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SOCIAL STARS: MODELING THE INTERACTIVE LIVES OF STARS IN DENSE CLUSTERS AND BINARY SYSTEMS IN THE ERA OF TIME DOMAIN ASTRONOMY

A dissertation submitted in partial satisfaction of the requirements for the degree of

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by

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Abstract

Social Stars: Modeling the Interactive Lives of Stars in Dense Clusters and Binary Systems in the Era of Time Domain Astronomy

by

Morgan Elowe MacLeod

This thesis uses computational modeling to study phases of dramatic interaction that intersperse stellar lifetimes. In galactic centers stars trace dangerously wandering orbits dictated by the combined gravitational force of a central, supermassive black hole and all of the surrounding stars. In binary systems, stars’ evolution – which causes their radii to increase substantially – can bring initially non-interacting systems into contact. Moments of strong stellar interaction transform stars, their subsequent evolution, and the stellar environments they inhabit.

In tidal disruption events, a star is partially or completely destroyed as tidal forces from a supermassive black hole overwhelm the star’s self gravity. A portion of the stellar debris falls back to the black hole powering a luminous flare as it accretes. This thesis studies the relative event rates and properties of tidal disruption events for stars across the stellar evolutionary spectrum. Tidal disruptions of giant stars occur with high specific frequency; these objects’ extended envelopes make them vulnerable to disruption. More-compact white dwarf stars are tidally disrupted relatively rarely. Their transients are also of very different duration and luminosity. Giant star disruptions power accretion flares with timescales of tens to hundreds of years; white dwarf disruption flares take hours to days. White dwarf tidal interactions can additionally trigger thermonuclear burning and lead to transients with signatures similar to type I supernovae.

In binary star systems, a phase of hydrodynamic interaction called a common envelope episode occurs when one star evolves to swallow its companion. Dragged by
the surrounding gas, the companion star spirals through the envelope to tighter orbits. This thesis studies accretion and flow morphologies during this phase. Density gradients across the gravitationally-focussed material lead to a strong angular momentum barrier to accretion during common envelope. Typical accretion efficiencies are in the range of 1 percent the Hoyle-Lyttleton accretion rate. This implies that compact objects embedded in common envelopes do not grow significantly during this phase, increasing their mass by at most a few percent. This thesis models the properties of a recent stellar-merger powered transient to derive constraints on this long-uncertain phase of binary star evolution.
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Chapter 1

Introduction

Stars spend most of their billion-year lifetimes steadily converting hydrogen to helium in their cores. A single star like the sun, in a galactic environment where the typical stellar spacing is 100 million times a star’s radius, almost never collides or directly interacts with other stars. Other stars inhabit very different systems, and these stars – particularly stars in dense cluster cores (where surrounding stars are packed a hundred times closer in every direction) and binary pairs (in which two stars orbit about their shared center of mass) – live very different lives.

Stars in binaries or clusters are subject to moments of dramatic interaction which transform their evolution, the stellar systems they inhabit, and can reveal properties invisible in their slowly-evolving rest states. As observational astronomy transforms from a science of mapping to one of cinematography, it is revealing a Universe more interactive, dynamic, and time-varying than we could have expected. This thesis focuses on understanding these interactive moments in stellar lifetimes through computational modeling.

The physical processes that govern the interactive and transient Universe are disparate and intertwined. When a transient flare from a stellar interaction briefly illuminates the night sky, the underlying physics lies at the intersection of how stellar
gas moves when stars interact, and how heat and light escape from the gas to create the electromagnetic radiation we can observe. Therefore, methodologically, this research synthesizes the tools of modern computational astrophysics – stellar evolution, stellar dynamics, hydrodynamics, and radiation transfer.

This thesis will focus on two particular categories of stellar interactions. The first occurs in galactic center environments which harbor supermassive black holes of thousands to billions of solar masses. These massive black holes tidally shred the occasional star whose orbit wanders too close. The second interaction channel in which this thesis focuses is a phase of dynamical interaction in binary star systems, known as a common envelope episode, which leads to either a complete merger of the two stars or a transformed and tightened binary pair.

1.1 Stars Around (and Eaten By) Massive Black Holes

Most massive black holes (MBHs), like Sgr A* at the center of the Milky Way, are quiescent, only accreting material from their surroundings at a very low rate. Although its mass is $\sim 4 \times 10^6 M_\odot$, Sgr A* has a bolometric luminosity of only $\sim 10^2 L_\odot$ – less than many giant stars! Yet, MBHs inhabit galactic galactic center environments of extreme stellar density. When the occasional star plunges too close to the black hole and is disrupted by tides, it fuels an extremely luminous accretion flare (Rees, 1988). Surveys are beginning to discover the optical and x-ray counterparts of these events at rates of tens per year (Komossa, 2015).

Disrupted stars include main sequence stars, but also giant stars and white dwarfs. Chapter 2 combines stellar evolutionary models of giant stars’ structures with hydrodynamic studies of the disruptive passages and dynamical modeling of the nuclear cluster environment of nearby galactic centers (MacLeod et al., 2012). This hydrodynamic modeling suggests that the envelopes of giant stars require tens of pericenter
passages to be completely removed. This prompted follow up work, presented in Chapter 3, which models the sequences of flaring episodes that would be generated by these multi-passage events using stellar evolutionary calculations of the perturbed star’s structure (MacLeod et al., 2013). These predictions have become more intriguing in light of recently-claimed repeated-flaring tidal disruption events, like that in IC 3599 (Campana et al., 2015).

White dwarfs are disrupted outside the horizon only for MBHs with mass \( \lesssim 10^6 M_\odot \), therefore these disruptions trace highly uncertain populations of relatively low to intermediate mass MBHs. The fallback of tidal debris well above the MBH’s Eddington-limit mass accretion rate is likely to launch relativistic jets following a white dwarf tidal disruption (e.g. De Colle et al., 2012; MacLeod et al., 2014). The beamed emission from these jets significantly outshines those of more common main-sequence star disruptions, so much so that white dwarf tidal disruptions should be the most frequently observed events arising from low-mass MBHs. Drawing on these ideas, Chapter 4, models the detection likelihood of these jetted transients by instruments like Swift’s BAT, and found that their properties bear strong similarities to the emergent class of ultra-long gamma ray bursts (Levan et al., 2014; Levan, 2015). Deep-passing helium or carbon/oxygen white dwarfs can be ignited by tidal compression, and Chapter 5 uses a radiative transfer synthesis of the tidal debris to predict the properties of these supernova-like thermonuclear optical transients (MacLeod et al., 2016a).

These studies share a theme of using transients from tidal disruption to trace out the stellar populations that surround MBHs of varying mass and location in the Universe.
1.2 Star-Eat-Star Universe: Common Envelope Episodes

A common envelope episode occurs when one star in a binary pair evolves to engulf its companion. Within their shared gaseous envelope, the two stellar cores are dragged inward, eventually ejecting the envelope and leaving behind a binary system transformed by processes of mass accretion and orbital tightening.

The common envelope process is not rare; the majority of stars are in binary systems that will interact. Tight binaries of black holes and neutron stars which can merge under the influence of gravitational radiation can only be created through a common envelope phase. These binaries are highly anticipated as the primary sources for ongoing laser interferometer gravitational wave detection experiments like LIGO. Without a detailed understanding of common envelope, it is impossible to accurately predict or interpret this population of merging compact binaries.

Even so, a detailed understanding of common envelope has proved elusive for the forty years since it was first suggested (Paczynski, 1976), primarily because the interaction is governed by intertwined physical processes on an enormous range of scales (e.g. Ivanova et al., 2013b). To illustrate the challenge of modeling common envelope, for a neutron star embedded within the envelope of its giant-star companion, the relevant length scales range from the $10^6$ cm radius of the neutron star to the $10^{13}$ cm radius of the envelope. The governing timescales range from the stellar evolutionary timescale that establishes the initial conditions of a binary interaction to the dynamical timescale of flow near an embedded object’s surface. These challenges imply that no single calculation can capture the range of physical processes that determine the outcome of a common envelope interaction.

Instead, this thesis makes use of simplified calculations which address individual aspects of the common envelope problem. The morphology of flows inside a common envelope remains under debate, and likely depends on binary mass ratio along
with common envelope structure (e.g. Ricker and Taam, 2008, 2012; Passy et al., 2012a; MacLeod and Ramirez-Ruiz, 2015a). These flow morphologies have significance in determining the rate of drag (and orbital inspiral) induced by the surrounding envelope as well as any possible accretion onto the embedded object.

Chapter 7 examines stellar structures at the onset of common envelope. It shows that the density gradient that spans the material focussed toward an embedded star is a principle property of common envelope structure; the typical scale is of order one density scale height in the gravitationally focussed material. The imposition of this density gradient breaks the symmetry that defines Hoyle-Lyttleton accretion from a homogeneous medium (Hoyle and Lyttleton, 1939). The flow morphology is distorted, net rotation is imposed, and angular momentum serves as a barrier in preventing material from accreting. Regardless of whether it is able to accrete, material is still gravitationally focused and dissipates its momentum, leaving the rate of drag on the surrounding fluid relatively unchanged (MacLeod and Ramirez-Ruiz, 2015a).

Chapter 8 applies these results on relative rates of drag and accretion to the assembly of double neutron star binaries. These binaries exhibit well-defined and constraining properties. Their masses are typically well-known through the measurement of post-Keplerian orbital parameters, and are found to be both low, and have a narrow dispersion $M_{\text{NS}} \approx 1.35 \pm 0.05 M_\odot$ (Özel et al., 2012). This suggests that these objects must have accreted little during the common envelope phase that lead to their assembly. Accretion flows onto neutron stars during common envelope are thought to cool by neutrino accretion and have been predicted to lead to $\gtrsim 1 M_\odot$ of mass gain (Chevalier, 1993). Our new understanding of the accretion limits imposed by flow morphology in common envelope instead suggests that neutron stars gain at most a few percent their own mass during common envelope (MacLeod and Ramirez-Ruiz, 2015b). This explains a long-standing empirical understanding that the observed double neutron star binaries require low mass accretion rates during common envelope.
Common envelope interactions occur frequently enough that they will be increasingly revealed in the optical and infrared transient night sky (Kochanek et al., 2014; Kasliwal et al., 2014). As an example, a promising source has been revealed in the past year, known as M31 LRN 2015 (Williams et al., 2015; Kurtenkov et al., 2015). This “luminous red nova” transient is thought to be powered by a stellar merger by association with similar transients like V838 Mon (which was studied in detail by the Hubble Space Telescope) and V1309 Sco (which was observed as an eclipsing binary before its outburst and a single object afterward) (Tylenda et al., 2011). The M31 source is particularly remarkable because Williams et al. (2015) discovered the pre-outburst system in Hubble Space Telescope imaging of that region of M31. With the distance and progenitor magnitude and color known, this system offers a unique avenue to link the transient outburst to the pre-merger binary properties. We pursue these details with modeling of the outburst light curve, the hydrodynamics of mass ejection, and the pathway to merger for the system that drove the transient outburst.

1.3 A Computational Modeling Ecosystem

The work presented in this thesis has shown that, in modeling the interactive lives of stars, stellar dynamics strongly intertwines with stellar structure and the hydrodynamics of the encounters themselves to determine transient events’ observable properties. In the era of time-domain survey astronomy, an ecosystem of computational modeling tools is needed to model from the initial conditions of an encounter to the emergent radiation. For stellar evolution, this work uses the highly-flexible, open-source Modules for Experiments in Stellar Astrophysics (MESA) implementations of single and binary stellar evolution (Paxton et al., 2011). For the stellar dynamics of large $N$ systems like star clusters, this thesis employs the GPU-accelerated implementation of NBODY6 (Aarseth, 2003; Nitadori and Aarseth, 2012). Finally, the work
presented here extensively uses the FLASH code, an adaptive mesh, finite volume code for compressible flows in astrophysical hydrodynamics (Fryxell et al., 2000). To compute the properties of emergent radiation in astronomical transients, including synthetic lightcurves and spectra, Chapter 5 uses the radiation transport code SEDONA (Kasen et al., 2006).

1.4 Outline of This Thesis

Chapters 2 through 5 describe the ways that a variety of stellar types can meet their demise under the influence of strong tides from a massive black hole. A strong theme that runs through the detailed individual studies presented in these chapters is the effort to understand the full scope of stellar digestion by massive black holes. What is the full range of stellar types destroyed by tidal interaction with massive black holes? What are the properties of the transient flares they produce? And what are the stellar-dynamical processes that regulate these interactions and determine their relative likelihoods?

Chapter 6 delves further into the properties of stellar systems surround massive black holes by reproducing them in gravitational N-body simulations. This chapter aims to characterize the stellar-dynamical histories of stars which find their way into close partnerships with black holes in dense stellar clusters. As the previous chapters reveal, these partnerships often don’t end well. This chapter looks statistically at the properties of disrupted stars and gravitational-wave inspiral events.

Chapters 7 through 9 focus on stars in binary systems. Chapters 7 and 8 model the transformation of stars during common envelope phases using hydrodynamic simulations, and focus particularly on mass accumulation by objects embedded inside a common envelope. These chapters aim to answer the question of whether common envelope phases change binary’s orbits alone or also the constituent objects themselves.
Chapter 9 traces a stunning episode of common envelope “caught in the act” during a recent transient from the Andromeda galaxy, M31.
Chapter 2

The Tidal Disruption of Giant Stars and Their Contribution to the Flaring Supermassive Black Hole Population

2.1 Chapter Abstract

Sun-like stars are thought to be regularly disrupted by supermassive black holes (SMBHs) within galactic nuclei. Yet, as stars evolve off the main sequence their vulnerability to tidal disruption increases drastically as they develop a bifurcated structure consisting of a dense core and a tenuous envelope. Here we present the first hydrodynamic simulations of the tidal disruption of giant stars and show that the core has a substantial influence on the star’s ability to survive the encounter. Stars with more massive cores retain large fractions of their envelope mass, even in deep encounters. Accretion flares resulting from the disruption of giant stars should last for tens to hundreds of years. Their characteristic signature in transient searches would not be the $t^{-5/3}$ decay typically associated with tidal disruption events, but a correlated rise over many orders of magnitude in brightness on months to years timescales. We calculate the relative disruption rates of stars of varying evolutionary stages in typical galactic
centers, then use our results to produce Monte Carlo realizations of the expected flaring event populations. We find that the demographics of tidal disruption flares are strongly dependent on both stellar and black hole mass, especially near the limiting SMBH mass scale of \( \sim 10^8 M_\odot \). At this black hole mass, we predict a sharp transition in the SMBH flaring diet beyond which all observable disruptions arise from evolved stars, accompanied by a dramatic cutoff in the overall tidal disruption flaring rate. Black holes less massive than this limiting mass scale will show observable flares from both main sequence and evolved stars, with giants contributing up to 10% of the event rate. The relative fractions of stars disrupted at different evolutionary states can constrain the properties and distributions of stars in galactic nuclei other than our own.

### 2.2 Introduction

Quasars are rapidly growing black holes lit up by the gas they accrete. They are the most dramatic manifestation of the more general phenomenon of active galactic nuclei (AGN) and they are among the most energetic objects in the universe. Several billion years after the Big Bang, the universe went through a quasar era when highly-luminous AGN were a standard feature of most massive galaxies (Kormendy and Richstone, 1995; Richstone et al., 1998). Since that time, AGN have been dying out and the only activity that still occurs in many nearby galactic nuclei is weak (Schawinski et al., 2010). All that should remain in the centers of local galaxies are the remnants of quasar-era exponential growth: quiescent SMBHs, now dim and starved of fuel (Ho, 2008). It is, therefore, not surprising that definitive conclusions about the presence of local SMBHs are typically drawn from very nearby galaxies with little to no AGN activity (Gebhardt et al., 2000; Ferrarese and Merritt, 2000). The centers of these galaxies are well resolved, revealing the region where the black hole dominates the stellar dynamics.

The question then arises of whether the postulated presence of SMBHs lurking
in the centers of most galaxies is consistent with the apparent quiescence of their nuclei. Quiescent black holes are, in fact, nearly black; they may only be lit up by the luminance of accreting matter. We do not directly know how much gas there is near most SMBHs, and there is no a priori reason why nuclear regions should be swept completely clean of gas. The distribution of stars in dense clusters that surround SMBHs, on the other hand, is much better constrained. These densely packed stars trace complicated and wandering orbits under the combined influence of all the other stars and the black hole itself. If a star wanders too close to the black hole it is violently ripped apart by the hole’s tidal field (e.g., Hills, 1975; Frank, 1978; Rees, 1988). About half of the debris of tidal disruption eventually falls back and accretes onto the SMBH. This accretion powers a flare which is a definitive sign of the presence of an otherwise quiescent SMBH and a powerful diagnostic of its properties (Rees, 1988). Tidal disruption events are expected to be relatively rare, on the order of one per $\sim 10^4$ years per galaxy (e.g., Magorrian and Tremaine, 1999; Wang and Merritt, 2004). Depending primarily on the structural properties of the disrupted star, an ultra-luminous transient signal could persist steadily for months to at most tens of years; thereafter the flare would rapidly fade. In a given galaxy, this luminous flaring activity would then have a short duty cycle. Quiescent SMBHs should greatly outnumber active ones. Observational constraints would not, therefore, be stringent until we had observed enough candidates to constitute a proper ensemble average. However, the long decay tails of these flares may account for an appreciable fraction of the total low-luminosity AGN activity in the local universe (Milosavljevic et al., 2006).

The critical pericenter distance for tidal disruption is the tidal radius,

$$r_t \equiv \left( \frac{M_{bh}}{M_*} \right)^{1/3} R_*, \quad (2.1)$$

where $M_*$ and $R_*$ are the stellar mass and radius, and $M_{bh}$ is the black hole mass.
The tidal radius is larger than the black hole's horizon, $r_s = 2GM_{bh}/c^2$, for solar type stars as long as the black hole is less massive than about $10^8$ solar masses. In encounters with more massive black holes, solar type stars may pass the horizon undisrupted and are effectively swallowed whole. Such events would leave little electromagnetic signature (although a portion of the gravitational wave signature would remain, Kobayashi et al., 2004). However, for a given stellar and black hole mass, evolved stars have larger tidal radii than main sequence (MS) stars and are therefore more vulnerable to tidal disruption. Furthermore, giant branch stars are the only stars that can produce observable tidal disruption flares in encounters with the most massive black holes $\gtrsim 10^8M_\odot$.

Motivated by these facts, we examine the importance of stellar evolution in the context of tidal disruption. A sun-like star spends the majority of its lifetime on the MS, $\tau_{ms} \sim 10^{10}$ years, followed by a relatively brief period of post-MS (giant branch) evolution, $\tau_g \sim 10^8$ years, once its central supply of hydrogen fuel is exhausted. As nuclear reactions slow, the stellar core loses pressure support, and, in approximately a thermal diffusion time, the star ascends the giant branch as its outer layers expand in response to the core's collapse. Giant-branch stars are much less common than MS stars in a typical stellar population due to the ratio of their lifetimes, which for a solar mass star is $\tau_g/\tau_{ms} \sim 10^{-2}$. However, their large radii imply that they are exceptionally vulnerable to tidal disruption during these brief periods. The contribution of giant stars to the tidal disruption event rate and the luminosity function of AGN will depend on the competing effects of their enhanced cross-section and their relative rarity.

Understanding the details of giant star disruption events and their contribution to the SMBH flaring population is the focus of this work. We use several methods to study this problem. In Section 2.3, we discuss the calculation of detailed stellar evolution models and outline the importance of stellar evolution in the context of tidal disruption. The non-linear dynamics of the encounters themselves must be understood.
through hydrodynamic simulations; this is particularly true for post-MS stars which are not well described by a simple single-polytrope model. In Section 2.4, we describe how we derive giant star initial models from our stellar evolution calculations, our methods of hydrodynamic simulation, and the results of our simulations of close encounters between giant-branch stars and SMBHs. In Section 2.5, we calculate the rates of tidal disruption that result from the two-body relaxation driven random walk of nuclear cluster stars in angular momentum space. We focus on the relative rates of disruption of stars in different evolutionary states. In Section 2.6, we combine our stellar evolution, hydrodynamic, and rate calculations of Sections 2.3 - 2.5 and present Monte Carlo realizations of flaring events. We discuss the demographics of tidal disruption-powered flaring events as a function of black hole mass, the contribution of giant stars to the luminosity function of local AGN, and the detection of flares due to the disruption of giant stars.

2.3 Stellar Evolution in the Context of Tidal Disruption

Stellar evolution naturally enriches the physics of tidal disruption and leads to a large diversity of accretion flare events. The effects of stellar evolution are most pronounced as stars evolve off the MS. Their radius can change by orders of magnitude, they often suffer some degree of mass loss, and the density contrast between the core and the envelope dramatically increases. In this section, we will outline the basic characteristics of post-MS stellar evolution and provide some intuition for how the capture rates and properties of the tidal disruption events might change as a stars evolve off the MS.
2.3.1 Stellar evolution models

To capture the intricacies of post-MS stellar evolution we use the open source Modules for Experiments in Stellar Astrophysics (MESA) stellar evolution code (version 3290, Paxton et al., 2011). MESA solves the stellar evolution equations within the Lagrangian formalism, closed by a tabulated equation of state which blends OPAL, SCVH, PC, and Helmholz equations of state to treat a wide range of fluid conditions and ionization states (Paxton et al., 2011; Rogers and Nayfonov, 2002; Saumon et al., 1995; Potekhin and Chabrier, 2010; Timmes and Swesty, 2000). MESA handles short evolutionary periods (for example, a helium core flash) by switching to a hydrodynamic solution which allows non-zero velocities at cell interfaces. This capability gives MESA the ability to efficiently evolve stars from their pre-MS collapse all the way to the formation of a white dwarf. For our purposes this is essential because it allows us to capture the entire lifetime of stars.

We compute models of solar metallically stars in the mass range of \(0.95 - 5 M_\odot\). To do so, we employ an extension to MESA’s basic hydrogen and helium nuclear burning network called agb.net. Stellar mass loss is modeled for red giant (RG) stars using Reimers’s formula,

\[
\dot{M}_R = 4 \times 10^{-13} \eta_R \left( \frac{L_*}{L_\odot} \right) \left( \frac{R_*}{R_\odot} \right) \left( \frac{M_*}{M_\odot} \right)^{-1} M_\odot \text{ yr}^{-1},
\]

(2.2)

with \(\eta_R = 0.5\) (Reimers, 1975). Mass loss on the asymptotic giant branch (AGB) is given by the Blöcker formula,

\[
\dot{M}_B = 1.932 \times 10^{-21} \eta_B \left( \frac{L_*}{L_\odot} \right)^{3.7} \left( \frac{R_*}{R_\odot} \right) \left( \frac{M_*}{M_\odot} \right)^{-3.1} M_\odot \text{ yr}^{-1},
\]

(2.3)

with \(\eta_B = 0.1\) (Bloecher, 1995). A switch between the wind schemes is triggered when the central hydrogen and helium are depleted. The efficiencies of wind mass loss rates are
Figure 2.1: Radius (left) and mean density (right) evolution of post-MS stars. In the left panel, stages are marked as: main sequence (MS), red giant (RG), horizontal branch (HB), and asymptotic giant branch (AGB). In the right panel, we plot the mean density of stars between $0.95$ and $5 \ M_\odot$ as a function of fractional lifetime. The mean density describes the stars’ vulnerability to tidal disruption (2.1). Periods of low density occupy an increasing portion of the lifetime of more massive stars.

relatively uncertain (e.g. Habing and Olofsson, 2003), and they affect some properties of giant stars. The efficiencies $\eta_R = 0.5$ and $\eta_B = 0.1$ are typical values used in the fiducial $1M_\odot$ models presented with the MESA code (Paxton et al., 2011).

### 2.3.2 Post-MS stellar evolution

Stars exhaust their central reserves of hydrogen fuel in about $\tau_{\text{ms}} \sim 10^{10}(M_*/M_\odot)^{-2.9}$ years (Hansen et al., 2004). After this, they begin to evolve off the MS. Nuclear reactions slow in the stellar core, which contracts under the influence of its self-gravity as it loses pressure support in approximately a thermal diffusion timescale. Hydrogen burns to helium off-center in a thick shell that continues to add mass to the now inert helium core (Iliadis, 2007). The star’s structure reacts to the core’s contraction through an expansion of the star’s outer layers. During this phase, the star’s luminosity increases greatly, while its effective temperature decreases causing it to ascend the RG
branch in the Hertzsprung-Russell (HR) diagram. The star develops an increasingly bi-
furcated structure with a dense, radiative core and a convective envelope. In stars less
than about $2.2M_\odot$, the helium core collapses to the point where it becomes degenerate.
When helium burning eventually ignites it does so in degenerate material, causing a
dynamic event known as the helium core flash. Above about $2.2M_\odot$, helium ignition
occurs off-center in non-degenerate material, and the star does not reach as extreme
radii while on the RG branch (Maeder, 2009).

Following helium ignition, the core expands and the star stably burns helium
to carbon and oxygen for a period of about $\tau_{\text{hb}} \sim 10^8$ years along the horizontal branch
(HB). During this time the core mass grows significantly as hydrogen continues to burn
to helium in a shell surrounding the core (Maeder, 2009). Finally, the star may ascend to
the tip of the giant branch one final time as its carbon oxygen core becomes degenerate.
Stars above (about) $9M_\odot$ will go through additional burning phases (Maeder, 2009). For
the intermediate mass stars considered here, at the tip of the AGB, the stars’ luminosity
becomes so great that the bulk of the envelope is driven off via a combination of strong
winds and thermal pulses, eventually exposing the bare carbon-oxygen core – a proto-
white dwarf (Habing and Olofsson, 2003).

In the left panel of Figure 2.3.2, we show typical evolutionary stages for stars
with $1.4M_\odot$ and $2.87M_\odot$ zero age MS (ZAMS) mass. For simplicity we define here the
transition from the MS to the sub-giant portion of the RG branch as being when a
star first exceeds 2.5 times its ZAMS radius. The post-MS evolution is a very sensitive
function of the initial stellar mass. In particular, the peak radii and timescales of the
RG and AGB phases vary considerably with mass. A common feature is that while
on the giant branch, stars with a wide range of masses spend considerable time having
radii in the range $\sim 5 - 20R_\odot$ and brief phases above $100R_\odot$. The tidal radius of a
star, equation (2.1), is a time-varying function that depends on the mean stellar density,
as $r_t(t) \propto M_*(t)^{-1/3}R_*(t) \propto \bar{\rho}_*(t)^{-1/3}$. The right panel of Figure 2.3.2 illustrates the
variation in stellar evolution profiles with initial mass and that these late phases of stellar evolution give rise to brief periods in which the star becomes orders of magnitude more vulnerable to tidal disruption than it is on the MS.

### 2.3.3 Tidal disruption basics applied to evolved stars

Tidal disruptions occur when stars are scattered into sufficiently low angular momentum orbits that they pass within the tidal radius at pericenter, \( r_t = (M_{\text{bh}}/M_*)^{1/3} R_* \).

We denote the impact parameter of an encounter in terms of the ratio of the tidal radius, \( r_t \), to the pericenter distance, \( r_p \), as \( \beta = r_t/r_p \), such that \( \beta \gg 1 \) signifies a deep encounter. Tidal disruptions are only observable if they occur outside the black hole’s Schwarzschild radius \( r_s = 2GM_{\text{bh}}/c^2 \). In terms of \( r_s \), the tidal radius may be rewritten as

\[
r_t/r_s \approx 23.5 \left( \frac{M_*}{M_\odot} \right)^{-1/3} \left( \frac{M_{\text{bh}}}{10^6 M_\odot} \right)^{-2/3} \left( \frac{R_*}{R_\odot} \right). \tag{2.4}
\]

For \( M_{\text{bh}} \gtrsim 10^8 M_\odot \), solar-type stars will be swallowed whole, producing no observable flare. The precise value of this black hole mass cutoff is almost certainly modulated to some extent by the black hole’s spin and innermost stable circular orbit (Kesden, 2012; Haas et al., 2012). Although stellar remnants such as white dwarfs are expected to be numerous in galactic center environments (e.g. Alexander, 2005), we do not consider them here since they will be swallowed whole by black holes \( M_{\text{bh}} \gtrsim 10^5 M_\odot \) (Luminet and Pichon, 1989b; Rosswog et al., 2008a,b).

A characteristic encounter timescale in tidal disruption events is the pericenter passage time. This timescale is equivalent to the stellar dynamical time, \( r_t/v_p \approx \sqrt{R^3/GM} = t_{\text{dyn}} \), for encounters with pericenter at the tidal radius, \( r_p = r_t \), regardless of stellar structure or evolutionary state. Considering encounters of varying impact parameter, the passage timescale becomes \( t_p \sim \beta^{-1} t_{\text{dyn}} \). During the encounter, material is stripped from the stellar core and spread into two tidal tails. The ma-
terial in one of the tails is unbound from the black hole and ejected on hyperbolic trajectories. The other is bound to the black hole and will return on a wide range of elliptical orbits. Following Rees (1988), we can derive the fallback time of the most bound material, $t_{fb}$, by assuming the star is initially on a parabolic orbit, $E_{orb} = 0$. Then, $E_{min} \approx -\beta^2 GM_*/R_*(M_{bh}/M_*)^{1/3}$, and the corresponding Keplerian period, $t_{fb} = 2\pi GM_{bh}(-2E_{min})^{-3/2}$, can be recast as

$$t_{fb} \approx 0.11\beta^{-3}\left(\frac{M_{bh}}{10^6 M_\odot}\right)^{1/2}\left(\frac{M_*}{M_\odot}\right)^{-1}\left(\frac{R_*}{R_\odot}\right)^{3/2} \text{ yr.}$$  \hspace{1cm} (2.5)

Accretion flares from tidally disrupted giant stars with $R_* \sim 10 - 100 R_\odot$ are thus expected to rise and fall on timescales of years to hundreds of years. Since the total mass lost from the star is similar, the peak accretion rate $\dot{M} \sim \Delta M/t_{fb}$ for these events will be correspondingly lower than for stars on the MS.

How often a star enters the zone of vulnerability delineated by the tidal radius depends on the stellar density and velocity dispersion. One can crudely estimate the rate of tidal disruption as

$$\dot{N}_{iso} \sim 10^{-4}\left(\frac{M_{bh}}{10^6 M_\odot}\right)^{4/3}\left(\frac{n_*}{10^5 \text{ pc}^{-3}}\right)\left(\frac{\sigma}{100 \text{ km s}^{-1}}\right)^{-1}\left(\frac{r_t}{100 R_\odot}\right) \text{ yr}^{-1},$$  \hspace{1cm} (2.6)

by assuming that the black hole inhabits an isotropic sea of stars such that $\dot{N}_{iso} \sim n_*\Sigma_t\sigma$, and making use of the gravitational focus limit of the tidal disruption cross-section $\Sigma_t$ (see Rees, 1988, equation 2). In equation (2.6), the tidal disruption rate is linearly proportional to the tidal radius, $\dot{N}_{iso} \propto r_t$. A star that spends most of its lifetime at $1R_\odot$, 10% of its lifetime at $10R_\odot$, and 1% at $100R_\odot$ would therefore have a similar likelihood of tidal disruption during each of these stages, even though their timescales span two orders of magnitude. However, as noted by Rees (1988), there are factors
which complicate this picture. In particular, the isotropic description is invalidated
by the facts that the number density of stars falls off sharply with distance from the
black hole and that stars within the black hole’s sphere of influence orbit the black hole
directly (in nearly Keplerian orbits). More detailed models find similar tidal disruption
rates but different scalings than those derived under the simplifying isotropic hypothesis
(Magorrian and Tremaine, 1999; Wang and Merritt, 2004). The implications of these
more accurate models will be discussed in more detail in Section 2.5.

2.4 Hydrodynamics of giant disruption

The hydrodynamics of tidal disruption have been studied in detail for main
sequence stars and planets, which are thought to be well described by polytropes. Initial
simulation efforts exploited semi-analytic formalisms that follow the distortion of a
sphere into a triaxial ellipsoid in the tidal field of the black hole (Carter and Luminet,
1982; Luminet and Marck, 1985; Luminet and Carter, 1986; Kosovichev and Novikov,
1992; Ivanov and Novikov, 2001). Later studies used both Lagrangian (Nolthenius and
Katz, 1982; Bicknell and Gingold, 1983; Evans and Kochanek, 1989; Laguna et al., 1993;
Kobayashi et al., 2004; Brassart and Luminet, 2008; Rosswog et al., 2009b; Ramirez-Ruiz
and Rosswog, 2009; Lodato et al., 2009; Antonini et al., 2011) and Eulerian formalisms
(Khokhlov et al., 1993a,b; Diener et al., 1997; Guillochon et al., 2009, 2011; Guillochon
and Ramirez-Ruiz, 2013). In this section, we will emphasize some of the characteristics
that make the disruption of giant stars unique.

2.4.1 Initial models for hydrodynamic simulations

We explore the typical characteristics of tidal disruption events involving a
\(10^6 M_\odot\) black hole and stars in the representative stages outlined in Section 2.3 and
shown in Figure 2.3.2. To this end, we define 4 initial models based on the MESA
Figure 2.2: In dashed black, we show profiles of enclosed mass calculated in MESA for a 1.4$M_\odot$ ZAMS mass star at characteristic stages along its post-MS evolution. From left to right, these are: ascending the RG branch (RG I), the tip of the RG branch (RG II), HB, and AGB. The orange and blue lines show how we approximate the stellar structure using a nested, two-fluid polytrope. We match the profile of the envelope as accurately as possible while allowing the core to be enlarged $R_{\text{core}} \sim R_*/20$. Unless the passage is so close as to be disruptive for the core ($\beta > 20$), this approximation of the stellar structure is valid and lessens the computational burden of our calculations.

evolution of a solar metallicity 1.4$M_\odot$ ZAMS mass star. We include two models for RG stars, one of which is ascending the RG branch (model RG I), and one at the tip of the RG branch (model RG II). A single model is used for each of the HB (model HB) and AGB (model AGB) phases of stellar evolution.

For our hydrodynamic calculations we approximate the core and envelope structure using a nested polytropic structure with two fluids of different mean molecular weights (Chandrasekhar, 1967). This structure is approximate, but, as seen in Figure 2.2, the fits to the enclosed mass profiles within the envelopes are quite good. We decided to enlarge the core radius relative to its true size for numerical convenience because it would be very restrictive to attempt to simultaneously resolve the physical and temporal scales of the true core ($R_{\text{core}} \approx 10^9$ cm and $t_{\text{dyn}} \approx 5$ s) while also following the evolution of the extended envelope ($R_{\text{env}} \approx 10^{13}$ cm and $t_{\text{dyn}} \approx 10^6$ s). In our models we choose the core radius to be $R_{\text{core}} \approx R_{\text{env}}/20$. This approximation imposes the restriction that we may consider only encounters for which the pericenter distance, $r_p$,
Table 1: $1.4M_\odot$ ZAMS initial models generated from MESA

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_*$</th>
<th>$M_{\text{core}}$</th>
<th>$R_*$</th>
<th>$n_{\text{core}}$</th>
<th>$n_{\text{env}}$</th>
<th>$\mu_{\text{core}}/\mu_{\text{env}}$</th>
<th>$R_{\text{core}}/R_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGI</td>
<td>1.4</td>
<td>0.28</td>
<td>12.5</td>
<td>1.5</td>
<td>1.7</td>
<td>8.0</td>
<td>0.048</td>
</tr>
<tr>
<td>RGII</td>
<td>1.32</td>
<td>0.44</td>
<td>110</td>
<td>1.5</td>
<td>1.8</td>
<td>7.0</td>
<td>0.053</td>
</tr>
<tr>
<td>HB</td>
<td>1.26</td>
<td>0.55</td>
<td>12.3</td>
<td>1.5</td>
<td>1.7</td>
<td>8.0</td>
<td>0.052</td>
</tr>
<tr>
<td>AGB</td>
<td>1.17</td>
<td>0.57</td>
<td>208</td>
<td>1.5</td>
<td>1.8</td>
<td>7.0</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Note. — Columns: (1) model name, (2) total mass [$M_\odot$], (3) core mass [$M_\odot$], (4) radius [$R_\odot$], (5) polytropic index of core, where $\Gamma = 1 + 1/n$, (6) polytropic index of envelope, (7) ratio of core to envelope mean molecular weights, (8) ratio of core radius to total radius.

satisfies $r_t(R_{\text{core}}, M_{\text{core}}) \ll r_p \lesssim r_t(R_{\text{env}}, M_{\text{env}})$. These are encounters that are disruptive for the envelope while leaving the enlarged core tidally unaltered. By comparison with equation (2.6), this range of impact parameters encompasses more than 90% of all tidal encounters between giant stars and SMBHs. In Table 1, we give the characteristics of our polytropic approximations to MESA models.

2.4.2 Methods

The calculations presented here have been performed with the tidal disruption code described in detail in Guillochon et al. (2009), Guillochon et al. (2011) and Guillochon and Ramirez-Ruiz (2013). Our formalism is based on the FLASH4 code (Fryxell et al., 2000), an adaptive mesh Eulerian hydrodynamics code. The compressibility of the gas is described with a gamma-law equation of state $P \propto \rho^\gamma$. For the envelope we take $\gamma = \gamma_{\text{ad}} = 5/3$, while we choose a stiff equation of state to describe the artificial core, $\gamma = 5$. Because the core is only weakly compressible, it does not expand significantly as mass is removed from the star’s envelope, ensuring that it makes no spurious impact on the dynamics of the tidal disruption. The flux between grid cells is solved using piecewise parabolic method (PPM) (Colella and Woodward, 1984; Woodward and
Colella, 1984), which incorporates second-order directional operator splitting (Strang, 1968). We base the grid mesh refinement on the value of the density, with successive cutoffs limiting the refinement by one level per decade in density.

Our calculations are performed in the frame of the star to avoid introducing artificial diffusivity (for a fixed resolution) by moving the star rapidly across the grid structure (Guillochon et al., 2009, 2011). The self-gravity of the star is computed using a multipole expansion about the center of mass of the star with $l_{\text{max}} = 10$. The orbit is then evolved based on the center of mass of the star and the position of the (Newtonian) point mass black hole. We refer the reader to Appendix A of Guillochon et al. (2011) for details of this algorithm. In our simulations the approximation of Newtonian gravity for the black hole is justified by the fact that the closest approach which any of the giant stars in our simulations make is about $110r_s$, well into the weak field regime.

The simulations presented here are all resolved by at least $R_*/\Delta r_{\text{min}} > 200$, where $\Delta r_{\text{min}}$ is the dimension of the smallest cells. However, by implementing density cutoffs in the peak refinement for a given block we allow the linear resolution on the limb of the star to be 2 or 3 levels lower than the peak resolution of the core region. This limits our total discretization of the computational domain to $(1-10) \times 10^6$ cells, constraining the computational intensity of the problem while allowing us to survey some degree of impact parameter and stellar evolution parameter space.

### 2.4.3 Pericenter passage and mass removal

The passage through pericenter is mainly characterized by the impact parameter $\beta = r_t/r_p$. Since stars are generally scattered to the black hole from apocenter distances around the SMBH’s sphere of influence (see Section 2.5), typical orbits leading to disruption follow nearly radial trajectories. Thus, we consider orbits which are initially parabolic.

As the star passes through pericenter, quadrupole distortions of the stellar sur-
Figure 2.3: Density across slices through the orbital plane during a passage through pericenter, $t=0$, for a $\beta = 1.5$ encounter between the RG I model ($1.4M_\odot, 12.5R_\odot$) and a $10^6M_\odot$ black hole. The top panels show an inset of the inner region shown in the lower panels, which have widths of 6 and 80 times the initial stellar radius, respectively. Visualization created with yt (Turk et al., 2011).

face reach an amplitude of order unity. Material is sheared from the stellar envelope as the time varying tidal field applies a gravitational torque to the distorted star, spinning stellar material to nearly its corotational angular velocity. A portion of this spun-up material is unbound from the star and ejected into two tidal tails. The star’s center of mass continues to follow a roughly parabolic trajectory, material in one of the tails is unbound and thus ejected onto hyperbolic orbits, and material in the other tail traces out elliptical orbits and will eventually fall back to the black hole. The tidal field of a $10^6M_\odot$ black hole is only $\approx 3\%$ asymmetric on the scale of a star at the tidal radius (the degree of asymmetry scales linearly with $\beta$). This symmetry is reflected in the formation of the two similarly shaped tidal tails as shown in Figure 2.3.

While the structures and tidal deformations seen in Figure 2.3 appear broadly consistent with those resulting from the disruption of MS stars (Guillochon and Ramirez-Ruiz, 2013), the quantitative nature of the distortions in the giant star case is marked by the gravitational influence of the dense core. Of course, MS stars have cores that are more dense than their envelopes, but the ratio of their central to average density is $\rho_{\text{core}}/\bar{\rho}_* \approx 100$, whereas for giant stars this ratio is $\gtrsim 10^6$. In the initial models we
Figure 2.4: Mass loss with varying impact parameter, $\beta$, for the RG I model. The left panel shows the star’s self-bound mass as a function of time, where $t = 0$ corresponds to pericenter. The center panel shows the net mass loss, as measured after $\sim 50t_{\text{dyn}}$, as a fraction of the envelope mass, $M_{\text{env}}$. As $\beta$ increases, the mass lost asymptotes to a value less than $M_{\text{env}}$. This can be contrasted to MS stars, which are fully disrupted at $\beta \sim 1.9$. The reason for this asymptotic behavior can be seen in the adiabatic response of the stellar envelope in the expanding background of the core-black hole potential. In the right panel, we plot the enclosed mass within the (time-varying) Hill radius of the core, equation (2.7). The increase in enclosed mass after pericenter shows that the envelope is contracting, becoming more dense, and effectively shielding itself from further mass loss.
construct, the density contrast is not as high due to numerical limitations, $\rho_{\text{core}}/\bar{\rho}_* \approx 2 \times 10^3$, but it is similarly marked by a discontinuous transition in the density profile. Because the dynamical time of the core is much less than that of the envelope, the core material is not perturbed, but its gravitational influence significantly modifies the rearrangement of the surrounding envelope material as the encounter progresses. The adiabatic response of centrally condensed, nested polytropes, like the ones constructed here, is to contract when they lose sufficient mass, thus shielding the star from further mass loss (Hjellming and Webbink, 1987). The ability of the stellar core to retain envelope gas can be seen in the upper panels in Figure 2.3. These panels show a close view of the region near the star’s core that leaves the passage tidally excited, but undisturbed. The core itself is completely unperturbed.

In Figure 2.4, we explore the causes of the mass loss in greater quantitative detail. The left panel of Figure 2.4 displays the star’s self-bound mass as a function of time for a set of encounters with $0.75 < \beta < 2.25$. Encounters of varying $\beta$ lead to different degrees of mass loss and varying speed of mass removal (characterized by the slope of the lines near pericenter, $t = 0$). The self-bound mass gradually decreases over many dynamical times, eventually converging to a net mass loss, $-\Delta M$, plotted as a fraction of the envelope mass $M_{\text{env}}$ in the center panel of Figure 2.4. The mass lost during the encounter asymptotes to less than the total envelope mass, with deeper encounters not being able to remove significantly more mass.

The reason for the observed asymptotic behavior of $-\Delta M$ with $\beta$ can be inferred from the right panel of Figure 2.4. The tidal force at pericenter increases with increasing $\beta$, removing a greater fraction of the envelope mass as $\beta$ increases. This increase in the rate of mass loss is drastically halted when the total envelope mass remaining approaches the mass of the core. Because the core is a significant fraction of the total stellar mass, the star’s structure responds to a drastic loss of envelope mass by contracting. As more mass is removed, the surrounding envelope contracts more
dramatically (Hjellming and Webbink, 1987; Passy et al., 2012b). The competition between the tidal mass stripping and the ensuing envelope contraction, both of which are seen to increase with $\beta$, leads to the asymptotic behavior with $\beta$ of $-\Delta M$. A subtlety lies in the fact that this contraction occurs not in an absolute sense, but in an expanding frame defined by the fluid trajectories in the combined potential of the black hole and the surviving core. This can be seen directly by measuring the enclosed mass within the Hill radius of the core, shown in the right panel of Figure 2.4. The Hill radius of the core varies as function of time through the encounter as

$$R_{h,\text{core}}(t) = \left( \frac{M_{\text{core}}}{M_{\text{bh}}} \right)^{1/3} r(t),$$

where $r(t)$ is the separation between the core and the black hole. $M_{\text{enc}}(< R_{h,\text{core}})$ decreases initially and reaches a minimum at pericenter ($t = 0$) where $R_{h,\text{core}} = \beta^{-1}(M_{\text{core}}/M_*)^{1/3} R_*$. After pericenter, the mass within this equipotential surface increases because envelope material contracts relative to the expanding frame of the disrupted star.

Each of the post-MS models described in Section 2.4.1 differs in the exact polytropic index of the envelope and the core to envelope mass ratio. However, the qualitative nature of mass removal from these models is very similar to that shown in detail for the RG I model in Figure 2.4. As can be seen in Figure 2.5, evolved stars retain an increasing fraction of envelope material at a given $\beta$ as their core to envelope mass ratio grows. This is because larger core mass fractions in evolved stars increase the strength of the envelope’s adiabatic response to contract with increasing mass loss (Hjellming and Webbink, 1987; Passy et al., 2012b). In all cases, a significant fraction of envelope mass remains bound to the core for even the deepest encounters we have explored. This stands in stark contrast to MS stars (Guillochon and Ramirez-Ruiz, 2013), which are fully disrupted beyond $\beta \sim 1.9$. We tested both more and less dense artificial cores and confirmed that the asymptotic behavior of the mass loss is in no way
affected by our enlarged core approximation at the impact parameters we consider.

The following fitting formulae provide a reasonable approximation to the degree of mass loss within the range of $\beta$ presented here

$$- \Delta M = \begin{cases} 
0.52 + 1.11\beta^{-1} - 1.95\beta^{-2} + 0.66\beta^{-3} & \text{RG I}, \\
0.57 + 0.61\beta^{-1} - 1.58\beta^{-2} + 0.62\beta^{-3} & \text{RG II}, \\
0.62 - 0.06\beta^{-1} - 0.30\beta^{-2} - 0.04\beta^{-3} & \text{HB}, \\
0.55 - 0.15\beta^{-1} - 0.32\beta^{-2} + 0.07\beta^{-3} & \text{AGB}.
\end{cases}$$ (2.8)

Our findings may be compared to those of authors who study giant star stellar collisions (e.g. Bailey and Davies, 1999; Dale et al., 2009). These studies similarly find that the dense core of the giant star retains a significant fraction of envelope mass. Dale et al. (2009) also note that the great majority of the star’s hydrogen envelope must be removed to appreciably alter the star’s evolution along the giant branch. Because they retain sufficient portions of their envelope mass, the remnants of the tidal disruption events discussed here do not appear as naked helium cores and instead are likely more similar in structure and appearance to their original, giant-star nature.

2.4.4 Evolution of unbound material

Material stripped from the surviving core forms two symmetric tidal tails (see Figure 2.3). The evolution of the material in the tails is well described by two characteristic stages: an initial period of hydrodynamic evolution, followed by a period of homologous expansion. Hydrodynamic interactions remain important when the local sound speed of the stellar material exceeds the relative expansion velocity of adjacent fluid elements. As the stellar material expands, $\rho$ decreases and the sound speed drops because $c_s = \sqrt{\gamma P/\rho} \propto \rho^{(\gamma-1)/2} \propto \rho^{1/3}$ for $\gamma = 5/3$. When these sound waves are unable to propagate as quickly as the distance between neighboring fluid parcels in-
Figure 2.5: Mass loss as a function of $\beta$ for the four post-MS models shown in Figure 2.2. Mass lost is shown as a fraction of the original envelope mass of a given star. Increasing core to envelope mass ratio dictates that that a smaller fraction of envelope mass is lost at a given impact parameter. However, in all cases, much less than the entirety of the envelope mass is lost, even in encounters for which $\beta > 1$. The solid lines are described by the fitting formulae presented in equation (2.8).
Figure 2.6: Temporal evolution of the specific binding energy of the tidal tail material, calculated here for the AGB model at an impact parameter of $\beta = 1$. The lines plotted range from $t = 5t_{\text{dyn}}$ (blue) after pericenter to $t = 35t_{\text{dyn}}$ (brown) after pericenter in intervals of $5t_{\text{dyn}}$ and include only the material which is unbound from the surviving core. In the left and center panels, specific binding energy relative to the black hole is shown on the x-axis. The star approaches the black hole on a parabolic orbit corresponding to $E = 0$. Material with $E > 0$ is ejected on hyperbolic orbits, while that with $E < 0$ eventually falls back to the black hole on a range of elliptical orbits that may be mapped to the projected fallback rate shown in the right panel using equation (2.9). The energy distribution of material near the tips of the tidal tails is frozen into homologous expansion at early times. Material closer to $E \sim 0$ evolves to fill in the cavity in $dM/dE$ for many $t_{\text{dyn}}$ as the surviving remnant becomes increasingly isolated from the black hole’s potential. The $dM/dE$ and $\dot{M}$ distributions shown in Figures 2.6 - 2.8 are plotted with 500 bins in energy or time.
creases, the material’s velocity distribution begins to freeze. Any additional evolution of the stellar fluid is then relatively well described by collisionless trajectories in the black hole’s gravitational field.

To determine the trajectories of the material in the tails, we calculate the evolution of the specific binding energies of all of the fluid elements relative to the black hole. This gives the spread of mass per unit energy within the disrupted star, $dM/dE$, where $E$ is the specific orbital binding energy relative to the black hole. In Figure 2.6, we plot the time evolution of $dM/dE$ with the self-bound material of the surviving stellar core excised. The majority of the material achieves homologous expansion only after a few dynamical timescales, with the structure of the material near the tips of the tidal tails being frozen in earliest. The distribution of material close to the the surviving core continues to evolve for some time (center panel of Figure 2.6), and does not fully converge until the surviving remnant is truly isolated from the black hole. This effect can also be seen in the left panel of Figure 2.4, where the self-bound mass of the surviving star decreases gradually as a function of time. The gentle evolution of the stream material at the force balance point between the remnant and the black hole occurs over a long timescale when compared to the pericenter passage time because the surviving star continues to be slowly depleted of mass that crosses the lowered potential barrier between the star and the black hole. Hence, the time-evolving gravitational influence of the surviving star has to be taken into account to accurately compute the resulting $dM/dE$ distribution (for a more detailed discussion, the reader is referred to Guillochon and Ramirez-Ruiz, 2013). While accurately computing the final mass distribution requires numerical calculation out to very late times, Dale et al. (2009) present a linear extrapolation method that is useful to estimate the degree of eventual mass loss by the remnant. Performing this extrapolation analysis on our data shows that while we present values of $\Delta M$ as measured at the end of our simulation time,

$^1$The authors linearly extrapolate the remnant mass in $t^{-1}$ to $t^{-1} = 0$.
these are likely within 1% of the expected total mass loss.

The material in the tails follows ballistic trajectories after sufficient time has elapsed. As a result, the spread in energy can be mapped to a return time to pericenter for the tail that is bound to the black hole,

\[ \dot{M} = \frac{dM}{dt} = \frac{dM}{dE} \frac{dE}{dt} = \frac{(2\pi GM_{bh})^{2/3}}{3} \left( \frac{dM}{dE} \right) t^{-5/3}. \]  

For a flat \( dM/dE \), one recovers the canonical \( t^{-5/3} \) power law predicted for the fallback mass return rate after a tidal disruption event (Rees, 1988; Lodato et al., 2009). The right panel in Figure 2.6 shows the mapping to \( \dot{M} \) from \( dM/dE \) plotted in the left two panels. The cavity in \( dM/dE \) that results from not including self-bound material leads to a similar cavity in the predicted fallback rate at very late times. As the remnant flies away from the black hole, becoming increasingly isolated from the black hole’s potential, we expect this distribution to fill in (but not necessarily completely flatten since this depends sensitively on the rate of freezing as compared to the rate of fallback).

### 2.4.5 Debris fallback and AGN flaring

Debris ejected into the tidal tail bound to the black hole returns to pericenter at a rate given by equation (2.9). The returning gas does not immediately produce a flare of activity from the black hole. First material must enter quasi-circular orbits and form an accretion disk (Ramirez-Ruiz and Rosswog, 2009). Once formed, the disk will evolve under the influence of viscosity (Cannizzo et al., 1990; Montesinos Armijo and de Freitas Pacheco, 2011). However, the viscosity would have to be extremely low for the bulk of the mass to be stored for longer than \( t_{fb} \) (equation 2.5) in a reservoir at \( r \approx r_t \). Taking typical values, the ratio of the viscous accretion timescale to the fallback timescale is

\[ \frac{t_v}{t_{fb}} \approx 10^{-3} \beta^{3/2} \left( \frac{\alpha_v}{10^{-2}} \right)^{-1} \left( \frac{M_{bh}}{10^6 M_\odot} \right)^{1/2} \left( \frac{M_*}{1M_\odot} \right)^{-1/2}, \]  

(2.10)
Figure 2.7: The fallback rate of bound material with varying $\beta$ for the RG I model, normalized to the Eddington accretion rate, $\dot{M}_{\text{edd}} = 0.02 \epsilon^{-1} (M_{\text{bh}}/10^6 M_\odot) M_\odot \text{yr}^{-1}$. This may be compared to Figure 2.4, which details the removal of mass through pericenter for this model. Differing impact parameter changes $\dot{M}_{\text{peak}}$ and $t_{\text{peak}}$ to some extent. In deeper encounters, the material most bound to the black hole falls back earlier and exhibits a more gradual rise to peak. After peak, all of the $\beta$'s shown here appear to fall off more steeply than $t^{-5/3}$ for many years. The other three post-MS models exhibit very similar qualitative behavior.
Figure 2.8: A comparison of the fallback accretion between the four evolved star models and the disruption of a sun-like MS star, at $\beta = 1.5$. Accretion flares from the disruption of giant stars are long-lived and peak at relatively lower $\dot{M}$. While the 100-1000 year typical timescales of stars disrupted at the tip of the giant branches will be difficult to distinguish from other long-term low-level AGN activity, the rise to peak resulting from the disruption of stars either ascending the red giant branch (RG I) or on the horizontal branch stars (HB) should be observable in current time domain surveys. We extrapolate the fallback beyond the position of the cavity in $\dot{M}$ (dashed lines) using the slope of $\dot{M}$ captured by our last simulation snapshot.
where we have made the standard assumption of a thick \((H/R) \sim 1\) disk (Shakura and Sunyaev, 1973). The accretion rate onto the black hole, and to a certain extent the bolometric luminosity of the resulting flare (i.e. \(L \propto \dot{M}c^2\)), is thus generally expected\(^2\) to be limited by the gas supply at pericenter \(\dot{M}\), equation (2.9), not by the rate at which the orbiting debris drains onto the black hole (Rees, 1988; Ulmer, 1999).

In the simulations of giant star tidal disruption presented here, we find sizable variations in the black hole feeding rate, \(\dot{M}\), with impact parameter \(\beta\). In Figure 2.7, we show \(\dot{M}\) as a function of \(\beta\) for the RG I model. The time of peak, \(t_{\text{peak}}\), is observed to vary only slightly with \(\beta\). Additionally, since \(\Delta M\) asymptotes at high \(\beta\), the peak accretion rates, \(\dot{M}_{\text{peak}}\), are similar for most deep encounters. The slope of the initial fallback after peak is, however, significantly steeper than \(t^{-5/3}\), particularly for the more grazing encounters. This variation in post-peak slope differentiates \(\dot{M}\) curves that overlap at peak.

The time of initial fallback, \(t_{\text{fb}}\), varies considerably with \(\beta\). Because the time of peak, \(t_{\text{peak}}\), is relatively constant, the slope of the early-time rise varies correspondingly, as can be seen in Figure 2.7. Observations of this portion of the \(\dot{M}\) curve could help break the relative degeneracy seen in the post-peak light curves. However, at these early times viscous and angular momentum redistribution processes might play some role in shaping the accretion luminosity. Further, because the early portion of \(\dot{M}\) represents only a small fraction of the total mass lost by the star, \(\Delta M\), it is sensitive in our simulations to the limited mass resolution of discretized material in the tidal tails. In Figure 2.6, we show that the early-time \(\dot{M}\) freezes into homologous expansion quickly, and is seen to accurately preserve its shape as the disruption ensues. We have studied the variations in the early time \(\dot{M}\) with increasing resolution and find no systematic variations. We do note, however, that the short-timescale bumps and variations in \(\dot{M}\) at

\(^2\)When the black hole is fed above the Eddington limit (\(\dot{M} > \dot{M}_{\text{Edd}}\)), the relationship between \(\dot{M}\) and disk luminosity is, however, less certain (e.g. Strubbe and Quataert, 2009, 2011).
$t \sim t_{fb}$ likely result from grid (de-)refinement events, which are unavoidable in keeping our computations feasible.

While variations in $\dot{M}$ with $\beta$ are present, major changes are observed in the feeding rates as stars evolve and their structures are dramatically altered. In Figure 2.8, we compare the black hole feeding rates from the disruptions of evolved stars with a typical MS disruption. The disruptions of giant stars lead to long-lived flaring events that peak at, or near, the Eddington rate for a $10^6 M_\odot$ black hole. The peak time of these events is, as expected, correlated with the radial extent of the evolving star. Our simulations show that while the timescale of peak accretion rate in RG I and HB star flares is several years, the rise toward peak $\dot{M}$ occurs over similar timescales to those seen in the disruption of sun-like stars, thus offering hope for detectability in surveys tailored to detect MS flares. On the other hand, tidal disruptions of stars at the tips of the giant branches (the RG II and AGB models) lead to flares that peak at hundreds of years timescales, which make them difficult to discern from other non-transient AGN feeding mechanisms. In Figure 2.9, we show how the peak $\dot{M}$ and fallback timescales are expected to scale with increasing black hole mass.

2.5 Loss cone theory and rates of giant disruption

Following the observation by Hills (1975) that stars passing within $r_t$ at pericenter are tidally disrupted, a series of studies focused on the rate at which stars are fed into disruptive orbits. This rate depends on the distribution of stars around the black hole and the rate at which small scatterings in orbital angular momentum allow stars to diffuse into nearly radial loss cone orbits that lead to disruption (Frank and Rees, 1976; Lightman and Shapiro, 1977). Cohn and Kulsrud (1978) present a formalism for treating this diffusion process by numerical integration of a Fokker-Planck equation. More recent work extends the Cohn and Kulsrud (1978) formalism to compute the flux of stars into
Figure 2.9: Encounters of $\beta = 1$ rescaled to the range of known supermassive black hole masses. With increasing black hole mass, flares become longer-lived but also peak lower relative to the Eddington rate for that black hole mass. The forbidden region shows where $r_t < r_s$ and stars enter the black hole’s horizon whole without being tidally disrupted.
the loss cones of observed galaxies (Magorrian et al., 1998; Magorrian and Tremaine, 1999; Wang and Merritt, 2004), with a key result being that stars are typically fed into the loss cone from weakly bound orbits with semi-major axes similar to the black hole’s sphere of influence, equation (2.11). Magorrian and Tremaine (1999) also considered the effect of an axisymmetric (rather than spherical) stellar distribution, while Wang and Merritt (2004) considered the effects of black hole mass on tidal disruption rates. Additional recent work has focused on the effects of black hole spin (Kesden, 2012), binary black holes (Ivanov et al., 2005; Chen et al., 2009), and numerical determination of the tidal disruption rate using Monte Carlo and N-body calculations of stellar orbits (Freitag and Benz, 2002; Brockamp et al., 2011).

Syer and Ulmer (1999) use a simplified treatment of both the loss cone formalism and stellar evolution to roughly account for the effects of evolved stars on the integrated disruption rate. We extend the work of previous authors by emphasizing the relative rates of disruption of stars in different evolutionary phases using our MESA stellar evolution models, but we otherwise follow the Fokker-Planck formalism closely (Cohn and Kulsrud, 1978; Magorrian and Tremaine, 1999; Wang and Merritt, 2004).

