
LIST OF TABLES

3.1 Discrete Approximate Inversion - 77 Hz Design . . . . . . . . . . . . . . . . . 39
3.2 Regulation Error RMS - 77 Hz . . . . . . . . . . . . . . . . . . . . . . . . . . 41
3.3 Tracking Error RMS - 77 Hz . . . . . . . . . . . . . . . . . . . . . . . . . . . 42
3.4 Discrete Approximate Inversion - 200 Hz Design . . . . . . . . . . . . . . . 45
3.5 Regulation Error RMS - 200 Hz . . . . . . . . . . . . . . . . . . . . . . . . . . 47
3.6 Tracking Error RMS - 200 Hz . . . . . . . . . . . . . . . . . . . . . . . . . . . 48
Acknowledgments

Thanks to Professor Tsu-Chin Tsao, Herrick Chang, Chris Kang, James Simonelli, and Sandeep Rai; this work would not have been possible without their help and guidance.
CHAPTER 1

Introduction

High-precision machining processes play a vital role in the advancement of many technologies. In industrial systems such as high-fuel efficiency vehicles and high-fidelity optics, performance may be driven directly by the precision of manufactured components. For these applications, identification and elimination of error sources in the tooling process are of critical importance. The primary sources of error in most cutting processes can be categorized as geometric, thermal, or cutting-force induced [RMP00]. Geometric errors arise from misalignment and manufacturing tolerances of the cutting apparatus itself. Thermal errors are caused by changes in machine geometry due to changes in ambient conditions, frictional heat generated by the motion of machine parts, and by the cutting process itself. Cutting-force induced error refers to deflection of the cutting apparatus due to the torsional and radial loads on the tool during the machining process. Increased cutting force also increases the rate of tool wear [ACL04], which contributes further error to the finished part. This research considers one alternative approach to a conventional machine tool system for the capability of increasing throughput and reducing error in high-precision machining processes [BGH94].

1.1 Active Magnetic Bearing Spindle Systems

Active Magnetic Bearings (AMB) consist of a series of electromagnets arranged in a ring to create a contact-free rotor bearing. By eliminating mechanical contact between the rotor and housing, AMBs enable frictionless operation of the rotor, save for air drag. Common problems afflicting conventional journal bearings such as heat generation and mechanical wear are non-issues for the AMB. AMB rotor systems have found industrial applications
including high-speed machining, vacuum operation, energy storage systems, and clean room environments [BGH94]. The Active Magnetic Bearing Spindle (AMBS) refers to a machine tool wherein the spindle which holds the cutting tool is supported by AMBs. The categories of thermal and cutting-force induced errors [ACL04] are addressed in particular by this type of spindle; recall that geometric errors are contributed primarily by the fixtures locating the spindle and workpiece, not the spindle itself. The use of AMBS systems has many advantages but also poses challenges for design and implementation.

Figure 1.1: Cross Section of a Typical AMBS

1.1.1 Advantages of AMBS

A primary advantage of the AMBS is the ability to achieve high spin rates as frictionless rotation eliminates thermal expansion and mechanical wear at the points of support. High-speed cutting is valuable to manufacturers for its ability to drastically reduce cycle time, yield a better surface finish, and reduce tool wear [BGH94].

Cutting-force measurement in machining processes is of interest because it provides information which may be used to provided error compensation, estimate tool wear, and infer the quality of a finished workpiece. In conventional machine tools, the spindle motion and cutting forces are typically decoupled through a large gear ratio. Cutting force measurements in these machines are difficult to obtain and require expensive sensors, such as dynamotor-accelerometer combos, which have limited bandwidth of around 200 Hz [ACL04].
The AMBS, however, is a direct-drive system in which the current in the coils of the electromagnets directly determines the force applied to the spindle. Therefore, when feedback control is employed to counter disturbances, the control command is strongly coupled with the forces developed in the cutting process. This allows for an indirect measurement of cutting force based on the AMB controller commands, which is provided at practically no cost, and is available up to the bandwidth of the controller [ACL04].

Using an AMBS, the spindle can be made to track trajectories within the air gap between the spindle and stator. This extra degree of freedom in manipulation of the cutting tool position is useful for cutting irregularly shaped features [BZ07].

1.1.2 Challenges of AMBS

All AMB systems are open-loop unstable. Therefore, the first challenge for using any AMB is to determine some stabilizing control law. Even with stability achieved, there are other issues which make precise control of an AMBS difficult. The design problems of disturbance
rejection and trajectory tracking drive specifications for both the AMB actuator and the control system.

High stiffness is an important characteristic for the spindle and the AMB support \cite{BGH94}. The same direct coupling that makes cutting force convenient to measure also implies that the disturbance generated by the cutting process will directly affect the spindle. This results in challenges to ensure that the AMB actuator force is sufficiently large to counter the disturbance forces and thus this complicates the feedback control design for rejecting the disturbance.

Another challenge arises from disturbance forces due to rotor imbalance. Rotor imbalance is a well-studied and ubiquitous phenomenon in rotating machinery. Even precision made rotors are not perfectly symmetric about their geometric axis of rotation. Consider the center of mass at each cross section taken along the rotor geometric axis of rotation. At any cross section where the center of mass is not perfectly aligned with this axis, some unbalance force will be generated when the rotor spins. There does exist, however, an inertial axis about which the rotor spins with no imbalance. This leads to two distinct applications for AMBs. One application is auto-balancing wherein the AMB stabilizes the levitation of the rotor, but allows it spin about the inertial axis. A second application is position regulation, wherein the rotor imbalance force is treated as a disturbance to be rejected and the rotor is regulated to spin about its geometric axis \cite{BGH94}. For the purposes of precision machining, the AMB will be used to reject imbalance disturbances, constrain the rotor to spin about its geometric center, and locate that central axis at points along a specified trajectory.

High-bandwidth tracking of mechanical motion is also a challenging problem in the system level and control design. Bandwidth limitations may be driven by the actuator saturation limits or the signal processing hardware.

Rotor imbalance, cutting forces, and reference tracking all introduce disturbances to the AMBS system which, taken together, are broadband with some peaks synchronous with the spindle frequency and its higher harmonics \cite{BGH94}.

