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"Menos y menos da más": Using Spanish as the Language of Instruction with English Learners in Algebra 1

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Diaz, Marco Antonio

Publication Date
2013

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“Menos y menos da más”:

Using Spanish as the Language of Instruction with

English Learners in Algebra 1

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy in Education

By

Marco Antonio Díaz

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2013
ABSTRACT OF THE DISSERTATION

“Menos y menos da más”: Using Spanish as the Language of Instruction with English Learners in Algebra 1

by

Marco Antonio Díaz

Doctor of Philosophy in Education

University of California, Los Angeles, 2013

Professor Megan L. Franke, Co-Chair

Professor Patricia Gándara, Co-Chair

This study examines the use of Spanish as the language of instruction with Spanish dominant EL students in a 9th grade Algebra 1 classroom. The study documents conceptions of mathematical symbols at the start and at the end of the academic year of two groups of students; one receiving primary language instruction and another receiving sheltered English instruction. Students receiving primary language instruction demonstrated a considerable amount of prior knowledge about the equal sign and basic conceptions of variable at the start of the study and were found to have a nuanced understanding of variable and showed greater growth in their understandings of the minus sign than did the students receiving sheltered English instruction.
The dissertation of Marco Antonio Díaz is approved.

Concepción M. Valadez

Uri Treisman

Megan L. Franke, Committee Co-Chair

Patricia Gándara, Committee Co-Chair

University of California, Los Angeles

2013
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VITA

1994  Bachelor of Arts, Mathematics
      Occidental College
      Los Angeles, CA

1995  Master of Arts, Teaching
      Occidental College
      Los Angeles, CA

1995-2001  High School Mathematics Teacher
           Los Angeles Unified School District
           Los Angeles, CA

2001-2006  Mathematics Specialist/Coordinator of Public Programs
            Lawrence Hall of Science, UC Berkeley
            Berkeley, CA

2006-2011  Graduate Research Assistant
            Civil Rights Project/Proyecto de Derechos Civiles
            University of California, Los Angeles
            Los Angeles, California

2006-2009  Mathematics Instructional Coach
            Benjamin Franklin High School
            Los Angeles, CA

2009-2010  Lecturer
            Michael Eisner School of Education
            California State University, Northridge
            Northridge, California

2010-2013  Adjunct Faculty
            Rossier School of Education
            University of Southern California
            Los Angeles, California
Chapter 1: INTRODUCTION

Secondary level mathematics teachers throughout the country are asking themselves how best to help students that are beginning English Learners (hereafter also referred to as EL or EL students) to succeed in their courses. While the population of English Learners is diverse in many aspects, at least 3 out of 4 ELs in the United States are Spanish speaking and standardized test data reveal a persistent pattern of alarmingly low achievement in mathematics among these students (Fry, 2007; Llagas, 2003). Studies show that many educators perceive proficiency in English as a pre-requisite to rigorous coursework (R. M. Callahan, 2005; Harklau, 1994; Minicucci & Olsen, 1992). As beginning ELs these students have basic oral fluency skills in English at best (Cummins, 1981). Considering their English proficiency and the importance of Algebra 1 in the secondary mathematics course sequence, should these students be required to learn English before they are taught algebra?

Enrollment of beginning ELs in 9th grade Algebra 1 raises several questions. Can these students learn English while simultaneously building on their knowledge of mathematics? In what ways are learning Algebra and second language acquisition related in the classroom context? How does the context of Algebra 1 in the 9th grade influence EL students’ experience? Is there a benefit to teaching beginning ELs Algebra 1 in their native language? Answers to these questions have proven to be elusive in the secondary mathematics classroom given beliefs about ELs, mathematics, language, & learning (Moschkovich, 2007b) as well as a lack of understanding about the relationship between language and mathematics. This study will explore in what ways mathematics instruction in Spanish influences the development of conceptions that support algebraic thinking in a
9th grade high school Algebra 1 class consisting of Spanish speaking students that are beginning ELs.

Given the pervasiveness of Spanish speaking students amongst the EL population in the United States, this study is concerned with how much of Spanish speaking EL students’ prior knowledge about mathematics is tied up in their knowledge of Spanish. I want to find out in what ways might beginning EL students, that are Spanish dominant, conceptualize symbols that are crucial to building deep conceptual understanding in Algebra. This is important since students’ initial conceptions of mathematical symbols strongly influence how they make sense of the concepts that these symbols represent in Algebra. Therefore, this study will examine how a group of EL students, taught in Spanish, develop their conceptions of the equal sign, the minus sign and variable and in what ways these conceptualizations might differ in EL students who have been taught using Specially Designed Academic Instruction in English (SDAIE).

1.2 ELs and the success of public schools

Spanish-speaking ELs are a significant part of the public school population in the United States. Changing demographics and the accountability system set up by the No Child Left Behind Act of 2002 (NCLB) have put ELs in the middle of a statistical tug of war that has no winner. Significant portions of Latinos in public schools are ELs (45%) and significant portions of ELs are Spanish speaking (79%). In addition, the Spanish speaking EL population is now present in states that have not been centers for immigrants in the past (Lazarin, 2006). This has added pressure on school districts across the country to scramble for solutions, since Spanish speaking ELs are a significant portion of three subgroups of students: Latinos, English Learners and Low Socio-Economic Status.
students (commonly defined by eligibility for Federal Free and Reduced Lunch Program) that are monitored by NCLB. This means that: “Latino student outcomes are tied to EL student achievement” (Lazarin, 2006) and that both of these are influenced by socio-economic status. This combination of factors ensures that schools with high EL populations almost inevitably end up on the list of Program Improvement (PI) schools. This results in severe outcomes for Latino/Spanish speaking EL students as research has shown that approximately 40% of Latino students attend high schools where graduation is not the norm (i.e. better than a 50/50 chance of graduating and less than 60% promoting rate) (Balfanz & Legters, 2004).

Academic Performance Index (API) is the measure by which schools are identified as a Program Improvement (PI) school. Placement on the PI list means reconstitution (i.e. replacement of the entire school staff and faculty) if a school does not improve over a set period of time. A significant factor in a school’s API is the student achievement on state standardized tests. California Standards Test (CST) results for 2011 for 9th graders in Algebra 1 reveal low achievement for ELs in comparison to other subgroups and overall.
Table 1.2-1
California Standards Test (CST), 2011
9th graders in Algebra 1

<table>
<thead>
<tr>
<th>Sub-Group</th>
<th>Students tested</th>
<th>% Enroll</th>
<th>Students w/scores</th>
<th>Mean Scale score</th>
<th>% Advanced</th>
<th>% Proficient</th>
<th>% Basic</th>
<th>% Below Basic</th>
<th>% Far Below Basic</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL</td>
<td>51,459</td>
<td>10.1%</td>
<td>51,300</td>
<td>286.0</td>
<td>2%</td>
<td>9%</td>
<td>20%</td>
<td>43%</td>
<td>27%</td>
</tr>
<tr>
<td>Latino</td>
<td>150,448</td>
<td>29.6%</td>
<td>150,032</td>
<td>301.5</td>
<td>3%</td>
<td>16%</td>
<td>25%</td>
<td>38%</td>
<td>19%</td>
</tr>
<tr>
<td>White</td>
<td>61,809</td>
<td>12.2%</td>
<td>61,675</td>
<td>325.3</td>
<td>6%</td>
<td>27%</td>
<td>29%</td>
<td>28%</td>
<td>11%</td>
</tr>
<tr>
<td>Black</td>
<td>20,145</td>
<td>4.0%</td>
<td>20,037</td>
<td>292.6</td>
<td>2%</td>
<td>13%</td>
<td>21%</td>
<td>41%</td>
<td>24%</td>
</tr>
<tr>
<td>Asian</td>
<td>11,500</td>
<td>2.3%</td>
<td>11,492</td>
<td>343.1</td>
<td>12%</td>
<td>32%</td>
<td>27%</td>
<td>22%</td>
<td>8%</td>
</tr>
<tr>
<td>Low SES</td>
<td>159,500</td>
<td>31.4%</td>
<td>159,023</td>
<td>301.3</td>
<td>3%</td>
<td>16%</td>
<td>24%</td>
<td>38%</td>
<td>19%</td>
</tr>
</tbody>
</table>

It should be noted that 9th grade test scores on Algebra 1 are influenced by the recent push by legislators in states such as California to make Algebra 1 an 8th grade level course (EdSource, 2009). This means that students who enroll in Algebra in the 9th grade tend to be students that were determined to not be ready for Algebra 1 in the 8th grade or failed and must repeat the course in the 9th grade. One explanation for EL test results is that in states, such as California, ELs are required to take standardized math tests in English. Research has shown that testing beginning ELs on content area subject matter in English puts these students at a significant disadvantage (Abedi, 2002; Abedi & Lord, 2001). However, testing of EL math proficiency in Spanish reveals similar results. The National Assessment of Educational Progress (NAEP) or “The Nation’s Report Card” uses Spanish versions of their mathematics assessment for Spanish dominant EL students. Even so the results appear to confirm the low level of proficiency in mathematic among Spanish speaking ELs. Recent reports from the NAEP on 4th and 8th grade achievement in mathematics show that the percent of students in each subgroup (i.e. non-low-income,
non-EL, white students) scoring at or above proficiency in mathematics is but a fraction of their mainstream counterparts (Lee, Grigg, & Dion, 2007).

Table 1.2-2: The Nation’s Report Card (NAEP), Mathematics 2007

<table>
<thead>
<tr>
<th>Subgroup</th>
<th>Percent at or above Proficient 4th grade</th>
<th>Percent at or above Proficient 8th grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>51</td>
<td>41</td>
</tr>
<tr>
<td>Hispanic</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Black</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>Asian</td>
<td>59</td>
<td>49</td>
</tr>
<tr>
<td>Eligible FRL</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Not Eligible FRL</td>
<td>53</td>
<td>42</td>
</tr>
<tr>
<td>EL</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>Non EL</td>
<td>42</td>
<td>33</td>
</tr>
</tbody>
</table>

By ethnicity the percent of Hispanics that are reported to have scored at or above proficiency is less than half that of whites in both the 4th and 8th grade. Similar to the results by race, the percent of those who were eligible for free or reduced lunch that scored at or above proficiency was less than half that of those who were not eligible (Income level is determined by eligibility in the federal free or reduced lunch program). Finally, the NAEP report includes data specifically on EL student achievement in mathematics. In this regard, the previously discussed results seem to be synthesized as the percent of ELs that scored at or above proficiency is less than 2/5ths that of non ELs in the 4th grade and less than 1/5 at the 8th grade.

This low achievement is of concern to schools because of possible low API. Of concern to this study is that EL achievement is low on CST as well as NAEP, or rather, EL students perform at low levels of proficiency whether tested in English or Spanish. As
previously mentioned, low achievement on CST, for example, could be explained by the questionable validity of testing EL students in English. What could explain low achievement of ELs on NAEP if they are tested in Spanish? Given current language policies in the United States there is little chance that these students were taught in Spanish. Therefore, this study seeks to understand what instruction in Spanish to Spanish dominant EL students provides that is not provided by instruction in English?

### 1.3 Algebra 1 is a significant gatekeeper for ELs in high school.

Algebra 1 is a pre-requisite to advanced courses in mathematics and has a documented influence on a students’ overall experience in high school and beyond. Socio-cultural studies show that low achievement in Algebra 1 creates a stigma that labels these students as non-college track material (K. Gutierrez, Asato, Santos, & Gotanda, 2002; R. Gutierrez, 2002; Moschkovich, 2007b). Success in mathematics courses has a direct impact on whether students experience success in other course work (Lager, 2004) and successful completion of Algebra 1 is a strong determinant of whether students move on to higher education (Lager, 2004; R. Moses, 1994; R. P. Moses & Cobb, 2001; Schoenfeld, 2002).

Algebra 1 is a gatekeeper because successful completion means access to advanced coursework in mathematics. One study showed that placement in minimum standards courses over advanced courses resulted in low achievement (Wang & Goldschmidt, 1999). Furthermore, this study found that, “the marginal effect of being classified as LEP changed depending upon whether the student was enrolled in a minimum standards mathematics course,” versus a high-level mathematics course (Wang & Goldschmidt, 1999, p. 108). This means that success in Algebra 1 has a particular
effect on ELs. Indeed, access to advanced coursework has a lasting effect. The highest level of mathematics a student experiences in high school has the strongest influence on whether or not that student completes their bachelor’s degree (Adelman, 1999).

1.4 Most secondary EL students are taught Algebra 1 in English

The current scenario in many schools is that beginning ELs have less access to Algebra and other A-G/College Prep coursework (R. Callahan, 2005; Callahan, Wilkinson, & Muller, 2010) and those EL students that are enrolled in Algebra 1 are taught in English where they must learn algebra while simultaneously acquiring English as a second language (Minicucci & Olsen, 1992; Rivera, Hafner, & LaCelle-Peterson, 1997; Zehler, Fleischman, Hopstock, Pendzick, & Stephenson, 2003). This study posits that instruction in English at best builds off of students’ social language in English. Using English for instruction hampers students’ ability to develop conceptual knowledge because beginning ELs do not have the English language skills to understand symbols and the language of mathematics. The result is procedural proficiency at best. Spanish language instruction leverages EL students’ native language skills to develop the conceptual knowledge of symbols that is important to learning algebra.

Teaching ELs in English means that EL students are expected to learn mathematical concepts through instruction that completely ignores both their limited English and their existing Spanish language abilities. This forces students to use their weakest skill set to make sense of topics that are dependent on a nuanced understanding of the instructional language and negates the potential importance of the academic language skills in Spanish. The decision to instruct EL students in English or their native
language must take into account the relationship between the language of instruction and learning Algebra.

1.5 Mathematics and Language

Learning Mathematics and language are related. Studies that examine socio-cultural issues in mathematics classrooms have identified a students’ primary language as a resource to be leveraged during mathematics instruction (R. Gutierrez, 2002; Gutstein, 1997). One factor that may explain this finding is the relationship between learning mathematics and language. Studies show that reasoning and problem solving in mathematics classrooms are related to language and dependent on strong vocabulary skills specific to math (Dale & Cuevas, 1992; Jarret, 1999). Scholars that work closely with classroom teachers have concluded that “It is evident that use of language is essential to mathematics learning, and mathematical activities provide opportunities to extend language skills” (Coggins, Kravin, Davila Coates, & Carroll, 2009). That is, the learning of mathematics and the development of language skills are interrelated in the context of learning. Standards published by the National Council of Teachers of Mathematics state this clearly: “The ability to read, write, listen, think, and communicate about problems will develop and deepen students’ understanding of mathematics” (NCTM, 2000). Furthermore, this perspective on language use in the math classroom has spurred many reform efforts that emphasize an increase in oral and written communication in order to support conceptual learning and deep understanding of mathematics (Franke, Kazemi, & Battey, 2007). These reform initiatives have particularly serious consequences for ELs, who by definition do not have sufficient mastery of English to function successfully without support in the mainstream English
classroom (Gandara, Rumberger, Maxwell-Jolly, & Callahan, 2003; Khisty & Morales, 2004; Moschovich, 1999). That is, while beginning EL students may be able to read, write and listen in English (to varying degrees), thinking and communicating are much more closely dependent on their existing language skills in Spanish.

What language skills do students, and especially ELs, need to learn mathematics? A growing body of scholarly work characterizes the language of content area classrooms in greater and greater detail. The notion of academic language is goes beyond vocabulary to focus on multiple linguistic factors and features of the specialized language in each content area. In the mathematics classroom a significant component of academic language is the use of symbols to communicate concept.

1.6 Language and math symbols

Research in mathematics education suggests that successful learning in Algebra 1 depends on the students’ ability to transition from arithmetic to algebraic thinking (Franke & Kazemi, 2001; Kieran & Chalouh, 1993; Tall & Vinner, 2002). This study posits that students make this transition at the same time that they move from using conversational to academic language in the mathematics classroom. Development of algebraic thinking must build off of students’ prior experience with arithmetic (Kieran, 1981; Vinner, 2007). That is, how they understand arithmetic influences how they learn Algebra. This is important with regards to ELs. Cognitive research shows that student conceptions of mathematical symbols are tied to the language in which they learn arithmetic (Sousa, 2008). Many beginning ELs in a High School Algebra 1 class learn arithmetic almost entirely in their native language.
Students arrive to the Algebra 1 classroom with different conceptualizations about fundamental symbols used in Algebra (Malisani & Spagnolo, 2009). Three of these symbols are variable, the equal sign and the minus sign. Students’ conceptualizations at the start of Algebra 1 are usually based on their experience with arithmetic in primary and middle school classrooms with some introduction (sometimes wide ranging) to algebraic concepts. Significant to learning algebra is to build on these conceptualizations so that they are conducive to algebraic concepts that seek to generalize rather than compute. Language in all its mediums (reading, writing, speaking and listening) is the primary mode of teaching and learning (Pimm, 1987). Therefore the type of language that is used to describe familiar arithmetic procedures is a factor in how conceptual understanding develops. That is, do students understand the equal sign to mean equality between two elements, as in \( 8+2 \) is the same as 10, or do they understand the equal sign to have an operational (arithmetic) meaning that directs one to compute (i.e. to produce an answer); \( 8+2 \) is 10? Is the minus sign a symbol that means “the opposite” or does it only mean to subtract? Is a variable a symbol for an unknown value (e.g. \( x+2=8 \)) or does it function as a parameter for relationships between quantities (e.g. \( x+2=y \))? If students’ conceptualizations of these symbols is developed in Spanish then requiring these students to make the transition to algebraic thinking in English presents an obvious obstacle to this development.