In this section, we begin by describing a simplified galactic nuclear cluster model. We outline the loss cone formalism and compute how the tidal disruption rate changes as a star’s tidal radius changes due to stellar evolution. We compare our findings based on the simple nuclear cluster model to a sample of observed nuclear cluster profiles to infer the effects of structural differences between galaxies. We are then able to use our calculation of how the tidal disruption rate scales with tidal radius to determine the stellar ingestion diet of SMBHs.

### 2.5.1 Simplified nuclear cluster model

To develop intuition for the relative rates of disruption of MS and evolved stars, we first explore a simplified nuclear cluster model. This model consists of a
homogeneous stellar population and is described by a Keplerian potential. The black
hole is the dominant influence on the stellar kinematics within a nuclear cluster of radius

\[ r_h = \frac{G M_{bh}}{\sigma_h^2} = 1.08 \left( \frac{M_{bh}}{10^6 M_\odot} \right)^{1/2.12} \text{pc,} \quad (2.11) \]

where \( \sigma_h \) is the external velocity dispersion of the greater galactic bulge. In the numerical
expression above we use the \( M_{bh} - \sigma \) relation (e.g. Ferrarese and Merritt, 2000;
Gebhardt et al., 2000; Tremaine et al., 2002; Gültekin et al., 2009), with fitting values
from Gültekin et al. (2009), \( \sigma_h = 2.43 \times 10^5 (M_\odot / M_{bh})^{1/4.24} \text{ cm s}^{-1} \).

At radii \( r \lesssim r_h \), the cluster potential is approximately Keplerian,

\[ \phi(r) = \frac{G M_{bh}}{r}, \quad (2.12) \]

where, following Magorrian and Tremaine (1999), we choose a positive sign convention
for the potential and the energies of bound orbits. The velocity dispersion at a given
radius is approximately \( \sigma^2(r) \sim \phi(r) \). We define stellar orbits based on their binding
energy to the black hole

\[ \varepsilon(a) = \frac{G M_{bh}}{2a} = \frac{1}{2} \phi(a), \quad (2.13) \]

where \( a \) is the orbital semi-major axis. The Keplerian orbital period of stars with a
given \( \varepsilon \) is \( P(\varepsilon) = 2\pi G M_{bh} (2\varepsilon)^{-3/2} \).

We take a singular isothermal sphere stellar number density profile,

\[ \nu_*(r) = \nu_h r^{-2}, \quad (2.14) \]

where \( \nu_h = \nu_*(r_h) \), as Wang and Merritt (2004) do. This is comparable to the steep
density cusp expected to form in a relaxed stellar distribution about a black hole (Bahcall
and Wolf, 1976; Bahcall and Wolf, 1977), while also being consistent with the roughly flat
velocity dispersion profiles observed in typical galaxies outside \( r_h \). For simplicity, a single stellar mass \( M_\star \) is commonly assumed, and, as a result the mass density is related to the number density by \( \rho_\star(r) = M_\star \nu(r) \). We choose \( \nu_h \) so that the mass of stars within the sphere of influence is twice the black hole mass, which gives \( \nu_h = (2\pi r_h^3)^{-1}(M_{bh}/M_\star) \).

We further assume that stellar orbits are isotropic in angular momentum space and therefore may be described by a distribution function solely in energy,

\[
f(\varepsilon) = (2\pi \sigma_h^2)^{-3/2} \frac{\Gamma(3)}{\Gamma(1.5)} \left( \frac{\varepsilon}{\sigma_h^2} \right)^{1/2},
\]

where the numerical factor \( \frac{\Gamma(3)}{\Gamma(1.5)} \approx 2.26 \) (equation 9 in Magorrian and Tremaine, 1999). Having described the properties and phase space distribution of stars in our cluster model, we can now determine how often stars are disrupted.

### 2.5.1.1 The full loss cone disruption rate

The angular momentum of a circular orbit is the maximum angular momentum allowed for a given \( \varepsilon \). It is

\[
J^2_c(\varepsilon) = \frac{G^2 M_{bh}^2}{2\varepsilon}.
\]

The angular momentum of a loss-cone orbit is

\[
J^2_{lc}(\varepsilon) = 2r_{\min}^2 \left( \frac{GM_{bh}}{r_{\min}} - \varepsilon \right) \approx 2GM_{bh}r_{\min},
\]

where \( r_{\min} \) is the maximum of \( r_t \) or \( r_s \).

The number of stars in a full loss cone per energy is then \( N_{lc}(\varepsilon) = 4\pi^2 f(\varepsilon)P(\varepsilon)J^2_{lc}(\varepsilon) \), and the total number is

\[
N_{lc} = \int_0^{\varepsilon_{\max}} N_{lc}(\varepsilon) d\varepsilon,
\]

where \( \varepsilon_{\max} \) is the energy corresponding to \( r_{\min} \). The flux of stars into the loss cone, if
we assume that it is continuously replenished, is simply

\[ F_{\text{full}}(\varepsilon) = \frac{N_{\text{lc}}(\varepsilon)}{P(\varepsilon)}. \]  

(2.19)

Similarly, the net rate at which stars are disrupted from a full loss cone is obtained by integrating over energy,

\[ \dot{N}_{\text{full}} = \int_0^{\varepsilon_{\text{max}}} \frac{N_{\text{lc}}(\varepsilon)}{P(\varepsilon)} d\varepsilon. \]  

(2.20)

However, the loss cone is not necessarily full for all \( \varepsilon \). We turn our attention to where \( \dot{N}_{\text{full}} \) is an appropriate estimate of the tidal disruption rate.

### 2.5.1.2 Repopulating loss cone orbits

The derivation of \( \dot{N}_{\text{full}} \) assumes that at all energies, \( \varepsilon \), stars destroyed by the black hole at pericenter are replaced on a timescale shorter than the orbital period, \( P(\varepsilon) \), thus ensuring that the phase space density of low \( J \) orbits remains undepleted. A typical timescale for stars to random walk in angular momentum by \( J_{\text{lc}} \) via the two-body relaxation process is

\[ t_1(\varepsilon) \sim \left[ \frac{J_{\text{lc}}(\varepsilon)}{J_c(\varepsilon)} \right]^2 t_r(a(\varepsilon)), \]  

(2.21)

where two-body relaxation time is

\[ t_r(r) = \frac{5.68 \times 10^{19} M_{\odot} 10^4 M_{\odot} \text{pc}^{-3}}{\ln \Lambda} \frac{\rho_*(r)}{M_*} \left( \frac{\sigma(r)}{10^7 \text{cm/s}} \right)^3 \text{s}, \]  

(2.22)

(equation 7-107 in Binney and Tremaine, 2008). The Coulomb logarithm is approximated as \( \Lambda \approx 0.4N_*(r < r_h) \), where \( N_*(r < r_h) \) is the number of stars within \( r_h \), or equivalently, the number of stars with \( \varepsilon > \varepsilon_h \).

Thus, in an orbital period, the typical change in angular momentum a star
receives via this random walk process is

\[ \Delta J^2(\varepsilon) = J_{lc}^2(\varepsilon)P(\varepsilon)/t_3(\varepsilon), \]  

(2.23)

or, written as a ratio to the loss cone angular momentum (Cohn and Kulsrud, 1978),

\[ q(\varepsilon) \equiv \frac{\Delta J^2(\varepsilon)}{J_{lc}^2(\varepsilon)} = \frac{P(\varepsilon)}{t_3(\varepsilon)}. \]  

(2.24)

We can imagine two limiting cases for the value of \( q \). The full loss cone rate is appropriate when \( \Delta J \gg J_{lc} \). This is the so-called pinhole limit where per-orbit scatterings are large compared to the target, here given by the size of the loss cone (Lightman and Shapiro, 1977). Loss cone orbits are easily repopulated in an orbital time in this limit, ensuring that \( F_{\text{full}} \) is satisfied. In the opposite limit, \( \Delta J \ll J_{lc} \), the loss cone refills much more gradually than it is depleted. Once in the loss cone, stars have little chance of being scattered out in an orbital period. They are disrupted as they pass through pericenter and the loss cone remains mostly empty. This is known as the diffusion limit, where stars must diffuse toward the loss cone over many orbital periods (Lightman and Shapiro, 1977).

The flux into the loss cone in the diffusion limit is less than the full loss cone rate, since the loss cone will be depleted. We define the parameter \( R_0 \) to specify the low angular momentum limit above which orbits are typically populated,

\[ \frac{R_0(\varepsilon)}{R_{lc}(\varepsilon)} = \begin{cases} \exp(-q) & \text{if } q(\varepsilon) > 1, \\ \exp(-0.186q - 0.824\sqrt{q}) & \text{if } q(\varepsilon) < 1, \end{cases} \]  

(2.25)

where \( R \equiv J^2/J_{lc}^2 \) (Magorrian and Tremaine, 1999). This allows us to calculate the loss cone flux in the transition between the two limiting cases, where loss cone orbits are partially repopulated in an orbital period. The flux of stars into the loss cone is given
by

$$F_{lc}(\varepsilon) = 4\pi^2 \Delta J^2(\varepsilon) \frac{f(\varepsilon)}{\ln(R_0^{-1})}$$

(Magorrian and Tremaine, 1999). In the pinhole limit, $R_0(\varepsilon) \to 0$, and $F_{lc}(\varepsilon) \to F_{\text{full}}(\varepsilon)$. For more tightly bound orbits corresponding to the diffusion limit we have $R_0(\varepsilon) \to R_{lc}(\varepsilon)$ and $F_{lc}(\varepsilon) \ll F_{\text{full}}(\varepsilon)$. Finally, the integrated flux of stars into the loss cone is obtained by integrating $F_{lc}(\varepsilon)$ over $\varepsilon$,

$$\dot{N} = \int_0^{\varepsilon_{\text{max}}} F_{lc}(\varepsilon) d\varepsilon. \quad (2.27)$$

Figure 2.10 shows the loss cone flux for the simplified nuclear cluster model described here. The peak of the flux comes from $\varepsilon \sim \varepsilon_h$, which corresponds to orbits with semi-major axes comparable to the black hole’s sphere of influence, $r_h$. At energies lower than the peak, the pinhole limit is applicable, and $F_{lc} \sim F_{\text{full}}$. At high energies, $F \ll F_{\text{full}}$ because the loss cone is only partially refilled each orbital period.

### 2.5.1.3 Modification of $F_{lc}$ by direct stellar collisions

In the crowded nuclear cluster environment, the large geometric cross-section of giant stars leaves them susceptible to destruction through direct collisions with other stars. The effect of these collisions on $F_{lc}$ is most pronounced in the diffusion limit, where stars must complete $N_{\text{orb}} \sim 1/q(\varepsilon) = t_J(\varepsilon)/P(\varepsilon)$ orbits to be able to diffuse a total $\Delta J = J_{lc}$ in angular momentum space. To estimate the importance of collisions, we compute the probability of survival, $P_{\text{surv}}$, during one angular momentum diffusion time, $t_J$. The collision cross section is

$$\Sigma(R_{\text{min}}, r) = \pi R_{\text{min}}^2 \left[ 1 + \frac{2G(M_1 + M_2)}{R_{\text{min}} \sigma^2(r)} \right], \quad (2.28)$$
Figure 2.10: Loss cone flux $F_{lc}$ of solar-type stars for the simplified nuclear cluster model of Section 2.5.1. Orbital binding energy is inversely proportional to semi-major axis $\varepsilon \propto 1/a$, and $\varepsilon_{bh}$ is the energy corresponding to the radius of influence of the black hole, in this case $r_{bh} \sim 1$ parsec. At low $\varepsilon$, $\Delta J \gg J_{lc}$, and the loss cone is refilled on an orbital timescale. In this limit, $F_{lc}$ follows the full loss cone rate. At higher $\varepsilon$, $\Delta J \ll J_{lc}$, and orbits with $J < J_{lc}$ are incompletely repopulated in an orbital period. In this limit, $F_{lc}$ is set by the diffusion rate in orbital angular momentum. At $\varepsilon \gg \varepsilon_{bh}$, a star must complete $N_{\text{orb}} \sim J_{lc}/\Delta J$ orbits to diffuse one loss cone angular momentum, $J_{lc}$. At each pericenter passage stars are vulnerable to collisions, leading to a collisional truncation of $F_{lc}$ where $N_{\text{orb}} \gg 1$. 

[Diagram showing loss cone flux $F_{lc}$ as a function of $\varepsilon$, with regions marked by $\Delta J \gg J_{lc}$ and $\Delta J \ll J_{lc}$, and a star orbit showing $\varepsilon_{bh} = 10^6 M_\odot$.]
where $R_{\text{min}} = R_1 + R_2$ and the two stars are represented by subscripts 1 and 2. We approximate low angular momentum orbits (which may diffuse to loss cone orbits within $t_J$) as radial paths, $s$, through the cluster from apocenter to pericenter and back again. The typical number of collisions suffered during $t_J$ is then

$$N_{\text{coll}}(t_J) = \frac{2}{q(\varepsilon)} \int_{r_p}^{2a(\varepsilon)} \Sigma(R_1 + R_2, s)\nu(s)ds,$$  \hspace{1cm} (2.29)

and the probability of survival is $P_{\text{surv}}(t_J) = \text{Max}[0, 1 - \eta N_{\text{coll}}(t_J)]$, where $\eta$ represents the efficiency of collisions in destroying stars.

The importance of collisions can be seen in Figure 2.10 (which considers solar-type stars). At high orbital binding energies, $\varepsilon$, $F_{\text{lc}}$ is truncated by the strong likelihood of stars in low $J$ orbits suffering a collision in $t_J$. In Figures 2.10 and 2.11, we take $\eta = 1$, meaning that all stars are destroyed by any direct collision. This is a conservative estimate, since studies of stellar collisions have shown that grazing collisions only result in fractional mass loss (Freitag and Benz, 2005; Dale et al., 2009). However, because the collisional break in $F_{\text{lc}}(\varepsilon)$ is so steep, varying the collisional efficiency has little effect on the loss cone flux.

An effect we do not consider is that strong encounters, even if they are not direct collisions, result in a large-angle scattering of stars either toward or away from the loss cone. These encounters deviate stars from their previous random walk as dictated by the sum of distant encounters in the Fokker-Planck formalism. In fact, the chaotic effects of the strongly collisional regime (Frank and Rees, 1976) are most realistically captured by Monte Carlo or N-body formalisms (Freitag and Benz, 2002; Brockamp et al., 2011). The result may be a slightly different feeding rate than the Fokker-Planck formalism predicts for tightly bound stars, but we operate under the assumption that as long as the stars themselves are not destroyed, they may still be fed toward the loss cone in a future scattering.
2.5.1.4 Scaling of the loss cone flux with tidal radius

To determine the relative contribution of different stellar evolutionary stages to the tidal disruption rate, we consider the per star (specific) tidal disruption rate $\dot{n}$. The specific rate, $\dot{n}$, varies over the stellar lifetime with changing tidal radius, and, because we consider a cluster of homogeneous stellar constituents, $\dot{n} = \dot{N}/N$. In Section 2.3.3, we show that for an isotropic distribution of stars, the flux into the loss cone scales linearly with the tidal radius. This is only true because the loss cone is full, thus the rate depends on the size of the loss cone, and $\dot{n} \propto J_{lc}^2 \propto r_t$. If the loss cone is empty, changes in its size do not have an effect on the rate, which is set instead by the diffusion process. In the terminology of Lightman and Shapiro (1977), these are the pinhole and diffusion limits, respectively. Taking the relevant limits of equation (2.26), we have

$$F_{lc}(\varepsilon) \propto \begin{cases} J_{lc}^2 \propto r_t & \text{for } q(\varepsilon) \gg 1 \text{ (pinhole)}, \\ \ln(J_{lc}^2) \propto \ln(r_t) & \text{for } q(\varepsilon) \ll 1 \text{ (diffusion)}. \end{cases}$$

The exact scaling of the tidal disruption rate, $\dot{n}$, must be some combination of the contribution from these two limiting cases because it is an integral quantity in $\varepsilon$ that spans both limits.

The effect of the superposition of the full and empty loss cone limits within a particular nuclear cluster is shown in Figure 2.11. The left panel shows the change in the loss cone flux, $F_{lc}$, as $r_t$ increases by a factor of 10 and 100. In the portion of the cluster where $F_{lc} \sim F_{\text{full}}$, $F_{lc}$ scales linearly with $r_t$. In the empty loss cone limit $F_{lc}$ is approximately constant with increasing $r_t$, as expected. Because $J_{lc}$ is increasing while $\Delta J$ remains the same, the energy corresponding to $q(\varepsilon) = 1$ (roughly the peak in $F_{lc}$) moves to lower $\varepsilon$. Therefore, as the loss cone increases in size, stars are typically fed from less bound orbits that originate further from the black hole.

Specific tidal disruption rates, $\dot{n}$, can be calculated via the integrated loss cone
Figure 2.11: Scaling of the tidal disruption rate with increasing tidal radius. The left panel shows that with increasing $r_t$ in the $\Delta J \gg J_{lc}$ limit, $F_{lc}$ scales linearly with $r_t$, while in the $\Delta J \ll J_{lc}$ limit, $F_{lc}$ scales only weakly with $r_t$. We assume that increasing stellar radius is the mechanism causing $r_t$ to increase (because in stellar evolution the changes in mass are small). As a result, for larger $r_t$ collisions become increasingly important. The right panel shows the integrated rate per star, $\dot{n}$. For a wide range of tidal radii, the scaling is well described by a power law with slope, $\alpha = 1/4$. At very large $r_t$ collisions become dominant and $\dot{n}$ decreases, while for small $r_t$ stars are typically swallowed whole by the black hole because $r_s > r_t$, and $\dot{n}$ no longer depends on $r_t$. The shaded regions describe the relative fractions of swallowed (grey) versus flaring (blue) events for a range of black hole masses between $10^6 M_\odot$ and $10^{10} M_\odot$.

2.5.2 Impact of the structural diversity of observed galactic centers

In Section 2.5.1, we compute the relative likelihood of tidal disruption for stars of varying tidal radii under the assumption that the stellar density distribution...
is accurately described by a singular isothermal sphere. However, the central regions of observed galaxies exhibit somewhat more complex structure. Out of this complexity arises some degree of galaxy to galaxy variation, which we illustrate here.

2.5.2.1 Galaxy sample

Observations resolving the central SMBH’s sphere of influence in nearby, early type galaxies have been performed by the Nuker team using the Hubble Space Telescope (see Lauer et al. (1995) for early work and Lauer et al. (2007) for a more recent review). The radial surface brightness profile in these galaxies is found to be bimodal (Faber et al., 1997). Many of the most massive galaxies exhibit a core or flattening of the surface brightness profile at small radii. The presence of such cores is thought to be lingering evidence of gravitational heating from the inspiral of a pair of black holes following a major galactic merger (Faber et al., 1997). The surface brightness profile of less massive galaxies typically continues to rise at smaller radii up to the resolution limit of the observations and is well described by a single power law. These galaxies are termed power-law and exhibit central stellar densities tens to thousands of times higher than those inferred for core galaxies (Lauer et al., 2007). Nuker galaxy surface brightness profiles are fit to a parameterized, smooth function (the Nuker law) that describes the asymptotic slopes of the surface density profile at large and small radii relative to a break radius \( r_b \) (Byun et al., 1996). The rates of tidal disruption events in these galaxies are computed by Magorrian et al. (1998) and Magorrian and Tremaine (1999) and revised by Wang and Merritt (2004) to accommodate changes in inferred black hole masses.

2.5.2.2 Scaling of \( \dot{n} \) with \( r_t \)

To investigate the influence of the bimodal nature of galactic center profiles on the scaling of \( \dot{n} \) with \( r_t \), we follow the formalism of Magorrian and Tremaine (1999)
Figure 2.12: Scaling of the tidal disruption rate in a sample of 41 early type galaxies whose central surface density profiles were observed with HST and fit with parameterized Nuker laws (for the details of the sample see Wang and Merritt, 2004). In the left panel, we show the mass density of stars as a function of the density at the break radius, $r_b$, a parameter of the two-power Nuker fit. Inside $r_b$, galaxies show bimodal structures, with those hosting the most massive black holes typically having low density cores, whereas those with less massive black holes typically have dense central regions and single power law profiles. In the center panel, we plot the scaling of the specific tidal disruption rate $\dot{n}$ with $r_t$ for each of these galaxies, normalized to the rate of solar type stars, $\dot{n}_\odot$. Where the tidal radius is less than the Schwarzschild radius we still compute the rate for illustrative purposes, but color the lines in gray rather than by galaxy structure. Finally, the right panel shows the slope of the scaling curves plotted in the center panel as a function of $r_t$. There is more substantial galaxy to galaxy variation in the slopes of these scaling curves than there is between structural class of galaxies. However, in most cases, $d\ln(\dot{n})/d\ln(r_t) \ll 1$ and in many cases is broadly consistent with the $\alpha = 1/4$ scaling derived in the simplified model of section 2.5.1.
and Wang and Merritt (2004). We use the galaxy sample from Wang and Merritt (2004), who used data from Faber et al. (1997) and fit black hole masses based on the $M_{\text{bh}} - \sigma$ relation of Merritt and Ferrarese (2001). We show the deprojected mass density profiles of the galaxy sample used in our analysis in the left panel of Figure 2.12. At radii less that the break radius, $r_b$, the mass density profiles of core and power-law galaxies separate dramatically. Black holes with the highest inferred masses are typically found in core galaxies, rather than power-law galaxies. The center panel of Figure 2.12 shows the scalings of the specific tidal disruption rate, $\dot{n}$, which is plotted scaled to the specific disruption rate of solar type stars $\dot{n}_\odot$. Lines are colored here by the galaxies classification by Faber et al. (1997) as core or power-law, and we ignore collisions for simplicity. Where the tidal radius is less than the Schwarzschild radius, we still compute the $\dot{n}(r_t)$, but color the lines in gray.

We find substantial variation between galaxies: a $100R_\odot$ star is between 2 and 10 times more likely to be disrupted than a solar-type star depending on the galaxy in which it resides. The dashed lines in the center panel of Figure 2.12 show power law scalings $\dot{n} \propto r_t^\alpha$ with $\alpha = 0, 0.25, 0.5, 0.75$, and 1. Many galaxies fall in the range of $\alpha \sim 0.2 - 0.5$, broadly consistent with the simplified model presented in Section 2.5.1, which found $\alpha = 1/4$. Another effect is a general trend toward shallower slopes $d\ln(\dot{n})/d\ln(r_t)$ as $r_t$ increases, as seen in the right panel of Figure 2.12. The change in slope is most significant in core galaxies with massive black holes. The peak in loss cone flux moves out in the cluster as $r_t$ increases, feeding stars from regions of lower stellar number density. In core galaxies, the peak in $F_{\text{lc}}$ may cross $r_b$ with significant effect on the scaling of $\dot{n}$ arising from the change in slope of the stellar number density at the break radius. In general, we find that the tidal disruption rate appears to vary more from galaxy to galaxy than between core and power-law classes. In what follows, we will use the relatively weak scaling $\dot{n} \propto r_t^{1/4}$ as a value that is representative of the majority of galaxies, irrespective of their structure.
2.5.3 The stellar diet of SMBHs

We have calculated how the specific tidal disruption rate $\dot{n}$ changes as a function of $r_t$. The tidal radius of a star is a function of the initial stellar mass and age (see Figure 2.3.2), so one must integrate over a stellar lifetime to find the relative disruption probability for stars during different evolutionary stages. Because $\dot{n}$ is a cluster-integrated rate, we are implicitly assuming that stars of different evolutionary stages are distributed isotropically in energy space. The expectation value for the number of disruption events a particular star will experience during an evolutionary stage lasting from $t_1$ to $t_2$ is given by the integral of $\dot{n}(r_t)$ across that time period,

$$n_{12} = \int_{t_1}^{t_2} \dot{n}(r_t) \, dt,$$  \hspace{1cm} (2.31)

where $r_t$ is a function of $M_*(t)$ and $R_*(t)$. Similarly, the total lifetime integral is

$$n_{\text{tot}} = \int_{t_{\text{ZAMS}}}^{t_{\text{max}}} \dot{n}(r_t) \, dt,$$  \hspace{1cm} (2.32)

where $t_{\text{ZAMS}}$ is the age of the star at the zero-age main sequence, and $t_{\text{max}}$ is the age at the end of the star’s lifetime. Therefore, the fractional likelihood of disruption during a given evolutionary stage as a function of initial stellar mass is given by

$$f_{\text{stage}}(M_{\text{ZAMS}}) = \frac{n_{12}}{n_{\text{tot}}}.$$  \hspace{1cm} (2.33)

We can consider the probability of being disrupted at different evolutionary stages to be divided in this manner as long as most stars of a given $M_{\text{ZAMS}}$ do not experience a disruption during their lifetime ($n_{\text{tot}} < 1$). The fact that we observe giant stars in our own galactic center is a testament to the fact that many stars must survive without being disrupted during their entire lifetime (Genzel et al., 2010). This requirement is most likely to be satisfied for stars that are only weakly bound to the black hole (of
Figure 2.13: The stellar ingestion flaring diet of SMBHs is shown for a range of SMBH masses, and two scalings of \( \dot{n} \) with \( r_t \), \( \dot{n} \propto r_t^{\alpha} \). The fractional likelihood of tidal disruption during different stellar evolutionary stages, \( f_{\text{stage}} \), is plotted as a function of initial stellar mass, \( M_{\text{ZAMS}} \). Stars more massive than about 1 solar mass evolve to the post-MS in less than a Hubble time and these evolved stars contribute a significant fraction of disruption events. As \( M_{\text{bh}} \) increases, an increasing fraction of MS stars are swallowed whole rather than being tidally disrupted. Flaring events on the most massive black holes consist exclusively of post-MS stars.

which there are many and for which the black hole is an extremely small target). In this case, which corresponds to the peak in the feeding rate of giant stars, a typical star can easily survive without being disrupted over its entire lifetime, and we have \( \frac{N}{\dot{N}} \gg t_{\text{max}} \), indicating that \( n_{\text{tot}} \ll 1 \). However, our assumption is less justified when considering stars well inside the sphere of influence. Within this smaller reservoir of stars there is a relatively high probability of disruption integrated over the star’s lifetime. In this case, there is a non-negligible chance that the star would have already suffered a tidal interaction with the black hole before reaching a given evolutionary stage – either by evolving onto the loss cone (Syer and Ulmer, 1999) or by the typical diffusion process.

The fractional likelihood of tidal disruption during different evolutionary stages, equation 2.33, is shown in Figure 2.13 for a range of initial stellar masses, \( M_{\text{ZAMS}} \). We show how this flaring diet depends on the SMBH mass and the scaling of \( \dot{n} \) with \( r_t \). In calculating \( f_{\text{stage}} \), we assume that the stellar population is old enough that stars are
able to reach each of these representative evolutionary stages. The top panels in Figure 2.13 show the relative fractions of disrupted stars at different evolutionary stages given a $\dot{n} \propto r_t^{1/4}$ scaling, which we found to be representative of most galactic centers. The bottom panels show the result given the optimistic scaling of $\dot{n} \propto r_t^{1/2}$. For black holes of relatively low mass (the sort that typically inhabit power-law galaxies), disrupted stars are typically MS stars with giant stars contributing 10-40% of all disruption events. As black hole mass increases, an increasing fraction of MS stars are consumed whole rather than disrupted. The flaring diet of the most massive black holes consists nearly exclusively of post-MS stars. The disruption fractions corresponding to a population of stars can be constructed using the estimates of $f_{\text{stage}}$ given in Figure 2.13 by weighting them with the stellar population mass spectrum and age distribution.

2.6 Discussion

We have shown that the tidal disruption of stars while they are on the giant branch gives rise to longer-lasting accretion flares than when disruption occurs during the MS (Figure 2.8). Additionally, the stellar ingestion flaring diet of the most massive SMBHs consists exclusively of evolved stars (Figure 2.13). The question remains whether these long duration accretion events occur frequently enough for flaring activity to be a discernible characteristic of the most massive SMBHs. In this section, we use Monte Carlo realizations of flaring events to illustrate the defining properties and expected rates of flaring events as a function of black hole mass, their contribution to the accretion luminosity of local AGN, and the prospects for detecting AGN flares powered by the disruption of evolved stars.
Figure 2.14: In the left panel, we show the fractional composition of stars scattered into the loss cone, $f_N$, for a range of black hole mass between $10^6 M_\odot$ and $10^{10} M_\odot$. We show a sharp cutoff in the flaring rate for black holes more massive than $10^8 M_\odot$, beyond which MS stars are swallowed whole rather than tidally disrupted. Post-MS stars contribute some flaring events above this cutoff, but the weak scaling of $\dot{n}$ with tidal radius (Figure 2.11) dictates that these events are few compared to the flaring rate of lower mass black holes. The right panel shows the demographics of the flaring fraction only. These plots are based on a Monte Carlo realization of a single stellar mass ($1.4 M_\odot$) cluster with a flat distribution in stellar ages, and an $\dot{n} \propto r_t^{1/4}$ scaling law for the likelihood of individual events.
2.6.1 A limiting SMBH mass scale for tidal disruption flares

Giant stars are the only stars that can light up the most massive SMBHs by tidal disruption, since MS stars are swallowed whole rather than being tidally disrupted and subsequently producing a luminous accretion flare. Giant disruptions are, however, relatively infrequent due to the combination of the short lifetimes of evolved stars and the weak scaling of the tidal disruption rate with tidal radius \( \dot{n} \propto r_t^{1/4} \). Therefore, despite their large radii, evolved stars are unable to compensate for the drop in flaring rate at black hole masses where MS stars are swallowed whole. As a result, we find a sharp cutoff in the tidal disruption flaring rate around \( M_{\text{bh}} \approx 10^8 M_\odot \). This is illustrated in Figure 2.14, where, at low SMBH masses, the typical event is a MS disruption with evolved star disruptions contributing about 15%. As the black hole mass grows, an increasing fraction of MS stars are swallowed whole rather than disrupted, eventually leading to the sharp cutoff in the flaring rate seen in Figure 2.14.

The limiting black hole mass scale of \( 10^8 M_\odot \) coincides with a change in the SMBH flaring diet. In the right panel of Figure 2.14, the typical disrupted star at high black hole masses is a giant branch star. At the very high mass end \( M_{\text{bh}} \gtrsim 10^{9.5} M_\odot \), and even HB stars are swallowed whole. The transition in the typical evolutionary state of disrupted stars as a function of black hole mass may also be inferred from Figure 2.15, in which the average tidal disruption mass feeding curve is plotted as a function of SMBH mass. For a population of SMBHs with \( M_{\text{bh}} \lesssim 10^7 M_\odot \), the average \( \dot{M} \) curve is dominated by MS disruptions with some perturbation at late times due to disruptions of post-MS stars. The representative \( \dot{M} \) curves remain relatively unchanged until the black hole mass reaches the characteristic scale of \( 10^8 M_\odot \), where they transition rapidly to being dominated by post-MS disruptions. While post-MS \( \dot{M} \) curves peak near the Eddington rate for low mass black holes, they fall well below the Eddington rate for the most massive SMBHs, as illustrated in the center panel of Figure 2.15. Supplying mass
Figure 2.15: The left panel and center panels show the average of Monte Carlo realized flaring events in physical units and as a ratio to the Eddington rate, respectively. For simplicity, we take all events to be $\beta = 1.5$ encounters of a $1.4M_\odot$ ZAMS mass star. We draw events along the star’s lifetime using an $\dot{n} \propto r_x^{-1/4}$ scaling assuming a flat distribution of stellar ages in the nuclear cluster, then scale to the nearest of our simulations in Figure 2.8. As black hole mass increases, the dominant flaring event goes from a MS disruption to a post-MS disruption, with the transition being particularly sharp around $M_{bh} \sim 10^8 M_\odot$. Post-MS disruptions feed gas to the black hole over longer durations than MS disruptions at correspondingly lower peak accretion rates. When normalized to the Eddington accretion rate of the black holes, $\dot{M}_{\text{edd}} = 0.02c_{\alpha,1}^{-1}(M_{bh}/10^6 M_\odot) M_\odot \text{yr}^{-1}$, this amounts to a precipitous drop in the luminosity of tidal disruption powered SMBH flaring as black hole mass increases. The right panel shows the duty cycle of flaring luminosity, taking the overall loss cone flux from Wang and Merritt (2004), $\dot{N} \approx 6.5 \times 10^{-4} (M_{bh}/10^6 M_\odot)^{-1/4}$. Tidal disruption flares may be expected to contribute to the luminosity function of AGN harboring low mass black holes. However, they only contribute at very low luminosities relative to Eddington and with a low duty cycle in more massive SMBHs.
near the Eddington rate of black holes $\gtrsim 10^8 M_\odot$ via tidal disruption is not possible because of the transition to disruptions of stars exclusively in evolved stages.

Our finding of a limiting SMBH mass scale is at odds with the results of Syer and Ulmer (1999) who compute rates of giant consumption by SMBHs in Nuker galaxies and predict a high rate of disruption of giant stars in the most massive galaxies. This is because Syer and Ulmer (1999) consider a process in which the evolution of closely approaching stars results in mass transfer events with the black hole when they evolve off the MS. Contrary to their assumption, however, this process results in many weak encounters with the black hole ($\beta \ll 1$) rather than a single disruptive encounter. The sum of many encounters with the black hole may produce an interesting population of tidally-heated stars (Alexander and Morris, 2003; Alexander and Hopman, 2003), but, is unlikely to produce luminous flares.

2.6.2 Contribution of giant star disruptions to local low-luminosity AGN

Having established a limiting SMBH mass scale for tidal disruption flaring events, we examine how this mass limit manifests itself in the cumulative, low-luminosity output of local AGN. Tidal disruption flaring events are observed to rise sharply and then decay over long timescales approximately following the widely discussed $t^{-5/3}$ power law. The contribution of the tidal disruption of solar-type stars to the local AGN luminosity function was investigated by Milosavljevic et al. (2006) who found that the long decay tails of tidal disruptions can contribute to the integrated luminosity of AGN in gas-starved galaxies. The exact value of the tidal disruption rate as well as its evolution with redshift (which is rather uncertain; although see Merritt, 2009) determines the time at which gas-mode feeding of active galaxies drops to sufficiently

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3The orbital period of trapped stars about the black hole, $P \sim 10^4$ years, is much less than even the post-MS stellar evolution timescale, $\tau_g \sim 10^8$ years, resulting in very small per-orbit changes in stellar radius and impact parameter, $\beta$. 
low levels for the stellar disruption component to take over (e.g. Heckman et al., 2004).

For example, Milosavljevic et al. (2006) find that with a tidal disruption event of \( \dot{N} \approx 6.5 \times 10^{-4} \left( \frac{M_{\text{bh}}}{10^6 M_\odot} \right)^{-1/4} \), the associated flaring luminosity is likely to be responsible for at least 10% of local \((z \leq 0.2)\) AGN activity.

Observational AGN surveys have emphasized a picture of cosmic downsizing of black hole growth. The most massive SMBHs grew during the quasar era (Yu and Tremaine, 2002), while the black holes believed to be actively growing now are much less massive, around \(10^7 M_\odot\) (Heckman et al., 2004). Our finding of a limiting mass scale for tidal disruption powered flaring is consistent with the observation that most local, massive SMBHs are quiescent. Tantalizingly, the distribution of local AGN-hosting galaxies is biased to about an order of magnitude lower SMBH mass than the general galaxy population (Greene and Ho, 2007), with a decrease in activity around \(M_{\text{bh}} \approx 10^{8.5} M_\odot\) that lies coincident with the limiting SMBH mass scale for tidal disruption flares.

The right panel of Figure 2.15 examines the duty cycle of tidal disruption powered flaring in local AGN. Despite their lower frequency of disruption, giant star flares make a sizable contribution to the average duty cycle at low luminosities because their power law decay tails have higher normalization than those of MS stars (see Figure 2.8). The combined effects of the decrease in stellar consumption rate (Wang and Merritt, 2004), the precipitous drop in the flaring fraction (Figure 2.14) and the change in the demographics of disrupted stars (Figures 2.14 and 2.15) all conspire to ensure that tidal disruption powered flares contribute well below Eddington and with a very low duty cycle (a small number in the active state) on the most massive SMBHs.

The relative contributions of MS and evolved stars to the cumulative, tidal disruption-fueled accretion disk luminosity of local AGN depends on the total amount of stellar mass fed to SMBHs at rates below their corresponding Eddington limits. Super-Eddington fallback results in an inefficient conversion of fallback mass into light, as the
accretion rate onto the black hole (and, correspondingly, the accretion disk luminosity) is capped at or near the Eddington limit. Because their disruption results in highly super-Eddington feeding of lower mass SMBHs around $10^6 M_\odot$, MS disruptions are rather inefficient at converting mass stripped from the disrupted stars into accretion luminosity. Giant stars disruptions, on the other hand, peak at lower accretion rates and feed more of their mass to the black hole below the Eddington limit. Figure 2.16 shows the fractions of accretion disk luminosity contributed by stars at different evolutionary states. Giant star disruptions make up for in efficiency what they lose in disruption frequency when compared to MS stars and, thus, contribute significantly to the overall power of tidal disruption-fueled AGN.

By contrast, the contribution of giant stars to the relativistically jetted emission resulting from tidal disruption events, as discussed recently by De Colle et al. (2012), is expected to be relatively small. Because the jet luminosity is thought to track the rate at which mass is fed to the black hole, observed jet luminosities greater than $M_{\text{edd}} c^2$ are possible. In a magnitude-limited sample of tidal disruption events, the lower frequency and reduced feeding rate of giant disruptions as compared to MS disruptions indicate that MS disruptions are likely the dominant channel for the generation of high-energy, beamed emission resulting from the tidal disruption of stars.

### 2.6.3 Detection of giant star tidal disruption flares

The primary characteristic that distinguishes the tidal disruption flares of evolved stars from those of MS stars is their comparatively long timescales. Tidal disruption events are typically characterized in transient surveys by their late-time $\sim t^{-5/3}$ decay (see Gezari et al., 2009; van Velzen et al., 2011; Saxton et al., 2012, for some recent examples). However, as seen in Figure 2.8, We are unlikely to observe the character-

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The super-Eddington fallback phase may, however, give rise to outflows (rather than accretion of mass with low radiative efficiency) and a conversion of mass to light at larger radius as the ejected debris expands (Strubbe and Quataert, 2009, 2011; Kasen and Ramirez-Ruiz, 2010).
Figure 2.16: The fractional distribution of total tidal disruption fed accretion disk light emitted, $E_{\text{disk}}$, calculated by a Monte Carlo realization of the mass fed to SMBHs below their Eddington limit, again for a $1.4M_\odot$ star and $\beta = 1.5$ encounters. At lower SMBH masses, MS disruptions peak at highly super Eddington black hole feeding rates. As a result, MS disruptions inefficiently convert stellar mass loss into accretion luminosity. However, giant stars feed more of their mass to SMBHs below the Eddington limit, resulting in a more efficient conversion to disk luminosity. Thus, while post-MS disruptions occupy about 15% of disruption events by rate, in terms of total disk emission, they contribute nearly 40% to the ensemble. As the SMBH mass increases to $10^8M_\odot$, both MS and post-MS disruptions are sub-Eddington, and the fractional contributions to $E_{\text{disk}}$ are similar to the event rate convolved with the mass loss per event (see Figures 2.14 and 2.5, respectively).
Figure 2.17: The power-law slope inferred from two successive detections of hypothetical MS and post-MS tidal disruption flare transients given different observing cadences. Observing cadences that initially detect MS disruption events in the plateau or decay phases will catch the rise to peak in post-MS disruption flares. This plot assumes that flares above an intrinsic luminosity of $10^{43}$ erg s$^{-1}$ may be detected. This is roughly equivalent to the bolometric luminosity at which the recent tidal disruption flare PS1-10jh at $z = 0.1696$ was initially detected (Gezari et al., 2012). At the Eddington limit, we assume the luminosity of these flares plateaus but is convolved with a normally distributed variability factor with $\sigma = 0.01(\Delta t/1\text{yr})^{1/2}$. The flares are all assumed to occur at an impact parameter of $\beta = 1.5$. The post-MS curve is the average of the RGI and HB models (see Figure 2.8). In both cases, the lines plotted are the median while the shading shows the $\pm 1\sigma$ region for a Monte Carlo realization of detection times relative to the observing cadence. The inset plots show the detectable fraction of events, $f_{\text{det}}$, defined as those typically visible with at least two observations above the $10^{43}$ erg s$^{-1}$ threshold given the cadence.
istic power-law decay in the evolved cases due to their extremely long characteristic decay times. Our simulations show that the rise up to peak luminosity in giant star flares occurs over similar timescales as the decay in MS disruption flares. Figure 2.17 illustrates that typical observing cadences will detect giant star disruptions during their rise to peak brightness. Shown in the post-MS category in Figure 2.17 are the average mass feeding rates of the RG I and HB disruption simulations. Tidal disruptions of stars at the tips of the giant branches (the RG II and AGB models) result in flares with such long timescales that they would be difficult to identify in transient surveys. The power law slope during the rise phase is quite steep, indicating that the brightness of a galactic nucleus would increase in a correlated fashion by many orders of magnitude over a period of a few years.

AGN are infamous for exhibiting variability at a range of timescales, and one might worry that this would frustrate the detection of giant disruption flares. However, the power spectrum of these variations is well below the amplitude expected from the disruption of a post-MS star. an Velzen et al. (2011) perform a detailed analysis of potential sources of tidal disruption event confusion, including an empirical analysis of the amplitude of AGN flaring events in their Sloan Stripe 82 sample of 1304 AGN. They find that the largest amplitude flaring events exhibit changes in flux of $\Delta F/F \lesssim 5$, far less than the amplitude of the correlated rise expected from the disruption of an evolved star. More detailed studies of AGN variability also find small correlated increases (Kelly et al., 2009, 2011; MacLeod et al., 2010), giving credence to the idea that the tidal disruption of an evolved star would imprint a unique signal in the AGN flaring history.

The prospects for detecting giant star flares in the near future are, in fact, quite promising. While these events are less common than MS disruptions, they should be commonly detected in future transient surveys, like LSST, which are expected to yield hundreds of MS disruption events per year (Strubbe and Quataert, 2009).
insets in Figure 2.17 show that finding giant star disruption events does not require short cadence surveys. It is even possible that the signatures of giant star disruptions are lurking in current data sets and may be detectable through binning the data at lower cadences. Tidal disruption events with well sampled light curves that facilitate detailed comparison with theoretical $\dot{M}$ profiles offer the best hope of characterizing the physics of tidal disruption.

2.6.4 Caveats and prospects

In this work we have assumed that when the tidal radius is less than the Schwarzschild radius stars are swallowed whole without producing a flare. The impact parameter at which this transition occurs, however, is likely sensitive to the orientation of the encounter relative to the spin of the black hole (Kesden, 2012; Haas et al., 2012). The exact fraction of events which are promptly swallowed or produce flares thus depends on a general relativistic description of the black hole’s potential.

A great deal of uncertainty lies in the properties of the nuclear star clusters from which stars are fed into disruptive orbits. We know very little about stellar kinematics in galactic centers other than our own (e.g. Genzel et al., 2010). Our work has made the usual assumptions of a spherical nuclear star cluster which feeds stars to the black hole by a two-body relaxation-driven random walk in angular momentum space. Theoretical work has emphasized a wide variety of effects which may enhance the rate at which stars are fed towards the black hole. A triaxial potential can feed stars to the loss cone collisionlessly through chaotic, boxy orbits (Magorrian and Tremaine, 1999; Merritt and Poon, 2004; Merritt and Vasiliev, 2011). Rings or disks of stars, if present, would feed stars to the black hole at an enhanced rate through secular instabilities (Madigan et al., 2011). Another secular effect is resonant relaxation, which almost certainly enhances the loss cone flux to some extent for the most bound stars (Magorrian and Tremaine, 1999; Hopman and Alexander, 2006; Gürkan and Hopman, 2007). Fi-
nally, a second massive body, like an insprialng black hole or a giant molecular cloud could easily induce large angle scatterings of stars (Ivanov et al., 2005; Perets et al., 2007; Chen et al., 2008, 2009, 2011; Wegg and Bode, 2011). All of these processes result in more completely filled loss cone than the conservative scenario we’ve outlined here.

Further, even the simplest characteristics of galactic center stellar populations, like the age and mass distributions of stars, are wildly uncertain. Stars are recycled from the galaxy in general as the interaction between tightly bound stars and the more weakly bound galactic stellar population lead to a time evolution of the nuclear cluster compactness and properties (Merritt, 2009). In fact, our own galactic center is marked by a striking and enigmatic population of massive, young stars in the central \( \sim 0.1 \) pc (e.g. Schödel et al., 2007) that lies in contrast to the underlying, old stellar population. There are various alternative lines of evidence further suggesting that the initial mass function (IMF) of stars surrounding our own galactic center may be very different from that representative of the bulk of the star formation (Maness et al., 2007; Bartko et al., 2010).

The dynamics of a cluster comprised of a spectrum of stellar masses is also expected to operate somewhat differently than that of a single mass cluster. In power law galactic centers, which are presumed to be dynamically relaxed (Bahcall and Wolf, 1976; Bahcall and Wolf, 1977), one expects that massive stars, binaries, and stellar remnants to segregate to tightly bound orbits over the cluster’s relaxation time. An illustrative example of these effects comes from simulations of globular clusters harboring intermediate mass black holes (IMBHs), which have been performed with direct N-body integrations (e.g. Baumgardt et al., 2004a,b; Trenti et al., 2007a) and Monte Carlo techniques (e.g. Gürkan et al., 2004). For example, Baumgardt et al. (2004b) consider a multi-mass stellar cluster and find that the massive stars segregate to form a tightly bound cusp around the IMBH, while the less massive stars relax to a shallower distribution. The scaling of the specific tidal disruption rate, \( \dot{n} \), is modified
by the varying radial profile from which stars are fed. In this case, more massive stars are fed into the loss cone preferentially as compared to low mass stars, and the rates of giant star disruption in their simulations are 15-20% of the total stellar disruption rate, a factor of $\sim 2$ greater than one derives from a simple weighting over the stellar mass spectrum.

The detection of tidal disruptions holds great promise for revealing the properties of quiescent SMBHs and the nuclear star clusters that surround them, which, as we have discussed, are rather unconstrained. The universe is now being explored in a panchromatic way over a range of temporal scales, leading toward a more complete and less biased understanding of its constituents. Flares from the disruption of stars in all evolutionary states will certainly be observed in the not-too-distant future. The recent observations of PS1-10jh presented by Gezari et al. (2012) offer a glimpse into disruptions of stars not on the MS. Taken in a statistical sense, the observed rates of tidal disruption and, in particular, the relative rates of disruptions of different stellar evolutionary stages, will hold tremendous distinguishing power in both the dynamical mechanisms typically operating in galactic centers and the properties of the populations of stars themselves. Will the most common tidal disruption flares originate from young, massive stars in disks? Or from old stars fed from the near the edge of the black hole’s sphere of influence? It is within the capacity of planned surveys to obtain lightcurves of giant star tidal disruption flares that are well resolved temporally. These lightcurves will challenge our understanding of the physics of tidal disruption, and through comparison with simulated events, offer a window into the extreme nature of close encounters between stars and SMBHs.
Chapter 3

Spoon-Feeding Giant Stars to Supermassive Black Holes: Episodic Mass Transfer From Evolving Stars and Their Contribution to the Quiescent Activity of Galactic Nuclei

3.1 Chapter Abstract

Stars may be tidally disrupted if, in a single orbit, they are scattered too close to a supermassive black hole (SMBH). Tidal disruption events are thought to power luminous but short-lived accretion episodes that can light up otherwise quiescent SMBHs in transient flares. Here we explore a more gradual process of tidal stripping where stars approach the tidal disruption radius by stellar evolution while in an eccentric orbit. After the onset of mass transfer, these stars episodically transfer mass to the SMBH every pericenter passage giving rise to low-level flares that repeat on the orbital timescale. Giant stars, in particular, will exhibit a runaway response to mass loss
and “spoon-feed” material to the black hole for tens to hundreds of orbital periods. In contrast to full tidal disruption events, the duty cycle of this feeding mode is of order unity for black holes \( M_{\text{bh}} \gtrsim 10^7 M_\odot \). This mode of quasi-steady SMBH feeding is competitive with indirect SMBH feeding through stellar winds, and spoon-fed giant stars may play a role in determining the quiescent luminosity of local SMBHs.

3.2 Introduction

The prevalence of quasar activity at early epochs provides evidence that supermassive black holes (SMBHs) must lurk in the centers of many galactic halos (Soltan, 1982). Yet, in the local universe the vast majority of galactic center SMBHs exhibit little activity. Recent study has revealed that many of these SMBHs are likely shining due to mass accretion, but only at a tiny fraction of their Eddington luminosities, \( L/L_{\text{Edd}} \ll 1 \) (Ho, 2009). To best understand the origin of the low observed Eddington ratios of local SMBHs, it is important to develop a census of the processes that combine to establish a minimum, “floor”, feeding level, \( \dot{M} \). This floor accretion rate determines the most typical level of SMBH activity and therefore gives rise to the quiescent luminosity of galactic nuclei, where \( L = \eta \dot{M} c^2 \). In galactic nuclei devoid of gas, any potential fuel comes solely from the dense stellar clusters that surround SMBHs, thus stars alone serve to establish a lower limit of SMBH activity. By constructing an accurate census of fuel sources arising from the stellar distribution, we can eventually constrain the accretion efficiency \( \eta \) in an effort to better understand the accretion flows onto these SMBHs.

Stars feed the black hole in two primary ways, directly and indirectly. Indirect feeding arises from processes that inject material into the nuclear cluster medium, as occurs with stellar winds and stellar collisions (Holzer and Axford, 1970; Coker and Melia, 1997; Loeb, 2004; Quataert, 2004; Cuadra et al., 2005, 2006, 2008; Freitag and Benz, 2002; Volonteri et al., 2011; Rubin and Loeb, 2011). To reach the SMBH, material
fed indirectly into the cluster medium must overcome a further barrier to accretion in the form of feedback from the stars themselves and the SMBH (e.g. Blandford and Begelman, 1999; Quataert, 2004; Shcherbakov et al., 2013).

Direct feeding of the SMBH results from tidal interactions between stars and the SMBH. Tidal interactions result in a dynamically assembled disk (e.g. Bogdanović et al., 2004; Guillochon et al., 2013), which is relatively invulnerable to the feedback processes which plague our understanding of indirectly fed accretion mechanisms. Stars passing within approximately a tidal radius of the SMBH, \( r_t \equiv \left( \frac{M_{\text{bh}}}{M_*} \right)^{1/3} R_* \), where \( M_* \) and \( R_* \) are the stellar mass and radius, will experience strong tidal distortions and may be partially or completely destroyed by the black hole’s tidal field (e.g. Hills, 1975; Rees, 1988). Half of the tidally stripped debris of tidal disruption eventually falls back to the SMBH, forms a disk, and viscously accretes. Full tidal disruptions of main-sequence (Guillochon and Ramirez-Ruiz, 2013) and giant stars (MacLeod et al., 2012) produce luminous flares (e.g. Evans and Kochanek, 1989; Strubbe and Quataert, 2009, 2011; Ramirez-Ruiz and Rosswog, 2009; Lodato et al., 2009; Guillochon et al., 2013), but the duration of flares is generally short compared to their repetition time, \( \sim 10^4 \) yr (Rees, 1988; Magorrian and Tremaine, 1999; Wang and Merritt, 2004). In quiescence, the accretion rate to the SMBH is determined by the late time fallback of tidal debris (Milosavljevic et al., 2006), which decays roughly as \( \dot{M} \propto t^{-5/3} \). While the average accretion rate is relatively large, \( \sim \frac{M_{\odot}}{t_{\text{repeat}}} \sim 10^{-4} M_{\odot} \text{yr}^{-1} \), the rapid decline in the fallback after peak results in a median accretion rate that is much lower, \( \sim \dot{M}_{\text{peak}} \left( \frac{t_{\text{repeat}}}{t_{\text{peak}}} \right)^{-5/3} \sim 10^{-9} M_{\odot} \text{yr}^{-1} \), assuming typical parameters for a main-sequence star.

In this paper, we study a mechanism that does not result in luminous flares but can fill in between the tails of tidal disruption events and result in much higher median accretion rates. This process is the mass transfer that ensues when a giant star grows, over the course of many orbital periods, such that its tidal disruption radius becomes...
comparable to its orbital pericenter distance. Because of the large disparity between $r_t$ for a main-sequence star and $r_t$ for a giant star, there exist many main-sequence star orbits that pass safely within the giant star tidal radius at pericenter. While on the main sequence, a star in such an orbit experiences little disturbance from the black hole’s tidal field. However, as the star evolves off of the main sequence it expands, and, as a result, its mean density drops. With each passing orbit, the star therefore feels the tidal forcing from the SMBH with increasing strength. Eventually, the star is distorted to the point that a fraction of its envelope mass is removed at pericenter.

As the star evolves up the giant branch, its recently developed dense core helps protect it against complete disruption (Hjellming and Webbink, 1987; MacLeod et al., 2012; Liu et al., 2013), and the surviving remnant therefore returns to pericenter after each orbital period. The adjustment of the star’s structure to the mass loss it undergoes determines the strength of these subsequent encounters and the number of orbits over which the giant’s envelope is depleted. Stars that undergo many passages by the SMBH are altered by these encounters (Alexander and Hopman, 2003; Alexander and Morris, 2003; Alexander, 2005; Li and Loeb, 2013), and the star’s history of encounters with the SMBH will determine the nature of the subsequent passages. It is worth noting that an orbital history where the star returns to pericenter many times is distinct from the single-passage encounters that have received the majority of focus in previous studies of tidal disruption in galactic nuclei. Recent studies of the tidal disruption of objects on eccentric orbits have looked at giant planets (Guillochon et al., 2011), repeating flares from stars deposited into tightly bound orbits through binary disruption (Antonini et al., 2011), and the fallback properties of tidal debris in eccentric disruptions (Hayasaki et al., 2012).

Giant stars that repeatedly transfer small amounts of their envelope mass to the SMBH (which we call “spoon-feeding”) do so over many orbital periods. As a result, this channel of SMBH feeding results in a quasi-steady feeding rate to the black hole, in
contrast to the highly peaked feeding due to the tidal disruption of stars. We find that as a result of effectively spreading the bulk of their mass over longer feeding timescales than typical tidal disruption events, spoon-fed giant stars may play a significant role in determining the quiescent luminosity of local SMBHs. The feeding that results from these mass-transferring stars is competitive with the amount of mass fed indirectly to the SMBH by stellar winds.

This paper is organized as follows. In Section 3.3, we discuss the onset of mass transfer resulting from the evolution of a giant star trapped in an elliptical orbit and the SMBH. In Section 3.4, we show that the star episodically spoon-feeds mass to the SMBH over the course of many pericenter passages. In Section 3.5, we estimate the expected population of these trapped stars and estimate the rate at which they evolve to feed mass to the SMBH. In Section 3.6, we discuss the effects of these mass-transfer events on the floor activity level and duty cycle of local, tidally-fed SMBHs. In Section 3.7, we conclude and offer prospects for future study.

3.3 Mass transfer from evolving stars

A main-sequence star in an orbit that passes within the maximum red giant tidal radius at pericenter may survive for a long time relatively unperturbed by the black hole. Eventually, the star leaves the main sequence and evolves up the giant branch, at which point its radius expands and its mean density drops. At each pericenter passage the evolving star feels the tidal force of the black hole with increasing strength. Finally, the star grazingly begins to lose mass at pericenter. In this section we present a hydrodynamical simulation of this first disruptive passage. We will use this simulation to study the effects of the encounter on the surviving stellar core and to motivate a semi-analytical model for the subsequent passages in Section 3.4.

Previous analytic work and numerical simulations of mass transfer episodes in
eccentric binaries have focused primarily on the context of stellar mass binaries (Regős et al., 2005; Sepinsky et al., 2007; Sepinsky et al., 2009; Lee et al., 2010; Sepinsky et al., 2010; Lajoie and Sills, 2011; East et al., 2012; East and Pretorius, 2012; Davis et al., 2013). Recently, Faber et al. (2005) and Guillochon et al. (2011) have numerically explored higher mass ratio eccentric encounters in the context of the orbital dynamics and disruption of giant planets in eccentric orbits about their parent stars. Antonini et al. (2011) have speculated about the fate of stars that are dynamically deposited on tightly bound orbits through binary star disruptions, while Hayasaki et al. (2012) and Dai et al. (2013) have numerically studied the fallback properties of tidal debris in eccentric disruptions.

In Figure 3.1, we present a simulation of a grazing encounter between a giant star and the SMBH preformed in the FLASH hydrodynamics code (Fryxell et al., 2000) using the method described in detail in MacLeod et al. (2012). Our formalism is based on the FLASH4 code in Newtonian gravity and follows the encounter in the frame of the star (Guillochon et al., 2009, 2011; MacLeod et al., 2012; Liu et al., 2013; Guillochon and Ramirez-Ruiz, 2013). Our initial stellar model is a nested polytrope representative of a 1.4$M_\odot$, 50$R_\odot$ red giant with a 0.3$M_\odot$ dense core. A core mean molecular weight of twice that of the envelope fluid produces a relatively inert core. The structure of both the core and envelope are $n = 1.5$ polytropes. The adiabatic fluid gamma is $\Gamma = 5/3$ for the envelope gas, and it is $\Gamma = 5$ to model the tidally unperturbed core. The star is initially resolved by 90 grid cells in radius. After the encounter, grid refinement adaptively follows the density of the stripped gas.

Even in a relatively grazing encounter, the star is subject to a rapidly time-varying potential at pericenter (Regős et al., 2005). It is non-linearly distorted and some portion of its envelope mass may be unbound from the stellar core. This material is ejected from the star in two tidal tails, one of which is bound to the black hole, while the other is ejected on hyperbolic trajectories. The amount of mass lost depends on
the impact parameter of the encounter, which is defined by the ratio of the star’s tidal radius to the pericenter of its orbit, \( \beta \equiv r_t/r_p = (R_*/r_p)(M_{bh}/M_*)^{1/3} \). The giant star in Figure 3.1 encounters the black hole with \( \beta = 0.6 \) and loses \( \Delta M \approx 10^{-2}M_\odot \). A linearized approach to determining the degree of mass loss at pericenter by calculating the degree to which the stellar envelope overflows its effective Hill sphere at pericenter, \( r_p(M_*/M_{bh})^{1/3} \), would suggest that no mass is lost at these grazing \( \beta \), where the star is still a factor of \( \sim 2 \) smaller than its Hill sphere. Thus it is extremely important to account for the non-linear distortion of the star, even when the degree of mass loss is very small. We discuss a simple model for the degree of mass loss as a function of \( \beta \) in Section 3.4.

Some of the material originally ejected into the tidal tails during the encounter will eventually fall back to the stellar surface. The insets of Figure 3.1 show the state of the remnant post-encounter. As a fraction of the stripped material falls back to the oscillating and rotating stellar envelope, spiral shocks are generated. These shocks heat a tenuous layer of envelope material with mass similar to the fallback mass (\( \lesssim \Delta M \)) that adiabatically expands to extend significantly beyond the initial stellar radius (upper inset panel). By contrast, the interior portion of the star, \( r \ll R_* \), is not heated by shocks in encounters where \( \Delta M \ll M_* \). Our adiabatic simulation does not capture the radiative cooling of this material which extends to beyond the initial stellar radius. The lower inset panel of Figure 3.1 shows that the local photon diffusion time (approximated as \( \tau_{\text{diff}} \sim \rho \kappa_{\text{es}} R_*^2 / c \)) through these outermost heated layers is very short, much less than an orbital period, which we will denote \( \tau_{\text{orb}} \). We therefore expect these outermost layers to cool effectively despite the heating due to interaction with the fallback from the debris streams. As a result, this fallback heating should be a small perturbation to the remnant’s structure.