As mentioned earlier, the spindle can be made to track trajectories within the air gap
between the spindle and stator for cutting irregularly shaped features. One example is boring of non-circular profiles of engine piston wrist pin holes. Extensive research has been carried out to prolong the life and increase performance of piston heads for internal combustion engines. A key feature of piston geometry is the wrist pin hole, which houses the wrist pin to form a revolute joint with the connecting rod. Stress concentration in this bore at the interface between the piston and the wrist pin is a primary contributor to fatigue in the piston head [Sil06]. The problem of piston fatigue is a central limitation to decreasing piston weight; which, in turn, decreases inertial loads in engines to improve fuel efficiency [Sil06]. Stress concentration and fatigue may be reduced by specifying non-circular, typically elliptical, geometries for the wrist pin hole which vary in cross section along the axis of the bore [WWW13]. This geometry cannot be created by conventional machining processes. Other methods, such as fast-tool servo have been used with some success [WWW13], but are limited in high-speed capabilities compared to the AMBS. With this application in mind, the work presented in this thesis investigates the modeling of an AMBS system and feedback control techniques for trajectory tracking and rejection of disturbances which arise from rotor imbalance.
CHAPTER 2

System and Modeling

This chapter provides an overview of the experimental system and descriptions of the hardware. The methodology for system identification is presented, along with experimental results. Plant models which will be used in the control design are fit to the system identification data.

2.1 System Overview

The experimental plant for this work is an industrial grade AMBS system. The plant was fabricated as a research platform for wrist pin hole tooling, however, the modeling and control techniques presented are generally applicable to AMB rotor systems.

2.1.1 Axis Orientations

The Active Magnetic Bearing Spindle system includes two radial AMBs, one axial AMB, and an induction motor. There are five degrees of freedom available for control: two translational degrees at each bearing, as well as axial translation. Spinning the shaft is handled by a commercial motor driver. Figure 2.1 shows the AMBS and the axes used.
The efforts of system identification and control design were directed at the 4-input 4-output system set up by the two radial AMBs. The Z axis is assumed to be decoupled from the radial motion and is regulated by PID control. As identified in Figure 2.1, the axes used for control are \([V_{13}, W_{13}, V_{24}, W_{24}]\). The axes pairs \([V_{13}, W_{13}]\) and \([V_{24}, W_{24}]\) represent each bearing, the axes pairs \([V_{13}, V_{24}]\) and \([W_{13}, W_{24}]\) represent the two planes of motion. There are hall effect distance sensors collocated with each axis which measure the gap between the rotor and housing.

### 2.1.2 Hardware

There are several components required for operation and control of the AMBS. The interconnection of hardware subsystems is outlined in Figure 2.2. MB Control, MB Research, and MB Scope are commercial products included in the system. For the purposes of this work,
feedback control was implemented on an xPC target at a sampling time of 10 kHz. The are three National Instruments cards in the target, a PCI-6733 for analog output, a PCI-6052E primarily for analog input, and a PCI-QUAD04 for encoder reading.

Figure 2.2: Experimental System Diagram

A few intermediate steps were necessary for interfacing with the AMBS system. The currents in the AMB electromagnet coils are driven by an amplifier which receives commands from MB Control. MB Control was used only to pass through control signals from the xPC target and relay sensor readings to the xPC target. The MB Scope software package is necessary to set MB Control to not influence the control signal. Analog signals relayed between the xPC target and MB Control pass through the MB Research breakout panel. Encoder signals were read directly into the xPC target by the NI PCI-QUAD04.

The spindle motor is controlled by a Bosch/Rexroth RD500 high-speed motor drive. The laboratory system also includes a linear motor and driver which actuate the workpiece fixture.
2.2 System Identification and Modeling

The first step toward controlling the AMBS was to obtain a mathematical model of the system dynamics. While analytical, or white-box, modeling is important for understanding the dynamics and gaining insight, it is of limited practical use for model-based design on a complex plant such as the AMBS. For this reason, the sine sweep method was used to obtain a black-box model which captures the spindle, AMB, and amplifier dynamics. System identification for this plant is complicated by the fact that it is open-loop unstable; some stabilizing controller must be employed during the identification process.
Figure 2.4: Block Diagram for System Identification

Figure 2.4 shows the loop configuration for system identification. A stabilizing controller which was inherited with the plant was used for this experiment. The reference, \( r \), is available for input and the control signal, \( u \), and the output, \( y \), are available for measurement. The sine sweep method relies on the basic fact that for some sinusoidal input a linear time invariant system produces a steady-state sinusoidal output of the same frequency with some alteration to the amplitude and phase. The gain and phase response values may be represented as complex numbers and, when known across the entire spectrum, completely characterize the system. So to identify a linear time invariant system, one simply needs to apply sinusoidal inputs at many frequencies and determine the gain and phase relationships to the steady-state output signals.

![Schematic of Signals for System Identification](image)

Figure 2.5: Schematic of Signals for System Identification

In the case of the AMBS, the signals \( r \), \( u \), and \( y \) are each 4 element vectors. Therefore, the response at a given frequency is characterized by a 4-by-4 complex matrix where each column describes the response of each output channel to a given input channel. The following
correlation method was used to determine the complex frequency response online, for scalar (single-channel) input $u_n$ to scalar output $y_m$

\[
H_{\text{real}} = \frac{1}{kT_s} \sum_{0}^{k} y_m(k)u_n(k)T_s
\]

\[
H_{\text{imag}} = \frac{j}{kT_s} \sum_{0}^{k} y_m(k)u_{n90}(k)T_s
\]

where $T_s$ is the time-step and $u_{n90}$ is the input sinusoid shifted by a 90 degree phase lead. Inspection of (2.1) shows that $H_{\text{real}}$ and $H_{\text{imag}}$ are the average of the sums on the right hand side. For correct results, $k$ must be allowed to become sufficiently large. The continuous-time derivation on which the relationship in (2.1) is based on shows that under ideal conditions, exactly one period of the input provides the correct frequency response. Experimentally, to avoid the trouble of extracting exactly one period (2.1) was allowed to converge over no fewer than 10 cycles of the input signal.

A quick examination of Figure 2.4 reveals that any noise added to the measurement at the output of the plant, $P$, will be correlated with the output of the stabilizing controller, $C$. For this reason, the frequency response of the plant cannot be obtained directly. Instead, the plant frequency response must be deduced from the available uncorrelated relationships from $r$ to $u$ and $r$ to $y$. Using the method described in (2.1) for each input-output pair, 4-by-4 complex frequency response matrices were built up in a piecewise manner at 103 frequencies for these two relationships. In the future, it could be useful to improve the resolution of the identified frequency response. The control loop was run at 10 kHz which allowed data collection up 5 kHz. For this experiment, data was collected up to 3 kHz primarily because of the poor quality of sine waves described by relatively few points at frequencies higher than this. Using the sine sweep data, the frequency response of the open-loop loop plant may be calculated as
\[ H_{ru} = C(I + PC)^{-1} \]  
\[ H_{ry} = PC(I + PC)^{-1} \]  
\[ H_{ry}H_{ru}^{-1} = PC(I + PC)^{-1}(C(I + PC)^{-1})^{-1} \]
\[ = PC(I + PC)^{-1}(I + PC)C^{-1} \]
\[ = P. \]

Where \( H_{ru} \) and \( H_{ry} \) are the complex 4-by-4 frequency response matrices from reference, \( r \), to control signal, \( u \), and reference, \( r \), to output, \( y \), respectively. \( C \) and \( P \) represent the frequency responses of the stabilizing controller and open-loop plant, respectively.