1.7 Purpose of Study

The challenge with academic language of mathematics for ELs is that it takes 4 to 7 years to develop the academic language necessary to function in mainstream classrooms while the transition from arithmetic to algebraic thinking requires significant
use (experience) with academic language. This research fills a void; the existing research has not looked at ways that conceptual learning differs between English and native language instruction. This study will build on a growing body of research that examines linguistic and communication factors in Latinos and ELs learning mathematics (Khisty & Morales, 2004; Morales, Khisty, & Chval, 2003; Moschkovich, 2007a, 2007c). This study seeks to find out in what ways beginning EL students, that are Spanish dominant, conceptualize symbols that are crucial to building a deep conceptual understanding of algebra. I will explore the ways in which EL’s, taught in Spanish, develop their conceptions that are useful to making the transition from arithmetic to algebraic thinking. Specifically, I will examine how a group of EL students that are taught in Spanish conceptualize the equal sign, minus sign and variable.

I hypothesize that Spanish dominant secondary EL students taught in Spanish will demonstrate more conceptual understandings as compared to arithmetical or procedural understandings of the equal sign, minus sign and variable. This study will examine how a class of EL students taught in Spanish, develop their conceptions of the equal sign, the minus sign and variable through classroom observations, a pre & post assessment and interviews. The goal is to find out in what ways might beginning EL students, that are Spanish dominant, conceptualize symbols that are crucial to building deep conceptual understanding in Algebra. Thus, an additional concern of this study is in what ways these conceptualizations might differ in EL students who have been taught in English?

By exploring the connection between the language of instruction and the conceptions EL students have of algebraic symbols this study aims to build on research that finds that a students’ primary language is a resource that can be leveraged during
classroom instruction. Secondarily, this study will explore in what ways instruction in Spanish develops emergent understandings that later facilitate acquisition of useful conceptions of algebraic symbols in the second language. In particular, this study will address the following questions:

1) In what ways does the language of instruction (i.e. the primary language used for instruction and among students) mediate conceptual understanding of mathematical symbols in a secondary Algebra 1 class of Spanish dominant EL students who receive instruction in Spanish?

2) How does Spanish language instruction influence secondary Spanish dominant EL students’ conceptualizations of fundamental Algebraic concepts in Spanish?

3) What linguistic resources are provided to EL students by instruction in Spanish?
Chapter 2: Literature Review

Few teachers receive training on instruction for ELs using primary language instruction or Specially Designed Academic Instruction in English/Sheltered English Instruction (SDAIE/SIE). Part of the reason is that these skills are not fully integrated into credential program curriculum. However, this is also due in part to the fact that research on ELs in bilingual classrooms is limited and still new. Secondary EL students in Algebra 1 are not simply learning math in a foreign language, they are learning to deal with the language of math in a foreign language. This is not an easy task given the linguistic demands of learning mathematics. In addition, beginning Algebra students (EL as well as non-EL) are asked to transition from thinking about numbers in the context of arithmetic computation to thinking about generalizations about numbers and processes. These initial conceptions of mathematical ideas are closely tied to the language in which they were learned. This may explain why it is that while Algebra 1 is a gatekeeper course for many students in U.S. public schools, and many native English speaking students struggle with making this transition from arithmetic to Algebra, these challenges have clearly manifested themselves as roadblocks for those students whose native language is not English.

Whether or not to use EL students’ native language for instruction has long been debated. The debate itself is easily problematized by political influences and challenges to research methodology. What is clear is that to use EL students’ native language for instruction is not inferior to English-only instruction and that to do so does not interfere with the students’ ability to acquire English (August & Shanahan, 2006). With regards to mathematics instruction the question of language is not limited to which language but
also how language will be used. What has been made clear in this case is that language in the form of structured discourse is a highly effective mechanism for learning mathematics.

Knowing that discourse can help students is helpful in identifying curricular activities that may be more effective due to their attention to discourse. However, the question of which language is more beneficial to use for instruction has been shelved in lieu of formal and informal policy that requires the large majority of teachers to use English as the language of instruction for EL students in secondary math classrooms. For teachers of EL students the primary question becomes: “How do I do this?” That is, how do we teach beginning EL students mathematics in English? This study posits that using students’ primary language for instruction is the most effective way to build academic language and that building academic language (in this case the language of mathematics) supports second language acquisition.

In the first section of this chapter I explicate the linguistic demands of learning mathematics and how language use is a factor in the transition from arithmetic to algebra and in developing conceptual understanding in order to contextualize the overall relationship between language and learning mathematics. This is followed by a brief review of the literature on the effectiveness of bilingual education and how a focus on classroom discourse can address the needs of EL students.

The second section of this chapter will discuss the three symbols that this study will use to examine how language choice and use influence the development of conceptual understanding. Special attention will be paid to how students in beginning Algebra conceptualize these symbols and which conceptualizations of these symbols are
important to future learning in Algebra and the secondary mathematics curriculum with the aim that this will inform the methodology of this study.

2.1 Linguistic demands of the mathematics classroom

Halliday (1975) provides a starting point for understanding the linguistic demands of the mathematics classroom when he defined the mathematics register as:

...a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a ‘mathematics register’, in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes.

This means that having students memorize vocabulary definitions is only one part of developing students’ fluency with the language needed to learn and understand mathematics.

In order to distinguish between everyday language and the mathematics register they must negotiate specific language structures. For example, Pimm (1987) points out that mathematics includes the use of dense noun phrases (e.g. A rectangle with a length a 5 cm and a width of 8 cm) as well as “speech orientations” (e.g. “listener oriented”, where the listener is expected to work out the conversational relevance versus “message oriented”, where the definitions are precise and explicit) as well as the complex and institutional use of metaphor (Pimm, 1987; Sfard, 2000).

Adding to the complexity is the semiotic nature of the language used in math classrooms (Monaghan, 2009; Pimm, 1987; Schleppegrell, 2001, 2007). Symbols communicate meaning. This is even more crucial in the context of beginning algebra where the concept of variable becomes a symbol with multiple meanings. For example, the expression: 3(x + 5) is read as “three times the sum of x and five.” That is, the use of
parentheses and the “+” symbol imply a specific grouping (which in turn can imply a particular order for the carrying out of calculations).

The language of mathematics makes use of multi-semiotic formations whereby meaning goes beyond the use of the symbol to include place (e.g. location of a constant term in an expression is conventional: “3x” versus “x3”), size (e.g. the use of subscripts to indicate a label as opposed to a variable such as \( R_i \)), and context (e.g. the use of \( i \) for imaginary numbers).

Linguistic demands are not the only factor influencing EL student success with Algebra. Beginning Algebra requires that all students shift their thinking in ways that are new and sometimes contrary to their experience with arithmetic.

### 2.2 Transition to Algebra & Language Use

Algebra 1 has a unique place in the trajectory of the secondary mathematics curriculum. It is the first in a series of courses that will ask students to think differently about numbers. Most students experience numbers both in school and out of school in the context of arithmetic. Mali\(\text{sani} \) & Spagnolo (2009) describe the process experienced when solving arithmetic problems:

> “solving arithmetic problems means to carry out one or more operations with specific data for reaching a solution (almost always unique), proceeding in a sequential way and using *fundamentally natural language...*” (p.20).

This means that language plays a part in how students understand arithmetic. This connection between language and arithmetic is also validated in other areas of research. Studies in cognitive science suggest that a strong connection exists between arithmetic and language abilities. For example, part of the brain that is responsible for language, Broca’s area, is used when one makes arithmetical calculations (S. Dehaene, Spelke,
Pinel, Stanescu, & Tsivkin, 1999). Other studies have shown that bilingual learners use their primary language when doing arithmetic, even when they have acquired a considerable amount of the second language (Sousa, 2008). This phenomenon has been confirmed by the experience of bilinguals (Moschkovich, 2005). The point is that for all students arithmetic is understood as part of their experience with numbers and this involves the use of language to make sense of these understandings.

It is important to be aware of the beliefs about numbers and arithmetic that beginning Algebra 1 students’ bring to the classroom. The study of Algebra requires students to think in ways that may appear backwards. For example, researchers argue that setting up the equation in Algebra requires an analytic mode of thinking that is exactly opposite to that used when solving a problem arithmetically (Kieran & Chalouh, 1993; Usiskin, 1999). Consider the following situation:

A fair charges $5 for admission and $2 for each ticket for the carnival rides.

a) How much money will be spent if you attend the fair and purchase 3 tickets?

b) Write an equation that describes the total amount spent in terms of the number of tickets purchased.

The first question appeals to students experience with numbers. Students are asked to perform a calculation. This may proceed in the following fashion: the price of admission ($5) plus $2 for each of three tickets ($2 x 3 tickets) will result in $11. Symbolically: $5 + 2(3) = 11$.

The second question asks students to demonstrate a different way of organizing the operations. A correct response may look something like this:
Let S represent the total amount of money spent.
Let T represent the number of tickets purchased.
Then, the total amount of money spent (S) is “equal to” two dollars times the number of tickets purchased (T) plus the admission price ($5).
So, \( S = 2T + 5 \).

Putting these two answers next to each other (i.e. \( 5 + 2(3) = 11 \) and \( S = 2T + 5 \)) shows how the thinking needed to set up an equation to model the situation can be seen as the reverse of making a calculation in the same context.

It is also important to note that although there are some variations on this equation that would be acceptable (e.g. \( S = 5 + 2T \) or \( 5 + 2T = S \)) producing the “correct” equation requires the understanding of mathematical conventions that defy real experience (Pimm, 1987; Usiskin, 1999). By convention, the term with the independent variable (T) in the above equation should be placed to the left of the constant term (5) resulting in \( S = 2T + 5 \). However, this defies the real experience. It would not be wrong to think of paying the admission and only then purchasing the tickets, resulting in an organization of the operations that makes sense (i.e. \( 5 + 2T \)).

How then does this affect EL students? The transition from arithmetic to algebra must negotiate linguistic factors. Everyday language for beginning ELs is in Spanish. This puts ELs at a disadvantage if they are introduced to the mathematics register in English. For example, everyday language in Spanish does not identify double negatives as illogical. The statement: “Yo no se nada” does not mean that the speaker does know something. In Spanish the “double negative” does not result in a positive (Stubbs, 2011).
This would imply that in order to successfully teach ELs in English would require that the instruction include activities that develop the conceptions and language of arithmetic in English before proceeding to introduce the Algebraic concepts. The alternative is that instruction must build off of prior knowledge and pay attention to language use in order to support students as they make sense of what they know about numbers and relationships between them.

2.3 Discourse

A growing body of research shows that structured discourse provides students the opportunity to develop their understanding of mathematics. Getting students to talk about mathematics is not an easy endeavor and the goal is not discourse for the sake of discourse (Pimm, 1987). Students must have the opportunity to “read, write and think about mathematical problems” and classroom instruction that has meaning (relevance) and approaches that make sense (Sousa, 2008; Stein, Engle, Hughes, & Smith, 2006). This requires that teachers understand students' abilities (linguistic as well as mathematical) and prior experience. Franke et. al. (2007, p. 237) explain that discourse is beneficial in the mathematics classroom because:

Teachers who know about their students’ mathematical thinking can support the development of mathematical proficiency. Knowing about students’ mathematical thinking supports opportunities for question asking linked to students’ ideas, eliciting multiple strategies, and so on. When teachers ask more questions and ask for more than recall, providing students’ opportunities to express their ideas and actions verbally, students’ develop understanding. When students are required to describe their strategies in detail and why they work, they develop understanding.

Structuring discourse in beneficial ways is a challenge for many mathematics teachers with ELs in their mainstream classrooms that use English as the language of instruction. Teachers must organize the opportunities for discussion so that EL students have an
opportunity to dialogue about the topics amongst themselves in order to allow them to make use of their native language. EL students would then require an opportunity to dialogue with non-EL students in order to develop their ability to verbalize their understandings in English. Finally, the teacher would need to facilitate a whole class discussion that summarizes the learning while paying close attention to students’ conceptual understandings by re-voicing students’ incomplete or incoherent responses during the discussion and by asking carefully crafted questions that clarify meanings and ask for justifications.

Researchers that look at Latino ELs in math class explain that solving problems in math class is a complex process when EL students are taught in English. They must listen to the initial problem, translate it into Spanish, understand the problem in Spanish, develop a solution in Spanish, translate the solution to English, and then verbalize their response; which by the time a student is ready to share their solution it is irrelevant given the time that has elapsed (Moschkovich, 2007c). Also, studies have shown that although students limited language skills in English may limit EL students’ ability to translate word problems this does not interfere with their ability to carry out the arithmetic (Bernardo, 2005; Rodriguez et al., 2001). Furthermore, much of the recent research on ELs in mathematics classrooms has centered on how a students’ conceptual understanding may be masked by their language abilities or the linguistic demands of the problem (Abedi & Lord, 2001; Khisty & Chval, 2002; Lager, 2006; Moschkovich, 2002). While non-EL students are in the process of learning the mathematics and the language of the mathematics classroom EL students are in the process of doing the same, but have an additional layer added: acquiring English language is indeed a factor for EL students
when learning mathematics given the multiple modes of learning that are occurring simultaneously.

Learning mathematics requires the negotiation of challenging linguistic demands (Barwell, 2005a, 2005b; Caglar, 2003; Monaghan, 2009; Pimm, 1987; Schleppegrell, 2001, 2007). These linguistic challenges can be insurmountable when it takes 3 to 5 years to acquire oral fluency and 4 to 7 years to acquire academic English (Hakuta, Goto, & Witt, 2000). Utilizing discourse as a pedagogical strategy has been shown to be a powerful tool in developing students understanding. It must be noted that the linguistic challenges of the math class and obstacles to engaging discourse are greater for ELs than for non-ELs (Brenner et al., 1997; Lager, 2006). Using Spanish as the language of instruction for Spanish dominant ELs should allow discourse to be a much more effective tool than it would otherwise. Spanish speaking EL students who have a considerable grasp of their own native language possess a skill set that can be leveraged in math classrooms.

### 2.4 Research on Bilingual Education

In their examination of the research base for policies that restrict the use of native language instruction (e.g. Proposition 227 in California, Proposition 203 in Arizona and Question #2 in Massachusetts) August et al (2010) review 7 meta-analyses of research on bilingual education. The first three analyses examined did not take into account program effect size. The criteria for inclusion in two of these analyses (Baker & de Kanter, 1981; Rossell & Baker, 1996) were based on strict scientific definitions of research (i.e. random sampling). These studies concluded that there was no basis on which bilingual education could be mandated as a superior instructional strategy. August et al (2010) point out that
these analyses do not conclude that transitional bilingual education is inferior to English Only (EO) instruction, only that it is not superior. Alternatively, a third analysis (Slavin & Cheung, 2005) found that a bilingual approach was superior to EO programs when it comes to English reading outcomes. Four analyses (Francis, Lesaux, & August, 2006; Greene, 1997; Rolstad, Mahoney, & Glass, 2005; Willig, 1985) took into account the effect size of each study. The criteria for inclusion in some of these analyses were much more refined than in Baker & de Kanter (1981) and Rossell & Baker (1996). For instance, studies had to measure effect size after at least 1 year 1 academic year. Also, studies that were done outside the U.S. or where instruction was outside the classroom were excluded. All four of these analyses found positive effect sizes for programs that use native language instruction.

Applying this theory to the mathematics classroom suggests that mathematics instruction in students primary language versus an L2, could not only leads to procedural skills in that language, but also to a deeper conceptual and mathematical proficiency, which is related to procedural fluency and general academic skills in the second language (Verhoeven, 1994). For instance, a beginning EL in Algebra may learn when to subtract a number from both sides of an equation when solving simple linear equations. This procedural fluency is valuable, but not as effective in the long run if the student does not understand that the concept framing the procedure involves equality. For example, students in Algebra 1 are often asked to use the equation for the area of a triangle ($A = \frac{bh}{2}$) to find the base when given the area and height of the triangle. It is much more useful if the student understands that the base can be represented in terms of the area and the height (conceptual understanding) than it is for the student to know that a constant
can be subtracted from both sides of an equation (procedural fluency). So, if all students are expected to increase their conceptual understandings of mathematical concepts (as per the numerous calls for deep understanding from organizations such as NCTM) instruction for ELs in particular must incorporate communication that is context reduced and language processes that require high cognitive demands. This study holds that in the mathematics classroom this means an increased attention to conceptual understanding while supporting procedural fluency.

Recent studies on EL’s in mathematics classrooms focus on procedural skills and or a “narrow” conception of mathematics (Moschkovich, 2008). This study intends to examine students’ conceptual knowledge of three symbols. The following section will describe what research tells us about how students conceptualize specific symbols that represent important mathematical themes that pervade the Algebra 1 curriculum and beyond.

2.5 Conceptualizations of Three Algebraic Symbols

How do students conceptualize the meaning of the symbols used for: variable, the equal sign, and the minus sign? Research in mathematics education has shown that students arrive to the Algebra 1 classroom with different conceptualizations about fundamental symbols used in Algebra (Malisani & Spagnolo, 2009). Three of these symbols are variable, the equal sign and the minus sign. Students’ conceptualizations at the start of Algebra 1 are usually based on their experience with arithmetic in primary and middle school classrooms with some introduction (sometimes wide ranging) to algebraic concepts. Significant to learning algebra is to build on these conceptualizations
so that they are conducive to algebraic concepts which seek to generalize rather than compute.

2.5.1 The Equal Sign

The equal sign is possibly the most familiar symbol to students who are starting Algebra 1. Although familiar it must take on a very different conceptualization than how it was viewed when students were doing arithmetic. When doing arithmetic students understand the equal sign to have an operational meaning whereby it means “to compute” or “find the answer.” This is not a surprise since middle school math textbooks have been found to overwhelmingly use an operational view of the equals sign (McNeil et al., 2006). In algebra, the goal is to get students to see that the equal sign means “is the same as.” This conceptualization is “relational” in that it is making a statement about a relationship. For instance, many students will quickly agree that $12 + 23 = 25$. However, when asked if $25 = 12 + 23$, many student will consider this false because the operation is on the left hand side of the equal sign. This is an operational understanding of the equal sign. Students who have a relational understanding of the equal sign are better prepared for future topics in Algebra 1 such as solving basic linear equations (Kieran, 1981; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005).

