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Study of $B^0 \rightarrow \pi^0 \pi^0$, $B^+ \rightarrow \pi^+ \pi^0$, and $B^+ \rightarrow K^+ \pi^0$ decays, and isospin analysis of $B \rightarrow \pi \pi$ decays

STUDY OF $B^0 \to \pi^0\pi^0\ldots$

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We present updated measurements of the branching fractions and CP asymmetries for $B^0 \to \pi^0 \pi^0$, $B^+ \to \pi^+ \pi^0$, and $B^- \to K^- \pi^0$. Based on a sample of $383 \times 10^6 \ Y(4S) \to B \bar{B}$ decays collected by the BABAR detector at the PEP-II asymmetric-energy $B$ factory at SLAC, we measure $B(B^0 \to \pi^0 \pi^0) = (1.47 \pm 0.25 \pm 0.12) \times 10^{-6}$, $B(B^+ \to \pi^+ \pi^0) = (5.02 \pm 0.46 \pm 0.29) \times 10^{-6}$, and $B(B^- \to K^- \pi^0) = (13.6 \pm 0.6 \pm 0.7) \times 10^{-6}$. We also measure the CP asymmetries $A_{\pi^0} = 0.09 \pm 0.35 \pm 0.05$, $A_{\pi^+ \pi^0} = 0.93 \pm 0.08 \pm 0.01$, and $A_{K^- \pi^0} = 0.030 \pm 0.039 \pm 0.010$. Finally, we present bounds on the Cabibbo-Kobayashi-Maskawa angle $\alpha$ using isospin relations.

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For candidates consisting of two EMC clusters or one cluster and a converted photon, the reconstructed \( \pi^0 \) mass is required to be between 110 and 160 MeV/c\(^2\), and the candidates are then kinematically fit with their mass constrained to the \( \pi^0 \) mass. We distinguish merged \( \pi^0 \) candidates from single photons and other neutral hadrons using the second transverse moment, \( S = \sum_i E_i \times (\Delta \alpha_i)^2 / E_i \), where \( E_i \) is the energy deposited in each CsI(Tl) crystal, and \( \Delta \alpha_i \) is the angle between the cluster centroid and the crystal. Because merged \( \pi^0 \)s are caused by two overlapping photon clusters, they have a larger \( S \) than solitary photons. We use a large sample of \( \pi^0 \)s from \( \tau^+ \to \rho^+ \nu \) decays to validate that our Monte Carlo simulation (MC) accurately simulates merged \( \pi^0 \)s and photon conversions, as well as our overall \( \pi^0 \) efficiency.

We use two kinematic variables to isolate \( B^0 \to \pi^0 \pi^0 \) and \( B^\pm \to h^\pm \pi^0 \) candidates from the background of \( e^+e^- \to q\bar{q} \) (\( q = u, d, s, c \)) continuum events. The first is the beam-energy-momentum mass \( m_{\text{ES}} = \sqrt{(s/2 + \mathbf{p}_i \cdot \mathbf{p}_B)^2 / E_i^2 - \mathbf{p}_B^2} \), where \( s \) is the total \( e^+e^- \) center-of-mass (CM) energy, \( (E_i, \mathbf{p}_i) \) is the four-momentum of the initial \( e^+e^- \) system, and \( \mathbf{p}_B \) is the \( B \)-candidate momentum, both measured in the laboratory frame. The second variable is \( \Delta E = E_B - \sqrt{s} / 2 \), where \( E_B \) is the \( B \) candidate energy in the CM frame. For \( B^\pm \to h^\pm \pi^0 \), we require \( m_{\text{ES}} > 5.22 \) GeV/c\(^2\) and \( -0.11 \) GeV < \( \Delta E < 0.15 \) GeV. We define the main signal region in the \( B^0 \to \pi^0 \pi^0 \) analysis as \( m_{\text{ES}} > 5.20 \) GeV/c\(^2\) and \( |\Delta E| < 0.20 \) GeV.

To further discriminate the signal from \( q\bar{q} \) backgrounds, we exploit the event topology variable \( \theta_S \): the angle in the CM frame between the sphericity axis of the \( B \) candidate’s decay products and that of the remaining neutral clusters and charged tracks in the rest of the event. Since the distribution of \( |\cos \theta_S| \) peaks at 1 for \( q\bar{q} \) events, we require \( |\cos \theta_S| < 0.8 \) (0.7) for events with a \( B^\pm \to h^\pm \pi^0 (B^0 \to \pi^0 \pi^0) \) candidate. To further improve background separation, we construct a Fisher discriminant \( \mathcal{F} \) from the sums \( \sum_i p_i \) and \( \sum_i p_i \cos^2 \theta_i \), where \( p_i \) is the CM momentum and \( \theta_i \) is the angle with respect to the thrust axis of the \( B \) candidate’s daughters, in the CM frame, of all tracks and clusters not used to reconstruct the \( B \) meson.