### 2.2.1 Experimental Frequency Response

This subsection presents the data obtained in the sine sweep system identification. Figure 2.6 shows the measured frequency response from \( r \) to \( y \). There is a strong resonant peak around 200 Hz. All corresponding input-output pairs (shown on the diagonal) are 20 to 40 dB higher than the cross-coupled (off diagonal) responses. The amplitude of the input signal had to be chosen so that displacement was not too large on any axis to avoid a collision with the housing. As a result, the signal to noise ratio in the off diagonal responses was significantly lower than in the diagonal responses. Phase calculations range from decent on the diagonal terms to virtually unusable on some of the off diagonal terms. Note that the response from Input \( W_{24} \) to Output \( V_{24} \), which had a particularly poor phase calculation, had a magnitude response about 50 dB lower than the corresponding diagonal term.
Figure 2.6: Reference to Output Raw Frequency Response Data

Figure 2.7 shows the measured frequency response from $r$ to $u$. Difficulties in obtaining the frequency response from reference to control signal are similar to those in the reference to output case. Diagonal input-output pairs have the cleanest calculation of gain and phase.
while off-diagonal pairs remain more uncertain.

Figure 2.7: Reference to Control Signal Raw Frequency Response Data

The open-loop plant response was calculated by (2.4) using the data in Figures 2.6 and 2.7 and is shown in the next section. To verify that the frequency response data obtained
was reasonably accurate, the open-loop plant response was used along with the closed-loop system response to back-calculate the response of the controller. The closed-loop transfer function for the system in Figure 2.4 may be written as

\[ H_{ry} = PC(I + PC)^{-1}. \] (2.5)

Using the data collected for \( H_{ry} \) along with the newly obtained frequency response for \( P \) and the known frequency response of \( C \), an estimate of \( C \) may be calculated as

\[ C_{est} = P^{-1}H_{ry}(I + PC) \] (2.6)

which does use some knowledge of the controller, \( C \), but is still strongly coupled with the experimental data. Figure 2.8 compares the estimated and actual response of the controller.
The estimated and actual responses for $C$ agree well. This is a promising result for the accuracy of the experimentally obtained plant frequency response.
2.2.2 Modeling

A model must be fit to the experimental data to move forward with control design. Although this will be a blackbox model, there are a few basic expectations that the model should fulfill. Each electromagnetic axis contributes one unstable pole, therefore, there are expected to be 4 unstable poles in modeled system. The process of fitting a model to the experimental data was carried out as follows: Single-input single-output (SISO) infinite impulse response (IIR) filters were fit to each of the 16 frequency responses. These 16 systems were then concatenated into a 4-by-4 multiple-input multiple-output (MIMO) high-order model. A balanced realization and truncation of states with small Hankel singular values was employed to reduce the model to a more manageable order. This method produced the Direct Fit model in Figure 2.10.

The controller used for system identification did not stabilize the Direct Fit model. A good model for control design should predict the behavior of the plant in simulation. The so-called Modified Fit model sought to utilize the information that an accurate plant model should be stabilized by the inherited controller. In order to enforce this requirement, a plant model was first generated in the same method as the Direct Fit. Then the following system, which is equivalent to Figure 2.4 for the purposes of stability, was considered.

![Figure 2.9: Block Diagram for Analyzing Model Closed Loop Stability](image)

Let the model and controller in Figure 2.9 be represented as \([A_p, B_p, C_p, D_p]\) and \([A_c, B_c, C_c, D_c]\),
respectively. The closed loop equations from $d$ to $y$ for this system are

\[
\begin{bmatrix} x_p(k + 1) \\ x_c(k + 1) \end{bmatrix} = \begin{bmatrix} A_p & -B_pC_c \\ B_c C_p & A_c \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_c(k) \end{bmatrix} + \begin{bmatrix} B_p \\ 0 \end{bmatrix} d(k)
\]

(2.7)

\[
y(k) = \begin{bmatrix} C_p & 0 \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_c(k) \end{bmatrix}
\]

which is unstable. The model may now be modified to be stabilized by the controller through a solution of the Linear-Quadratic-Regulator (LQR) problem. The LQR problem and solution are described in detail in the next chapter. For stabilization, the weighting matrix $Q$ was chosen to be the zero matrix and $R$ to be the identity. The state feedback gains provided by these parameter selections provide the minimal "push" (control effort) to move the system eigenvalues to locations which will be stabilized by the controller. The state matrix of the system stabilized by the optimal state feedback gain $K$ is

\[
\begin{bmatrix} A_p & -B_pK_{np} \\ B_c C_p & A_c \end{bmatrix} - \begin{bmatrix} B_p \\ 0 \end{bmatrix} K
\]

(2.8)

where $(B_pK)_{np}$ and $(B_pK)_{np}$ are appropriately dimensioned partitions of the product $B_pK$. The state matrix in (2.8) is guaranteed to be stable. Therefore, with no modification to the inherited controller, the state and input matrices of the Modified Fit model, described by $[\bar{A}_p, \bar{B}_p, C_p, D_p]$, may be written as

\[
\bar{A}_p = A_p - (B_pK)_{np}
\]

(2.9)

\[
\bar{B}_p = -(B_pC_c - (B_pK)_{nc})C_cC_c^T(C_cC_c^T)^{-1}
\]

(2.10)

and exist for $C_c$ full rank, which is satisfied by this system. The frequency responses of the Direct Fit and Modified Fit are compared with the data calculated directly from the system identification in Figure 2.10.
The Direct Fit model has 4 inputs, 4 outputs, and 55 states. The Modified Fit model has 4 inputs, 4 outputs, and 45 states. As expected, and without any direct coercion, both identified...
systems contain exactly 4 unstable eigenvalues. The Modified Fit model is a visually poorer fit to the experimental data than the Direct Fit model. It is included here, however, because it did prove useful for controller design, as will be described in the next chapter. From Figure 2.10 it is clear that the strongest relationships are those between corresponding input-output pairs. Off diagonal responses are all about 20 dB below the diagonal responses at low-frequencies. This observation could motivate the design of 4 independent SISO controllers. However, at higher frequencies, resonant modes in the cross-coupled systems are large enough to require consideration. Particularly around 1100 Hz, several of the cross-coupled responses are comparable in magnitude to the corresponding direct input-output pairs. The strong coupling at high-frequencies motivates the design of a MIMO control strategy.
In this section, stabilizing feedback controllers are designed using the models obtained in the system identification. The stabilizing controllers are implemented on the AMBS and some experimental results are shown and compared. An internal model principle type controller, the plug-in resonator, is introduced. A control design task for the AMBS is established and a plug-in resonator is designed. The plug-in is implemented as an addition to one of the stabilizing controllers. Experimental results are presented to demonstrate the performance with and without the resonator.