Additional heat may be deposited into the stellar interior through the dissipation of oscillation energy or interaction with gas in the circum-black hole medium.
Figure 3.1: Mass stripping from a star in an eccentric orbit around a SMBH. The main panel shows the formation of black hole bound and unbound tidal tails. Bound material streams back pericenter where it will circularize and drain into the SMBH. The upper inset shows a layer of stellar envelope heated by spiral shocks that originate from the remnant’s interaction with material falling back from the tidal tails. The lower inset shows that despite this heating, the photon diffusion time through these tenuous layers is short enough that the envelope cools in much less than a typical orbital period, $\tau_{\text{orb}}$. Our initial stellar model is a nested polytrope representative of a $1.4M_\odot$, 50$R_\odot$ red giant with a 0.3$M_\odot$ dense core. The simulation shown was computed at a smaller mass ratio ($M_{bh} = 10^4M_\odot$) and with lower eccentricity $e = 0.8$ than the encounters described in the text in order to illustrate the fallback and circularization processes. The scalebar is in units of $R_\ast = 50R_\odot$ in this case.
However, the tidal heating effect has been studied in detail by Li and Loeb (2013) and was shown to be a small perturbation to stellar structure for timescales comparable to the star’s red giant branch lifetime. This is partially because heat deposited into a giant star’s envelope (rather than its core) is easily radiated due to the short diffusion time through the envelope (McMillan et al., 1987). Interactions between the star and the remnant disk at pericenter may also heat the stellar envelope through shocks (e.g. Armitage et al., 1996; Dai et al., 2010). In the case considered here, this effect is likely to be of small importance because the disk mass will typically only be of order $10^{-2} M_\odot$ (See Figure 3.2), spread to extremely low density over the tidal sphere. These factors suggest that the state of the star in its subsequent encounters with the SMBH will be dominated by the stellar structure’s response to the mass loss experienced, rather than the effects of extra heating or orbital evolution.

Following a passage by the SMBH, changes in the remnant’s orbit will alter the properties of subsequent encounters. Of particular importance in determining the strength of the encounter is the orbital angular momentum, which determines the pericenter distance. There are several effects which can potentially modify the orbital energy and angular momentum of the remnant. First, due to the cumulative effect of encounters with other stars in the stellar cusp around the SMBH, the remnant’s orbit undergoes a random walk in orbital energy and angular momentum. In Section 3.5, we define the phase space of stellar orbits for which this random walk is small.

Second, any asymmetry in the mass ejection between the tidal tails results in a change in the orbital energy of the surviving remnant (Faber et al., 2005; Guillochon et al., 2011; Liu et al., 2013; Cheng and Evans, 2013; Manukian et al., 2013). This change in energy maximizes around the star’s own specific binding energy, $E_s \approx GM_*/R_*$, for coreless stars and deep encounters (Cheng and Evans, 2013; Manukian et al., 2013), but it is strongly limited by the presence of a stellar core, as is the case in giant planets (Liu et al., 2013).
Finally, non-radial oscillations are excited in the remnant following the encounter leading to a transfer of orbital energy and angular momentum into stellar oscillation energy and angular momentum. The magnitude of these perturbations are typically a fraction of the star’s binding energy or breakup angular momentum. This result was analytically predicted by Press and Teukolsky (1977), and has more recently been numerically explored in the case of objects without (Guillochon et al., 2011; Cheng and Evans, 2013) and with (Liu et al., 2013) cores. In the case of giant stars interacting with SMBHs on bound orbits, the star’s orbital energy and angular momentum are both large compared to the giant star’s binding energy and maximum rotational angular momentum. Typical values for these ratios of orbital binding energy to stellar binding energy are

$$\frac{E_{\text{orb}}}{E_*} \approx \frac{R_*}{a} \frac{M_{\text{bh}}}{M_*} \approx 11 \left( \frac{R_*}{50R_\odot} \right) \left( \frac{a}{1\text{pc}} \right)^{-1} \left( \frac{M_{\text{bh}}}{10^7M_*} \right),$$

where $a$ is the orbital semi-major axis, and $E_{\text{orb}} = GM_{\text{bh}}/(2a)$. By a similar analysis, the ratio of the orbital angular momentum, $J_{\text{orb}} \approx \sqrt{2GM_{\text{bh}}r_p}$, to breakup rotational angular momentum of the star, $J_* \approx \sqrt{GM_*R_*}$, is of order

$$\frac{J_{\text{orb}}}{J_*} \approx \left( \frac{M_{\text{bh}}}{M_*} \right)^{2/3} \approx 5 \times 10^4 \left( \frac{M_{\text{bh}}}{10^7M_*} \right)^{2/3},$$

if the substitution that the pericenter distance equals the tidal radius, $r_p = r_t$ is made (which leads to the lack of dependence on the stellar radius, $R_*$, in the above expression). Since the orbital quantities are much larger than the maximum reservoir of binding energy or rotational angular momentum available in the giant star, tidal excitation cannot induce substantial changes in the orbit.

Interestingly, all of these processes result in only small perturbations to the giant star’s orbit. The orbital parameters of the bound giant star remnant are therefore essentially unchanged following the passage by the SMBH. As a result, in subsequent orbits the star will return to the same pericenter distance with a similar orbital period.
3.4 Episodic flares over many pericenter passages

In this section, we model the encounter history of a giant star on a bound orbit with the SMBH after the onset of mass transfer. Previously, we argued that the remnant’s orbit is essentially unchanged by the encounter with the SMBH. Therefore, we can determine the mass lost each pericenter passage by calculating the changes in the stellar structure and comparing the pericenter of the star to its new tidal radius through the impact parameter, $\beta$. We adopt a model that combines an analytic description of the degree of mass loss and its return to the black hole with a stellar evolution calculation of the adjustment of the mass-losing star’s structure. This approach is necessary to explore these multiple passage encounters because the range of timescales between the star’s dynamical time and a typical orbital period make a full hydrodynamic calculation prohibitively computationally expensive. Recent work by Zalamea et al. (2010) has similarly adopted an analytic model to study runaway flares from the progressive disruption of a white dwarf by an intermediate mass black hole.

3.4.1 Mass Stripping

To predict the degree of mass loss at each passage as a function of pericenter distance, we adopt a simple approximating formula motivated by simulation results from MacLeod et al. (2012) and Guillochon and Ramirez-Ruiz (2013),

$$\Delta M(\beta) = f(\beta) \left( \frac{M_s - M_c}{M_s} \right)^2 M_s,$$

(3.3)
where $M_c$ is the core mass and

$$f(\beta) = \begin{cases} 
0 & \text{if } \beta < 0.5, \\
\beta/2 - 1/4 & \text{if } 0.5 \leq \beta \leq 2.5, \\
1 & \text{if } \beta > 2.5.
\end{cases} \quad (3.4)$$

This parameterization captures two critical features of the simulation results. First, convective stars with condensed cores begin to lose mass around $\beta \sim 0.5$ – a much more grazing encounter than linear models of the mass loss would predict. Yet, the tidal stripping does not reach its maximum until much deeper encounters, around $\beta \sim 2.5$. Second, while only the envelope material is susceptible to disruption, the increased influence of the core makes it more difficult to remove envelope material when the core is a larger mass-fraction of the star (thus the squared dependence on $M_{\text{env}}/M_c$, see Figure 5 of MacLeod et al., 2012).

Each mass loss episode results in a readjustment of the star’s structure and therefore a new effective impact parameter with each pericenter passage. The importance of the adjustment of the mass-losing star’s structure in the context of extreme mass ratio circular binaries has been demonstrated by Dai et al. (2013) and Dai and Blandford (2013). We calculate the changes to the stellar properties using the MESA stellar evolution code (Paxton et al., 2011, 2013). Our stellar models are non-rotating, and the only source of mass loss is the interaction with the black hole. In the MESA models, we allow the star to adjust to the mass loss continuously by applying an effective stellar wind that carries away the outermost envelope material at a rate $\dot{M} = \Delta M/\tau_{\text{orb}}$, recalculated each pericenter. Timesteps are chosen such that each orbital period, $\tau_{\text{orb}}$, is resolved by ten steps, but our results are not sensitive to this choice.

Figure 3.2 shows our results for a star that is $1.4M_\odot$ and $50R_\odot$ at the onset

\footnote{version 4849}
of mass loss. Due to the giant star’s isentropic envelope, it first becomes less dense
upon losing mass, resulting in a runaway process in which successive encounters are
increasingly disruptive. The lower panel shows the corresponding $\Delta M$ at each pericenter
passage. Eventually, when much of the star’s hydrogen envelope has been stripped, the
core becomes the dominant gravitational force in the star’s structure, and the mean
density of the object increases again with subsequent mass loss episodes. This quenches
the runaway mass loss, and $\Delta M$ decreases in subsequent passages. Stars in longer
orbital periods lose mass with decreasing adiabaticity. These stars evolve farther up
the giant branch each orbit, resulting in a slow increase in the hydrogen shell-burning
luminosity at the core-envelope interface. This evolution drives these stars to undergo
stronger encounters with the SMBH and leads to their envelopes being stripped in fewer
orbital periods.

Also in Figure 3.2, we make a direct comparison with the mass loss history
that would be realized for the adiabatic evolution of a nested polytrope of $1.4M_\odot$ and
$50R_\odot$ with a $0.3M_\odot$ condensed core. Here we compute the star’s mass-radius relation
as $R_* / R_0 = (M_* / M_0) \xi_{\text{ad}}$, where $\xi_{\text{ad}}$ is given by an approximate formula from Hjellming
and Webbink (1987),

$$\xi_{\text{ad}} \approx \frac{1}{3 - n} \left( 1 - n + \frac{m_c}{M_* - m_c} \right). \quad (3.5)$$

The contours for $n = 1.5 - 2.5$ in intervals of 0.25 reveal that this expression does
not provide a reasonable description for the mass loss history from the star. $n = 1.5$
corresponds to a convective envelope, higher $n$ indicate an increasing degree of radiative
transport in the giant’s envelope. The stellar evolution models are therefore essential
to capture the realistic adjustment of the mass-losing star. Equation (3.5) provides a
poor fit for two primary reasons. First, the assumption is that the readjustment of the
stellar structure is adiabatic. However, through the differences between the MESA mass
loss histories with different orbital periods, we see that this is not the case. Second,
this expression assumes that the giant’s envelope has constant $n$ as a function of time, whereas in the stellar evolution models the radial extent of convective and radiative zones evolves as the star loses mass.

### 3.4.2 Return to the Black Hole

For each portion of mass removed from the star, about half, $\Delta M/2$, returns to the black hole. The other half is ejected on hyperbolic orbits. This is formally true if the initial orbit is parabolic, but we will demonstrate that this approximation is reasonable for a wide range of orbital parameters. The accretion rate onto the black hole, $\dot{M}$, is then determined by the rate at which stellar material falls back to the vicinity of the black hole (Rees, 1988),

$$\dot{M} \approx \dot{M}_{\text{peak}} \left( \frac{t}{\tau_{\text{peak}}} \right)^{-5/3},$$

(3.6)

where, by requiring that the $\int \dot{M} dt = \Delta M/2$,

$$\dot{M}_{\text{peak}} \approx \frac{1}{3} \frac{\Delta M}{\tau_{\text{peak}}},$$

(3.7)

and the time of peak, $\tau_{\text{peak}}$, is similar to the fallback time of the most bound material,

$$\tau_{\text{peak}} \sim t_{\text{fb}} \approx 120 \left( \frac{M_{\text{bh}}}{10^7 M_\odot} \right)^{1/2} \left( \frac{M_*}{M_\odot} \right) \left( \frac{R_*}{50 R_\odot} \right)^{3/2} \text{ yr.}$$

(3.8)

This formulation, which treats encounters as nearly parabolic, is a good approximation as long as the spread in energy across the star at pericenter is large compared to the star’s orbital binding energy, satisfied for

$$\frac{a}{r_h} \gtrsim 5 \times 10^{-3} \left( \frac{M_{\text{bh}}}{10^7 M_\odot} \right)^{0.123} \left( \frac{M_*}{M_\odot} \right)^{2/3} \left( \frac{R_*}{50 R_\odot} \right).$$

(3.9)
Figure 3.2: Episodic mass transfer from a giant star to the SMBH, plotted here for a $1.4M_\odot$ giant star that is $50R_\odot$ at the onset of mass loss. The mean density of the star decreases initially upon mass loss then begins to increase again when the bulk of the star’s hydrogen envelope is depleted. The response of the star’s structure to mass loss determines the impact parameter of the next encounter with the black hole and the quantity of mass removed at pericenter (lower panel). As a result of the changing stellar structure, no single one of the analytic adiabatic response curves can provide an adequate description of the mass loss history. The contours shown are from Equation (3.5), shown for $n = 1.5 \text{--} 2.5$ in intervals of 0.25 (with $n = 1.5$ producing the shallowest profile). Stars in long orbital periods continue to evolve between passages, which drives the mass transfer episode to completion in fewer orbits. The lines terminate when the star’s envelope mass decreases to $5 \times 10^{-3}M_\odot$.  

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where $a$ is the star’s orbital semi-major axis and $r_h$ is the radius of the black hole sphere of influence, Equation (3.11). The resultant scalings thus derive partially from the definition of $r_h$, which is based on the SMBH $M - \sigma$ relation and discussed in Section 3.5. Another condition for these expressions to be applicable is that the viscous accretion time should be short relative to $t_{fb}$. For mass to be stored in a reservoir at its circularization radius, $\sim 2r_t$, for longer than $t_{fb}$ (Equation 3.8), the viscosity would have to be extremely low. Taking typical values, the ratio of the viscous accretion timescale to the fallback timescale is

$$\frac{\tau_\nu}{t_{fb}} \approx 3 \times 10^{-3} \left( \frac{\alpha_\nu}{10^{-2}} \right)^{-1} \left( \frac{M_{ph}}{10^7 M_\odot} \right)^{1/2} \left( \frac{M_*}{1 M_\odot} \right)^{-1/2} \left( \frac{H}{R} \right)^{-2},$$  

(3.10)

where we have assumed an $\alpha$-viscosity disk (Shakura and Sunyaev, 1973).

In some cases, the condition expressed in Equation (3.9) is not satisfied. For these more bound orbits, both tidal tails may be bound to the black hole and the most bound material falls back to the SMBH extremely rapidly (Hayasaki et al., 2012). In these cases, the nearly impulsively assembled disk is accreted as mediated by viscosity. The timescales and temporal evolution of this accretion are complex and remain a subject of debate. For example, see the self-similar solution of Cannizzo et al. (1990) as compared to recent numerical studies by Montesinos Armijo and de Freitas Pacheco (2011) and Shen and Matzner (2014). The common finding of these studies is that turbulent disk viscosity spreads the accretion of material out over longer timescales than pure fallback. The resulting median accretion rates are thus closer to the average accretion rate than in the pure fallback case. For simplicity, we adopt $\dot{M} = \langle \dot{M} \rangle = \Delta M/\tau_{orb}$ such that a pile-up of material in the accretion disk does not occur between subsequent stellar orbits. However, in Section 3.5 we argue that mass transfer from giant stars in such tight orbits is probably rare due to the destructive effects of direct stellar collisions.
Figure 3.3: Profile of a repeating flaring episode due to the episodic mass transfer of a giant star to the SMBH. The figure shows a flare from a $1.4M_\odot$, $50R_\odot$ star in a $10^{3.5}$ year orbit about a $10^7M_\odot$ black hole. The gray shaded region shows the overall envelope of the flaring episode, while the dashed line shows the median accretion rate. The inset, with axes in the same units as the main figure, shows that the entire envelope is made of individual flaring episodes with $t^{-5/3}$ decay tails.
Figure 3.3 applies the flaring model described by Equations (3.6), (3.7), and (3.8) to a $1.4 M_\odot$ star orbiting a $10^7 M_\odot$ black hole with an orbital period of $\tau_{\text{orb}} = 10^{4.5}$ years. The star begins to lose mass to the black hole when it reaches $50 R_\odot$. The shaded region in the main panel shows the overall envelope of the flaring event and the dashed line shows the median $\dot{M}$. The inset panel reveals each flare to be made up of a short peak and power-law decay phase. Because the star interacts with the black hole only once per orbit at each pericenter passage, the orbital period sets the repetition timescale for the individual flaring episodes. The total duration of the repeating flare is several hundred orbital periods, or approximately $10^6$ yr.

The remnants of spoon-feeding episodes are giant stars stripped of their hydrogen envelopes. The cores of these objects are white dwarfs of helium or carbon/oxygen composition, depending on the mass of the original giant star. Because the white dwarf core is relatively immune to the SMBH’s tidal field, this population of remnants are not readily destroyed by the black hole. This population of objects may eventually circularize through dissipation of orbital energy in tides or gravitational waves (Frank and Rees, 1976; Rees, 1988; Khokhlov et al., 1993b). This possibility emphasizes the still poorly constrained role that the SMBHs may play in shaping the stellar populations that surround them.

3.5 Estimating the population of mass-transferring stars

In this section, we estimate the orbital phase space and number of actively mass-transferring giant stars in a stellar cusp surrounding the SMBH. Syer and Ulmer (1999) have considered the rate at which stars evolve to reach their tidal disruption radius as a component of the tidal disruption rate. This approach is problematic because, as we have shown, stars that evolve to transfer mass to the black hole do so over many orbital periods, rather than a single, fully disruptive encounter. With the context of
repeating flares in mind, we first outline a simple stellar cusp model and the relevant timescales of the stellar dynamical system. We then outline the phase space in which giant stars might be expected to evolve to transfer mass to the SMBH upon reaching their "loss cone" in angular momentum space (Lightman and Shapiro, 1977).

In what follows, we will use a simplified model of a nuclear star cluster consisting of a power-law stellar density profile $\nu_*(r) \propto r^{-\alpha}$, normalized such that there is a black hole mass of stars within the black hole’s sphere of gravitational influence,

$$r_h = \frac{GM_{\text{bh}}}{\sigma_h^2} = 5.16 \left( \frac{M_{\text{bh}}}{10^7 M_\odot} \right)^{0.54} \text{pc},$$

where $\sigma_h$ is the external velocity dispersion of the greater galactic bulge. In the numerical expression above we use the $M_{\text{bh}}-\sigma$ relation (e.g. Ferrarese and Merritt, 2000; Gebhardt et al., 2000; Tremaine et al., 2002; Gültekin et al., 2009; Kormendy and Ho, 2013), with fitting values from Kormendy and Ho (2013), $\sigma_h = 2.3 \times 10^5 (M_{\text{bh}}/M_\odot)^{1/4.38} \text{ cm s}^{-1}$.

Interior to $r_h$, the black hole is the dominant gravitational influence on stellar orbits, while outside $r_h$, stellar orbits are primarily determined by the collective gravitational influence of all of the other stars.

Observations of the centers of early-type galaxies with the Hubble Space Telescope (HST) have shown that the stellar surface brightness profiles in these galactic centers are bimodal (Faber et al., 1997). Some galaxies exhibit a ‘cuspy’ core that is defined by a power-law rise in surface brightness to the resolution limit of the HST imaging. Others exhibit a shallower ‘core’ profile with a break radius that is typically similar to the inferred black hole sphere of influence (Faber et al., 1997). Given these observed stellar distributions, one could, in principle, analyze the populations of mass-transferring giant stars expected in those galaxies and compare to their SMBH activity on a case by case basis. As a simple first step, we instead illustrate the expected populations for two representative stellar profiles, $\nu_*(r) \propto r^{-\alpha}$ with $\alpha = 2$ to represent
cuspy galactic center profiles and \( \alpha = 1 \) to represent the shallower core profiles (for an illustration of the stellar density profiles, see Figure 12 of MacLeod et al., 2012). These profiles were chosen to capture two extreme cases for the stellar distribution in observed galactic centers in order to illustrate the range of possibilities for the rates of mass-transfer interactions.

Stars in galactic nuclei live in orbits of period \( \tau_{\text{orb}} \) whose energy and angular momentum change on timescales \( \tau_\varepsilon \) and \( t_J \), respectively. \( \tau_\varepsilon \) is the cluster’s local relaxation time (Binney and Tremaine, 2008; Alexander, 2005). The angular momentum of loss cone orbits, \( J \sim J_{lc} \approx \sqrt{2GM_b r_t} \), is typically much less than the circular angular momentum for a given orbital semi-major axis \( a \), \( J_c \approx \sqrt{GM_b a} \). In other words, \( r_p \approx r_t \ll a \), and typical loss cone orbits are very eccentric. Thus, the angular momentum change time for orbits that approach the loss cone,

\[
\tau_J \equiv \left( \frac{J_{lc}}{J_c} \right)^2 \tau_\varepsilon, \tag{3.12}
\]

is much less than that for energy, \( \tau_\varepsilon \), because the loss-cone angular momentum is much less than the circular angular momentum, \( J_{lc} \ll J_c \).

Stars are also susceptible to collisions after a time \( \tau_{\text{coll}} \equiv \tau_{\text{orb}}/N_{\text{coll}}(\tau_{\text{orb}}) \), where the number of collisions per orbit is an integral along the orbital path,

\[
N_{\text{coll}}(\tau_{\text{orb}}) = 2 \int_{r_p}^{2a(\varepsilon)} \Sigma(s) \nu_*(s) ds, \tag{3.13}
\]

where \( \Sigma \) is the collision cross section of a star, approximately the geometric cross-section \( \Sigma \approx \pi R^2_\ast \) in the high velocity dispersion central regions of the cluster. For nearly radial loss-cone orbits and power-law density profiles, collisions are dominated by the orbital pericenter approach if \( \alpha > 1 \). If there were to be a break in the power-law profile at small radii, the number of collisions would therefore be reduced. Here we will also
assume that collisions are always destructive, even though this may not be the case for
high velocity collisions involving giant stars (Bailey and Davies, 1999; Dale et al., 2009).
In this way, we calculate an upper limit to the collisional destruction of giant stars that
might otherwise go on to spoon-feed the black hole.

Finally, the tidal radius of a star changes as a function of time due to stellar
evolution according to a timescale

\[
\tau_{\text{evol}} \equiv \frac{r_t}{r_t} \approx \frac{R_s}{\dot{R}_s}.
\] (3.14)

\(\tau_{\text{evol}}\) is long on the main sequence and shortens as the star evolves off the giant branch.
Because this change happens over the course of the red giant branch, this timescale is
roughly the length of the red giant branch, \(\langle \tau_{\text{evol}} \rangle \sim \tau_{\text{rgb}}\). We adopt \(\tau_{\text{evol}} = 10^7\) yr in
the following analysis.

The population of stars that can evolve to reach the loss cone is bounded to
have angular momenta that are low enough that the black hole’s tides will impinge on
the star at some point during the star’s giant-branch evolution, thus

\[
J < J_{\text{lc}}(M_*, R_{\text{max}}),
\] (3.15)

where \(R_{\text{max}}\) is the maximum stellar radius reached during the red giant phase. This
condition implies \(r_p < r_t(M_*, R_{\text{max}})\). This population of stars is also is bounded in
orbital energy space by several considerations based on the timescales of various drivers
of evolution in the star’s orbital parameters. The first condition is that stellar evolution,
rather than the star’s orbital random walk, must drive the star to reach the loss cone,
\(\tau_{\text{evol}} < t_J\). Second, the orbital random walk must be small during the flare duration,
\(\tau_{\text{flare}} < t_J\). Because the peak of the flaring event takes \(\sim 10^2\) orbits, we make the
approximation \(\tau_{\text{flare}} \approx 10^2\tau_{\text{orb}}\). These conditions define the least bound objects, which
have shorter $t_4$. The most bound objects exhibit very nearly Keplerian orbits, but since they pass through the densest central regions of the stellar cusp $\tau_{\text{evol}}/\tau_{\text{orb}}$ times during their giant branch lifetime, they may be extremely vulnerable to collisions. Figure 3.4 shows these constraints on the phase space of trapped stars in the upper two panels.

The shading in Figure 3.4 shows the distribution of orbits in orbital energy $\varepsilon$, and in orbital period $\tau_{\text{orb}}$. These distributions are computed by considering the number of stars per unit energy that satisfy the low angular momentum condition of Equation (3.15). This number is $dN/d\varepsilon = 4\pi^2 f(\varepsilon)\tau_{\text{orb}}(\varepsilon)J_c(M_*, R_{\text{max}})$, where $f(\varepsilon)$ is the distribution function of stars in orbital energy (Magorrian and Tremaine, 1999). Following Magorrian and Tremaine (1999) we have assumed that the distribution function is isotropic in $J^2$ and therefore only depends on energy. The distribution function has scaling $f(\varepsilon) \propto \varepsilon^{\alpha - 3/2}$, while from Kepler’s law, $\tau_{\text{orb}}(\varepsilon) \propto \varepsilon^{-3/2}$. As a result, $dN/d\varepsilon \propto \varepsilon^{\alpha - 3}$ (Magorrian and Tremaine, 1999). Thus, for an $\alpha = 2$ stellar density profile, orbits within the loss cone are distributed in energy as $dN/d\varepsilon \propto \varepsilon^{-1}$. The total number of stars trapped in this phase space that will lead to mass transfer with the SMBH is then an integral over $dN/d\varepsilon$,

$$N_{\text{trapped}} = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} \left(\frac{dN}{d\varepsilon}\right) d\varepsilon,$$

where $\varepsilon_{\text{min}}$ and $\varepsilon_{\text{max}}$ are limits on the energy based on the comparisons of timescales described above. For $\alpha = 2$, different orbital energies contribute equally to the integrated number of stars trapped within the giant star loss cone, because $dN/d\varepsilon \propto \varepsilon^{-1}$ or $dN/d\log \varepsilon = \text{constant}$. These flares have equal likelihood of occurring with any pericenter distance because of the isotropic angular momentum distribution of the stellar cluster, thus $dN/dJ^2$ and $dN/dr_p$ are both constant. These flat distributions imply that a mass transfer episode involving a star of a given mass between 10 and 11 $R_\odot$ is equally likely as when that star is between 100 and 101 $R_\odot$. The average stellar radius on encountering the SMBH is therefore $\approx R_{\text{max}}/2$, where $R_{\text{max}}$ is the maximum radius.
Figure 3.4: Phase space of stars that can evolve to transfer mass to the SMBH. The top two plots assume a cuspy stellar density profile $\nu_*(r) \propto r^{-2}$, as does the blue line in the third panel, while the red, core, line in the lower panel takes $\nu_*(r) \propto r^{-1}$ (e.g. Faber et al., 1997). The phase space for tightly bound stars is limited by collisions. Two considerations limit the more weakly bound population: first, that the star reach the loss cone through evolution rather than under the influence of a random walk in angular momentum, $\tau_{\text{evol}} < t_J$, and, second, that once mass transfer begins, the flaring event transpires more rapidly than the random walk timescale, $t_{\text{flare}} < t_J$. The lower panel shows the number of mass-transferring stars that might be expected at any given time, estimated as $\langle N_{\text{MT}} \rangle = 10^2 \langle \tau_{\text{orb}} \rangle \Gamma_{\text{evol}}$, where $\langle \tau_{\text{orb}} \rangle$ is the orbital period averaged over the distribution of stars in binding energy.
of stars on the red giant branch.

The rate at which the $N_{\text{trapped}}$ trapped stars evolve to reach the loss cone is given by the mean lifetime of the stars. We find,

$$\Gamma_{\text{evol}} \approx \frac{N_{\text{trapped}}}{\tau_{\text{life}}} = 10^{-6} \left( \frac{\tau_{\text{life}}}{1 \text{ Gyr}} \right)^{-1} \left( \frac{N_{\text{trapped}}}{10^3} \right) \text{yr}^{-1},$$

(3.17)

where $\tau_{\text{life}}$ is the age of the stellar population. By comparing the duration of a typical flare, approximated as $10^2 \tau_{\text{orb}}$, with the rate above, we can estimate the number of stars expected to be actively mass transferring with the black hole at any given time. To do so, we must incorporate knowledge of the orbital period distribution of mass-transferring stars. Figure 3.4 shows the distributions of trapped stars in orbital binding energy and period. In the lower panel of Figure 3.4, we show the resultant number of actively mass-transferring stars, computed as $N_{\text{MT}} = 10^2 \langle \tau_{\text{orb}} \rangle \Gamma_{\text{evol}}$, based on the average orbital period, $\langle \tau_{\text{orb}} \rangle$. Black holes with masses greater than approximately $10^7 M_\odot$ might then be expected to host an actively mass-transferring population of trapped giant stars. The nuclei of lower-mass black holes are characterized by higher stellar number densities, and thus have shorter relaxation and collision times. In these nuclei ($M_{\text{bh}} \lesssim 10^7$) mass transfer events do occur but with a low duty-cycle.

3.6 Significance to the duty cycle of tidally-fed SMBHs

In this section, we discuss the role of direct tidal feeding episodes in the duty cycle of low-level SMBH activity. We illustrate these processes with Monte Carlo realizations of black hole accretion histories. We make use of the stellar cluster properties described in Section 3.5 with $\nu_* \propto r^{-2}$ and assume a stellar mass of $1.4 M_\odot$, corresponding to a main-sequence turnoff age of approximately 4 Gyr. While this choice is meant to be illustrative rather than exact, similar stellar ages are seen in the nuclear region of M32 by Seth (2010). Such stars have a giant-branch lifetime of $\sim 4 \times 10^8$ years, and
thus spend $\sim 10\%$ of their lifetime on the giant branch. The effect of stellar population age on the rate at which stars evolve to reach the loss cone and begin to transfer mass can be seen in equation (3.17), thus the expected event rate would be lower by a factor of three if the main-sequence turnoff age were 12 Gyr, and higher by a factor of four if it were instead 1 Gyr.

Accretion histories (a subset) and duty cycles are shown for $M_{bh} = 10^7 M_\odot$, $10^{7.5} M_\odot$, and $10^8 M_\odot$ in Figure 3.5. We draw the stellar parameters of tidal disruption flares according to the methods described in MacLeod et al. (2012) and scale the profiles derived from hydrodynamic simulations in timescale and $\dot{M}$. The relative likelihood of disruption of different stellar types scales with their occurrence in the stellar population and $t_4^{1/4}$ (Wang and Merritt, 2004; Milosavljevic et al., 2006; MacLeod et al., 2012). We draw the giant star mass transfer events from the same stellar cluster distribution. Using the distributions in Figure 3.4, we populate the stars’ orbital periods and assume that the mass transfer profile follows one of the profiles from Figure 3.2, choosing the nearest in logarithmic space to the orbital period. We then compare the resultant feeding directly to observed Eddington ratio distributions in local quiescent galaxies and to the indirect SMBH feeding by stellar winds.

In this section, we normalize accretion rates with respect to black hole mass to a fiducial Eddington accretion rate $\dot{M}_{\text{Edd}} = 0.02(M_{bh}/10^6 M_\odot)M_\odot \text{yr}^{-1}$, which corresponds to a radiative efficiency of $\eta = 0.1$ where $L = \eta \dot{M} c^2$ and thus $\dot{M}_{\text{Edd}} = L_{\text{Edd}} \eta^{-1} c^{-2}$. The normalized mass accretion rates thus show what the bolometric accretion luminosity, $L/L_{\text{Edd}}$, of a SMBH would be given a certain radiative efficiency, and may be scaled to different values of the radiative efficiency according to $(\eta/0.1)$.

### 3.6.1 Tidal disruption flares

At the highest Eddington ratios, main-sequence flares dominate the SMBH feeding (Figure 3.5). At lower accretion rates main-sequence and giant-star flares con-
Figure 3.5: Monte Carlo realizations of the duty cycle of tidally fed black hole activity for SMBH masses of $10^7 M_\odot$, $10^{7.5} M_\odot$, and $10^8 M_\odot$. The blue line shows the tidal disruption of main-sequence stars, labeled TDEs (MS); the red line shows giant branch stars, labeled TDEs (Giants). These tidal disruption components only include the fraction of events fed from the large angle scattering regime where $\Delta J > J_{lc}$. A fraction $r_s/r_t$ of events are promptly swallowed by the SMBH, here we plot only those that pass outside $r_s$ and produce a flare. The purple line shows the contribution from episodically mass-transferring giant stars, labeled MT. For black hole masses $\gtrsim 10^7 M_\odot$ the number of actively mass-transferring stars is $\gtrsim 1$. These mass transfer episodes dominate above the decay tails of tidal disruption events at low Eddington ratios. We expect these mass-transferring stars to be the dominant contribution to tidally fed SMBH activity at $\dot{M}/\dot{M}_{\text{Edd}} \lesssim 10^{-4}$. The dashed lines show two estimates of the degree to which stellar winds may feed the SMBH. The total stellar wind injection with the sphere of influence is an upper limit and is marked as $\dot{M}_w (< r_h)$. The winds found to actually accrete in models of Sgr A* are a small fraction of that, $\dot{M}_w(MHD)$ from simulations of the accretion flow by Shcherbakov and Baganoff (2010); Shcherbakov et al. (2013). $\dot{M}/\dot{M}_{\text{Edd}}$ is computed assuming $\dot{M}_{\text{Edd}} = 0.02 (M_{\text{bh}}/10^6 M_\odot) M_\odot\text{yr}^{-1}$, which corresponds to a radiative efficiency of $\eta = 0.1$. 

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tribute similarly to the duty cycle despite the lower rate of giant-star tidal disruption events because the decay timescale $t_{fb}$ is longer for giant-star disruptions (Equation (3.8), and MacLeod et al., 2012). The late-time power law decay tails of tidal disruption events ($\sim t^{-5/3}$) give rise to the power law seen in the duty cycle at lower Eddington ratios. In particular if the late time decay follows Equation (3.6), then the duty cycle can be written

$$f_{on}(\dot{M}) = \frac{t_{fb}}{t_{repeat}} \left( \frac{\dot{M}}{\dot{M}_{peak}} \right)^{-3/5},$$

where $t_{repeat}$ is generally the inverse of $\Gamma_{TDE}$, the rate of tidal disruption events. While the simulation results used in Figure 3.5 differ slightly in late-time power law slope, these basic scalings give intuition for the low Eddington ratio duty cycle that results from tidal disruption events. One consequence is that since typically $t_{fb} \ll t_{repeat}$, the accretion rate at which $f_{on} \sim 1$ is far less than $\dot{M}_{peak}$. In Figure 3.5, the steady state accretion rate between flares (for which the duty cycle is of order unity) is only achieved at $\dot{M} \sim 10^{-9} \dot{M}_{edd}$. A very similar analysis has been performed by Milosavljevic et al. (2006) who further show that the resulting luminosity function may explain $\sim 10\%$ of local AGN activity. While our analysis is in agreement with previous work that has shown that tidal disruption flares cannot explain the entire local AGN luminosity function, we do find that quasi-steady feeding from mass-transferring giant stars may provide a significant contribution to the median accretion rate between luminous flaring episodes.

### 3.6.2 Episodic mass transfer from evolving stars: Spoon-feeding

Giant stars that grow through stellar evolution to episodically transfer mass at pericenter over the course of many orbital periods to the SMBH. As a result, the SMBH feeding from this spoon-fed material is smeared over long timescales by the episodic nature of the individual flaring episodes. Mass-transferring stars therefore generally
feed the SMBH at low Eddington ratios but at quasi-steady rates. This can be seen mathematically through inspection of Equation (3.18). Mass transfer episodes repeat every orbital period, reducing \( t_{\text{repeat}} \), and therefore achieve a duty cycle of order unity at higher \( \dot{M} \) than full tidal disruption flares. These events can be expected to establish a minimum accretion rate above that established by the tails of tidal disruption events in systems where the number of actively mass transferring stars (the lower panel of Figure 3.4) is of order unity or greater, \( \langle N_{\text{MT}} \rangle \gtrsim 1 \). In systems where \( \langle N_{\text{MT}} \rangle < 1 \) stars do occasionally grow to the loss cone and feed the black hole, but they are not able to establish a quasi-steady floor accretion rate with \( f_{\text{on}} \sim 1 \).

Episodic flares from evolving stars are represented with the purple line in Figure 3.5. In the upper panel, we show a representative timeseries of accretion rate to the SMBH, several interesting features appear. First, it is obvious that while they never reach luminosities similar to the peaks of tidal disruption flares, the mass transfer from spoon-fed giant stars does fill in the "gaps" between tidal disruption flaring events. The quasi-steady accretion rate that results ranges between \( \dot{M} \sim 10^{-4} - 10^{-6} \dot{M}_{\text{Edd}} \).

The structure in the curve results from the modulation of repeating flares of different orbital periods. Stars in relatively long period orbits have longer time between episodes and relatively deeper troughs between peaks. This overall curve is mediated by shorter timescale variation from flares with shorter repetition times. At some times, like near 0.4 Myr, it is possible to see the characteristic structure of an entire flaring episode with relatively short orbital period. Here the flare and decay cycles are so rapid that they appear blurred in the Figure, and instead we see the shape of the overall flaring envelope, as in Figure 3.3.

The dominance of mass transfer episodes over the tails of tidal disruption events can be seen in the lower panels of Figure 3.5. The relatively steep profile of these duty-cycle curves arises from the small range of \( \dot{M} \) achieved in spoon-feeding events, as seen both in the upper panel of Figure 3.5 and the single event shown in Figure 3.3. We find
that mass-transferring stars are the dominant contribution to tidally fed SMBH activity at \( \dot{M} \lesssim 10^{-4} \dot{M}_{\text{Edd}} \), particularly for relatively massive black holes \( M_{\text{bh}} \gtrsim 10^7 M_\odot \). A similar behavior is realized at slightly higher black hole masses in the case of galactic nuclei with shallow cores. In these galactic centers, spoon-fed stars might be expected to make a meaningful contribution to the duty cycle in systems with \( M_{\text{bh}} \gtrsim 10^{7.5} M_\odot \). This behavior can be inferred directly from the lower panel of Figure 3.4, in which we show the averaged duty cycle for the two extreme cases of the galactic nucleus structure. As discussed in Section 3.5, shallower core-like profiles are probably most relevant for massive black holes \( M_{\text{bh}} \gtrsim 10^8 M_\odot \), thus we expect that spoon-fed giant stars will feed these very massive black holes but according to the core galaxy duty cycle shown in Figure 3.4. SMBHs with mass \( M_{\text{bh}} \gtrsim 10^8 M_\odot \) will swallow main sequence stars whole rather than tidally disrupting them (e.g. MacLeod et al., 2012), but giant stars might still be expected to feed the SMBH through the spoon-feeding and tidal disruption channels.

### 3.6.3 Diffusion to the loss cone

The gradual diffusion of main-sequence stars in angular momentum space to the loss cone will have little effect on the SMBH activity duty cycle. In general, these events contribute a fraction of the flux of stars into the loss cone. They will not result in a single, strongly disruptive passage by the SMBH. Only objects that are fully convective such as the lowest mass \( (M \lesssim 0.4 M_\odot) \) and very high mass \( (M \gtrsim 20 M_\odot) \) main-sequence stars exhibit a mass-radius relationship that allows for a runaway response to mass loss over multiple orbits. Sun-like objects (which may be modeled by \( n = 3 \) polytropes) exhibit a strongly protective initial response to mass loss (Hjellming and Webbink, 1987). Further, the critical radius that defines the transition between scattered orbits and diffusing orbits moves to tighter binding energies for stars with small \( J_{\text{lc}} \) like main-sequence stars (Frank and Rees, 1976; Lightman and Shapiro, 1977).
The orbital periods of these objects become short (thus they contribute less to the duty cycle of SMBH activity). Additionally, in cuspy nuclei that are dynamically relaxed, mass segregation likely leads to a relatively small number of low mass stars that are very tightly bound (e.g. Alexander, 2005). Secular effects like the Schwarzschild barrier also become important for such tightly bound orbits (Merritt et al., 2011). Main-sequence stars that are partially disrupted by the SMBH are likely kicked out by the resultant mass loss asymmetry, which increases their specific orbital energy by a factor $\sim GM_*/R_*$, which is greater than the orbital binding energy for typical orbits when evaluated for the main-sequence mass and radius (Manukian et al., 2013).

Giant branch stars, if they are able to diffuse to reach the loss cone, likely undergo a spoon-feeding encounter history similar to that described in this paper. The population of objects able to do so may be limited. Collisions are particularly damaging for giants in galactic nuclei (Davies et al., 1998; Bailey and Davies, 1999; Dale et al., 2009) and may be even more severe for giants on very eccentric orbits that pass through the densest regions of the stellar cluster every orbit (MacLeod et al., 2012). In high mass systems, $M_{bh} \gtrsim 10^7 M_\odot$, evolution to the loss cone dominates, as shown in Figure 3.4. In systems with lower mass black holes, $M_{bh} \sim 10^6 - 10^7 M_\odot$, diffusion to the loss cone may be possible. Only a few percent of diffusing stars that reach the loss cone will be giants because in the diffusion limit there is only a logarithmic enhancement in the loss cone rate with tidal radius, $\Gamma_{\text{diff}} \propto \ln(r_t)$ (Lightman and Shapiro, 1977). A careful consideration of the importance of these stars would need to include the orbital random walk in the stars’ mass transfer histories and is an interesting future application of the episodic mass transfer model presented in this paper.

### 3.6.4 Comparison to stellar wind feeding

Stellar winds feed material indirectly to SMBHs by ejecting material into the cluster medium (e.g. Holzer and Axford, 1970). Simulations of stellar wind feeding onto
massive black holes typically find two key features of the accretion flow. First, only a small amount of the material ejected into the cluster medium ever reaches the SMBH. The low inflow rate arises because winds inject both mass and kinetic energy into the cluster. Young star winds, in particular, may have super-virial velocities, leading to the ejection of the bulk of the mass (e.g. Quataert, 2004; Rockefeller et al., 2004; Cuadra et al., 2005, 2006, 2008; De Colle et al., 2012). Second, the accretion flows tend to have low net angular momentum because they result from the combined contribution of many stars. In a spherical nucleus almost perfect cancelation of angular momentum results, and the accretion flow circularizes only at very small radii (Cuadra et al., 2008). The low angular momentum of the accretion flow may contribute to the inferred low radiative efficiency of accretion fed by stellar winds (Baganoff et al., 2003; Ho, 2009), particularly if the circularization radius is inside the transition to an advection dominated accretion flow (ADAF) (Fabian and Rees, 1995; Narayan et al., 1998; Quataert and Narayan, 1999; Blandford and Begelman, 1999).

In Figure 3.5, we include several direct comparisons to models of stellar wind feeding. Shcherbakov and Baganoff (2010) and Shcherbakov et al. (2013) use magneto-hydrodynamic models of the accretion flow onto Sgr A* to conclude that an inflow rate of $\sim 10^{-8} M_\odot \text{yr}^{-1}$ is consistent with the feeding. This corresponds to an Eddington ratio $\dot{M}_w(\text{MHD})/\dot{M}_{\text{Edd}} = 1.25 \times 10^{-7}$ and is shown in the panels of Figure 3.5 as scaling with mass to maintain a constant Eddington ratio. Considerably more mass is available to the SMBH within the nuclear cluster environment. As a strict upper limit, we take a cluster of $1.4 M_\odot$ stars that lose mass at rate $\langle \dot{M}_* \rangle \approx 0.8 M_\odot/4 \text{Gyr} = 2 \times 10^{-10} M_\odot \text{yr}^{-1}$ based on the initial-final mass relation and lifetime of these stars (Kalirai et al., 2008). Multiplying by the number of stars within the SMBH’s sphere of influence gives a crude upper limit to the mass contributed by stellar winds that is potentially available to the black hole of $\dot{M}_w(< r_h) \lesssim 10^{-2} \dot{M}_{\text{Edd}}$. Considerable uncertainty exists between these two limiting cases, in particular with respect to the degree to which material reaches the
black hole, and the radiative efficiency with which it will shine. By contrast, material spoon-fed to the SMBH is injected at small radii (comparable to the tidal radius), making it less susceptible to feedback or outflows as it accretes than are stellar winds, which are dominantly injected at the sphere of influence. We see from Figure 3.5 that spoon-feeding from mass-transferring giant stars can lead to accretion rates within the range of those those implied by stellar wind feeding in galactic nuclei of $M_{\text{bh}} \sim 10^7 - 10^8 M_\odot$.

3.6.5 Implications for low-luminosity active galactic nuclei

We compare our predictions about the duty cycle of spoon-fed SMBHs to distributions of $L/L_{\text{Edd}}$ from Ho (2009). Using data from the Palomar sample of local active galactic nuclei (AGN), Ho (2009) finds that most galactic nuclei shine at a very small fraction of their Eddington limit. The range of median luminosities for AGN classes computed by Ho (2009) span several orders of magnitude. From low to high $L/L_{\text{Edd}}$ these are Absorption (2.2 × 10$^{-7}$), Transition (1.5 × 10$^{-6}$), Liner (6 × 10$^{-6}$), and Seyfert (1.1 × 10$^{-4}$) nuclei. Some uncertainty lies in the determining whether accretion luminosity is the true source of nuclear activity at such low $L$. Potential sources of contamination include low-mass x-ray binaries (Miller et al., 2012) or perhaps even diffuse emission (e.g. Soria et al., 2006). Thus, the constant Hα and x-ray to bolometric corrections assumed by Ho (2009) are suggested to carry an error for individual sources $\lesssim$ 0.7 dex. Ho (2009) uses these data to conclude that we must be primarily observing the signatures of radiatively inefficient ($\eta \ll 1$) accretion of a relatively large amount of material supplied by stellar winds. However, there remain large uncertainties in the degree to which gas within the black hole sphere of influence is able to accrete (due to either feedback processes or outflows). Thus, both the radiative efficiency, and the efficiency with which material reaches the SMBH are parameterized in this conversion from $\dot{M}$ to $L$.

At low black hole masses, $M_{\text{bh}} \lesssim 10^7 M_\odot$, stellar winds likely set the minimum
accretion floor in gas-deprived galaxies. For higher black hole masses in the range $M_{\text{bh}} \sim 10^7 - 10^8 M_\odot$ plotted in Figure 3.5, we suggest that the inferred $L/L_{\text{Edd}}$ Eddington ratios in some local galactic nuclei may also be consistent with the digestion of mass from spoon-fed stars given efficiencies $\eta \sim 10^{-3} - 10^{-1}$. Because stellar wind feeding and direct tidal feeding of SMBHs potentially result in different morphologies of the resulting accretion structure, one might not necessarily expect that $\eta$ is fixed for a given fuel supply, $\dot{M}$. A more complete picture of the properties of the lowest luminosity AGN may provide some information about their fueling mechanism, especially when coupled with more detailed modeling of the accretion flows themselves. In particular, parallel constraints on the activity level of SMBHs and the stellar distributions that surround them may offer insight into the feeding mechanism and, in turn, the morphology and radiative efficiency of the resulting accretion flow.

### 3.7 Conclusions and Future Work

To understand of the nature of the accretion flows in quiescent galactic nuclei, we must make certain our census of potential fuel sources is complete. To this end, we study the SMBH feeding that arises when stars trapped in eccentric orbits evolve to transfer mass episodically to the SMBH. We call this process spoon-feeding giant stars to SMBHs. About half the mass stripped from the star falls back towards the SMBH, lighting it up in an accretion flare. The remnant returns to the SMBH to transfer mass once per orbital period. We show that the thermal evolution of the remnants determines the magnitude of the subsequent mass-loss episodes. We compute orbital histories in which we self-consistently compute the adjustments to the the stellar structure in the MESA stellar evolution code. A typical low-mass giant branch star may transfer mass for $\sim 10^2$ orbits before leaving behind a helium white dwarf remnant. We estimate that a steady-state population of these mass-transferring stars is likely to
exist in galactic nuclei hosting SMBHs more massive than approximately $10^7 M_\odot$. In nuclei with lower density cores, this transition happens at somewhat higher black hole mass $M_{bh} \gtrsim 10^{7.5} M_\odot$. Using Monte Carlo realizations of SMBH accretion histories, we show that this population of mass-transferring stars contributes significantly to the duty cycle of low-level SMBH activity. The feeding from these stars may exceed that from the decay tails of tidal disruption events at $\dot{M} \lesssim 10^{-4} \dot{M}_{\text{Edd}}$.

In this work, we have presented a preliminary formalism for modeling the mass transfer that occurs when giant stars in eccentric orbits grow to reach their loss cone and spoon-feed material to the SMBH. This approach may be extended to explore several interesting questions. In particular, we have considered the idealized case in which the star’s orbit does not evolve in the course of the encounter. Stars that diffuse in angular momentum undergo a random walk in angular momentum due to two-body relaxation (Lightman and Shapiro, 1977), while stars in non-isotropic structures like rings, disks, or triaxial clusters are subject to torques that can drive coherent orbital evolution (Magorrian and Tremaine, 1999; Merritt and Vasiliev, 2011; Madigan et al., 2009; Madigan et al., 2011; Vasiliev and Merritt, 2013; Antonini and Merritt, 2013). These differing cases of orbit evolution likely imprint themselves on the light curves of the resulting mass-transfer flares in unique ways, and in turn the duty cycle of SMBH activity, perhaps offering an avenue to explore the dynamical processes at play in distant galactic nuclei (e.g. Rauch and Ingalls, 1998). In the giant star case we have analyzed, the encounter itself does not change the orbital parameters significantly. This is partially because of the star’s stratified core-envelope structure (Liu et al., 2013) and partially because the star’s specific orbital energy $\sim GM_{bh}/a$ is larger than the star’s specific binding energy, $GM_*/R_*$ (Manukian et al., 2013). For main-sequence stars and compact stellar remnants with higher escape velocities, this is not necessarily the case. The orbital energy change imprinted by mass loss becomes an important factor in determining the star’s encounter history with the SMBH.
We further assume in this work that the primary change in stellar structure is due to the change in envelope mass. As we demonstrate in Section 3.3, this is almost certainly the case for giant stars where surface heating is counteracted by rapid radiative cooling (McMillan et al., 1987). However, in other classes of objects, particularly those for which their thermal adjustment time is long compared to an orbital period, the heat deposited into the star may play a dramatic role in modifying the star’s structure and mass transfer history with the SMBH. Thus, the additional heat deposited deep within a star may not be able to be radiated effectively in an orbital period. If this is the case, the star will adiabatically expand (Podsiadlowski, 1996) and more strongly feel the tidal force of the SMBH in its subsequent passages. This process can easily lead to a runaway, as has been found by Guillochon et al. (2011) in the case of giant planets interacting with their host stars. The rotation excited by the encounter with the SMBH is an additional effect on the stellar structure we do not address here. The expected rotation may be a significant fraction of the star’s breakup rotation, leading to mixing and other effects that can modify stars’ subsequent evolution (Alexander and Kumar, 2001), while also reducing their mean density. This may make stars more prone to disruption in subsequent passages.

In Section 3.6, we illustrate how single-mass stellar clusters might imprint their presence on the accretion history and thus also the luminosity function of low-luminosity AGN. The stellar cusps around SMBHs are certainly not single stellar mass, but our knowledge of stellar populations in galactic nuclei other than our own (e.g. Schödel et al., 2007) is weakly constrained. The detailed nature of these stellar populations plays a significant role in determining the relative rates of different classes of tidal interactions between stars and SMBHs (MacLeod et al., 2012). As we outline above, further complexities in the orbital distribution and orbital evolution of the population of stars residing close to SMBHs in galactic centers will also imprint themselves on the luminosity function of low-level AGN. Thus, direct comparisons between observed
distributions of SMBH activity and predictions based on various feeding mechanisms will offer a statistical window into the more general nature of the stellar dynamics and populations of galactic center stellar clusters.
Chapter 4

Illuminating Massive Black Holes With White Dwarfs: Orbital Dynamics and High Energy Transients from Tidal Interactions

4.1 Chapter Abstract

White dwarfs (WDs) can be tidally disrupted only by massive black holes (MBHs) with masses less than $\sim 10^5 M_\odot$. These tidal interactions feed material to the MBH well above its Eddington limit, with the potential to launch a relativistic jet. The corresponding beamed emission is a promising signpost to an otherwise quiescent MBH of relatively low mass. We show that the mass transfer history, and thus the lightcurve, are quite different when the disruptive orbit is parabolic, eccentric, or circular. The mass lost each orbit exponentiates in the eccentric-orbit case leading to the destruction of the WD after several tens of orbits. We examine the stellar dynamics of clusters surrounding MBHs to show that single-passage WD disruptions are substantially more common than repeating encounters. The $10^{49}$ erg s$^{-1}$ peak luminosity of these events...
makes them visible to cosmological distances. They may be detectible at rates of as many as tens per year by instruments like Swift. In fact, WD-disruption transients significantly outshine their main-sequence star counterparts, and are the most likely tidal interaction to be detected arising from MBHs with masses less than $10^5 M_\odot$. The detection or non-detection of such WD-disruption transients by Swift is, therefore, a powerful tool to constrain lower end of the MBH mass function. The emerging class of ultra-long gamma ray bursts all have peak luminosities and durations reminiscent of WD disruptions, offering a hint that WD-disruption transients may already be present in existing datasets.

4.2 Introduction

Tidal disruption events have been studied theoretically since their prediction by Hills (1975). They are expected to produce luminous but short-lived accretion flares as debris from the disrupted star streams back to the massive black hole (MBH) (Rees, 1988). The characteristic pericenter distance for tidal disruption to occur is the tidal radius, $r_t = (M_{bh}/M_*)^{1/3} R_*$, which is defined by the average density of the star and by the MBH mass. The tidal radius scales differently with MBH mass than the MBH’s Schwarzschild radius, $r_s = 2GM_{bh}/c^2$. As a result, for a given black hole mass, some stellar types may be vulnerable to tidal disruption while others would instead pass through the MBH horizon whole. Of particular interest to this study is the fact that MBHs more massive than $\sim 10^5 M_\odot$ swallow typical white dwarfs (WDs) whole, while those of lower mass can produce tidal disruptions of WDs.

Tidal disruptions of WDs, therefore, uniquely probe the long-debated existence of MBHs with masses less than $10^5 M_\odot$. The kinematic traces of such black holes are difficult to resolve spatially due to their relatively small radii of gravitational influence, even with the Hubble Space Telescope, which has proven a powerful tool for probing
more massive nuclei (e.g. Lauer et al., 1995; Seth, 2010). While current observational constraints suggest that black holes are ubiquitous in giant galaxies (Richstone et al., 1998), their presence is more uncertain in dwarf galaxies (although, see Reines et al., 2011). Determination of the galactic center black hole mass function has traditionally focused on active galaxies (e.g. Kelly and Merloni, 2012; Miller et al., 2015), for which we can directly infer the black hole mass, and work by Greene and Ho (2004), Greene and Ho (2007); Greene and Ho (2007) and Reines et al. (2013) has shown intriguing possibilities for active galactic center black holes with masses similar to $10^5 M_\odot$. But, observations of tidal disruption events probe the mass function of otherwise quiescent black holes, offering a powerful check on mass functions derived from their active counterparts (Gezari et al., 2009).

With this motivation, predicting the signatures of tidal interactions between WDs and MBHs has been the subject of substantial effort. Studies find that the resulting mass transfer nearly always exceeds the MBH’s Eddington limit mass accretion rate, $\dot{M}_{\text{Edd}} = 2 \times 10^{-3}(M_{\text{bh}}/10^5 M_\odot) M_\odot \text{ yr}^{-1}$, where we have used $L_{\text{Edd}} = 4\pi G M_{\text{bh}} m_p c / \sigma_T$ and a 10% radiative efficiency, $L = 0.1\dot{M}c^2$. Accretion disk emission from these systems is at most $\sim L_{\text{Edd}}$, which is increasingly faint for smaller MBHs (e.g. Beloborodov, 1999). However, we expect that these systems also launch relativistic jets as a result of the extremely rapid mass supply (Giannios and Metzger, 2011; Krolik and Piran, 2012; De Colle et al., 2012; Shecherbakov et al., 2013). The observed luminosity of these jetted transients is likely to be proportional to $\dot{M}c^2$ (De Colle et al., 2012) and thus may greatly exceed $L_{\text{Edd}}$ when $\dot{M} \gg \dot{M}_{\text{Edd}}$. While disk emission may peak at ultraviolet or soft x-ray frequencies (Ramirez-Ruiz and Rosswog, 2009; Rosswog et al., 2009b), the jetted emission can be either produced by internal dissipation or by Compton-upscattering the disk photon field to higher frequencies (e.g. Bloom et al., 2011; Burrows et al., 2011). We turn our attention to these luminous high-energy jetted transients arising from WD-MBH interactions in this paper.
Despite the general feature of high accretion rates, theoretical studies predict a wide diversity of signatures depending on the orbital parameters with which the WD encounters the MBH. Single, strongly-disruptive passages are thought to produce quick-peaking lightcurves with power-law decay tails as debris slowly falls back to the MBH (Rosswoag et al., 2009b; Haas et al., 2012; Cheng and Evans, 2013; Shcherbakov et al., 2013). It has also been suggested that these sufficiently deeply-passing encounters may result in detonations of the WD (Luminet and Pichon, 1989a; Rosswoag et al., 2009b; Haas et al., 2012; Shcherbakov et al., 2013; Holcomb et al., 2013), and thereby accompany the accretion flare with a simultaneous type I supernova (Rosswoag et al., 2008a). Multiple passage encounters result in lightcurves modulated by the orbital period (Zalamea et al., 2010; MacLeod et al., 2013), and recent work by Amaro-Seoane et al. (2012), Hayasaki et al. (2013), and Dai et al. (2013) has shown that the mass fallback properties from eccentric orbits should be quite different from those in near-parabolic encounters. Krolik and Piran (2011) have suggested that tidal stripping of a WD might explain the variability in the lightcurve of Swift J1644+57 (Levan et al., 2011; Bloom et al., 2011; Burrows et al., 2011). Finally, Dai et al. (2013) and Dai and Blandford (2013) have shown that the Roche lobe overflow of a WD in a circular orbit around a MBH will produce stable mass transfer, and a long-lived accretion flare. Transients in which the WD completes many orbits are of particular interest as they are persistent gravitational radiation sources with simultaneous electromagnetic counterparts (Sesana et al., 2008).

We review the properties of transients produced by tidal interactions between WDs and MBHs, with particular emphasis on the role that the orbit may play in shaping the ensuing mass transfer from the WD to the MBH in Section 4.3. We focus on cases where the supply of material to MBH is above the hole’s Eddington limit and launches a relativistically-beamed jet component. In Section 4.4, we discuss our assumptions about the nature of stellar clusters surrounding MBHs. We model the tidal and gravitational
wave-driven capture of WDs into bound orbits in order to predict the orbital distribution and rates of eccentric and circular mass transfer scenarios in Section 4.5. We find that these events are likely outnumbered by single-passage disruptions. In Section 4.6, we illustrate that although they are rare, WD disruptions may sufficiently outshine MS disruptions in jetted transients that they should be easily detectible. In Section 4.7, we argue that the detection or non-detection of these transients should place strong limits on the existence of MBHs with masses less than $10^5 M_\odot$. Finally, we show that WD-MBH interaction transients bear similarities in peak luminosity and timescale to the newly-identified ultra-long gamma ray bursts (GRBs; Levan et al., 2014).

4.3 Phenomenology of White Dwarf Tidal Interactions

We can distinguish between WD-MBH encounters based on the orbit with which the WD begins to transfer mass to the MBH. This section reviews some of the expected signatures of these encounters, emphasizing the role of the orbital eccentricity at the onset of mass transfer.

In Figure 4.1, we show representative light curves, calculated by assuming that $L = 0.1 M c^2$ for each of the orbital classes we will consider below. Transients produced range from a slow, smooth decline for Roche lobe overflow to multiple short-timescale flares in the eccentric tidal stripping case. We presume a WD mass of $0.5 M_\odot$ and a MBH mass of $10^5 M_\odot$ in Figure 4.1 (see e.g. Kepler et al., 2007; Maoz et al., 2012, for discussions of the single and binary WD mass distributions). In all of the following we will assume that the WD mass radius relationship is described by

$$R_{wd} = 0.013 R_\odot \left( \frac{1.43 M_\odot}{M_{wd}} \right)^{1/3} \left( 1 - \frac{M_{wd}}{1.43 M_\odot} \right)^{0.447},$$

(4.1)

from Zalamea et al. (2010). Where relevant, we will further assume that the internal structure of the WDs is described by that of a $n = 3/2$ polytrope (e.g. Paschalidis et al.,
Figure 4.1: Accretion-powered flares that result from tidal interactions between $0.5M_\odot$ WDs and a $10^5M_\odot$ MBH, calculated assuming that $L = 0.1\dot{M}c^2$. A tidal disruption event with $r_p = r_t$ is shown in blue, a repeating flare due to tidal stripping of the WD in an eccentric orbit is shown in green, and Roche lobe overflow (RLOF) and the ensuing stable mass transfer is shown in red. For comparison, the gray line shows disruption of a sun-like star and the dashed line shows the Eddington luminosity for a $10^5M_\odot$ black hole. Tidal disruption $\dot{M}(t)$ curves are from Guillochon and Ramirez-Ruiz (2013, available online at astrocrash.net). A wide diversity of flare characteristics are achieved with differing orbital parameters.