2.5.2 Variable

A symbol that may be new to beginning Algebra 1 students is the variable. Variables are usually indicated by the use of letters. Due to various reform efforts, many elementary and middle school students have been exposed more and more to the concept of “the unknown.” While this is helpful in that students will need to interpret variables in this way in Algebra 1, they will also need to understand variable in other ways (Usiskin,
For instance, in the equation $x + 8 = 23$ the variable $x$ has only one solution. Understanding this means the student conceptualizes variable as an unknown. However, if we ask students what numbers solve the equation $x + y = 23$ students must understand the relationship between two variables and that values for one variable can be found by inputting values for the other. Understanding variable in this way is to understand variable as a functional parameter. Some scholars argue that conceptualizing variable as an unknown can interfere with developing an understanding of variable as a functional parameter (Knuth et al., 2005).

### 2.5.3 Minus Sign

A third symbol that students encounter in beginning Algebra is the minus sign. While familiar to students from their earliest experiences with arithmetic the minus sign takes on a very conceptual significance in Algebra 1. In arithmetic the minus sign indicates that subtraction be performed between two numbers (e.g. $8 - 3 = ?$). This is an operational conceptualization of the minus sign. Between 6th grade and 8th grade students are confronted with the task of making sense of subtraction problems where the second number is large than the first (e.g. $5 - 8 = ?$). This is the mathematical motivation for the use of integers. This conceptualization of the minus sign is sufficient for dealing with addition and subtraction of integers to a certain point. Sooner or later, other operations must be defined in this context. How do we multiply and divide these integers? Making sense of these operations requires that students understand the minus sign as “negative of something” or the “the opposite.” This conceptualization of the minus sign is “predictive” (Vlassis, 2004). Studies have shown that adults commit more errors in calculation when simple problems are recast using negative numbers. For example, while $8 + 3 = ?$ is no
problem for most adults, recasting the problem with $8 - (-3) = ?$ caused many to make simple errors (Das, LeFevre, & Penner-Wilger, 2010). Students in beginning algebra often ask “is this a negative or a minus sign?” as they point to either of the two “-“ symbols. Some suggest that it is a lack of conceptual knowledge about symbols and that students have been introduced to the symbols without experiencing the need for themselves (Vlassis, 2004).
Chapter 3: Methods

This study hypothesizes that primary language instruction for ELs supports the development of students’ algebraic understandings of the equal sign, the minus sign and variable over arithmetic interpretations. In order to explore this connection between primary language instruction and the development of conceptual knowledge in Algebra 1 this study addresses the following questions:

1) In what ways does the language of instruction (i.e. the primary language used for direct instruction and among students) mediate conceptual understanding of mathematical symbols in a secondary Algebra 1 class of Spanish dominant EL students who receive instruction in Spanish?
2) How does Spanish language instruction influence secondary Spanish dominant EL students’ conceptualizations of fundamental Algebraic concepts in Spanish?
3) What linguistic resources are provided to EL students by instruction in Spanish?

The preceding two chapters provide context for this study. This chapter will summarize this context and impetus of the study, describe the sites & participants and outline the research methods used.

3.1 Impetus of Study and Context

This study was done as part of a larger research project named Project Secondary Online Learning (Project SOL) that was using online Spanish language curriculum in Math and Science to increase the number of college preparatory coursework completed by beginning ELs at four high schools in California. One math teacher and one science teacher at each of four schools participated in the project. Initial observations during the first year of the project revealed that while some collaborative group work was utilized most teachers still used lecture as the primary mode of instruction. The most typical
format for instruction was to provide a lecture that might ask students to read some text from the curriculum followed by review of some examples and then concluded by asking students to complete a problem set. The lecture, in addition to some procedural explanations, consisted of mostly IRE (Initiate, Response, Evaluate) interactions.

After observing all four of the math teachers it was apparent that lectures in one class were more consistently interactive than in the other three classes. Mr. Franco at East Valley High School used instructional strategies that made lectures engaging and in some cases were intended to elicit discourse. Mr. Franco was observed using prompts, sentence starters, as well as advanced organizers. It should be noted that all of these strategies have been identified as strategies that address the needs of ELs in the math class (Carr et al., 2009; Coggins, Kravin, Davila Coates, & Carroll, 2007). Beyond these specific strategies Mr. Franco was also observed taking the time to explain concepts and making explicit connections between vocabulary and concepts. Mr. Franco does not simply lecture, he speaks with and listens to students and waits for the appropriate time to ask questions that facilitate discussion, he speaks with groups of students who turn to each other after his consultation, and he uses hand gestures and body language to communicate concepts. Video recorded lessons and in person observations of Mr. Franco’s classroom revealed that students clearly verbalized their mathematical understandings.

These observations led me to want to study how ELs experienced the development of algebraic concepts and how this was connected to language use in the classroom. I wanted to see how L1 instruction leveraged students’ prior knowledge of Spanish as a source of social as well as academic language. Given that a key factor was that the language of instruction was in Spanish as well as the materials and that Mr.
Franco paid explicit attention to academic language development I chose Mr. Franco’s class at East Valley High School for the study. In order to explore a comparative approach I also chose to observe a group of EL students at a similar school in the neighboring community named San Felipe HS (SFHS).

3.2 Research Sites

East Valley High (EVHS) is located in a semi-suburban environment on the outskirts of a large metropolitan city in southern California. The campus is nestled in a quiet residential neighborhood that consists of predominantly single-family homes. The school population of 3485 students (in grades 9 through 12) is drawn from a community that was approximately 76% Hispanic (or of Latino origin) as indicated by 2010 U.S. Census data. 69.9% of the students at EVHS qualify for Federal Free and Reduced Lunch Program (FRL). Census data on Housing Population statistics show that the median household yearly income for the community is $54,289 with an average of $47,744.

In 2010-2011 the student population at EVHS was approximately 94% Latino, 2% African American, 2% white (non-Latino), and less than 1% Asian. Of the 3485 students approximately 23.8% are ELs. Not surprisingly, 76.69% of the ELs at EVHS are Spanish speaking. Data from the state department of education shows that EVHS reported providing the following services to their EL population; 51% receive ELD and Specially Designed Academic Instruction in English (SDAIE), 16.5% receive ELD & SDAIE with primary language support and 6% Receive ELD and academic subjects through the primary language.

The second site, San Felipe High (SFHS), is in the same district and less than 4 miles from EVHS. The campus is located in a residential neighborhood with the back of
the school facing a busy street that is less than a mile from a major highway junction. The school serves over 3212 students that come from a community that was approximately 96.3% Hispanic (or of Latino origin) as indicated by 2010 U.S. Census data. Students from SFHS as a whole school population come from a slightly lower socio-economic community than EVHS. 85.5% of the students at SFHS qualify for Federal Free and Reduced Lunch Program (FRL) and Census data on Housing Population statistics reveal that the median household yearly income for the community is $49,899 with an average of $40,398.

In 2010-2011 the student population at SFHS was approximately 96.7% Latino, less than 2% African American, less than 1% white (non-Latino), and less than 1% Asian. Of the 3212 students approximately 29% are ELs. Like EVHS the portion of ELs at SFHS that are Spanish speaking is very high at 83.53%. Data from the state department of education shows that EVHS reported providing the following services to their EL population; 69% receive ELD and Specially Designed Academic Instruction in English (SDAIE), 1% receive ELD & SDAIE with primary language support and just under 2% Receive ELD and academic subjects through the primary language.

Using SFHS to explore a comparative approach fit well as the two schools are very similar. The schools are within a few miles of each other and serve a very similar population in terms of size and demographics. The only differences are slightly lower SES among students at SFHS according to the number of students on FRL and U.S. Census data on household income.

3.3 Teachers
The participants in this study were three Algebra 1 classrooms and their teachers. The focus of the study was on one teacher, Mr. Franco, and 23 EL students in his Algebra 1 class. Mr. Franco uses Spanish as the language of instruction as well as Spanish language materials. The secondary participants consist of 22 students across two classes with two teachers, Mr. Estevez and Mr. Valdez, at SFHS. Both classes at SFHS were designated as using Sheltered English Instruction (SEI) and both used English language materials. Mr. Estevez only used English to instruct while Mr. Valdez used English with Spanish as a support.

At the time of this study Mr. Franco was a third year high school math teacher at EVHS. He taught Algebra 1 and Geometry using Sheltered English Instruction and was in his second year as a Project SOL teacher using Spanish as the language of instruction. Mr. Franco speaks Spanish having spent his early childhood in El Salvador. He spent the second and third grade as an EL student in the United States after his family immigrated to New York City before moving to California where he was programmed into the mainstream program. He attended the local state university after high school where he changed his major twice before choosing mathematics and then deciding to teach in 2007. Generally his instruction is relatively traditional with lecture, guided and independent practice as core components. What he adds to this is consistent use of collaborative groups (either pairs or groups of 4) and a focus on student talk. His goal is to “get students to understand why the rules work and why it’s important to learn.” Students spend as much time reading, writing and speaking mathematics as they do listening to Mr. Franco.
Mr. Estevez is a first year high school math teacher. He was teaching Algebra 1 and Geometry classes that are all designated as SEI this year. Mr. Estevez speaks Spanish. He was born in California to immigrant parents from Mexico and spoke Spanish at home until he entered elementary school as an English Learner and then transitioned to mainstream English classes by the third grade. He had a very positive experience with mathematics during middle and high school and found himself inspired by his Calculus BC teacher to become a math teacher himself. Choosing to stay close to home and family after high school he attended the local state university where he eventually majored in math after trying out engineering. His commitment to his community led him back to his alma mater where he now teaches. Mr. Estevez is a dynamic instructor that spends a great deal of time developing curriculum, in collaboration with his colleagues, aimed at building conceptual knowledge and problem solving skills. He frequently makes use of collaborative group work and he uses very structured lectures that engage students at the board in front of the class as well as with each other.

Mr. Valdez was in his seventh year teaching at the time of this study. He is bilingual and was formally educated in Peru through his first year of university until immigrating to California. Mr. Valdez made his way through adult school and community college as an EL and found himself at the local state university where he majored in mathematics. Mr. Valdez was drawn to teaching when he became more involved with his community as a tutor. The needs of so many people, that were not unlike his own, drew him to leverage his experience and skills in the field of education. Mr. Valdez uses a traditional lecture format followed by independent and guided practice throughout entire class. Collaborative group work was very informal in that students were
allowed to work together but rarely given a specific task as a group. Lectures are highly planned and consistently delivered through the use of PowerPoint presentations. Although Mr. Valdez uses Spanish to instruct all the language in PowerPoint and textbook are in English.

The two teachers at SFHS are similar to Mr. Franco at EVHS. All three speak Spanish in addition to English. They all live & work in the community and all three earned their teaching credential at the same local state university. The goal was to explore a comparative approach such that the assessment results of students in Mr. Franco’s class could be juxtaposed with results from students experiencing SEI.

3.4 Students

Mr. Franco’s class at EVHS consisted of 23 students at the start of the 2010-2011 school year. Students were primarily 9th graders with a few exceptions (21 ninth graders, 1 tenth grader and 1 twelfth grader). All of the students indicated that Spanish was the primary language spoken at home. All 23 of students were identified as English Learners with 15 at ELD-1 and 8 at ELD-2. All 23 students had California English Language Development Test (CELDT) scores. 20 students had an overall score of 1 and three students had an overall score of 2.

Mr. Valdez’ class at SFHS consisted of 24 students at the start of the 2010-2011 school year. Students were primarily 9th graders with a few exceptions (19 ninth grade, 4 tenth grade, 1 eleventh grade). Twenty-two (22) of the students were identified as English Learners with 10 at ELD-1 and 12 at ELD-2. Twenty-two (22) of the students had California English Language Development Test (CELDT) scores; 19 students had an overall score of 1 and three students had an overall score of 2.
Mr. Estevez’ class at SFHS consisted of 38 students at the start of the 2010-2011 school year. Grade levels were slightly more varied than the other two classes with 25 students in the ninth grade, 5 students in tenth grade and 5 in the eleventh grade. All of the students indicated that Spanish was the primary language spoken at home. 27 of the 38 students were identified as Limited English Proficient (LEP) with 11 identified as LEP and 16 as Re-designated Fluent English Proficient (RFEP).

One challenge was to account for the change in the composition of each class. Over the course of the year 23 students across all three classes dropped the course and 19 students added the course. This resulted in a limited number of students that took both the pre-assessment as well as the post-assessment. Specifically, 16 students from Mr. Franco’s class at EVHS and 22 students from both classes at SFHS took both assessments.

3.5 Instruments and data collected

The study started with classroom observations to confirm the type of instruction and student population in each class. During this time data on students were collected. After initial meetings formal interviews with each teacher were set up to get a clearer picture of the instructors. Additionally, lessons were video recorded in all three classrooms in order to get a picture of the type of language that was used by the teacher. Finally, in order to assess students’ conceptual understandings of the three target symbols a pre- assessment was administered at the start of the year and a post-assessment at the end of the year.
3.5.1 Classroom observations

Classroom observations and field-notes were used to characterize each class in terms of instruction and level of primary language support. Classes were observed bi-weekly for 8 weeks. Observations were aimed at identifying factors that influence students’ language and conceptions of the target symbols. Guiding Questions for these observations were as follows: 1) What strategies elicit student verbalizations of their own conceptualizations? 2) In what ways do students communicate these conceptualizations? 3) In what ways do lessons done in English differ from those done in Spanish? 4) How are the linguistic demands negotiated by students?

3.5.2 Teacher interviews

Bilingualism is complicated. Speaking more than one language comes from a very diverse set of experiences. While all three teachers reported that they spoke Spanish it was important to find out how their language abilities came to be. Two interviews were set up with each teacher. The first interview was aimed at gathering basic biographical information such as place of birth, schooling and teaching experience. The second interview was aimed at gathering the teachers’ thoughts on: language, EL students, and students’ language use. Questions on language included: “Do you consider yourself bilingual? If so can you elaborate on the reasons why you think so?” and, “What helped you most when learning a second language?” Questions about students and their language use included: “What types of language do students use?” “In what ways do students speak about their own language use?” “In which language do your EL students best communicate with and about mathematics?” and “In what ways do you develop your EL students’ ability to communicate with mathematics in Spanish and/or in English?”
3.5.3 Video

After the initial 8 week observation period classroom lessons were video recorded to capture language used by the teacher and students in natural classroom setting (Jacobs, Kawanaka, & Stigler, 1999). The guidelines for recording a lesson included: 1) instruction where the teacher is explaining, and 2) students have the opportunity to speak with or about mathematics. Over ten hours of instruction were captured across a semester and a half of instruction. Video recordings were logged and documented. Lessons were described and any extensive discourse was transcribed. The video logs were then coded and examined for patterns of language use.

3.5.4 Mathematics assessment

A researcher designed mathematics pre-assessment consisting of 15 free response items was administered to all of the students during the first two weeks of the 2010-2011 school-year. This pre-assessment was aimed at characterizing students’ conceptions of the equal sign, the minus sign and variable when they came into the class. Three items (1-3) assessed notions of the equal sign, seven items (4-11) assessed students’ notions of variable and four items (12-15) assessed student notions of the minus sign. Responses were coded and then categorized by type of conceptualization. A post-assessment with the same items was administered to explore any changes in conceptions at the end of the observation and video recording period. The post-assessment was aimed at identifying growth in knowledge as well as growth in types of conceptualizations overall after receiving instruction in Spanish or Sheltered English Instruction.

A subset of the items on the assessment were used to identify how students understand variable in terms of Kuchemann’s (1978) six levels “for describing the
different ways that letters can be used.” The following table lists a description for each level and notes the item on the assessment used in this study that assess that particular level.

**Table 3.5.4-1: Six levels for describing the different ways letters can be used (Kucheman, 1981)**

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letter EVALUATED</td>
<td>Items where the value of the letter can be determined immediately and there are no intermediate steps involving the unknown.</td>
<td>8a and 8b</td>
</tr>
<tr>
<td>Letter IGNORED</td>
<td>Items where a solution can be determined by ignoring the letters. Additional information such as placement of terms allows for one to one comparisons.</td>
<td>6</td>
</tr>
<tr>
<td>Letter as OBJECT</td>
<td>Items where the letter represents the name of an object as opposed to numbers.</td>
<td>9</td>
</tr>
<tr>
<td>Letter as SPECIFIC UNKNOWN</td>
<td>Items where the letter represents a number that cannot be evaluated.</td>
<td>11</td>
</tr>
<tr>
<td>Letter as GENERALIZED NUMBER</td>
<td>Items where the letter represents a set of numbers as opposed to one number.</td>
<td>7, 8c and 14</td>
</tr>
<tr>
<td>Letter as VARIABLE</td>
<td>Items where a second order relationship between two expressions must be found as the value of the letter varies.</td>
<td>10</td>
</tr>
</tbody>
</table>
The following section will list each assessment item and describe how that item assessed students’ conceptions of the symbols targeted by this study. (Note: Items are listed in the following section are in English for clarity. Students in Mr. Franco’s and Mr. Valdez’s class used Spanish versions of the assessments.)

