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Theoretical Aspects of Underwater Photography

I. GRAPHICAL-NUMERICAL CALCULATION OF THE LUMINANCE FIELD ABOUT A SUBMERGED POINT SOURCE

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PREFACE

Fundamental studies of underwater photography by means of artificial light currently in progress in the Visibility Laboratory have revived interest in an internal memorandum written in 1953 in the course of solving a specific Navy photographic problem. A reproduction of that memorandum is hereby issued as a Laboratory report. An added appendix indicates applications of the memorandum to related but different underwater photographic problems, and extends the discussion to include effects due to multiply scattered light.
ABSTRACT

The primary luminance distribution was calculated about a given source submerged in an homogeneous hydrosol with a non-isotropic volume scattering function. The computations were carried out for a given set of distances from the source, and at each distance the primary luminance was computed for a given range of angles of sight from the observer-source direction. These computations allowed certain specific recommendations to be made as to what film speeds were necessary in order that the field luminance about the given point source be recorded on a photographic plate.
GRAPHICAL–NUMERICAL CALCULATION OF THE PRIMARY LUMINANCE FIELD ABOUT A SUBLIMED POINT SOURCE

R. W. Preisendorfer

VISIBILITY LABORATORY 11 NOVEMBER 1953
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1. STATEMENT OF THE PROBLEM

Let $M^*$ be an homogeneous hydrosol with attenuation coefficient $\alpha'$ and volume scattering coefficient $\sigma(\delta)$. Let a sphere of small* but finite diameter $D$ be immersed in $M^*$ with center at $s$. Let the sphere have a self-luminous Lambert surface with luminance $B_s$. If an observer is at a point $o$ in $M^*$ which is at a distance $R_o$ from $s$, and the line of sight of the observer is directed at an angle $\theta$, $0 \leq \theta < \pi$, with the line $os$, required: the primary luminance $B'(\theta)$ of the field,** as seen from $o$ in the direction $\theta$.

2. DERIVATION OF FINITE SUMMATION FORMULA FOR PRIMARY LUMINANCE

2.1 Relation between $B_s$ and the Total Luminous Flux Output $F_s$ of Sphere.

Some sources of luminous energy are defined by giving their total luminous flux output instead of their inherent luminance. A relation between $F_s$ and $B_s$ of the given spherical source is needed for use in the complete solution of the stated problem. There are two basic ways to obtain the relation between $F_s$ and $B_s$. The first proceeds as follows: Let $R = D/2$ be the radius of the sphere. Then

$$B_s = \Delta I_s / \Delta A \cos \theta \quad (2.1.1)$$

* Small compared to the distance $R_o$, i.e., $5D \leq R_o$. See Summary Sheet No. 25 of ((3)).

** Primary Luminance is defined in Summary Sheet No. 74 of ((3)).
where $\Delta I_\theta$ is the luminous intensity in the direction $\theta$ to the normal of an element of area $\Delta A$ of the surface of the given sphere. Since $\Delta I_\theta$ obeys Lambert's law we may write

$$\Delta I_\theta = \Delta I_0 \cos \theta$$

(2.1.2)

so that the element of luminous flux $\Delta F_\theta$ emitted by $\Delta A$ is

$$\Delta F_\theta = \frac{\pi}{2} \int_{\theta=0}^{\pi/2} \Delta I_0 \cos \theta \sin \theta \, d\theta$$

(2.1.3)

$$= \pi \Delta I_0.$$  

(2.1.3)