3.1 Stabilizing Control

A stabilizing controller may be designed using either of the 4-input 4-output models obtained from the system identification. A Linear-Quadratic-Gaussian (LQG) design was selected because this method expands trivially from SISO to MIMO systems and provides an explicit method to deal with uncertainty in the plant models. Integral action is added to the control effort to enable reference tracking. The LQG-plus-integral controller is referred to as LQGi. To simplify the design and analysis of the plug-in resonator in the next section, the LQGi controller is implemented in error-feedback form.

3.1.1 Design of Stabilizing Control

This section presents the formulation of the LQGi stabilizing controllers. For the discrete-time system
\[ x(k + 1) = Ax(k) + Bu(k) + Gw(k) \]  
\[ y(k) = Cx(k) + v(k) \]  

where \( y(k) \) is the measured output and \( w \) and \( v \) are Gaussian white noise processes with covariance \( W \) and \( V \), respectively. The LQG control law utilizes the separation principle to independently design an observer which minimizes a quadratic cost function of the estimation error and a feedback gain matrix which minimizes a quadratic cost function of the states and control effort. The optimal compensator can be realized by a controller of the form

\[ \hat{x}(k + 1) = A\hat{x}(k) + Bu(k) + \bar{L}[y(k) - C\hat{x}(k)] \]  
\[ u(k) = -\bar{K}\hat{x}(k). \]

Furthermore, this controller can be placed into an error-feedback form by the substitution \( y = r - e = -e \) where \( r \) is the reference, set to zero for the regulation problem, and \( e \) is the tracking error, \( r - y \). So that

\[ \hat{x}(k + 1) = [A - B\bar{K} - \bar{L}C]\hat{x}(k) - \bar{L}e(k) \]  
\[ u(k) = -\bar{K}\hat{x}(k). \]

First, the observer feedback gain, \( \bar{L} \) is considered. The steady-state error covariance

\[ J_{\text{obs}} = \lim_{k \to \infty} E[[e(k) - E[e(k)]][e(k) - E[e(k)]]^T] = M_\infty \]  

is to be minimized in the mean-square sense. This is achieved by choosing \( \bar{L} \) to be the steady-state Kalman filter gains for the system (3.1). That is

\[ \bar{L} = AM_\infty C^T(CM_\infty C^T + V)^{-1} \]
where $M_{\infty}$ is calculated as the solution to the discrete-time Riccati equation

$$M_{\infty} = A(M_{\infty} - M_{\infty}C^T(CM_{\infty}C^T + W)^{-1}$$ (3.6)

Before calculating the optimal state feedback gain, $\bar{K}$, the system must be augmented with an integrator state, $x_i$. Integral action adds tracking capability to the controller. The augmented system may be written as

$$\dot{x}(k+1) = \begin{bmatrix} x(k+1) \\ x_i(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x_i(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) = \dot{\tilde{x}}(k) + \tilde{B}u(k)$$ (3.7)

$$y(k) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x_i(k) \end{bmatrix} = \tilde{C}\tilde{x}(k).$$ (3.8)

The optimal feedback gain is selected to minimize the quadratic cost function

$$J_{sfb}(\tilde{x}(0), u) = \sum_{k=0}^{\infty} [\tilde{x}^*(k)Q\tilde{x}(k) + u^*(k)Ru(k)]$$ (3.9)

where $Q$ is symmetric positive semi-definite and $R$ is symmetric positive definite. The weighting matrix $Q$ can be used to penalize arbitrary linear combinations of the states or outputs, including the augmented integrator states. The weighting matrix $R$ penalizes linear combinations of the control input. Increasing the magnitude of $Q$ relative to $R$ leads to more aggressive control designs; while increasing the magnitude of $R$ relative to $Q$ leads to more conservative designs with lower control effort. Once $Q$ and $R$ have been selected for a design, the optimal feedback gain matrix may be written as

$$\bar{K} = (R + \tilde{B}^TP\tilde{B})^{-1}\tilde{B}^TP\tilde{A}$$ (3.10)
where \( P \) is the solution to the yet another discrete-time Riccati equation

\[
P = \tilde{A}^T[P - P\tilde{B}(R + \tilde{B}^T P \tilde{B})^{-1}\tilde{B}^T P]A + Q. \tag{3.11}
\]

The resulting gain matrix can be partitioned into the sections affecting the system and integrator states, respectively, as

\[
\tilde{K} = [\tilde{K}_x \quad \tilde{K}_i]. \tag{3.12}
\]

Now, the optimal compensator with integral action may be realized in error-feedback form as

\[
\begin{bmatrix}
\hat{x}(k+1) \\
x_i(k+1)
\end{bmatrix} =
\begin{bmatrix}
A - B\tilde{K}_x - LC & -B\tilde{K}_i \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{x}(k) \\
x_i(k)
\end{bmatrix} +
\begin{bmatrix}
-L \\
I
\end{bmatrix} e(k) \tag{3.13}
\]

\[
u(k) =
\begin{bmatrix}
\tilde{K}_x & \tilde{K}_i \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{x}(k) \\
x_i(k)
\end{bmatrix} \tag{3.14}
\]

### 3.1.2 Analysis of Stabilizing Control

Stabilizing LQGi controllers were designed for both the Direct Fit and Modified Fit plant models presented in the previous chapter. The design strategy, common to both controllers, was to penalize the integrator states and control effort heavily in relation to the model states. Low weighting on the model states and a large weighting on the control signals were necessary to avoid control signal saturation when large errors appeared, such as when the rotor must be lifted off the housing at the beginning of levitation. Heavy weighting on the integral states was used to reclaim some of the performance aspects, such as short convergence time, that were lost by the small weighting on the model states. The LQGi controller was implemented in error-feedback form as shown in Figure 3.1.
Figure 3.1: Implementation of the LQGi Controller

If the controller is represented by the state space matrices \([A_c, B_c, C_c, D_c]\) with the state vector \(x_c\), and the open-loop plant by \([A_p, B_p, C_p, D_p]\) with the state vector \(x_p\), then the closed-loop dynamics are described by

\[
\begin{bmatrix}
x_p(k+1) \\
x_c(k+1)
\end{bmatrix} =
\begin{bmatrix}
A_p & B_p C_c \\
-B_c C_p & A_c
\end{bmatrix}
\begin{bmatrix}
x_p(k) \\
x_c(k)
\end{bmatrix} +
\begin{bmatrix}
0 \\
B_c
\end{bmatrix} r(k) \tag{3.15}
\]

\[
y(k) =
\begin{bmatrix}
C_p & 0
\end{bmatrix}
\begin{bmatrix}
x_p(k) \\
x_c(k)
\end{bmatrix}.
\]