This is strictly most relevant at low WD masses, but because low-mass white dwarfs are the most common (Maoz et al., 2012) and also those most vulnerable to tidal interactions, we suggest this may be a reasonable approximation for most astrophysically relevant cases.

4.3.1 Near-parabolic orbit tidal disruption

Typical tidal disruption events occur when stars are scattered by two-body relaxation processes in orbital angular momentum into orbits that pass close to the black hole at pericenter. We will parameterize the strength of the encounter with $\beta \equiv r_t/r_p$, such that higher $\beta$ correspond to deeper encounters as compared to the
tidal radius. Simulations of WD disruptions have been performed recently by several authors (Rosswog et al., 2008a, 2009b; Haas et al., 2012; Cheng and Evans, 2013), and we describe some of the salient features here.

The vast majority of these orbits originate from quite far from the MBH, where star-star scatterings become substantial (Frank, 1978; Merritt, 2013). As a result, typical orbits are characterized by \( e \approx 1 \). The aftermath of a disruption has been well determined in the limit that the spread in binding energy across the star at pericenter is large compared to its original orbital energy (Rees, 1988). The critical orbital eccentricity above which the parabolic approximation holds is (Amaro-Seoane et al., 2012; Hayasaki et al., 2013)

\[
e > e_{\text{crit}} \approx 1 - \frac{2}{\beta} \left( \frac{M_*}{M_{\text{bh}}} \right)^{1/3}.
\]

For a \( \beta = 1 \) encounter between a \( 0.5M_\odot \) WD and a \( 10^5M_\odot \) MBH, \( e_{\text{crit}} \approx 0.97 \).

If \( e > e_{\text{crit}} \), about half of the debris of tidal disruption is bound to the MBH, while the other half is ejected on unbound orbits (Rees, 1988; Rosswog et al., 2008b). The initial fallback of the most bound debris sets the approximate timescale of peak of the lightcurve, which scales as \( t_{\text{fb}} \propto M_{\text{bh}}^{1/2}M_*^{-1}R_*^{3/2} \). The peak accretion rate, which is proportional to \( \dot{M}_{\text{peak}} \propto \Delta M/t_{\text{fb}} \), thus scales as \( \dot{M}_{\text{peak}} \propto M_{\text{bh}}^{-1/2}M_*^{2}R_*^{-3/2} \) (Rees, 1988).

The fallback curves typically feature a fast rise to peak, and then a long, power-law decay with asymptotic slope similar to \( t^{-5/3} \) (though, see Guillochon and Ramirez-Ruiz, 2013). Since the orbital time at the tidal radius is much shorter than that of the most bound debris, it is usually assumed that the accretion rate onto the MBH tracks the rate of fallback (Rees, 1988).

In Figure 4.2, we estimate typical properties for encounters between WDs of various masses and MBHs of \( 10^4 \) and \( 10^{4.5}M_\odot \). To construct this Figure, we draw on results of hydrodynamic simulations of tidal disruption of \( n = 3/2 \) polytropic stars performed by Guillochon and Ramirez-Ruiz (2013). We plot colored lines corresponding
to ten different impact parameters, where the WD would lose a fraction $0.1 - 1$ of its mass in intervals of 0.1 in an encounter with a $10^{4.5} M_\odot$ MBH. We plot a single dot-dashed line for a 50% disruptive encounter between a WD and a MBH. All of these events fuel rapid accretion to the MBH with typical accretion rates ranging from hundreds to thousands of solar masses per year. Typical peak timescales for the accretion flares are hours. The long-term fallback fuels accretion above the Eddington limit for a period of months, after which one might expect the jet to shut off, terminating the high-energy transient emission (De Colle et al., 2012).

In the upper left panel, we compare pericenter distance to both $r_s$ and $r_{\text{ISCO}} \approx 4r_s$. Simulations of tidal encounters in general relativistic gravity, for example those of Haas et al. (2012) and Cheng and Evans (2013) indicate that if the pericenter distances $r_p \sim r_s$, relativistic precession becomes extremely important and free-particle trajectories deviate substantially from Newtonian trajectories. We expect, therefore, that encounters with $r_p \lesssim r_{\text{ISCO}}$ will experience strong general relativistic corrections to the orbital motion of tidal debris. The result is likely to be prompt swallowing of the bulk of the tidal debris rather than circularization and a prolonged accretion flare. For that reason we will use $r_{\text{ISCO}}$ as a point of comparison for determining when stars are captured whole or produce a tidal disruption flare in this paper (as suggested, for example in chapter 6 of Merritt, 2013). Future simulations of these extreme encounters will help distinguish where the exact cutoff between capture and flaring lies.

### 4.3.2 Tidal stripping in an eccentric orbit

From an eccentric orbit, if $e < e_{\text{crit}}$, equation (4.2), all of the debris of tidal disruption is bound to the MBH (Amaro-Seoane et al., 2012; Hayasaki et al., 2013; Dai et al., 2013). If it is only partially disrupted, the remnant itself will return for further passages around the MBH and perhaps additional mass-loss episodes (Zalamea et al., 2010). We explore the nature of the accretion that results from the progressive tidal
Figure 4.2: The properties of tidal disruptions of WDs with masses $0.2 - 1.0 M_{\odot}$ encountering MBHs with masses of $10^4$ and $10^{4.5} M_{\odot}$. Colored lines represent encounters with a $10^{4.5} M_{\odot}$ MBH in which the WD loses a fraction between 0.1 and 1 of its total mass, in intervals of 0.1. Dot-dashed lines represent encounters in which half of the WD mass is stripped in an encounter with a $10^4 M_{\odot}$ MBH. The upper left panel shows that disruptive encounters occur outside the MBH’s Schwarzschild radius for the range of masses considered, but many close passages have $r_p < r_{\text{ISCO}}$, which may be a more appropriate cutoff for determining whether an accretion flare or prompt swallowing results from a given encounter. The remaining panels draw on simulation results from Guillochon and Ramirez-Ruiz (2013) for $n = 3/2$ polytropes to show the peak $\dot{M}$, timescale of peak, $t_{\text{peak}}$, and time spent above the Eddington limit, $t_{\text{Edd}}$. 
stripping of a WD in this section. In Sections 4.4 and 4.5, we will elaborate on the stellar dynamical processes that can lead a WD to be captured into such an orbit.

Tightly-bound orbits around the MBH are very well described by Keplerian motion in the MBH’s gravitational field and the WD should be only scattered weakly each orbit. In other words, its orbital parameters diffuse slowly in response to any background perturbations (see Section 4.4 for more discussion of the stellar dynamics of tightly-bound stars) (e.g. Hopman and Alexander, 2005). Thus, when such a WD first enters the tidal radius, it would do so only grazingly, losing a small fraction of its mass. We suggest in Sections 4.4 and 4.5 that a more typical process may be progressive tidal forcing to the point of disruption. In this picture, a WD in an initially non-disruptive orbit is eventually disrupted by the build up of tidal oscillation energy. Over many passages orbital energy is deposited into $l = 2$ mode oscillation energy of the WD (Baumgardt et al., 2006). Eventually, the oscillation energy exceeds the WD’s gravitational binding energy and mass is stripped from the WD envelope.

After the onset of mass transfer between the WD and the MBH, the WD will expand in radius and decrease in density following equation (4.1). The strength of subsequent encounters increases until the WD is completely destroyed by the MBH. At each encounter, we calculated the new $\beta$ parameter based on the adjusted mass, and in turn the corresponding $\Delta M$. The exact extent of mass loss may be modulated through the superposition of the WD’s oscillation phase and tidal forcing at pericenter (Mardling, 1995a,b; Guillochon et al., 2011). Unlike, for example, a giant star being tidally stripped (e.g. MacLeod et al., 2013), as long as degeneracy is not lifted the internal structure of the WD remains polytropic. We find that, over the course of tens of orbits, the mass loss episodes escalate from $< 10^{-2} M_\odot$ until the remaining portion of the WD is destroyed. This is in contrast to the calculation of Zalamea et al. (2010), who, as a result of using a more approximate formula for $\Delta M(\beta)$ with shallower $\beta$-dependence, predict that the tidal stripping episode will persist for $\sim 10^4$ orbits. If additional heating occurs near
the surface of the WD due to interaction between oscillations and marginally-bound material, as, for example, observed in simulations of WD (Cheng and Evans, 2013) and giant-star disruptions (MacLeod et al., 2013), the degeneracy of the outermost layers of the WD may be lifted, leading to an even more rapid exponentiation of mass-loss episodes.

The example in Figure 4.1 shows a WD being stripped in an orbit with a period of $10^{4.5}$ seconds. This timescale sets the repetition time of the flares, and corresponds to $e \approx 0.97 \approx e_{\text{crit}}$. One consequence of orbits with lower eccentricity is that the fallback of the bound material happens on very rapid timescales, potentially more rapidly than material that circularizes at the tidal radius may be viscously accreted. The ratio of fallback time (here estimated by the orbital period) to viscous time at pericenter is approximately,

$$\frac{t_{\text{visc}}}{t_{\text{orb}}} \approx 2 \left( \frac{1 - e}{0.03} \right)^{3/2} \left( \frac{\alpha_{\nu}}{0.01} \right)^{-1} \left( \frac{H/R}{0.5} \right)^{-2},$$

(4.3)

where $\alpha_{\nu}$ is the Shakura-Sunayev viscosity parameter (Shakura and Sunyaev, 1973).

When the viscous time is longer than the fallback time, the lightcurve will represent the viscous accretion of the nearly-impulsively assembled torus of tidal debris. To illustrate the accretion rate that may be expected, we employ a super-Eddington disk model proposed by Cannizzo and Gehrels (2009) and employed by Cannizzo et al. (2011) to describe super-Eddington accretion in the case of Swift 1644+57. In this simple model, the accretion rate of an impulsively assembled disk is mediated by the rate of viscous expansion of the material. Quoting from Cannizzo et al. (2011), the peak accretion rate is

$$\dot{M}_0 = 273 \left( \frac{\Delta M}{10^{-2} M_{\odot}} \right) \left( \frac{M_{\text{bh}}}{M_{\odot}} \right)^{1/2} \times \left( \frac{r_p}{10^{11} \text{cm}} \right)^{-3/2} \left( \frac{\alpha_{\nu}}{0.01} \right) M_{\odot} \text{ yr}^{-1}.$$  

(4.4)
The behavior in time is then

\[ \dot{M}(t) = \dot{M}_0 \left( \frac{t}{t_0} \right)^{-4/3}, \]  

(4.5)

where \( t_0 = 4/9\alpha^{-1} r_p^{3/2} (GM_{\text{bh}})^{-1/2} \), approximately the viscous time at pericenter for a thick disk. The \( t^{-4/3} \) proportionality is similar to that derived in the case of zero wind losses for impulsively assembled disks by Shen and Matzner (2014). Shen and Matzner (2014) go on to show if the fraction of material carried away in a wind is, for example, \( 1/2 \), then the time proportionality steepens to \( -5/3 \). This indicates that the \( t^{-4/3} \) power-law decay plotted in Figure 4.1 is at the shallow end of the range of possible behavior. Any degree of wind-induced mass loss from the disk would steepen the slope of the falloff in time, further reducing the luminosity between flaring peaks. Coughlin and Begelman (2014) have recently proposed a new thick disk and jet launching model for super-Eddington accretion phases of tidal disruption events. Their ZEBRA model can capture the rise and peak phases in the lightcurve, not just the late-time decay behavior. These characteristics will be essential in making constraining comparisons to potential future observations.

### 4.3.3 Roche-lobe overflow from a circular orbit

If the WD reaches the tidal radius in a circular orbit, mass transfer will proceed stably (Dai and Blandford, 2013). The rate at which gravitational radiation carries away orbital angular momentum,

\[ \dot{J}_{\text{GR}} = -\frac{32}{5} \frac{G^3 M_{\text{bh}} M_{\text{wd}} (M_{\text{bh}} + M_{\text{wd}})}{a^4} J_{\text{orb}}, \]  

(4.6)

is balanced by the exchange of mass from the WD to MBH, and the corresponding widening of the orbit (e.g. Marsh et al., 2004). The resulting equilibrium mass transfer
rate is then given by

\[ \dot{M}_{wd} = \left[ 1 + \frac{\zeta_{wd} - \zeta_{rl}}{2} - \frac{M_{wd}}{M_{bh}} \right]^{-1} \frac{\dot{J}_{GR}}{J_{\text{orb}}} M_{wd} \]  

(4.7)

where \( \zeta_{wd} \) and \( \zeta_{rl} \) are the coefficients of expansion of the WD and Roche lobe, respectively, in response to mass change, \( \zeta = d \ln r / d \ln M \), (eq. 19 in Marsh et al., 2004). For low mass WDs, \( \zeta_{wd} \approx -1/3 \), while the Roche lobe is well described by \( \zeta_{rl} \approx 1/3 \).

As a result of the stability, mass transfer between a WD and a MBH would persist above the Eddington limit for multiple years. They would also radiate a persistent gravitational wave signal, with frequencies of order \( \sim \left( \frac{GM_{bh}}{r_t^3} \right)^{1/2} \), or about 0.2Hz for a \( 10^5 M_\odot \) MBH and 0.5 \( M_\odot \) WD. Such frequencies would place these objects within the sensitivity range of the proposed LISA and eLISA missions (e.g. Hils and Bender, 1995; Freitag, 2003; Barack and Cutler, 2004; Amaro-Seoane et al., 2007).

### 4.4 Stellar Clusters Surrounding MBHs

The properties of the stellar systems that surround MBHs determine the the orbital parameters with which WDs encounter MBHs. The nature of the stellar systems that surround MBHs with masses less than \( 10^6 M_\odot \) remains observationally unconstrained. However, dense stellar clusters appear to almost universally surround known galactic center MBHs, which typically span the mass range of \( 10^6 \text{-} 10^9 M_\odot \). Even in galaxies that lack nuclear activity, dense stellar clusters in galactic nuclei with centrally-peaked velocity dispersion profiles strongly suggest the presence of central massive objects (e.g. Lauer et al., 1995; Byun et al., 1996; Faber et al., 1997; Magorrian et al., 1998). That MBHs should be surrounded by stars is not entirely unexpected. With a mass much greater than the average mass of surrounding stars, a MBH sinks rapidly to the dynamical center of the stellar system in which it resides (Alexander, 2005). There may also exist a population of nearly “naked” MBHs only surrounded by a hyper-compact
stellar cluster (Merritt et al., 2009; O’Leary and Loeb, 2009, 2012; Rashkov and Madau, 2014; Wang and Loeb, 2014). Such systems originate in dynamical interactions that lead to the high velocity ejection of MBHs from their host nuclei.

4.4.1 A Simple Cluster Model

In what follows, we adopt a simplified stellar cluster model in which the gravitational potential is Keplerian (dominated by the black hole), and the stellar density is a simple power-law with radius. Our approach is very similar to that of MacLeod et al. (2012). In Figure 4.3, and in the following paragraphs, we introduce the relevant scales that describe the orbital dynamics of such a system.

MBHs embedded in stellar systems are the dominant gravitational influence over a mass of stars similar to their own mass. At larger radii within the galactic nucleus, the combined influence of the MBH and all of the stars describes stellar orbits. Keplerian motion around the MBH is energetically dominant within the MBH’s radius of influence

\[
r_h = \frac{GM_{bh}}{\sigma_h^2} = 0.43 \left( \frac{M_{bh}}{10^5 M_\odot} \right)^{0.54} \text{pc},
\]

(4.8)

where \( \sigma_h \) is the velocity dispersion of the surrounding stellar system. We will assume that the velocity dispersion of stellar systems surrounding MBHs can be approximated by the \( M_{bh} - \sigma \) relation (e.g. Ferrarese and Merritt, 2000; Gebhardt et al., 2000; Tremaine et al., 2002; Gültekin et al., 2009; Kormendy and Ho, 2013). This assumption, by necessity, involves extrapolating the \( M_{bh} - \sigma \) relation to lower black hole masses than those for which it was derived. We’ve adopted \( \sigma_h = 2.3 \times 10^5 (M_{bh}/M_\odot)^{1/4.38} \text{ cm s}^{-1} \) (Kormendy and Ho, 2013).

To normalize the mean density of the stellar cluster, we assume that the enclosed stellar mass within \( r_h \) is equal to the MBH mass, \( M_{\text{enc}}(r_h) = M_{bh} \). Despite the uncertainty in extrapolating the \( M_{bh} - \sigma \) relation (e.g. Graham and Scott, 2013), this
exercise can provide a telling estimate of the order of magnitude rates of interactions between WDs and MBHs should the $M_{bh} - \sigma$ relation actually extend to lower masses. This calculation more robustly constrains the WD interaction rate relative to other interactions that are also based on the density of the stellar cluster, like main-sequence star disruptions.

In energetic equilibrium, stars within this radius of influence distribute according to a power-law density profile in radius. We will show following equation (4.12) that the energetic relaxation time for stellar clusters (of the masses we consider) is short compared to their age. Thus, the assumption of an equilibrated stellar density profile is realistic. The slope of this power-law depends on the mass of stars considered as compared to the average stellar mass (Bahcall and Wolf, 1976; Bahcall and Wolf, 1977). We adopt a stellar number density profile $\nu_* \propto r^{-\alpha}$ with $\alpha = 3/2$ in Figure 4.3 and the examples that follow. As a result, the enclosed mass as a function of radius is $M_{enc} = M_{bh} (r/r_h)^{3/2}$. If we assume that the angular momentum distribution is isotropic, then this radial density profile also defines the distribution function of stars in orbital binding energy, $\varepsilon$, (Magorrian and Tremaine, 1999),

$$ f(\varepsilon) = (2\pi\sigma_h^2)^{-3/2} \nu_*(r_h) \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha - \frac{1}{2})} \left( \frac{\varepsilon}{\sigma_h^2} \right)^{\alpha - \frac{3}{2}}. \tag{4.9} $$

This density profile also sets the local one-dimensional velocity dispersion,

$$ \sigma^2 = \frac{GM_{bh}}{(1 + \alpha)r}, \tag{4.10} $$

in the region $r \ll r_h$.

If the outermost radius of the cluster is defined by the radius of influence, then the characteristic inner radius is the distance from the MBH at which the enclosed stellar mass is similar to the mass of a single star. This scale provides insight into the
expected binding energy of the most bound star in the system. As a result, the radius that encloses a single stellar mass is

\[ r_{\bar{m}_*} = \left( \frac{\bar{m}_*}{M_{bh}} \right)^{2/3} r_h, \]  

(4.11)

where \( \bar{m}_* \) is the average stellar mass. For simplicity, we adopt \( \bar{m}_* = 1M_\odot \). In reality, mass segregation will create a gradient in which the average mass may vary substantially as a function of radius. It is possible that objects as large as \( 10M_\odot \), for example stellar-mass black holes, may be the dominant component at very small radii. However, because we adopt a radius-independent value of \( \bar{m}_* \), we take \( 1M_\odot \) as representative of the turnoff mass of a \( \sim 12 \) Gyr-old stellar population (see Alexander, 2005, for a more thorough discussion). We plot the radii that enclose 1, 10, and 100 \( M_\odot \) in Figure 4.3.

4.4.2 Orbital Relaxation

Within a dense stellar system, stars orbit under the combined influence of the MBH and all of the other stars. As a result, their orbital trajectories are constantly subject to perturbations, and deviate from closed, Keplerian ellipses. The magnitude of these perturbations may be estimated by comparing the orbital period, \( P \), to the orbital relaxation time, \( t_r \). For most stars, two-body relaxation drives orbital perturbations

\[ t_{NRR} = \frac{0.34\sigma^3}{G^2\bar{m}_*\rho \ln \Lambda}, \]  

(4.12)

equation 3.2 of Merritt (2013), also see (Binney and Tremaine, 2008; Alexander, 2005). We adopt a value for the Coulomb logarithm of \( \ln \Lambda = \ln (M_{bh}/\bar{m}_*) \), the natural log of the number of stars within the sphere of influence. Under these assumptions, a cluster with a \( M_{bh} = 10^5M_\odot \) MBH and \( \bar{m}_* = 1M_\odot \) would have undergone approximately 160 relaxation times within the age of the universe. Only when \( M_{bh} \approx 10^{6.75} \) does
Figure 4.3: Characteristic scales for WD-MBH interactions in stellar cusps surrounding MBHs given the cluster properties described in Section 4.4.1. Shown, from bottom to top, are: 1) the Schwarzschild radius, $r_s$, and the radius of the Innermost Stable Circular Orbit $\sim 4r_s$ (black solid and dotted, respectively), 2) the tidal radius, $r_t$ for WDs ($0.5M_\odot$) and MS (sun-like) stars (yellow solid and dashed), 3) the radii that enclose 1, 10, and 100 $M_\odot$ (gray dot-dashed), 4) The characteristic orbital semi-major axis that marks the transition from the empty (smaller $a$) to the full (larger $a$) loss cone regimes (red solid and dashed lines labeled $\Delta J = J_{lc}$), and 5) the MBH radius of influence, $r_h$. Filled shading denotes the region that is, on average, populated by stars. Filling colors denote the primary orbital relaxation mechanism with general-relativistic resonant relaxation (purple), mass-precession resonant relaxation (cyan), and finally non-resonant relaxation (green) being dominant from small to large radii, respectively.
the relaxation time equal the Hubble time, suggesting that the choice of a relaxed, power-law distribution of stellar number density is appropriate for the MBH masses we consider.

Tightly bound stellar orbits are also perturbed by secular torques from the orbits of nearby stars. An important aspect of estimating the “resonant relaxation” evolution time of a star’s orbit in response to these torques is estimating the coherence time of the background orbital distribution. The coherence time is the typical timescale on which neighboring orbits precess, and thus depends on the mechanism driving the precession. When this coherence time is determined by Newtonian advance of the argument of periastron, or mass precession, the incoherent resonant relaxation time is a factor of $M_{\text{bh}}/\bar{m}_*$ greater than the orbital period,

$$t_{\text{RR},M} = \left( \frac{M_{\text{bh}}}{\bar{m}_*} \right) P,$$  \hspace{1cm} (4.13)

and $t_{\text{coh},M} = M_{\text{bh}}P/M_{\text{enc}}$ (equations 5.240 and 5.202 of Merritt, 2013). The orbital period $P$ is defined as $P(a)$, the Keplerian orbital period for a semi-major axis, $a$, equal to $r$. Where general-relativistic precession determines the coherence time, the incoherent resonant relaxation time is

$$t_{\text{RR},GR} = \frac{3}{\pi^2} \frac{r_g}{a} \left( \frac{M_{\text{bh}}}{\bar{m}_*} \right)^2 \frac{P}{N_{\text{enc}}},$$  \hspace{1cm} (4.14)

for a coherence time of $t_{\text{coh},GR} = aP/(12r_g)$, where $r_g = GM_{\text{bh}}/c^2$ and $N_{\text{enc}} = M_{\text{enc}}/\bar{m}_*$ (equations 5.241 and 5.204 of Merritt, 2013). Either equation (4.13) or (4.14) determines the resonant relaxation timescale, $t_{\text{RR}}$ depending on which coherence time is shorter.

We take the relaxation time, $t_r$, to be the minimum of the resonant and non-resonant relaxation timescales,

$$t_r = \min(t_{\text{NRR}}, t_{\text{RR}}).$$  \hspace{1cm} (4.15)
The background shading in Figure 4.3 shows the dominant relaxation mechanism as a function of semi-major axis. First general relativistic resonant relaxation (4.14), then mass-precession resonant relaxation (4.13), and finally non-resonant relaxation (4.12) dominate from small to large radii.

4.4.3 Scattering to the Loss Cone

Within a relaxation time, stellar orbits exhibit a random walk in orbital energy and angular momentum. Orbits deviate by of order their energy and the corresponding circular angular momentum in this time (e.g. Lightman and Shapiro, 1977; Cohn and Kulsrud, 1978; Merritt, 2013). Thus, the root-mean-square change in angular momentum per orbit is \( \Delta J = J_c \sqrt{P/t_r} \). The characteristic angular momentum of orbits that encounter the black hole is the loss cone angular momentum, \( J_{lc} \approx \sqrt{2GM_{bh}r_p} \), where \( r_p \) is the the larger of the tidal radius, \( r_t \) and the black hole Schwartzschild radius \( r_s \) (Lightman and Shapiro, 1977). This loss cone angular momentum is significantly smaller than the circular angular momentum, \( J_{lc} \ll J_c \). Thus, the timescale for the orbital angular momentum to change of order the loss cone angular momentum is typically much less than the relaxation time. As a result, stars tend to be scattered into disruptive orbits via their random walk in angular momentum rather than in energy (Frank, 1978).

A comparison between the loss cone angular momentum, \( J_{lc} \), and the mean scatter, \( \Delta J \), gives insight into the ability of orbital relaxation to repopulate the phase space of stars destroyed through interactions with the black hole. Where \( \Delta J \gg J_{lc} \) the loss cone is often described as full (Lightman and Shapiro, 1977). Orbital relaxation easily repopulates the orbits of stars that encounter the black hole. Conversely, where \( \Delta J \ll J_{lc} \), the loss cone is, on average, empty (Lightman and Shapiro, 1977). The transition between the full and empty loss cone regimes is typical of the semi-major axes from which most stars will be scattered to the black hole (e.g. Syer and Ulmer,
1999; Magorrian and Tremaine, 1999; Merritt, 2013). From Figure 4.3, we can see that this radius of transition lies well within the MBH radius of influence, where the enclosed mass is small.

The flux of objects into the loss cone, and thus their disruption rate, is calculated based on these criteria of whether the loss cone is full or empty at a given radius (or equivalently, orbital binding energy, $\varepsilon$). The number of stars in a full loss cone is $N_{lc}(\varepsilon) = 4\pi^2 f(\varepsilon) P(\varepsilon) J_{lc}^2(\varepsilon) d\varepsilon$ (Magorrian and Tremaine, 1999). The rate at which they enter the loss cone is mediated by their orbital period defines a loss cone flux $F_{lc}(\varepsilon) = F_{\text{full}}(\varepsilon) = N_{lc}(\varepsilon)/P(\varepsilon)$. In regions where the loss cone is not full, somewhat fewer objects populate the loss cone phase space and $F_{lc}(\varepsilon) < F_{\text{full}}(\varepsilon)$ (Cohn and Kulsrud, 1978; Magorrian and Tremaine, 1999; Merritt, 2013). The exact expressions we use for $F_{lc}$ in the empty loss cone regime are not reprinted here for brevity but are given in equations (24-26) of MacLeod et al. (2012) and come from the model of Magorrian and Tremaine (1999). Once the loss cone flux as a function of energy, $F_{lc}$, is defined, the overall rate may be integrated

$$\dot{N}_{lc} = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} F_{lc}(\varepsilon) d\varepsilon,$$

where we take the limits of integration to be the orbital binding energy corresponding to $r_h$ ($\varepsilon_{\text{min}}$) and that corresponding to the $M_{\text{enc}} = 10M_\odot$ radius ($\varepsilon_{\text{max}}$).

For MBH masses that can disrupt WDs (where $r_t > r_{\text{ISCO}} \approx 4r_s$), most of the typically populated orbits are in the full loss cone limit. Most WDs, therefore, are scattered toward MBHs with mean $\Delta J \gtrsim J_{lc}$. In Figure 4.4, we plot the cumulative distribution function of $\dot{N}_{lc}$ with respect to $\Delta J/J_{lc}$. This shows that for MBHs with $M_{\text{ph}} < 10^5M_\odot$, all WDs that reach the loss cone have mean $\Delta J > 0.3J_{lc}$. This has implications for the ability of objects to undergo multiple passages with $J \approx J_{lc}$ as we discuss in the next section.
Figure 4.4: Fraction of the total loss cone flux for which the mean $\Delta J < x$, or the cumulative distribution function of loss cone flux with respect to $\Delta J$. This is computed following equation (4.16) where we set $\varepsilon_{\text{max}}$ based on $\Delta J$ and we assume $0.5 M_\odot$ WDs. We plot lines for MBH masses between $10^3$ and $10^5 M_\odot$. For all black hole masses shown, 100% of the loss cone flux originates from regions in the cluster where $\Delta J \gtrsim 0.3 J_{lc}$, and the bulk of $\dot{N}_{lc}$ originates from the full loss cone regime where $\Delta J \gg J_{lc}$. Because stars receive substantial per-orbit scatter, it is unlikely that they will complete multiple close passages by the MBH with $J \approx J_{lc}$. 

\[
\text{log}(M_{\text{BH}}/M_\odot) \\
3 \quad 3.5 \quad 4 \quad 4.5 \quad 5
\]
4.5 WD Capture and Inspiral

Here, we focus on the stellar dynamics of the capture of WDs into tightly bound orbits from which they can transfer mass to the MBH. We show that WDs are placed into tightly bound orbits primarily through binary splitting by the MBH (Miller, 2005). These orbits then evolve under the influence of tides and gravitational radiation until the WD begins to interact with the MBH. In modeling this process, we adopt aspects of the pioneering work by Ivanov and Papaloizou (2007).

4.5.1 Binary Splitting and WD Capture

A key requirement for stars to undergo multiple-passage interactions with a MBH is that the per orbit scatter in angular momentum be sufficiently small that the pericenter distance remains similar between passages. A star in the full loss cone limit ($\Delta J \gg J_{lc}$) that survives an encounter is very likely to be scattered away from its closely-plunging orbit before it undergoes another encounter. As we demonstrate in the previous section and in Figure 4.4, most WDs are in regions where the per-orbit scatter is large relative to $J_{lc}$.

Instead, in this section, we focus on the disruption of binary stars scattered toward the MBH, which can leave one star tightly bound to the MBH while the other is ejected on a hyperbolic orbit (Hills, 1988). Disruptions of binary stars lead WDs to be deposited into orbits from which they are hierarchically isolated from the remainder of the stellar system (Amaro-Seoane et al., 2012). These hierarchically isolated objects have an orbital semi-major axis that is smaller than the region that typically contains stars, $a < r_{m_*}$. This is the region inside the shaded region of Figure 4.3 as determined by equation (4.11). Given such an initial orbit, the WD may undergo many close passages with the MBH without suffering significant scattering from the other cluster stars.

To estimate the rates and distribution of captured orbits, we follow the an-
alytic formalism of Bromley et al. (2006), which is motivated by results derived from three-body scattering experiments. Bromley et al. (2006) equations (1)-(5) describe the probability of splitting a binary star as a function of impact parameter, as well as the mean and dispersion in the velocity of the ejected component. We use these expressions to construct a Monte Carlo distribution of binary disruptions. We let the binaries be scattered toward the black hole with rate according to their tidal radius, 

$$r_{t, \text{bin}} = \left( \frac{M_{\text{bh}}}{m_{\text{bin}}} \right)^{1/3} a,$$

where $m_{\text{bin}}$ is the mass of the binary. We use WD masses of $0.5 M_\odot$ and companion masses of $1 M_\odot$ in this example. We let the binaries originate from the same stellar density distribution described in Section 4.4, with a radially-constant binary fraction of $f_{\text{bin}} = 0.1$. For simplicity, we distribute this population of binaries such that there is an equal number in each decade of semi-major axis $dN/da \propto a^{-1}$, within a range $-3 < \log(a/\text{au}) < -1$, although see Maoz et al. (2012) for a more detailed consideration of the separation distribution of field WD binaries. In our simulations, the most tightly bound binaries contribute most to the population that evolves to transfer mass the the MBH, and thus this limit most strongly affects the normalization of our results. The distribution of pericenter distances is chosen given a full loss cone of binaries, such that $dN/dr_p = \text{constant}$, and we ignore the small fraction of events with $r_p < a$.

A sampling prior is placed based on the likelihood of a particular encounter occurring. This is estimated by integrating the flux of binaries to the loss cone from the portion of the cluster for which the full loss cone regime applies. This rate is $f_{\text{bin}} f_{\text{WD}}$ times the nominal loss-cone flux, $F_{\text{lc}}$, integrated from the radius of transition between the full and empty loss cone regimes for a given binary separation outward to $r_h$. This calculation is done following equation (4.16) with $\varepsilon_{\text{max}}$ determined by the binding energy at which $\Delta J = J_{\text{lc}}$. Binaries that diffuse toward the black hole gradually from the empty loss cone regime are more likely to undergo a complex series of multiple encounters, the outcome of which is less easily predicted in an analytical formalism (e.g. Antonini et al., 2012).
Therefore we do not include the diffusion of binaries toward the black hole from the empty loss cone regime in our estimate.

If the captured star has sufficiently small semi-major axis, it will be hierarchically separated from the rest of the stellar cluster as the most bound star. The requirement for this condition is that

$$a < r_{\bar{m}_*},$$

where $r_{\bar{m}_*}$ is given by equation (4.11) and $\bar{m}_* = 1M_\odot$. When selecting orbits that may undergo many passages, we require them to be hierarchically isolated following equation (4.17). As can be seen in Figure 4.3, less bound stars (in the full loss cone regime) are subject to major perturbations $\Delta J \gtrsim J_{lc}$ each orbit, and thus could not undergo a multiple passage encounter with the MBH. The most tightly bound star, by contrast, evolves in relative isolation from the rest of the cluster until it is subject to a major disturbance.

Another criteria we place on WD-capture orbits is that their gravitational radiation inspiral time be less than their isolation time. A chance close encounter is possible, but more likely is that another star is captured into a similarly tightly bound orbit. This unstable configuration can persist only as long as the stellar orbits avoid intersection. Eventually, this out-of-equilibrium configuration is destroyed, and one or both stars are scattered to more loosely bound orbits (or perhaps even a tidal disruption). Thus, we take the isolation time for a captured WD to be the inverse of the rate at which new binaries are split and deposit WDs into orbits with $a < r_{\bar{m}_*}$. This is, of course, an approximation and Nbody simulations offer the possibility to determine the time between exchanges of most-bound cluster stars – with the effects of mass segregation almost certainly playing a role (Gill et al., 2008). We therefore make a final cut that requires $t_{\text{insp}} < t_{\text{iso}}$, where $t_{\text{iso}}$ is the isolation time as described above,
and $t_{\text{insp}}$ is approximated as

$$t_{\text{insp}} \approx \frac{a}{\dot{a}} \approx \frac{e^5 a^4 (1 - e^2)^{7/2}}{G^3 M_{\text{bh}} M_{\text{wd}} (M_{\text{bh}} + M_{\text{wd}})},$$

the order of magnitude gravitational wave inspiral time (Peters, 1964). Gravitational radiation is the relevant loss term (as opposed to, for example, tides) because the orbits limited by this criteria are in the gravitational wave dominated regime of pericenter distance (Figure 4.5).

The combination of these limits on the captured WD population ensures that these WDs will interact primarily with the MBH over the course of their orbital inspiral. In the next subsections, we describe how interactions with the MBH transform the captured distribution.

### 4.5.2 Modeling the Evolution of Captured WD orbits

To model the subsequent evolution of the WD orbits under the influence of both tides and gravitational radiation, we have developed an orbit-averaged code that can rapidly trace these inspirals. The effects of gravitational radiation from the orbit are applied following the prescription of (Peters, 1964), which is equivalent to the 2.5-order post-Newtonian approximation. Tidal excitation is computed following the model of Mardling (1995a,b). Mardling (1995a) shows that the exchange between orbital and oscillation energy depends on the amplitude and phase of the WD’s oscillation as it encounters the MBH. This process leads to a “memory” of previous interactions, and orbits that evolve chaotically as a given interaction can lead to either a positive or negative change in orbital energy and angular momentum. To model the fiducial change in orbital energy (for an unperturbed star) we follow the prescription given by Ivanov and Papaloizou (2007)

$$\Delta E_t = 0.7 \phi^{-1} G m_{\text{wd}}^2 / r_{\text{wd}},$$

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Figure 4.5: Phase space of encounters between WDs and MBHs. Gravitational waves are the dominant orbital evolution term above the solid lines (shown for $M_{\text{wd}} = 0.2$, 0.6, and 1.0 $M_\odot$). Tidal excitation is the dominant orbital-energy loss term for pericenter distances below the solid line. To the right of the dashed lines, the pericenter distance is within the MBH's Schwarzschild radius and the WD would be swallowed whole. The gray shaded area (valid to the left of the dashed lines) shows the region in which tidal forcing at pericenter is strong enough to produce mass loss. Progressive shadings show the onset of mass loss, and $\Delta M = 10\%$, 50\% and 100\% of the WD mass, from top to bottom, respectively.
where $\phi = \eta^{-1} \exp(2.74(\eta - 1))$, with the dimensionless variable $\eta$ a parameterization of pericenter distance $\eta^2 = (r_p/r_{wd})^3 (m_{wd}/M_{bh})$. This expression is a fit to results computed following the method of Press and Teukolsky (1977) and Lee and Ostriker (1986), where the overlap of $l = 2$ fundamental oscillation mode with the tidal forcing is integrated along the orbital trajectory. We compared Equation (4.19) with numerical results derived computing such an integral and found at most a few percent difference as a function of $r_p$, and thus we adopt this simplifying form. The orbital energy lost through tides goes into the quadrupole fundamental mode of the WD, which oscillates with an eigenfrequency $\omega_f \approx 1.445GM_{wd}R_{wd}^3$ (Ivanov and Papaloizou, 2007). The angular momentum exchange with oscillations is related to the energy loss,

$$\Delta L_t = 2\Delta E_t/\omega_f.$$  \hfill (4.20)

Finally, we allow gravitational radiation to carry away oscillation energy from the tidally-excited WD. The luminosity of gravitational radiation scales with the oscillation energy (Wheeler, 1966), resulting in a constant decay time of

$$t_{dec} = 1.5 \times 10^2 \text{yr} (M_{wd}/M_{\odot})^{-3} (R_{wd}/10^{-2}R_{\odot})^4,$$  \hfill (4.21)

which corresponds to $t_{dec} = 6447$ yr for the $0.5M_{\odot}$ WD example used here (Ivanov and Papaloizou, 2007).

We terminate the evolution when one of several criteria are reached:

1. The pericenter distance is less than the radius at which mass loss occurs ($r_p < 2r_t$).

2. The accumulated oscillation energy of the WD exceeds its binding energy,

$$E_{osc} > \frac{3}{10 - 2n} \frac{GM_{wd}^2}{R_{wd}}.$$  \hfill (4.22)

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with \( n = 3/2 \).

3. The orbit circularizes. In the code this is when \( e < 0.1 \). Further evolution is traceable via the gravitational wave inspiral as impulsive excitation of tidal oscillations no longer occurs.

These termination criteria correspond roughly to the categories of interactions between WDs and MBHs outlined in Section 4.3. When criteria 1 is met, either a single-passage tidal disruption or multiple passage mass transfer episode can be initiated depending on the orbital eccentricity, \( e \). When criteria 2 is met, eccentric-orbit mass transfer ensues. When criteria 3 is met, the WD’s orbit evolves to eventual Roche-lobe overflow. In the next subsection, we use these termination criteria to examine the distribution of orbits at the onset of mass transfer – after they have been transformed by their tidal and gravitational wave driven inspiral.

### 4.5.3 Distributions at the Onset of Mass Transfer

In Figure 4.6, we show how captured 0.5\( M_\odot \) WDs from split binaries are eventually disrupted by a 10\(^5\)\( M_\odot \) MBH. The distribution of captured orbits is shown filled, while the final distribution is shown with lines. We find that in all cases, the deposition of orbital energy into tidal oscillation energy of the WD eventually reaches and exceeds the WD binding energy. We terminate our calculations at this point (termination criteria 2) as this represents the onset of tidally-forced mass loss from the WD to the MBH (Baumgardt et al., 2006). We find no cases of complete circularization in our simulations (termination criteria 3). Circularization without tidal disruption requires larger initial pericenter distances, where tidal excitation is minimal, and correspondingly longer isolation times in order to allow the gravitational wave inspiral to complete (e.g. Gültekin et al., 2004). The circularization and inspiral times are similar under the influence of gravitational radiation. As a result, we find no cases of termination criteria
Figure 4.6: Orbital distributions of WDs captured from split binaries at the onset of mass transfer to the MBH. Initial distributions are shown filled, final distributions are shown as lines. The upper panel shows the semi-major axis (blue) and pericenter distance (red) along with their corresponding initial distributions. The middle panels show the corresponding eccentricity and orbital period distributions. Orbits are evolved under the influence of gravitational waves and tidal excitation until the $l = 2$ oscillation energy grows to reach the WD binding energy, at which point mass will be stripped from the WD envelope. The lower panel shows the number of orbits the WD survives before the onset of mass transfer, $N_{\text{orb}}$. 
1 in our isolated evolutions. Instead, when the pericenter distance drops to within the
radius at which tides are the dominant $\Delta E$ (Figure 4.5), the tidal oscillation energy
tends to rapidly grow to exceed the WD’s binding energy leading termination criteria
2 to be met.

The number of orbits elapsed after capture and before the onset of mass trans-
fer and termination is shown in the lower panel of Figure 4.6. Following the onset
of mass loss, tidal stripping and eventual disruption over repeated pericenter passages
proceeds as described in Section 4.3. We find that most WDs are disrupted with mod-
erate eccentricity and a broad range of orbital periods between $10^3$ and $10^6$ s. The
eccentricity distribution shows no nearly-circular orbits, but many orbits with $e < e_{\text{crit}}$, equation (4.2). The orbital period is particularly important in the case of eccentric
encounters because it sets the timescale for repetition between subsequent pericenter
mass-stripping episodes. After the onset of mass transfer, the WD can be expected to
survive for at most tens of passages (Section 4.3), thus the repetition time also fixes the
range of possible total event durations.

4.6 Detecting High Energy Signatures of White Dwarf
Disruption

In this Section, we compare the relative rates and expected luminosities of
different classes of transients associated with WD-MBH interactions to discuss their
detectability. We show that although rare, WD transients should outnumber their main-
sequence counterparts in high energy detections because of their substantially higher
peak luminosities. We then calculate that the rate of these events is sufficiently high to
allow their detection by instruments such as Swift.

Main sequence disruptions significantly outnumber WD disruptions and mass
transfer interactions. In the upper panel of Figure 4.7, we compare the rate of main-
sequence star tidal disruptions to that of WD tidal disruptions, and to repeating flares resulting from mass transfer from captured WDs. The disruption rates of stars and binaries are computed by integrating the flux into the loss cone given the cluster properties outlined in 4.4, equation (4.16). In the case of repeating transients, our disruption rate calculation is supplemented by the Monte Carlo simulation that traces orbits to the onset of mass transfer, described in 4.5. To compute the values shown in the Figure, we assume a binary fraction of \( f_{\text{bin}} = 0.1 \), and a WD fraction of \( f_{\text{wd}} = 0.1 \) that applies both within the cluster and within binaries. We represent the remaining stars as main sequence stars that are sun-like, with \( R_\star = R_\odot \) and \( M_\star = M_\odot \).

White dwarf interactions display a cut-off in MBH mass where most events transition from producing flares (if \( r_p \gtrsim r_{\text{ISCO}} \approx 4r_\star \)) to being consumption events with little or no electromagnetic signature. For the 0.5\( M_\odot \) WDs plotted, this cutoff occurs at black hole masses very near \( 10^5M_\odot \). Interestingly, the progressive disruption of WDs in eccentric orbits extends to slightly higher MBH masses, since the WD is disrupted gradually, over a number of orbits, without actually penetrating all the way to the tidal radius. These limits in black hole mass are flexible depending on the spin parameter and orientation of the MBH's spin, since the general relativistic geodesic deviates substantially from a Newtonian trajectory in such deeply-penetrating encounters (Kesden, 2012). If oriented correctly with respect to a maximally rotating Kerr hole, a 0.5\( M_\odot \) WD could, marginally, be disrupted by a \( 10^6M_\odot \) black hole. A realistic spectrum of WD masses would also contribute to softening this transition from flaring to consumption. While the lowest mass WDs are expected to be rare in nuclear clusters due to the effects of mass segregation (e.g. Alexander, 2005), they are less dense than their more massive counterparts and could be disrupted by slightly more massive black holes. For example, a 0.1\( M_\odot \) WD could be disrupted by a \( 3 \times 10^5M_\odot \) black hole.

Although rare, relativistic WD transients significantly outshine their main sequence counterparts (Ramirez-Ruiz and Rosswog, 2009). In the lower panel of Figure
4.7, we combine the relative rates of different tidal interactions with their expected peak luminosities as a function of MBH mass. We allow the beamed luminosity of all of these jetted transients to trace the mass supply to the black hole, $L \propto \dot{M}c^2$, as in Figure 4.1 and assume that the degree of collimation is similar for each of the different classes of events. Given a population of MBHs with masses $M_{\text{bh}} \lesssim 10^5 M_\odot$, WD tidal disruptions should be more easily detected than main sequence disruptions. Eccentric disruptions over the course of multiple orbits favor slightly higher black hole masses. Their rarity compared to single-passage WD tidal disruptions implies that although they have similar peak luminosities they represent a fractional contribution to the range of detectible events. This result suggests that WD disruptions, rather than main sequence disruptions, should serve as the most telling signpost to MBHs with masses less than $10^5 M_\odot$.

In the following subsection, we discuss how high energy emission can be produced in these transients.

4.6.1 Dissipation and Emission Mechanisms

Internal dissipation leading to a non-thermal spectrum, to be most effective, must occur when the jet is optically thin. Otherwise it will suffer adiabatic cooling before escaping, and could be thermalized (e.g. Ramirez-Ruiz, 2005). The comoving density in the jet propagating with a Lorentz factor $\Gamma$ is $n' \approx L_j/(4\pi r^2 m_p c^3 \Gamma^2)$, and using the definition of the Thomson optical depth in a continuous outflow $\tau_j \approx n' \sigma_T(r/\Gamma)$ we find the location of the photosphere

$$r_\text{r} = \frac{\dot{M} \sigma_T}{4 \pi m_p c \Gamma^2} = 10^{13} \left( \frac{L_j}{10^{49} \text{ erg/s}} \right) \left( \frac{\Gamma}{10} \right)^{-3} \text{ cm.}$$

(4.23)

If the value of $\Gamma$ at the jet base increases by at least a factor 2 over a timescale $\delta t$, then the later ejecta will catch up (De Colle et al., 2012) and dissipate a significant
Figure 4.7: Rates of different interaction channels per galaxy, $\dot{N}_{\text{gal}}$, as a function of $M_{\text{bh}}$. The black line is the disruption of sun-like stars. Blue is the disruption of WDs, Green is the capture of WDs by split binaries into inspiralling orbits. Top: The disruption of MS stars per galactic center greatly outnumbers that of WDs. WD disruptions peak at lower $M_{\text{bh}}$ and are consumed whole by MBHs with masses $M_{\text{bh}} \gtrsim 10^5 M_{\odot}$. Repeating flares extend to slightly higher $M_{\text{bh}}$ because they are disrupted progressively with pericenter distances moderately outside the tidal radius. Bottom: When weighted by their relative luminosities, disruptions of WDs appear more common than disruptions of MS stars. This panel is normalized to the MS value, and assumes similar $f_{\text{beam}}$ for all classes of events. Repeating flares are also quite luminous, but their relative rarity implies that they should make only a fractional contribution to the population of relativistic MS disruptions.
fraction of their kinetic energy at some distance given by (Rees and Meszaros, 1994)

\[ r_t \approx c \delta t \Gamma^2 = 3 \times 10^{13} \left( \frac{\delta t}{10 \text{ s}} \right) \left( \frac{\Gamma}{10} \right)^2 \text{ cm.} \]  

(4.24)

Outside \( r_\tau \), where radiation has decoupled from the plasma, the relativistic internal motions in the comoving frame will lead to shocks in the gas (De Colle et al., 2012). This implies the following lower limit on

\[ \Gamma \gtrsim \Gamma_c = 7.5 \left( \frac{L_j}{10^{49} \text{ erg/s}} \right)^{1/5} \left( \frac{\delta t}{10 \text{ s}} \right)^{-1/5}. \]  

(4.25)

When \( \Gamma \leq \Gamma_c \), the dissipation occurs when the outflow is optically thick and an almost thermal transient is expected to emanate from the jet’s photosphere (e.g. Goodman, 1986). When \( \Gamma \geq \Gamma_c \), dissipation takes place when the jet is optically thin. In the presence of turbulent magnetic fields built up behind the internal shocks, the accelerated electrons within this region can produce a synchrotron power-law radiation spectrum similar to that observed in GRBs (Ramirez-Ruiz and Lloyd-Ronning, 2002; Pilla and Loeb, 1998; Meszaros et al., 2002). The resulting non-thermal flare from an internal shock collision will arrive at a detector at a time \( \Delta t_{\text{obs}} \approx r_t/(c\Gamma^2) \approx \delta t \) (Rees and Meszaros, 1994). Thus, the relative time of flare variability at the detector will have a close one-to-one relationship with the time variability within the jet.

Alternatively, high-energy emission can be produced as the jet propagates through the accretion disk region while interacting with very dense soft photon emission with typical energy \( \Theta_{\text{disk}} = kT_{\text{disk}}/(m_e c^2) \). A fraction \( \approx \min(1, \tau_j) \) of the photons are scattered by the inverse Compton effect to energies \( \approx 2\Gamma^2 \Theta_{\text{disk}} \), where we have assumed that a constant \( \Gamma \) has been attained. Each seed photon is boosted by \( \approx \Gamma^2 \) in frequency, yielding a boosted accretion disk spectrum (Bloom et al., 2011). The observed variability time scale, in this case, is primarily related to changes in the accretion disk.
luminosity (De Colle et al., 2012). Due to relativistic aberration, the scattered photons propagate in a narrow $1/\Gamma$ beam. The Compton drag process can be very efficient in extracting energy from the jet and can limit its maximum speed of expansion so that $\Gamma^2 L_{\text{Edd}} \lesssim L_j$ (Phinney, 1982; Ramirez-Ruiz, 2004). Typical bulk Lorentz factors range from $\Gamma \approx 10$ in quasars (Begelman et al., 1984) to $\Gamma > 10^2$ in GRBs (Lithwick and Sari, 2001; Gehrels et al., 2009). Transients that have so far been associated with tidal disruptions of stars have been mildly relativistic, with typical Lorentz factors of a few. In the case of Swift J1644+57, Zauderer et al. (2011) and Berger et al. (2012) inferred $\Gamma \approx 2.2$. Cenko et al. (2012) find the that $\Gamma \gtrsim 2.1$ is required in Swift J2058+05. In both cases, the observed spectrum can be explain by both internal dissipation and Compton drag (see e.g. Bloom et al., 2011).

4.6.2 Event Rates

We can estimate the detectable event rate by considering the space density of dwarf galaxies that might host these black holes. We estimate that a lower limit on the number density of dwarf galaxies is $\sim 10^7$ Gpc$^{-3}$ (Shcherbakov et al., 2013) although recent work has shown that it may be up to a factor of $\sim 30$ higher (Blanton et al., 2005). If we assume that the MBH occupation fraction of these galaxies is $f_{\text{MBH}}$, and adopt a per MBH rate of $\dot{N}_{\text{gal}} \sim 10^{-6}$ yr$^{-1}$, then the rate of WD tidal disruptions per volume is $\dot{N}_{\text{vol}} \sim 10 f_{\text{MBH}}$ Gpc$^{-3}$ yr$^{-1}$. Note that this rate is approximately a factor of 100 smaller than the rate estimate of Shcherbakov et al. (2013), because they adopt a higher $\dot{N}_{\text{gal}}$ that is derived by combining the tidal disruption rate normalization of an isothermal sphere ($\nu_\ast \propto r^{-2}$) (Wang and Merritt, 2004) with the fraction of disrupted WDs from N-body simulations of globular clusters (Baumgardt et al., 2004a,b).

Considering their high luminosity, these transients may be detected out to cosmological distances. As an example, the annual event rate for transients with $z < 1$
is $\dot{N}_{z<1} \sim 1500 f_{\text{MBH}} \text{ yr}^{-1}$, where we have used the fact that in an $\Omega_m = 0.3$, $H_0 = 70$ cosmology, $z < 1$ encloses a comoving volume of approximately $150 \text{ Gpc}^3$ (Wright, 2006). Because the emission is beamed, only a fraction $f_{\text{beam}}$ are detectable from our perspective due to the random orientation of the jet column. Thus we arrive at a potentially observable event rate of

$$\dot{N}_{z<1,\text{obs}} \sim 1500 f_{\text{beam}} f_{\text{MBH}} \text{ yr}^{-1}. \quad (4.26)$$

If $f_{\text{beam}} = 0.1$, then of order $150 f_{\text{MBH}}$ events are theoretically detectable per year. The fraction of these that would have triggered Swift in the past is still not completely understood. From Figure 4.2, typical peak timescales are thousands of seconds. Levan et al. (2014) suggest that $\sim 10\%$ of exposures have sufficiently long-duration trigger applied to detect a longer event duration event like a WD-MBH interaction (see Zhang et al., 2014, for another discussion of the detection of events in this duration range). Assuming that $10\%$ of the theoretically observable events are found ($f_{\text{Swift}} = 0.1$), that leaves a Swift rate of $\dot{N}_{\text{Swift}} \sim 15 f_{\text{MBH}} \text{ yr}^{-1}$. This rate is low compared to the typical GRB rate detected by Swift, but potentially high enough to build a sample of events over a several year observing window with some long-cadence observations tailored to trigger on transients of this duration.

### 4.7 Discussion

#### 4.7.1 The MBH mass function

For MBH masses of $\lesssim 10^5 M_\odot$, jetted transients associated with WD tidal disruptions are extremely luminous and fuel the black hole above the Eddington limit for nearly a year. These events offer a promising observational signature of quiescent black holes in this lower mass range due to their high luminosities. Unbeamed emission
from the accretion flow is roughly Eddington-limited (e.g. Guillochon et al., 2014b), and therefore will be at least three orders of magnitude fainter than the beamed emission (Haas et al., 2012; Shcherbakov et al., 2013). While previous Swift trigger criteria catered to much shorter-duration events (Lien et al., 2014), with increasing focus on long duration events recently (e.g. Levan et al., 2011; Cenko et al., 2012; Levan et al., 2014), the fraction of transients that would trigger Swift, $f_{\text{Swift}}$, is likely to increase or at least become better constrained in future observations.

With a Swift detection rate of order of

$$\dot{N}_{\text{Swift}} \sim 15 \, f_{\text{MBH}} \, (f_{\text{Swift}}/0.1) \, (f_{\text{beam}}/0.1) \, \text{yr}^{-1},$$

it should be possible to constrain the occupation fraction, $f_{\text{MBH}}$. More than one event per year would result if $f_{\text{MBH}} \gtrsim 0.1$, and thus it is most likely possible to constrain $f_{\text{MBH}}$ to that level or larger. In placing such a limit, there remains some degeneracy, for example, $f_{\text{MBH}} = 0.1$ in the above expression could either mean that 10% of dense nuclei harbor MBHs, or that 10% of MBHs are surrounded by stellar systems. Even so, with knowledge of the expected signatures, the detection or non-detection of WD-disruption transients can place interesting constraints on the population of MBHs in this mass range with current facilities. Non-detections of events, therefore, would argue against the presence of MBHs or the presence of stellar cusps for this mass range.

### 4.7.2 Ultra-long GRBs as WD Tidal Disruptions?

There is tantalizing evidence that tidal disruptions of WDs by MBHs have already been detected, under the guise of ultra-long GRBs (Shcherbakov et al., 2013; Jonker et al., 2013; Levan et al., 2014). Levan et al. (2014) elaborate on the properties of several members of the newly emerging class of ultra-long GRBs: GRB 101225A, GRB 111209A, and GRB 121027A. All of these GRBs reach peak X-ray luminosities of
∼ 10^{49}\text{erg} \text{s}^{-1} \) and non-thermal spectra reminiscent of relativistically beamed emission. At times greater than 10^4 seconds all of these bursts exhibit luminosities that are more than a factor of a hundred higher than typical long GRBs. Astrometrically, the two bursts for which data is available (GRB 101225A are GRB 111209A) are coincident with their host galaxy’s nuclear regions, suggesting compatibility with the idea that these transients originated through interaction with a central MBH. However, it is worth noting that if these events are associated with dwarf or satellite galaxies, they might appear offset from a more luminous central galaxy despite being coincident with the central regions of a fainter host, a clear-cut example being the transient source HLX-1 (Farrell et al., 2009). Jonker et al. (2013) discuss a long-duration x-ray transient, XRT 000519, with a faint optical counterpart and quasi-periodic precursor emission. The source is located near M86. If it is at the distance of M86, the luminosity is similar to the Eddington limit of a 10^4M_\odot MBH. If it is, instead, a background object, the emission could be beamed and have a luminosity of up to ∼ 10^{48} \text{erg} \text{s}^{-1}.

Might such events be tidal disruptions of WDs by MBHs? Further evidence is certainly needed to ascertain the origin of these bursts, but the properties, including luminosities and decay timescales are in line with those we have reviewed for disruptions of WDs by MBHs. Figure 4.8 augments the phase space diagram of Levan et al. (2014), showing characteristic luminosities and decay times for single-passage tidal disruptions of WDs and MBHs (blue shaded region). In Figure 4.8, we plot the peak timescale and luminosity of peak for the disruptions, for MBH masses from 10^3 to 10^5M_\odot, and WD masses of 0.25 - 1M_\odot. Other relevant timescales include t_{\text{Edd}}, the time above the MBH’s Eddington limit, plotted in Figure 4.2, and t_{90}, as plotted for the GRB and soft gamma-ray repeater (SGR) sources, which is a factor of ≈ 30 greater than t_{\text{peak}}.

The peaks in the lightcurve of Swift J1644+57 (e.g. Saxton et al., 2012) have been associated with periodic spikes in the mass supply from a gradually disrupting WD in an eccentric orbit by Krolik and Piran (2011). The suggested repetition time
is $P \sim 5 \times 10^4$ s (Krolik and Piran, 2011). In our $M_{\text{bh}} = 10^5 M_\odot$, $M_{\text{wd}} = 0.5 M_\odot$ example of Figure 4.6, $\sim 40\%$ of the captured population initiates mass transfer with orbital periods $10^4 < P < 10^5$ s, thus, reproducing this repetition time does seem to be possible. Our inspiral simulations suggest that such repeating encounters are approximately an order of magnitude less common than their single-passage WD-disruption counterparts. More importantly for determining the origin of Swift J1644+57, by comparison to Figure 4.7 we expect that repeating encounters with these sorts of repetition times would be detected at $\sim 10\%$ the rate of jetted main-sequence disruptions from these same MBH masses. However, single-passage WD disruptions, repeating encounters, and main-sequence disruptions each originate from different range of characteristic MBH masses (as shown in the lower panel of Figure 4.7). If there is a strong cutoff in the low end of the MBH mass function we might expect this to truncate one class of events but not another.

One remaining mystery is the shape of the lightcurve of Swift J1644+57 during the plateau phase. Variability could originate in modulated mass transfer (Krolik and Piran, 2011) or from the accretion flow and jet column itself, as described in Section 4.6, (and by De Colle et al., 2012). If the jetted luminosity traces the mass accretion rate, $L \propto \dot{M} c^2$, as we have assumed here, we would expect the peaks in Swift J1644+57’s lightcurve to trace the exponentiating mass loss from the WD – instead of the observed plateau. If, however, this simplifying assumption proves incorrect (or incomplete) it does appear to be possible to produce events with plateau and super-Eddington timescales comparable to Swift J1644+57 with multi-passage disruptions of WDs. Detailed simulations of disk-assembly in multi-passage encounters offer perhaps the best hope to further constrain the electromagnetic signatures of these events.