**Item 1**

Tell if the following number sentences are true or false:

<table>
<thead>
<tr>
<th>Item</th>
<th>T or F ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13 + 5 = 18</td>
</tr>
<tr>
<td>B</td>
<td>17 + 24 = 41</td>
</tr>
<tr>
<td>C</td>
<td>44 = 25 + 19</td>
</tr>
<tr>
<td>D</td>
<td>38 = 27 + 16</td>
</tr>
<tr>
<td>E</td>
<td>37 = 37</td>
</tr>
<tr>
<td>F</td>
<td>23 + 36 = 23 + 36</td>
</tr>
<tr>
<td>G</td>
<td>23 + 37 = 35 + 25</td>
</tr>
<tr>
<td>H</td>
<td>22 + 36 = 23 + 35</td>
</tr>
<tr>
<td>I</td>
<td>37 + 18 = 18 + 37</td>
</tr>
<tr>
<td>J</td>
<td>36 + 19 = 18 + 38</td>
</tr>
<tr>
<td>K</td>
<td>13 + 15 = 14 + 14</td>
</tr>
</tbody>
</table>

**Item 2**

Tell if the following number sentences are true or false:

<table>
<thead>
<tr>
<th>Item</th>
<th>V o F ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>19 + 25 = 44</td>
</tr>
<tr>
<td>B</td>
<td>19 + 25 = 44 + 0</td>
</tr>
<tr>
<td>C</td>
<td>19 + 25 = 44 + 25</td>
</tr>
<tr>
<td>D</td>
<td>19 + 25 = 0 + 44</td>
</tr>
<tr>
<td>E</td>
<td>19 + 25 = 25 + 19 + 0</td>
</tr>
<tr>
<td>F</td>
<td>19 + 25 = 43 + 1</td>
</tr>
</tbody>
</table>

Items 1 and 2 consist of what are called "arithmetic identities." The goal of these items is to see if comparisons can be made between both sides of the equal sign. If appropriate comparisons are made this will suggests a "relational" understanding of the equal sign in this context. If appropriate comparisons are not made then this means
students have the conception of the equal sign as a "do something signal." The arithmetic identities in item 1 start with familiar arithmetic relationships where the equal sign is on the right and with the answer placed to its right. Then identities that place the equal sign on the left are presented to see if this placement influences the interpretation of the equal sign. Finally, identities with operations on both sides are presented in order to see if students are ready to make sense of algebraic equations with operations on both sides. Kieren (1981) points out that "extending the notion of the equal sign within the framework of arithmetic equalities prior to the introduction of algebraic equations [is] considered essential in the construction of meaning for non-trivial equations." That is, arithmetic identities that are structured with the equal sign on the right (i.e. 1+3=4) can help make sense of equations of the similar form (i.e. x+3=4). Having operations on both sides requires that sense be made of equations with operations on both sides (i.e. x+3=2x+5). The identities in item number 2 are structured in the same manner as item 1 except that there is a focus on the use of zero and placement.

**Item 3**

What number(s) can be put in the box (□) to make the following number sentence true?

\[ 23 + 18 = \square + 12 \]

Item 3 is also designed to assess students’ understanding of the equal sign. Use of multiple operations on both sides and a box to the right of the equal sign (see item 3) provides students the opportunity to decide if the result to the operation on the left takes precedence over conceptualizing the relationship as a balance. If students place the sum of 23 and 18 in the box (i.e. 41) then this means there is an arithmetic notion of the equal sign (i.e. the equal sign means “do something.”). If the difference between 41 and 12 is
placed in the box then a relational notion of the equal sign is being applied. Previous studies that have used a similar item have shown that most beginning algebra students place the sum of the left hand side or the sum of all of the numbers in the box (Kieran & Chalouh, 1993; Sáenz-Ludlow & Walgamuth, 1998).

**Item 4**

What numbers can go in the box (□) to make the following number sentence true?

\[28 - □ = 12\]

Item 4 focuses on variable as unknown and is connected to item 3. If students correctly answer item 3 then there is a relational understanding of the equal sign and item 4 should also be correct. If students answer item 3 incorrectly then an arithmetic notion of the equal sign is being used. If so, item 4 assesses whether or not this notion of the equal sign is being obscured by a misconception of variable.

**Item 5**

List all possible values of the variable \(m\). Explain.

\[4 + m + m = m + 10\]

Items 5 and 7 assess students’ conception of variable. Item 5 assesses students’ belief that the same letter represents only one value? The variable can take on the value of six or a range of values (e.g. 2, 8 and 6) to make the sentence true. Either answer of these types will demonstrate appropriate understanding of the equal sign. However, only an answer of six will demonstrate an understanding of variable in the algebraic sense.

**Item 6**

If \(a + b = 43\); Then \(a + b + 2 = \square\).
Determine which number goes in the box.

Studies have shown that few students conceive of variables (letters in algebra) as generalized numbers, but that most students are able to interpret letters as specific unknowns. For example, Kuchemann (1981) noted in his research that although most students could not handle questions such as item 10 they had no difficulty with questions such as item 6. Item 10 requires that students consider a range of values while item 6 asks students to think of one value.

In terms of Kuchemann’s levels of understanding variables item 6 address understanding letters as ignored. Item 6 provides students the opportunity to ignore the variable(s). That is, the variables do not represent much more than place-holders. Comparing both equations shows that adding 2 is the only difference. An answer of 45 demonstrates that the student understands “a+b” to take on the value of 43 not as unknowns to be found but as specific unknowns.

**Item 7**

<table>
<thead>
<tr>
<th>Two numbers add up to ten. This relationship is represented by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ x + y = 10 ]</td>
</tr>
</tbody>
</table>

What sets of numbers \((x \& y)\) make this number sentence true?

Fujii (2003) found that items similar to 5 and 7 function as a pair. If students hold the misconception that *different letters stand for different numbers* then item 5 was correct but item 7 was incorrect. On the other hand, if students have the misconception that *the same letter stands for the same number* then item 5 was incorrect and item 7 was correct. Fujii (2003) noted that it was rare for any students to get both item correct.
**Item 8**

Consider the following equation:  \( y = 3 + x \)

a. What is the value of \( y \) if \( x = 1 \)?

b. What is the value of \( x \) if \( y = 23 \)?

c. If we want the values of \( y \) to be greater than 3 and less than 10, which values can \( x \) take on?

Item 8a is intended to prompt students to think of a variable as representative of a unique value. Item 8b requires that students conceive of the variable as an unknown in an equation. Finally, item 8c requires that variable be thought of as a functional parameter.

Items 8a and 8b assess student understanding of variables as “letter evaluated” on Kuchemann’s (1978) levels of understanding variables. Item 8c assesses student conceptions of variable in terms of “letter as generalized number”.

**Item 9**

There are six times as many students as professors at the local university. Write an equation that represents this situation.

Item 9 has been used by many studies to investigate student conceptions of symbols in Algebra (see Crowley, Thomas, & Tall, 1994; Laborde, 1993) and assessed students’ understanding of variable in terms of “letter as object” on Kuchemann’s levels of understanding variable. The item requires that words be translated into symbols that represent the relationship that is described. Crowley et. al (1994) found that 37% of college students in their study answered incorrectly with two thirds of them writing \( 6S = P \), rather than \( S = 6P \). This reversal error is said to occur because the letters are believed to represent objects as opposed to a number of object. Kaput (1987) argues that this is
because incorrect responses attempt to follow the word order from the original problem.

In another study (Rosnick, 1981) 22% of students in study believed that $S = \text{professors}$.

**Item 10**

Which is larger $2n$ or $n + 2$?

Item 10 was intended to assess students’ conceptions of variable. The challenge with this item is that when $n=2$ the expressions are equal. That is, the relationship between $2n$ and $n+2$ changes as the value of $n$ varies. This is the second order relationship referred to in Kuchemann’s (1978) description of understanding a “Letter as variable.”

**Item 11**

When is the following true: Always, Never, or Sometimes? Explain.

$L + M + N = L + P + N$

Item 11 has been used in several studies (Booth, 1984; Fujii, 2003; Kuchemann, 1981; Olivier, 1988) in order to assess whether or not students believe that different letters *must represent different values*. A response that indicates that this equation is true when $P$ is equal to $M$ demonstrates that students have greater flexibility in their conception of variable. Kucheman (1981) examined middle school students and found that 51% answered “never.” Shortly thereafter, Booth (1984) found that 14/35 thirteen to fifteen year olds (40%) in his study answered “never.” In a third study, Olivier (1988) reported that 74% of 13 year olds answered “never.” Item 11 addresses student notions of “letter as specific unknown” on Kuchemann’s levels of understanding variable. That is, the letters represent numbers that cannot be evaluated.
**Item 12**

Solve.

a. $3 + 4 = \square$

b. $3 - (-4) = \square$

Items 12a and 12b are intended to assess students’ conceptions of the minus sign. In their study Das et. al. (2010) highlighted how the minus sign is interpreted by using re-casted arithmetic problems. They argue that it is the process of subtraction that has a larger influence on processing arithmetic problems than does the negative number itself. When this occurs operations with negative numbers are viewed by students as problems that are solved by finding a difference without regard for the meaning of the minus sign as opposite.

Item 12a functions as a scaffold that is intended to prompt comparison with a familiar arithmetic operation. Item 12b is re-cast in order to highlight the meaning of the minus sign. An answer of -1 or 1 suggests that the minus sign has been interpreted as subtraction. A correct answer of 7 suggests that a componential view, where the meaning of the minus sign is derived through its context, was taken to determine the intended operation.

**Item 13**

Consider the equation: $x - y = 7$

a. The arrow above points to a symbol. What is the name of the symbol?

b. What does the symbol mean?

c. Can the symbol mean anything else? If yes, please explain.
Items 13a through 13c are modeled after an item that was used by Knuth et al. (Knuth et al., 2005) to assess students’ conceptions of the equal sign. The structure of the questions used by Knuth et al. provided a framework for item 13 in order to investigate students’ conceptions of the minus sign. Item 13a (What is the name of the symbol?) was designed to preempt students from using the name of the symbol in their response to item 13b (What does the symbol mean?). Item 13c (Can the symbol mean anything else?) was intended to provide students the opportunity to consider an alternative interpretation of the minus sign.
**Item 14**

Which expression (right or left) is larger if $k$ is a negative number?

<table>
<thead>
<tr>
<th></th>
<th>&gt;, &lt; or =</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-k$</td>
<td>$k$</td>
<td></td>
</tr>
<tr>
<td>$3k$</td>
<td>$-4k$</td>
<td></td>
</tr>
<tr>
<td>$k + 2$</td>
<td>$k - 2$</td>
<td></td>
</tr>
<tr>
<td>$2k - 1$</td>
<td>$2k + 1$</td>
<td></td>
</tr>
<tr>
<td>$k^2$</td>
<td>$k$</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>$k + k$</td>
<td></td>
</tr>
<tr>
<td>$-k$</td>
<td>$(-k) + (-k)$</td>
<td></td>
</tr>
</tbody>
</table>

Item 14 was intended to assess students’ conceptions of the minus sign as well as variable. Correct comparisons made between the two expressions demonstrated students understanding of the minus sign as meaning “the opposite.” Accurate comparisons are also dependent on students understanding that the letter in each item represents a set of numbers as opposed to one number. Item 14 therefore assesses students’ understandings of “letters as generalized numbers” according to Kucheman’s (1978) levels of understanding letters in algebra.

**Item 15**

Simplify the following expressions:

a) $20 + 8 - 7n - 5n$

b) $6 - 5a - 3 - 4a$

c) $4 - 6n - 4n$

Vlassis (2004) describes making sense of the minus sign as becoming flexible in ‘negativity.’ Vlassis investigated the meaning that was given to the minus sign by 8th graders by analyzing how they simplified polynomial expressions. Student work was
analyzed and the researchers found that most of them applied conceptions of the minus sign that “attest to early manipulations with natural numbers.” That is, the reasoning used included knowledge from their experience with arithmetic. Item 15 is modeled after these items and is intended to assess students’ conceptions of the minus sign in the context of simplifying expressions. Correct answers demonstrate an understanding of the minus sign to mean “negative.”

Responses were examined for the types of reasoning outlined by Vlassis (2004) including Operating from right to left (where the expression \(-4n-5n\) results in an answer of \(-1n\)), Brackets reasoning (where grouping symbols are misinterpreted), Signs Rule (Over application of the rule for multiplying two negative numbers where anytime two negatives appear the result is always positive regardless of the operation), and switching the order of terms (where students apply the associative property of addition by grouping positive and negative terms).

3.5.5 Data Analysis

The goal was to characterize how students understood the three symbols and not to give an overall score. Given this, the student results were organized by class, by symbol and subtopic. The primary focus of the analyses was a close examination of what the students in the different classrooms were making sense of in relation to the symbols and concept of variable assessed. The following tables
### Table 3.5.6-1: Coding of assessment responses for equal sign

<table>
<thead>
<tr>
<th>Item(s)</th>
<th>Concept &amp; Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a, 1b, 2a</td>
<td>Correct response to all three coded as familiar with arithmetic relationships</td>
</tr>
<tr>
<td>1c, 1d, 1e</td>
<td>Correct response to all three coded as placement of equal sign does not influence interpretation of the equal sign.</td>
</tr>
<tr>
<td>1f-1k, 2f</td>
<td>Correct responses to all five items coded as familiar with arithmetic relationships with operations on both sides.</td>
</tr>
<tr>
<td>2b, 2d, 2e</td>
<td>Correct response to all three coded as familiar with arithmetic relationships with operations on both sides including zero.</td>
</tr>
<tr>
<td>2c</td>
<td>Response of “F” coded as relational thinking, response of “T” coded as arithmetic.</td>
</tr>
<tr>
<td>3</td>
<td>Response of “29” coded as relational, response of “41” coded as arithmetic.</td>
</tr>
<tr>
<td>4</td>
<td>Response of “6” coded as relational, response of “10” coded as arithmetic.</td>
</tr>
<tr>
<td>3, 4</td>
<td>Incorrect responses to both items coded as notion of equal sign obscured by misconception of variable.</td>
</tr>
</tbody>
</table>
Table 3.5.6-2: Coding of assessment responses for variable

<table>
<thead>
<tr>
<th>Item(s)</th>
<th>Concept &amp; Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 7</td>
<td>Correct response to item 5 and incorrect response to 7 coded as holding misconception that different letters equal different numbers.</td>
</tr>
<tr>
<td>5, 7</td>
<td>Incorrect response to item 5 and correct response to item 7 coded as holding misconception that the same letter must equal the same value.</td>
</tr>
<tr>
<td>6</td>
<td>Response of “45” coded as understanding variable as letter IGNORED</td>
</tr>
<tr>
<td>7</td>
<td>Correct response coded as understanding letter as GENERALIZED NUMBER</td>
</tr>
<tr>
<td>8a, 8b</td>
<td>Correct response to both items coded as understands letter EVALUATED</td>
</tr>
<tr>
<td>8c</td>
<td>Correct response coded as understanding letter as GENERALIZED NUMBER</td>
</tr>
<tr>
<td>9</td>
<td>Response of “6S = P” coded as holds misconception of letter as OBJECT</td>
</tr>
<tr>
<td>10</td>
<td>Correct response coded as understanding letter as VARIABLE</td>
</tr>
<tr>
<td>11</td>
<td>Response of “Never” coded as holding misconception that different letters must equal different numbers.</td>
</tr>
</tbody>
</table>

Table 3.5.6-3: Coding of assessment responses for minus sign

<table>
<thead>
<tr>
<th>Item(s)</th>
<th>Concept &amp; Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Correct response coded as understands meaning of minus sign through its derived context</td>
</tr>
<tr>
<td>13</td>
<td>Responses coded as either 1) understands minus sign in context of subtraction as well as a negative, 2) understands minus sign only reference to procedure or 3) understands minus sign to only mean subtract.</td>
</tr>
<tr>
<td>15a-15c</td>
<td>Correct responses to all three items coded as indicating “flexible notion” of the minus sign</td>
</tr>
<tr>
<td>15a-15c</td>
<td>Specific responses coded for type rationale used: Variables Ignored, Operating from Right to Left, Brackets reasoning, Signs rule, Sign selecting, and Switching Order of Terms.</td>
</tr>
</tbody>
</table>
Responses for three items (items 10, 13 and 14) were omitted because responses could not be clearly analyzed. Most students did not respond to item 10 and many of those that did only provided an answer without any additional explanation. Item 13 did elicit responses from the majority of students, however, the responses were inconsistent in ways that did not make sense. Item 14 was also a challenge to analyze. Responses were almost random and it appeared as though the use of inequalities may have obscured students’ interpretation of the integers.

Responses to all other items were grouped according to codes listed above. I examined the proportion of students who responded in particular ways and compared the proportion on the pre-assessment to the proportion on the post-assessment using a Chi square test.
Chapter 4: Assessment Results and Findings

The pre and post algebra assessments examine how students conceptualized the equal sign, the minus sign and variable. Students that took both the pre and post assessments were divided in two groups. The BIL group consisted of 16 students that received instruction in Spanish and 22 students received instruction that used Sheltered English instruction. In the following sections I will use assessment results to describe the student performances by group and note any misconceptions held about the three symbols at the start of the study. I then discuss the differences from the pre to the post assessment in both groups. This will be followed by highlighting some of the more nuanced findings from the student assessments.

4.1 Pre-assessment Results: BIL Group

Students in the BIL group brought a considerable amount of prior knowledge to their Algebra class at the start of the academic year. Results showed that at the start of the study a considerable portion of students in the BIL group demonstrated knowledge about the equal sign and a grasp of basic conceptual knowledge about variable while few demonstrated conceptions of the minus sign that are beyond a symbol that means to subtract. Also, It is important to note that there were several misconceptions that students did not hold at the start of the study (See TABLE 4.1.5 MISCONCEPTIONS NO STUDENTS HELD in Appendix).

What did all students in the BIL group know at the start of the study?

There were several areas where all students (i.e. 100%) in the BIL group demonstrated favorable conceptions of the three symbols (See Table 4.1.1). With regards to the equal sign, all students demonstrated familiarity with arithmetic relationships and correctly interpreted the meaning of the equal sign when operations are on the right side
of the equal sign and the answer is on the left (as assessed by items 1c, 1d and 1e). All students also demonstrated a relational understanding of the equal sign. With regards to variable, almost all students (15/16) could determine the value of \( m \) in the equation in item 5 (i.e. \( 4 + m + m = m + 10 \)) as well as demonstrate understanding letter IGNORED (as assessed by item 6) as described in chapter 3 by Kucheman’s (1981) levels of understanding variable described in chapter 3.