Figures 3.2 and 3.3 compare the frequency response of the open-loop plant models with the response of the system when the loop is closed by the LQGi controllers.
Figure 3.2: Frequency Response of the Stabilized System - Direct Fit
The response is shaped in a similar manner by both controllers, which is not surprising as the models and weighting matrices chosen were similar for each design. Both of these controllers were successful in stabilizing the actual plant. A comparison between the simulated and experimental step response for the Direct Fit model and controller is shown in Figure 3.5. The simulation to experiment comparison for the Modified Fit, omitted here, is comparable to that of the Direct Fit.
To visualize cross coupling effects, all channels are shown in each step response of Figure 3.5. Note that the step is not applied to all channels at the same time; each plot represents a distinct trace where the step has been applied to a single input. Inspection of the responses reveals that the simulation captures the rise time of the system relatively well. The accurate prediction of convergence time is expected, as the convergence is governed primarily by the integral terms in the controller. Note that the effects of cross-coupling are under-estimated by the simulation.

The experimental step responses of the Direct Fit LQGi controller, Modified Fit LQGi controller, and the inherited controller used for system identification are shown in Figure 3.5.
Again, the step inputs are not applied to all four channels simultaneously, each plot represents a distinct time trace. The Direct Fit controller has the best step response performance of the three controllers. All have a similar rise time, however, the Modified Fit has substantially greater overshoot and the inherited controller exhibits ringing and a relatively slow convergence time. In the step response experiments the spindle was not rotating. Recall that rotation of the spindle introduces disturbances due to imbalance which are synchronous with rotational speed. For a complete comparison of the controllers, performance during spindle rotation must be examined. The ability of the stabilizing controller designs to regulate spindle position during rotation are compared in Figure 3.6.
Figure 3.6: LQGi Regulation RMS Error

Figure 3.6 shows the root-mean-square (RMS) error for a 10 second period of steady-state spindle regulation with the spindle rotating at a given frequency. In the no spin case, the two LQGi designs are comparable in regulation performance. Through the 100 to 300 Hz range, the controller designed using the Direct Fit Model outperforms the Modified Fit design. Then the tables are turned for the 400 Hz case, and performance is again comparable at 500 Hz; with the Modified Fit design having a slight advantage. This data alone does not lead directly to any clear cut decision about which controller has done a better job. To analyze regulation performance further, the control effort RMS values corresponding to the experiments in Figure 3.6 are plotted in Figure 3.7.
Inspection of Figure 3.6 shows that for all spin rates beyond 100 Hz the control effort of the Modified Fit design is approximately equal or less than that of the Direct Fit design. The difference between the two becomes more pronounced at higher frequencies. The control objectives discussed in the following sections will require that the system track references of around 400 Hz and will strain the limits of the actuators. Control schemes using the Direct Fit controller proved difficult to stabilize when operated under these conditions.

For these reasons, the stabilizing controller used in subsequent design processes and in the experimental implementation will be the LQGi controller designed using the Modified Fit model.

### 3.2 Plug-In Resonator

The plug-in resonator is developed as an add-on to the stabilizing LQGi control to improve performance in disturbance rejection and trajectory tracking. This controller utilizes the internal model principle to achieve asymptotic regulation of the error signal at a chosen set
of frequencies [WCT09]. The internal model principle states that if a model of a disturbance is placed in the feedback path of the error, components of the error signal described by that model will be asymptotically driven to zero [FW76]. This principle holds for both disturbances and tracking error, as these share the same feedback path. The plug-in structure allows performance gains to be achieved without modifying an existing nominal control loop. Figure 3.8 show the structure of the controller.

![Figure 3.8: Structure of the Plug-in Peak Resonator](image)

The plug-in controller, Cr, is comprised of the peak filter, L, in a positive feedback loop to produce a resonator, and the inversion filter, F. Design of the filters L and F is outlined in the following sections.

### 3.2.1 Resonator

The disturbance due to rotor imbalance manifests as a sinusoidal disturbance synchronous with the rotor speed, along with higher harmonic components. Likewise, tracking error for a periodic trajectory will have fundamental components at the frequency of the reference and its higher harmonics. According to the internal model principle, these are the frequencies which should be modeled in the feedback path; this is the motivation for the design of the resonator [KT14]. Figure 3.9 shows the structure of the resonator system.
The peak filter, $L$, must be designed so that this loop has very high gain near the resonant frequencies and gain nearly equal to zero away from those frequencies. That is

$$L(e^{j\omega T_s}) = 1 \text{ for } \omega = \omega_k$$  \hspace{1cm} (3.16)

and

$$L(e^{j\omega T_s}) \approx 0 \text{ for } \omega \neq \omega_k.$$  \hspace{1cm} (3.17)

A realization of $L$ may be obtained in the following fashion: First a series of second-order notch filters are designed at the frequencies to be targeted by the resonator. Each notch filter is specified by

$$H = \prod_{k=1}^{p} \frac{1 - 2\beta_k \cos(\omega_k T_s)z^{-1} + \beta_k^2 z^{-2}}{1 - 2\rho_k \cos(\omega_k T_s)z^{-1} + \rho_k^2 z^{-2}}$$

where $\omega_k$ is the notch frequency and $\beta$ and $\rho$ tune the depth and bandwidth of the notch, respectively. The notch filters are then cascaded and inverted to produce the peak filter. There are two primary advantages to this design method: First, frequency specification is straightforward and not constrained to be a multiple of the sampling time. Secondly, the depth and bandwidth of each notch can be tuned independently, so long as the notches are sufficiently spread across the band. The inversion to produce the peak filter
is carried out simply as

\[ L = 1 - H \]  \hspace{1cm} (3.18)

and then \( L \) is wrapped in a positive feedback loop to produce the resonator

\[ R_{imp} = L(I - L)^{-1}. \]  \hspace{1cm} (3.19)

If the notch parameter \( \beta \) is chosen to be equal to unity, then the resonator system, \( R_{imp} \), will be marginally stable with poles on the unit circle. Designs using \( \beta = 1 \) will produce infinite gain at the targeted frequencies.