Since

$$E_s = \Delta I_0 / \Delta A$$

(2.1.4)

we may write

$$F_s = \pi E_s \Delta A$$

(2.1.5)

so that the total luminous flux output of the sphere is

$$F_s = \pi^2 \Delta A E_s.$$  

(2.1.6)
The second basic method of deriving the relation between $F_s$ and $B_s$ is as follows: Let $\Delta \omega_s$ be the solid angle subtended by the sphere at $o$. Then

$$\Delta \omega_s = \frac{\pi D^2}{4R_o^2}. \quad (2.1.7)$$

The illuminance $\Delta E_s$ produced at $o$ by the luminous flux from the given sphere on a surface normal to the line $os$ (imagining $\beta' = 0$ in $M^*$) is:

$$\Delta E_s = F_s / 4\pi R_o^2. \quad (2.1.8)$$

Since

$$B_s = \frac{\Delta E_s}{\Delta \omega_s} \quad (2.1.9)$$

we have

$$B_s = \frac{\frac{F_s}{4\pi R_o^2}}{\frac{\pi D^2}{4R_o^2}} = \frac{F_s}{\frac{2}{\pi} D^2}. \quad (2.1.10)$$
2.2 Formula for $B^{(1)}(\psi)$.

For the geometrical relations used in the following derivation refer to Figures 2.2.1 and 2.2.2.

Let $p$ be a point in $M^*$ at which the unattenuated luminous flux from $s$ is scattered through an angle $\Theta$ to $o$. The unattenuated illuminance $E^{(o)}$ from $s$ at $p$ on a surface normal to the line $sp$ is:

$$E^{(o)} = F_s e^{-\Phi R_1 / 4\pi R_1^2} . \tag{2.2.1}$$

We now determine a region of $M^*$ about $p$ of finite volume $dv$ by means of an elementary rectangular solid angle centered on the line $sp$. Let the vertical angular height of the solid angle be $\alpha_v$ and the horizontal angular width be $\alpha_h$ measured in radians. The volume $dv$ is determined by the intersection of this solid angle and a solid angle of suitable dimensions $\alpha_v, \alpha_h$ with vertex at $o$. It is easy to see that

$$dv = R_1^2 \alpha_h \alpha_v dR_1 . \tag{2.2.2}$$

where $dR_1$ is the increment of length of $R_1$ cut off by the solid angle with vertex at $o$. See Figure 2.2.2. Some of the unattenuated luminous flux at $p$ is redirected by scattering through an angle $\Theta$ to $o$. The primary luminous intensity $dI^{(1)}_{\Theta}$ of $dv$ in the direction $po$ as induced by the $E^{(o)}$ given in (2.2.1) is:

$$dI^{(1)}_{\Theta} = E^{(o)} \sigma(\Theta) dv \tag{2.2.5}$$

by definition of $\sigma(\Theta)$. Hence
As seen from $o$, the element of volume $dv$ is a rectangle of height $dR_1 \sin \theta$ and width $a \cdot R_1$, so that the solid angle $d\Omega_o$ subtended at $o$ by $dv$ is:

$$d\Omega_o = a \cdot R_1 \ dR_1 \ \sin \theta / R_2^2.$$  \hspace{1cm} (2.2.5)

The unattenuated illuminance $dE_2^{(1)}$ produced at $o$ (on a surface whose normal lies along $p_0$) by this unattenuated luminous flux from the illuminated element of volume $dv$ is:

$$dE_2^{(1)} = dI_o^{(1)} \ e^{-\beta R_2 / R_2^2}.$$  \hspace{1cm} (2.2.6)

The contribution $B_p^{(1)}$ to the primary luminance $B^{(1)}(\theta)$ by the illuminated element of volume $dv$ is then:

$$B_p^{(1)} = \frac{dE_2^{(1)}}{d\Omega_o} = \frac{dI_o^{(1)} \ e^{-\beta R_2 / R_2^2}}{a \cdot R_1 \ dR_1 \ \sin \theta / R_2^2}$$  \hspace{1cm} (2.2.7)

that is,

$$B_p^{(1)} = \frac{(F_0 / 4\pi) \ a \ \sigma(\theta) \ e^{-\beta(R_1 + R_2)} \ dR_1}{a \cdot R_1 \ dR_1 \ \sin \theta}$$
or
\[ B_p^{(1)} = F_s a_v \sigma(\theta) e^{-\beta'(R_1 + R_2)} / (4\pi R_1 \sin \theta). \]  

(2.2.8)

Now
\[ R_0 \sin \psi = R_1 \sin \theta \]  