That almost all students could correctly answer item 5 alone only shows they can solve an equation where the variable appears multiple times but in conjunction with item 7 these results demonstrate a misconception, and, only a few could subsequently provide the sets of numbers \((x & y)\) that make the relationship \( x + y = 10 \) true.

Table 4.1-1: What did all (93% to 100%) students in the BIL group know at the start of the study?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign:</td>
<td>familiar arithmetic relationships where the equal sign is on the right hand side:</td>
<td>16/16=100%</td>
</tr>
<tr>
<td>1a, 1b &amp; 2a</td>
<td>e.g. True or False? 17 + 24 = 41</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>correct interpretation of the equal sign when placed on the left hand side of an arithmetic equation;</td>
<td>15/16=93.75%</td>
</tr>
<tr>
<td>1c, 1d &amp; 1e</td>
<td>e.g. True or False? 44 = 25 + 19</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>relational understanding of the equal sign;</td>
<td>15/16=93.75%</td>
</tr>
<tr>
<td>2e</td>
<td>e.g. True or False? 19 + 25 = 44 + 25</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>relational understanding of the equal sign;</td>
<td>16/16=100%</td>
</tr>
<tr>
<td>4</td>
<td>e.g. What numbers can go in the box (□) to make the following number sentence true?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28 – □ = 12</td>
<td></td>
</tr>
<tr>
<td>Symbol Item(s)</td>
<td>Topic</td>
<td>Pre</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Variable 5</td>
<td>understands when the same letter equals the same value;</td>
<td>15/16=93.75%</td>
</tr>
<tr>
<td></td>
<td>e.g. List all possible values of the variable m. Explain.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 + m + m = m + 10</td>
<td></td>
</tr>
<tr>
<td>Variable 6</td>
<td>variable as Letter IGNORED;</td>
<td>15/16=93.75%</td>
</tr>
<tr>
<td></td>
<td>e.g. If a + b = 43; Then a + b + 2 = □. Determine which number goes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>in the box.</td>
<td></td>
</tr>
</tbody>
</table>

What did most students in the BIL know at the start of the study?

Assessment responses were examined to see what most (between 65% and 95%) students knew at the start of the study (see Table 4.1.2). Most or all students demonstrated a relational understanding of the equal sign as indicated by all three items (items 2c, 3 and 4) that assessed this understanding, as well as familiarity with arithmetic operations on both sides of the equation. Both of these understandings are important to developing conceptual knowledge of algebra.
Table 4.1-2: What did most (65% to 95%) students in the BIL group know?

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
</table>
| Equal Sign | 1f-1k & 2f | Correct response to all three items demonstrates student is familiar with arithmetic equations with operations on both sides;  
  e.g. True or False? \(37 + 18 = 18 + 37\) | 11/16=68.75% |
| Equal Sign | 2b, 2d & 2e | Correct response to all three demonstrates student is familiar with arithmetic equations with operations on both sides including zero;  
  e.g. True or False? \(19 + 25 = 0 + 44\) | 13/16=81.25% |
| Equal Sign | 3 | Response of 29 indicates a relational understanding of the equal sign;  
  e.g. What number(s) can be put in the box (□) to make the following number sentence true? \(23 + 18 = □ + 12\) | 12/16=75% |

What did only a few students in the BIL group know at the start of the study?

At the start of the study few (between 10% and 65%) students demonstrated knowledge of variable and the minus sign (see Table 4.1.3). In each case it is less than 44% of the students. Few students demonstrated understanding the meaning of the minus sign in the context of the operations that surround it and few students demonstrated a “flexible notion” of the minus sign.
Table 4.1-3: What did a few (10% to 65%) students in BIL group know?

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item: Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 8a &amp; 8b</td>
<td>Correct response to both items demonstrates student understands letter EVALUATED;</td>
<td>7/16=43.75%</td>
</tr>
<tr>
<td></td>
<td>e.g. Consider the following equation: $y = 3 + x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) What is the value of $y$ if $x = 1$?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) What is the value of $x$ if $y = 23$?</td>
<td></td>
</tr>
<tr>
<td>Variable 8</td>
<td>A response of $x = 1, 2, 3, 4, 5, 6$ demonstrates student understands letter as GENERALIZED NUMBER;</td>
<td>5/16=31.25%</td>
</tr>
<tr>
<td></td>
<td>e.g. Consider the following equation: $y = 3 + x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If we want the values of $y$ to be greater than 3 and less than 10, which values can $x$ take on?</td>
<td></td>
</tr>
<tr>
<td>Variable 9</td>
<td>Provided $6P=S$ or $S=6P$; Answered students-professors problem correctly;</td>
<td>2/16=12.5%</td>
</tr>
<tr>
<td></td>
<td>e.g. There are six times as many students as professors at the local university. Write an equation that represents this situation.</td>
<td></td>
</tr>
<tr>
<td>Minus Sign 12</td>
<td>Correct response to both items demonstrates student understands meaning of minus sign derived through context;</td>
<td>4/16=25%</td>
</tr>
<tr>
<td></td>
<td>e.g. Solve.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) $3 + 4 = \square$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) $3 - (-4) = \square$</td>
<td></td>
</tr>
<tr>
<td>Minus Sign 15a, 15b &amp; 15c</td>
<td>Correct response to all three items demonstrates “flexible notion” of the minus sign;</td>
<td>2/16=12.5%</td>
</tr>
<tr>
<td></td>
<td>e.g. Simplify the following expressions:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) $20 + 8 - 7n - 5n$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) $6 - 5a - 3 - 4a$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) $4 - 6n - 4n$</td>
<td></td>
</tr>
</tbody>
</table>
Results show that what the BIL group didn’t know at the start of the year was typical of beginning Algebra students; few students can handle simple equations, few to no students know letter as variable (i.e. in its multiple contexts) and a very small number of students know how to deal with negative numbers in specific contexts. Few students knew how to handle simple equations such that just under half of the students know letter as Evaluated, where they can substitute and solve for a variable in a simple equation and about a third know variable as GENERALIZED NUMBER (item 8c) where they provide a range of values for the variable. Item 7 was a bit more complex with regards to letter as GENERALIZED NUMBER such that only one student could provide the full range of sets of numbers that make $x+y = 10$ true. The highest levels of understanding variable were not part of this group of students’ prior knowledge. Only two students provided the correct equation in item 9. The students-professors problem in item 9 requires that variables not be treated as objects such that the equation follows the order of the words and how to use variables to represent a situation. Furthermore, No students demonstrated knowing letter as variable as assessed by item 10. No students could compare two expressions while allowing the value of the variable to vary. Finally, knowledge of negative numbers in the context of re-cast problems (i.e. item 12) and simplifying expressions (item 15) was only found among very few students. Overall, like many students in their first Algebra class this group of students did not demonstrate knowledge of the more contextual definitions of variable and negative numbers.
Table 4.1-4: What did very FEW TO NO (0 to 10%) students in the BIL group know?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 7</td>
<td>Letter as GENERALIZED NUMBER; e.g. Two numbers add up to ten. This relationship is represented by: $x + y = 10$. What sets of numbers ($x$ &amp; $y$) make this number sentence true?</td>
<td>1/16=6.25%</td>
</tr>
<tr>
<td>Variable 10</td>
<td>Letter as VARIABLE; e.g. Which is larger $2n$ or $n + 2$?</td>
<td>0%</td>
</tr>
</tbody>
</table>

What misconceptions did students in the BIL group hold at the start of the study?

The pre assessment also assessed misconceptions about the equal sign, the minus sign and variable. Students in the BIL group demonstrated holding only three out of twelve misconceptions that were assessed. These misconceptions were in regards to the minus sign and variable.

Table 4.1-5: Misconception held by most (65% to 95%) students in the BIL group.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Topic: Items</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>holds the misconception that different letters must equal different values (as indicated by correct answer to item 5 and incorrect answer to item 7)</td>
<td>14/16=87.5%</td>
</tr>
</tbody>
</table>
There was one misconception that most students in the BIL group demonstrated. Almost 90% of the students indicated holding the misconception that different letters must equal different values. This misconception shows that students have some experience with variables. They are able to solve a linear equation where the variable appears three times (item 5) but can’t produce all of the solution pairs when they see two variables in a relationship because they don’t see a difference between (2,8) and (8,2) (item 7). It should be noted that item 11 also assessed for this misconception

**Item 11**

When is the following true; Always, Never, or Sometimes? Explain.

\[ L + M + N = L + P + N \]

A response of “never” demonstrates student holds the misconception that different letters must equal different values (letter as SPECIFIC UNKNOWN). In this case 0/16=0% of students in the BIL group responded “never.”

Table 4.1-6: Misconceptions held by few (10% to 65%) students in the BIL group

<table>
<thead>
<tr>
<th>Symbol Item</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minus Sign 12</td>
<td>response of 12 or -12 indicates misinterpretation of parentheses obscures understanding of minus sign; e.g. Solve. a) [ 3 + 4 = ] b) [ 3 - (- 4) = ]</td>
<td>4/16=25%</td>
</tr>
<tr>
<td>Minus Sign 12</td>
<td>Response of -7 indicates applying minus sign to previous term</td>
<td>6/16=37.5%</td>
</tr>
</tbody>
</table>
A few students demonstrated misconceptions in two areas regarding the minus sign. In one case a few students misinterpreted parenthesis to mean multiplication (item 12). In the second case a few students applied the minus sign to the term that precedes it.

**Summary: What did students in the BIL group know at the start of the study?**

Summarizing these results paints a picture of a group of students that has significant grasp of the equal sign, is developing their knowledge of variable and is only just starting to experience the minus sign in its many contexts; also, the majority of the misconceptions assessed were not found amongst any students in the BIL group. The only misconception present amongst most students in the BIL group was that different letters must equal different values.

Most or all students demonstrated understanding all items that assessed the equal sign. Only one student demonstrated holding a misconception about the equal sign. All students knew letter as IGNORED and just under half of the students understood letter EVALUATED. The more advanced conceptions of variable were not as prevalent among the BIL group and most students demonstrated holding one misconception about variable. Approximately one third demonstrated understanding letter as GENERALIZED NUMBER and no students demonstrated understanding letter as VARIABLE. The only misconception found to be held by most students was that different letters must equal different values. Finally, few students demonstrated knowledge of the minus sign. In addition, two out of three misconceptions that a few students demonstrated were about the minus sign.

**4.2 Pre-assessment Results: SEI Group**

**What did all or most students in the SEI group know at the start of the study?**
Most students demonstrated knowledge of several basic understandings of the equal sign. Almost all (21/22 = 95.45%) students demonstrated a relational understanding of the equal sign. Most showed that they were familiar with conventional arithmetic relationships as well as those presented with the equal sign on the left hand side. Most students were also familiar with arithmetic relationships that have operations on both sides which is important to conceptualizing equations in beginning algebra. Also, similar to the BIL group, most students demonstrated understanding variable as letter IGNORED (as assessed by item 6).

Table 4.2-1: What did most (65% to 95%) students in the SEI group know?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign</td>
<td>familiar with arithmetic relationships where the equal sign is on the right hand side;</td>
<td>19/22=86.36%</td>
</tr>
<tr>
<td>1a, 1b &amp; 2a</td>
<td>e.g. True or False? 17 + 24 = 41</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>correct interpretation of the equal sign when placed on the left hand side of an arithmetic equation;</td>
<td>18/22=81.82%</td>
</tr>
<tr>
<td>1c, 1d &amp; 1e</td>
<td>e.g. True or False? 44 = 25 + 19</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>familiar with arithmetic equations with operations on both sides including zero;</td>
<td>15/22=68.18%</td>
</tr>
<tr>
<td>2b, 2d &amp; 2e</td>
<td>e.g. True or False? 19 + 25 = 0 + 44</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>a relational understanding of the equal sign;</td>
<td>21/22=95.45%</td>
</tr>
<tr>
<td>2c</td>
<td>e.g. True or False? 19 + 25 = 44 + 25</td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>a relational understanding of the equal sign</td>
<td>13/22=59.09%</td>
</tr>
<tr>
<td>3</td>
<td>e.g. What number(s) can be put in the box (□) to make the following number sentence true?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23 + 18 = □ + 12</td>
<td></td>
</tr>
<tr>
<td>Symbol Item(s)</td>
<td>Topic</td>
<td>Pre</td>
</tr>
<tr>
<td>---------------</td>
<td>-------</td>
<td>-----</td>
</tr>
<tr>
<td>Equal Sign 4</td>
<td>a relational understanding of the equal sign; e.g. What numbers can go in the box ( □ ) to make the following number sentence true?</td>
<td>19/22=86.36%</td>
</tr>
<tr>
<td>Variable 6</td>
<td>variable as Letter IGNORED; e.g. If a + b = 43; Then a + b + 2 = □. Determine which number goes in the box.</td>
<td>15/22=68.18%</td>
</tr>
</tbody>
</table>

What did a few (10% to 65%) students in the SEI Group know at the start?

Similar to the BIL group students in the SEI group showed that they are still developing their knowledge of variable and the minus sign. Unlike the BIL group, the SEI group only had a few students that were familiar with arithmetic equations with operations on both sides (items 1f through 1k and 2f).

Table 4.2-2: What did few (10% to 65%) students in the SEI group know?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign 1f - 1k &amp; 2f</td>
<td>familiar with arithmetic equations with operations on both sides; e.g. True or False? 37 + 18 = 18 + 37</td>
<td>9/22=40.91%</td>
</tr>
<tr>
<td>Variable 5</td>
<td>Response indicates understanding of when the same letter equals the same value; e.g. List all possible values of the variable m. Explain. 4 + m + m = m + 10</td>
<td>4/22=18.18%</td>
</tr>
<tr>
<td>Symbol Item(s)</td>
<td>Topic</td>
<td>Pre</td>
</tr>
<tr>
<td>---------------</td>
<td>-------</td>
<td>-----</td>
</tr>
</tbody>
</table>
| Variable 8a & 8b | letter EVALUATED; e.g. Consider the following equation: \( y = 3 + x \)  
| | a) What is the value of \( y \) if \( x = 1 \)?  
| | b) What is the value of \( x \) if \( y = 23 \)? | 12/22=54.55% |
| Variable 9 | Answered students-professors problem correctly Provided response of \( 6P=S \) or \( S=6P \); e.g. There are six times as many students as professors at the local university. Write an equation that represents this situation. | 2/22=9.09% |
| Minus Sign 12 | Correct response to both items demonstrates student understands meaning of minus sign derived thru context; e.g. Solve.  
| | a) \( 3 + 4 = \Box \)  
| | b) \( 3 - (-4) = \Box \) | 11/22=50% |
| Minus Sign 15a, 15b & 15c | Correct response to all three items demonstrates “flexible notion” of the minus sign; e.g. Simplify the following expressions:  
| | a) \( 20 + 8 - 7n - 5n \)  
| | b) \( 6 - 5a - 3 - 4a \)  
| | c) \( 4 - 6n - 4n \) | 4/22=18.18% |

Few students understood arithmetic equations with operations on both sides (items 1f through 1k and 2f). However, most students demonstrated understanding the same type of equations including zero (e.g. \( 2 + 3 = 5 + 0 \)). This is because this does not require the
same understanding of the equal sign in that the conception of the equal sign that the answer comes next still applies here.

Similar to the BIL group few students in the SEI group demonstrated conceptions of the minus sign that support learning Algebra. In both cases the percentage of students was slightly higher than those in the BIL group. 50% of the SEI group demonstrated understanding the meaning of the minus sign as derived through its context (item 12) and 19% of the SEI group demonstrated a “flexible notion” of the minus sign.

Table 4.2-3: What did very few to no (0-10%) students in the SEI group know?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 7</td>
<td>Letter as GENERALIZED NUMBER; e.g. Two numbers add up to ten. This relationship is represented by: ( x + y = 10 ). What sets of numbers (( x &amp; y )) make this number sentence true?</td>
<td>1/22=4.55%</td>
</tr>
<tr>
<td>Variable 10</td>
<td>Letter as VARIABLE; e.g. Which is larger ( 2n ) or ( n + 2 )?</td>
<td>0/22=0%</td>
</tr>
</tbody>
</table>

As was the case with the BIL group the higher levels of understanding variable were not very accessible to the SEI group. Only one student demonstrated understanding letter as GENERALIZED NUMBER and no student demonstrated understanding letter as VARIABLE.

**What misconceptions did students in the SEI group hold at the start of the study?**

A few students (between 13% and 45%) in the SEI group demonstrated holding misconceptions across all three symbols. The most prevalent misconception among students in the SEI group was the misconception that different letters must equal different values. A small number of students (3 or less) demonstrated: an arithmetic notion of the
equal sign, the misconception that the same letter must always equal the same number, the understanding of letter as OBJECT and that operations with negative numbers always means that a difference must be calculated. Finally, two misconceptions regarding the minus sign (Operating from right to left and Sign selecting) were not held by any student in the SEI group.

Table 4.2-4: Misconceptions held by few (from 10% to 65%) students in the SEI group.

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign 3</td>
<td>demonstrated an arithmetic notion of the equal sign; e.g. What number(s) can be put in the box (□) to make the following number sentence true? 23 + 18 = □ + 12</td>
<td>5/22=22.73%</td>
</tr>
<tr>
<td>Variable 5 &amp; 7</td>
<td>holds the misconception that different letters must equal different values (as indicated by a correct answer to item 5 and incorrect answer to item 7). e.g. <strong>Item 5:</strong> List all possible values of the variable ( m ). Explain. 4 + ( m + m = m + 10 ) <strong>Item 7:</strong> Two numbers add up to ten. This relationship is represented by: ( x + y = 10 ). What sets of numbers (( x &amp; y )) make this number sentence true?</td>
<td>4/22=18.18%</td>
</tr>
<tr>
<td>Variable 11</td>
<td>Response of “never” demonstrates student holds the misconception that different letters must equal different values (letter as SPECIFIC UNKNOWN) e.g. When is the following true; Always, Never, or Sometimes? Explain. ( L + M + N = L + P + N )</td>
<td>10/22=45.55%</td>
</tr>
<tr>
<td>Symbol</td>
<td>Item(s)</td>
<td>Topic</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Minus Sign 12</td>
<td>Response of 1 or -1 demonstrates understanding operations with negative numbers as finding a difference; e.g. Solve.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a) $3 + 4 = \square$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b) $3 - (-4) = \square$</td>
</tr>
<tr>
<td></td>
<td>Minus Sign 12</td>
<td>Response of -7 indicates applying minus sign to previous term</td>
</tr>
</tbody>
</table>

The misconceptions that a few students in the SEI group held at the start of the study included one area in equal sign, two in variable and two in minus sign. Some students demonstrated an arithmetic notion of the equal sign by assuming that the space to the right of the equal sign must be the result of the operation to the left and responding with an answer of 41.