### 3.2.2 Discrete Approximate Plant Inversion

Consider the following block diagram, which is equivalent to Figure 3.10

![Figure 3.10: Equivalent Block Diagram of the Resonator and Closed-Loop Plant](image)

The resonator developed in the previous section satisfies the internal model principle, however, direct application as a plug-in to the closed-loop system, \( G \), does not result in a stable system in general. Furthermore, modifications to the stabilizing controller, \( C \), to stabilize the new loop which includes the resonator may not be straightforward. Avoiding this complicated design problem and maintaining the functionality of a plug-in style controller motivates the design of the stabilizing filter, \( F \). The Nyquist stability criterion for MIMO systems may be conservatively applied as a sufficient stability condition for the loop in Figure 3.10.
First, the closed-loop transfer function may be derived as

\[ y = GFL(I - L)^{-1}(I + GFL(I - L)^{-1})^{-1}r \]
\[ y = GFL(I - L + GFL)^{-1}r \]
\[ y = GFL(I + (GF - I)L)^{-1}r. \]  \hfill (3.20)

By inspection of equation (3.20), the closed-loop characteristic polynomial is

\[ (I + (GF - I)L)^{-1}. \]  \hfill (3.21)

For stable filters \( G, F, \) and \( L \) the Nyquist stability criterion states that the closed-loop system is stable if and only if the contour of

\[ \det[I + (G(e^{j\omega T_s})F(e^{j\omega T_s}) - I)L(e^{j\omega T_s})] \]  \hfill (3.22)

in the z-plane completes zero encirclements of the origin. Noting that this contour rests at the point \((1,0)\) when loop gain is zero, the following sufficient condition for stability may be derived

\[ \sup_{\omega} \| \det(I + (G(e^{j\omega T_s})F(e^{j\omega T_s}) - I)L(e^{j\omega T_s})) - 1 \| < 1 \]  \hfill (3.23)

which guarantees zero encirclements of the origin by restricting the contour to remain within a unit circle centered at \((1,0)\). Now consider the effect of \( L \) on this condition. By design, if \( \omega \neq \omega_p \), then \( L \approx 0 \) so that

\[ \sup_{\omega} \| \det(I + (G(e^{j\omega T_s})F(e^{j\omega T_s}) - I)L(e^{j\omega T_s})) - 1 \| \approx 0 < 1. \]  \hfill (3.24)

Also, if \( \omega = \omega_p \), then \( L = I \) so that

\[ \sup_{\omega_p} \| \det(I + (G(e^{j\omega_p T_s})F(e^{j\omega_p T_s}) - I)L(e^{j\omega_p T_s})) - 1 \| \]  \hfill (3.25)

which is minimized by \( G(e^{j\omega_p T_s})F(e^{j\omega_p T_s}) = I. \)

These results motivate an inversion of the closed-loop plant, \( G \). Common SISO inversion methods such as zero-phase error tracking are not easily extended to MIMO systems. Also,
A realization of the discrete approximate inversion may be obtained as follows: Let \( \omega_p \) be the set of frequencies targeted by the resonator. Then at each of these frequencies the response of the plant may be represented by a 4-by-4 complex matrix \( G(e^{j\omega_p(k)T_s}) \). The response of a system which inverts \( G \) at these frequencies can be determined as \( F(e^{j\omega_p(k)T_s}) = G(e^{j\omega_p(k)T_s})^{-1} \). Each element of these matrices represent the desired response for a given input-output pair at each targeted frequency. Finally, 16 finite impulse response (FIR) filters, one for each input-output pair, are fit to satisfy the requirements of \( F(e^{j\omega_pT_s}) \) and then concatenated into a MIMO FIR inversion filter for the plant \( G \). The FIR filters are fit to minimize the cost function

\[
J_{fit} = \sum_{k=1}^{n} w_t(k) \| I - G(e^{j\omega_p(k)T_s})F(e^{j\omega_p(k)T_s}) \|_2^2 \tag{3.26}
\]

where the weighting \( w_t \) is an additional design tool to allow for some flexibility to tune filter behavior over the spectrum [Kan14].

### 3.3 Implementation and Experimental Results

For implementation on the AMBS, the following design task was considered: Track a sinusoidal reference signal synchronous with twice the spindle rotational speed. This task is applicable to wrist pin hole machining wherein feed rate must be increased twice per revolution at either end of the major and minor axes of the elliptical geometry. This control objective will require the plug-in controller to show its capabilities for both tracking and disturbance rejection. Two spindle rates will be considered. First, the spindle will rotate at 77 Hz, this relatively slow speed allows for a demonstration of the plug-in resonator performance while remaining comfortably within the plant capabilities and using the low frequency portion of the plant model which is expected to describe the physical system well. After-
wards, the spindle will be spun at 200 Hz which strains the limits of the actuators and relies on higher frequency portions of the plant model where the fit is more uncertain.

### 3.3.1 Spindle Rate of 77 Hz

In this experiment the spindle is spun at a rate of 77 Hz. Disturbance rejection for the regulation problem and tracking of sinusoidal trajectories synchronous with twice the spindle speed, around 154 Hz, are examined. The peak filter frequencies were chosen to target the rotor speed and its first three harmonics, which, because the reference is synchronous, includes the reference speed and its first harmonic. Figure 3.11 shows the design of the peak filter.

![Figure 3.11: Frequency Response of the Peak Filter, 77 Hz Design](image)

It should be noted that Figure 3.11 shows the response of a SISO system. For this
design, the same peak filter was used for all channels, although the formulation does allow for different peak filters on each channel. The filter $L$ has been designed such that all poles of the resonator lie on the unit circle.

The discrete approximate inversion fit used 65\textsuperscript{th} order FIR filters to characterize each input-output relationship. The SISO filters were concatenated into a 4-by-4 MIMO system and a minimal realization was computed to produce $F$. The term which was used to show a sufficient stability condition in \((3.23)\), $|\det(I + (GF - I)L) - 1|$ is plotted for all frequencies in Figure 3.12.

![Figure 3.12: Magnitude of Shifted MIMO Nyquist Contour, 77 Hz Design](image)

In Figure 3.12 the dashed grey line represents the bound that the plotted term must stay beneath to sufficiently ensure stability. Crossing this bound does not imply an unstable system. Tradeoffs are allowed to reduce fit complexity at the cost not strictly satisfying \((3.23)\), which is the case in this design. Stability can be determined to within the accuracy of the model by simply examining the closed loop eigenvalues. Triangular markers indicate the location of peak frequencies and the value at those frequencies are recorded in Table 3.1.
Table 3.1: Discrete Approximate Inversion - 77 Hz Design

| Frequency, Hz | \(|\det(I + (GF - I)L) - 1|\) |
|--------------|-------------------------------|
| 77.1         | 0.0009                        |
| 154.2        | 0.0064                        |
| 231.2        | 0.0029                        |
| 308.4        | 0.0005                        |

The magnitude of the sensitivity function for this design is shown in Figure 3.13.

\[ L_g = PC(I + C_r) \]  
\[ S \triangleq (I + L_g)^{-1} \]  
\[ S = (I + PC(I + C_r))^{-1} \]

The sensitivity function is derived to analyze the expected performance in disturbance rejection and tracking. For the block diagram in 3.8

\[ L_g = PC(I + C_r) \] (3.27)  
\[ S \triangleq (I + L_g)^{-1} \] (3.28)  
\[ S = (I + PC(I + C_r))^{-1} \] (3.29)

Figure 3.13: Magnitude of the Sensitivity Function, 77 Hz Design
In Figure 3.13 it is clear that the plug-in resonator has had a strong effect in reduction
the sensitivity of the error to disturbances and references at the targeted frequencies, as is
expected. Frequencies nearby these notches have been amplified in some places. This is
the so-called waterbed effect, a consequence of the Bode Integral Theorem; pushing down
the sensitivity at one frequency will cause it to rise at another to satisfy conditions on the
integral of the function [Moh90]. An advantage of the plug-in resonator is that it may be
designed to affect only narrow frequency bands. Targeting narrow bands reduces the impact
of the high-gain control on the integral of the sensitivity function and is used to keep the
inter-harmonic amplification from becoming too large.