We use an extended, unbinned maximum likelihood (ML) fit to determine the number of signal events and the associated asymmetries. The probability density function (PDF) \( P_i(x_j; \alpha_i) \) for event \( j \) and signal or background hypothesis \( i \) is the product of PDFs for the variables \( x_j \), given the set of parameters \( \alpha_i \). The likelihood function \( \mathcal{L} \) is

\[
\mathcal{L} = \exp\left(-\sum_{i=1}^{M} n_i \prod_{j=1}^{N} P_i(x_j; \alpha_i)\right),
\]

where \( N \) is the number of events, \( n_i \) is the PDF coefficient for hypothesis \( i \), and \( M \) is the total number of signal and background hypotheses.

In the \( B^0 \to \pi^0 \pi^0 \) fit, the variables \( x_j \) are \( m_{\text{ES}}, \Delta E, \) and \( \mathcal{F} \). In addition to the signal and \( q\bar{q} \) background, we expect background events from the charmless decays \( B^\pm \to \rho^\pm \pi^0 \) and \( B^0 \to K^0_S \pi^0(K^0_S \to \pi^0 \pi^0) \) to contribute \( 61 \pm 7 \) events in the signal region, as determined from MC, so we include an additional component in the fit to account for this \( B\bar{B} \) background. For the \( B^0 \to \pi^0 \pi^0 \) signal and the \( B\bar{B} \) background, we observe a correlation coefficient between \( m_{\text{ES}} \) and \( \Delta E \) of \( 0.2 \), so a two-dimensional PDF, derived from MC simulation, is used to parametrize these distributions. The \( q\bar{q} \) background PDF is described by an ARGUS threshold function [12] in \( m_{\text{ES}} \) and a polynomial in \( \Delta E \). We divide the \( \mathcal{F} \) distribution from signal MC into ten equally populated bins, and use a parametric step function to describe the distribution for all of the signal and background hypotheses. We fix the relative size of the \( \mathcal{F} \) bins for the signal and \( B\bar{B} \) background to values taken from MC. These values are verified with a sample of fully reconstructed \( B \) meson decays. Continuum \( \mathcal{F} \) parameters are free in the fit.

In order to measure the time-integrated \( CP \) asymmetry \( C_{\varphi^0,\varphi^0} \), we use the remaining tracks and clusters in a multivariate technique [13] to determine the flavor (\( B^0 \) or \( B^0 \)) of the other \( B \) meson in the event (\( B_{\text{Bkg}} \)). Events are assigned to one of seven mutually exclusive categories \( k \) (including untagged events with no flavor information) based on the estimated mistag probability \( w_k \) and on the source of the tagging information. The PDF coefficient for \( B^0 \to \pi^0 \pi^0 \) is given by

\[
n_{\varphi^0,\varphi^0,k} = \frac{1}{f_k N_{\varphi^0,\varphi^0}} \left[ 1 - s_j (1 - 2 \chi_{\varphi^0}) (1 - 2 w_k) C_{\varphi^0,\varphi^0} \right],
\]

where \( N_{\varphi^0,\varphi^0} \) is the total number of \( B^0 \to \pi^0 \pi^0 \) decays.

### Table 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>( N_S )</th>
<th>( N_{\text{cont}}(10^3) )</th>
<th>( N_{\text{Bkg}} )</th>
<th>( \varepsilon ) (%)</th>
<th>( \mathcal{B}(10^{-6}) )</th>
<th>Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \to \pi^0 \pi^0 )</td>
<td>154 ± 27</td>
<td>17.67 ± 0.13</td>
<td>61 ± 7</td>
<td>27.3</td>
<td>1.47 ± 0.25 ± 0.12</td>
<td>-0.49 ± 0.35 ± 0.05</td>
</tr>
<tr>
<td>( B^0 \to \pi^0 \pi^0 )</td>
<td>627 ± 58</td>
<td>58.75 ± 0.24</td>
<td>69 ± 3</td>
<td>32.5</td>
<td>5.02 ± 0.46 ± 0.29</td>
<td>0.03 ± 0.08 ± 0.01</td>
</tr>
<tr>
<td>( B^0 \to K^0_S \pi^0 )</td>
<td>1364 ± 57</td>
<td>25.07 ± 0.17</td>
<td>9 ± 2</td>
<td>26.6</td>
<td>13.6 ± 0.6 ± 0.7</td>
<td>0.030 ± 0.039 ± 0.01</td>
</tr>
</tbody>
</table>
\( \chi_d = 0.188 \pm 0.004 \) [14] is the time-integrated mixing probability, and \( s_j = +1(-1) \) when the \( B_{\text{tag}} \) is a \( B^0 (B^0) \). The fraction of events in each category, \( f_k \), and the mistag rate are determined from a large sample of \( B^0 \to D^{(*)}(n\pi)\pi \) decays.