**Item 3**

What number(s) can be put in the box (□) to make the following number sentence true?

$$23 + 18 = \square + 12$$

This is surprising that students would have this misconception. The results from the BIL group on this item and in general the overall positive results on equal sign make me question the validity of this result. However, the item is not multiple-choice. Students had a wide range of possible answers to submit.

Finally, a few students also demonstrated holding the misconception that different letters must equal different numbers. Two separate items (items 5 & 7 and item 11) assessed this misconception. While a small number of students (4/22=18.18%)
demonstrated holding this misconception as indicated by item 5 & 7, almost half of the group (10/22 = 45.55%) did so on item 11.

**Summary: What did students in the SEI group know at the start of the study?**

At the start of the study most students in the SEI group demonstrated the great majority of understandings about equal sign with one exception; only a few students demonstrated familiarity with arithmetic operations on both sides of the equal sign. In addition, one student demonstrated an arithmetic notion of the equal sign.

With regards to variable more than two thirds of the students in the SEI group demonstrated understanding letter as IGNORED, just over half demonstrated understanding letter as EVALUATED and no students demonstrated understanding letter as GENERALIZED NUMBER or letter as VARIABLE. A few students did demonstrate holding the misconception that different letters must equal different values on two separate items.

Finally, only a few students demonstrated experience with the minus sign beyond subtraction and a few students held misconceptions about the minus sign. Only a few demonstrated understanding the minus sign through its derived context and only a few demonstrated a “flexible notion” of the equal sign while simplifying polynomial expressions. A few students held the misconception that the minus sign only means difference and a few students applied the minus sign to the term that precedes it with an expression.

**4.3 Where did students in the BIL group demonstrate growth?**

Both groups are similar in what they know (most of the equal sign and some basic understandings of variable). The main difference was that the BIL Group **topped out** in
more areas than did the SEI group and the BIL group showed significantly greater growth with regards to the minus sign than did the SEI group.

The BIL group showed growth in all three categories. Students solidified their understandings of the equal sign, a few more students demonstrated understanding letter as EVALUATED and most students demonstrated understanding the minus sign where there were only a few at the start of the study.

Table 4.3-1: Areas where students in the BIL group demonstrated growth.

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign</td>
<td>familiar with arithmetic equations with operations on both sides</td>
<td>18.75%</td>
</tr>
<tr>
<td>1f-1k and 2f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>Relational understanding of the equal sign</td>
<td>19.75%</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Letter as GENERALIZED NUMBER;</td>
<td>25%</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>letter EVALUATED</td>
<td>25%</td>
</tr>
<tr>
<td>8ab</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minus Sign</td>
<td>Correct response to both items demonstrates student understands</td>
<td>50%</td>
</tr>
<tr>
<td>12</td>
<td>meaning of minus sign derived through context</td>
<td></td>
</tr>
<tr>
<td>Minus Sign</td>
<td>Correct response to all three items demonstrates “flexible notion” of the minus sign</td>
<td>62.5%</td>
</tr>
<tr>
<td>15abc</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

While there was little room to improve on equal sign more students (18.75%) demonstrated being familiar with arithmetic equations with operations on both sides and a relational understanding of the equal sign.

With regards to variable students in the BIL group demonstrated growth in two areas; the number of students that demonstrated understanding letter as EVALUATED
(items 8a and 8b) and letter as GENERALIZED NUMBER (item 7) increased by approximately 25%.

The greatest growth demonstrated by the BIL group was in regards to the minus sign. While few students demonstrated understanding the meaning of the minus sign as derived through its context or a “flexible notion” of the minus sign on the pre-assessment, most students (more than 50% more) did so on the post-assessment. This growth in understanding the minus sign will be more closely examined in section 4.5.4.

Table 4.3-2: Misconceptions held by few students in the BIL group at the end of the study.

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minus Sign 12</td>
<td>response of 12 or -12 indicates misinterpretation of parentheses obscures understanding of minus sign; e.g. Solve.</td>
<td>4/16=25%</td>
<td>2/16=12.5%</td>
</tr>
<tr>
<td></td>
<td>a) 3 + 4 = □</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) 3 – ( - 4) = □</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minus Sign 12</td>
<td>Response of -7 indicates applying minus sign to previous term</td>
<td>6/16=37.5%</td>
<td>2/16=12.5%</td>
</tr>
</tbody>
</table>

There were two misconceptions held by only a few students about the minus sign at the start of the year. In both cases even fewer students demonstrated these misconceptions on the post assessment.
### Table 4.3-3: Misconceptions held by most students in the BIL group at the end of the study

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item: Topic</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>5 &amp; 7: holds the misconception that different letters must equal different values (as indicated by a correct answer to item 5 and incorrect answer to item 7).</td>
<td>14/16=87.5%</td>
<td>9/16=56.25%</td>
</tr>
<tr>
<td></td>
<td>e.g.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Item 5</strong>: List all possible values of the variable ( m ). Explain.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 4 + m + m = m + 10 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Item 7</strong>: Two numbers add up to ten. This relationship is represented by: ( x + y = 10 ). What sets of numbers (( x ) &amp; ( y )) make this number sentence true?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The only misconception that most students held at the start of the study persisted on the post-assessment. Most (87.5%) BIL students demonstrated holding the misconception that different letters must equal different numbers on the pre-assessment. This misconception was still present in more than half of the students on the post-assessment. Although the decreases in misconceptions on the minus sign were slight it is notable that all three misconceptions were less prevalent at the end of the study.

### 4.4 Where did students in the SEI group demonstrate growth?

Students in the SEI growth showed growth across all three categories. Two areas in regards to equal sign left room for improvement on the pre-assessment and in both cases 18% or more students demonstrated these understandings on the post-assessment. More students also demonstrated understandings in three areas assessing variable on the post-assessment. Finally, there was notable growth in regards to students understanding of the minus sign.
Table 4.4-1: In what areas did students in the SEI group demonstrate growth?

<table>
<thead>
<tr>
<th>Symbol Item(s)</th>
<th>Topic</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign 2b, 2d &amp; 2e</td>
<td>Correct response to all three demonstrates student is familiar with arithmetic equations with operations on both sides including zero; e.g. True or False? 19 + 25 = 0 + 44</td>
<td>18.18%</td>
</tr>
<tr>
<td>Equal Sign 3</td>
<td>Relational understanding of the equal sign; e.g. What number(s) can be put in the box (□) to make the following number sentence true? 23 + 18 = □ + 12</td>
<td>27.27%</td>
</tr>
<tr>
<td>Variable 5</td>
<td>Response of 6 indicates understanding of when the same letter equals the same value; e.g. List all possible values of the variable m. Explain. 4 + m + m = m + 10</td>
<td>45.46%</td>
</tr>
<tr>
<td>Variable 7</td>
<td>Letter as GENERALIZED NUMBER; e.g. Two numbers add up to ten. This relationship is represented by: x + y = 10. What sets of numbers (x &amp; y) make this number sentence true?</td>
<td>36.36%</td>
</tr>
<tr>
<td>Variable 8ab</td>
<td>letter EVALUATED; e.g. Consider the following equation: y = 3 + x a) What is the value of y if x = 1? b) What is the value of x if y = 23?</td>
<td>22.72%</td>
</tr>
</tbody>
</table>
The growth in the number of students demonstrating they are familiar with arithmetic equations with operations on both sides including zero (items 2b, 2d and 2e) and relational understanding of the equal sign (item 3) is a considerable part of the picture of what students know about the equal sign. At the end of the study most students in the SEI group demonstrated all assessed understandings about the equal sign.

The greatest growth students demonstrated was with regards to variable where 35% more demonstrated understanding letter as GENERALIZED NUMBER and more than 20% more students demonstrated letter EVALUATED.

There was also notable growth in the number of students that knew how to interpret the meaning of minus sign as derived through its context as well as in those that demonstrated a “flexible notion” of the minus sign.
Table 4.4-2: Misconceptions held by few (10% to 65%) students in the SEI group at the end of the study

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item(s)</th>
<th>Topic</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign</td>
<td>3</td>
<td>demonstrated an arithmetic notion of the equal sign;</td>
<td>5/22=22.73%</td>
<td>0/22=0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. What number(s) can be put in the box (□) to make the following number sentence true?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>23 + 18 = □ + 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>5 &amp; 7</td>
<td>holds the misconception that different letters must equal different values (as indicated by a correct answer to item 5 and incorrect answer to item 7).</td>
<td>4/22=18.18%</td>
<td>7/22=31.82%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Item 5:</strong> List all possible values of the variable (m). Explain.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4 + m + m = m + 10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Item 7:</strong> Two numbers add up to ten. This relationship is represented by: (x + y = 10). What sets of numbers ((x &amp; y)) make this number sentence true?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>11</td>
<td>Response of “never” demonstrates student holds the misconception that different letters must equal different values (letter as SPECIFIC UNKNOWN)</td>
<td>10/22=45.55%</td>
<td>9/22=40.91%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. When is the following true; Always, Never, or Sometimes? Explain.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(L + M + N = L + P + N)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minus Sign</td>
<td>12</td>
<td>Response of -7 indicates applying minus sign to previous term</td>
<td>4/22=18.18%</td>
<td>3/22=13.64%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>e.g. Solve.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>a) (3 + 4 = □)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>b) (3 - (-4) = □)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Combining the growth in students demonstrating understandings about the equal sign with the decrease in the only misconception held by a few students completes a picture of a group of students where most understand the equal sign and only a very few hold one misconception. The only misconception that students held about the equal sign (as assessed by item 3) decreased to 0%.

The only other notable misconception that was present among students in the SEI group on the pre-assessment was that different letters must equal different values (as assessed by items 5 & 7). The number of students holding this misconception slightly increased on this pair of items and only decreased slightly as assessed by item 11.

**Discussion on variable**

When juxtaposing the two groups understanding of variable shows that students in the BIL group have a nuanced understanding of this symbol. Items 5 and 7 are modeled after a study done by Fuji et al (2007). In their study they found that students who answered item 5 correctly and provided an incorrect response to item 7 hold this misconception.

The SEI group had a few students that held this misconception at the start of the year (18.18%) and this slightly increased on the post-assessment (31.2%). Most students in the BIL group (87.5%) held this misconception as assessed by the combination of these items on the pre-assessment. The number decreased on the post-assessment to just over half (56.25%).

Another case where the context of the variable changes is when the letter cannot be evaluated such as in item 11. Item 11 has been used in several studies (Fujii, 2009, Kuchemann, 1981, Booth, 1984, Olivier, 1988) in order to assess whether or not students
believe that different letters must represent different values. A correct response of “sometimes” the equation is true when P is equal to M demonstrates that students have greater flexibility in their conception of variable. An answer of “never” suggests that the student did not conceive of two different variables having the same value.

Table 4.4-3

<table>
<thead>
<tr>
<th>Group</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEI</td>
<td>10/22 = 45.55%</td>
<td>9/22 = 40.91%</td>
</tr>
<tr>
<td>BIL</td>
<td>0/16 = 0%</td>
<td>0/16 = 0%</td>
</tr>
</tbody>
</table>

With regards to item 11 almost half of students (10/22=45.55%) in the SEI group demonstrated holding the misconception that different letters must equal different values as assessed by this item on the pre-assessment. This only decreased by one student (9/22=40.91%) on the post-assessment. While amongst students in the BIL group only no students demonstrated holding the misconception on the pre-assessment or the post-assessment.

4.5 Summary of Findings

Overall there are three important findings within the results from the pre and post assessments as they relate to the issues of language addressed in this study. First, the most unexpected finding is the amount of prior knowledge students in the BIL group had at the start of the study with regards to the equal sign and variable. Second, the BIL students have a nuanced understanding of variable; many have a grasp of basic conceptions of variable but some hold misconceptions about variable. Third, with regards
to the minus sign, the BIL group demonstrated greater growth than the SEI group and the difference was statistically significant.

4.5.1 Prior knowledge of Equal Sign

Although it is often noted that EL students are very low skilled, the EL students in the BIL group in this study started the school year with a considerable amount of prior knowledge about the equal sign. I expected some students would be beyond some of the misconceptions on the equal sign but I did not expect all students to demonstrate this knowledge across all items assessing conceptions of the equal sign. They particularly understand the equal sign in ways that prepared them for the study of Algebra. All or most students showed that they conceived of the equal sign in ways that support the learning of Algebra and overall very few misconceptions were found to be held by students.

4.5.2 BIL group conceptions of variable

The second finding given the results from the pre and post assessments is that students in the BIL group have a nuanced understanding of variable given the responses to items assessing their conceptions and misconceptions about this symbol. At the start of the study students demonstrated a grasp of basic conceptions of variable. While there was change in their conceptions of variable by the end of the study there was somewhat conflicting results with regards to conceiving of letter as SPECIFIC UNKNOWN.

There was little change in how students understand variable by the end of the study. Also, students understand letter as SPECIFIC UNKNOWN (i.e. the misconception that different letters must equal different numbers) according to items 5 and 7 but do not
according to item 11 where no students responded with “never” on item 11 while 40% to 70% of students in other studies and among the SEI group respond with “never.”

4.5.3 Significant difference in growth on items assessing minus sign.

The third finding that resulted from the pre and post assessment results is the growth in students’ ability to deal with operations involving negative numbers and negative numbers in the context of simplifying expressions. Both groups showed growth with regards to how they understand the minus sign in these contexts. Students in the BIL group showed greater growth than the students in the SEI group and this growth was statistically significant. As discussed in section 4.1, students’ conceptions of the minus sign were assessed by items 12 and 15. Item 12 assessed if students understand the meaning of the minus sign as derived through the context of the operations that surround it and item 15 assessed to what extent students have a “flexible notion of the minus sign” and if they hold any misconceptions about the symbol.

Table 4.5-1: SEI & BIL – ASSESSMENT RESULTS ON ITEMS 12 AND 15a-c

<table>
<thead>
<tr>
<th>Item: Topic</th>
<th>SEI Pre</th>
<th>SEI Post</th>
<th>SEI Growth</th>
<th>BIL Pre</th>
<th>BIL Post</th>
<th>BIL Growth</th>
<th>Sig: p&lt;.05?</th>
</tr>
</thead>
<tbody>
<tr>
<td>12: understands meaning of minus sign derived through context.</td>
<td>11/22 = 50%</td>
<td>14/22 = 63.64%</td>
<td>13.64%</td>
<td>4/16 = 25%</td>
<td>12/16 = 75%</td>
<td>50%</td>
<td>YES</td>
</tr>
<tr>
<td>15a, 15b and 15c: Demonstrated “flexible notion” of the minus sign.</td>
<td>4/22 = 18.18%</td>
<td>9/22 = 40.91%</td>
<td>22.73%</td>
<td>2/16 = 12.5%</td>
<td>12/16 = 75%</td>
<td>62.5%</td>
<td>YES</td>
</tr>
</tbody>
</table>

The portion of students in the SEI group that respond to item 12 with an answer of 7 went from 50% on the pre-assessment to 63.64% on the post-assessment. The portion
of students in the BIL group that responded with an answer of 7 went from 25% to 75%. The difference in growth of proportions is statistically significant (p<0.05).

**Item 12**

Solve.

a) \(3 + 4 = \square\)

b) \(3 - ( - 4) = \square\)

The portion of students that correctly simplified all three expressions in item 15 in the SEI group went from 18.18% on the pre-assessment to 40.91% on the post-assessment. The portion of students in the BIL group that correctly simplified all three expressions in item 15 went from 12.5% on the pre-assessment to 75% on the post-assessment. The difference in growth of proportions is statistically significant (p<0.05).

**Item 15**

Simplify the following expressions:

a) \(20 + 8 - 7n - 5n\)

b) \(6 - 5a - 3 - 4a\)

c) \(4 - 6n - 4n\)

While using these two sets of items provided the opportunity to assess student misconceptions of the minus sign they did not provide sufficient understanding of how students conceptualize the minus sign.

**Closing**

Most or all students in the BIL group demonstrated a large amount of prior knowledge about the equal sign and basic conceptions of variable at the start of the study. Secondly, the BIL students have a nuanced understanding of variable where many have a grasp of basic conceptions of variable with mixed results regarding misconceptions about
variable. Finally, while students in both groups showed considerable growth in flexible notion of the equal sign and understanding the meaning of the minus sign through its derived context; the students in the BIL group had greater growth that was statistically significant.
Chapter 5 Nuances of instructional language in Spanish

This chapter addresses instructional language in Mr. Franco’s class to understand the nuances of language use particularly as they relate to understanding any advantages provided by using Spanish as the language of instruction. Specifically, the expectation was to identify language practices that can help the field begin to understand what to look for as we continue to study language in the context of the mathematics classroom. The study was interested in how Spanish was used as a support to learning mathematics. Very specific factors made the instructional practice of the other two teachers very different. In one case the use of English language materials limited the instructional language, albeit in Spanish, to translating text. In the second case, the use of English for instruction as well as English language materials provided only data on student conceptions. Therefore the focus of analysis is on Mr. Franco’s practice.