To examine the performance of the plug-in controller, the regulation problem is considered
first. In Figure 3.14 the rotor is spun at 77 Hz and begins with the LQGi controller only;
the plug-in resonator is turned on at around 1.2 seconds.

![Figure 3.14: Disturbance Rejection at Spindle Speed of 77 Hz](image)

The plug-in significantly reduces the error caused by the rotor imbalance during spin.
The error RMS values for the LQGi controller and LQGi with the plug-in resonator are
organized in Table 3.2

40
Table 3.2: Regulation Error RMS - 77 Hz

<table>
<thead>
<tr>
<th></th>
<th>LQGi</th>
<th>Plug-in Resonator</th>
<th>No Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>V13</td>
<td>4.12</td>
<td>0.503</td>
<td>0.400</td>
</tr>
<tr>
<td>W13</td>
<td>2.97</td>
<td>0.488</td>
<td>0.352</td>
</tr>
<tr>
<td>V24</td>
<td>2.33</td>
<td>0.850</td>
<td>0.800</td>
</tr>
<tr>
<td>W24</td>
<td>3.02</td>
<td>0.447</td>
<td>0.368</td>
</tr>
</tbody>
</table>

The spectral composition of the error during regulation is shown in Figure 3.15. The peaks due to rotor imbalance disturbances are clearly visible at the fundamental frequency of 77 Hz and each of its harmonics. The plug-in resonator significantly reduces the error components at each targeted frequency. Note that the error component at the 4th harmonic, which was not included in the peak filter design, remains unchanged.

Figure 3.15: Error Spectrum During Regulation at Spindle Speed of 77 Hz

In the next experiment, the spindle is commanded to track a 20 µm peak-to-peak sinusoid synchronous with twice the spindle speed. Time traces of the steady-state tracking error with
and without the plug-in resonator are shown in Figure 3.16.

![Figure 3.16: Steady-State Error for Tracking 154Hz Sinusoid at Spindle Speed of 77 Hz](image)

The steady-state RMS error values are included in Table 3.3.

<table>
<thead>
<tr>
<th></th>
<th>LQGi</th>
<th>Plug-in Resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td>V13 [µm]</td>
<td>19.8</td>
<td>0.716</td>
</tr>
<tr>
<td>W13 [µm]</td>
<td>14.6</td>
<td>0.896</td>
</tr>
<tr>
<td>V24 [µm]</td>
<td>8.78</td>
<td>0.842</td>
</tr>
<tr>
<td>W24 [µm]</td>
<td>16.6</td>
<td>0.810</td>
</tr>
</tbody>
</table>

Table 3.3: Tracking Error RMS - 77 Hz

Choosing the reference sinusoids to be 90 degrees out of phase between the V and W axes results in an elliptical spindle trajectory. This shape is representative of the wrist pin hole machining process. Figure 3.17 shows a scatter plot of spindle position inside each bearing and projected normal to the nominal axis of rotation. In the experiment, the reference creates an elliptical spindle trajectory with a major axis of 60 µm along V and a minor axis...
of 40 µm along W. "Outboard" refers to the end of the spindle to which the cutting tool is attached. "Inboard" refers to the end opposite the cutting tool.

The plug-in resonator drastically improves the tracking ability of the nominal closed-loop system for this elliptical trajectory.

### 3.3.2 Spindle Rate of 200Hz

For the following experiments, spindle rotational speed is increased from 77 Hz to 200 Hz. This faster spindle speed is now in the range of what would be considered high-speed machining. Challenges associated with increasing the spindle speed are increased disturbance forces due to imbalance, greater model uncertainty due to relying on higher frequency portions of the model, greater model uncertainty due to changes in the rotor dynamics from the no spin condition of the system identification, and the danger of control saturation when tracking references at frequencies where the plant response has rolled off significantly. At this spindle rate, tracking will be done by the Outboard AMB only. The Inboard AMB is used solely
for regulation. Operating in this manner is consistent with practical implementation of the AMBS in machining processes, wherein a single point cutting tool at the end of the spindle traces out a three-dimensional envelope within the workpiece.

The peak filter is designed separately for the Inboard and Outboard AMBs. Channels $V_{13}$ and $W_{13}$, which are used solely for regulation, utilize a peak filter which targets the spindle rate and its first harmonic. Channels $V_{24}$ and $W_{24}$, which will be used for tracking, use peak filters which target the fundamental frequency and the first two harmonics. Stabilization of the experimental system becomes difficult for peaks placed at the fourth harmonic and beyond; likely due to discrepancies between the stationary identified model and actual spinning plant causing the inversion at high frequencies to become poor. The peak filter designs for both AMBs are shown in Figure 3.18.

![Figure 3.18: Frequency Response of the Peak Filter, 200 Hz Design](image)

The discrete approximate inversion was built up using 75th order FIR filters. The inversion filter was then calculated as a minimal realization of the 4-input 4-output concatenation of the FIR filters for each input-output pair. In Figure 3.19, the term $|\det(I+(GF-I)L)-1|$ from the sufficient stability condition in (3.23) is plotted over all frequencies.
Figure 3.19: Magnitude of Shifted MIMO Nyquist Contour, 200 Hz Design

As in the previous design, the bound for sufficient stability is not strictly satisfied here. This is due to design tradeoffs and does not imply instability. Peak frequencies in Figure 3.19 are marked with triangles, the values at these frequencies are recorded in Table 3.4.

| Frequency, Hz | \(|\text{det}(I + (GF - I)L) - 1|\) |
|--------------|-----------------------------------|
| 199.4        | 0.0319                            |
| 398.7        | 0.0948                            |
| 598.1        | 0.0078                            |

Table 3.4: Discrete Approximate Inversion - 200 Hz Design

The sensitivity function of the system with this plug-in design is shown in Figure 3.20. Note that inter-harmonic amplification is more pronounced in the \(V_{24}\) and \(W_{24}\) channels where wider peaks were chosen to improve tracking performance for small deviations in spindle frequency.
Figure 3.20: Magnitude of the Sensitivity Function, 200 Hz Design

As in the previous section, the regulation problem will be considered first. Figure 3.21 shows the time traces for regulation with and without the plug-in resonator. In these plots, the resonator is turned on at time = 1 second.