(2.2.9)

which allows (2.2.8) to be written
\[ B_p^{(1)} = F_s a_v \sigma(\theta) e^{-\beta'(R_1 + R_2)} / (4\pi R_0 \sin \psi). \]  

(2.2.10)

Equation (2.2.10) thus gives the characteristic form for the component \( B_p^{(1)} \) of \( B^{(1)}(\psi) \). If a set \( dv_1, dv_2, \ldots, dv_k \) of volumes at points \( p_1, p_2, \ldots, p_k \) along the line of sight each subtended a vertical angular height \( a_v \) at \( s \) and if the scattering angle \( \theta_1 \) is associated with \( p_1 \), and if \( R_{11} \) and \( R_{21} \) are the distances \( sp_1 \) and \( p_1o \) respectively, and if the resulting luminance component of \( B^{(1)}(\psi) \) associated with \( p_1 \) is denoted by \( B_{p_1}^{(1)} \), then we may approximate \( B(\psi) \) by writing
\[ B^{(1)}(\psi) = \sum_{i=1}^{k} B_{p_i}^{(1)} = \frac{F_s a_v}{4\pi R_0 \sin \psi} \sum_{i=1}^{k} \sigma(\theta_1) e^{-\beta'(R_{11} + R_{21})}. \]  

(2.2.11)

The computation of \( B^{(1)}(\psi) \) for given \( F_s, \psi \) and \( R_0 \) is reduced to a finite summation of the products \( \sigma(\theta_1) e^{-\beta'(R_{11} + R_{21})} \). The choice of the angle \( a_v \) depends on how fine the subdivisions of the line of sight are to be chosen. The dimensions of the physical situation should suggest an appropriate choice of \( a_v \). See Figure 2.2.3.
3. **PHYSICAL AND GEOMETRICAL DATA FOR NUMERICAL EXAMPLE.**

This report is based on a request to the Visibility Laboratory for an answer to a specific problem which had the form of the statement given in section 1. For the specific problem submitted optical constants for the hydrosol $M^*$ were chosen as $\beta' = 0.051 / \text{ft}$ which is equivalent to an horizontal hydrological range of 77 feet. The volume scattering function $\sigma(\theta)$ was taken from graphical data for ocean water given in the figures on page 71 in [1]. The source was assumed to have a diameter of 0.20 feet and a luminous energy output of $1.2 \times 10^5$ lumen-seconds which is the luminous energy output of a #60 photoflash. A luminous energy output was assumed instead of a luminous flux (which is energy per unit of time) since the detector of the field luminance $B(\psi)$ was to be a photographic plate of suitable speed. Consequently, if $F_s$ has units of lumen-seconds, $B^{(1)}(\psi)$ as given in (2.2.11) will have units of luminance-seconds and is so recorded in TABLE 4.1 and plotted in Figure 5.1. The calculations were performed for the following values of $R_0$: 40, 100, 200, and 300 feet; and for the following values of $\psi$: 0°, 2.5°, 5°, 10°, 20°, 25°, and 30°.
4. RESULTS TABULATED

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<th>$\psi$</th>
<th>$R_o$</th>
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<th>$R_0$</th>
<th>$R_0$</th>
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<td></td>
<td>40 ft</td>
<td>100 ft</td>
<td>200 ft</td>
<td>300 ft</td>
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<td>$1.37 \times 10^1$</td>
<td>$9.3 \times 10^{-2}$</td>
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<td>$2.50 \times 10^{-6}$</td>
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<td>$5.00 \times 10^{-6}$</td>
<td>$1.78 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
6. SUMMARY AND CONCLUSIONS

The primary field luminance about a submerged point source has been calculated in a given hydrosol for various distances from the given point source and for various angular distances about the line connecting the source with the observer. For details, see section 3. The results are tabulated in TABLE 4.1 and Fig. 5.1. For the physical and geometrical conditions defined in section 3, some statements about the film speeds needed for film plates exposed to the luminance field may be made:

For \( R_o = 40 \) ft, the source and aureole are expected to be recorded on Eastman Panatomic X or Eastman Portrait Panchromatic.