For the \( B^+ \to h^+ \pi^0 \) fit, along with \( m_{\text{ES}}, \Delta E, \) and \( \mathcal{F} \), we include the Cherenkov angle \( \theta_C \) to measure the \( B^+ \to \pi^+ \pi^0 \) and \( B^+ \to K^+ \pi^0 \) yields and asymmetries simultaneously. The difference between the expected and measured Cherenkov angle, divided by the uncertainty, is described by two Gaussian distributions. The values for \( m_{\text{ES}} \) and \( \Delta E \) are calculated assuming the track is a pion, so a \( B^+ \to K^+ \pi^0 \) event will have \( \Delta E \) shifted by a value dependent on the track momentum, typically \(-45 \text{ MeV} \). For the signal, the \( m_{\text{ES}} \) and \( \Delta E \) distributions are modeled as Gaussian functions with low-side power-law tails. The means of these distributions and the \( \Delta E \) width are determined in the fit, while the \( m_{\text{ES}} \) width is determined by MC simulation. We expect \( 69 \pm 3 \) background events in the \( B^+ \to \pi^+ \pi^0 \) signal region from other \( B \) meson decays, mainly from the same \( B \) decays as in the \( B^0 \to \pi^0 \pi^0 \) case. For the \( B^+ \to K^- \pi^0 \) signal region we expect \( 9 \pm 2 \) events from \( B \to X_s \gamma \) and \( B^0 \to \rho^+ K^- \). The PDFs for the \( B \bar{B} \) backgrounds, the \( q\bar{q} \) background, and the signal \( \mathcal{F} \) are all treated the same as in the \( B^0 \to \pi^0 \pi^0 \) case. The PDF coefficient for \( B^0 \to h^+ \pi^0 \) is given by \( n_i = \frac{1}{2} N_i (1 - q_j \mathcal{A}_j) \), where \( A_j \) is the charge asymmetry, and \( q_j = \pm 1 \) is the charge of the \( B \) candidate.

The results from the \( B^0 \to \pi^0 \pi^0 \) and \( B^+ \to h^+ \pi^0 \) ML fits are summarized in Table I. In a total of 17 881 events, we find \( 154 \pm 27 \) \( B^0 \to \pi^0 \pi^0 \) decays and an asymmetry \( C_{\pi^0 \pi^0} = -0.49 \pm 0.35 \). For the \( B^+ \to h^+ \pi^0 \) fit, we find \( 627 \pm 58 \) \( B^+ \to \pi^+ \pi^0 \) and \( 1364 \pm 57 \) \( B^+ \to K^+ \pi^0 \) events in a total of 85 895 events. All of the correlations among the signal variables are less than 5%. In Fig. 1 we use the event weighting and background subtraction method described in Ref. [15] to show signal and background distributions for \( B^0 \to \pi^0 \pi^0 \) events. Signal and background distributions for \( B^+ \to h^+ \pi^0 \) events are shown in Fig. 2 using the same method.

In order to account for a small bias in the \( B^+ \to h^+ \pi^0 \) asymmetries arising from the difference in the \( \pi^+ \) and \( \pi^- \) reconstruction efficiencies and the \( K^+ \) and \( K^- \) hadronic interaction cross sections in the BABAR detector, the

**FIG. 1** (color online). Distributions made with the event weighting and background subtraction method described in Ref. [15] and PDF projections for the likelihood fit variables in the \( B^0 \to \pi^0 \pi^0 \) fit. Shown are \( m_{\text{ES}} \) [(a),(b)], \( \Delta E \) [(c),(d)], and \( \mathcal{F} \) [(e),(f)] for signal [(a),(c),(e)] and continuum background [(b),(d),(f)].
The $B^\pm \rightarrow \pi^\mp \pi^0$ asymmetry is corrected by $+0.005 \pm 0.004$ and the $B^\pm \rightarrow K^\pm \pi^0$ asymmetry is corrected by $+0.008 \pm 0.008$. We determine the $\pi^\pm \pi^0$ bias from a study of $\tau^\pm \rightarrow \rho^\pm \nu$ decays and verify it using the continuum background in data. For the $B^\pm \rightarrow K^\pm \pi^0$ charge asymmetry bias, we use the continuum background and combine the results of the $\pi^\pm \pi^0$ asymmetry study and the $K^\pm \pi^0$ asymmetry study in Ref. [6]. After the bias correction we find $\mathcal{A}_{\pi^\pm \pi^0} = 0.03 \pm 0.08$ and $\mathcal{A}_{K^\pm \pi^0} = 0.030 \pm 0.039$.