Figure 3.21: Disturbance Rejection at Spindle Speed of 200Hz
The steady-state RMS error values for the regulation problem are given in Table 3.5.

<table>
<thead>
<tr>
<th></th>
<th>LQGi</th>
<th>Plug-in Resonator</th>
<th>No Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>V13 [µm]</td>
<td>4.01</td>
<td>0.723</td>
<td>0.400</td>
</tr>
<tr>
<td>W13 [µm]</td>
<td>3.65</td>
<td>0.517</td>
<td>0.352</td>
</tr>
<tr>
<td>V24 [µm]</td>
<td>2.06</td>
<td>0.984</td>
<td>0.800</td>
</tr>
<tr>
<td>W24 [µm]</td>
<td>2.43</td>
<td>0.670</td>
<td>0.368</td>
</tr>
</tbody>
</table>

Table 3.5: Regulation Error RMS - 200 Hz

The spectral components of steady-state error for the regulation problem are shown in Figure 3.22.

![Figure 3.22: Error Spectrum During Regulation at Spindle Speed of 200 Hz](image)

All targeted frequencies show significantly reduced error, although the error peaks around 200 and 400 Hz on $V_{13}$ and $W_{13}$ have not been attenuated completely. This is due to the narrow design of peaks on these channels being sensitive to slight mismatches in design frequency and actual rotor frequency. Narrow peaks on these channels were necessary for
stable implementation. The error components at targeted frequencies on channels \(V_{24}\) and \(W_{24}\), however, have been attenuated to the noise floor. Much wider peak designs were realizable on these channels, as shown in Figure 3.18.

In the next experiment, spindle position is regulated on axes \(V_{13}\) and \(W_{13}\) while 20 \(\mu\)m peak-to-peak sinusoidal trajectories, synchronous with twice the rotor speed, are tracked on axes \(V_{24}\) and \(W_{24}\). Time traces of the steady-state error in this tracking problem are shown in Figure 3.23.

Figure 3.23: Steady-State Error for Tracking 400 Hz Sinusoid at Spindle Speed of 200 Hz

The RMS values of steady steady-state tracking error are given in Table 3.6.

<table>
<thead>
<tr>
<th></th>
<th>LQGi</th>
<th>Plug-in Resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_{13}) [(\mu)m]</td>
<td>4.66</td>
<td>0.680</td>
</tr>
<tr>
<td>(W_{13}) [(\mu)m]</td>
<td>4.76</td>
<td>0.670</td>
</tr>
<tr>
<td>(V_{24}) [(\mu)m]</td>
<td>9.57</td>
<td>1.09</td>
</tr>
<tr>
<td>(W_{24}) [(\mu)m]</td>
<td>11.6</td>
<td>0.564</td>
</tr>
</tbody>
</table>

Table 3.6: Tracking Error RMS - 200Hz
The control signal during the tracking experiment is shown along with the control saturation limits in Figure 3.24.

![Figure 3.24: Saturation of the AMB Control Signal](image)

The control signal on Channel $V_{24}$ has just begun to enter the saturation region, showing that for this controller design the actuator’s limit of performance has been reached. The AMBS will not be able to track larger amplitude or faster trajectories on this axis without, at the very least, significant controller design changes.

In Figure 3.25 the spindle position is shown as a scatter plot viewed along the nominal rotation axis as the Outboard AMB tracks a circular trajectory and the Inboard AMB regulates.
Similarly to the 77 Hz case, the plug-in resonator corrects the mostly incoherent tracking efforts of the nominal closed loop system to track a path that is centered about the reference trajectory.
CHAPTER 4

Conclusion

This thesis has presented the modeling of an AMBS, feedback control design for regulation and reference tracking, and implementation results on the experimental system. System identification was carried out using the sine sweep method. Two MIMO models were fit to the experimental identification data which prioritize different merits of fit: One model sought primarily to fit the collected data well, and the other sought to make use of the information within an inherited stabilizing controller. Stabilizing controllers were designed for each of the models by the LQGi method and the performance of these controllers were compared. An internal model principle type controller, the plug-in resonator, was proposed for rejection of disturbances due to rotor imbalance and the tracking of periodic trajectories. This control strategy was implemented for spindle rates of 77 and 200 Hz.

The system identification models were both sufficiently accurate to produce stabilizing controllers. These models correctly predicted the 4 unstable poles expected in the system. The implementation of the feedback controllers designed based on the models shows that the rotating spindle maintains stability for speeds up to 500 Hz. Plant inversions based on the identified model worked well up to about 600 Hz and for spindle speeds up to about 200 Hz. Attempts to invert the plant at frequencies greater than 600 Hz when rotation rate was 200 Hz or more revealed deficiencies in the model. Future works should focus on obtaining a model with closer fidelity to the actual system for different spin rates; or, at least, several models at different spin rates.

The plug-in resonator worked well for rejecting disturbances within the capabilities of the actuator. In the case of the spindle rotating at 77 Hz, the regulation error RMS is reduced, on average, by a factor of 5.9 down to within 120% of the error RMS in the no spin case.
In the 200 Hz rotation rate case, the regulation error RMS is reduced, on average, by a factor 4.7 down to within 160 % percent of the error RMS in the no spin case. The nominal closed-loop system with LQGi, which was incapable of tracking trajectories at these spindle rates, was greatly improved by the plug-in resonator. Tracking error was reduced so that it remained within less than twice the error bound for no spin regulation.

Limitations on performance with the plug-in resonator stemmed primarily from plant model uncertainty and actuator saturation limits. Uncertainty in the plant model forced the design of relatively narrow peaks in the peak filter, this caused performance to fall off quickly for even a small mismatch between the design and actual rotor speed. When tracking a sinusoidal reference of 400 Hz at an amplitude of 20 \( \mu \text{m} \) peak-to-peak the actuator of channel \( V_{24} \) just entered the saturation range. The plant was able to maintain stability and decent performance, but this prevented any further reduction of the tracking error and discouraged further experiments at higher spindle rotation rates.

The experimental results show that the plug-in resonator may significantly improve the performance of a control system without any modification to a known stabilizing controller. The resonator may be easily designed to target any frequency and turned on and off at will during operation. The discrete approximate inversion of the closed loop plant simplifies the typically difficult inversion task and the entire plug-in may be designed to have minimal impact on high frequency dynamics which may be poorly modeled.
References


