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$\pi + n \rightarrow 2\pi + n$
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EFFECT OF THE $\pi-\pi$ RESONANCE IN THE REACTION
$\pi + N \rightarrow 2\pi + N$

Yongduk Kim

February 5, 1960
EFFECT OF THE $\pi-\pi$ RESONANCE IN THE REACTION

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Experiments are now being contemplated to measure $\pi-\pi$ cross sections by the method of Chew and Low.\(^1\) In particular, the two reactions

\[ \pi^+ + p \rightarrow \pi^+ + \pi^+ + n \quad (1) \]

and

\[ \pi^- + p \rightarrow \pi^+ + \pi^- + n \quad (2) \]

are suitable for studying the $\pi^+ - \pi^+$ and $\pi^+ - \pi^-$ cross sections, respectively. Also under consideration are the processes

\[ \pi^- + p \rightarrow \pi^- + \pi^0 + p, \quad (3) \]

\[ \pi^+ + p \rightarrow \pi^+ + \pi^0 + p, \quad (4) \]

which involve the $\pi^- - \pi^0$ and $\pi^+ - \pi^0$ cross sections.

The method is based on the conjecture that there exists a pole in the scattering amplitude at $(p_2 - p_1)^2 = -\mu^2$ where $p_2$ and $p_1$ are the four momenta of final and initial nucleons, respectively, and $\mu$ is the pion mass. The formula for the "pole part" of the cross section in Processes (1) and (2) above is
\[
\frac{d^2 \sigma_p}{dp^2 d\omega^2} = \frac{r^2(p^2/\mu^2)\omega}{m_{1L}^2 (p^2 + \mu^2)^2} \left( \frac{\omega^2}{4} - \mu^2 \right)^{1/2} \sigma_{\pi\pi}^\omega(\omega),
\]

where \(p^2_{2m}\) is the recoil kinetic energy of the neutron (in the laboratory system), \(q_{1L}\) the incident momentum of the pion, and \(\omega\) the total energy of the two outgoing pions (in their center-of-mass system); \(\sigma_{\pi\pi}\) refers to \(\sigma_{\pi^+\pi^-}\) for the Process (2) and to \(\sigma_{\pi^0\pi^0}\) for Process (1).

This formula represents the complete cross section only in the unphysical immediate neighborhood of the pole, but if \(\sigma_{\pi\pi}\) is sufficiently large the pole may be expected to dominate that part of the physical region where \(p^2 \sim \mu^2\). The formulas for Processes (3) and (4) are similar, except that the right-hand side is multiplied by 1/2 because the virtual target particle is now a neutral pion.

The purpose of this letter is

A. To estimate the above cross sections by use of the theoretical resonance prediction for \(\sigma_{\pi\pi}\) in the \(I = 1\) state made by Frazer and Fulco.\(^2\)

B. To consider the same processes according to the statistical model.

C. By comparing the two results, to find suitable regions in the phase space for the experiments to be performed.

So far there is no clear-cut theoretical prediction concerning the \(I = 0\) and \(I = 2\) states of the \(\pi\pi\) system except that no resonance is expected. On the assumption that the \(I = 1\) resonance is dominant we shall set the \(\pi\pi\) cross sections in these states equal to zero; the
cross sections appearing in the residues of the poles for Processes (1) to (4) then stand in the ratio: 0:1:1:1.

We begin by calculating the pole part of the cross section for Process (2) according to two possibilities for $\sigma^{\pi^+\pi^-}$ which are obtained by Frazer and Fulco, corresponding to positions of the resonance at $\sqrt{q_r^2} \left( \frac{\omega}{\mu} \right)^2 = 1 = 1.5$ and 2 (Fig. 1). For each of the two $\sqrt{q_r}$'s the calculation was carried out for three values of momentum for the incoming pion:

$q_{1L} = 1.75$ Bev/c, $1.4$ Bev/c, and $1.14$ Bev/c.

At these energies the phase space available for single pion production is shown in Fig. 2 in terms of the two dimensionless variables $\left( \frac{p}{\mu} \right)^2$ and $t \equiv \left( \frac{q}{\mu} \right)^2$. The resulting differential cross section (5) is plotted in Fig. 3a and b at fixed $\left( \frac{p}{\mu} \right)^2$. These curves clearly show the effect of the resonance. Then the differential cross section is integrated in $\left( \frac{p}{\mu} \right)^2$ from its minimum to 5, which is a plausible guess for the range in which the pole term may be dominant. (The next nearest singularity in $p^2$ is a branch point at $p^2 = -9\mu^2$ corresponding to the three-pion component of the nucleon cloud.) The results are shown in Figs. 4a and b. The total cross section for this part of the phase space is then obtained by integrating over $t$ and is shown in Fig. 5 for various upper limits on $t$. The corresponding cross section for Processes (1), (3) and (4) are obtained by multiplying the
result by 0, 1/2, and 1/2, respectively.