In WD disruptions, the jetted component is significantly more luminous than the Eddington-limited accretion disk component (about a thousand times more-so than in the main sequence case; De Colle et al., 2012; Guillochon and Ramirez-Ruiz, 2013),
and thus we have pursued the beamed high-energy signatures of these events in this paper. With the advent of LSST, however, detecting the corresponding disk emission signatures may become more promising. In a fraction of events that pass well within the tidal radius (e.g. Carter and Luminet, 1982; Guillochon et al., 2009), a detonation might be ignited upon compression of the WD (Luminet and Pichon, 1989a; Rosswog et al., 2009b; Haas et al., 2012; Shcherbakov et al., 2013). In this scenario, maximum tidal compression can cause the shocked white dwarf material to exceed the threshold for pycnonuclear reactions so that thermonuclear runaway ensues (Holcomb et al., 2013). The critical $\beta$ appears to be $\gtrsim 3$ (Rosswog et al., 2009b), so perhaps $\lesssim 1/3$ of the high-energy transients plotted in Figure 4.8 are expected to be accompanied by an optical counterpart in the form of an atypical type I supernova.

Robustly separating ultra-long GRBs into core collapse and tidal disruption alternatives remains a challenge (see e.g. Gendre et al., 2013; Boër et al., 2015; Stratta et al., 2013; Yu et al., 2013; Levan et al., 2014; Zhang et al., 2014; Piro et al., 2014). The central engines of ultra-long GRBs are essentially masked by high-energy emission with largely featureless spectra, revealing little more than the basic energetics of the relativistic outflow (Levan et al., 2014). Several distinguishing characteristics are, however, available. Variability timescales should be different (as they would be associated with compact objects of very different mass, see Section 4.6). Significantly, the evolution of the prompt and afterglow emission at high energy and at radio wavelengths would be expected to deviate from that of a canonical impulsive blast wave in tidal disruption events due to long-term energy injection from the central engine (De Colle et al., 2012; Zauderer et al., 2013). Disk emission, if detected in optical or UV observations, would present strong evidence of tidal disruption origin. While the bulk of WD disruptions would lack a coincident supernova, a minority would be accompanied by atypical type I supernovae. Optical signatures of a core-collapse event are uncertain, perhaps involving emission from the cocoon (Ramirez-Ruiz et al., 2002; Kashiyama et al., 2013; Nakauchi
Figure 4.8: Luminosity versus duration adapted from Levan et al. (2014). The WD+MBH region is the region of peak timescale and luminosity for a range of WD-MBH single-passage disruptive encounters. In the shaded region, MBH masses range from $10^3$ to $10^5 M_\odot$, while the WD masses plotted are 0.25-1$M_\odot$. For the GRB and SGR sources, $t_{90}$ is plotted. If $L \propto \dot{M}$ in the WD disruptions, $t_{90}$ is a factor $\approx 30$ greater than $t_{\text{peak}}$. The timescales and durations of WD-MBH interactions are well removed from typical long GRBs, but coincide with those of the emerging class of ultra-long GRBs, such as GRB 101225A, GRB 111209A, and GRB 121027A.

et al., 2013), accretion disk wind (MacFadyen and Woosley, 1999; Pruet et al., 2004; Lopez-Camara et al., 2009), or type IIP-like lightcurves (Levan et al., 2014) but the detection of hydrogen lines in an accompanying supernova spectrum would point to a core-collapse origin. Levan et al. (2014) emphasize that one way to tackle these observational challenges in the near term is looking statistically at the astrometric positions of ultra-long bursts and whether they coincide with galactic centers.

4.7.3 Prospects for simultaneous Electromagnetic and Gravitational Wave Detection

A primary source of interest in WD-MBH interactions has been their potential as sources of both electromagnetic and gravitational wave emission (Ivanov and Papaloizou, 2007; Rosswog et al., 2008a,b; Sesana et al., 2008; Rosswog et al., 2009b;
Zalamea et al., 2010; Haas et al., 2012; Dai and Blandford, 2013; Cheng and Evans, 2013), especially as these events, if observed, would constrain the MBH mass function at low masses (e.g. de Freitas Pacheco et al., 2006). Chirp waveforms have been computed for single, disruptive passages (e.g. Rosswog et al., 2009b; Haas et al., 2012) and should be detectible only if the source is within $\sim 1$Mpc given a $10^5 M_\odot$ MBH (Rosswog et al., 2009b).

Potentially less restrictive are longer-lived periodic signals (though, see Berry and Gair, 2013). The longest-lived transient, and that with the most uniform periodicity, would occur if a WD were overflowing its Roche lobe and transferring mass to the MBH from a circular orbit (e.g. Dai and Blandford, 2013). However, we see no such circularization events in our orbit evolution simulations. Instead, the build-up of tidal oscillation energy in the WD leads to its disruption before the orbit circularizes, even in cases where gravitational radiation is the dominant term in the orbit evolution. In these eccentric cases, the gravitational wave signature reminiscent would be of a series of roughly-periodically spaced chirps associated with the pericenter passages. It is worth noting that these passages should not be strictly periodic because the orbital period wanders chaotically as successive passages pump energy into and out of the WD oscillations depending on the oscillation phase with which it encounters the MBH (Mardling, 1995a,b).

4.8 Summary

In this paper we have discussed the role that orbital dynamics plays in shaping the transients that result from interactions between WDs and MBHs. WDs most commonly encounter black holes in single passages. Multiple passages from an eccentric orbit are about an order of magnitude less common, but would have characteristic repetition timescales of $10^4 - 10^6$ s. The relative paucity of repeating events in our
calculations, combined with the small range of MBH masses in which they appear to occur, suggests that the likelihood that \textit{Swift} J1644+57 could form via the repeating disruption channel, as outlined shortly after the event by Krolik and Piran (2011), is \( \lesssim 10\% \). We find no instances of mass transfer from a circular orbit. The consequence of these encounters is a mass supply that greatly exceeds the MBH’s Eddington limit. We expect the resulting thick accretion flow should amplify a poloidal magnetic field and launch a jet. The relativistically beamed emission from these events may be more readily detectable than beamed emission from disruptions of main sequence stars. We therefore argue that the best prospects to constraining the lower-mass end of the MBH mass function lie in searching for the high-energy signatures of WD disruption events. The possibility of collecting a sample of such events in coming years with \textit{Swift} appears promising (e.g. Shcherbakov \textit{et al.}, 2013; Jonker \textit{et al.}, 2013; Levan \textit{et al.}, 2014). The detection or non-detection of these transients should offer strong constraints on the population of MBHs with masses \( M_{\text{bh}} \lesssim 10^5 M_\odot \) and the nature of the stellar clusters that surround them.
Chapter 5

Optical Thermonuclear Transients From Tidal Compression of White Dwarfs as Tracers of the Low End of the Massive Black Hole Mass Function

5.1 Chapter Abstract

In this paper, we model the observable signatures of tidal disruptions of white dwarf (WD) stars by massive black holes (MBHs) of moderate mass, \( \approx 10^3 - 10^5 M_\odot \). When the WD passes deep enough within the MBH’s tidal field, these signatures include thermonuclear transients from burning during maximum compression. We combine a hydrodynamic simulation that includes nuclear burning of the disruption of a 0.6\( M_\odot \) C/O WD with a Monte Carlo radiative transfer calculation to synthesize the properties of a representative transient. The transient’s emission emerges in the optical, with lightcurves and spectra reminiscent of type I SNe. The properties are strongly viewing-angle dependent, and key spectral signatures are \( \approx 10,000 \) km s\(^{-1} \) Doppler shifts due to the orbital motion of the unbound ejecta. Disruptions of He WDs likely produce large
quantities of intermediate-mass elements, offering a possible production mechanism for Ca-rich transients. Accompanying multiwavelength transients are fueled by accretion and arise from the nascent accretion disk and relativistic jet. If MBHs of moderate mass exist with number densities similar to those of supermassive BHs, both high energy wide-field monitors and upcoming optical surveys should detect tens to hundreds of WD tidal disruptions per year. The current best strategy for their detection may therefore be deep optical follow up of high-energy transients of unusually-long duration. The detection rate or the non-detection of these transients by current and upcoming surveys can thus be used to place meaningful constraints on the extrapolation of the MBH mass function to moderate masses.

5.2 Introduction

Stars that pass too close to a massive black hole (MBH) can be disrupted if the MBH’s tidal gravitational field overwhelms the star’s self-gravity. The approximate pericenter distance from the MBH for this to occur is the tidal radius, \( r_t = (M_{\text{bh}}/M_*)^{1/3} R_* \) (Rees, 1988). For the star to be disrupted, rather than swallowed whole, the tidal radius must be larger than the horizon radius of the MBH. For non-spinning MBHs, this implies \( r_t > r_s = 2GM_{\text{bh}}/c^2 \). This paper examines transients generated by the tidal disruption of white dwarf (WD) stars, whose compactness means that the condition that \( r_t > r_s \) is satisfied only when \( M_{\text{bh}} \lesssim 10^5 M_\odot \) (e.g. Kobayashi et al., 2004; Rosswog et al., 2009b; MacLeod et al., 2014; East, 2014). Transient emission generated by tidal encounters between WDs and MBHs would point strongly to the existence of MBHs in this low-mass range.

Identifying the distinguishing features of WD tidal disruptions as they might present themselves in surveys for astrophysical transients is of high value. These transients could help us constrain the highly-uncertain nature of the black hole (BH) mass
function for BH masses between stellar mass and supermassive, just as main sequence star disruptions can do for supermassive BHs (Magorrian and Tremaine, 1999; Stone and Metzger, 2016). This paper focuses on the generation and detection of transients, and in so doing, it expands on a growing body of literature exploring the tidal disruption of WDs by MBHs (e.g. Luminet and Pichon, 1989a; Kobayashi et al., 2004; Rosswog et al., 2008a,b, 2009b; Zalamea et al., 2010; Clausen and Eracleous, 2011; Krolik and Piran, 2011; Haas et al., 2012; Shcherbakov et al., 2013; Jonker et al., 2013; MacLeod et al., 2014; Cheng and Bogdanović, 2014; East, 2014; Sell et al., 2015). Two primary avenues for generation of bright transients have emerged from this work. First, dynamical thermonuclear burning can be ignited by the strong compression of the WD during its pericenter passage, producing radioactive iron-group elements. Secondly, in analogy to other tidal disruption events and their associated flares, the disruption of the WD feeds tidal debris back toward the MBH fueling an accretion flare.

Thermonuclear transients from tidal compression of WDs can be generated in encounters where the WD passes well within the tidal radius at pericenter. The sequence of events in such an encounter is outlined in Figure 5.1. In these deep encounters, WD material is severely compressed in the direction perpendicular to the orbital plane (phase II in Figure 5.1), while at the same time being stretched in the plane of the orbit (Carter and Luminet, 1982; Luminet and Marck, 1985; Brassart and Luminet, 2008; Stone et al., 2013). As portions of the star pass through pericenter, they reach their maximum compression and start to rebound (phase III in Figure 5.1 and, e.g. the hydrodynamic simulations of Rosswog et al., 2008a, 2009b; Guillochon et al., 2009). The thermodynamic conditions in the crushed WD material can be such that the local nuclear burning timescale is substantially shorter than the local dynamical timescale (Brassart and Luminet, 2008; Rosswog et al., 2009b). This implies that runaway nuclear burning can occur, injecting heat and synthesizing radioactive iron-group elements in the tidal debris. The radiation takes some time to emerge from the debris, and the
resultant transients are supernova (SN) analogs, with emergent radiation peaking at optical wavelengths, phase VI in Figure 5.1 (Rosswog et al., 2008a, 2009b).

In the meantime, bound debris falls back to the MBH. In order for accretion-fed transients to be generated, the tidal debris stream must fall back to the MBH and self-intersect to produce and accretion disk, shown as phase IV in Figure 5.1 (Kochanek, 1994; Ramirez-Ruiz and Rosswog, 2009; Dai et al., 2013; Hayasaki et al., 2013; Bonnerot et al., 2016; Hayasaki et al., 2015; Shiokawa et al., 2015; Guillochon and Ramirez-Ruiz, 2015; Piran et al., 2015; Dai et al., 2015). If compact disks (with their proportionately short viscous draining times) do not form efficiently in all cases, then some events will produce luminous accretion flares while others do not (Guillochon and Ramirez-Ruiz, 2015). The inferred accretion rates onto the MBH drastically exceed the MBH’s Eddington accretion rate \( \dot{M}_{\text{Edd}} = 4 \times 10^{-3} (M_{bh}/10^5 M_\odot) M_\odot \text{yr}^{-1} \), where we have used \( L_{\text{Edd}} = 4\pi G M_{bh} c / \kappa_{\text{es}} \) and a 10% radiative efficiency, \( L = 0.1 \dot{M} c^2 \) (\( \kappa_{\text{es}} \) is the electron scattering opacity, 0.2 cm\(^2\) g\(^{-1}\) for hydrogen-poor material). This highly super-Eddington accretion flow, shown as phase V in Figure 5.1, may present an environment conducive to launching jets, implying that the observed signatures of accretion flare would be strongly viewing-angle dependent (Strubbe and Quataert, 2009; Bloom et al., 2011; De Colle et al., 2012; Metzger et al., 2012a; Tchekhovskoy et al., 2013; Kelley et al., 2014; MacLeod et al., 2014). Along a jet axis an observer would see non-thermal beamed emission, with isotropic equivalent luminosity similar to proportional to the accretion rate. Away from the jet axis, thermal emission would be limited to roughly the Eddington luminosity. In both cases, the emission is likely to emerge at high energies with X-ray and gamma-ray transients (e.g. Komossa, 2015) setting a precedent for their detection (MacLeod et al., 2014).

This paper draws on simulations published by Rosswog et al. (2009b) to present detailed calculations of the emergent light curve and spectra from the radioactively powered thermonuclear transients. We present the hydrodynamic simulation method
I. Approach

II. Compression & Burning

III. Decompression of Ejecta

IV. Circularization

V. Disk & Jet Production

VI. Radiation from Unbound Ejecta

Figure 5.1: The sequence of events during a deep-passing WD tidal disruption event. A fraction of WD tidal disruption events pass sufficiently deeply within the tidal radius to ignite thermonuclear burning within the crushed WD material (Rosswog et al., 2009b). As bound debris falls back to the MBH and forms an accretion disk and jet, the unbound remnant material expands until radiation emerges due to decay of radioactive iron group elements in synthesized in the WD core.
used and initial model in Section 5.3, and the emergent optical lightcurves and spectra in Section 5.4. We discuss the process of WD debris fallback, accretion disk formation, and the associated accretion-fueled signatures of WD disruption in Section 5.5. We then use our multiwavelength picture of transients emerging from close encounters between WDs and MBHs to discuss the prospects for the detection of thermonuclear transients at optical wavelengths and accretion signatures at X-ray and radio wavelengths by current and next generation surveys in Section 5.6. The primary uncertainty in the prevalence of these transients is the highly uncertain nature of the MBH mass function at low MBH masses. In section 5.7 we discuss the implications of our findings with the ultimate goal of using WD tidal disruption transients as means to constrain the MBH mass function in the range of $10^3 - 10^5 M_\odot$. In Section 5.8, we summarize our findings and conclude.

### 5.3 Hydrodynamic Simulation

Rosswog et al. (2009b) performed detailed hydrodynamics-plus-nuclear-network calculations of the tidal compression of WDs with a particular focus on those systems that lead to runaway nuclear burning. Our examination of the emergent lightcurves and spectra that define these transients is based on this work. The simulations of Rosswog et al. (2009b) were carried out with a smoothed particle hydrodynamics (SPH) code (Rosswog et al., 2008b). The initial stars were constructed as spheres of up 4 million SPH particles, placed on a stretched close-packed lattice so that equal-mass particles reproduce the density structures of the WDs. For both the initial WD models and the subsequent hydrodynamic evolution the HELMHOLTZ equation of state (Timmes and Swesty, 2000). It makes no approximation in the treatment of electron-positron pairs and allows to freely specify the nuclear composition that it can be conveniently coupled to nuclear reaction networks. The calculations used a minimal, quasi-equilibrium reduced reaction network (Hix et al., 1998) based on 7 species (He, C, O, Mg, Ne,
Si-group, Fe-group). Numerical experiments performed in the context of WD-WD collisions (Rosswog et al., 2009a) showed that this small reaction network reproduces the energy generation of larger networks to usually much better than 5% accuracy.

The initial WD of 0.6 $M_\odot$ is cold ($T = 5 \times 10^4$ K) and consists of 50% carbon and 50% oxygen everywhere (Model 9 of Rosswog et al., 2009b). This model is chosen since 0.6$M_\odot$ WDs are among the most common single WDs (Kepler et al., 2007). The model WD is subjected to a $\beta \equiv r_t/r_p = 5$ encounter with a 500$M_\odot$ MBH. Rosswog et al. (2009b) show that sufficiently deep passages lead to extreme distortions of the WD as it passes by the MBH. In deep passages, the pericenter distance and the size of the WD may be comparable. As a result, the entire WD is not compressed instantaneously, instead a compression wave travels along the major axis of the distorted star (see, for example, Figure 6 of Rosswog et al., 2009b). For guidance, analytical scalings for polytropic stars suggest that at this point of maximum vertical compression, $z_{\text{min}}/R_{\text{WD}} \sim \beta^{-2/\left(\gamma-1\right)}$, where $\gamma$ is the polytropic index (Stone et al., 2013). Thus, inserting $\gamma = 5/3$ gives the scaling $z_{\text{min}}/R_{\text{WD}} \sim \beta^{-3}$ (Luminet and Carter, 1986; Brassart and Luminet, 2008). Rosswog et al. (2009b)’s three dimensional simulations show that the realities of a more complex equation of state and three dimensional geometry imply some departure from these scalings, but they nonetheless provide a useful framework in interpreting the compression suffered by close-passing WDs.

This compression results in nuclear burning along the WD midplane. Following the encounter, the inner core of 0.13$M_\odot$ has been burned completely to iron group elements, and is surrounded by a shell of intermediate mass elements (IME) and an outer surface of unburned C/O. The compositional stratification resembles that of standard SN Ia models. The explosion has released an energy of $10^{50.62} \text{ erg}$ in excess of the gravitational binding energy of the initial WD. The synthesized 0.13$M_\odot$ of iron-group elements in the WD core are presumably in large part composed of radioactive $^{56}\text{Ni}$.

Lacking detailed nucleosynthetic abundance calculations, however, we must interpolate
the abundances in order to compute spectra and light curves. In doing so, we are guided by nucleosynthesis yields in calculations of SNe Ia explosions by Khokhlov et al. (1993). We assume that iron group material consists of 80% radioactive $^{56}$Ni, with the remaining 20% being stable isotopes (primarily $^{54}$Fe and $^{58}$Ni). In regions of IME we applied the compositions of the ‘O-burned’ column of Table 1 of Hatano et al. (1999). Unburned carbon and oxygen material was assumed to be of solar metallicity.

Following the encounter, the subsequent hydrodynamical evolution of the unbound material is computed for an additional 13.9 minutes as energy deposited by nuclear burning and gravitational interaction with the MBH shape the debris morphology. We show the structure of the unbound debris of the disrupted remnant in Figure 5.2. Although the nuclear energy injection is significant, it does not dominate the energetics of the WD’s passage by the MBH. The orbital kinetic energy at pericenter,

$$E_{k,p} = \frac{1}{2} M_{WD} v_p^2 = \frac{\beta G M_{WD}^2}{2 R_{WD}} \left( \frac{M_{bh}}{M_{WD}} \right)^{2/3},$$

is so large that even the $\sim 10^{51}$ ergs of explosion energy (similar to the WD binding energy) represents a perturbation because this energy is larger than the WD’s binding energy by a factor $\sim (M_{bh}/M_{WD})^{2/3}$. Because the orbital energy is so important, the morphology of the tidal debris largely retains its elongated shape seen in Figure 5.2, and the explosion is far from spherical. The energy injection does, however, shape the binding energy distribution of material to the MBH. The final mass of unbound material is 0.4 $M_\odot$, with the remaining 0.2 $M_\odot$ of material expected to be accreted onto the MBH (see Figure 4 of Rosswog et al., 2008a).

The unbound remnant material moves along its hyperbolic orbital path with a bulk velocity of 9140 km s$^{-1}$. The velocity of the least-bound debris following the tidal encounter depends on the MBH mass, but only weakly, $v_{max}/v_{esc} \approx (M_{bh}/M_{WD})^{1/6}$, where $v_{esc}$ is the WD escape velocity, $\sqrt{2GM_{WD}/R_{WD}}$. The nuclear and tidal energy
Figure 5.2: Visualization of the unbound remnant of the WD tidal disruption event. Red shading illustrates the density structure of the $0.4M_\odot$ of ejecta, while the cyan contour shows the iron group element distribution. Coordinate directions are marked on one corner of the rendered volume (not at the coordinate origin). Angles in the plane of the orbit are defined as follows: an observer at $\phi = 0$ is aligned along the $\hat{x}$-direction, an observer along the $\hat{y}$-direction is at $\phi = \pi/2$. The initial pericenter approach is in the $-\hat{y}$-direction relative to the MBH, this is at $\phi = 3\pi/2$. In the orbital plane, viewing angles along the $y$-axis, $\phi = \sim \pi/2$ and $3\pi/2$, are surface area minimizing, while angles of $\sim 0$ and $\pi$ (along the $x$-axis) maximize the projected surface area. These differences are traced out in the observed brightness in Figure 5.3.
injection lead to a maximum expansion velocity relative to the center of mass of about 12,000 km s$^{-1}$. At the end of the computation time (13.9 minutes after pericenter passage) the mean gravitational (due to interaction with the MBH) and internal energy densities are small ($\lesssim 1\%$) relative to the kinetic energy density, and the velocity structure is homologous ($v \propto r$) to within a few percent, indicating that the majority of the remnant had reached the phase of approximate free-expansion.$^1$

5.4 Optical Thermonuclear Transients

We calculate the post-disruption evolution of the WD remnant using a 3-dimensional time-dependent radiation transport code, SEDONA, which synthesizes light curves and spectral time-series as observed from different viewing angles (Kasen et al., 2006). SEDONA uses a Monte Carlo approach to follow the diffusion of radiation, and includes a self-consistent solution of the gas temperature evolution. Non-grey opacities were used which accounted for the ejecta ionization/excitation state and the effects of $\approx 10$ million bound-bound transitions Doppler broadened by the velocity gradients. We calculate the orientation effects by collecting escaping photon packets in angular bins, using 30 bins equally spaced $\cos \theta$ and 30 in $\phi$, where $\theta, \phi$ are the standard angles of spherical geometry, defined relative to the plane of orbital motion (Figure 5.1). This provides the calculation of observables from 900 different viewing angles, each with equal probability of being observed.

The diagnostic value of the synthetic spectra and light curves in validating the ignition mechanism is striking. Light curve observations constrain the energy of the explosion, the total ejected mass, and the amount and distribution of $^{56}\text{Ni}$ synthesized in the explosion. Spectroscopic observations at optical/UV/near-IR wavelengths constrain

$^1$In reality, despite the fact that the large majority of the material has obtained free expansion, an increasingly small portion of the marginally unbound material (binding energy near zero) continues to gravitationally interact with the MBH and we ignore this interaction in our analysis.
the velocity structure, thermal state and chemical stratification of the ejected matter. Gamma-ray observations constrain the degree of mixing of radioactive isotopes. Given the complexity of the underlying phenomenon, no single measure can be used to determine the viability of an explosion model; instead, we will consider together a broad set of model observables.

5.4.1 Light Curves

The optical light curve of the remnant is powered by the radioactive decay chain $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ which releases $\approx 1 \text{ MeV}$ gamma-rays. We model the emission, transport, and absorption of gamma-rays which determine the rate of radioactive energy deposition. For the first 20 days after disruption, the mean free path to Compton scattering is much shorter than the remnant size and, despite the highly asymmetrical geometry, more than 90% of gamma-ray energy is absorbed in the ejecta. This energy is assumed to be locally and instantaneously reprocessed into optical/UV photons.

In the upper panel of Figure 5.3 we show the bolometric light curve as seen from various viewing angles. The bolometric luminosity reaches a peak about 10–12 days after the disruption. The rise time, which depends little on the viewing angle, reflects the time-scale for optical photons to diffuse from radioactively heated regions to the remnant surface. The diffusion time is shorter than for a Type Ia SNe (rise time of $\approx 18$ days) because of the lower total ejected mass and also the geometric distortion, which creates a higher surface area to volume ratio than for spherical ejecta.

The asymmetry of the remnant leads to strongly anisotropic emission. The lower panel of Figure 5.3 maps the peak brightness (in units of $10^{42} \text{ erg s}^{-1}$) onto the $\cos \theta$, $\phi$ viewing angle plane. The peak luminosity varies by a factor of about 10 depending on orientation, with the model appearing brightest when its projected surface area is maximized. From such surface–maximizing orientations, anisotropy enhances the brightness by a factor of several relative to a comparable spherical model. The relatively
Figure 5.3: The top panel shows the bolometric light curve of the model as observed from different viewing angles. Due to the asymmetric geometry of the ejecta (as seen in Figure 5.2), the lightcurve peak and shape vary substantially with different viewing orientation. The lower panel shows the peak brightness as a function of viewing angle. The coordinates are defined such that the orbital plane is defined by $\cos \theta = 0$. Since this is the plane in which the ejecta distribution and velocity changes with viewing angle, the peak luminosity changes by a factor $\sim 10$ with respect to changing $\phi$, as mapped by the ejecta distribution shown in Figure 5.2.
short diffusion time also promotes a higher luminosity. The model therefore predicts maximum peak luminosities of $\sim 8 \times 10^{42}$ erg s$^{-1}$, similar to some dim SNe Ia making nearly 3 times as much $^{56}$Ni. For the less common surface–minimizing orientations, the model luminosity is significantly reduced ($\sim 8 \times 10^{41}$ erg s$^{-1}$). These orientations may be compared to the coordinate system defined in Figure 5.2.

In Figure 5.4, we decompose the bolometric lightcurve into Johnson UBVRIJ photometric bands by convolving the model spectra with filter transmission profiles. The absolute magnitudes are shown in Vega units. This figure shows several striking features of the thermonuclear transients. As one expects for typical thermonuclear SNe, the U and B bands are the most rapidly evolving. These peak earliest when the ejecta temperature is still quite high, and decline most rapidly from peak. The reddest bands, like I and J, do not show the strong second maximum caused by recombination that is observed in more luminous type Ia SNe (e.g. Kasen et al., 2006). These lightcurves also provide a quantitative measure of the dispersion in brightness in various filters with viewing angle. In general, the dispersion is broadest near peak and narrows at late times as the ejecta become increasingly spherical. Additionally, the viewing angle dependence is strongest in the U, B bands, due to differences in the degree of line blanketing of the blue part of the spectrum. We will explore this effect in more detail by examining the spectra in Section 5.4.2.

Another useful photometric diagnostic of the multicolor lightcurves is the evolution in relative colors. We examine B-V, V-R and V-I lightcurves of our models (again sampled over 100 viewing angles) in Figure 5.5. For comparison, we overplot differential colors from P. Nugent’s lightcurve templates\(^2\) for SNe of normal type Ia, 1991bg-like (Nugent et al., 2002), and SN Ib/c (Levan et al., 2005) categories. All colors show consistent reddening in time over the duration for which we have model data. The B-V color shows the most variability with viewing angle, with a spread of $\sim 1$ magnitude.

\(^2\)https://c3.lbl.gov/nugent/nugent_templates.html
Figure 5.4: Multiband lightcurves of the model thermonuclear transients realized from different viewing angles. 10 samples of $\cos \theta$ and 10 of $\phi$ are plotted, or a total of 100 different viewing angles. The U, B bands peak earliest, and show the strongest viewing angle dependence in time of peak and peak magnitude. Thus these blue colors best probe the early evolution, when the remnant is most asymmetric. At later times, the dispersion between viewing angles narrows somewhat as the ejecta become increasingly spherical. The I, J bands do not show the strong decay then secondary maximum caused by recombination around day $\sim 40$ of typical, luminous, type Ia SNe.
Figure 5.5: Differential color evolution for our model thermonuclear transients at 100 different viewing angles evenly sampled in $\cos \theta$ and $\phi$. We compare to a selection of SN templates from P. Nugent described in the text. A progressive reddening is observed in all colors as a function of time. The B-V curves show the broadest color spread with different viewing angles. Some angles show similar B-V evolution to typical Ia. V-R and V-I colors depart significantly from a standard type Ia, showing the lack of color inversion $\sim 10$ days after B maximum. In general the color evolution is more consistent with less luminous events like the 1991 bg SNe and the type Ib/c. However, the V-R color of the WD disruption transient is $\sim 0.4$ magnitudes bluer than the more standard SNe, and evolves to redder colors more rapidly.
shortly after peak (e.g. 0–10 days in Figure 5.5). By contrast, the V-R and V-I colors show relatively narrow dispersion, despite the ejecta anisotropy. This can likely be traced to differences in the opacity source for these respective wavelengths. While the V, R, and I bands are largely electron-scattering dominated, the B-band is subject to strong Fe-group absorption features. The WD disruption transient’s colors deviate from those of more common SNe in the V-R color for 1991bg events and type Ib/c and in both the V-R and V-I colors for normal type Ia.

In Figure 5.6, we examine the lightcurve shape and how it varies with viewing angle by plotting the B-band peak magnitude and \( \Delta m_{15} \) (Phillips, 1993). We also plot the Phillips et al. (1999) relation (with a solid line) for normal type Ia and the Taubenberger et al. (2008) relationship for 91bg like SNe (dashed line). Overplotted on this space are a selection of type Iax, 91bg like, and Ib/c SNe (with sample data provided by M. Drout from Figure 5 of Drout et al., 2013). Many of the tidal thermonuclear transients occupy similar phase space to SN type Iax and Ib/c – exhibiting similar decline rates but lower luminosities as compared to normal Ia. They are photometrically distinct however, from normal Ia and 91 bg SNe. The fastest-declining viewing angles, with \( \phi \sim 4 \), extend to the tail of the quick-declining tail of the normal SNe distributions.

5.4.2 Spectra

In this section we explore the distinctive features of synthetic spectra of the WD tidal disruption thermonuclear transient. The model spectra of a tidally disrupted WD broadly resembles those of SNe, with P-Cygni features superimposed on a pseudo-blackbody continuum.

Figure 5.7 compares representative model spectra at day 20 to those of several characteristic SNe classes. We plot the spectrum perpendicular to the orbital plane, and in the orbital plane along the directions of maximum and minimum brightness (as mapped in the lower panel of Figure 5.3). The model spectra show common features
Figure 5.6: Position of model thermonuclear transients in width-luminosity phase space (Phillips, 1993). Model transients exhibit a wide range of lightcurve peak magnitudes and $\Delta m_{15}$ in the B-band. The most luminous with $M_B \approx -18$ exhibit a range of $1 \lesssim \Delta m_{15} \lesssim 1.9$. Standard, luminous Ia's are plotted as a black line showing the Phillips et al. (1999) relationship. Fast and underluminous Ia SNe are shown as a dashed line with the relationship of Taubenberger et al. (2008). Data for a sample of representative 91bg-like, Iax, and Ib/c SNe are shown for comparison (sample data from Drout et al., 2013, Figure 5). The tidal thermonuclear transients exhibit similar widths to standard Ia, despite their fainter peak magnitude. This puts them in similar width-luminosity phase space to SNe like Ib/c, and Iax.
of intermediate mass elements, in particular Si II and Ca II, and a broad absorption near 4500 Å due to blended lines of Ti II, which is characteristic of cooler, dimmer SNe (e.g. Nugent et al., 2002). Figure 5.7 also shows that the spectrum of the model varies significantly with orientation. The absorption features are weaker and narrower from “face-on” viewing angles (relative to the distorted ejecta structure), as the velocity gradients along the line of sight are minimized. These angles include perpendicular to the orbital plane, and in the orbital plane where the surface area is maximized (orientations of maximum brightness). Line absorption features are stronger from pole-on views. Similarly, the degree of line blanketing also varies with viewing angle, which affects the color of the transient. In general, the models have more blanketing in the blue and UV and so look redder from pole-on views.

Immediate distinctions between the representative SNe categories and the WD tidal disruption model spectra are visible in Figure 5.7. The model spectra share relatively narrow lines of SN Ia and SN 91 bg (data from P. Nugent’s online templates, which have original sources of Nugent et al., 2002; Levan et al., 2005). But the orientations perpendicular to the orbital plane and along the direction of maximum luminosity show much weaker line signatures than do our model spectra, which, at these orientations, only show weak line features. The orientation of minimum luminosity shares more spectral commonalities with the SN 91 bg spectra, especially the blended absorption features seen in the blue. The model spectra are, however, quite distinct from SN Ib/c spectra and the high velocity Ib/c spectra typically associated with gamma ray bursts (GRBs). Both of these Ib/c template spectra show significantly broader features than all but the most extreme (and minimum luminosity) WD disruption transient spectrum.

Figure 5.8 shows the spectral evolution of the remnant from two viewing angles. The viewing angle in the left panel is in the plane of WD orbit about the MBH, \( \cos \theta = 0 \). We choose the azimuthal angle of maximum brightness, \( \phi \approx 5.5 \). The right-hand panel shows the spectral evolution as viewed nearly perpendicular to the orbital plane, where
Figure 5.7: Spectra at $t = 20$ days viewed in the orbital plane ($\cos \theta = 0$) (black, lower two spectra), the upper spectrum is near the viewing angle of maximum brightness $\phi \approx 5.5$, while the lower spectrum is near the minimum brightness $\phi \approx 1$. A spectrum viewed perpendicular to the orbital plane is also plotted. The blue wavelengths of the minimum brightness spectrum are heavily blanketed by broad and strongly blue-shifted lines. By contrast, the upper spectrum, from the brighter viewing angle is bluer and exhibits very narrow line features near their rest wavelength. The narrow lines as seen from this orientation create a much lower effective opacity in this direction and higher photosphere effective temperature as the observer sees deeper into the ejecta. Model spectra are smoothed with a 50 Angstrom rolling average. These spectra are compared to a spectral templates at $t = 22$ days for SN type Ia, SN 91 bg, SN Ib/c and a high-velocity SN Ib/c (from P. Nugent’s online templates, original sources: Nugent et al., 2002; Levan et al., 2005). The minimum brightness spectrum in the orbital plane shares some similarities with SNe of type 1991 bg, which share relatively low nickel masses. However, it is quite distinct from the type Ib/c and high velocity type Ib/c events that are often associated with gamma ray bursts.
Figure 5.8: Time series of spectra for two different viewing angles. On the left, \( \cos \theta \approx 0, \ \phi \approx 5.5 \). On the right, \( \cos \theta \approx 0.84, \ \phi \approx 2.8 \). Thus, the left timeseries is viewed in the orbital plane near the orientation of maximum brightness, while the right is viewed close to face on. The timeseries consists of spectra at times of \( t = 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 36, \) and 40 days (violet to red). The basic evolution from blue to redder continuum is accompanied by the development of strong and broad P-Cygni lines. Solid lines show the rest wavelength of the SiII 6355 feature, while dashed and dotted lines show a \( 10^4 \) km s\(^{-1} \) and \( 2 \times 10^4 \) km s\(^{-1} \) velocity offsets, respectively. Model spectra show some Monte Carlo noise, which is smoothed with a 50 Å rolling average.
\[ \cos \theta = 0.84, \text{ or } \theta \approx 30^\circ. \] At this second orientation there should be less variation with azimuthal angle. Spectra are plotted at times of \( t = 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 36, \) and 40 days after pericenter passage (as the line color goes from violet to red). This timeseries shows the progressive reddening of the spectral energy distribution as well as the gradual emergence of blended absorption lines in the spectrum. Strong absorption features due to intermediate mass elements only appear around day \( \sim 14, \) several days after peak brightness, while the early spectra are relatively featureless. This can be contrasted to typical type Ia, which show strong P-Cygni lines near peak. Model spectra also show some Monte Carlo noise, which we have smoothed by adopting a 50 Å rolling average of the model data’s 10 Å bins.

When viewed from altitude orientations other than \( \cos \theta = \pm 1, \) the projection of the hyperbolic orbital motion of the unbound debris of the tidal disruption event plays a role in shaping the spectra with respect to viewing angle. Figure 5.9 shows the variance in spectra with viewing angle \( \phi, \) the azimuthal angle in the plane of the orbit with \( \cos \theta = 0.5 \) at \( t = 20 \) days. One can immediately see that the orbital motion of the WD offers an important signature for tidal disruption events. The velocity dispersion due to expansion from the center of mass produces the typical broad P-Cygni absorption profile, with a line minimum blueshifted by \( \sim 10,000 \text{ km s}^{-1}. \) However, the bulk orbital motion contributes an additional overall Doppler shift of comparable magnitude. From viewing angles in which the remnant is receding from the observer, the two velocity components nearly cancel and the absorption feature minima occur near the line rest wavelength. From viewing angles in which the remnant is approaching the observer, the two velocity components add and the blueshifts reach \( \sim 20,000 \text{ km s}^{-1}. \)

In Figure 5.9 we explore these Doppler shifts by marking the point of maximum absorption in the SiII 6355 feature with a point. One can see this makes a characteristic “S” curve with semi-amplitude of \( \sim 10^4 \text{ km s}^{-1} \) as seen from this viewing angle and time. In Figure 5.10, we zoom in on the region around the SiII 6355 feature, and plot the line
Figure 5.9: Spectra at $t = 20$ days, as viewed from $\theta = 60^\circ$ ($\cos \theta = 0.5$), and $\phi$ ranging from 0 to $2\pi$. As viewing angle varies, the Doppler shift of the P-Cygni line features varies significantly with viewing angle. The ejecta is maximally blue-shifted near $\phi \approx 1$ and maximally red-shifted near $\phi \approx 4$, offset by a factor of $\pi$ in viewing angle. Spectra are smoothed with a 30 Å rolling average. A point marks the minimum of the SiII 6355 feature as viewed from each azimuthal angle. This can be compared with the dashes in background which show $10^4$ km s$^{-1}$ and $2 \times 10^4$ km s$^{-1}$ velocity offsets from the rest wavelength.
Figure 5.10: Normalized model spectra near the SiII 6355 feature from the same time and viewing angles ($t = 20$ days, $\theta = 60^\circ$) as Figure 5.9. In this panel we plot all 30 viewing angles sampled by the radiative transfer calculation. The x-axis shows velocity offset from the rest wavelength of the SiII feature (6355 Angstroms). The maximum absorption of the line feature shifts from +5000 km s$^{-1}$ (redshift) to -18,000 km s$^{-1}$ (blueshift) depending on azimuthal viewing angle. These spectra are each normalized by an approximate continuum constructed through a 1000 Å blend of the model spectrum, then smoothed with a 40 Å rolling average.
profiles in velocity space for all 30 azimuthal viewing angles with \( \cos \theta = 0.5 \). In this panel we plot spectra that are normalized by the continuum constructed by a 1000 Å blend of the spectrum. In these line profiles, one sees an absorption feature with width \( \sim 8000 - 10,000 \) km s\(^{-1}\). As we scan through viewing angle, the line’s P-Cygni profile shifts relative to its rest wavelength with the bulk orbital motion of the WD. Some variation in line strength and shape is also visible as we view the highly asymmetric ejecta structure from a variety of angles.

We can conclude that the discovery of a transient with reasonably broad SN-Ia like absorptions, but with spectral systematically off-set from the host-galaxy redshift by 5000-10,000 km s\(^{-1}\) would be strong evidence of tidal disruption event.

5.5 Multi-wavelength Accretion Counterparts

In addition to the photometric and spectroscopic features described above, thermonuclear transients arising from tidal disruptions of WDs are unique among the diverse zoo of stellar explosions in that they are accompanied by the accretion of some of the tidal debris onto the MBH. In this section we describe predictions for the accompanying accretion signatures.

5.5.1 Debris Fallback, Circularization, and Accretion

Following a tidal disruption of a star by a MBH, a fraction of the material falls back to the vicinity of the hole. This gravitationally bound material can power a luminous accretion flare (Rees, 1988). In the case of tidally triggered thermonuclear burning of a WD, the injection of nuclear energy into the tidal debris does change the mass distribution and fraction of material bound to the MBH by tens of percent (Rosswog et al., 2009c). The traditional thinking has long been that material falling back to the vicinity of the MBH will promptly accrete subsequent to forming a small
accretion disk with size twice to the orbital pericenter distance (Rees, 1988). This long-held assumption has recently been called into question by work that has examined the self-intersection of debris streams in general relativistic orbits about the MBH (Dai et al., 2013; Hayasaki et al., 2013; Bonnerot et al., 2016; Hayasaki et al., 2015; Shiokawa et al., 2015; Guillochon and Ramirez-Ruiz, 2015; Piran et al., 2015; Dai et al., 2015). Self-intersection, and thus accretion disk formation, are found to be much less efficient than originally believed, potentially slowing the rise and peak timescales of many tidal disruption flares (Guillochon and Ramirez-Ruiz, 2015), while also potentially modifying their characteristic flare temperatures (Dai et al., 2015).

We explore the effects of debris circularization on flare peak accretion rates and timescales for WD tidal disruptions in Figure 5.11. We use the same approach as Guillochon and Ramirez-Ruiz (2015) and perform Monte Carlo simulations of tidal disruptions and their circularization, but we use parameters appropriate for WD disruptions. In this model, the star is assumed to be initially on a zero-energy (i.e. parabolic) orbit, with its periapse distance being drawn assuming a full loss cone, \( P(\beta) \propto \beta^{-2} \) (Magorrian and Tremaine, 1999; MacLeod et al., 2014). We sample WD masses from a distribution around 0.6\( M_\odot \), \( P(M_{\text{WD}}) \propto \exp(-0.5[(M_{\text{WD}}/M_\odot - 0.6)/0.2]^2) \) (e.g. Kepler et al., 2007). Because the distribution of MBH masses is uncertain for \( M_{\text{bh}} \lesssim 10^6 M_\odot \) (Greene et al., 2008), \( M_{\text{bh}} \) is drawn from a flat distribution in log \( M_{\text{bh}} \) between \( 10^2 M_\odot \) and \( 10^7 M_\odot \). Black hole spin \( a_{\text{spin}} \) is drawn from a flat distribution ranging from zero to one. We reject combinations of WD and MBH mass that would lead to the WD being swallowed whole by the MBH by redrawing \( M_{\text{WD}} \), \( \beta \) and \( a_{\text{spin}} \); in cases which fail to produce a valid disruption after 100 redraw attempts, a new MBH mass is chosen (this allows us to include in our sampling the small fraction of events that are not swallowed by MBHs with \( M_{\text{bh}} \gtrsim 10^6 M_\odot \)).

The peak accretion rate \( \dot{M}_{\text{peak}} \) and time of peak \( t_{\text{peak}} \) are determined for each disruption using the fitting formulae of Guillochon and Ramirez-Ruiz (2013), assuming
that WDs with mass $M_{WD} < M_\odot$ are $\gamma = 5/3$ polytropes and $M_{WD} > M_\odot$ are $\gamma = 4/3$ polytropes. In Figure 5.11, the underlying grey points show the distributions in these quantities realized by the prompt fallback rate to the MBH and the timescale of peak relative to pericenter passage. Purple points show full and partial disruptions with $\beta < \beta_{\text{thermo}} = 3$. Green points show events that will also lead to the thermonuclear burning of the WD, where $\beta > \beta_{\text{thermo}}$.

Comparing the fallback and accretion rates suggests that tidal disruption flares resulting from WD encounters with MBHs are slowed in timescale and reduced in peak accretion rate by up to one order of magnitude due to circularization effects (As seen in the upper panel of Figure 5.11). The typical “dark period,” the time between disruption and when the stream first strikes itself, is found to be small, with a median value of just $1.6 \times 10^3$ s. This is roughly the amount of time the most-bound debris takes to complete one orbit, which is different than what is typical for main-sequence star disruptions by supermassive BHs where the stream can wrap a dozen times around the BH before striking itself (Guillochon and Ramirez-Ruiz, 2015). While WD disruptions are likely to be just as relativistic as their main sequence disruption cousins (Ramirez-Ruiz and Rosswog, 2009), the debris stream size is larger compared to the pericenter distance due to the smaller mass ratio of star to MBH, this means that the small deflection introduced by MBH spin is less capable of facilitating a long dark period.

With the factor of ten reduction in accretion rate due to inefficient circularization, WD disruption flares are somewhat less luminous than one would determine from the fallback rate. However, the lower panel of Figure 5.11 demonstrates that nearly all WD tidal disruptions generate super-Eddington peak accretion rates, with many leading to accretion many orders of magnitude above the Eddington limit, even after accounting for the viscous slowdown of the flares. Events that lead to tidal thermonuclear transients nearly generate accretion flows onto the MBH that range from $10^5 - 10^{10}$ times the MBH's Eddington rate.
Figure 5.11: Peak timescales and accretion rates for WD disruptions by MBHs with masses $10^2 M_\odot < M_{bh} < 10^7 M_\odot$. WD masses are chosen from a distribution around $0.6 M_\odot$, and combinations in which the WD would be swallowed whole are rejected. In the lower panel, accretion rates are plotted relative to the MBH’s Eddington rate, and in solar masses per year on the right-hand axis of the upper panel. Illustrative jet luminosities are generated assuming an efficiency of 10%, $L_j = 0.1 M c^2$ (e.g. De Colle et al., 2012; Krolik and Piran, 2012) on the left-hand axis of the upper panel. Grey points show the intrinsic fallback rates (Guillochon and Ramirez-Ruiz, 2013), while purple points show the timescales slowed or delayed by the debris stream’s self-intersection time upon return to pericenter (Guillochon and Ramirez-Ruiz, 2015). Green points highlight events with $\beta > \beta_{thermo}$ that would lead to thermonuclear ignition and the associated transient. Those events that lead to thermonuclear transients are preferentially prompt and generate highly super-Eddington accretion rates with peak timescales of $10^2 - 10^4$ s after pericenter passage, offering promising prospects for precursor emission.
5.5.2 Accretion Disk and Thermal Emission

Despite the peak mass fallback rate greatly exceeding the Eddington limit, we expect the thermal disk luminosity to be similar to Eddington at most viewing angles. The timescale for accretion to exceed the Eddington limit was found to be months to a year for WD tidal disruptions by MacLeod et al. (2014). Thus, during this Eddington limited phase, the implied disk luminosities are $L_d \sim L_{\text{Edd}} \approx 2.5 \times 10^{42} (M_{\text{bh}}/10^4 M_\odot)$ erg s$^{-1}$. And maximum temperatures are

$$T_{\text{max}} \sim 10^6 K \left( \frac{\kappa}{0.2 \text{ cm}^2 \text{ g}^{-1}} \right) \left( \frac{\eta}{0.1} \right) \left( \frac{M_{\text{bh}}}{10^4 M_\odot} \right)^{-1/4},$$

(5.2)

where $\eta$ is the radiative efficiency of the disk $L = \eta \dot{M} c^2$ (Miller, 2015), also see Haas et al. (2012) for a similar estimate derived from a slim disk model. Further, Dai et al. (2015) have argued that deeply plunging tidal disruption events might preferentially form compact disks with characteristically high effective temperatures as a result of their larger relativistic precession of periapse. This temperature implies peak emission frequencies in the soft X-ray ($\nu_{\text{peak}} \sim 6 \times 10^{16}$ Hz $\sim 0.25$ keV). If the spectrum is characterized by a blackbody, the optical $\nu L_\nu$ is lower by a factor of $\sim 10^5$ than the peak, suggesting that thermal emission in the optical band is likely very weak.

If an optically thick reprocessing layer forms from tidal debris that is scattered out of the orbital plane (Loeb and Ulmer, 1997; Ulmer et al., 1998; Bogdanović et al., 2004; Strubbe and Quataert, 2009; Guillochon et al., 2013; Coughlin and Begelman, 2014), the photosphere temperature might be lower, bringing the peak of the spectral energy distribution into the ultraviolet. As noted by Miller (2015), this is found to be the case in optically-discovered tidal disruption transients (van Velzen et al., 2011; Gezari et al., 2012; Chornock et al., 2014; Arcavi et al., 2014; Holoien et al., 2014; Vinkó et al., 2015). Miller (2015) has also suggested that disk winds may lower the peak disk temperature by an order of magnitude by reducing the flux originating from
the innermost radii of the disk. If either a lower temperature photosphere enshrouds
the disk (Guillochon et al., 2013) or strong disk winds lower the peak temperature, then
the ultraviolet or even optical luminosity could conceivably reach brightness similar to
the disk luminosity and MBH Eddington limit.

Clausen and Eracleous (2011) have shown that the irradiated unbound debris
of a WD tidal disruption can produce notable emission line spectral signatures when
irradiated by photons from the accretion disk. Emission lines of carbon and oxygen
are found to be particularly strong, with CIV at 1549 Å able to persist at $\sim 10^{38}$ erg
s$^{-1}$ for hundreds of days following the disruption. Similarly, [OIII] lines at 4363 and
5007 Å should be quite strong $\sim 10^{36} - 10^{37}$ erg s$^{-1}$ over timescales of hundreds of
days. During the phase in which the thermonuclear transient is near peak these lines
would be overwhelmed. But, as the transient fades, deep observations could look for the
signatures of these emission lines which would be expected to slowly fade to the host
galaxy level.

5.5.3 Jet Production and Signatures

Fallback and accretion rates realized by WD tidal disruptions regularly exceed
the MBH’s Eddington limit by a large margin. Thus, although thermal, accretion disk
emission is limited to $\sim L_{\text{Edd}}$, many of these systems should also launch relativistic jets
(e.g. Strubbe and Quataert, 2009; Zauderer et al., 2011; De Colle et al., 2012). These
jets carry power which may depend on MBH spin and available magnetic flux as in
the Blandford and Znajek (1977) mechanism or they can even be powered solely by
the collimation of the radiation field (Sadowski and Narayan, 2015). In either case, the
isotropic equivalent jet power is expected to be a fraction of $\dot{M}c^2$ (e.g. Krolik and Piran,
2012; De Colle et al., 2012; Tchekhovskoy et al., 2013; Sadowski and Narayan, 2015).
As a result, for viewers aligned with the jet axis, the isotropic equivalent of the beamed
emission from jetted transients can substantially outshine the thermal emission because
The jet power, the radiative efficiency, and the relativistic beaming factor along the line of sight are all thus uncertain to some degree (See e.g. Krolik and Piran, 2012, section 3.1, for a description of these uncertainties in the context of tidal disruption jets).

In the upper panel of Figure 5.11 we map the accretion rate to a peak beamed jet luminosity following a simple accretion rate scaling, \( L_j = 0.1 \dot{M} c^2 \). Under these assumptions, typical peak jet luminosities for events that lead to explosions are in the range of \( \sim 10^{47} - 10^{50} \) erg s\(^{-1}\) with rise timescales of \( \sim 10^2 - 10^4 \) seconds. These powerful jets will result in short-duration, luminous, flares with the bulk of the jet’s isotropic equivalent luminosity Doppler boosted to emerge in high energy bands (MacLeod et al., 2014). At viewing angles along the jet beam, the observer would see high energy emission arising either from Compton-upscattering of the disk’s photon field (which peaks in the UV/soft X-ray, see equation 5.2) or by internal dissipation within the jet (van Velzen et al., 2011, 2013; Krolik and Piran, 2012; De Colle et al., 2012; Tchekhovskoy et al., 2013; MacLeod et al., 2014). In the cases of transients thought to be relativistically beamed tidal disruption events (Swift J1644+57, J2058+05, and J1112-8238) the spectrum can be explained by either alternative (Bloom et al., 2011; Burrows et al., 2011; Cenko et al., 2012; Pasham et al., 2015; Brown et al., 2015). The observed luminosity of these transients suggest that X-ray efficiencies \( L_X = \eta_X L_j \) of order unity may be typical (Metzger et al., 2012a). Further, the jet itself can be relatively long-lived, with accretion exceeding the MBH’s Eddington limit for timescales of months to a year (e.g. Figure 2 of MacLeod et al., 2014).

If the Blandford and Znajek (1977) mechanism supplies the jet power, there may be a strong jet power (and thus luminosity) dependence on the energy source – the MBH’s spin and the magnetic field in the disk midplane (e.g. Tchekhovskoy et al., 2013). Our illustrative assumption of 10% efficiency is applicable for nearly maximally rotating MBHs. McKinney (2005) show, through numerical simulations, a steep dependence of
jet power on spin, where ∼7% of $\dot{M}c^2$ is ejected in jet power for Kerr spin parameter of $a/M = 1$, while for $a/M = 0.9$, efficiencies of 1% result. Thus, if the typical MBH in this mass range has a low spin parameter, we might expect systematically less luminous jets than if MBHs are universally rapidly-spinning.

Both on and off of the beaming axis, radio emission generated from interaction between the relativistic outflow and the external medium should trace the jet activity. Along the jet axis, the fundamental plane of BH activity (Merloni et al., 2003) relates X-ray and 5GHz Radio luminosity to BH mass. Ignoring uncertainties, $\log L_R = 6.32 + 0.82 \log M_{bh} + 0.62 \log L_X$ (Merloni et al., 2003). Given a flare accretion rate, we can also use the fundamental plane to estimate characteristic radio luminosities. For a boosted luminosity of $L_X = 10^{48}$ erg s$^{-1}$ we might infer an intrinsic luminosity of $L_X = 10^{46}$ erg s$^{-1}$ given $\Gamma_j = 10$ (e.g. Metzger et al., 2012a; De Colle et al., 2012; Mimica et al., 2015).

Then, with $\log M_{bh} = 4$, the predicted 5GHz radio luminosity would be $L_R \sim 10^{38}$ erg s$^{-1}$. This characteristic radio luminosity may be de-boosted to off-axis viewing angles with the transformation $L_\nu = L_{\nu,0}(1 - \beta_j \cos(i_{\text{obs}}))^{\alpha_s - 2}$, where $L_\nu$ is the observed and $L_{\nu,0}$ is source luminosity density at frequency $\nu$. The jet velocity is $\beta_j = v_j/c$ and $\cos(i_{\text{obs}})$ accounts for the observer’s viewing angle. The spectral index is $\alpha_s$. This expression suggests a difference of a factor of ∼ 40 for on vs off axis viewing angles ($\cos(i_{\text{obs}})$ of 0 or 1) with a jet Lorentz factor $\Gamma_j = 10$ and $\alpha_s = 1.3$ (Zauderer et al., 2011). With $\alpha_s = 1$, a factor of 200 can easily arise.

To illustrate the time dependent properties of this radio emission, we compare to afterglows of decelerating jets calculated by (van Eerten et al., 2010). The kinetic energy of these jets is $E_K = 2 \times 10^{51}$ erg, which implies a fraction of the rest energy $\eta_j = E_K/M_{\text{acc}}c^2 \approx 6 \times 10^{-3}$ where the accreted mass $M_{\text{acc}} = 0.2 M_\odot$ goes into powering the jet. The jet opening angle is assumed to be $\theta_j = 0.2$. The isotropic equivalent power is $\approx 10^{53}$ erg, implying a beaming fraction of $\approx 0.02$ and a mean Lorentz factor of $\Gamma_j \sim 7$. The surrounding medium number density is assumed to be $n = 1$ cm$^{-3}$. We might
expect that the jet-medium interaction dynamics would proceed somewhat differently in the case of a tidal disruption event, where the jet is not an impulsively powered blastwave. However, the jet power is dominated by the time of peak accretion, which comes long before the afterglow peaks at longer wavelengths. Thus, the impulsively powered blastwave provides a reasonable zeroth-order approximation of the afterglow properties.

Our results are visualized in Figure 5.12, which represents the multi-wavelength and multi-viewing angle perspective of a deep-passing WD tidal disruption event. Properties that can be observed only from along the jet axis (here $0^\circ$), are drawn with dot-dashed lines. Properties visible to off-axis observers are shown in solid lines. Colors denote the wavelength of the emission plotted. We show X-ray (0.2 -10 keV), Optical (V-band) as well as 1 and 10 GHz radio emission. Efficiencies in these bands for the disk and jet components are computed assuming that the disk spectrum is approximately a blackbody at $T_{\text{max}}$, equation (5.2), and the jet spectrum is boosted by $\Gamma_j^2$ in frequency and apparent luminosity. For an observer at $0^\circ$, a luminous X-ray jet proceeds an optical afterglow, which is followed by the thermonuclear transient. The radio afterglow follows as the thermonuclear transient begins to fade in the optical. The X-ray jet is depicted with a shaded region to represent the high level of variability observed in Swift J1644+57, J2058+05, and J1112-8238 (e.g. Saxton et al., 2012; Pasham et al., 2015; Brown et al., 2015). For an off axis observer, early time emission is dominated by the accretion disk, whose peak is in the soft X-ray. Disk optical and UV are not plotted here because they are $10^5$ and $10^4$ times less luminous, respectively (assuming blackbody emission from the disk). At off-axis angles, the thermonuclear transient is substantially brighter (and shorter duration) than the optical afterglow. The radio afterglow follows and peaks with timescales of 0.1 - 1 year, depending on viewing angle.
Figure 5.12: This diagram depicts the observed multiband lightcurve of a hypothetical deep-passing WD tidal disruption by a $10^3 M_\odot$ MBH. We have assumed that a jet is launched carrying kinetic energy of $6 \times 10^{-3}$ of the accreted rest energy of $0.2 M_\odot$ from the disruption of a $0.6 M_\odot$ WD, or $2 \times 10^{51}$ erg. Line styles denote different viewing angles with respect to this jet axis. Dot-dashed lines are along the jet axis, while solid lines denote an off-axis perspective. Colors show different wavelengths of emission from X-ray to 1 GHz radio. For a viewer along the jet axis, a luminous jet may precede an optical-wavelength thermonuclear transient. The thermonuclear transient significantly outshines both jet afterglow and disk thermal emission at optical wavelengths. For off-axis events detected in the optical, accompanying X-ray emission from the accretion disk along with a radio afterglow complete the multiwavelength picture, and significantly distinguish these thermonuclear transients from other SNe.
5.6 Event Rates and Detectability of WD Tidal Disruption Signatures

In this section, we use our estimates of the observable properties of WD tidal disruption events to study the rate and properties of events detectable in current and upcoming high energy monitors and optical surveys.

5.6.1 Specific Event Rate

Following the calculation presented in MacLeod et al. (2014), we estimate that the specific (per MBH) event rate of WD tidal disruptions is of order $\dot{N}_{\text{MBH}} \sim 10^{-6} \text{ yr}^{-1}$. This estimate is based on the method of Magorrian and Tremaine (1999). In so doing, we assume that the MBH mass-velocity dispersion relation can be extrapolated to MBH masses of $\sim 10^3 - 10^5 M_\odot$. This extrapolation suggests that the MBH is energetically dominant within a region of radius $r_h = GM_{\text{bh}}/\sigma_h^2 = 0.43 \left(M_{\text{bh}}/10^5 M_\odot\right)^{0.54} \text{ pc}$, where $\sigma_h$ is the velocity dispersion, adopted from the $M_{\text{bh}} - \sigma$ relation, $\sigma_h = 2.3 \times 10^5 \left(M_{\text{bh}}/M_\odot\right)^{1/4.38} \text{ cm s}^{-1}$ (Kormendy and Ho, 2013). We assume that the stars within this radius are distributed in a steep cusp with $\nu_\ast \propto r^{-3/2}$, as representative of a relaxed stellar distribution (Frank and Rees, 1976; Bahcall and Wolf, 1976; Bahcall and Wolf, 1977). We normalize the density of stars in the central cluster (at radii less than $r_h$ from the MBH) by assuming that the enclosed stellar mass is equal to the MBH mass (Frank and Rees, 1976). While these assumptions are motivated by observed distributions of stars around supermassive BHs (e.g. Lauer et al., 1995; Faber et al., 1997; Magorrian et al., 1998; Syer and Ulmer, 1999; Kormendy and Ho, 2013), there is significant uncertainty in their extrapolation to low mass. Finally, we assume that the fraction of stars that are WDs is $f_{\text{WD}} = 0.1$, and that they are distributed in radius proportionately with the rest of the stellar distribution. For more discussion of the derivation of this rate we refer the reader to MacLeod et al. (2014).
Another possible host system for lower or intermediate mass MBHs is globular clusters. The tidal disruption event rate (of main sequence stars) in these clusters is relatively low, $\sim 10^{-7}$ yr$^{-1}$ (Ramirez-Ruiz and Rosswog, 2009). A smaller fraction still of these events would be WD tidal disruptions. If 1% of the events are WD disruptions, the event rate per globular cluster MBH would be $\sim 1$ Gyr$^{-1}$ (e.g. Baumgardt et al., 2004a,b; Haas et al., 2012; Shcherbakov et al., 2013; Sell et al., 2015). Each galaxy hosts many globular cluster systems, but not all of them will necessarily host a MBH. By contrast, evidence from the Milky Way’s globular cluster system is that most globular clusters do not host MBHs at the present epoch (Strader et al., 2012). If, for example, one in 100 clusters hosted a MBH, then this would mitigate the potential enhancement of a given galaxy hosting up to $\sim 100$ globular clusters. Thus, with the evidence present, we suggest that MBHs in dwarf galaxies may be the primary source of WD tidal disruption events.