In this chapter I will discuss three characteristics that framed Mr. Franco’s instruction as it relates to language that emerged in classroom observations and interviews. I first differentiate the instructional approach of the three teachers and discuss Mr. Franco’s beliefs about EL students’ language development and abilities. Second, I describe observed strategies used to support academic language development. Finally, I examine the instances where the students and the teacher choose to use English.

5.1 Beliefs about EL students

Perhaps because all three teachers in this study attended the same credential program instruction appeared to be similar in all three classrooms with some slight differences. Mr. Valdez, who taught in Spanish and English with English language materials, is traditional and lecture oriented with almost exclusively informal
opportunities for students to work together. Mr. Estevez, who taught in English, is also lecture oriented and provides multiple formal and informal opportunities for students to work collaboratively. The major difference was how language was used and beliefs about EL students. Mr. Franco expressed his thoughts on his experience as a teacher and on EL students in two separate interviews on 08/11/10 and 01/02/11 and during informal observations. In his responses to interview questions Mr. Franco makes it clear that that he believes “[EL students] are able to express themselves better in Spanish [compared to English] and grasp concepts a lot better when [presented] in Spanish.” Through his responses to interview questions it was apparent that Mr. Franco understands the diversity among EL students with regards to language and education and that academic language development occurs over time.

Mr. Franco recognizes the diversity of EL students in social and linguistic terms. Mr. Franco can relate to the different ways of speaking in Spanish. He explains that this difference depends on the student’s country of origin and even the specific region a student comes from within a country. For example, when asked how students speak about language he responded by describing how students that come from Mexico will use the term “voz” to describe or reference the variety of Spanish used by students from Central America. Along the same lines he noticed that students that come from the region in and around Mexico City speak in rhythmic tones described in Spanish as “cantado.”

Mr. Franco knows students come with a range of experience with academic language. When asked how students communicate with each other he says: “You can tell when high level students and low level students don’t understand each other because high level students come with more academic words. For example, some students do not
understand that ‘eje’ is the Spanish word for axis.” In his experience a portion of his EL students are comfortable using academic terms in Spanish and that they possess strong math skills. He believes that this is because many students have attended schools in their country of origin.

Mr. Franco believes that language development happens over time. He explains that as the year progresses “all of [the students] can articulate what they mean much better” in Spanish. Additionally, in Mr. Franco’s experience students with more developed academic language abilities in Spanish learn to support those with less developed abilities. He has observed students adjusting their language when they are in this situation: “They explain to each other and they try to use more simple words.”

In sum, Mr. Franco’s beliefs about his students are non-deficit oriented. Academic language development is embedded within instruction. His goal is to leverage students’ home language as an asset. Regardless of the level at which students come to his class he believes language develops over-time. Simply put, Mr. Franco believes in his students.

5.2 Support for Academic Language Development

Mr. Franco’s instruction is framed around making verbal and written language comprehensible. In more explicit terms academic language development is supported through the guided reading sessions that provide opportunities to ask students to re-voice their thoughts and for the teacher to model what it means to clarify the objective of a problem or question.

5.2.1 Essential support for language

Mr. Franco employed strategies that support academic language development to begin a lesson, to guide instruction and to close a lesson. When asked how he addresses
academic language Mr. Franco’s immediate response was that he “front loads vocabulary.” That is, he starts his lessons by discussing definitions of vocabulary specific to the topic being covered. He says that his goal is “to be clear about what [he is] talking about.” This perspective guides his choice in strategies during instructional time. For example, during instructional time multiple colors are used to write on the whiteboard during direct instruction to provide emphasis on specific symbols or concepts. Learning in Mr. Franco’s class also involves speaking and listening to peers. When students engage in guided or independent practice they are almost always in pairs or groups. Collaborative group work is sometimes informal where students are invited to work with a partner or group or where students are assigned into groups with a specific goal such as preparing a presentation. Students are guided by sentence frames or sentence starters when discussing the text or rewording what they read. However, articulating mathematical knowledge is not only left to the students. Mr. Franco consistently follows direct instruction or the days’ class by a summarizing the important points.

Mr. Franco sees front loading vocabulary, the use of collaborative groups and summarizing a lesson as effective instruction. When it comes to academic language development Mr. Franco is much more explicit. He uses the online Spanish language curriculum as the primary source of mathematical text that students encounter. At least twice a week over the course of the year Mr. Franco guides students through reading about the topic that is the focus of the lesson.

### 5.2.2 Guided Reading

Guided reading sessions are an integral component of Mr. Franco’s instruction. Students read from the online curriculum that is simultaneously projected on the
interactive whiteboard and on individual laptops. This is how students engage with written and oral academic language in Spanish. Typically, a student volunteer reads a passage from the Colegio curriculum and then the teacher facilitates a discussion about the passage.

The guided reading sessions provide the opportunity to check for understanding. For example, on 03.21.2011 Mr. Franco checks for understanding by asking for a volunteer to explain what was just read aloud by another student. Students are reading about solving systems of equations from the online curriculum. The student that responds explains that he understood the paragraph to mean that there are various solutions ("varias soluciones") to a system of linear equations. Mr. Franco responds with an encouraging response ("Tienen la idea, pero hay otro detalle."), and then clarifies that the paragraph was speaking about the three methods for solving a system of linear equations. Checking to see if students understood what they read or hear is one important way in which the reading sessions were guided. Additional guidance was provided by instances where they were asked to explain their understandings.

5.2.2a Students are asked to explain

Defining vocabulary terms is routine in Mr. Franco’s class. For example, on 05.01.2011 while discussing the instructions for a problem set, from an English language textbook that asks students to factor a difference of squares Mr. Franco asked the class: “La palabra ‘factoring’, que significa esto?” (The word factoring, what does it mean?) and also asked: “Que significa la palabra diferencia?” (What does the word difference mean?)

T: “La palabra ‘factoring’, que significa esto?”

Ss: “Sacar factores.”
T: “Que significa la palabra diferencia?”
Ss: “Minus.”

It would be easy as an instructor to simply state the definition of a “difference of squares” and move on. However, Mr. Franco allows students to reflect on the definitions of “factoring” and “difference.”

Mr. Franco’s instruction with regards to language goes beyond stating definitions of vocabulary terms. Students are asked to explain what they understand. Mr. Franco is consistent in using this strategy. What was unexpected was the different ways that he asked students to do so. Mr. Franco varies how he asks students to explain. On 01.17.2011 Mr. Franco asks a student to read the instructions for the first problem that is projected on the screen. Mr. Franco asks students to clarify the type of answer they are looking for. In this case he asks: “In other words what do they want for number, or, letter a?” (“En otras palabras que es lo que quieren para el numero, o, la letra a?”). On 03.21.2011 Mr. Franco asks a student to read. After she is done, the teacher asks for someone to explain “in your own words” (“Explica en tus propias palabras…””) what the paragraph just read means. On 05.0.2011 Mr. Franco is helping students understand the phrase difference of squares. He starts by asking students to define difference. When students offer the “minus” sign as a response he encourages students to explain in mathematical terms or in terms of mathematical operations.

T: “Que significa la palabra diferencia?”
S: “Minus”

T: “En terminos de matematicas. (Pause) En terminos de operaciones: sumar, restar…”
S: “Restar.”
Finally on 05.11.2011 the class is working on graphing quadratic equations. The teacher asks a student to read from the curriculum. After the student reads the teacher reiterates that, in a quadratic equation, if the leading coefficient, $a$, is smaller than zero, then the equation is negative, but if $a$ is larger than zero, then the equation is positive. If the equation is positive, the image curves upward, if it is negative, it curves downward. Afterwards, Mr. Franco checks for understanding by asking in Spanish: “What does that mean, in terms that we use?” Students respond, in English, “happy face or sad face.”

T: “Que es lo que significa mayor de zero y menos de zero? Mayor que zero?”

Ss: “Positvo.”

T: “Menos que zero?”

Ss: “Negativo.”

T: “Y la representacion grafica significa que abre para arriba o abre para ..?”

Ss: “Abajo.”

T: “Que significa eso en los terminos que nosotros ponemos?”

Ss: “Happy face y sad face.”

T: “Aunque lo están leyendo en español, lo tenemos que traducir en terminos que nosotros entendemos”

While it was expected that students would be asked to define vocabulary terms it was not expected that Mr. Franco would ask students to re-voice (i.e. to explain their understandings verbally) in so many different ways. In Mr. Franco’s class students are asked to define terms and to explain “in other words,” in their “own words,” “mathematical terms” and in “terms [they] use in class.”

5.2.2b Using Subject Specific Discourse
Although it is not easy to determine what effect the strategies Mr. Franco employed had on students' language abilities one practice specific to the language of mathematics was apparent. On several occasions Mr. Franco takes time to question students on what they are being asked to do. That is, he asks them to verify their objective. This is done consistently enough such that by the second semester students were heard checking their own understanding of their objective.

During the lesson On 01.17.2011 mentioned above when Mr. Franco asks students “….what do they want for number, or, letter a?” (“…qué es lo que quieren para el número, o, la letra a?”) he is also modeling a subject specific discourse practice. On the same day Mr. Franco checks to make sure students know what he wants them to see when solving a system of linear equations. He asks: “What do I want you to see?” (“qué es lo que quiero que vean?”). The following day (On 01.18.2011) Mr. Franco presents the class with a linear equation. He asks students “What do they want us to do?” (“…Que es lo que quieren que hagamos?”).

These instances of Mr. Franco guiding students to reflect on their objective served as a model such that students demonstrated applying the same strategy. On 02.27.2011 the following exchange between two students was observed:

S: “x-method. Seis arriba, cinco abajo, dos y tres” (while pointing to the x symbol)
S: “Estás segura que esta así se queda? Allí dice ‘solution’ este es factor.”
S: “Sí, es cierto.”

TRANSLATION
S: “X-method. Six above, five below, two and three.”
S: “Are you sure it stays like that? It says solution right here, this is a factor.
S: “Yes, that’s true.”
While solving a quadratic equation by factoring two students clarify the objective. These two students realize that they have not fulfilled the objective of the problem. They have only factored the quadratic equation but the problem is asking for a solution.

5.3 Two language one mathematics: student and teacher use of English

While instruction in this class was in Spanish there were several instances where the students and Mr. Franco use English. Students used English in multiple contexts while Mr. Franco used English for one specific purpose. Interestingly, when English could have been leveraged Mr. Franco chose to use Spanish.

5.3.1a Students use English when encountering Cognates

Students used English when encountering cognates (i.e. words that have similar spellings and pronunciations in both languages). This was the case when Mr. Franco’s class was studying polynomials and operations on polynomials. Additionally, the use of cognates provided Mr. Franco the opportunity to model the use of precise language. While utilizing a choral response structure he notices that students are pronouncing names of polynomials in English even though they are discussing the topic in Spanish. Over two days Mr. Franco notices the students pronounce monomial and trinomial in English. On 01.24.2011 Mr. Franco asks students, in Spanish, ‘Si tiene ____ terminos, se llama____?’ When students respond in English by saying “binomial,” the teacher notes a pronunciation difference when saying “binomial” in Spanish versus English. The following day (on 01.25.2011) Mr. Franco is guiding students through the procedure for multiplying polynomials. He starts by asking students to name the polynomials. Again students respond in English and he addresses this.

T: “Primero qué es el nombre de este de acá? Mono…”
Ss: “Monomial.”
T: “Y lo estoy multiplicando por un….”
Ss: “Trinomial.”

Students were able to respond appropriately but they did so in English and Mr. Franco takes time to correct students’ pronunciation. Although seemingly trivial at first Mr. Franco sees the opportunity to support students in using precise language. Students’ academic language development was not only addressed in phonological terms (i.e. pronunciation) but also modeled the socio-linguistic component of the language of mathematics by supporting the use of precise language (i.e. precise language is a characteristic of the language of mathematics).

5.3.1b Students use English when it is common practice

Another instance when students opted for English is when the vocabulary is a part of the language of the class. That is, the language of the mathematics classroom that was shared by students and their teacher. The exchange between Mr. Franco and his students on 05.11.2011 is a case in point.

T: “Qué es lo que significa mayor de zero y menos de zero? Mayor que zero?”
Ss: “Positvo.”
T: “Menos que zero?”
Ss: “Negativo.”
T: “Y la representación grafica significa que abre para arriba o abre para ..?”
Ss: “Abajo.”
T: “Que significa eso en los términos que nosotros ponemos?”
Ss: “Happy face and sad face.”
T: “Aunque lo estan leyendo en espanol, lo tenemos que traducir en terminos que nosotros entendemos”

In this instance when students are prompted, in Spanish, to use “terms we use [in class]” to identify an upward or downward facing parabola students respond, in English by saying “happy face or sad face.” At the end of the exchange Mr. Franco acknowledges that while the text they are reading is in Spanish they must still use terms familiar to the class because these are terms that “we” understand. It seems that regardless of which language is used, English or Spanish, what is important is the language of mathematics in the context of this particular classroom.

5.3.1c Students use English to define

Having two languages provides additional ways of navigating the language of the mathematics classroom. There can be multiple ways of defining a vocabulary term. For example, when asked in Spanish for a definition or a meaning of a term EL students sometimes answer with the translation of the term in question to English.

T: “La palabra ‘factoring’, que significa esto?”
Ss: “Sacar factores.”

T: “Que significa la palabra diferencia?”
Ss: “Minus”

T: “En terminos de matematicas. (Pause) En terminos de operacion: sumar, restar…”
S: “Restar.”

T: “Cual es la diferencia entre siete y cuatro?”
Ss: “tres”

T: “que es lo que hiciste? Que operación?”
Ss: “restar”
On this occasion students are asked to define “difference” (“diferencia”), several students respond by saying “minus.” While students’ response is not necessarily incorrect Mr. Franco sees the need to guide students further by suggesting the context of subtraction.

5.3.1d Students use English to avoid redundancy

Finally students were observed using English in an attempt to avoid redundancy and communicate effectively. Having two languages to speak mathematically is an advantage when making sense of concepts that seemingly overlap such as: “less than,” “negative” and “minus.” Conceptually these terms refer to slightly different concepts. This is problematized by the fact that in Spanish the terms “minus” and “less than” are very similar (“menos” and “menos que”). On 01.18.2011 when a student dictates the following inequality: 5X - 2 (7X + 1) <-14X. The student states: “Cinco equis menos dos, parentesis, siete equis mas uno, parentesis, luego ‘less than’ menos catorce equis.” The student uses “less than” (instead of “menos que”) to distinguish from the negative fourteen that follows (“menos catorce”). That is, the inequality could have been read as: “…menos que menos catorce equis.” In effect the students in this case avoided the redundancy by using English.

In sum, observations of classroom lessons revealed that students use of English occurs in several contexts: Students use English when encountering cognates, when it is a common practice in class, to define terms in Spanish and in one case to avoid redundancy.
5.3.2 Preparing for the state exam

Unlike his students Mr. Franco only had one reason to use English. Mr. Franco uses English to prepare students for state testing. Mr. Franco feels that certain vocabulary terms are important to helping students make sense of the items on the exam. Preparing students for the state exam involves not only understanding the mathematics in the exam but also the English vocabulary that is helpful in making sense of the exam. For example, on 01.25.2011 Mr. Franco is discussing problems that ask students to derive a linear equation given slope and two points. Mr. Franco emphasizes that end of year state tests require they be careful and know how to figure out slope when it is not given to them. He says: “Hay que tener cuidado con la ecuación que nos dan. Hay veces que les dan ‘el slope’ y hay veces que les dan los dos puntos.” Mr. Franco says that although his students know that “pendiente” is the formal Spanish term for the slope of a line “my kids will say ‘slope’ because that is how we refer to it.” Referring to specific terms in English is a way to prepare students for the state exam but the use of these terms has more to do with the language of Mr. Franco’s class than it does with the fact that the term is in English.

5.3.3 The Language of Technology

The use of technology was an additional factor in Mr. Franco’s classroom. In this case, Mr. Franco was intentional about choosing to use Spanish to instruct. Use of Spanish was vital to making the curriculum accessible for students. The Spanish language curriculum was online and students needed to learn how to use laptops and navigate the website to access the curriculum. Given that this was a 9th grade class of ELs would
suggest that students’ experience with technology was limited. Using technology to teach and learn requires that instruction focus on the technology as well as the content. Mr. Franco did this from day one. He used Spanish to instruct students on technology even though it may have been easier for him in English. While Mr. Franco is bilingual he is more experienced with written English and with the language of technology in English. Using Spanish to instruct on technology was only one component of his strategy to get students comfortable with the curriculum.

Mr. Franco’s approach to teaching technology was similar to his instruction of mathematics in that he frontloads vocabulary and takes time to make language meaningful. Instruction on the use of the laptops and navigating the website was gradual. Video recordings captured Mr. Franco teaching students how to navigate through the website on at least five separate occasions. The last of which was two-thirds of the way into the semester. Visuals including the projection of the curriculum on the front screen as well as on laptops provided the opportunity to point to symbols (such as where to check on the wireless signal).

Mr. Franco used Spanish to instruct students on technology. This could have been easier for him to do so in English since most of the language on laptops is in English. Also, students would navigate to the curriculum website on an English language browser. What resulted was that like his instruction of mathematics his instruction on technology used Spanish to explain concepts, procedures and directions. This was supported by the structures and strategies mentioned above and the use of English in specific cases.

Mr. Franco’s experience with Spanish includes formal education during his early childhood and then primarily in social contexts with friends and family. That is to say,
like many people he has a grasp of everyday language in Spanish. He does not have experience with the language of technology in Spanish. Like the academic language of any content area the language of technology is very specialized. He learned the terms as he went along. He was learning the language of technology in Spanish at the same time students were doing so.