The systematic uncertainties on the branching fraction and asymmetry measurements are summarized in Tables II and III, respectively. The largest systematic errors for the $B^0 \rightarrow \pi^0 \pi^0$ and $B^\pm \rightarrow h^\pm \pi^0$ branching fractions are from uncertainties in the $\pi^0$ reconstruction efficiency, signal selection efficiencies, $\mathcal{F}$ parameters, and $B\bar{B}$ background yields. We evaluate these systematic errors using either control samples of $\pi^0$ mesons from $\tau^\pm \rightarrow \rho^\pm \nu$ decays and fully reconstructed $B$ meson decays from data or by varying fixed parameters and refitting. We simulate radiative effects using the PHOTOS simulation package [16] and assign a systematic error equal to the difference between PHOTOS and the scalar QED calculation in Ref. [17]. For the $B^\pm \rightarrow h^\pm \pi^0$ analysis, we also include as a systematic a small ($<2\%$) fit bias due to correlations among fit varia-

**TABLE II. Systematic errors on the branching fractions for $B^0 \rightarrow \pi^0 \pi^0$, $B^\pm \rightarrow K^\pm \pi^0$, and $B^0 \rightarrow \pi^0 \pi^0$.**

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta B(\pi^\pm \pi^0)$</th>
<th>$\Delta B(K^\pm \pi^0)$</th>
<th>$\Delta B(\pi^0 \pi^0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$ efficiency</td>
<td>3.0%</td>
<td>3.0%</td>
<td>6.0%</td>
</tr>
<tr>
<td>$\mathcal{m}_{ES}$ and $\Delta E$ PDF</td>
<td>1.7%</td>
<td>1.7%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Selection efficiency</td>
<td>2.8%</td>
<td>3.0%</td>
<td>2.7%</td>
</tr>
<tr>
<td>$\mathcal{F}$ PDF</td>
<td>2.5%</td>
<td>0.7%</td>
<td>1.7%</td>
</tr>
<tr>
<td>$B\bar{B}$ backgrounds</td>
<td>0.2%</td>
<td>&lt;0.1%</td>
<td>2.1%</td>
</tr>
<tr>
<td>PHOTOS</td>
<td>1.9%</td>
<td>1.1%</td>
<td>...</td>
</tr>
<tr>
<td>Fit bias</td>
<td>1.7%</td>
<td>1.2%</td>
<td>...</td>
</tr>
<tr>
<td>Luminosity</td>
<td>1.1%</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.5%</td>
<td>0.5%</td>
<td>...</td>
</tr>
<tr>
<td>Total</td>
<td>5.8%</td>
<td>5.0%</td>
<td>8.2%</td>
</tr>
</tbody>
</table>
TABLE III. A summary of the systematic errors on the asymmetries $A_{\pi^-\pi^0}$, $A_{K^-\pi^0}$, and $C_{\pi^+\pi^-}$. All values are expressed in units of $10^{-2}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta(A_{\pi^-\pi^0})$</th>
<th>$\Delta(A_{K^-\pi^0})$</th>
<th>$\Delta(C_{\pi^+\pi^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B\bar{B}$ backgrounds</td>
<td>0.2</td>
<td>&lt;0.1</td>
<td>3.4</td>
</tr>
<tr>
<td>Tagging</td>
<td>...</td>
<td>...</td>
<td>2.5</td>
</tr>
<tr>
<td>Tag-side interference</td>
<td>...</td>
<td>...</td>
<td>1.6</td>
</tr>
<tr>
<td>PDF parameters</td>
<td>0.8</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>Detector asymmetry</td>
<td>0.4</td>
<td>0.8</td>
<td>...</td>
</tr>
<tr>
<td>Measured $\chi_d$ error</td>
<td>...</td>
<td>...</td>
<td>0.8</td>
</tr>
<tr>
<td>Total</td>
<td>0.9</td>
<td>1.0</td>
<td>4.7</td>
</tr>
</tbody>
</table>

The largest systematic uncertainties in the measurement of $C_{\pi^+\pi^-}$ are from the uncertainty on the $B$ background $CP$ content, tag-side interference, and the tagging fractions and asymmetry of $B_{tag}$. The major contributions to the systematic error on $A_{k^-\pi^0}$ are from the detector charge asymmetry and the $\Delta E$ and $f$ PDF parametrization.