Only a fraction of disruption events pass close enough to the MBH to lead to a thermonuclear transient. We can estimate the fraction of disruption events that lead to a thermonuclear transient based on the relative impact parameters needed. We denote critical impact parameter leading to thermonuclear ignition as $\beta_{\text{thermo}}$. When the phase space of the loss cone is full because it is repopulated efficiently in an orbital period, the fraction of tidal disruption events with $\beta > \beta_{\text{thermo}}$ is $(\beta_{\text{thermo}}/\beta_{\text{ml}})^{-1}$, where $\beta_{\text{ml}}$ is the impact parameter at which the WD would start lose mass. This is expected to be the case for WDs in clusters around MBHs (See Figure 4 of MacLeod et al., 2014). For WDs, $\beta_{\text{ml}} \approx 0.5$ (Guillochon and Ramirez-Ruiz, 2013; MacLeod et al., 2014) and $\beta_{\text{thermo}} \approx 3$ (Rosswog et al., 2009b), so we can expect a fraction $f_{\text{thermo}} \approx 1/6$ of events to result in a thermonuclear transient. When the MBH mass becomes too large $\gtrsim 10^5 M_\odot$, the deeply plunging orbital trajectories will lead to the WD being swallowed by the MBH with no possibility to produce a flare or thermonuclear transient (MacLeod et al., 2014).
5.6.2 The MBH Mass Function: Estimating the Volumetric Event Rate

To convert this specific event rate to a volumetric rate, we must then consider the space density of MBHs hosting dense stellar clusters. Sijacki et al. (2015) find a MBH number density per unit MBH mass of \( \Phi(M_{\text{bh}}) \approx 10^7 \text{ Gpc}^{-3} \text{ dex}^{-1} \) for \( M_{\text{bh}} \sim 10^6 \) and redshifts \( z \lesssim 2 \) in the Illustris simulation. In the following, we illustrate the range of possibilities using three examples for the extrapolation of this mass function present themselves.

1. One possibility is that the mass function for \( M_{\text{bh}} < 10^6 M_\odot \) is flat, and \( \Phi(M_{\text{bh}}) = 10^7 \text{ Gpc}^{-3} \text{ dex}^{-1} \).

2. A second possibility is that the slope of approximately \( \Phi(M_{\text{bh}}) \propto M_{\text{bh}}^{-1/2} \) observed in the MBH mass function for MBHs \( 10^7 M_\odot < M_{\text{bh}} < 10^9 M_\odot \) (ie \( 10^7 M_\odot \) MBHs are approximately \( 10 \times \) more common than \( 10^9 M_\odot \) MBHs) extends to masses below \( 10^6 M_\odot \). In this case we normalize the distribution to \( \Phi(10^7 M_\odot) \approx 5 \times 10^6 \text{ Gpc}^{-3} \text{ dex}^{-1} \). If the occupation fraction of MBHs in dwarf galaxies is of order unity, it would indicate a rising MBH mass function to lower masses (e.g. work by Blanton et al., 2005, constrains the density of dwarfs in the local volume).

3. A third possibility is that no MBHs below \( \sim 10^6 M_\odot \) exist, although new detections of MBHs with masses \( \sim 10^4 M_\odot \) make this option appear unlikely (Farrell et al., 2009; Reines et al., 2013; Baldassare et al., 2015).

Each of these mass functions can be integrated over MBH mass to give the volume density of MBHs in the that disrupt WDs, \( n_{\text{MBH}} \). We adopt limits of \( 10^3 - 10^5 M_\odot \) here, since lower-mass MBHs are less likely to host tightly-bound stellar clusters, and higher mass MBHs often swallow WDs whole rather than disrupting them (e.g. MacLeod et al., 2014).
Adopting the flat extrapolation of the MBH mass function to lower masses implies \( n_{\text{MBH}} \approx 2 \times 10^7 \text{ Gpc}^{-3} \). If we instead assume that the mass function continues to rise to lower MBH masses, the volume density of \( 10^3 - 10^5 M_\odot \) is \( n_{\text{MBH}} \approx 4 \times 10^8 \text{ Gpc}^{-3} \). The volumetric event rate of thermonuclear transients accompanying WD disruptions in dwarf galaxies can then be estimated as

\[
\dot{N}_{\text{vol}} \approx 1.7 \left( \frac{\dot{N}_{\text{MBH}}}{10^{-6} \text{yr}^{-1}} \right) \left( \frac{n_{\text{MBH}}}{10^7 \text{Gpc}^{-3}} \right) \left( \frac{f_{\text{thermo}}}{1/6} \right) \text{yr}^{-1} \text{Gpc}^{-3}. \tag{5.3}
\]

where a factor of 2 higher gives the appropriate scaling for a flat extrapolation of the MBH mass function, and a factor of 40 gives the appropriate scaling for a MBH mass function that rises to lower masses. We will use these examples to illustrate most of the remainder of our analysis. In Section 5.7.4, we allow for a MBH mass function with free power-law index below \( 10^7 M_\odot \).

### 5.6.3 Detecting Thermonuclear Transients with LSST

With an estimate of the volumetric event rate as guidance, we can now estimate the detection rate and detectable distributions of events in upcoming optical surveys. We will focus on the Large Synoptic Survey Telescope (LSST), which, when operational, will survey south of +10 degrees declination and have an approximately 3 day cadence, making it well suited to catching these relatively rapid transients (Collaboration et al., 2009). The anticipated R-band limit for a single 15 second LSST exposure is 24.5 magnitude (Collaboration et al., 2009).

We calculate that the brightest R-band viewing angles (\( M_R \approx -18 \)) could be detected by LSST out to a luminosity distance of 2.0 Gpc (redshift, \( z = 0.37 \)), if we adopt a typical R-band extinction, \( A_R \), of 1 magnitude (we assume a WMAP9 Cosmology, Hinshaw et al., 2013). This extinction implies \( A_V \sim 1.25 \) (e.g. Hendricks et al., 2012). This is higher than most observed type Ia SNe discovered (Holwerda,
Figure 5.13: A Monte Carlo realization of thermonuclear transients captured by a magnitude limited transient optical survey. For this figure we adopt a limiting magnitude of 24.5, the R-band single exposure limit for LSST. Black lines show the intrinsic distributions of event properties, including absolute magnitude, viewing angles, color and photosphere velocity evaluated near R-band peak at $t = 16$ days. Blue histograms show how these intrinsic properties are mapped to the detected events. Detectible events exhibit a preference for particular viewing angles, and the detected absolute magnitude distribution is centered around the brighter events. The photosphere velocity is assessed by the doppler offset of the minimum of the SiII 6355 Å line and scales as $(M_{bh}/M_{WD})^{1/6}$. The event rate of detection given these survey properties and a flat extrapolation of the MBH mass function to masses below $10^6 M_\odot$ is $\approx 14 \text{ yr}^{-1}$.
2008), but we expect that some lines of sight into galactic nuclear regions would be heavily extincted (Strubbe and Quataert, 2009). However, the thermal tidal disruption flare discovered by Gezari et al. (2012) exhibits relatively low reddening, consistent with $A_V \approx 0.25$ for $R_V = 3.1$. Given this maximum viewing distance we use equation (5.3) to infer an event rate of $\approx 44$ events per year within the 13.3 Gpc$^3$ volume inclosed by $z = 0.37$ for MBH mass function option (1) in Section 5.6.2, a flat extrapolation to lower MBH masses. We find $\approx 890$ events per year for MBH mass function (2), which rises to lower MBH masses.

There is, however, substantial variation with viewing angle seen in the lightcurves. We therefore perform a Monte Carlo sampling of events in viewing angle and redshift with viewing angles sampled isotropically and redshifts sampled according to the co-moving volume. In this simple model, we assume all events have the lightcurves of the representative tidal disruption event described in this paper. In our Monte Carlo simulations, we find an LSST-detectable event rate of

$$\dot{N}_{LSST} \approx 14 \ (290) \ yr^{-1}$$

for MBH mass function options (1) and (2), respectively. This rate scales with the same physical parameters as the volumetric rate of equation (5.3), and with the adopted extinction as $10^{-3 A_R/5}$ (ie, with no extinction, the detectable event rate would be a factor of $\sim 4$ higher than with $A_R = 1$). This calculation suggests that this distribution of events in distance and in peak brightness implies that $\sim 60\%$ of events are detected within the maximum volume based on the observed sky area, a conclusion that also holds for other survey limiting magnitudes.

In Figure 5.13, we compare intrinsic distributions of events to those that could be observed with LSST. In black, we plot the intrinsic distributions of source properties, while the shaded blue histograms show the distributions of detected events. The left two
panels look at source absolute and apparent magnitude distributions. The detectable distribution is biased toward the viewing angles which generate the brightest transients. This can be seen both in the absolute magnitude distribution and in the center panels which show the viewing angle distributions. The two right-hand panels quantify some of the ways that this viewing angle preference propagates into source properties evaluated at $t = 16$ days, near R-band peak. We find that detected events are slightly biased toward lower $B - V$ (bluer) colors at day 16. Lower photosphere velocities are also preferred, with those blue-shifted by more than 15,000 km s$^{-1}$ somewhat less likely to be observed. In summary, though, these selection effects are mild, and do not suggest than one particular viewing angle, color, or photospheric velocity is strongly preferred.

### 5.6.4 Detecting Beamed Emission with High-Energy Monitors

In Section 5.5.3 we outlined a case for the production of jets and beamed emission in tidal disruptions of WDs. Here, we examine the detection of this jetted emission by high-energy monitors like the Swift Burst Alert Telescope (BAT) (Krimm et al., 2013). The BAT is sensitive in the hard X-ray channel ($\sim 15-150$ keV) and is thus well suited to detection beamed emission generated by a tidal disruption-fed accretion flow. We can use the BAT threshold $\sim 2 \times 10^{-10}(t/20\text{ks})^{-1/2}$ erg s$^{-1}$ cm$^{-2}$ (5 sigma) for an exposure time $t$.\(^3\) This implies that a jetted transient for which the BAT-band luminosity is $10^{48}$ erg s$^{-1}$ can be observed to a luminosity distance of $2 \times 10^{28}$ cm (6.5 Gpc) or, with our assumed WMAP9 cosmology, a redshift of 0.96. If the jet luminosity reaches $10^{49}$ erg s$^{-1}$, then it may be detected to a redshift of 2.45.

These redshifts imply that the $10^{48}$ erg s$^{-1}$ and $10^{49}$ erg s$^{-1}$ transients can be observed within cosmological volumes 11 and 65 times larger than the LSST volume for the thermonuclear transient. Following from the volumetric event rate, we expect the BAT detection rate of WD disruption events that also produce a thermonuclear transient to be...
transient to be of the order of

\[ \dot{N}_{\text{BAT}} \approx 12 \text{ yr}^{-1}, \quad (5.5) \]

if we assume \(10^{49} \text{ erg s}^{-1}\) transients with a beaming fraction of \(1/50\) (\(\Gamma_j \sim 7\)), that the BAT monitors 20\% of the sky (Krimm et al., 2013), and a flat MBH mass function, option (1) in Section 5.6.2. If the typical transient is instead \(\sim 10^{48} \text{ erg s}^{-1}\), but other properties are similar, then the BAT rate is \(\approx 2 \text{ yr}^{-1}\) based on the smaller accessible volume. If the MBH mass function rises to lower masses, option (2) in Section 5.6.2, then we infer a detection rate of

\[ \dot{N}_{\text{BAT}} \approx 230 \text{ yr}^{-1} \quad (5.6) \]

for \(10^{49} \text{ erg s}^{-1}\) transients and \(\approx 40 \text{ yr}^{-1}\) for \(10^{48} \text{ erg s}^{-1}\) transients. The rates above are for transients that produce a thermonuclear transient. Less deeply-plunging tidal disruptions of WDs occur \(f_{\text{thermo}}^{-1} \approx 6\) times more frequently (MacLeod et al., 2014).

This calculation suggests that despite their substantially different peak luminosities at different frequencies, WD tidal disruption transients may be currently detectable by BAT at rates similar to what LSST will allow for in the future in the optical. Because high-energy emission precedes the thermonuclear transient, deep optical follow-up for candidate disruption flares is highly desirable, and it offers the best present-day strategy for detecting these transients. The thermonuclear transients detected via follow-up of beamed emission would lie along the jet-launching axis, perhaps lying perpendicular to the original orbital plane (although significant MBH spin could torque the debris stream or the inner accretion disk out of its original plane, changing the orientation of the jet/disk relative to the thermonuclear transient).
5.7 Discussion

5.7.1 A Diversity of Thermonuclear Transients from WD Tidal Disruption

In this work, we have examined in detail one model thermonuclear transient generated by the tidal compression and disruption of a carbon/oxygen WD. A primary caveat in extending the conclusions reached through examination of this model is that the tidal disruption process is likely to generate a diversity of transients. However, there is little reason to expect MBH-mass dependence on the generation of thermonuclear transients via tidal disruption. To linear order, the forces acting on the WD and timescale of passage can all be written in terms of the dimensionless impact parameter $\beta$. Whether or not runaway burning takes place depends on the ratio of dynamical timescale to burning timescale. The passage timescale is not a function of MBH mass, because for $\beta = 1$ encounters it is always equal to the WD dynamical timescale. Similarly, the bulk velocity of the unbound ejecta scales only weakly with MBH mass, $v_{\text{max}} \propto (M_{\text{bh}}/M_{\text{WD}})^{1/6}$. These simple scalings can be modified when encounters have pericenter distances similar to $r_s$ and relativistic effects are important. Gafton et al. (2015) found an increased effective impact parameter (stronger compression and mass loss than the Newtonian limit) for strongly relativistic encounters.

While strong dependence on MBH mass is not expected, the degree of burning should scale with both WD mass and impact parameter, $\beta$, as has been demonstrated by the simulations of Rosswog et al. (2008a) and Rosswog et al. (2009b). At the extremes, these range from no burning in weakly-disruptive encounters to strong burning in deep encounters. In particular, varying amounts of iron-group elements synthesized will affect the peak brightness of optical transients – which are powered primarily through radioactive decay. Table 1 of Rosswog et al. (2009b) provides a summary of the nuclear energy release and iron-group synthesis in their runs. The mass of iron group elements in
explosive events ranged between $\sim 0.01 - 0.7 M_\odot$. In cases where burning is incomplete, we might expect partial burning in some portions of the WD to manifest itself as large amounts of incompletely burned $\alpha$-chain elements in the spectrum. The iron-group mass is largest in deeper encounters, and in those involving massive white dwarfs. For example, a $0.2 M_\odot$ WD in a $\beta = 12$ encounter produces $0.034 M_\odot$ of iron-group material, but a $1.2 M_\odot$ WD in a $\beta = 1.5$ orbit produces a similar quantity. Rosswog et al. (2009b) also find the degree of iron-group synthesis to be a strong function of $\beta$; a $1.2 M_\odot$ WD in a $\beta = 2.6$ orbit produces $0.66 M_\odot$ of iron group elements. This range of iron-group masses produces a spread in lightcurve peak brightnesses (and varying WD mass likely to a range of peak timescales, with lower masses resulting in more rapid peak) across the range of possible disruptions. Interestingly, however, the viewing-angle diversity explored in this paper is of a similar magnitude, and may encompass some of the diversity in possible WD-MBH combinations.

The simulation presented in this paper used a C/O WD, but many lower-mass WDs are composed primarily of helium. Helium is significantly easier to burn than carbon and oxygen, with detonations being possible at lower temperature and densities (Seitenzahl et al., 2009; Holcomb et al., 2013), and thus perhaps at more grazing impact parameters, decreasing $\beta_{\text{thermo}}$ and increasing $f_{\text{thermo}}$. Helium WDs are also significantly lower in density than C/O WDs, enabling their disruption by higher-mass MBHs before they are swallowed whole, with disruptions by $M_{\text{bh}} \gtrsim 10^6 M_\odot$ being possible. Holcomb et al. (2013) show that incomplete burning with large mass fractions of $^{40}\text{Ca}$, $^{44}\text{Ti}$, $^{48}\text{Cr}$ are a common outcome of He detonations in conditions typical of WD - MBH encounters (Rosswog et al., 2008a, 2009b). Thus, such disruptions were recently proposed by Sell et al. (2015) to explain a calcium-rich gap transients (Kasliwal et al., 2012), faint Ia-like events with large velocities, fast evolution, and occurring preferentially in the outskirts of giant galaxies where unseen dwarf hosts (which would potentially harbor moderate-mass MBHs) may lie. Foley (2015) emphasizes that kinematic evidence may be key in
disentangling the origin of calcium-rich transients, and argues that those discovered so far have line-of-sight velocities suggestive of being ejected from disturbed galactic nuclei – but only exploding much later. Although we predict strong line-of-sight velocities, at face value Foley (2015)’s position-dependent kinematic evidence is not consistent with a prompt optical transient following an encounter with a MBH.

Although a systematic study is beyond the scope of the present paper, we intend to explore a range of encounters and the diversity of possible optical thermonuclear transients in future work.

5.7.2 Are ULGRBs WD Tidal Disruptions?

Shcherbakov et al. (2013) offer the tidal disruption of a WD as an explanation for the underluminous, long GRB 060218 and its accompanying SN 2006aj. While our calculations show that the properties of the high-velocity SN 2006aj is not consistent with the tidal disruption of a WD, this suggestion put forward the intriguing possibility that high energy flares from WD tidal disruption may already be lurking in existing data sets. Levan et al. (2014) and MacLeod et al. (2014) followed up on this suggestion but considered instead the emerging class of ultralong gamma ray burst (ULGRB) sources which has been reviewed in detail by Levan et al. (2014) and Levan (2015). At present, it remains uncertain whether these events form a distinct population of high-energy transients or the tail of the long gamma ray burst duration distribution (Levan, 2015). If these objects do, in fact, represent a distinct class of transients, several possible scenarios present themselves to explain their emission. Suggested progenitor models include collapsars, the collapse of giant or supergiant stars, and beamed tidal disruption flares, particularly those from WD disruption (Levan et al., 2014; MacLeod et al., 2014; Levan, 2015). In addition to their long duration, the ULGRBs are highly variable in a manner reminiscent of the relativistically beamed tidal disruption flares Swift J1644+57 and Swift J2058+05 (e.g. De Colle et al., 2012; Saxton et al., 2012; Pasham et al., 2015).
Of particular relevance to our study are searches for accompanying SN emission in the optical and infrared (which are reviewed in detail by Levan, 2015, in their Section 3.6). In the case of GRB 130925A at $z = 0.35$, the afterglow appeared as highly extinguished and no optical afterglow emission or SN signatures were detected. SN emission from GRB 121027A at $z \approx 1.7$, would be challenging to detect. In the cases of GRB 101225A and 111209A red excess is observed by Levan et al. (2014) in the afterglow’s lightcurve evolution. Figure 6 of Levan et al. (2014) compares this to a SN 1998 bw template. GRB 101225A shows a mild reddening only about a half magnitude brighter than the 1998 bw template and a significantly bluer SED than the 1998 bw template at order 10 days after the outburst (Figure 7 of Levan et al., 2014).

The lightcurve of the SN accompanying GRB 111209A was recently published by Greiner et al. (2015). The SN is extremely bright, $M_{\text{bol}} \sim -20$, which rules out a thermonuclear transient from WD tidal disruption on the basis of the required nickel mass in Greiner et al. (2015)’s best fit for a thermonuclear power source, $\approx 1M_\odot$ in $3M_\odot$ of ejecta. This transient is $\sim 3\times$ brighter than even the Ic’s typically associated with GRBs, leading Greiner et al. (2015) to suggest a magnetar central engine model. It is also an order of magnitude brighter than the brightest WD tidal disruption transients studied here, suggesting that a tidal disruption is unlikely to be responsible for GRB 111209A. In theory, the explosion of a stripped core of a more-massive star could explain the energetics of the event (Gezari et al., 2012; MacLeod et al., 2013; Bogdanović et al., 2014), but the occurrence rate of deep encounters of tidally stripped stars is unknown.

Optical and infrared SN searches for future transients in the ULGRB category should offer further valuable constraints on whether WD tidal disruption is consistent with their origin. In particular, we now offer better description of the expected tidal thermonuclear lightcurves and spectra. We expect only a fraction $f_{\text{thermo}} \approx 1/6$ to be accompanied by any SN-like transient. Even a sample of $\sim 10$ events with follow-up observations could be used to provide strong evidence for or against a a WD tidal
disruption origin based on the fraction accompanied by thermonuclear transients.

5.7.3 Strategies for Identifying WD Tidal Disruptions

In this paper we have identified the photometric and spectroscopic properties that distinguish WD tidal disruption transients from more-common SNe. Even so, it is worthwhile to compare the volumetric rate of thermonuclear transients generated by tidal disruptions to the most commonly observed thermonuclear explosions of WDs, type Ia SNe. Ia SNe are far more common, with event rates $\approx 5 \times 10^4 \text{ yr}^{-1} \text{ Gpc}^{-3}$ (e.g. Graur et al., 2014) as compared to $\sim \text{few yr}^{-1} \text{ Gpc}^{-3}$. This comparison makes apparent the degree of challenge that will be faced by next generation surveys in identifying and recovering these and other exotic transients from amongst a vast quantity of SNe.

Rather than relying on detection of only the thermonuclear transient, we suggest that the multi-wavelength signatures of WD tidal disruptions is what makes these transients truly unique. These signatures are generated, as described in Section 5.5 and Figure 5.12, through a combination of nuclear burning and accretion power. Further, our examination of the relative sensitivity and luminosities of present high-energy and next-generation optical instrumentation and transients suggests that survey efforts may uncover WD tidal disruptions at similar rates in these two wavelengths.

The best present-day strategy to successfully uncover and firmly identify WD tidal disruption transients is to search for beamed emission with high energy monitors like Swift’s BAT. Optical follow-up is a critical component of this strategy and could be used to constrain or detect the presence of a thermonuclear transient. If each high-energy-detected event were a tidal disruption, we would expect $f_{\text{thermo}} \approx 1/6$ to be accompanied by a thermonuclear transient.

When LSST comes online, a second potential strategy will emerge. This could involve searching for optical transients with photometric and spectroscopic properties similar to those described here, and following-up viable candidate events at X-ray and
radio wavelengths for accretion signatures. For example, the thermal emission from the accretion disk (eg. \( L \sim 10^{42} \text{ erg s}^{-1} \)) would be visible in a 10 ks XMM-Newton exposure at \( z \lesssim 0.3 \) (Watson et al., 2001), offering a valuable constraint on the MBH accretion that accompanies the transient. If the accompanying accretion flow launches a jet, we expect a radio afterglow to follow the optical transient, as described in Section 5.5.

5.7.4 Uncovering the Mass Distribution of Low-Mass MBHs

This paper has examined the multiwavelength characteristics of transients that arise from WD-MBH tidal interactions. We find that the transients accompanying these interactions should be luminous and detectable in the optical and at high energies to redshifts of \( z \sim 0.35 \) (LSST, thermonuclear transients) or \( z \gtrsim 1 \) (BAT, beamed emission from accretion flow). We showed in Section 5.6.2 that the largest uncertainty in estimating the detection rate is the uncertainty in the MBH mass function’s extrapolation from well-known MBHs with masses of \( 10^6 - 10^9 M_\odot \) down to MBHs with masses of \( \sim 10^3 - 10^5 M_\odot \). We computed event rates for two possibilities, these are that we either extrapolate the value, or the slope of the MBH mass function, \( \Phi(M_{\text{bh}}) \), to lower masses.

If we extrapolate the slope of the MBH mass function to lower masses, a large number density of MBHs in the intermediate mass range will exist in the volume probed by WD disruption transients. Under this assumption WD disruption transients would be discovered at rates up to hundreds per year both by LSST and by high energy wide-field monitors like BAT. If instead the mass function remains relatively flat below \( 10^6 M_\odot \), we should detect tens of events per year. Finally, if no intermediate-mass MBHs exist, we should expect very few WD tidal disruption events and their associated signatures. Those that occurred would arise from rare encounters between maximally spinning MBHs with mass \( \sim 10^6 \) in which the orbital plane is aligned with the spin plane (Kesden, 2012). Deep encounters, those than can produce thermonuclear transients, would be rarer still as they require an impact parameter well inside the tidal radius.
(β ≳ 3) and thus require even more finely tuned conditions of MBH spin and orbital orientation.

In Figure 5.14, we illustrate the influence of the low-mass MBH mass function on the detection rate of WD tidal disruption transients associated with deeply-plunging WD tidal disruptions. We assume a simple power-law mass function in which \( \Phi(M_{\text{bh}}) \propto M_{\text{bh}}^{\alpha} \), approximately normalized to the volume density of \( 10^{7} M_\odot \) MBHs in the local universe (e.g. Sijacki et al., 2015). The slope of the extrapolation of the MBH mass function, \( \alpha \), has a dramatic effect on the number density \( n_{\text{MBH}} \) of MBHs with masses of \( 10^{3} - 10^{5} M_\odot \) (which we assume can ignite WDs in tidal encounters). We assume the volumetric event rate of equation (5.3). A remaining caveat is the unknown typical jetted-transient beamed luminosity, its dependence on MBH spin, and whether all super-Eddington tidal disruption accretion flows successfully launch jets (see, e.g. Krolik and Piran, 2012; De Colle et al., 2012; Tchekhovskoy et al., 2013; van Velzen et al., 2013, for further discussion). We marginalize over these uncertainties by showing two possible jet luminosities in Figure 5.14, and we note that our formalism can be easily extrapolated to other assumptions. The variation in volume density of MBHs implies dramatic differences in the detection rate of transients associated with deeply-passing WD tidal disruptions. These differences are sufficiently significant that a few years of monitoring with high energy monitors like BAT or in the optical with LSST should produce a catalog of transients that can be used to place meaningful constraints on the number density of low-mass MBHs in the universe.

5.8 Summary and Conclusion

In this paper we have examined the properties of thermonuclear transients that are generated following the ignition of nuclear burning in a deep tidal encounter between a WD and a MBH. This burning produces iron group elements in the core of
Figure 5.14: Effect of the slope of the MBH mass function below $10^7 M_\odot$ on the detection rate of WD tidal disruption transients. This diagram assumes a simple, power law MBH mass function, $\Phi(M_{bh}) \propto M_{bh}^\alpha$, shown in the figure. The power law slope of the low-mass MBH mass function determines the number density of MBHs with masses of $10^3 - 10^5 M_\odot$, whose density, $n_{MBH}$, is shown on the upper x-axis. With detection rates ranging from tens to hundreds per year for flat to rising mass functions, it should be possible to place meaningful constraints on the allowed extrapolation of the MBH mass function to low masses within several years. The BAT transients plotted are only those deep passages that generate an accompanying thermonuclear transient (a fraction $f_{thermo} \sim 1/6$ of the total events). The more common non-thermonuclear disruptions may also produce jets and high-energy transients, as described in Section 5.5.
the unbound debris of the tidal disruption event, surrounded by intermediate mass ele-
ments and unburned material. As this debris expands, an optical-wavelength transient
with appearance similar to an atypical type I SN emerges. The peak brightness and
color of the transient’s lightcurve are highly viewing angle dependent as a result of the
asymmetric distribution of the expanding tidal debris. A strong spectral signature of
these transients is P-Cygni lines strongly offset from their rest wavelength by the orbital
motion of the unbound debris (see Figures 5.9 and 5.10).

These transients should be accompanied by accretion signatures driven by
relatively prompt bound-debris stream self-interaction and accretion disk formation.
Accretion signatures range from thermal emission of the accretion disk, expected to
peak in the soft X-ray, to harder X-ray non-thermal beamed emission along a jet axis.
Optical and radio afterglow emission may trace the launching of a jet at viewing angles
away from the jet axis. We analyze the relative detection rates of WD tidal disruption
transients at high energy and optical frequencies given high-energy wide field monitors
like Swift’s BAT and LSST in the optical. Our results suggest that detection rates
may be similar with these disparate survey strategies, and we suggest that the most
constraining events may be those in which multiple counterparts of the disruption event
are observed. In particular, the most promising present-day strategy is probably deep
optical follow-up of high-energy flares of unusually long duration or variability, like the
ULGRBs (Levan et al., 2014; Levan, 2015).

In closing, we note that the existence of these transients remains uncertain (e.g.
Sell et al., 2015), just as the existence of the MBHs of intermediate masses \( \sim 10^3 - 10^5 M_\odot \)
remains uncertain (e.g. Reines et al., 2013; Baldassare et al., 2015). With detailed
estimates of the potential properties of these transients, either their detection or non-
detection should be able to be used to place meaningful constraints on the prevalence
of intermediate mass MBHs in the universe.
Chapter 6

The Close Stellar Companions to Intermediate Mass Black Holes

6.1 Chapter Abstract

When embedded in dense cluster cores, intermediate mass black holes (IMBHs) acquire close stellar or stellar-remnant companions. These companions are not only gravitationally bound, they tend to hierarchically isolate from other cluster stars through series of multibody encounters. In this paper we study the demographics of IMBH companions in compact star clusters through direct $N$-body simulation. We study clusters initially composed of $10^5$ or $2 \times 10^5$ stars with IMBHs of 75 and 150 solar masses, and follow their evolution for 6-10 Gyr. A tight innermost binary pair of IMBH and stellar object rapidly forms. The IMBH has a companion with orbital semi-major axis at least three times tighter than the second-most bound object over 90% of the time. These companionships have typical periods of order years and are subject to cycles of exchange and destruction. The most frequently observed, long-lived pairings persist for $\sim 10^7$ yr. The demographics of IMBH companions in clusters are diverse; they include both main sequence, giant stars, and stellar remnants. Companion objects may reveal
the presence of an IMBH in a cluster in one of several ways. Most-bound companion
stars routinely suffer grazing tidal interactions with the IMBH, offering a dynamical
mechanism to produce repeated flaring episodes like those seen in the IMBH candidate
HLX-1. Stellar winds of companion stars provide a minimum quiescent accretion rate
for IMBHs, with implications for radio searches for IMBH accretion in globular clusters.
Finally, gravitational wave inspirals of compact objects occur with promising frequency.

6.2 Introduction

Globular clusters (GCs), dense young star clusters, and the compact nuclear
star clusters of low-mass galaxies are environments of extreme stellar density. Their
typical sizes of parsecs and masses $\gtrsim 10^5 M_\odot$ imply stellar number densities between
$10^3$ and $10^6$ times that of the solar neighborhood and velocity dispersions of tens of km
s$^{-1}$ (Heggie and Hut, 2003; Neumayer and Walcher, 2012). With similar ratios of stellar
mass to system mass, stellar escape velocity to velocity dispersion, and stellar lifetime
to system relaxation time, these dense stellar systems have broad dynamical similarities
despite their disparate environments. These dense clusters are environments of intense
stellar interactivity, where single and binary stellar evolution and gravitational dynamics
intertwine to shape the long-term evolution of the cluster as a whole (Heggie and Hut,
2003; Aarseth, 2003; Kalirai and Richer, 2010).

There has long been speculation that these dense stellar systems might harbor
intermediate mass black holes (IMBHs) with masses between that of the known popu-
lations of stellar mass and supermassive black holes (BHs) (e.g. Lightman and Shapiro,
1978; Miller and Hamilton, 2002). If the $M_{BH} - \sigma$ relation of supermassive BH mass and
velocity dispersion were to extrapolate to objects as small as GCs and nuclear clusters,
it would imply that these stellar systems should host BHs in the IMBH mass range
(e.g. Ferrarese and Merritt, 2000; Lützgendorf et al., 2013c; Baldassare et al., 2015).
In young clusters of low metallicity, evolving massive stars might preferentially leave behind massive remnant BHs, with masses \( \gtrsim 10^2 M_\odot \) (Mapelli et al., 2013c; Spera et al., 2015). Through dynamical interaction these BHs could acquire and accrete mass from companion objects (Mapelli et al., 2013c; Mapelli and Zampieri, 2014; Ziosi et al., 2014) implying a self-consistent channel by which the BH could grow into the IMBH mass range as the cluster evolves.

There is growing evidence that some nuclear star clusters in low-mass galaxies host BHs in the IMBH mass range (e.g. Neumayer and Walcher, 2012). These sources have been discovered primarily in the fraction of systems where the central BH is active. For example, Seth et al. (2008) study the coincidence of BH activity and nuclear star clusters, and find that a large fraction of all nuclear star clusters show signs of BH activity (> 50% of those with masses \( \gtrsim 10^7 M_\odot \)). Seth et al. (2010) found a BH mass of \( \sim 5 \times 10^5 M_\odot \) in the NGC 404 nucleus. By studying the active BH fraction in a large sample of Sloan Digital Sky Survey dwarf galaxies, Reines et al. (2013) find a \( \sim 0.5\% \) active fraction in their sample of \( 10^{8.5} - 10^{9.5} M_\odot \) dwarfs with median BH masses of \( \sim 2 \times 10^5 M_\odot \). Reines et al. (2013) note that sensitivity limits their ability to detect active BHs of less than \( 10^5 M_\odot \), even if they are emitting near their Eddington luminosities. Even so, their data suggest a similar active fraction among these low mass BHs to that of \( 10^7 M_\odot \) BHs. More recently, Baldassare et al. (2015) report a \( 5 \times 10^4 M_\odot \) IMBH in the dwarf galaxy RGG 118 accreting at 1% its Eddington luminosity. The fact that this BH lies along the extrapolation of the \( M_{BH} - \sigma \) relation to low masses suggests that BHs in this mass range may be common in dwarf galaxies, with their detection hampered only by their low characteristic luminosities.

The evidence for the presence or absence of IMBHs in GCs remains contested. An IMBH, unless illuminated through accretion, would be be dark (e.g. Strader et al., 2012; Ramirez-Ruiz and Rosswog, 2009; Volonteri et al., 2011). This leaves only the IMBH’s gravitational interaction with surrounding stars as a tracer of its presence.
Searches for peaks in the central velocity dispersion or surface brightness of GCs due to orbital motion around an unseen IMBH have returned contested results (e.g. Gebhardt et al., 2005; Lanzoni et al., 2007; Noyola et al., 2008; Miocchi, 2010; Noyola et al., 2010; Noyola and Baumgardt, 2011; Lützgendorf et al., 2011; Jalali et al., 2012; Lützgendorf et al., 2012b,a, 2013b; Feldmeier et al., 2013; Kamann et al., 2014; Lützgendorf et al., 2015), with a fundamental challenge being that unless the IMBH is a large fraction (e.g. $\gtrsim 1\%$) of the cluster mass very few stars will trace orbits dominated by the IMBH's gravity (e.g. Vesperini and Trenti, 2010; Umbreit and Rasio, 2013). This challenge has lead to theoretical and observational efforts to identify other dynamical tracers of the presence of IMBHs in GC systems (e.g. Baumgardt et al., 2005; Heggie et al., 2007; Hurley, 2007; Gill et al., 2008; Pasquato et al., 2009; Pasquato, 2010; van der Marel and Anderson, 2010; Beccari et al., 2010; Samra et al., 2012; Leigh et al., 2014).

In this paper, we consider a stellar dynamical implication of the possible presence of IMBHs in dense cluster cores: stars bound tightly to the IMBH. IMBHs in clusters acquire a number of gravitationally bound stars that make up their sphere of influence (Peebles, 1972; Bahcall and Wolf, 1976; Preto et al., 2004; Baumgardt et al., 2004a,b). These stars orbit under the combined influence the IMBH and all of the other bound stars (Frank and Rees, 1976). The stellar dynamics of the relatively small number of stars in the sphere of influence region is complex, and relies on the interface of stars’ orbital, dynamical relaxation, and evolution timescales. In simulation, the most accurate (but also computationally expensive) approach to treating the multiphysics sphere of influence dynamics is direct $N$-body evolution of the stars’ orbital motion (Aarseth, 2003). Companion objects to IMBHs have been mentioned in previous studies (e.g. Baumgardt et al., 2004b; Blecha et al., 2006; Baumgardt et al., 2006; Konstantinidis et al., 2013; Leigh et al., 2014; Giersz et al., 2015), however, in this paper we present the first systematic study of IMBH companions in a dense cluster context that takes advantage of the accuracy of direct $N$-body integration.
Stellar objects or stellar remnants tightly bound to an IMBH are of extreme astrophysical interest because of their ability to reveal the presence of the otherwise dark BH. The category of ultraluminous X-ray (ULX) sources is particularly interesting because their X-ray luminosities of $\gtrsim 10^{39}$ erg s$^{-1}$ preclude an Eddington limited stellar mass BH source (e.g. Humphrey et al., 2003; Pasham et al., 2014). In dense clusters, stars can be scattered into close, disruptive encounters with IMBHs, powering luminous but brief flares of ULX luminosity (Ramirez-Ruiz and Rosswog, 2009). If, instead, stars are captured into tight orbits and induced to Roche lobe overflow, persistent ULXs could conceivably be the very high mass analogs X-ray binaries (Hopman et al., 2004; Li, 2004; Portegies Zwart et al., 2004; Blecha et al., 2006; Patruno et al., 2006; Baumgardt et al., 2006). If less tightly bound to the IMBH, a stellar companion could still fuel accretion through its stellar wind (Cuadra et al., 2005, 2006, 2008; Miller et al., 2014).

The source HLX-1 (Farrell et al., 2009) is particularly dramatic, because its lightcurve shows repeated flaring episodes during which the source undergoes state transitions, strongly suggesting an IMBH as the central engine (Godet et al., 2009; Farrell et al., 2011; Davis et al., 2011; Servillat et al., 2011; Webb et al., 2012, 2014; Farrell et al., 2014). The roughly periodic flaring behavior has also lead to an association of the behavior with a IMBH plus star system in an eccentric orbit – where close pericenter passages might account for the brightening (Lasota et al., 2011; Soria, 2013; Miller et al., 2014; Godet et al., 2014). The association of the object with a compact stellar system strengthens the interpretation that the accretion flares might be powered by a stellar companion (Wiersema et al., 2010; Farrell et al., 2012; Mapelli et al., 2012, 2013b,a; Soria et al., 2013). An avenue for these repeated, grazing tidal interactions with supermassive BH hosts has been suggested by MacLeod et al. (2013), where stars evolve to ‘spoon-feed’ portions of their envelope to the supermassive BH over many passages. In the galactic center context, Guillochon et al. (2014a) have proposed this channel as a dynamical mechanism to produce the tightly-bound G2 gas cloud. However, the
dynamical processes that lead an IMBH, like the putative BH in HLX-1, to acquire then strip mass from a tightly bound companion remain poorly understood.

This paper studies the dynamical processes by which IMBHs in dense clusters acquire, retain, and lose close companion stars. We find that a stellar object among the sphere of influence members tends to segregate to substantially tighter separations than the other bound stars. These companion objects form a hierarchically separated binary with the IMBH, and persist until they are replaced in an exchange interaction or destroyed through direct interaction with the IMBH. We characterize the orbital demographics and statistics of these companion objects using the $N$-body simulations presented in this paper. In Section 6.3 we describe our direct $N$-body simulation method and parameter choices. In Section 6.4 we explore the stellar dynamics, populations, and processes governing tightly-segregated companions to the IMBH. In Section 6.5 we examine the dependence of our simulated populations of companion on some key cluster parameters. In Section 6.6 we discuss some astrophysical implications of our results, particularly the role that these tightly bound companions might play in revealing the presence of otherwise hidden IMBHs. Finally, in Section 6.7 we conclude.

6.3 Direct $N$-body Simulations of Clusters Hosting IMBHs

6.3.1 Simulation Method

In this work we use the direct-summation $N$-body star cluster dynamics code NBODY6 (Aarseth, 1999, 2003) in a version enabled for GPU-accelerated calculations (Nitadori and Aarseth, 2012)\(^1\) to follow the evolution of star clusters in a realistic tidal field following the implementation of Heggie et al. (2006) and subsequent papers (Trenti et al., 2007b,a; Trenti, 2008; Trenti et al., 2008; Gill et al., 2008; Pasquato et al., 2009; Trenti et al., 2010; Vesperini and Trenti, 2010; Trenti and van der Marel, 2010).

\(^1\)Version: August 1, 2014.
Simulations containing $N$ stellar particles are performed in $N$-body units where $G = M_T = -4E_T = 1$, in which $M_T$ and $E_T$ are the total mass and energy of the cluster initial conditions (Heggie and Mathieu, 1986). The corresponding dynamical time unit is

$$t_d = GM_T^{5/2}/(-4E_T)^{3/2}. \quad (6.1)$$

The stellar cluster’s dynamical state relaxes over approximately a relaxation time,

$$t_{rh} = 0.138N_{r_h}^{3/2}/\ln(0.11N), \quad (6.2)$$

defined using the half mass radius, $r_h$ (Spitzer, 1987; Trenti et al., 2010). The external tidal field of the host galaxy is implemented following Trenti et al. (2007b) and the reader is directed to section 2 of that paper for details.

The NBODY6 distribution embeds the SSE and BSE codes of Hurley et al. (2000, 2002) as a simplified, on-the-fly implementation of stellar evolution\(^2\). The binaries in our clusters evolve with the standard BSE implementations of binary evolution. NBODY6 simulations include default binary evolution processes implemented in BSE including tidal circularization, magnetic braking, Roche Lobe overflow, and binary coalescence (Hurley et al., 2002).

We initialize our models with an IMBH of mass $M_{bh}$ at rest at the system center of mass. This IMBH particle then evolves identically to the remaining particles. In this sense, our simulations do not self-consistently account for the formation of such an IMBH, instead we aim to simulate its subsequent dynamical evolution in the cluster context. Because the IMBH is more massive than any of the stellar particles it sinks toward the cluster center, motivating our initial condition. We find within the first

\(^2\)Compared to the more recent stellar evolution calculations of Spera et al. (2015), the mass distribution of stellar remnants in our approach is favors lower values, especially at low metallicities, with a potential impact on the dynamics being the number of objects with mass much greater than the average stellar mass (e.g., see Mapelli and Zampieri 2014).
core relaxation time the BH acquires non-zero energy relative to the cluster center of mass through strong interactions with cluster core stars – rapidly erasing any imprint of this initial condition. We include gravitational wave energy and angular momentum losses from any very compact binaries following the Peters (1964) formulation, which is particularly relevant for the IMBH and its companion (e.g. Miller and Hamilton, 2002; Gültekin et al., 2004, 2006; Leigh et al., 2014).

Close interactions with the IMBH can lead to the tidal destruction of a star or the inspiral and merger of a compact object. We handle those events as follows in our simulations. For stars, we delete any star passing within one tidal radius,

$$r_t = \left(\frac{M_{\text{bh}}}{M_*}\right)^{1/3}R_*,$$

(6.3)
of the BH at pericenter. In a tidal disruption event, the star is often on a weakly-bound or near-parabolic orbit (Frank and Rees, 1976; Lightman and Shapiro, 1977). In this case approximately half of the tidal debris remains bound to the BH, while the other half is ejected (Rees, 1988). We model this behavior by adding half of the disrupted star’s mass to the IMBH’s mass following a disruption event. An ideal implementation of the stellar tidal disruption process would account for the stellar structure in determining the degree of mass loss, and would also allow for partial tidal disruptions (Guillochon and Ramirez-Ruiz, 2013). Partial tidal stripping is particularly relevant to evolved stars which contain a dense, tidally invulnerable core (MacLeod et al., 2012, 2013; Bogdanović et al., 2014). However, in this work we adopt the simplified, full-disruption model described above. We implement gravitational wave inspiral and merger events through the inclusion of gravitational wave corrections to the two-body equation of motion of tight pairs of compact objects (Peters, 1964). Rather than allowing merging particles to spiral to arbitrarily small separations, we instead merge pairs of compact objects whose inspiral time becomes shorter than 1 $N$-body time unit, equation (6.1).
Table 6.1: Table of $N$-body simulation groups A-D, the number of simulations run per group, as well as their parameters and initial conditions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number</th>
<th>$N$</th>
<th>$W_0$</th>
<th>$r_{h,0}$</th>
<th>$M_{bh,0}$</th>
<th>$t_{\text{max}}$</th>
<th>$\sigma_k/\sigma_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>100k</td>
<td>7</td>
<td>2.3</td>
<td>150</td>
<td>6</td>
<td>2.5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>100k</td>
<td>7</td>
<td>2.3</td>
<td>150</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>100k</td>
<td>7</td>
<td>2.3</td>
<td>75</td>
<td>9</td>
<td>2.5</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>200k</td>
<td>7</td>
<td>2.3</td>
<td>150</td>
<td>10.4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

We do not apply a kick to the remnant $M_{bh}$, although such a kick may be relevant for some mergers (Holley-Bockelmann et al., 2008; Konstantinidis et al., 2013).

We have implemented several diagnostics to allow us to better study close companions to the IMBH. We record all three-body encounters in which a third object approaches with pericenter distance within three times the semi-major axis of the most-bound companion to the IMBH. We also include the parameters of all tidal disruption and merger events described above. For close tidal passages that do not lead to disruption, those with pericenter distances $r_t < r_p < 10^3 r_t$ we also record the system parameters. While we record full-cluster mass, position, and velocity snapshots every 10 $N$-body time units, we record the IMBH’s position and velocity every 0.1 $N$-body time unit. Every $N$-body time unit we record the parameters of the most tightly bound partner to the IMBH.

### 6.3.2 Numerical Simulations

In this paper, we present 4 groups of simulations with differing initial parameters. Simulations within each group have statistically different realizations of the initial conditions. Table 6.1 defines the properties these simulation groups. We use the McLus-ter code (Küpper et al., 2011) to generate the initial stellar distribution following King (1966) models with $W_0 = 7$. Our simulations include a IMBH which is initialized at the center of mass with zero velocity and $N = 1 - 2 \times 10^5$ stars. The size of our clusters is
constrained by the computational expense of the direct $N$-body method. These clusters are thus at the low mass end of the spectrum of GC masses, and a factor of 10-100 less massive than typical nuclear star clusters. In later sections we consider how our results might scale to larger $N$ clusters.

The stars in our initial conditions follow a Kroupa (2001) initial mass function (IMF), within the mass range $0.1M_\odot - 30M_\odot$, with no binaries. The metalliclicity is one-tenth solar. We initialize our clusters as tidally-underfilling by a factor of 1.5. In $N$-body units, the initial tidal radius is 10.455 rather than 6.95 for $W_0 = 7$ in our simulations (compare to Table 1 of Trenti et al., 2007b). This allows for some cluster expansion due to mass loss before stars begin to escape. We consider cases of different IMBH mass, varied between 0.13% and 0.26% of the initial cluster mass.

Finally, we consider two different cases for the velocity kicks imparted to stellar remnant NSs and BHs (for a recent discussion, see Contenta et al., 2015). Rather than choosing absolute kick velocities based on observational constraints, here we choose velocities relative to the cluster velocity dispersion in order to produce different retention fractions of stellar remnants. This strategy is needed because the clusters we are able to simulate with the direct $N$-body method are small relative to realistic dense star clusters. In both cases remnants are given a kick drawn from a Maxwellian distribution with sigma of either 1 or 2.5 times the initial cluster velocity dispersion, $\sigma_* = \sqrt{GM_T/r_h}$. These kick distributions imply that 19.8% or 1.6%, of kicked objects have kick velocities less than $\sigma_*$, which in the absence of binaries, is proportional to the fraction of retained remnants.

6.3.3 Methodological Comparison to Previous Work

Some previous work has mentioned or studied close companion stars to IMBHs in GCs. Here we describe the methodological differences in our new simulations as they compare to this work. Baumgardt et al. (2004b) run direct $N$-body simulations
of clusters containing IMBHs and up to 131K stars. They plot several sequences of semi-major axis of tightly bound stars as a function of time from their simulations containing IMBHs (their Figures 14 and 15). These sequences show a long-lived most-bound companion in several cases with semi-major axis considerably smaller than the other stars and hint at the possible universality of IMBH companions. Baumgardt et al. (2006) focus on the production of mass-transferring binary pairs containing the IMBH and a star. They run $N$-body simulations including approximate tidal dissipation and gravitational wave radiation from stellar orbits and study the statistics of tidally-captured stars in young ($\leq$12 Myr) clusters as possible ULX progenitors. Konstantinidis et al. (2013) also model small, young (10 Myr age) clusters initially containing $N=32k$ stars using direct $N$-body integration. The clusters a massive IMBH with $M_{bh} = 500 - 1000 M_\odot$. They study one simulation in detail, in which the IMBH captures a stellar BH companion, goes through several cycles of exchange, and eventually inspirals and merges. Konstantinidis et al. (2013) argue that the merger generates sufficient gravitational wave recoil to eject the IMBH from their low-mass simulation cluster. Mapelli et al. (2013c), Mapelli and Zampieri (2014), and Ziosi et al. (2014) study the dynamics of massive stellar-mass BHs (up to $10-100 M_\odot$) in a large sample of $N$-body calculations of young star clusters with a range of metallicities and $N = 5500$ stars.

Leigh et al. (2014) also run $N$-body simulations containing IMBHs with between $N=32k$ and $N=131k$ stars, but they do not include gravitational radiation from compact binaries. After the formation of an IMBH-BH compact binary, the eccentricity of this innermost binary varies over time. The binary passes many times through orbital configurations that should have driven it to merger. As a result, Leigh et al. (2014)’s simulations show artificially long-lived companion objects, and a single companion persisting throughout the cluster evolution. Recently, Giersz et al. (2015) have used the MOCCA Monte Carlo Cluster simulation code to study accretion-fed IMBH growth over GC lifetimes. Their simulations, particularly their “slow” scenario with a
single massive BH, show the cycles of replacement of the innermost object missing from earlier $N$-body work. However, as Giersz et al. (2015) describe, the Monte Carlo code is not well-suited to studying the detailed dynamics small number of stars tightly bound to the IMBH, nor is it well-suited to the inclusion of a single, massive object.

Blecha et al. (2006) adopt a slightly different approach. Rather than simulate an entire GC, they use a binary evolution code to evolve the IMBH and any companions with a fixed background of scatterers drawn from a GC core distribution function. While this approach facilitates modeling a larger number of IMBH pairs and a detailed treatment of IMBH-star interactions, it does not capture the dynamical feedback of the IMBH and its partner on the host cluster.

The simulations presented in this paper were designed specifically to study close companion objects to the IMBH in realistic GCs. This involved combining some of the best aspects of the previous work described above. In particular, we take advantage of the accurate dynamics of the $N$-body methods, and implement specific interaction outcomes for close interactions between the IMBH and stellar objects like tidal disruptions and gravitational wave inspirals. We model an IMBH which is a realistically small fraction ($\sim 0.3\%$) of the total cluster mass. This approach limits our study to relatively small GCs with $N_0 \sim 10^5$, with their proportionately low-mass IMBHs, $M_{\text{bh}} \sim 10^2 M_\odot$, but allows us to follow their long-term evolution over 6-10 Gyr without approximation. In Section 6.5, we discuss some considerations in scaling the lessons learned from our simulations to more massive clusters or IMBHs.

6.3.4 Cluster Global Evolution

Our simulated clusters exhibit global evolution similar to those of others studied in detail combining IMBHs and a tidal field (e.g. Trenti et al., 2007a; Gill et al., 2008; Trenti et al., 2010; Lützgendorf et al., 2013a; Giersz et al., 2015). Thus, we comment only briefly on the generic properties of our model clusters to set the stage for
our consideration of the dynamics of their IMBH-hosting cores. Clusters in a tidal field slowly dissolve as (typically low-mass) stars escape from the system. Our models of simulation groups A and B are run for 6 Gyr, during which time the total number of stars decreases to $\approx 2 \times 10^4$. Simulation groups C and D are run to the point of dissolution in the tidal field, which occurs after 9 and 10.4 Gyr, respectively. Mass loss from stellar evolution rapidly decreases the cluster mass by $\sim 30\%$ in the first 50 Myr, then proceeds increasingly gradually as time goes on. The combination of these effects dictate that the mass range of typical cluster stars narrows over the course of the GC’s evolution. At early times, stars are the most-massive objects other than the IMBH, while at later times, for example $\gtrsim 1$ Gyr, the turnoff mass has decreased to $< 3M_\odot$, and stellar remnants are the most massive objects. Lützgendorf et al. (2013a)’s Figure 1 gives an excellent overview of the time-evolution of $N$-body models of GCs containing IMBHs.

The IMBH sinks nearly to the cluster center of mass, as for example, can be seen in Figure 2 of Konstantinidis et al. (2013). Once in the core, dynamical interactions with the IMBH and its companion object act as a heat source for the cluster, stalling core collapse or causing the system to expand (e.g. Quinlan, 1996; Yu and Tremaine, 2003; Trenti et al., 2007a; Lützgendorf et al., 2013a). Figure 6.1 shows an example cluster evolution from simulation group A. The half-mass and core radii expand initially in response to stellar evolution mass loss and mass loss from kicked supernova remnant objects. These characteristic radii then stabilize for the remainder of the simulation and the cluster neither expands nor contracts as energy loss balances dynamical heating. Over time, the IMBH grows by accreting the debris of tidally disrupted stars and by swallowing inspiralling compact objects. The individual sequences are, of course, variable, but the IMBHs in our clusters grow by $\sim 30\%$ over the evolutionary span of the cluster lifetime.
Figure 6.1: Characteristic radii from the IMBH in an example cluster evolution (from simulation group A). This diagram shows the evolution of the half-mass, $r_h$, and core, $r_c$, radii, along with the distance that encloses a IMBH-mass of stars, $r_m$. Interior to these radii, we plot the semi-major axes of the 5th most-bound object, $a_4$, the second most bound, $a_1$, and the most bound, $a_0$. We plot the pericenter distance of the most-bound object, $r_{p,0}$, as well. Coloring of the most-bound object denotes its stellar-evolutionary type, labeled in the key on the right-hand side. Stellar types include main sequence (MS) and giant stars. The remnant types include white dwarfs (WD), neutron stars (NS) and black holes (BH).
6.4 Close Companions to the IMBH

In this section, we examine the properties of close companions to a cluster IMBH using our fiducial simulation group A described in the previous section and in Table 6.1. The qualitative behavior of each of the simulation groups is similar, and we defer to Section 6.5 to consider differences in our results between the cluster parameters represented in simulation groups B-D.

6.4.1 Stars Bound to the IMBH

The process of capture of a close companion begins with the two body relaxation of the cluster. The stellar cluster's dynamical state relaxes over approximately a relaxation time, $t_{rh}$ (Spitzer, 1987). This relaxation drives the stellar distribution surrounding the IMBH to higher and higher densities, until a steady-state distribution is reached (Peebles, 1972; Bahcall and Wolf, 1976; Frank and Rees, 1976; Lightman and Shapiro, 1977). Because the IMBH mass is much greater than the average stellar mass, many stars can be gravitationally bound to the IMBH at a given time. For these stars,

$$\varepsilon = -\frac{GM_{bh}}{r_{bh}} + \frac{1}{2}v_{bh}^2 < 0,$$

where we've written their orbital energy per unit mass $\varepsilon$ with respect to IMBH in terms of their instantaneous separation $r_{bh}$ and velocity $v_{bh}$ relative to the IMBH. As such, these are the stars that occupy a phase space where the IMBH is energetically dominant in their orbits. In general, these stars satisfy,

$$a \lesssim r_h = \frac{GM_{bh}}{\sigma_c^2},$$

where $a = GM_{bh}/2\varepsilon$ is the semi-major axis of the bound star, and $\sigma_c$ is the core velocity dispersion. While many stars may exist in such bound orbits, we would not describe
each of these as a companion to the IMBH.

Instead, we will consider the IMBH to have a companion when its most-bound star is significantly more tightly bound than the second most bound. One way to express this is by comparison of those star’s semi-major axes (or equivalently orbital energies) with respect to the IMBH. The condition can be written \( a_0/a_1 \ll 1 \) which implies \( \varepsilon_0/\varepsilon_1 \gg 1 \). Here the coefficient 0 denotes the most bound and coefficient \( i \) denotes the \((i + 1)\)-th most tightly bound object. In these cases, the most-bound object has hierarchically separated from the rest of the cluster and sphere of influence stars. The remaining stars then orbit in the combined potential of the IMBH and its companion. The hierarchically isolated companion can exist relatively unperturbed until it suffers a strong encounter.

### 6.4.2 Companion Capture & Orbital Hardening

How does the most-bound star end up in an orbit hierarchically isolated from the other cluster stars? We explore this question visually in Figure 6.1. This diagram shows characteristic radii during a timeseries of one of the case A simulations. From top to bottom, the radii plotted are the cluster half-mass radius, \( r_h \), the core radius, \( r_c \), the radius enclosing a IMBH-mass of stars \( r_m \). Next we show the characteristic radii of some of the most-bound companions to the IMBH. These include the semi-major axis of the 5th-most bound star \( a_4 \), the 2nd-most bound star, \( a_1 \), and the most bound, \( a_0 \). We also show the pericenter distance of the most-bound object, \( r_{p,0} \). We color the most-bound object depending on its stellar-evolutionary type.

The larger radii, those that describe the bulk of the cluster, like the core and the half-mass radii are nearly constant in time. As the radii get smaller, and describe fewer and fewer particles, a higher degree of variability is seen. The semi-major axis of the most-bound object however, exhibits qualitatively different behavior than the other radii, or even the other tightly bound objects (as shown by \( a_1 \) and \( a_4 \)). The most-
bound star goes through a cyclic behavior of tightening semi-major axis followed by replacement with a new object. This cycle is seen to repeat throughout the timeseries of Figure 6.1.

The objects energetically bound to the IMBH all represent pairings that are ‘hard’ relative to the cluster background energy (Heggie, 1975; Hut and Bahcall, 1983). In clusters, binaries that are hard tend to get harder through interactions with remaining cluster stars (Heggie, 1975). This process can be understood as a statistical trend toward energy equipartition between the binary and the background as kinetic energy tends to transfer from the binary components to the perturber, ejecting it faster than it entered (Hills, 1975b,a). In this picture, each encounter leaves the IMBH plus companion pair (on average) somewhat more tightly bound, and ejects the perturbing object with high velocity (Saslaw et al., 1974).

Among the multiple stars bound to the IMBH, only the most-tightly object is ‘hard’ relative to the other bound stars. The orbits of less-bound objects cannot easily perturb the innermost orbit. The converse implies that the less-bound stars, despite being hard relative to the cluster core energy, are not hard relative to the more tightly bound companion to the IMBH. These less bound stars exchange energy with the innermost binary in a manner which prevents their orbits from tightening significantly. Because strong perturbations drive the binary hardening process, the continuous hardening of the most-bound star’s orbit can be approximated relative to the properties of the cluster background as

\[ \frac{da}{dt} \propto \frac{G \rho_c}{\sigma_c} a_0^2, \quad (6.6) \]

where \( \rho_c \) is the mass density of the cluster core environment surrounding the binary (Quinlan, 1996; Heggie and Hut, 2003). This hardening rate implies that the timescale for change of the semi-major axis, \( a_0/\dot{a}_0 \) scales as \( a_0^{-1} \) because \( \dot{a}_0 \propto a_0^2 \). These scalings
for the time-evolution of the most-bound object’s semi-major axis suggest the qualitative behavior seen in Figure 6.1, where $a_0$ first decreases rapidly while $a_0$ is large, then more slowly as $a_0$ decreases.