In this chapter I discussed three characteristics that framed Mr. Franco’s instruction as it relates to language that emerged from classroom observations and interviews. I first discussed Mr. Franco’s beliefs about EL students’ language development and abilities. Then, I described strategies I observed used to support academic language development and I examined the instances where the students and the teacher chose to use English. What I found was that Mr. Franco believes in his students. He believes they can communicate with and about mathematics in the language that they know best and that they develop academic language over time. Mr. Franco supports academic language development through instruction that’s framed around making verbal and written language comprehensible. Academic language development is supported through the guided reading sessions that provide opportunities to ask students to re-voice their thoughts and for the teacher to model what it means to clarify the objective of a problem or question. In sum, I found that using Spanish for instruction matters and Mr. Franco accepts and understands the subtle differences in language. Finally, I found that the use of English is strategic and not done for the purpose of learning the language itself but rather to communicate within and amongst each other in class. What emerges is that Mr. Franco is using Spanish as a learning tool and not just as a way to connect vocabulary to their native language.
Chapter 6  Discussion

The question of whether and how a student’s first language should be used in an instructional program has been a focus of discussion for many years. Some studies have found no advantage when a student’s first language is used for instruction while others have found an advantage in small to moderate ranges (August & Pease-Alvarez, 1996; Baker & de Kanter, 1981; Greene, 1997; Rolstad et al., 2005; Rossell & Baker, 1996; Slavin & Cheung, 2005; Willig, 1985). Drawing on Moschovich’s (2004) review of empirical research on Latinos, mathematics and language, which called for broader notions of language and mathematics, I studied the intersection of primary language instruction and conceptual knowledge in a secondary Algebra 1 class. In this chapter I first discuss the limitations of this study and implications for future research. I then conclude by discussing the contributions this study makes to research on ELs in mathematics classrooms.

Section 6.1 Limitations and suggestions for future research

Research on using students’ primary language for instruction with ELs is growing body of work. This is especially so with regards to secondary mathematics classrooms serving ELs. Research in this area is challenging due to various factors including clear definitions of the type of Bilingual Program used, setting up sufficient control groups, and large sample size. Given that I wanted to study the interaction between using L1 for instruction, academic language development and conceptual mathematics knowledge, I chose an exploratory approach knowing that there would be limitations to how the investigation would proceed. This study had several limitations that emerged as the
research was conducted: a small sample size, limited data on students’ language use, limited data on students’ thinking and their rationale behind responses given on the assessments and insufficient items on the assessments.

6.1.1 Sample Size

One limitation of this study was the sample size. Demographic changes and restrictive state bilingual education policy at the school sites in addition to a transient EL population resulted in a small sample size. The small sample size, with much variety in student language, prevented this study from making comparisons and limited the options in regards to understanding student learning and its relationship to language.

Over the six years previous to the study there has been a slight decline in the number of ELs at both schools. At SFHS the population went from 1640 ELs in 2006 to 925 in 2010. At EVHS the EL population went from 1155 in 2006 to 832 in 2010. Additionally, since 1998 when California voters approved proposition 227 (and other states) there are few schools that explicitly allow teachers to use Spanish for instruction. This legislation prompted the limited the number of classes that used primary language instruction. Finally, the sample size decreased even more due to students that dropped or added the course throughout the year. Out of 82 students across the three classes 23 students dropped the course and 19 students added the course before the end of the study. The effect was that only a subset of each class took the pre and the post assessment.

6.1.2 Data on student language

Another limitation of this study was the lack of data on student language. Video of classroom lessons provided teacher language but captured very limited examples of
student language. The goal was to capture the teachers’ instructional language and language used by students when speaking with each other about mathematics. I didn’t want contrived language. Therefore no set lessons or classroom structures were provided to teachers. The result was video capturing mostly direct instruction where the dominant voice was that of the teacher. It was evident that I could not connect assessment results with language practices.

Given the constraints, among suggestions for future studies would be the use of microphones and specific lesson formats which could render more data on student language. One way to capture student language is to set up microphones while students work during guided or independent practice sessions. This approach could be supported by specific lessons or tasks could be assigned to increase the opportunities for students to engage in discourse. Tasks such as instances of Think-Pair-Share, ask pairs to explain something to each other, have groups re-write a passage from a textbook in everyday terms.

6.1.3 Student interviews

Another limitation of this study was the lack of interviews with students. Interviews with individual students were planned but were not finalized due to logistical issues. The plan was to use a think-aloud protocol to gather student language. While most assessment items were free response students did not provide much in terms of written language in their responses. The result was a lack of data on students’ rationale for their responses. Individual interviews with a protocol that asked them to explain their
approach to specific items could have revealed a clearer picture of how they made sense of the symbols.

6.1.4 Additional Assessment Items

While the assessments used in this study yielded valuable information about student learning they could have revealed more. Drawing from research on conceptual knowledge in beginning algebra (see Kieran, 1981; Kuchemann, 1978; Linchevski & Williams, 1999; Malisani & Spagnolo, 2009; McNeil & Alibali, ; Vlassis, 2004) items were chosen to examine students’ conceptual understandings of the equal sign, the minus sign and variable. The intent was to only use items that had already been piloted and had a specific analysis attached to it. The result was an assessment consisting of 15 items. While this set of items assessed all three symbols, small gaps became evident after analyzing the results. For example, one result was that the BIL group demonstrated a misconception about variable in one case and did not do so in another. Another result was that some items had a higher response rate than others that could be addressed by using item formats that elicited higher response rates. Finally, it was clear that students in the BIL group made progress with regards to how they understand the minus sign it was unclear what exactly was different at the end of the year and how this understanding is connected to language data.

In one case a sub-group of BIL students demonstrated having a misconception on one item but not another that assessed the same misconception. Items 5 and 7 were grouped for analysis such that a correct response to item 5 and an incorrect response to
item 7 suggests that students hold the misconception that different letters must equal different numbers. Item 11 also assessed for the same misconception.

**Item 5**
List all possible values of the variable $m$. Explain.

\[4 + m + m = m + 10\]

**Item 7**
Two numbers add up to ten. This relationship is represented by: $x + y = 10$. What sets of numbers ($x \& y$) make this number sentence true?

**Item 11**
When is the following true?; Always, Never, or Sometimes?
Explain.

\[L + M + N = L + P + N\]

Many students only provided a partial list of pairs of numbers to item 7. It was not clear if students misunderstood the context of the equation or if they truly could not think of all of the pairs that add up to ten. In this case, the format of the item may be adjusted. Some item formats proved easier for analysis and simply told me more. For example, the use of blanks (or boxes to represent variables) elicited a very high response rate. This format could be combined with item 7 to elicit more responses but also to gain clarity on what students know.

Fill in the blanks in the following:

\[__ + ___ = 10\]

\[__ + ___ = 10\]

\[__ + ___ = 10\]

\[__ + ___ = 10\]
What other pairs of numbers add up to ten?

List the pairs of numbers from above that make the following equation true:

\[ x + y = 10 \]

The EL students in this study showed significant growth with regards to the minus sign. What was unclear after reviewing data on language use in the classroom was how students understand concepts that are not easy to verbalize. For example, items could be developed that might give insight into how students understand a double negative by using True/False number sentences such as:

\[ -(-2)(4) = 8 \]
\[ -(-2)(4) = -8 \]
\[ -(2)(-4) = -8 \]

This could help understand data on student language that showed that a double negative (at least verbally) is a challenge to communicate. Using the format of true/false number sentences would be strategic as these items were valuable in this study but what is also important is the connection to language use. This suggests that it may be helpful to document language use first and then select or develop items that assess concepts around the concepts that students verbalize.

6.2 Contributions to the field of study

This study makes several contributions to the field of mathematics education research. First, assessment results from the study provide support for developing a non-
deficit perspective of EL students. Second, the EL students in this study demonstrated significant growth in their understanding of the minus sign. Finally, a set of nuanced approaches emerged from the analysis of Mr. Franco’s practice.

6.2.1 Using conceptual knowledge to develop a non-deficit perspective of ELs

An important contribution this study makes to the field of mathematics education is that it provides additional evidence to support a non-deficit perspective of EL students. The focus on how students understand math symbols revealed that EL students in this study brought a considerable amount of knowledge to their first Algebra class. What we know about EL student achievement in mathematics is by and large a result of reports and studies on state exams. However, these reports can be considered questionable given that in some states these exams must be taken in English. Regardless, the general picture held regarding EL students in secondary math classes is that they demonstrate very little knowledge of mathematics.

The EL students in this study demonstrated considerable understanding of the equal sign and some notions of variable at the start of Algebra 1. The EL students in this study demonstrated a relational understanding the equal sign, which is to say, as more than just a signal to calculate. They also demonstrated some important notions of the equal sign such as variable as letter ignored where equations include variables that do not represent much more than place-holders such as item 6 in the assessment. The EL students in this study did not see these variables as unknowns to be found but as specific unknowns. Additionally, some common misconceptions about variable such as assuming that different letters must equal different values were not found amongst the BIL group.
What EL students know is not being revealed through the assessments currently used. This is a more robust picture of what EL students know about math, adding important insights towards a non-deficit perspective of EL students.

6.2.2 Growth in understanding the minus sign

A second contribution this study makes to the field of math education research is the seemingly simple result that EL students, taught in Spanish using Spanish language materials demonstrated significant growth in their understanding of the minus sign. The students in the BIL group that could derive the meaning of the minus sign through its context grew by 50% and those that demonstrated a “flexible notion” of the minus sign grew by more than 60%. This is considerable and significant growth that was not observed in the SEI group. It’s important to understand the significance of the minus sign. All three symbols that framed the assessment in this study were selected because they are critical to developing an algebraic manner of thinking and all three symbols carry multiple meanings that permeate the entire secondary curriculum. The ideas of equality (or sameness), negativity (or opposite) and unknown quantities are foundational algebraic ideas; and more importantly significant to making the transition from arithmetic to algebraic thinking which is what students in beginning Algebra are doing.

Developing an understanding of the minus sign and all of it’s meanings is a challenge for any student. Specifically much time is spent trying to understand why the procedures make sense. For example, why is the product of two negatives always positive? Time must be spent discussing the ideas such as opposites. In contrast, EL students in classrooms that use English for instruction spend their class time translating terms such “subtract” and “integer.” The latter of which may or may not be a new term
for them. It may be that teaching the EL students in this study in Spanish allowed them the opportunity to enter into the discourse of opposites and therefore resulted in greater flexibility with the meaning of the minus sign such that it is more than just an operation between two numbers for them.

6.2.3 **Nuanced use of language as a learning tool**

The final contribution this study makes to the field of mathematics education research is a set of nuanced approaches to support pedagogy in mathematics classrooms. This study found that Mr. Franco consistently asks students to verify their objective and to explain their understandings in very specific ways, but, more importantly intentionally developed language specific to the their class and used Spanish as a tool to do so. In Mr. Franco’s class students are asked to define terms and to explain “in other words,” in their “own words,” “in mathematical terms” and in “terms [they] use in class.” Building on this engagement Mr. Franco also develops the idea of language specific to himself and his students. While debriefing during a guided reading session Mr. Franco points out that the class has language held in common in regards to the graph of a parabola: “Aunque lo estan leyendo en espanol, lo tenemos que traducir en terminos que nosotros entendemos” (Translation: “Eventhough you are reading in Spanish we need to translate in terms that we understand.”). The students respond in chorus: “Happy face… sad face.” Students understood what common language Mr. Franco referred to and that the answer wasn’t necessarily English but rather a common linguistic practice in this classroom. Having two lanaguage at their disposal created an environment where students
had multiple representations and words to articulate what they understand and engage in mathematical practice.

In Mr. Valdez’ class, where instruction was in Spanish but materials were in English, the use of Spanish was sometimes limited to translation. However, in Mr. Franco’s class students negotiated the language of the mathematics classroom using Spanish and even English as the language of this particular math class developed. Mr. Franco had students engage the language of mathematics. In neither of the SEI classes were students encouraged to use their primary language to actually develop an understanding of the concepts. In one case they are forced into a second language, no matter how under developed, and in the other they were guided away from using their primary language because materials were still in English. Thus, in both cases there was an underutilization of their linguistic assets to engage and understand the mathematics at a conceptual level. This study shows that the development of conceptual understanding was aided by using the entire portfolio of linguistic assets that EL students have available.
## APPENDIX

### Table 4.1.5: Misconceptions held by no (0 to 10%) students in the BIL group.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item: Topic</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign</td>
<td>3: Response of 41 indicates an arithmetic understanding of the equal sign;</td>
<td>1/16=6.25%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td></td>
<td>e.g. What number(s) can be put in the box ( \square ) to make the following number sentence true?</td>
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<td></td>
<td>( 23 + 18 = \square + 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>4: Incorrect response demonstrates an arithmetic notion of the equal sign;</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td></td>
<td>e.g. What numbers can go in the box ( \square ) to make the following number sentence true?</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>( 28 - \square = 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>3 &amp; 4: Incorrect response to both items means notion of the equal sign is obscured by misconception of variable;</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td>Variable</td>
<td>5 &amp; 7: holds the misconception that the same letter must equal the same number (as indicated by an incorrect answer to item 5 and a correct answer to item 7);</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
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<td></td>
<td>e.g.</td>
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<td></td>
<td><strong>Item 5:</strong> List all possible values of the variable ( m ). Explain.</td>
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<td></td>
<td>( 4 + m + m = m + 10 )</td>
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<tr>
<td></td>
<td><strong>Item 7:</strong> Two numbers add up to ten. This relationship is represented by: ( x + y = 10 ). What sets of numbers (( x ) &amp; ( y )) make this number sentence true?</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td>Symbol</td>
<td>Item: Topic</td>
<td>Pre</td>
<td>Post</td>
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<td>------------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>Variable</td>
<td>9: Letter as OBJECT; provided response of 6S=P;</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td></td>
<td>e.g. There are six times as many students as professors at the local university. Write an equation that represents this situation.</td>
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<tr>
<td>Variable</td>
<td>11: Response of “never” demonstrates student holds the misconception that different letters must equal different values (letter as SPECIFIC UNKNOWN)</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
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<tr>
<td></td>
<td>e.g. When is the following true; Always, Never, or Sometimes? Explain.</td>
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<td></td>
<td>L + M + N = L + P + N</td>
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<tr>
<td>Minus Sign</td>
<td>12: Response of 1 or -1 demonstrates understanding operations with negative numbers as finding a difference;</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
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<td></td>
<td>e.g. Solve.</td>
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<td></td>
<td>a) 3 + 4 = □</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>b) 3 – (-4) = □</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minus Sign</td>
<td>15: Operating from right to left; indicates use of arithmetic presuppositions;</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
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<td></td>
<td>e.g. Simplify the following expressions:</td>
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<tr>
<td></td>
<td>a.) 20 + 8 – 7n – 5n</td>
<td></td>
<td></td>
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<td></td>
<td>b.) 6 – 5a – 3 – 4a</td>
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<td></td>
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<tr>
<td></td>
<td>c.) 4 – 6n – 4n</td>
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<tr>
<td>Symbol</td>
<td>Item: Topic</td>
<td>Pre</td>
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<td>----------</td>
<td>----------------------------------------------------------------------------</td>
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<tr>
<td>Minus Sign</td>
<td><strong>15:</strong> Brackets Reasoning; indicates use of arithmetic presuppositions and mixed models.</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td>Minus Sign</td>
<td><strong>15:</strong> Signs Rule; indicates use of arithmetic presuppositions and mixed models.</td>
<td>0/16=0%</td>
<td>0/16=0%</td>
</tr>
<tr>
<td>Minus Sign</td>
<td><strong>15:</strong> Sign selecting; indicates use of arithmetic presuppositions and mixed models.</td>
<td>1/16=6.25%</td>
<td>0/16=0%</td>
</tr>
</tbody>
</table>
Table 4.2.4: Misconceptions held by no (0 to 10%) students in the SEI group.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item: Topic</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Sign</td>
<td>4: demonstrated an arithmetic notion of the equal sign;</td>
<td>1/22=4.55%</td>
<td>0/22=0%</td>
</tr>
<tr>
<td></td>
<td>e.g. What numbers can go in the box (□) to make the following number sentence true?</td>
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<tr>
<td></td>
<td>28 – □ = 12</td>
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<td></td>
</tr>
<tr>
<td>Equal Sign</td>
<td>3 &amp; 4: Incorrect response to both means notion of the equal sign is obscured by misconception of variable;</td>
<td>1/22=4.55%</td>
<td>0/22=0%</td>
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<tr>
<td></td>
<td>e.g.</td>
<td></td>
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<tr>
<td></td>
<td>Item 3: What number(s) can be put in the box (□) to make the following number sentence true?</td>
<td></td>
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<tr>
<td></td>
<td>23 + 18 = □ + 12</td>
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<td>Item 4: What numbers can go in the box (□) to make the following number sentence true?</td>
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<td>28 – □ = 12</td>
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<tr>
<td>Variable</td>
<td>5 &amp; 7: holds the misconception that the same letter must equal the same number (as indicated by an incorrect answer to item 5 and a correct answer to item 7)</td>
<td>1/22=4.55%</td>
<td>2/22=9.09%</td>
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<tr>
<td>Variable</td>
<td>9: Letter as OBJECT; provided response of 6S=P; e.g. There are six times as many students as professors at the local university. Write an equation that represents this situation.</td>
<td>1/22=4.54%</td>
<td>1/22=4.54%</td>
</tr>
<tr>
<td>Minus Sign</td>
<td>12: response of 12 or -12 indicates misinterpretation of parentheses obscures understanding of minus sign; e.g. Solve. a) 3 + 4 = b) 3 – ( - 4) =</td>
<td>1/22=4.55%</td>
<td>3/22=13.64%</td>
</tr>
<tr>
<td>Minus Sign</td>
<td>15: Operating from right to left; indicates use of arithmetic presuppositions. e.g. Simplify the following expressions: a.) 20 + 8 – 7n – 5n b.) 6 – 5a – 3 – 4a c.) 4 – 6n – 4n</td>
<td>0/22=0%</td>
<td>0/22=0%</td>
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BIBLIOGRAPHY