We extract information on $\Delta\alpha \equiv \alpha_{\text{eff}} - \alpha$ and $\alpha$ using isospin relations [3] that relate the decay amplitudes of $B \rightarrow \pi \pi$ decays and measurements of the branching fraction and time-dependent $CP$ asymmetries in the decay $B^0 \rightarrow \pi^+ \pi^-$ from BABAR [6,7]. For each of the six observable quantities required to calculate $\alpha$ [$B(B^0 \rightarrow \pi^+ \pi^-)$, $B(B^+ \rightarrow \pi^+ \pi^0)$, $B(B^0 \rightarrow \pi^0 \pi^0)$, $S_{\pi^-\pi^+}$, $C_{\pi^+\pi^-}$, and $C_{\pi^+\pi^-}$], we generate an ensemble of simulated experiments with uncorrelated Gaussian distributions where the width on each distribution is the sum in quadrature of the statistical and systematic errors of that measurement. Sets of generated experiments that result in an unphysical asymmetry or violate isospin are removed from the sample. Using the resulting distributions for $\Delta\alpha$ and $\alpha$, we calculate a confidence level (C.L.) for each solution and plot the maximum value of 1 minus C.L. of the various solutions in Fig. 3. One can further constrain $\alpha$ by using the fact that the penguin amplitude contribution to $B \rightarrow \pi \pi$ decays must be very large if $\alpha$ is near 0 or $\pi$. We obtain a bound on the magnitude of the penguin amplitude from the branching fraction of the penguin-dominated decay $B_s \rightarrow K^+ K^-$ [18] by making the conservative assumption of $SU(3)$ breaking at less than $\sim 100\%$ [19]. In Fig. 3 we also show bounds on $\alpha$ when the size of the penguin amplitude is constrained by this assumption.

In summary, we measure the branching fractions and $CP$ asymmetries in $B^0 \rightarrow \pi^0 \pi^0$, $B^+ \rightarrow \pi^+ \pi^0$, and $B^\pm \rightarrow K^\pm \pi^0$ decays reconstructed from a sample of approximately $383 \times 10^6 B\bar{B}$ pairs. While all of these measurements represent significant improvements of the previous BABAR measurements, the error on $C_{\pi^+\pi^-}$ shows the largest improvement. All results are consistent with previously published results from BABAR [4] and the Belle experiment [5], and supersede the previous BABAR results. For the $B \rightarrow \pi \pi$ decays, we find $B(B^0 \rightarrow \pi^0 \pi^0) = (1.47 \pm 0.25 \pm 0.12) \times 10^{-6}$, $B(B^\pm \rightarrow \pi^\pm \pi^0) = (5.02 \pm 0.46 \pm 0.29) \times 10^{-6}$, $C_{\pi^+\pi^-} = -0.49 \pm 0.35 \pm 0.05$, and $A_{\pi^-\pi^0} = 0.03 \pm 0.08 \pm 0.01$. We constrain $\Delta\alpha$ to be less than 39° and exclude the range [25°, 66°] in $\alpha$ at 90% confidence level. If we consider only the preferred solution [20], we find $\alpha = (96.6^{+10}_{-6})^\circ$. For the $B^\pm \rightarrow K^\pm \pi^0$ decay, we find $B(B^\pm \rightarrow K^\pm \pi^0) = (13.6 \pm 0.6 \pm 0.7) \times 10^{-6}$ and $A_{K^\pm \pi^0} = 0.030 \pm 0.039 \pm 0.010$. The difference between $A_{K^\pm \pi^0}$ and $A_{K^\pm \pi^0} = -0.107 \pm 0.019$ [6] indicates that the effect of color-suppressed tree and electroweak-penguin amplitudes are significant.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (U.S.A.), Nserc (Canada), CEA and CNRS-IN2P3 (France), BMBF and
STUDY OF $B^0 \rightarrow \pi^0 \pi^0$, \ldots

DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), MEC (Spain), and STFC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation.

[2] Unless specifically stated, conjugate decay modes are assumed throughout this paper.
[12] The function is $f(x) \approx x \sqrt{1 - x^2} \exp[-\zeta(1 - x^2)]$, where $x = m_{ES}/m_0$, $m_0$ is the $m_{ES}$ endpoint, and $\zeta$ the shape parameter.