### 6.4.3 Companion Orbital Properties

In this section, we examine some of the orbital properties that define the population of most-bound companions to the IMBH. Figure 6.2 highlights some of the key orbital demographics of these companions. The lower left panel of Figure 6.2 displays the cumulative distribution function of the log of ratio of the semi-major axis of the most-bound star to the second most bound $\log(a_0/a_1)$. This diagram reveals that, in our simulations, the IMBH has a companion that is hierarchically isolated from the rest of the cluster stars a large fraction of the time. 90% of the time, the companion has $a_0 < \frac{1}{3}a_1$. Approximately 25% of the time, $a_0 < 10^{-2}a_1$.

The other panels of Figure 6.2 examine the distributions of orbital characteristics in physical units. In the upper left panel, we show the orbital semi-major axis distribution of most-bound companions. The largest separations are around $10^3$ AU. This distance becomes similar to the average distance of the less-bound stars within the IMBH’s sphere of influence (see, for example Figure 6.1), and, as a result, determines the maximum separation at which an object is able to persist as most-bound without exchange. The peak of the distribution, lies in the range of $10^1 - 10^2$ AU, and the minimum separation observed in our simulations is a few AU. The upper right panel shows the eccentricity distribution of companion objects. It is nearly thermal for most of the range of $e$, with some decrement observed near $e \sim 1$, because objects that enter into orbits that are too eccentric interact directly with the IMBH at orbital pericenter. The orbital separation distribution, along with the object masses, map to an orbital period distribution, shown in the lower right panel of Figure 6.2.

As can be inferred from an inspection of Figure 6.1, companions to the IMBH
Figure 6.2: Orbital properties of closest companions to the IMBH sampled at equal time intervals. We plot semi major axis, $a_0$, eccentricity, $e_0$, and period $P_0$, of the most-bound companion. We also show the ratio, $a_0/a_1$, which describes the compactness of the innermost companion’s orbit compared to the 2nd most-bound star. The $a_0/a_1$ distribution shows that the IMBH has a close companion that is significantly more tightly bound than any other objects a large fraction of the time. Approximately 90% of the time, $a_0/a_1 < 1/3$. 25% of the time, the companion is 100× more tightly bound than any other star. Typical orbital semi-major axes range from a few to a thousand AU, and the period distribution is also broad, peaking around 10 yr. The eccentricity distribution is nearly thermal, with some a slight decrement arising at $e_0 \sim 1$ due to the possibility of tidal disruption by the IMBH.
do not persist indefinitely. Instead these objects undergo cycles of replacement following
their termination – either through exchange or direct interaction with the IMBH. In Fig-
ure 6.3, we show the residence time of a given object as the companion to the IMBH. In
the grey histogram, we measure the persistence time of each individual partnership with
the IMBH. With this diagnostic, we can examine how long a typical close partnership
between the IMBH and a companion lasts. Figure 6.3 reveals a bimodal distribution
of persistence times of partnerships between the IMBH and other objects. One peak of
this distribution lies around $10^2 - 10^4$ yr, the second peaks around $10^7$ yr. A tail to-
ward longer residence times connects the two peaks. The origin of the first peak in this
distribution can be traced back directly to the orbital period of the typical most-bound
companion, as show in the right-hand panel of Figure 6.2. The longest orbital periods
of the most-bound companion occur when $a_0 \sim a_1$, and are $10^2$ yr for the parameters
used here. The short residence time branch can thus be associated with objects that
persist as partner to the IMBH for only one orbit. Near the peak of the distribution,
the grey histogram in Figure 6.3 records objects that last for a small integer number
of orbits. Associating the peak at $\sim 10^3$ yr in Figure 6.3 with $N$ orbits of of an object
near the maximum period for most-bound companions, we find that $\sim 10$ orbits is a
typical residence time before an exchange of hierarchy. A tail to longer residence times
represents objects whose orbits harden and thus persist for many more orbital periods.

The origin of the secondary peak at $\sim 10^6 - 10^8$ yr arises from objects whose
orbits harden sufficiently to become significantly hierarchically isolated from the remain-
ing cluster. This timescale thus becomes comparable to the cluster relaxation timescale
of $\sim 0.6$ Gyr. In the blue histogram of Figure 6.3, we show the distribution of residence
times for most-bound companion objects sampled at even time intervals (therefore it
is possible to count the same long-lived companion at multiple sampling times in this
case). If we observe the cluster at any given time, we are very likely to see a companion
with residence time of $10^6 - 10^8$ yr. This impression is confirmed visually in Figure
where the populations of objects that we see in the timeseries last, on average, for timescales of fractions of a Gyr or more.

6.4.4 Companion Stellar Properties

The color coding of stellar evolutionary state in Figure 6.1 shows that the demographics of the most-bound objects are remarkably diverse. This outcome is similar to the results from Mapelli and Zampieri (2014) for systems undergoing Roche Lobe overflow. Companion objects include BHs (1-2 Gyr), as well as MS and giant-branch stars, and WDs of various masses. In some cases, a single object will persist as the closest companion for long enough that it evolves and its stellar evolutionary state changes. One example of this is seen between $\sim 2.2 - 2.4$ Gyr, where a single most-bound star evolves from the MS to the giant branch, and eventually tidally interacts with the IMBH. In this section, we examine the statistical distributions of stellar evolutionary type, mass, and radius of the most-bound companion objects.

The diversity of companion types seen in Figure 6.1 is reflected quantitatively in Figure 6.4. The upper left panel of this figure shows the companions’ stellar evolutionary type. The most prevalent companions to the IMBH are MS stars, but compact objects like WDs, NSs, and BHs all represent significant contributions to the total demographics. The companion’s radius distribution, which we show in the lower left panel of Figure 6.4, directly reflects the stellar evolutionary status of the companions. We show the range of radii encompassing WDs to giant stars in this panel. The primary peaks of the radius distribution lie at $10^{-2}R_{\odot}$, and and $\gtrsim 1R_{\odot}$, reflecting the WD and massive MS populations, a smaller third peak is seen between $10^{1} - 10^{2}R_{\odot}$ that includes stars at various phases along the giant branch.

The objects’ mass distribution, shown in the right hand panels of Figure 6.4, exhibits a broad main peak centered at slightly more than a solar mass that is made up of MS stars, giants, massive WDs, and NSs. This peak lies substantially above the
Figure 6.3: Residence time of closest-partner status for companions to the IMBH. The grey distribution shows the time span of each individual closest-partnership formed with the IMBH (samples per unit partnership). The blue distribution samples residence time of the IMBH’s parter at equal time intervals (samples per unit time). Thus the grey distribution shows how long partnerships last, while the blue distribution shows the residence time likely to be seen by an observer seeing the cluster at a random time. An examination of the grey distribution shows that many partnerships with the IMBH suffer a breakup after \( \sim 10^3 \) yr, a timescale associated with one to several orbital periods. A tail of longer-residence time objects exists, though, followed by a secondary maximum at timescales around \( 10^7 \) yr, representative of the objects that hierarchically isolate from the remaining cluster stars. The blue distribution shows that these long-lasting partnerships form a dominant contribution to the typically observed IMBH companions. This qualitative conclusion can be inferred visually from Figure 6.1, which shows shows that the IMBH typically has a long-lasting companion (with residence time \( \gtrsim 10^6 \) yr).
Table 6.2: Demographics of stellar type of the most-bound companions to the IMBH. Tabulated by percentage as sampled at equal time intervals.

<table>
<thead>
<tr>
<th>Name</th>
<th>MS [%]</th>
<th>Giant [%]</th>
<th>WD [%]</th>
<th>NS [%]</th>
<th>BH [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40.3</td>
<td>9.2</td>
<td>29.9</td>
<td>6.5</td>
<td>14.1</td>
</tr>
<tr>
<td>B</td>
<td>11.5</td>
<td>2.3</td>
<td>12.5</td>
<td>6.4</td>
<td>67.3</td>
</tr>
<tr>
<td>C</td>
<td>27.6</td>
<td>6.0</td>
<td>49.6</td>
<td>1.6</td>
<td>15.2</td>
</tr>
<tr>
<td>D</td>
<td>21.9</td>
<td>3.2</td>
<td>43.6</td>
<td>4.0</td>
<td>27.4</td>
</tr>
</tbody>
</table>

mean mass of cluster stars, which is initially $\sim 0.58 M_\odot$. We also observe a tail toward much higher companion masses (up to $\sim 10 M_\odot$) made up mostly of MS stars, giants, and stellar-mass BHs. That the mass of most of the IMBH’s companions is significantly larger than that of a typical cluster star is not surprising. The trend toward energy equipartition in two-body interactions causes stars more massive than the mean mass to sink toward orbits deep in the cluster core, where they are very likely to come into direct interaction (and potentially partnership) with the IMBH (e.g. Goswami et al., 2012; Gürkan et al., 2004; Leigh et al., 2014). Further, more massive secondary objects are likely to replace an existing companion if they do undergo a three-body interaction (Fullerton and Hills, 1982; Sigurdsson and Phinney, 1993; Samsing et al., 2014; Mapelli and Zampieri, 2014). In the lower right panel of Figure 6.4 we show the un-normalized distribution of companion masses in order to demonstrate that there is strong evolution in the companion mass distribution with stellar population age. Young ages, $< 2$ Gyr, where the turnoff mass is still high, contribute the bulk of the companions $\gg 1 M_\odot$. This temporal dependence in the companion mass can be contrasted to the approximately time-independent companion orbital parameter distributions described in the previous sub-section.
Figure 6.4: Stellar properties of closest companions to the IMBH sampled at equal time intervals. The upper-left panel shows companion stellar evolutionary type (for a key see the legend of Figure 6.1), the upper-right panel shows companion masses $M_0$, and the lower-left panel their radii, $R_0$. Companions include comparable fractions of stars (both MS and giants) and stellar remnants (WDs, NSs, and BHs). The typical mass of BH companions is substantially higher than the initial cluster mean mass of $0.58 M_\odot$. However, a broad distribution suggests that even relatively low mass companions $\sim 0.3 M_\odot$ occasionally find themselves tightly bound to the IMBH. The lower-right panel shows how the mass distribution evolves in time. At early times in the simulation, shown in blue, the mass distribution is weighted toward high masses, and includes both massive stars and massive stellar remnants. Late times, when the cluster turnoff mass is lower, contribute substantially to the lower-mass portion of the distribution.
6.4.5 Termination of Close Partnerships

Close partnerships to the IMBH can be terminated through one of several channels. In this section, we describe the possible channels and explore their relative likelihoods.

6.4.5.1 Channels

Three possible channels of termination of a close partnership with the IMBH exist. They are shown in the diagram of Figure 6.5 and explained below.

- **Exchange:** Exchanges can occur when a third body’s pericenter distance from the IMBH becomes similar to the semi-major axis of the binary companion $a_0$. When this occurs, the system undergoes a strong three-body interaction, during which the gravitational attraction between both objects and the IMBH is similar. Either star may emerge from the three-body interaction as the new most-bound object. Statistically, there is also a chance that the IMBH could be the ejected object, leaving behind a binary of two stars. However, since the mass ratio is large, this exchange outcome is extremely unlikely (Heggie et al., 1996). Throughout the duration of our simulations, we record all perturbers to the central binary with pericenter distance less than 3 times $a_0$. This allows us to track the statistics of these exchange interactions. We find that the bulk of exchanges are due to strong three-body interactions, where the previous perturber emerges from the encounter as the new companion. A small fraction, of order a few percent, are the result of more complex multi-body strong interactions where an object other than the strongest perturber emerges as the companion immediately following the encounter.

- **Tidal Disruption:** An examination of Figure 6.1 reveals that despite having the highest binding energy to the IMBH, the orbit of the most-bound star is still
Figure 6.5: Interaction channels that can bring about the end of a close partnership between the IMBH and another object. Possible outcomes of a close partnership include three and multi-body exchanges, gravitational wave inspirals and tidal disruptions. The bulk of partnerships are terminated through the exchange channel, but a fraction of long-lasting binary companions may end up tidally disrupted or undergoing an inspiral. If the inspiralling object is a BH or NS, the initial gravitational wave capture leads to an IMRI (Amaro-Seoane et al., 2007; Mandel et al., 2008; Mandel and Gair, 2009; Konstantinidis et al., 2013), while if the object is a WD, tidal heating of the object likely leads to disruption while from an eccentric orbit and repeated flaring episodes (MacLeod et al., 2014).
subject to perturbations. Not only does its orbit tighten through interactions with other cluster-core stars, but its orbital angular momentum wanders in magnitude and direction, as can be seen through the orbital pericenter distance, $r_{p,0}$. As a result, the most-bound star will be occasionally perturbed into an orbit which leads it to pass within a tidal radius, $r_t$, of the IMBH. In our simulations, we record these tidal disruption events and delete the particle from the simulation, as described in Section 6.3. In reality, some stars, especially those with condensed cores, can lose part of their envelope gas without being completely disrupted (MacLeod et al., 2012, 2013). The remnants of these partial tidal stripping events could then go on to further interaction with the IMBH (Hopman and Portegies Zwart, 2005; Bogdanović et al., 2014).

- **Gravitational Wave Inspiral:** Just as the orbital wandering of the most-bound object permits the occasional tidal disruption event, if the companion to the IMBH is a compact object it may also undergo an intermediate mass ratio gravitational wave inspiral (IMRI) (Amaro-Seoane et al., 2007; Mandel et al., 2008; Sesana et al., 2008; Mandel and Gair, 2009; Konstantinidis et al., 2013). In our simulations, these are flagged as occurring when the gravitational wave inspiral time becomes less than one $N$-body time unit, and the particle is removed from the simulation (see Section 6.3). Holley-Bockelmann et al. (2008) and Konstantinidis et al. (2013) explore the retention of IMBHs suffering a gravitational wave recoil following an IMRI event. IMBHs that merge with stellar mass BHs with mass ratio $q \gtrsim 0.1$ have a significant probability of being ejected from the cluster. Smaller kicks would temporarily offset the IMBH from the cluster core. These inspirals typically occur when the companion is in a very eccentric orbit, and are driven primarily by the close pericenter distance, rather than, for example a close semi-major axis, much like the tidal disruptions (and in agreement with the findings of Gültekin
et al., 2004). The N-body simulations of Baumgardt et al. (2004b) and Leigh et al. (2014) lack this merger channel and Leigh et al. (2014) demonstrate that this leads to unrealistically long-lived central partnerships. If the companion is a BH or NS it will be swallowed as it merges with the IMBH. However, in some cases where the companion is a WD, it will tidally disrupt before merging with the IMBH (e.g. Casalvieri et al., 2006; Ivanov and Papaloizou, 2007; Sesana et al., 2008; Zalamea et al., 2010; Krolik and Piran, 2011; MacLeod et al., 2014). This can proceed in one of two ways. In most cases, tidal heating likely leads to the disruption of the WD while it is still in an eccentric orbit. However, in some cases the orbit may fully circularize before the onset of Roche Lobe overflow.

### 6.4.5.2 Termination Demographics

The termination of partnerships with the IMBH, as well as other close encounters, are displayed in Figure 6.6. In this figure, we show the same timeseries plotted in Figure 6.1. Here, we extend the range of the plot include radii very close the the IMBH, including the IMBH’s Schwarzschild radius, $r_s$. We also include tidal radii of typical WDs, MS stars and giant stars. Overset on this timeseries we highlight the orbital semi-major axis (solid symbol) and pericenter distance (open symbol) of encounters leading to either tidal disruption or gravitational wave inspiral. Tidal disruptions are shown with circular points colored blue for MS star disruptions and red for giant star
Figure 6.6: Characteristic radii measured with respect to the IMBH in the same example timeseries as Figure 6.1, from simulation group A. Here we expand the domain to include the IMBH’s Schwarzschild radius, $r_s$, and estimated tidal radii of typical WDs (brown), MS stars (blue), and giants (red). Overset on this timeseries we plot the semi-major axis (solid symbols) and pericenter distance (open symbols) of tidal disruption events (circular points) and gravitational wave inspirals (stars). These direct interactions with the IMBH often lead to the termination of long-standing partnerships with the IMBH. On visual inspection, several categories of events emerge. Gravitational inspirals are particularly likely to come at the end of a long partnership when the orbital SMA has decreased. Tidal disruptions, by contrast, often come in times of multi-body interaction that scatter stars toward the IMBH. These are particularly likely to occur when $a_0 \sim a_1$, either because $a_0$ is large or $a_1$ is smaller than average. In several cases represented here a most-bound star evolves off the MS and up the giant branch until it eventually reaches the point of tidal disruption due to its growing radius.
disruptions. Gravitational wave inspirals are marked with green stars.

Figure 6.6 reveals the broad demographics of changes of partnership with the IMBH. Many changes of partnership appear to arise from strong interactions between the companion to the IMBH and other cluster stars – often including the second most-bound star, whose semi-major axis, $a_1$, is occasionally similar to $a_0$. When $a_0 \sim a_1$, changes of partnership are frequent, and as a result the constantly-changing companion’s orbit does not have the opportunity tighten significantly. An example of this scenario is seen in the 2.5 – 4.5 Gyr span of Figure 6.6. Close encounters with the IMBH often accompany strong encounters – many tidal disruption events either involve the most-bound companion or stars scattered into a similar orbit. The chaotic three-body dynamics of the IMBH, companion, and perturber system then cause a fraction of stars to be scattered directly toward the IMBH. An exception to this requirement for scatterings comes in the form of giant star disruptions, which can occasionally occur when the star evolves to become tidally vulnerable to the IMBH rather than being strongly scattered (‘spoon-feeding’ the IMBH: MacLeod et al., 2013).

In Table 6.3, we examine the termination of close partnerships with the IMBH as well as the rates of close encounters with the IMBH. The most common, by far, means for changes of partnership with the IMBH is the exchange interaction, contributing approximately 98% of the changes in partnership. These exchanges dominate the large peak of IMBH companions that have residence times in the range of $1 - 10^6$ years, as shown in Figure 6.3. The mean exchange rate is $2.5 - 5 \times 10^{-7} \text{ yr}^{-1}$. Of these exchanges, ~98% are simple three-body exchanges in which the strongest perturber becomes the the new most-bound companion. The remaining 2% are more complex multi-body interactions. These may involve more than three objects, or they may involve the tidal disruption or inspiral of one or more objects. Gravitational wave inspirals and tidal disruption events contribute to the remaining termination of partnerships with the IMBH. The tidal disruption rate is of order $10^{-8} \text{ yr}^{-1}$ and the inspiral rate is of
order $10^{-9}$ yr$^{-1}$ in our simulations. In Section 6.6, we compare these event rates to previous predictions and mention their potential significance for electromagnetic and gravitational transients.

As can be inferred from Figure 6.6, the longer a companion object persists, the more likely it is that the pair will be split by direct interaction with the IMBH rather than exchange. Longer lived companion pairs segregate to smaller orbital semi-major axes, reducing the cross-section for encounters or exchange in proportion to $a_0$. Meanwhile, interaction with the IMBH becomes progressively more likely at smaller $a_0$ because the orbital eccentricity needed to produce a close pericenter passage becomes smaller. Thus, many long-lived companions appear to only be replaced when they are scattered to their disruption, or when they undergo a gravitational wave inspiral. These terminations contribute to the large fraction of tidal disruption events and gravitational wave inspirals arising from the most-bound companion in Table 6.3.

6.5 Dependence on Cluster Properties

Having outlined the basic process of IMBH companion formation, evolution, and destruction through reference to simulation group A in the previous section, here we consider the dependence of our results on the chosen simulation parameters. We run several sets of simulations A-D in Table 6.1, which allow us to systematically explore the differences in the properties of close companions to the IMBH with varying cluster properties. These variations prove useful in understanding the physics underlying companion object dynamics. Simulation group B has a lower kick velocity dispersion that our fiducial case, A, which we have up until now analyzed. This lower kick velocity allows a larger fraction of remnant NSs and BHs to be retained in or near the cluster core. Simulation group C has a lower IMBH mass than the fiducial case by a factor of two – it’s initial mass is $75M_\odot$ rather than $150M_\odot$. Finally, simulation group D has
Figure 6.7: Comparative timeseries of first 6 Gyr of cluster evolution of examples from simulation groups B-D. This figure is analogous to Figure 6.1, which plots the evolution of one of the simulation group A models. As in Figure 6.1, we color hard-binary companions to the IMBH based on their evolutionary type. Table 6.1 describes the differences between simulation groups B-D as compared to A.
a larger total number of stars (and mass) by a factor of 2 ($2 \times 10^5$ rather than $10^5$).

By exploring these systematic differences, we can also consider the extrapolation of our results to higher mass clusters or IMBHs.

### 6.5.1 Companion Objects in Cluster Evolution

In Figure 6.7, we plot comparative timeseries, analogous to that shown in Figure 6.1, for examples from simulation groups B-D (which may be compared directly to Figure 6.1 for simulation group A). Even looking at the examples in this figure, dynamical and demographic differences and similarities distinguish themselves. The pattern of orbital tightening and replacement of a most-bound companion is repeated throughout the panels, despite their varying cluster and IMBH parameters. The global evolution of the clusters is largely similar as well: cluster core and half mass radii are relatively constant in time, just as with simulation group A.

Differences in the stellar dynamics and demographics of the companion objects also become apparent on closer inspection of Figure 6.7. Simulation group B, with a larger population of retained BHs, shows a larger quantity of BHs joining the IMBH as most-bound companions. The cycles of orbital hardening experienced by these BHs are longer-lived than those of most of their stellar counterparts. By contrast, simulation group D, with a larger total number of stars than the fiducial case shows more frequent exchanges of partnership, but a diversity of companion-object stellar evolutionary types.

These BHs preferentially enter into partnerships with the IMBH because of their significantly higher-than-average mass, which causes them to sink in the cluster potential due to dynamical friction. When these stellar-remnant BHs interact with the IMBH and its companion, their high mass makes them very likely to replace an existing companion. Heggie et al. (1996) has shown that the exchange cross section depends quite strongly on the mass of the tertiary, perturbing body, even in the case where the mass ratio between the binary components is large. In Figure 6.8, we plot the fraction
Figure 6.8: Fraction of close encounters between perturbers of mass $M_p$ and the IMBH and companion pair that result in an exchange. The exchange fraction depends sensitively on the perturber to companion mass ratio, allowing massive objects to exchange into the inner binary easily, and then to resist being exchanged out by subsequent encounters with lighter objects. By contrast, we see little dependence of the exchange fraction on other cluster properties, like the IMBH mass. Errorbars are poisson errors on individual bins in the exchange cross section.

of encounters that result in an exchange given a particular perturber to companion mass ratio, $M_p/M_0$. This fraction is computed by counting the fraction of encounters in which the perturber comes within $a_0$ of the binary center of mass at pericenter that result in an exchange. While the exchange fraction depends strongly on perturber mass ratio, it does not depend nearly as strongly on other cluster properties. This plot implies that massive objects can exchange into the inner binary with relative ease and that, as companions, they can undergo many encounters, each hardening their orbit before one leads to an exchange.

6.5.2 Companion Demographics

Figure 6.9 further traces how companion demographics propagate to orbital distributions. The histogram of orbital semi-major axis is shown for each of the simulation groups A-D, and is broken down by companion type. Visually from Figure
Figure 6.9: Distributions of companion object semi-major axis, $a_0$, subdivided to show companion demographics. Panels show simulation groups A-D outlined in Table 6.1. These panels illustrate how differing companion demographics between simulation groups propagate to different distributions of companion orbital properties. The distributions of compact object (BH and WD especially) semi-major axes peak at smaller values than the overall distribution, so varying quantities of compact object companions propagate to different overall distributions of companion orbits.
6.7, and here quantitatively, we can infer that types of companions have different sub-
distributions. When combined in varying percentages, these demographic differences
lead to variations in the overall semi-major axis distribution of IMBH companions. The
most obvious example are BHs. Based on the histograms of Figure 6.9, BHs exist in
orbits which are preferentially among the most tightly bound. The distribution is broad
and over the course of the simulation there were BHs at all separations. We can un-
derstand this distinction because BHs are preferentially difficult to replace in exchange
interactions due to their high mass, and are not prone to tidal disruption when the the
companion has orbit with $e \sim 1$.

By contrast, giant stars tend to exist in the least-bound orbits in Figure 6.9. These objects are especially tidally vulnerable, and so are subject to tidal disruption by
the IMBH at larger orbital separations for a given eccentricity than are more compact
MS stars or WDs. The cases of MS stars and WDs lie intermediate to that of giants
and BHs. MS stars’ innermost semi-major axes are also limited by tidal disruption as
their orbits cycle through high-eccentricity phases (note the rapidly varying eccentricity
of the innermost orbit in Figure 6.1). WDs are less tidally vulnerable, but are not as
massive as the BHs, so their distribution peaks at somewhat larger separations. The
case of the NSs is interesting and somewhat more subtle. Although these remnants are
$\sim 3 \times$ the average stellar mass in the cluster, they are subject to kicks on formation and
they do not re-segregate to the cluster core as efficiently as the more-massive BHs.

Figure 6.10 quantitatively explores how these demographic differences propa-
gate to the other properties of the most-bound companions to the IMBH. The upper
left panel shows companion object demographics, which are also tabulated in Table
6.2. Qualitatively, the demographics remain unchanged in that the companions include
stellar remnants like WDs, BHs, and NSs, along with MS and giant stars. As seen in
Figures 6.7 and 6.9, the most dramatic difference arises with simulation group B, and
the retention of a higher fraction of BHs in the cluster’s central regions. Simulation

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group D, with its larger total particle number, also exhibits a mild enhancement in the fraction of BH companions. This difference can be traced to the larger total number of BHs retained in these simulations (despite the small retention fraction). Companion demographics shape the companion mass distribution, shown in the upper right panel of Figure 6.10. The enhanced BH fraction is reflected in the higher peak at companion masses $\gtrsim 3M_\odot$ for simulation groups B and D.

### 6.5.3 Companion Dynamics

In addition to demographic changes in the typical properties of IMBH companions, there are also dynamical differences we can observe across our simulations. We begin by examining how the long-lasting tail of the residence time distribution of IMBH companions is affected by changing cluster properties in the central panel of Figure 6.10 (the residence time plotted is counted per-partnership as is the grey distribution in Figure 6.3). This tail of long-lived companion objects is shaped by processes of companion exchange and disruption. As a result, it is influenced by a combination of cluster, IMBH, and companion demographic properties. An increased fraction of BH companions (as in simulation case B) implies more long-lasting partnerships because the IMBH’s companion is considerably more immune to exchange interactions (see Figure 6.8) and to tidal disruption. In fact, we can observe this directly in the termination statistics of Table 6.3, where we see that exchanges are significantly less frequent in simulation group B (because of the mass sensitivity of the exchange cross-section, Figure 6.8). Tidal disruptions are also nearly a factor of two less common than in case A, with a smaller fraction arising from the most-bound companion. With lower IMBH mass, as evaluated in simulation group C, weaker gravitational focusing of surrounding stars leads to the lower exchange rate tabulated in Table 6.3. This propagates to more objects with long residence times in the $10^6 - 10^8$ yr range. The larger number of stars present in simulation group D, by contrast, implies a larger encounter and exchange
Figure 6.10: In this figure, we show a general comparison of IMBH companion properties with varying simulation initial parameters. We include four groups of simulations in this diagram, A-D, described in Table 6.1. Simulation group B, has a lower kick velocity than group A, as a result, more stellar remnant BHs remain in the cluster core. These BHs are more massive than the typical stellar companion, and therefore have longer residence times in companionship with the IMBH because more massive innermost binaries are more difficult to disrupt. As a result of the increased time for the binary to harden, the $a_0/a_1$ distribution extends to smaller separations. Simulation group C includes a lower IMBH mass of $75M_\odot$ instead of $150M_\odot$, this has only a mild effect on the stellar properties of companions, but does allow companions to segregate to tighter separations in $a_0/a_1$, and thus also shorter median orbital periods. Finally, simulation group D includes twice as many initial particles as the other simulations, $N = 200k$. The increased particle number implies shorter residence times as the IMBH's most-bound companion because there are more potential perturbing bodies, and also shorter orbital periods because there are, on average, a larger number of tightly-bound stars.
rate and shorter typical residence times.

The lower panels of Figure 6.10 explore differences in companion periods, $P_0$, and hierarchies, $a_0/a_1$. Here distinctions arise both in the form of companion demographics and IMBH mass. All of simulation groups B-D exhibit shorter typical orbital periods than simulation group A. The peak in the orbital period distribution shifts by about 1 dex from $\sim 10$ yr, to $\sim 1$ yr, but the distribution remains broad in all cases. Companion hierarchy, as measured by $a_0/a_1$, is also affected both by companion demographics and IMBH mass. The fiducial case, simulation group A, shows companions which are less segregated than groups B-D. In the case of simulation group B, the tighter companion distribution can be traced to the increased BH fraction and correspondingly longer companion residence time (which allows the IMBH-companion pair more time to harden prior to disruption). Simulation group D’s tightly-bound tail can be similarly explained by a mildly-increased fraction of BH companions.

The IMBH mass effect on companion hierarchy can also be observed in the lower-left panel of Figure 6.10. The median $a_0/a_1$ shifts by a factor of approximately 3 between simulation groups A and C, which have a factor of 2 difference in IMBH mass. The origin of this distinction can be understood through a consideration of the dynamics of orbital hardening and exchange for IMBH companions. A typical close passage of a perturbing body of mass $M_p$ through the IMBH-companion pair carries away an energy of order

$$\Delta E_p \approx \frac{GM_0M_p}{2a_0},$$

when it passes with orbital pericenter within the semi-major axis of the inner binary (Yu and Tremaine, 2003). As compared to the orbital energy, $E_0 = GM_{bh}M_0/2a_0$, this typical per-encounter change of energy is

$$\frac{\Delta E_p}{E_0} \approx \frac{M_p}{M_{bh}}.$$
So the fractional, per-encounter hardening rate of the IMBH and companion pair scales inversely with the IMBH mass. But, the rate of encounters also changes with IMBH mass. The encounter cross section is dictated by gravitational focusing onto the inner binary, and is \( \Sigma_{\text{enc}} \approx 2\pi GM_{\text{bh}}a_0/v^2 \). Associating a typical velocity with the cluster core velocity dispersion, the encounter rate, \( \dot{N}_{\text{enc}} \propto n_c\sigma_c\Sigma_{\text{enc}} \), where \( n_c \) is the core number density, thus scales as,

\[
\dot{N}_{\text{enc}} \propto n_c\sigma_c^{-1}M_{\text{bh}}a_0,
\]

(6.9)

proportional to both \( M_{\text{bh}} \) and \( a_0 \), as long as the low-angular momentum phase space of encountering orbits is efficiently replenished (e.g. Frank and Rees, 1976). As a result, although each encounter with a perturber carries away less energy \( \propto M_{\text{bh}}^{-1} \), the encounters are more common \( \propto M_{\text{bh}} \) and the innermost binary hardens at an identical rate regardless of IMBH mass.

Although the hardening rate remains unchanged with differing IMBH masses, the residence time, as shown in the central panel of Figure 6.10, does change. In particular, when the encounter rate goes up, so does the exchange rate of IMBH companions. This is because the exchange fraction in our simulations does not appear to depend strongly on IMBH mass, as shown in Figure 6.8, instead it is sensitive primarily to the perturber-companion mass ratio. As IMBH masses go up, their companions harden at the same rate, but are exchanged with increasing frequency, restarting a new hardening cycle. This result implies, when extrapolated to larger IMBH masses, that only IMBHs of moderate mass ratio to their surrounding stars (perhaps those in the range \( \sim 10^2 - 10^4 M_\odot \)) will ever exhibit tightly segregated companions.

### 6.6 Discussion

In this section we draw on our results concerning the properties of IMBH companions to explore how these objects can occasionally reveal the presence of an
otherwise invisible IMBH in the cluster core.

### 6.6.1 IMBH Companions in Old Clusters

We have focused so far on IMBH companions across a range of stellar system ages. However, most GCs are very old stellar systems, with ages $> 10$ Gyr (e.g. Marín-Franch et al., 2009; Forbes et al., 2015; Trenti et al., 2015). How might the companions to IMBHs in these very old stellar systems differ? To examine this question we look at several evolutionary sequences of the last several Gyr of cluster evolution of the models in simulation group D. These evolutionary tracks are shown in Figure 6.11. We find that by these late epochs interactions with the IMBH (either mergers or ejections) have exhausted the supply of stellar-mass BHs which might fall into partnership with the IMBH (Leigh et al., 2014). If realistic GCs begin with much larger masses and $N$ than their present-day properties (or our smaller-still simulated systems), they would retain a similar fraction but larger number of BHs to late times (Morscher et al., 2015). If these BHs remain in the cluster we would expect them to mass-segregate toward the cluster core and, perhaps, play a role in shaping the time-dependence of the companion mass function, show in Figure 6.4. At 10 Gyr, the turnoff mass is now $\sim 1M_\odot$, and is therefore less than the mass of typical stellar remnants like WDs and NSs. As a result, the most common companions for the IMBH in our simulations at these times are massive WDs, those with masses $\gtrsim 1M_\odot$, which are therefore more massive than the MS stars in the cluster core. NSs similarly form a fraction of late-time companions, because those that were retained in the cluster (but perhaps kicked to the outskirts) have had sufficient time to re-segregate into the cluster core.
Figure 6.11: Evolution of representative members the four simulation group D models from 6 Gyr to their tidal dissolution at $\sim 10.5$ Gyr. The lines plotted here are a simplified set of those presented in Figure 6.1, from bottom to top in each panel: $a_0$ (color), $a_1$ (grey), $r_m$, $r_c$, $r_h$. This Figure may be compared with the first 6 Gyr of evolution shown in Figures 6.1 and 6.7. IMBH companions in old clusters are qualitatively similar to those at earlier epochs, but contain fewer BHs and a larger fraction of WDs. Sequences for other realizations of a given simulation group are quantitatively different but qualitatively similar to the models reproduced here.
6.6.2 Dynamical Mechanisms for Repeated Tidal Disruption Flares by IMBHs

One motivation for performing this study was to access whether close companions to the IMBH are ever driven into multiple-passage tidal interactions. Such multi-passage tidal disruption flaring interactions can be difficult to achieve. For multiple passages of similar strength to be sustained, the star must remain bound to the IMBH with its orbit relatively unchanged (in energy, or more importantly, angular momentum). Further, the object undergoing the passages must expand upon mass loss either through its adiabatic response to mass-loss or heating due to the violent interaction. These requirements suggest that orbits leading to multi-passage disruptions must exist in a phase space where the per-orbit scatter due to the remaining cluster stars is small (MacLeod et al., 2013). For MS stars or WDs in clusters with as few stars as we consider here, the only star in such an orbit is the most-bound companion to the IMBH. Thus, if this most-bound star enters into a weak tidal encounter with the IMBH the stripping may persist for several passages.

In our simulations, we find that a reasonable fraction (30-50%) of tidal disruption events involve the most-bound star (Table 6.3). Although we track only full disruptions dynamically in our simulations, many of these would, in fact lead to partial tidal disruptions over multiple orbits. Stars with condensed cores relative to their envelopes, in particular evolved stars and those with radiative envelopes are particularly able to retain a bound core despite losing part of their envelope (MacLeod et al., 2012; Guillochon and Ramirez-Ruiz, 2013). We find that some giant stars evolve to the point of mass transfer with the IMBH (MacLeod et al., 2013). WDs that undergo gravitational wave inspiral are also very likely to disrupt before their orbits circularize, leading to episodic mass transfer and repeated flaring (MacLeod et al., 2014). Many of the disrupted most-bound stars reach the tidal radius through a secular interaction.
with other tightly-bound objects – opening the possibility of a large number of passages with similar pericenter distance. In the ensuing repeated tidal stripping episodes the individual flares would be qualitatively similar to other tidal disruption flares albeit with less total fluence. The repetition time between flares would be governed by the orbital period distribution of the most-bound companion.

This leads us to the case of the extraordinary repeated-flaring transient HLX-1 (Farrell et al., 2009) by a putative IMBH with \( M_{\text{bh}} \sim 10^4 M_\odot \) (e.g. Davis et al., 2011). If this object is powered by the partial tidal stripping of a star in an eccentric orbit the object must have gone through \( \gtrsim 10 \) passages based on the lightcurve, with orbital period near a year. The IMBH mass predicted for HLX-1 is nearly a factor of 100 larger than those we are able to simulate. However, with the scalings of section 6.5 as guidance, we can speculate whether these properties appear plausible given our results.

First, having tightly bound stars in orbit around the IMBH appears likely given the IMBH (rather than supermassive) nature of the putative HLX-1 BH. Although scalings to a more massive system would modify them, our distributions of orbital periods easily extend to a orbital periods in the years to tens of years range. From a pericenter distance slightly outside the tidal radius, the donor star could have been tidally excited until it started transferring mass to the IMBH (Hopman et al., 2004; Baumgardt et al., 2006; Blecha et al., 2006). Secondly, a large fraction of tidal disruption events in our low-density, small-N clusters originate from the most-bound companion to the IMBH (as compared to the supermassive BH in a galactic center context upon which most tidal disruption literature is defined). We observe that the most-bound stars that tend to interact with the IMBH in our simulations arrive in disruptive orbits through secular interaction, rather than single, strong scattering events. Thus, it would not be entirely surprising if a tightly-bound star in the HLX-1 system ended up in a grazing, rather than fully-disruptive, interaction with the IMBH. In conclusion, a scenario involving a hierarchically isolated companion star being driven to interact with
the IMBH is a scenario that qualitatively explains the properties of a repeated flaring source like HLX-1, and extrapolation of our results indicates that such a configuration can be established through dynamical interactions of stars with the IMBH.

6.6.3 Revealing IMBHs in GCs

6.6.3.1 Luminous Companions to a Dark IMBH

The calculations of this paper have demonstrated that IMBHs residing in clusters acquire and retain close companions. In rare periods in which the IMBH disrupts and accretes its companion, a luminous accretion signature will mark the presence of the IMBH. We discuss this possibility further in the following section, but here we consider the ability of companion objects to reveal the IMBH in periods of relative quiescence. Among the companion objects, approximately half are luminous stars like MS stars and giants, while the other half are WDs, NSs and smaller BHs. Can this luminous population of stars serve to reveal the presence of otherwise dark IMBHs in nearby clusters?

First, the binary nature of the IMBH plus companion pair could potentially reveal the IMBH’s presence. The typical companion orbits found in our simulations (semi-major axes of $\sim 10^2$ AU and periods of $\sim 10$ yr, shown in Figure 6.2) imply typical orbital velocities of hundreds of km s$^{-1}$. If spectroscopically detected, this orbital motion would strongly point to a massive companion because of the high velocity given combined with relatively long orbital period (compared to a stellar-mass binary). This approach extends work already being done to explore radial velocities of stars bound to potential IMBHs in cluster cores (e.g. Noyola et al., 2010; Lützgendorf et al., 2013b). But we argue that among the tightly bound stars, it would be reasonable to expect a single very compact companion orbit in a large fraction of clusters hosting a IMBH.

Luminous stars also shed substantial stellar winds. With the star tightly bound
Figure 6.12: The distribution of accretion rates fed by the stellar wind of the companion to the IMBH. The series of lines labeled $\dot{M}_0$ is the wind mass loss rate of IMBH companions estimated with Reimers (1975)'s formula. The lower series of lines use the wind velocity and orbital properties to estimate the fraction captured by the IMBH. The shaded region denotes the range of possible Bondi accretion rates onto a $150M_\odot$ IMBH with ambient gas density of $10^{-24}$ g cm$^{-3}$ and sound speed $c_s = 10^6 - 10^7$ cm s$^{-1}$ (Naiman et al., 2012, 2013), where $\dot{M}_{\text{bondi}} = 4\pi(GM_{\text{bh}})^2\rho/c_s^3$. Stellar winds are only captured by the IMBH less than 10% of the time. The typical wind-fed accretion rates during this $\sim 5\%$ duty cycle are competitive with Bondi accretion from the background of cluster-core stars but only strongly dominant if our more conservative estimate of the gas properties ($c_s \sim 10^7$ cm s$^{-1}$) applies. Thus, we do not expect companions to IMBHs to impose strong limits on the accretion-fed luminosity of cluster IMBHs.
to the IMBH we examine whether and how these winds might contribute to fueling accretion activity onto the IMBH. The relevant scale for comparison here is the Bondi accretion rate at which the IMBH could accrete gas from the cluster-core interstellar medium, $\dot{M}_{\text{bondi}} = 4\pi (GM_{\text{bh}})^2 \rho / c_s^3$ (Volonteri et al., 2011). This interstellar medium is shaped by the interacting stellar winds of all the stars in the cluster core. Taking typical values of $10^{-25}$ g cm$^{-3}$ and sound speed $c_s = 10^6 - 10^7$ cm s$^{-1}$, we find $\dot{M}_{\text{bondi}} \approx 10^{-11} - 10^{-15} M_\odot$ yr$^{-1}$ (e.g. Naiman et al., 2012, 2013).

In Figure 6.12, we compare this nominal accretion rate from the background of stars to that fed by the companion. Here we plot the duty cycle (fraction of time) spent accreting above a given level. The upper series of lines in this figure shows $\dot{M}_0$, the mass loss rate of the companion star, estimated using Reimers (1975) mass loss formula, $\dot{M} = 4 \times 10^{-13} \eta_R (L_0 M_0 / R_0) / (L_\odot M_\odot / R_\odot)$ with $\eta_R = 0.5$ (McDonald and Zijlstra, 2015). We then estimate the fraction of this mass lost which is bound to the IMBH using the formulae of Miller et al. (2014, described in their section 4.1) The bound fraction depends on the orbital properties through the semi-major axis and eccentricity, which determine the velocity of the wind-shedding star relative to the IMBH. It also depends on the terminal velocity of the winds themselves, which we take to be the star’s escape speed, $\sqrt{2GM_0/R_0}$. The slow, massive winds of giant stars, for example, are particularly easy to capture. We plot the portion of companion wind material that is bound to the IMBH as $\dot{M}_{\text{bh}}$ in Figure 6.12. Most of these captured winds come when the IMBH hosts a giant star companion. These cumulative distributions show that the IMBH captures the winds of its companion at rates comparable to $\dot{M}_{\text{bondi}}$ of order 5% of the time in our clusters.

As a result, we can expect that the $\gtrsim 50\%$ of stellar IMBH companions should illuminate their host BH through their winds, providing a minimum accretion rate onto the IMBH. However, the fact that the IMBH has a companion does not imply that most IMBHs should accrete substantially above the Bondi accretion rate from the
ambient medium. Of course, how these alternative fuel sources transfer into accretion luminosity represents a significant uncertainty. Media captured from the cluster core has low net angular momentum, and likely does not efficiently form a disk (e.g. Cuadra et al., 2008), perhaps contributing to a low radiative efficiency. There is still more uncertainty in how much of the wind-deposited gas might reach a IMBH in scenarios with realistic feedback (Naiman et al., 2009; Park and Ricotti, 2011, 2012, 2013) and what fraction of the resultant accretion energy would contribute to radio and X-ray luminosities (Maccarone, 2004; Maccarone et al., 2005; Strader et al., 2012; Wrobel et al., 2015). Given these uncertainties, wind feeding from the companion implies a minimum activity level for the fraction of IMBHs hosting giant star or MS companions.

6.6.3.2 Electromagnetic and Gravitational Transients

Stellar tidal disruption events lead to a stream of debris from the disrupted star accreting onto the IMBH. The ensuing accretion flare may be extremely luminous (Ramirez-Ruiz and Rosswog, 2009). For the relatively low-mass clusters with low-mass IMBHs we have considered through our direct $N$-body integrations, the rate of tidal disruptions is relatively low, $\sim 10^{-8}$ yr$^{-1}$. More massive IMBHs and denser clusters both imply higher event rates, with $\dot{N}_{\text{tde}} \propto M_{\text{bh}}^{4/3} n_c$ (Rees, 1988). The events themselves, however, are likely short, with the flare characteristic timescales of order weeks to months (e.g. Ramirez-Ruiz and Rosswog, 2009; MacLeod et al., 2012; Guillochon and Ramirez-Ruiz, 2013) and observable signatures lasting at most for years or tens of years through nebular emission (Clausen and Eracleous, 2011; Clausen et al., 2012). The implied duty cycle of these flaring episodes is thus extremely small and the fraction of cluster IMBHs expected to be in a flaring state is $\sim 10^{-8}$. Tracing this number of clusters is not inconceivable, optical surveys for tidal disruption flares routinely cover $\sim 10^5$ galaxies (e.g. van Velzen et al., 2011), each of which might host $10^2 - 10^3$ globular clusters (Brodie and Strader, 2006). But this scaling argument is hampered by the fact
that the Eddington-limited luminosity of low mass IMBHs is much lower than their supermassive counterparts. Thus, this argument only suggests a reasonable fraction of detectable events if the occupation fraction of IMBHs in local-universe clusters is of order unity.

Relativistically beamed jet emission from tidal disruption events may be visible in some cases, and for low mass BHs, the implied luminosities are large enough to suggest that these events could be observed to cosmological distances (Bloom et al., 2011; Zauderer et al., 2011; Berger et al., 2012; De Colle et al., 2012; Zauderer et al., 2013). Because of their high apparent luminosity these high-energy transients, especially those arising from rare but luminous WD tidal disruptions, could dominate the observable population of tidal disruption flaring events arising from IMBHs (Krolik and Piran, 2011; MacLeod et al., 2014, 2016a). With a population of these transients we might eventually hope to understand the cosmological demographics and temporal evolution of IMBHs in clusters. But given their extreme rarity, these events do little to probe any potential local-group population of IMBHs in GCs.

Gravitational wave inspirals do offer a promising new probe of the presence or absence of IMBHs in nearby clusters (e.g. Miller and Hamilton, 2002; Gültekin et al., 2004; Hopman and Portegies Zwart, 2005; Amaro-Seoane et al., 2007; Ivanov and Papaloizou, 2007; Sesana et al., 2008; Mandel et al., 2008; Mandel and Gair, 2009; Konstantinidis et al., 2013). With laser interferometer experiments like advanced LIGO entering a phase of enhanced sensitivity, the possible detection of gravitational radiation from a merging binary appears promising. How might gravitational wave inspirals of compact objects into cluster IMBHs compare? Because of their higher masses, IMRI events enter the LIGO band only near the end of the inspiral (Miller and Hamilton, 2002). For example, the gravitational wave frequency at the innermost stable circular orbit of a 100\(M_\odot\) IMBH is \(\sim 40\) Hz, with an amplitude that might allow its observation to \(z \sim 0.1\) (Mandel et al., 2008). Similarly, Konstantinidis et al. (2013) show that a
LISA-like space-based detector could capture lower-frequency emission from IMRIs at times several years prior merger out to $z \sim 0.7$.

The event rate of IMRIs compared to standard binary mergers is much more uncertain. In our simulations, we find similar IMRI rates per GC, $\sim 1$ Gyr$^{-1}$, as estimated previously by Mandel et al. (2008). For context, we can assume a galaxy density of $10^7$ Gpc$^{-3}$ and $10^2$ globular clusters per galaxy (Sijacki et al., 2015; Brodie and Strader, 2006). If we conservatively take the inspiral rate from our low-mass clusters as representative, then an inspiral rate of order $1$ yr$^{-1}$ Gpc$^{-3}$ is expected if every cluster hosts a IMBH. This is a factor of $\sim 100$ lower than the double NS inspiral rate estimated recently (e.g. Dominik et al., 2013, 2015), but the large chirp masses of these IMBH binaries make their inspirals observable in the larger volumes mentioned above that might imply similar detection rates of both classes of source (Miller and Hamilton, 2002; Mandel et al., 2008) These constraints are compelling enough to offer hope that the upcoming era of gravitational wave detections will either offer guiding detections or constraining non-detections to pin down the IMBH occupation fraction in low-redshift clusters.

6.6.4 Extensions and Future Work

We have explored the question of close companion objects to IMBHs in dense cluster environments using direct $N$-body numerical simulations in this paper. This chosen approach carries the advantage of direct integration of each particle’s equation of motion, ensuring accurate dynamics. However, it necessitates some approximations due to the computational expense of each individual simulation. In particular, in our analysis we consider clusters with initial mass of $\sim 5 \times 10^4 M_\odot$ or $10^5 M_\odot$, by the time these clusters evolve, they are less massive (and thus less dense) than the the typical Milky Way GC. We similarly include IMBHs of relatively low initial mass, 75 or 150$M_\odot$. More massive clusters might harbor more-massive still IMBHs, changing the IMBH to
star mass ratio. While these constraints are shared by every direct \( N \)-body simulation method, they necessitate a consideration of how the stellar dynamics will scale and extrapolate to more massive systems found in Nature. We have qualitatively explored the response and dependence of our results to dependencies in cluster mass and density and IMBH mass in Section 6.5. To extend our simulated dynamics into the regimes that found in more massive IMBH environments, an \( N \)-body algorithm particularly well suited to study more extreme mass ratios, like that recently published by Karl et al. (2015), may offer a promising numerical approach.

We have included no primordial binaries in the simulations presented in this work. This simplification is not realistic; binary fractions are thought to be of order a few percent in typical GCs (Heggie and Hut, 2003). In future work, we intend to include a realistic binary fraction and study the role these primordial binaries may play in interacting with the IMBH and its close companions (Pfalz, 2005). Before addressing this question with numerical simulations, we can make a few analytic estimates of their potential importance. In a cluster environment, only hard binaries survive the interactive cluster core environment (Heggie, 1975). To order of magnitude, these binaries have separation \( GM \text{bin}/\sigma^2 \). If such a binary passes close to the IMBH and is split by the Hills (1988) mechanism, the one star can remain bound to the IMBH while the other is ejected. The bound star has a minimum typical semi-major axis of

\[
a_{\text{min}} \sim \left( \frac{M_{\text{bh}}}{M_{\text{bin}}} \right)^{2/3} a_{\text{bin}},
\]

plugging in a few typical numbers for our simulation clusters, this might indicate \( a_{\text{min}} \sim 3 \times 10^{15} \) cm, for marginally-hard source binaries. This is near the peak of the semi-major axis distribution of IMBHs companions in our case A simulations (as seen, for example, in Figure 6.2). Thus, in some cases, these split binaries would leave behind stars with similar semi-major axis to the most-bound IMBH companion. Additional
strong interactions between the newly-bound star and the companion would ensue with the possibility of an exchange of hierarchy.

Trenti et al. (2007a) run direct N-body simulations of star clusters with primordial binaries and a central IMBH under idealized conditions ($N = 16384$ equal-mass particles) and find that the IMBH presence increases the binary disruption rate in qualitative agreement with the theoretical predictions by Pfahl (2005). However, the large majority of the binaries are split by interactions with other stars in the dense central stellar cusp surrounding the IMBH, rather than by the IMBH tidal field (Trenti et al., 2007a, Figure 6). This hints that a majority of binaries in relatively low angular momentum orbits may be split before reaching the IMBH, and that those that survive as bound pairs and experience a close encounter with the IMBH are likely to be preferentially harder binaries. The earlier idealized experiments of IMBH-binary interactions also highlight the importance of resolving the dynamics of the sphere of influence exactly through direct N-body simulations. We plan to explore these effects more fully with the aid of future simulations.

6.7 Summary & Conclusion

IMBHs residing in clusters acquire close companions through strong dynamical interactions with their surroundings. We use full N-body calculations including stellar evolution to present a systematic study of the demographics, dynamics, and observable properties of these IMBH companions. This work confirms suggestions that IMBHs in dense clusters should have close companions for the majority of their evolution (Blecha et al., 2006; Mapelli et al., 2013c; Mapelli and Zampieri, 2014). We find that $\sim 90\%$ of the time, companion objects have semi major axes less than a third that of the next-most bound object, $a_0 < 1/3a_1$. The hierarchy can be substantially more extreme, with a few percent of companion configurations having $a_0 \sim 10^{-3}a_1$ (Figure 6.2). The most-
bound object demographics are broad, including a wide variety of stars and compact objects.

We show that most-bound partners to the IMBH go through cycles of orbital hardening followed by destruction, illustrated in Figure 6.1. The typically-observed pairings are long-lived, with residence times of $10^6 - 10^8$ yr, shown in Figure 6.3. These pairings are terminated through exchange or close interaction with the IMBH (gravitational wave inspiral or tidal disruption). We diagram the possible channels through which a partnership can be terminated in Figure 6.5, and show the destructive close encounters with the IMBH on the cluster evolution timeseries in Figure 6.6.

In Section 6.5, we varied several key cluster parameters to consider the sensitivity of IMBH companions to cluster and IMBH parameters. We considered clusters with a higher stellar remnant retention fraction, smaller IMBH mass, and larger particle number as compared to our fiducial parameter set. These comparisons show varying compositions of companion stellar type demographics (Figure 6.10). In particular, the retention of more NSs and BHs leads to a substantially larger fraction of BH companions as these objects mass-segregate to interaction with the IMBH (e.g. Leigh et al., 2014). These demographic differences propagate to minor changes in typical companion orbital semi-major axis (or period) as shown in Figure 6.9, where we break up the semi-major axis distribution of companions by stellar type. We show that companion hierarchy (as defined by $a_0/a_1$) depends on IMBH mass. Lower-mass IMBHs show companions that are more highly segregated from the remaining stellar system than do more massive IMBHs. This suggests that the capture of hierarchically segregated companions is a unique property of the IMBH mass range.

The segregation of close companions to IMBHs, as a property unique to BHs in the intermediate mass range, has dramatic implications for the production of repeated flaring episodes in tidal interactions between stars and IMBHs, as seen in the remarkable object HLX-1 (Farrell et al., 2009). A combination of stellar dynamical segregation,
secular perturbation from less bound stars, and tidal interaction with the IMBH (e.g. Hopman et al., 2004; Baumgardt et al., 2006; Blecha et al., 2006) could place a star in an orbit similar to the 1 yr period observed for HLX-1. Continued interactions with the cluster background could then account for the perturbation of this orbit into a disruptive configuration.

In closing, we note that we may be able to leverage detailed knowledge of IMBH companions to reveal or constrain the still-uncertain presence of IMBHs in dense clusters including GCs, young clusters, and compact galactic nuclear clusters. IMBHs themselves are notoriously tricky to uncover, but our analysis suggests that they may reveal themselves several transient channels. Gravitational waves generated by IMRI events should carry large enough amplitudes to be detectable to moderate redshifts. Our results indicate that these events occur, even in the modeled low-mass cluster, at rates similar to analytic predictions (Mandel et al., 2008), offering substantial promise for their detection by either ground or space based laser interferometers. Electromagnetic emission might arise through steady-state accretion from companion winds. Our calculations, shown in Figure 6.12, suggest that this may exceed the IMBHs Bondi accretion rate from the cluster medium only a small fraction, $\sim 5\%$, of the time, but it nonetheless suggests that a lower limit accretion rate exists for IMBHs in dense clusters. Finally, we discuss extremely luminous transient accretion occurring in rare episodes of tidal disruption of stars by the IMBH. Especially if the IMBH launches jets in these events, these transients could trace the presence of IMBHs in clusters to cosmological distances.
Chapter 7

Asymmetric Accretion Flows within a Common Envelope

7.1 Chapter Abstract

This paper examines flows in the immediate vicinity of stars and compact objects dynamically inspiralling within a common envelope (CE). Flow in the vicinity of the embedded object is gravitationally focused leading to drag and potential to gas accretion. This process has been studied numerically and analytically in the context of Hoyle-Lyttleton accretion (HLA). Yet, within a CE, accretion structures may span a large fraction of the envelope radius, and in so doing sweep across a substantial radial gradient of density. We quantify these gradients using detailed stellar evolution models for a range of CE encounters. We provide estimates of typical scales in CE encounters that involve main sequence stars, white dwarfs, neutron stars, and black holes with giant-branch companions of a wide range of masses. We apply these typical scales to hydrodynamic simulations of 3D HLA with an upstream density gradient. This density gradient breaks the symmetry that defines HLA flow, and imposes an angular momentum barrier to accretion. Material that is focused into the vicinity of
the embedded object thus may not be able to accrete. As a result, accretion rates drop dramatically, by 1-2 orders of magnitude, while drag rates are only mildly affected. We provide fitting formulae to the numerically-derived rates of drag and accretion as a function of the density gradient. The reduced ratio of accretion to drag suggests that objects that can efficiently gain mass during CE evolution, such as black holes and neutron stars, may grow less than implied by the HLA formalism.

### 7.2 Introduction

As stars evolve off of the main sequence their radius grows dramatically. This expansion has major consequences for stars in close binaries. A common envelope (CE) phase occurs when one star grows to the point that it engulfs its more compact companion within a shared envelope (Paczynski, 1976). This process is not rare (Kochanek et al., 2014); the majority of stars exist in binary or multiple systems (e.g. Duchêne and Kraus, 2013), and interactions or mergers should mark the evolution of ≳ 30% of massive stars (Sana et al., 2012; de Mink et al., 2014). During the CE phase, orbital energy and angular momentum are shared with the surrounding gaseous envelope tightening the orbit of the embedded binary cores (Paczynski, 1976; Iben and Livio, 1993; Taam and Sandquist, 2000; Ivanova et al., 2013b). Whether the surrounding envelope is ejected, and the binary survives the CE phase, depends on the efficiency with which energy (Webbink, 1984; Iben and Livio, 1993; Han et al., 1995; Tauris and Dewi, 2001) and angular momentum (Nelemans et al., 2000; van der Sluys et al., 2006; Toonen and Nelemans, 2013) can be harnessed to expel envelope material.

CE episodes, and their outcomes, are therefore critical in shaping populations of close binaries (e.g. Belczynski et al., 2002; Stairs, 2004; Kalogera et al., 2007; Toonen and Nelemans, 2013) as well as their merger products (Sana et al., 2012; de Mink et al., 2013, 2014). Substantial progress has been made by constraining the efficiencies of CE
ejection compared to the change in orbital energy via the parameter $\alpha_{CE}$ (Webbink, 1984; Iben and Livio, 1993), or the change in orbital angular momentum with the parameter $\gamma_{CE}$ (Nelemans et al., 2000). These binary population synthesis methods are powerful but also very sensitive to the details of stellar envelope structure at the onset of the CE event (e.g. Han et al., 1995; Tauris and Dewi, 2001; De Marco et al., 2011; Soker, 2013), as well as the specific post-CE binary system under consideration; there do not appear to be universal answers with respect to the outcome of CE events (Ivanova et al., 2013b).

Theoretical efforts to constrain the physics and properties of CE dispersal have recently been extended to include three-dimensional hydrodynamic simulations of the early inspiral process (Livio and Soker, 1988; Terman et al., 1994; Taam et al., 1994; Terman et al., 1995; Terman and Taam, 1996; Sandquist et al., 1998; Ricker and Taam, 2008; Passy et al., 2012b; Ricker and Taam, 2012). By necessity, the embedded object in these simulations is described only by its gravitational influence on the gas, and the region that would represent the companion is, at best, resolved by a few numerical cells in Ricker and Taam (2012) and Passy et al. (2012b). Determining flow properties in the immediate vicinity of an embedded object remains an unmet challenge because of the huge range of spatial and temporal scales described by a CE episode (Taam and Ricker, 2010).

The traditional approach to understanding flows around an embedded object during CE evolution has focused on analytic work by Hoyle and Lyttleton (1939), Bondi and Hoyle (1944), and Bondi (1952). These studies developed an analytic understanding of the nature of accretion onto a gravitational source moving supersonically through its surrounding medium (see Edgar, 2004, for a recent review). Semi-analytical work has used these prescriptions to estimate the inspiral and accretion experienced by an object embedded in an envelope. In some cases, the co-evolution of the envelope and accretor are treated in 1D (e.g. Taam et al., 1978; Meyer and Meyer-Hofmeister, 1979; Taam
et al., 1978; Shankar et al., 1991; Kato and Hachisu, 1991; Siess and Livio, 1999a,b; Passy et al., 2012c), while in others, the initial envelope properties are used as motivation for the relevant parameters for the embedded object (e.g. Livio and Soker, 1988; Chevalier, 1993; Iben and Livio, 1993; Bethe and Brown, 1998; Metzger et al., 2012b). An issue, of course, is that stellar envelopes are not uniform density media, an effect that can be seen particularly clearly in recent 3D simulations by Passy et al. (2012b) and Ricker and Taam (2008, 2012).