For \( R_o = 100 \) ft, the source and aureole are expected to be recorded out to the angular distance of 2.5° on Panatomic X.

For \( R_o = 200 \) ft, the source and aureole are expected to be recorded out to 1.25° on Panatomic X.

For \( R_o = 300 \) ft, only source should make an impression on Eastman Super XX.

The above recommendations are based on the following relation:

\[
\text{WESTON FILM SPEED} = 0.8 f^2 / Bt
\]

(6.1)

as given on pages 8-10 in ((2)), where

- \( f \) is the diaphragm opening
- \( B \) is the luminance of the scene
- \( t \) is the time of exposure to luminance field or flash duration.
The characteristics of the lens for the recording camera were assumed to be:

focal length -- 12 inches
lens diameter -- 4.8 inches
diaphragm opening -- 2.5

The product $B_t$ was taken from the data shown in Figure 5.1; the film speed and hence the type of film were determined by using (6.1).
APPENDIX

We append three general observations to the present work. First of all, we note that even though the results and conclusions of the present report are restricted to a specific problem of underwater photography (See Sec. 6), the method of approach considered here is amenable to wider applications. In particular, equation (2.2.11) summarizes a useful approximate method for the determination of the primary component of the steady state luminance distribution generated by an isotropic point source in an homogeneous optical medium with a general volume scattering function $\sigma$. Furthermore, equation (2.2.11) can be extended to the case of nonhomogeneous media with anisotropic point sources thus:

$$B^{(1)}(\psi) = \frac{A_s}{R_0 \sin \psi} \sum_i B_s(\theta_i - \psi) \sigma_i(\theta_i) T_i,$$

where $T_i$ is the beam transmittance of the path $C_i p_i S$, $A_s$ is the area of a great circle on the non-isotropic spherical source, $B_s(\theta_i - \psi)$ is the luminance of the source in the direction $(\theta_i - \psi)$ (See Figs. 2.2.1 and 2.2.3), and $\sigma_i(\theta_i)$ is the value of $\sigma$ at $\rho_i$ for the scattering angle $\theta_i$.

Secondly, knowledge of the primary luminance distribution allows one to compute such auxiliary photometric quantities as the primary component of the illuminance on arbitrarily oriented plane receivers, or primary scalar illuminance (associated with spherical receivers). Thus, the tabulations of luminance distributions are potentially more useful than tabulations of illuminance quantities and, for this reason, should always be given first priority in the initial phases of planning of any numerical tabulation project.
Finally, we observe that the primary component of the luminance distribution, which is the main object of study of the present report, is of course only one of a denumerably infinite number of scattering-order components of the observable luminance distribution at point \(0\). The extent to which \(B^{(1)}(\psi)\) contributes to the observable luminance at \(0\) depends principally on the ratio of the total scattering coefficient \(\Delta = \int \tau d\Omega\) to the volume attenuation coefficient \(\beta'\), namely on \(\omega_o = \Delta / \beta'\). An order of magnitude estimate of the observable luminance \(B(\psi)\) may be made by noting that

\[
B^{(j+1)}(\psi) \cong \omega_o B^{(j)}(\psi), \quad j = 1, 2, \ldots,
\]

where the symbol \(\cong\) denotes "is of the order of magnitude of." It follows that for \(\psi > 0\),

\[
B(\psi) = \sum_{j=1}^{\infty} B^{(j)}(\psi) \cong \frac{B^{(1)}(\psi)}{1 - \omega_o}.
\]

Thus, for the present medium, in which \(\omega_o\) is approximately 0.4, we estimate that \(B(\psi) \cong 1.7 B^{(1)}(\psi)\).

R. W. Preisendorfer
May 1959
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GEOMETRY OF DERIVATION OF $B^{(1)}(\psi)$

Fig. 2.2.1
DETAIL OF $dV$

*Fig. 2.2.2*
SHOWING EQUI-ANGULAR SUBDIVISION OF LINE OF SIGHT

Fig. 2.2.3