For the statistical model calculation we need the phase-space integral for two pions and one neutron of a given total energy, which can be shown to be

\[
\mathcal{J} = \int \delta(q_3^2 + \mu^2) \delta(q_4^2 + \mu^2) \delta(p_2^2 + m^2) \\
\delta(q_3 + q_4 + p_2 - p_1 - q_1) \, dp_2 \, dq_3 \, dq_4
\]

\[
= \frac{\pi^2}{m q_{1L}} \int \frac{\omega^{2/4} - \mu^2}{\omega} \, d\omega \, d\omega \, dp^2.
\]

Thus the differential cross section for single pion production in the statistical model is

\[
\frac{d^2 \sigma_s}{dt \, d(p/\mu)^2} = \frac{a}{4} \sqrt{\frac{1}{4} - \frac{1}{t}}
\]  

where \(a\) is a constant to be determined by normalizing to the experimental total cross section at a given initial energy for the reaction in question.

The experimental data\(^3\) for the cross section for Process (2) show a plateau at about 7 ± 2 mb in the range of \(q_{1L}\) between 1 and 1.5 Bev/c. Using the value 7 mb, we obtain for \(a\) the values 0.030 mb at \(q_{1L} = 1.75\) Bev/c, 0.054 mb at \(q_{1L} = 1.4\) Bev/c, and 0.019 mb at \(q_{1L} = 1.14\) Bev/c. The total cross sections for Processes (1), (3), and (4) are not reliably known at these energies but are presumed to be somewhat smaller.

Based on the above values for \(a\), the results for the statistical model are shown in Figs. 3(a,b), 4(a,b) and 5, where they may be compared with the pole part of the cross section. The corresponding comparisons
for Processes (3) and (4) will be similar. It may be noted from the
curve of Fig. 5 that the integrated pole cross section $\sigma_p$, is
1.5 mb at $q_L = 1.75$ Bev/c for $t_{\text{max}} = 26$ and $(\frac{p}{q})^2_{\text{max}} = 5$;
this is a factor of five greater than $\sigma_s = 0.28$ mb from the statistical
model. The enhancement is of course manifested most strongly for $t$
near the resonance energy, where the ratio of $\frac{d\sigma_p}{dt}$ to $\frac{d\sigma_s}{dt}$ is
nearly 10, as seen in Fig. 4. Finally, Fig. 3 shows the expected
effect that the smaller the value of $p^2/\mu^2$ the more the pole is favored.
Experiments with incident pions between 1 and 2 Bev should therefore
establish the existence of the resonance without much difficulty even
without extrapolating to $p^2 = -\mu^2$, if the position and width are
roughly as predicted by Frazer and Fulco.

As $q_L$ decreases to less than 1 Bev/c, the available phase
space for $(\frac{p}{q})^2 \leq 5$ dwindles rapidly (going to zero at $q_L \sim 0.7$ Bev/c),
while the total phase space becomes very large compared with the inter-
esting region for $q_L > 2$ Bev/c. Both these effects may be seen in
Fig. 1.

Recently Bonsignori and Selleri\textsuperscript{4} have considered the same problem
at 0.960 Bev/c incident pion kinetic energy. They, however, made no
attempt to exploit the strong $t$ dependence of the cross section when
a resonance occurs in the $\pi-\pi$ cross section.

I wish to thank Professor G. F. Chew for his suggestion of this
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REFERENCES


FIGURE CAPTIONS

Fig. 1. The resonance part of the $\pi^+\pi^-$ cross section according to Frazier and Fulco, with two possible positions for the resonance. The corresponding resonance contribution to the cross sections for $\pi^+\pi^0$ and $\pi^-\pi^0$ scattering is the same.

Fig. 2. Phase space for the reaction $\pi^+n \rightarrow \pi^+\pi^+ + n$ for three values of laboratory momentum. The region for small $t$ and small $(p_t^2)$ is also shown in a larger scale.

Fig. 3a. The differential cross section, Eq. (5), as a function of $t$ with fixed $(p_t^2)$ for $\sqrt{s} = 2.0$. The same differential cross section calculated from statistical model is also shown.

Fig. 3b. The differential cross section, Eq. (5), as a function of $t$ with fixed $(p_t^2)$ for $\sqrt{s} = 1.5$. The same differential cross section calculated from statistical model is also shown.

Fig. 4a. The differential cross sections, Eq. (5) and (6), integrated over $(p_t^2)$ from the minimum to 5 for $\sqrt{s} = 1.5$.

Fig. 4b. The differential cross sections, Eq. (5) and (6), integrated over $(p_t^2)$ from the minimum to 5 for $\sqrt{s} = 2.0$.

Fig. 5. The "pole part" of the total cross section $\sigma_p$ for $\pi^- + p \rightarrow \pi^+ + \pi^- + n$, compared with the statistical-model prediction $\sigma_s$ for various values of $t_{\text{max}}$ and $q_{\perp}$. The interval in $(p_t^2)$ is the same as in Fig. 4 (a,b).
Fig. 1
Fig. 2
Fig. 3a

$q_{IL} = 1.75 \text{ Bev/c} \\
(\eta_f = 1.5)$

\[ \frac{d^2 \sigma_p}{d\left(\frac{L^2}{L}\right)} \text{ (mb)} \]

\[ \begin{array}{c}
\left(\frac{p}{L}\right)^2 \\
1 \\
2 \\
3 \\
4 \\
5
\end{array} \]

Statistical

\[ t \]
Fig. 3b

\[ q_{ll} = 1.75 \text{ Bev/c} \]

\( (v_f = 2.0) \)
Fig. 4a
Fig. 4b
Fig. 5
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