In this paper, we adopt an idealized approach to perform numerical simulations that explore the behavior of flows in the immediate vicinity of the embedded object. In particular, accretion structures, as we discuss in Section 7.3, typically span a large portion of the stellar radius (e.g. Iben and Livio, 1993). They impinge on a large portion of the envelope, and sweep across a huge radial gradient of envelope density (Ricker and Taam, 2012). We extend traditional simulations of three-dimensional Hoyle-Lyttleton accretion (HLA) to consider the effects of these substantial density gradients on flow patterns, drag force on the object’s motion, and the rate of mass and angular momentum accretion by the embedded object.

This work builds on analytic considerations (Dodd and McCrea, 1952; Illarionov and Sunyaev, 1975; Shapiro and Lightman, 1976; Davies and Pringle, 1980) and simulations of non-axisymmetric flow conditions in HLA (Soker and Livio, 1984; Soker et al., 1986; Livio et al., 1986a; Fryxell and Taam, 1988; Ruffert, 1999). However, since much of this work was motivated by inhomogeneities in wind-capture binaries, typical density gradients tend to be much milder than those experienced by an embedded object within a CE\(^1\). We will compare our results in detail to conceptually similar but 2D simulations by Armitage and Livio (2000) to discuss the significant effect of 3D flow geometries. Of course, this idealized approach carries many compromises in that local

\(^{1}\)For example, 3D calculations involving mild density gradients in HLA are currently being performed by Rayner (2014).
simulations do not capture the full geometry, gravitational potential, eccentric orbital motions (Passy et al., 2012b), microphysics of energy sources or sinks (Iben and Livio, 1993; Ivanova et al., 2013b), or disturbed background flow present in true CE events (Ricker and Taam, 2012). Even so, we will argue that the effects of a density gradient on HLA-like flows are so dramatic that a detailed understanding of these idealized cases carries important implications for CE evolution.

The remainder of this paper is organized as follows. In Section 7.3, we parameterize typical flow characteristics around an object embedded in a CE. Typical properties range from one to several density scale heights per characteristic accretion radius. In Section 7.4 and 7.5 we describe the methods and results, respectively, of 3D hydrodynamic simulations of planar flow with an imposed density gradient past a gravitating object. We derive fitting formulae for the drag force and for rates of accretion of mass and angular momentum as a function of density gradient in these simulations. In Section 7.6, we discuss the implications of our findings for CE events, and in Section 7.7 we conclude.

7.3 Stellar Properties, Typical Scales and Gradients

At the onset of a CE episode, there is a dynamical phase following the loss of corotation between the envelope and the binary in which one object becomes embedded within the envelope of its companion and begins to spiral inward (Podsiadlowski, 2001). In our simplified analysis, we will suppose that the properties of the companion’s envelope are unperturbed by the presence of the embedded object. We note that this assumption is most justified when the embedded object is a small fraction of the total mass (Iben and Livio, 1993), but we defer the reader to Section 7.6.3 for a discussion of the potential implications of a perturbed envelope. In this section, we analyze the characteristic dimensionless scales that parameterize the flows that arise during this
phase of dynamical flow.

### 7.3.1 Characteristic scales

The first characteristic scale of the upstream flow is the Mach number,

$$
M_\infty = \frac{v_\infty}{c_{s,\infty}},
$$

(7.1)

where $c_{s,\infty}$ is the sound speed and $v_\infty$ is the relative speed of motion through the gas. In the context of a stellar envelope, the relevant sound speed is that evaluated at the radius of the embedded object within the star. The speed of the incoming flow is approximately the circular velocity of the orbiting stellar cores at that radius,

$$
v_\infty = v_{\text{circ}} = \frac{GM_*}{a} + \frac{M}{a}
$$

where $m_*(a)$ is the enclosed stellar mass at a given separation $a$, and is some fraction of the total stellar mass $M_*$. To be embedded, $a$ must be less than the envelope radius, $R_*$. As we will show in the next subsection, typical Mach numbers are mildly supersonic, $M_\infty \sim 1.5 - 5$.

The theory of gravitational accretion onto a supersonically moving point mass was first elaborated by Hoyle and Lyttleton (1939). It is useful here to frame the accretion flows that are set up within a stellar envelope in the context of those realized in HLA. The accretion radius, $R_a$, is a gravitational cross section for material in supersonic motion to be focused toward the accretor,

$$
R_a = \frac{2GM}{v_\infty^2}.
$$

(7.2)

This scale defines the material whose kinetic energy is sufficiently small that it will be focused to a line of symmetry downstream of the massive object and accrete (Blondin and Raymer, 2012). The accretion radius defines the material with which the embedded object can be expected to interact.
The accretion radius and the properties of the upstream flow suggest a characteristic accretion rate onto the embedded object. The HLA accretion rate is defined by the flux of material with impact parameter less than the accretion radius,

\[ \dot{M}_{\text{HL}} = \pi R_a^2 \rho_\infty v_\infty, \quad (7.3) \]

where \( \rho_\infty \) is the density of the upstream flow.

Within a stellar envelope, the radial density gradient of the star ensures that the accretor experiences an upstream density gradient of incoming material. The key difference that this work will emphasize is the role of this gradient in shaping flow structures around the embedded object. We will characterize these density gradients in terms of the local density scale height, \( H_\rho \). The density scale height is defined as

\[ H_\rho = -\rho \frac{dr}{d\rho}, \quad (7.4) \]

and is evaluated at the radius of the embedded object within the stellar envelope. It will be useful to define a parameter that describes the number of density scale heights swept by the accretion radius,

\[ \epsilon_\rho \equiv \frac{R_a}{H_\rho}. \quad (7.5) \]

In this context, traditional HLA flow is characterized by upstream homogeneity, \( \epsilon_\rho = 0 \). The density profile may then be expanded around a point, \( r_0 \), as

\[ \rho \approx \rho(r_0) \exp \left( \epsilon_\rho \frac{r_0 - r}{R_a} \right). \quad (7.6) \]

An example of this expansion is shown for the radial run of density in a stellar model in Figure 7.1. We show exponential approximations, equation (7.6), of the local density profile that extend \( \pm R_a \) from embedded radii of 0.2, 0.5, and 0.8\( R_* \). These expansions
show that an object embedded at those radii encounters significant density inhomogeneity, which can be expected to play a key role in shaping the flow.

7.3.2 MESA Simulations

We perform a set of stellar evolution calculations to explore the time evolution and range of typical values for the characteristic dimensionless scales outlined in the previous subsection. Our simulations use the MESA stellar evolution code, version 5527 (Paxton et al., 2011, 2013). We evolve stars of 1-16$M_\odot$ from the zero-age main sequence to their giant-branch expansion, during which a CE phase may be initiated.

7.3.2.1 Time Evolution

In Figure 7.2, we show the time-evolution of a 1$M_\odot$ star ascending the red giant branch (RGB). Under the simplifying assumption that the envelope structure is not immediately perturbed by the presence of the embedded object, we plot the upstream Mach number, the accretion radius as a fraction of the stellar radius, and the number of density scale heights per accretion radius, $\epsilon_\rho$. These panels display the conditions assuming the object is embedded at a separation $a$ from the center of its companion. The left panels show separation in units of the solar radius, the right panels normalize the radius to a fraction of the stellar envelope radius $R_\ast$.

The immediate conclusion we can draw from Figure 7.2 is that although the envelope expands dramatically during the course of the star’s giant branch evolution, when normalized to a fraction of the radius, $a/R_\ast$, the basic upstream conditions for the flow are remarkably consistent. In Figure 7.2, we color individual models based on their radius, which maps to the binary separation at the onset of CE. Typical Mach numbers range from $\mathcal{M}_\infty \sim 2$ in the deep interior to $\mathcal{M}_\infty \gtrsim 5$ near the stellar limb. Although various local features develop due to opacity transitions and equation of state effects, this qualitative trend holds across the entire giant branch expansion. The accretion
Figure 7.1: Density gradients encountered by objects embedded within the envelope of a giant star (as drawn schematically in the inset image). In the inset cartoon, an embedded star orbits within its companions envelope, shocking the surrounding material. In the main panel, we show the radial density profile of a $16M\odot$ red supergiant (RSG), and assume the embedded object is a $1.4M\odot$ neutron star (NS). The overplotted, dashed, lines show local exponential fits to the density profile following equation (7.6). The dashed lines extend for $\pm R_a$ as evaluated at the position of the embedded object (shown with points). We show examples of objects embedded at $0.2R_*$, $0.5R_*$ and $0.8R_*$. Because it is a significant fraction of the envelope radius, $R_a$ subtends a substantial density gradient implying that density asymmetries play an important role in shaping flow morphologies in CE evolution.
Figure 7.2: Time evolution of characteristic flow parameters within the envelope of a $1M_\odot$ star as it evolves up the RGB, with the embedded object mass assumed to be $0.7M_\odot$. The left series of figures show orbital separation in units of the solar radius, showing the expansion of the star as it ascends to the tip of the RGB and its maximum radius. The right-hand panels normalize the radius to the current stellar radius. From top to bottom, we plot Mach number, accretion radius as a fraction of stellar radius, and density gradient. Flow parameters vary with normalized radius, but are remarkably consistent over the RGB evolution. This implies that regardless of where on the RGB the onset of CE occurs, we can expect relatively consistent hydrodynamic conditions.
radius is a relatively constant fraction of the separation such that $R_a/R_*$ is roughly proportional to $a/R_*$, as shown in the center-right panel of Figure 7.2. The density gradient, as parameterized by $\epsilon_\rho$, is the parameter that exhibits the most temporal variation. In particular, near the stellar limb, the density gradient is extremely steep, $\epsilon_\rho \gtrsim 10$, in the early RGB phases, while maintaining more moderate values of $\epsilon_\rho \lesssim 8$ near the tip of the RGB. As an object spirals to smaller $a/R_*$, the density gradient becomes shallower.

That these dimensionless scales remain relatively constant throughout the evolution implies that the global flow morphology in a wide variety of encounters can be sufficiently well described by a relatively small set of typical numbers. In the next subsection, we evaluate the conditions found in a representative set of CE encounters.

### 7.3.2.2 Typical Scales in a Variety of Encounters

In this subsection we examine a small variety of representative CE events including those involving embedded planets, main sequence stars, and compact objects. In so doing, we hope to capture some of the diversity of CE events and explore how it manifests itself in the range of typical flow parameters.

We provide estimates for the properties of the following systems:

- **Jupiter + 1$M_\odot$ RGB star**: With increasing evidence that many low-mass stars host planetary systems – including those with giant planets close to their host stars – it is interesting to consider the final fate of these systems (e.g. Siess and Livio, 1999a,b; Sandquist et al., 1998; Sandquist et al., 2002; Metzger et al., 2012b; Passy et al., 2012c). A CE with extreme mass ratio may be initiated as the host star evolves up the giant branch. In these cases, the geometric cross section is very similar to the accretion radius, and the gravitational focusing of material from the upstream flow (as discussed in the remainder of this paper) is not a strong effect on the flow properties. Sandquist et al. (2002) has shown that more relevant effects
include entrainment of planetary material into the star, and possible consequences for the pollution of the host star.

- $0.7M_\odot$ white dwarf (WD) + $1M_\odot$ red giant branch (RGB) star: This low-mass CE scenario might precede the formation of a double-WD binary, where the CE is initiated as the lower-mass star evolves off of the main sequence. Nelemans et al. (2000) and van der Sluys et al. (2006) model the formation of these double WD systems, with typical progenitor masses in the 1-3$M_\odot$ range. Hall et al. (2013) have recently studied the structure and evolution of the low-mass stripped giant that would arise from such a system, and how this evolution would imprint itself during an ensuing planetary nebula phase.

- Sun + $2M_\odot$ asymptotic giant branch (AGB) star: In this case, a main sequence star of solar mass and radius is interacting with a CE donated by a slightly more massive AGB star. This scenario would lead to the formation of a close WD – Main sequence binary. Broadly-defined, this main sequence with giant branch scenario is expected to be relatively common and was invoked by Paczynski (1976) in the original description of the CE scenario. A cataclysmic variable state is among the possible outcomes from this channel if the post-CE binary is sufficiently close that it can be drawn into resumed mass transfer (Paczynski, 1976). Variants of this scenario have been studied in the Double Core Evolution series of papers (Taam and Bodenheimer, 1989; Taam and Bodenheimer, 1991; Yorke et al., 1995; Terman and Taam, 1996). A population synthesis of post-CE WD – main sequence binaries has been recently undertaken by Toonen and Nelemans (2013).

- Neutron star (NS) + $2, 8, 16 M_\odot$ giants: Following these scenarios in which a NS becomes embedded within the envelope of its companion, possible outcomes include close binaries consisting of a NS and either a WD (in the lower mass companion cases) or He-star (in the higher companion-mass cases). The He-star
could then undergo a core collapse supernova which might leave behind a double
NS or NS-BH binary (Postnov and Yungelson, 2014). The hydrodynamics of this
scenario have been considered by (Taam et al., 1978; Bodenheimer and Taam,
1984). Accretion onto the NS during this phase is relatively efficient as neutrinos
provide a cooling channel (Houck and Chevalier, 1991; Chevalier, 1993, 1996; Fryer
et al., 1996), leading to the suggestion that the NS might grow to collapse to a BH
in some, but not all, cases (Chevalier, 1993; Brown, 1995; Bethe and Brown, 1998;
Fryer and Woosley, 1998; Belczynski et al., 2002; Kalogera et al., 2007; Chevalier,
2012).

- **BH + 16 M☉ red supergiant (RSG):** In this scenario, a stellar mass BH interacts
  with its massive companion. This scenario might be realized under several circum-
  stances. If, as suggested above, accretion-induced collapse from NS to BH ever
  occurs during CE evolution, that would leave a BH interacting with a massive-
  star envelope mid CE event. Perhaps more simply, if a massive star evolves and
  produces a stellar mass BH remnant, the post supernova orbit may lead to CE
  evolution (Postnov and Yungelson, 2014).

Figure 7.3 is analogous to the upper and lower right-hand panels of Figure 7.2,
where we plot typical Mach numbers and gradients as a function of normalized radius.
Rather than showing a time-evolution, we plot representative models for each encounter
combination in 7 different representative CE pairings of objects. Mach numbers vary
by at most a factor of a few at a given radius. This is not entirely unexpected because
the highest Mach numbers are realized when the two objects are close to equal mass,
while the lower limit occurs when the giant dominates the total mass. The gradients
differ somewhat more significantly. In the extreme limit shown of a Jupiter-like planet
embedded in a 1M☉ giant, the gradient across the accretion radius is nearly zero. In
general, gradients are steepest near the limb of the envelope because energy diffusion
dictates that the density scale height becomes very steep at the photosphere. The most-nearly equal mass cases exhibit the steepest gradients, on average, in large part because the accretion radius tends to be a larger fraction of the stellar radius and thus encompasses a broader range of densities.

The properties of these encounters are further summarized in Figure 7.4. In the upper panel we plot the density gradient as parameterized by $\epsilon_\rho$ against the radius of the embedded object compared to the accretion radius. The ratio $R_{\text{obj}}/R_a$ effectively compares the geometric ($\sim \pi R_{\text{obj}}^2$) to the gravitational ($\sim \pi R_a^2$) cross section. In the case of an embedded Jupiter-like planet, these scales are similar. However, for embedded stars and compact objects the accretion radius is typically $10^2$ to $10^7$ times larger than the object radius. The symbols plotted show the values evaluated at $a = 0.2, 0.5$, and $0.8R_\star$. The lower panel of Figure 7.4 shows the other two characteristic parameters, the Mach number, $M_\infty$, and the size of of the accretion radius as a fraction of the stellar radius, $R_a/R_\star$. The mass ratio of the embedded object to its companion is significant. In cases where the embedded object’s mass is small compared to the envelope mass, the accretion radius is a small fraction of the total radius. In this case, fewer density scale heights are subtended by the accretion radius, leading to weaker density gradients, $\epsilon_\rho$. However, when the objects are relatively similar in mass, the accretion radius is of similar order of magnitude to the envelope radius, and it can therefore sweep across many density scale heights. These results are summarized in Table 7.1, in which we give the numerical values evaluated at a single separation, $a = 0.5R_\star$.

### 7.4 Methods

Our numerical approach is to perform idealized simulations based on the phase space of physically motivated flow parameters derived in the previous section. Our simulations follow closely in the tradition of simulations of HLA. We set up our experiment
Figure 7.3: Characteristic Mach numbers and density gradients for a variety of CE object pairings. Mach numbers are consistently highest and density gradients are steepest near the limb of stars, where energy diffusion dictates that the density scale height becomes small. Typical density gradients span a wider range than Mach numbers do, but for separations within the inner 50% of $R_*$, $0.5 \lesssim \epsilon_\rho \lesssim 4$ are representative values for the embedded star and compact object cases.
Figure 7.4: Typical flow parameters plotted for the set of CE scenarios shown in Figure 7.3. Points evaluate the conditions in a given model at \(0.2R_*\), \(0.5R_*\) and \(0.8R_*\) (with circles, squares, and diamonds, respectively). Colors are the same from upper to lower panels. The upper panel shows that embedded stars and compact objects are typically many times smaller than their gravitational capture radius. Only for an embedded Jupiter-like planet are the gravitational and geometric cross-sections similar. The more massive objects are subject to significant density gradients, parameterized by \(\epsilon_\rho\), while the planet is the only model with a mild gradient. The lower panel compares the local accretion radius to the stellar radius. While this ratio depends on the structural properties of the star, it is primarily determined by the mass ratio of the embedded object to its companion. Hydrostatic balance of the envelope structure ensures that typical Mach numbers are consistent, and mildly supersonic. These results are tabulated for \(a = 0.5R_*\) in Table 7.1.
Table 7.1: Typical Encounter Properties from MESA Simulations (evaluated at $0.5R_*$)

<table>
<thead>
<tr>
<th>Objects</th>
<th>Radius $R_a/R_*$</th>
<th>Accretion Rate $\dot{M}$</th>
<th>$R_{a*}/R_*$</th>
<th>$R_{obj}/R_a$</th>
<th>$\epsilon_\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter + $1M_\odot$ RG</td>
<td>103</td>
<td>1.84</td>
<td>1.22(-3)</td>
<td>0.82</td>
<td>8.64(-3)</td>
</tr>
<tr>
<td>0.7$M_\odot$ WD + $1M_\odot$ RG</td>
<td>103</td>
<td>2.53</td>
<td>0.47</td>
<td>2.50(-4)</td>
<td>3.34</td>
</tr>
<tr>
<td>Sun + $2M_\odot$ AGB</td>
<td>226</td>
<td>2.26</td>
<td>0.41</td>
<td>1.09(-2)</td>
<td>2.54</td>
</tr>
<tr>
<td>1.4$M_\odot$ NS + 2$M_\odot$ AGB</td>
<td>226</td>
<td>2.44</td>
<td>0.49</td>
<td>1.69(-6)</td>
<td>3.06</td>
</tr>
<tr>
<td>1.4$M_\odot$ NS + 8$M_\odot$ AGB</td>
<td>562</td>
<td>1.77</td>
<td>0.23</td>
<td>1.44(-6)</td>
<td>1.09</td>
</tr>
<tr>
<td>1.4$M_\odot$ NS + 16$M_\odot$ RSG</td>
<td>807</td>
<td>1.69</td>
<td>0.12</td>
<td>1.99(-6)</td>
<td>0.56</td>
</tr>
<tr>
<td>4$M_\odot$ BH + 16$M_\odot$ RSG</td>
<td>807</td>
<td>1.86</td>
<td>0.27</td>
<td>7.69(-8)</td>
<td>1.33</td>
</tr>
</tbody>
</table>

under many of the same premises to examine the effect of a substantial exponential density gradient in the upstream flow. In the following subsections, we first mention the most relevant previous numerical work on which our simulation builds, then discuss our numerical setup, and finally the simulation parameters chosen in our suite of runs.

### 7.4.1 Previous Numerical Studies of HLA

This work extends a long tradition of numerical study of HLA that began with the work of Hunt (1971). The reader is directed to Edgar (2004) a recent review, and to Foglizzo et al. (2005) for a detailed comparison of published simulations that attempts to synthesize results with respect to flow stability as it depends on Mach number, accretor size, geometry, and equation of state. Rather than restating their work here, we review only a few of the most relevant and recent studies.

A benchmark series of 3D simulations with high spatial resolution and a variety of flow parameters was published beginning with Ruffert (1994a). These simulations adopted homogeneous upstream boundary conditions. They employ nested grids to resolve a small region surrounding a point mass, from which a sphere is excised and a vacuum boundary condition is applied. Within this framework, flow morphologies, stability conditions, and accretion rates for different flow Mach numbers and equations
of state were studied. Ruffert and Arnett (1994) examined flows with adiabatic $\gamma = 5/3$ and Mach number $M_\infty = 3$. They found that accretion rates in steady state were reasonably approximated by the HLA formula, but that after the flow developed in the first few crossing times, some loss of axisymmetry occurred. This imparted time variability to the accretion rate, but the dramatic flip flop instabilities seen in lower-dimensionality simulations were, strikingly, not observed in 3D (Foglizzo et al., 2005).

Subsequent work (Ruffert, 1994b, 1995, 1996) confirmed the more-stable configurations of 3D flows, and found that more violently unstable scenarios occurred with smaller accretion boundaries, higher Mach numbers, and more compressible equations of state.

These simulations were recently revisited in 2D (Blondin and Pope, 2009; Blondin, 2013) and 3D (Blondin and Raymer, 2012; Naiman et al., 2011; Toropina et al., 2011; Lee et al., 2014). Again, major differences in flow morphology are found between 2D and 3D geometry. In particular, rotationally supported flows develop in 2D planar geometries while these are not observed in 3D. Blondin and Raymer (2012) study two different accretor sizes with $\gamma = 5/3$ and $M_\infty = 3$ to find extremely stable accretion with the larger accretor (with radius 5% of the accretion radius), but a breathing mode instability and some variation in accretion rate with an accretor of only 1% of $R_a$. Very little loss of axisymmetry is observed in these simulations, however, with differences as compared to Ruffert and Arnett (1994) potentially attributable to differences in grid meshing and resolution.

Of particular importance to this study is work that has extended the traditional HLA problem to look at background gradients of velocity and density. Livio et al. (1986b), Soker et al. (1986), Livio et al. (1986a) studied accretion from an inhomogeneous medium using a 3D particle in cell hydrodynamics method. Although the resolution around the accretor with this method is limited, they found that the introduction of gradients lead to the accretion of only a small amount of the angular momentum available in the upstream flow. Fryxell and Taam (1988) and Taam and Fryxell (1989)
studied density and velocity gradients, respectively, in 2D planar accretion with a grid-based hydrodynamics method. This approach vastly improved the ability to study the flow just outside the accretor. Fryxell and Taam (1988) adopted \( \gamma = 4/3 \), and found that small gradients of density lead to a flip-flop instability (Livio et al., 1991) of rotation around the accretor as vortices are shed into the wake. With steeper gradients, Fryxell and Taam (1988) displaced structures in the wake and very limited accretion rates. Armitage and Livio (2000) carry out relatively similar simulations except with a hard surface rather than inflow central boundary condition and evaluate the degree of rotational support of material bound to the accretor. Ruffert (1997) and Ruffert (1999) applied a mild upstream gradient which varied by 3% or 20% in either velocity or density was applied to their 3D accretion setup of Ruffert (1994a). In these simulations, the inhomogeneous upstream conditions lead to unsteady rotation of the flow but not to the formation of steady disks surrounding the accretor with \( \gamma = 5/3 \). Some accretion of angular momentum from the post shock region resulted, but a dramatic modification in the average accretion rate was not observed.

7.4.2 Numerical Approach and Simulation Setup

While previous work has examined the effects of mild upstream gradients in velocity and density on the HLA problem, we have shown in Section 7.3 that in many cases the relevant density gradients may be substantially stronger than those studied by Ruffert (1999). Thus we extend this work and perform 3D hydrodynamic simulations of accretion flows using the FLASH code (Fryxell et al., 2000). We solve the fluid equations using FLASH’s directionally split Piecewise Parabolic Method Riemann solver (Colella and Woodward, 1984). We make use of a gamma-law equation of state, and in most cases use \( \gamma = 5/3 \). A 3D cartesian grid is initialized surrounding a point mass that is fixed at the coordinate origin.

These simulations are performed in dimensionless units, where \( R_a = v_\infty = \)
<table>
<thead>
<tr>
<th>Name</th>
<th>$M$</th>
<th>$\epsilon$</th>
<th>$\gamma$</th>
<th>$R_s/R_a$</th>
<th>$\dot{M}/\dot{M}_{HL}$</th>
<th>$F_{d,x}/(\pi R_a^2 \rho_\infty v_\infty^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>0.05</td>
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<tr>
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<td>5/3</td>
<td>0.05</td>
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</tr>
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<td>5/3</td>
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</tr>
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<td>0.55</td>
<td>5/3</td>
<td>0.05</td>
<td>12.8</td>
<td>1.27(-1)</td>
</tr>
<tr>
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<td>2.69(-1)</td>
</tr>
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<td>T</td>
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<td>5/3</td>
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<td>5/3</td>
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<tr>
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<td>5</td>
<td>1.1</td>
<td>0.01</td>
<td>10.2</td>
<td>1.03(-1)</td>
</tr>
</tbody>
</table>

$\rho_\infty = 1$. In these units, the characteristic time is $R_a/v_\infty = 1$, and the characteristic accretion rate $\dot{M}_{HL} = \pi$, equation (7.3). The $-x$ boundary feeds a wind of material into the box and past the point mass. A simulation is then parameterized by the upstream Mach number, $M_\infty$, measured at $y = z = 0$. We also allow for a planar density gradient in the $\hat{y}$ direction, parameterized by $\epsilon_\rho = R_a/H_\rho$. The $\pm y$ and $\pm z$ boundaries are outflow boundaries positioned at $\pm 10 R_a$ (10 in code units). On the downstream, $+x$, boundary (positioned at $+4 R_a$) we apply a diode boundary condition that allows for outflow but not inflow.

We create a spherical absorbing boundary condition “sink” surrounding the
central point mass, with radius $R_s$. The potential of the point mass is smoothed, but only well within this sink, at a radius of $\approx R_s/10$. Thus, this smoothing does not affect the flow outside the excised region. Each time step, the average pressure and density are computed within a shell that extends from $R_s$ to $2R_s$. Then, the pressure and density inside the sink are set to $10^{-3}$ of these surrounding values to create a vacuum that does not impinge on the surrounding flow. This vacuum condition represents accretion with no feedback on the surrounding flow. Material in the vicinity of the sink is allowed to expand into it at the sound speed and thus be absorbed (Ruffert, 1994a).

Before the pressure and density are overwritten within the sink, we integrate the accreted mass and angular momentum. The mass accretion rate is then defined as the accreted mass divided by the time step, $\dot{M} = \delta m/\delta t$; the components of the angular momentum accretion rate are defined similarly. As a consistency check, we performed tests with sink density and pressure pre-factors between $10^{-1}$ and $10^{-5}$ with no visible difference in the accretion of mass or angular momentum. However, a qualitatively different central boundary condition, particularly one that did apply a feedback on the flow, would likely result in different mass and angular momentum accumulation rates. We explore the effects of such a boundary condition in Section 7.6.1.

The initial conditions are of constant velocity in the $\hat{x}$ direction, where the $x$-velocity is $v_x = v_\infty$. The initial density field, $\rho_i$, is a function of the $y$ position as

$$\rho_i = \rho_\infty \exp(\epsilon \rho y), \quad (7.7)$$

applied only within $-2 < y < 2$ to limit the total range of density in the background material of the computational volume. Within this planar gradient of densities, high densities are found at $+y$ coordinates. We turn the point mass on progressively over the first time unit, so that it is fully active after $R_a/v_\infty = 1$.

The simulations employ the PARAMESH library to provide adaptive mesh
refinement to resolve small features around the accretor within the large simulation box (MacNeice et al., 2000). Adaptive refinement is based on the second derivative of pressure. The box is initialized with 7 blocks (of \(8^3\) cells) in the \(x\) direction, and 10 each in the \(y\) and \(z\) directions. We then allow for between 6-9 levels of adaptive refinement of those coarsest blocks. To avoid devoting all of the computational effort to features far from the accretor, we force the maximum level of refinement allowed for a given block to drop in proportion to the radius from the coordinate origin (where the accretor resides). Blocks with size less than \(\alpha r\), where we adopt \(\alpha = 0.3\), are not allowed to be further refined. The first decrement in refinement occurs at \(r \approx 3R_s\), and drops one level further each time the radius doubles (See Couch and O’Connor, 2014, for another astrophysical problem in which this refinement criteria is applied in FLASH).

### 7.4.3 Simulation Parameters

The primary effect we explore in this paper is the inclusion of a significant density gradient to the upstream flow in supersonic accretion. To that end we perform a series of simulations with increasing density gradient \(\epsilon_\rho\). We adopt a Mach number of \(\mathcal{M}_\infty = 2\) for these simulations, and a gas ratio of specific heats, \(\gamma = 5/3\), that is representative of an ideal gas flow in which cooling is ineffective – like that embedded deep within a stellar interior. In one series, simulations A–H, we adopt a sink boundary condition size of \(R_s = 0.05R_a\). In simulations I–P, we reduce the sink size to \(R_s = 0.01R_a\). These \(R_s = 0.05R_a\) simulations are run from \(t = 0\) to \(t = 80R_a/v_\infty\). The \(R_s = 0.01R_a\) are started from checkpoints of the \(R_s = 0.05R_a\) simulations with the same upstream conditions at \(t = 20R_a/v_\infty\) and run for \(10R_a/v_\infty\). We perform a resolution study, simulations Q–S, which all have \(\mathcal{M}_\infty = 2\), \(R_s = 0.05R_a\) to demonstrate the robustness of our derived accretion parameters. We perform some simulations with lower and higher Mach number, \(\mathcal{M}_\infty = 1.1\) (T,U) and \(\mathcal{M}_\infty = 3\) (V,X) to test the sensitivity of flow morphology to upstream Mach number. In simulation X, we adopt \(\gamma = 1.1\) to
represent a flow which is can cool more effectively than the adiabatic conditions.

7.5 Results

The introduction of an upstream density gradient breaks the symmetry that defines classical HLA. In the following subsections we explore the effects of this symmetry-breaking on the morphology, accretion rates, sink-size effects, rotation, and drag realized our hydrodynamic simulations.

7.5.1 Flow Morphology

The introduction of upstream density gradients introduce dramatic changes to the morphology of the flow around objects embedded within a CE. Figure 7.5 displays these changing flow morphologies for upstream gradients of $\epsilon_\rho = 0, 0.3, 1, 3$ and sink sizes of $R_s = 0.05R_a$ and $R_s = 0.01R_a$. We show two slices through the simulation domain, one in the $x-y$ plane, the same plane as the imposed $\hat{y}$ density gradient, and one in the $x-z$ plane, perpendicular to the imposed gradient.

In the zero gradient, HLA-case, the $x-y$ and $x-z$ slices are nearly identical. A high degree of symmetry is preserved in this case of homogeneous upstream conditions. Flow lines converge toward a stagnation region in the wake of the accretor and some material reverses to fall into the sink from this accretion column (e.g. Edgar, 2004). Most of the accretion, therefore, occurs in the downstream hemisphere of the accretor (Blondin and Raymer, 2012).

The symmetry of the $\epsilon_\rho = 0$ case is broken by the introduction of an upstream gradient. Although the sink itself is small with respect to the density scale height, $R_s \ll H_\rho$, the bow shock sweeps through a large density contrast, affecting the flow even at small scales near the accretor. With 0.3 density scale heights per accretion radius, ($\epsilon_\rho = 0.3$, the second panel in Figure 7.5) the flow morphology is distorted and
it presents a tilted bow shock structure to the upstream flow. As the density gradient steepens further, the bow shock continues to rotate to face the flux of densest material. As a result, the bow-shock is nearly reverse-facing by the time the gradient steepens to $\epsilon_\rho = 3$, the right-hand panel of Figure 7.5. Further, the single shock interface of the symmetric case is replaced by multiple nested shocks at different rotation angles with respect to the accretor. In the $\epsilon_\rho = 1, 3$ cases with $R_s = 0.05$, a one-sided trailing shock facing the high density material extends inward to the surface of the accretor. When the sink size is reduced to $R_s = 0.01 R_a$, a low density cavity forms surrounding the accretor and the tail shock does not remain attached.

In these steep gradient cases, material of high and low density are both focused toward the wake of the accretor from positive and negative $y$ coordinates, respectively. The momenta of these fluid parcels do not cancel, however, as occurs in the case of homologous upstream conditions. Thus, the density gradient also introduces a net angular momentum swept up by the bow shock, and material in the post shock region carries net rotation around the accretor. This effect is observed in the flow lines overplotted on Figure 7.5. A downstream accretion column cannot exist in this scenario, because flow in the wake region is moving tangentially with respect to the accretor.

In the absence of velocity cancelation, flow lines show that much of the dense material is never focused into the accretor. This result was anticipated by Dodd and McCrea (1952) who, in analytically calculating the capture cross section of the accretor given a linear upstream density gradient, note that captured material need not go on to accrete. Instead, they state that only the material whose angular momentum can be redistributed could be expected to fall into an accretor. Traces of this process are seen in Figure 7.6. In particular, only material whose tangential velocity is partially canceled in the nested shock structures can fall into the sink.

Figure 7.6 examines the flow Mach numbers for the same set of simulation snapshots as Figure 7.5. An extended subsonic region trails in the wake in the symmetric
Figure 7.5: Comparison of flow morphologies realized with different upstream density gradients. The upper series of panels shows simulations with $R_s = 0.05R_a$ (simulations A, D, F, and H). The lower series has $R_s = 0.01R_a$ (simulations I, L, N, and P). Density is plotted in terms of $\rho_\infty$, the density at $y = 0$, while axis labels are in units of $R_a$. The upstream Mach number is $M_\infty = 2$ in all simulations, and snapshots are shown at a time $30R_a/v_\infty$. As gradients are introduced, increasing asymmetry develops in the imposed plane of rotation ($x$-$y$). Bow shock structures migrate from symmetric to tilted to finally wrapping nearly completely around the accretor into the wake for $\epsilon_\rho = 3$. 
Figure 7.6: Flow Mach number for the same frames shown in Figure 7.5 (simulations A, D, F, and H). Steepening gradients (and smaller sink sizes) lead to increasingly turbulent flows. Boundary layers are of shock heated, and therefore low-$M_\infty$, material divide regions of flow moving in different directions. Angular momentum is redistributed in these shocks with some plumes of material falling inward extending inward to the accretor, while other material is swept outside of $R_a$. In the zero-gradient case, some spurious features representative of the cartesian discretization arise. These artificial features are swept away by continuous fluid motion as soon as any gradient is introduced. Note the axis scale is a factor of two closer here than in Figure 7.5.
HLA case, while a sonic surface inside the bow shock marks the reacceleration of stalled material toward the accretor (Foglizzo and Ruffert, 1997, 1999; Foglizzo et al., 2005; Blondin and Raymer, 2012). In our $\epsilon_\rho = 0$ panel of Figure 7.6, some numerical artifacts can be seen originating from grid interfaces at the bow shock. These flaws highlight the difficulty of simulating a quasi-steady flow with a cartesian grid mesh. With symmetry broken as the upstream gradient is introduced, these artifacts disappear as the flow is no longer so near to steady state.

With the introduction of a density gradient and the corresponding lack of cancelation of tangential motion, material in the post-bow shock region carries some rotational support. This rotation leads to the development of regions surrounding the accretor that retain supersonic velocities. In contrast to the $\epsilon_\rho = 0$ simulation, where the leading face of the accretor remains in sonic contact with the bow shock, for $\epsilon_\rho > 0$, the sonic surface largely detaches from the accretor. The cavity of subsonic material in which sound waves can propagate becomes increasingly restricted in the steeper gradient simulations. The flow asymmetry and instability observed in the steep density gradient cases cannot therefore be due to acoustic perturbations cycling from the bow shock to the accretor, as is thought to drive instability for sufficiently high Mach number flows with homologous upstream conditions and $\gamma = 5/3$ (Foglizzo et al., 2005). Instead, the observed instability appears to be a vortical instability seeded in flow near the embedded object that carries too much angular momentum to accrete, and instead impinges on the surrounding material. This is related to the instability observed in isothermal flows with nearly-homologous upstream boundary conditions, which also tend to develop significant rotation (Foglizzo et al., 2005). Shear layers and density inversions with respect to the accretor’s gravity can be seen to develop within these flow structures and these effects also seed instability and vorticity in the post bow shock region. In general, as the gradient steepens and the sink size decreases, more unstable and turbulent flow is exhibited surrounding the accretor.
It is worthwhile here to examine some of the ways that these flows may be compared to previous numerical work with inhomogenous upstream density. Flow that breaks axisymmetry and develops significant rotation is observed in simulations with both velocity and density gradients by Ruffert (1997) and Ruffert (1999). Our simulation in the second panel of Figure 7.5 ($\epsilon_\rho = 0.3$) may be compared morphologically with model "NS" of Ruffert (1999), which is the steepest-gradient model explored and has $R_a/H_\rho = 0.2$ (see their Table 1 for parameters, and Figure 2 for a plot of the density distribution and flow vectors). These models exhibit very similar morphology, and in particular, similar flow patterns in the post shock region. As a point of comparison, it is worth noting that in our setup higher densities are at positive $y$ values, while they are at negative $y$ values in Ruffert (1999). By contrast, 2D cylindrical simulations with upstream density gradients by Fryxell and Taam (1988) and Armitage and Livio (2000) show qualitatively different behavior. Even with mild density gradients these 2D flows develop small, rotationally supported disks (Armitage and Livio, 2000) trailed by an attached wake that is unstable in the transverse sense and sheds vortices from the accretion region (Fryxell and Taam, 1988). We explore this difference further in Section 7.5.4.

### 7.5.2 Effects of Sink Size

A comparison of the upper and lower panels of Figures 7.5 and 7.6 highlights some differences in the large-scale flow that result from the size of the accretor. This effect is explored further in Figure 7.7 for the $\epsilon_\rho = 0.3$ case of simulations D and L. While rotating flow is relatively laminar in the $R_s = 0.05R_a$ simulation, vortices dominate the region in the $R_s = 0.01R_a$. At the root of this difference is that, with the introduction of a density gradient in the upstream flow, there is an angular momentum barrier as well as an energetic barrier to accretion.

We can estimate the radius at which material with impact parameter $< R_a$
Figure 7.7: Comparison of the $z$-component of the vorticity in the inner regions of simulations for which $R_s = 0.05R_a$ and $R_s = 0.01R_a$ (simulations S, top, and L, bottom). Both have otherwise identical parameters, with a gradient of $\epsilon_\rho = 0.3$, and identical maximum linear resolution $\delta_{\text{min}} \approx 10^{-3}R_a$. Imposing the sink boundary in the flow at smaller radii allows material to penetrate deeper into the accretor’s gravitational potential without being subsumed. While the flow for $R_s = 0.05R_a$ is mostly laminar, strong local vorticity develops surrounding the accretor when $R_s = 0.01R_a$. The extent of turbulent flow extends well beyond the accretor size in either simulation and becomes an important feature of the large-scale flow.
will circularize if it completely inelastically redistributes its momentum. Under these assumptions, the specific angular momentum (in the $\hat{z}$ direction) specified by the density distribution at infinity is $l_{z, \infty} = \dot{L}_{z}(< R_{a})/\dot{M}(< R_{a})$, where

$$\dot{M}(< R_{a}) = v_{\infty} \int_{< R_{a}} \rho(y) dA. \quad (7.8)$$

Here the area element can be re-written $dA = 2\sqrt{R_{a}^{2} - y^{2}} dy$ with limits $y = \pm R_{a}$. This expression reduces to $\dot{M}_{HL}$ when $\epsilon_{\rho} = 0$. Similarly,

$$\dot{L}_{z}(< R_{a}) = 2v_{\infty}^{2} \int_{-R_{a}}^{R_{a}} y\rho(y)\sqrt{R_{a}^{2} - y^{2}} dy. \quad (7.9)$$

Given equation (7.7) for $\rho(y)$, we can solve analytically for $l_{z, \infty}$ and find

$$l_{z, \infty} = \frac{I_{2}(R_{a}\epsilon_{\rho})}{I_{1}(R_{a}\epsilon_{\rho})} R_{a} v_{\infty} \quad (7.10)$$

where $I_{n}$ are the modified Bessel functions of the first kind,

$$I_{n}(z) = \frac{1}{\pi} \int_{0}^{\pi} e^{z \cos \theta} \cos(n\theta) d\theta, \quad (7.11)$$

for integer $n$ (Abramowitz and Stegun, 1972). The radius at which material with this angular momentum will be rotationally supported is $R_{\text{circ}} = l_{z, \infty}^{2}/GM$. In our adopted units, $R_{a} = 1$ and $v_{\infty} = 1$, $GM = 1/2$, we thus have

$$R_{\text{circ}} = 2 \left[ \frac{I_{2}(\epsilon_{\rho})}{I_{1}(\epsilon_{\rho})} \right]^{2}. \quad (7.12)$$

This quantity represents the radius at which the fluid with impact parameter less than the accretion radius will circularize if it has the opportunity to perfectly cancel momenta with opposing flow.

If $R_{\text{circ}} < R_{s}$, material can enter the accretor directly, sweeping through at
most one orbit. Visually, this is what we observe in the $R_s = 0.05 R_a$ case of Figure 7.7. Material that is drained from the accretion region acts as an effective cooling source on the flow in that accretion removes material from the vicinity on of the accretor so that it does not proceed to impinge on newly-inflowing material. Instability arises and vorticity appears to grow in our simulations when $R_{\text{circ}} > R_s$. In this case, there is an angular momentum barrier to accretion, and material is trapped in orbit around the accretor. This scenario becomes inherently unstable when coupled to the continuous infall of fresh material from the upstream region. The new material collides with the old as it tries to penetrate to the accretor. Shearing layers and density inversions that result near the accretor lead to instabilities that amplify such that turbulence dominates the post-shock region. Numerically, equation (7.12) suggests a circularization radius of $R_{\text{circ}}(\epsilon_\rho = 0.3) = 0.011$ for the example in Figure 7.7. This is mildly outside $R_s = 0.01$ and substantially inside $R_s = 0.05$ indicating the significance of the transition observed in Figure 7.7. We should, therefore, not expect convergence of the flow behavior with respect to sink size in flows where angular momentum plays a role in shaping flow morphology. This is particularly true in cases with $R_{\text{circ}} \sim R_s$.

### 7.5.3 Accretion of Mass and Angular Momentum

In our simulations we track the accumulation of mass and angular momentum into the central sink boundary. Each time step the accumulated quantities above the floor state are integrated over the sink volume before the pressure and density are rewritten. The rate implied is calculated each time step by $\dot{X} = \Delta X/dt$, where $X$ is an arbitrary quantity, $\Delta X$ is the integrated new material, and $dt$ is the time step.

In Figure 7.8, we show the time-dependent accretion of mass (upper panel) and angular momentum (lower panel) for the series of simulations (A–II), for which $R_s = 0.05 R_a$. We run these simulations for $80 R_a/v_\infty$, but accretion rates relax to their steady-states within the first domain-crossing time $\approx 10 R_a/v_\infty$ as found, for example
by Ruffert (1994a). As has been demonstrated in previous numerical simulations, in the zero-gradient case $\dot{M}_{HL}$ provides a good order-of-magnitude estimate of the accretion rate (e.g. Ruffert, 1994a; Ruffert and Arnett, 1994; Naiman et al., 2011; Blondin and Raymer, 2012). As the upstream density gradient steepens, the steady state accretion rate drops precipitously. Interestingly, the early time accretion rate in all cases is similar to that of the zero gradient case. The accretion rate tracks that of the homogeneous case and breaks off only when the bow shock has swept wide enough to trace out substantial upstream inhomogeneity and angular momentum. In cases where transient flows exist in which the bow shock is less than fully developed, the effective density gradient is thus reduced in proportion to the bow shock’s extent. As the upstream density gradient steepens, the accretion rate also becomes increasingly variable. The variability seen is chaotic and there is no single apparent periodicity or driving timescale in a Fourier decomposition of $\dot{M}(t)$ (e.g. Edgar, 2005).

The lower panel of Figure 7.8 compares the accretion rate of angular momentum between simulations with differing density gradients. Here we normalize our results to a characteristic angular momentum accretion rate, $\dot{M}_{HL}R_\alpha v_\infty$. The zero-gradient case preserves symmetry to better than 1 part in $10^4$, and provides a gauge for the fidelity of the other cases. As the gradient appears ($\epsilon_\rho < 1$), the accreted angular momentum at first increases because of rotation imparted to the post bow shock flow. For steeper gradients, the accreted angular momentum actually decreases again because $\dot{M}$ decreases in the steepest-gradient cases. For these combinations, the limiting of $\dot{M}$ appears to outweigh the increasing angular momentum content of the upstream flow.

To disentangle the accretion rate of mass and angular momentum, we turn our attention to the specific angular momentum of accreted material. In Figure 7.9, we plot histograms of the specific angular momentum content of accreted material. The specific angular momentum, $l = \sqrt{\dot{L}_x^2 + \dot{L}_y^2 + \dot{L}_z^2}/\dot{M}$, is normalized to the angular momentum of material in Keplerian orbit at the sink radius, $l_{\text{kep}} = \sqrt{GM R_\alpha}$. With $R_\alpha = v_\infty = 1$
Figure 7.8: Accretion of mass (top) and angular momentum (bottom) as a function of time in simulations with a sink size $R_s = 0.05 R_a$ (simulations A-H). Accretion rates are normalized to $\dot{M}_{HL}$ (for mass, equation [7.3]) and to $\dot{M}_{HL} R_a v_\infty$ (for angular momentum). The upper panel shows that the introduction of an upstream gradient not only dramatically decreases the accreted mass but also leads to increased chaotic time-variability compared to the median value. The variability observed can be attributed to turbulence in the flow seeded by the upstream gradient. The accretion rate of angular momentum is lowest in the $\epsilon_\rho = 0$ simulation, and provides a measure of the high degree to which our simulations preserve symmetry intrinsic to the setup. The $\epsilon_\rho = 0.1 - 0.55$ cases are able to accrete substantially more angular momentum than the stronger-gradient cases because $\dot{M}$ is not so highly impeded. Rotation is always found in the sense imposed by the upstream gradients.
Figure 7.9: Specific angular momentum of accreted material as compared to the angular momentum of material in Keplerian rotation at the sink radius in simulations A-H (top) I-P (bottom). Here $l_{\text{kep}} = \sqrt{GMR_s}$, in the dimensionless units of the simulations, $GM = 1/2$ and this becomes $l_{\text{kep}} = \sqrt{R_s/2}$. A transition can be seen in both panels as the density gradient steepens. The angular momentum content of accreted material at first increases, but then maximizes at a mean value of $|l|/l_{\text{kep}} \approx 0.5$. In the steeper-gradient cases ($\epsilon_\rho > 0.55$) the width of the distribution also broadens. However, accreted material always displays substantially sub-Keplerian rotation. The transition noted in Figure 7.7 can be seen above by comparing the $\epsilon_\rho = 0.3$ case in the upper and lower panels. In the upper panel, where $R_s = 0.05R_a$, the accreted angular momentum forms a narrow distribution, similar to that exhibited in the milder-gradient cases. In the lower panel, in which $R_s = 0.01R_a$, the distribution is broader, nearly joining the family of curves from the steep-gradient cases. As mentioned in the caption of Figure 7.7, this transition takes place material circularizes inside (for $R_s = 0.05R_a$) or outside (for $R_s = 0.01R_a$) the sink radius.
Figure 7.10: Accretion of mass and angular momentum summary with respect to gradient, $\epsilon_\rho$ in simulations A-H and I-P. The points show the median, while the error bars show the 5% and 95% percentile bounds of the time series data for different simulations, calculated for times $t > 20R_a/v_\infty$ when the simulations are in steady-state. The dashed lines show the flux of either mass or angular momentum through a surface with impact parameter at infinity, $b < R_a$. Although the available mass and angular momentum increase as an exponential density gradient is introduced, accretion is dramatically inhibited by the asymmetric flow geometry that develops. The smaller-sink simulations exhibit lower mass and angular momentum accretion rates with larger variability.
as used in the simulations, \( GM = 1/2 \) and this becomes \( \ell_{\text{kep}} = \sqrt{R_s/2} \). Even in the gradient cases, we find that the angular momentum accretion rate is much less than Keplerian. The specific angular momentum content of accreted material is also highly variable, as visualized by the broad histograms in Figure 7.9. Interestingly, \(|l|\) is similar in \( \epsilon_{\rho} = 0.55 - 3 \) simulations, despite the increased flux of angular momentum from the upstream conditions. The milder gradient simulations peak at somewhat lower typical angular momentum content \(|l|/\ell_{\text{kep}} \approx 0.1 - 0.4\) than the steeper-gradient simulations which peak at \(|l|/\ell_{\text{kep}} \approx 0.5\). This transition in behavior occurs when the circularization radius is outside the sink, \( R_{\text{circ}} \gtrsim R_s \), as opposed to when the net angular momentum allows circularization inside the sink. The lack of a tail extending to \(|l|/\ell_{\text{kep}} \approx 1\) indicates that none of the simulations ever reach a state of accretion from a nearly-Keplerian flow. It is worth contrasting this, briefly, to recent 2D planar simulations in which, after an initial growth phase, the specific angular momentum of accreted material is nearly always within a few percent of the Keplerian value (Blondin and Pope, 2009; Blondin, 2013). This difference in accretion modality, therefore, appears to lie in the geometry of the simulations.

Figure 7.10 summarizes the accretion of mass and angular momentum in our simulations. We have plotted the accreted mass and angular momentum for simulations with \( R_s = 0.05 R_a \) and \( R_s = 0.01 R_a \). We plot the flux of either mass or angular momentum through an upstream cross section with impact parameter \( b < R_a \), equations (7.8) and (7.9). The mass and angular momentum available in the flow both increase with gradient. With \( \epsilon_{\rho} = 0.3 \), the flux of mass with \( b < R_a \) is \( 1.01 \dot{M}_{\text{HL}} \), with \( \epsilon_{\rho} = 1 \) it is \( 1.13 \dot{M}_{\text{HL}} \), while with \( \epsilon_{\rho} = 3 \) it increases to \( 2.64 \dot{M}_{\text{HL}} \). The mass that reaches the sink and accretes decreases dramatically as the density gradient steepens. This limiting of accretion, despite the increased availability of material, must be attributed to the change in flow structure and angular momentum barrier to accretion described in the previous subsections.

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The comparison between $R_s = 0.05R_a$ and $R_s = 0.01R_a$ series results depends strongly on the upstream gradient imposed. The mass accretion rates decrease by a factor of a few with the small sink as compared to the large for the strongest-gradient cases, yet only by $\sim 10\%$ for zero-gradient. This difference points explicitly to the role of angular momentum in limiting the amount of material that is able to actually accrete. Figure 7.9 supports this conclusion; the angular momentum content of accreted material nicely follows the normalization with respect to $l_{\text{kep}}$, which depends explicitly on the sink radius.

### 7.5.4 Disk Formation?

With the accretion of angular momentum apparent in simulations with an upstream gradient, a natural question that arises is whether the rotation imposed in the flow leads to the formation of persistent accretion disks. Disk formation is, for example, apparent in accretion flows that develop from wind-capture in widely spaced binaries (see e.g. Zarinelli et al., 1995; Blondin, 2013; Huarte-Espinosa et al., 2013, for numerical simulations). When disk structures form, viscosity from magnetorotational instability (Balbus and Hawley, 1991) can transfer angular momentum to funnel disk material toward the central body. In these cases, the flow is primarily optically thin, and can cool effectively through line emission and the blackbody continuum. The effective polytropic index of the equation of state is assumed to be close to one, $\gamma \sim 1$, and as a result the flow is nearly isothermal. With effective cooling, rotationally supported structures are quickly assembled. When cooling is not effective, as is the case when embedded deep within a shared stellar envelope, the adiabatic index of the gas remains close to $\gamma = 5/3$. In this case, the adiabatic build-up of pressure is significant as gas moves to smaller radii. Without a means to dissipate this internal energy, the flow remains in large part pressure supported.

Figure 7.11 explores the rotational motion of material close to the accretor in
Figure 7.11: Rotation imposed by an upstream density gradient in simulations L, N, and P. In the upper panel, shown perpendicular to the plane of rotation, color shows inward (red) or outward (blue) motions compared to the local Keplerian velocity. The extent of the rotating material becomes smaller as the gradient increases and inflowing high-density material impinges on the orbiting material. The lower panels look at the same frames and the magnitude of tangential motion in the orbital plane. These show that the rotational flow that develops is not strictly disk-like. Rotating material in the orbital plane is interspersed with cavities of material with pressure, rather than rotational, support. Plumes of material with little rotational support extend to the sink surface, feeding accretion.
simulations with an imposed gradient. The lower panels of this Figure look at tangential motion in the orbital plane, while the upper panel plots motion into and out of the perpendicular plane. In the $\epsilon_\rho = 0.3$ simulation, rotational motion is sub-Keplerian nearly everywhere. In the larger gradient simulations there are pockets of material rotating with the Keplerian velocity. However, these pockets are interspersed with shocked material with little rotational support. Rather than exhibiting steady circular flow patterns, streamlines are highly elliptical in the orbital plane, being first flung upstream then wrapping around to encounter incoming material. These flow patterns define a cavity of material bounded by shocks in which angular momentum is redistributed. As visualized by flow streamlines, some material encountering these boundary layers is deviated back toward the accretor, but much of the rest is advected away and shed in the wake. That persistent orbits do not exist indicates that material does not have the opportunity to be viscously accreted before it is advected away from the sink region.

The lack of a dense built-up disk feature is contrary to what has been observed in 2D cylindrical coordinates with a $\gamma = 5/3$ equation of state. Fryxell and Taam (1988) and Armitage and Livio (2000) find rotationally-supported flows with the introduction of upstream density gradients in their 2D simulations. Blondin and Pope (2009) and Blondin (2013) look specifically at disk formation in 2D simulations of HLA. They find that even without any imposed gradients the flow is unstable to the development of quasi-Keplerian disks. As mentioned earlier, accreted material in these simulations typically carries specific angular momentum close to the Keplerian value ($|l|/l_{\text{kep}} \approx 1$), which may be contrasted to Figure 7.9, in which we find $|l|/l_{\text{kep}} \approx 0.5$ to be much more representative. In both of these sets of 2D simulations spiral shocks appear to mediate the transport of angular momentum that allows material in the disk to accrete (Blondin, 2000, 2013).

Pressure certainly plays a role in distinguishing 2D and 3D simulations. Because of the difference in radial dependence of the volume element, material is com-
pressed to differing degrees in 2D and in 3D. The standoff shock seen in 2D simulations is entirely rotationally developed, being a consequence of the flip-flop instability saturating and wrapping around the accretor into the upstream flow (Blondin and Pope, 2009). By contrast, a bow shock forms promptly in 3D simulations with $\gamma = 5/3$ solely due to compression of convergent flow. This offers some explanation of why 2D adiabatic and 3D isothermal simulations show similar properties. The 2D volume element $dV_{2D}/dr = 2\pi r$, while in 3D, $dV_{3D}/dr = 4\pi r^2$. If we examine the adiabatic increase in pressure of accreting gas in three cases in which simulations have been performed, $P \propto \rho^\gamma \propto V^{-\gamma}$. With $\gamma = 1$ in 3D, $P \propto r^{-3}$, and disk formation is apparent. In 2D with $\gamma = 5/3$ disks are again apparent, and $P \propto r^{-10/3}$. While in 3D with $\gamma = 5/3$ purely-Keplerian disks do not appear, and $P \propto r^{-5}$.

We illustrate this point with a comparison between simulations V and X in Figure 7.12, which share $M_{\infty} = 3$, $\epsilon = 5$, and $R_s = 0.01R_a$ but differ in compressibility of the equation of state. Simulation V has $\gamma = 5/3$ while simulation X has $\gamma = 1.1$. While the $\gamma = 5/3$ case forms the same cavity seen in the other steep gradient simulations, the $\gamma = 1.1$ case forms a persistent thin disk. This disk is much denser than the surrounding material, and it sets up in steady rotation around the accretor. Notably, the scale height of the disk is thicker in the upstream direction as it is impinged on by incoming material. The $\gamma = 1.1$ equation of state indicates that some cooling occurs in the gas, and thus is not physically realized deep within a CE. Early in the CE phase, though, near the surface layers of a star, the cooling time may be short and $\gamma < 5/3$ is possible.

### 7.5.5 Drag

The rate of momentum dissipation due to gravitational focusing of the surrounding gas sets the drag force felt by the embedded body. In turn, this corresponds to the rate of orbital energy dissipation into the CE and the rate that the embedded
Figure 7.12: A comparison between $M_\infty = 3$, $\epsilon_\rho = 5$ simulations with different equations of state. These simulations (V and X in Table 2), have $\gamma = 5/3$ and $\gamma = 1.1$, respectively. Just as in the other $\gamma = 5/3$ simulations disk formation is not apparent in simulation V, despite the steep density gradient. In simulation X ($\gamma = 1.1$), the more compressible equation of state permits the formation of thin, dense, and persistent disk.
object inspirals to tighter orbital separations.

We estimate the momentum dissipation realized in our simulations along the direction of orbital motion ($\hat{x}$) as follows,

$$F_{d,x} = \int \rho v_x (\vec{v} \cdot \hat{r}) dS$$  \hspace{1cm} (7.13)

where $F_{d,x}$ is the $x$-component of the drag vector, $\hat{r}$ is the vector normal to the surface of a unit sphere, and the surface of integration $dS$ is a sphere with the gravitational radius of the accretor, $R_a$ (after Ricker and Taam, 2008, equation 3). We calculate this flux using the marching cubes surface reconstruction algorithm in yt (Turk et al., 2011). In this way, we measure the momentum deposition rate by the inflowing gas across the gravitational focus cross section, $\pi R_a^2$. In our simulations, performed in the frame of the accretor, we observe a slowing and pileup of background gas due to gravitational interaction with the point mass. In the frame of the fluid, this represents a continuous decrease in relative velocity of the accretor, or a gravitational drag force (Ostriker, 1999). Integrating along the orbital path, this drag force produces the CE inspiral. Thus the rate of orbital energy decay due to drag should be $\dot{E}_d \approx F_{d,x} v_\infty$, under the assumption that it takes many flow crossing times to dissipate the accretor’s kinetic energy, $E/\dot{E}_d \gg R_a/v_\infty$.

We find that to within a factor of $\sim 4$, the drag is similar with changing density gradient. We plot the drag rates realized in simulations A-P in Figure 7.13. We normalize our results to the drag expected in HLA theory, $\pi R_a^2 \rho_\infty v_\infty^2$, for which the corresponding energy dissipation rate is $\dot{E}_{HL} = \pi R_a^2 \rho_\infty v_\infty^3$. In simulations with a mild gradient ($\epsilon_\rho \lesssim 1$) we find slightly lower drag than the $\epsilon_\rho = 0$ case. This decrease can be

\footnote{This quantity can be differentiated from the aerodynamic or sometimes called hydrodynamic drag generated by a non-gravitating sphere’s geometric cross section. In many cases, the geometric cross section $\pi R_{obj}^2 \ll \pi R_a^2$ (See Figure 7.4) and the drag generated through gravitational convergence of the background flow dominates (Ostriker, 1999; Passy et al., 2012b). If the evaluating surface in Equation (7.13) were similar to the object’s size, we could instead measure the aerodynamic drag of the sink through the gas as is done by Ricker and Taam (2008, 2012).}
attributed to a trade-off between stronger shocking in the $x$-$y$ plane, but weaker in the $x$-$z$ plane with the introduction of a density gradient, as can be seen visually in Figure 7.5. As the gradient steepens to $\epsilon_\rho = 3$, the focusing of dense material in the $x$-$y$ plane dominates and the drag force again increases with respect to the nominal value.

Our simulations are not perfectly suited to measure the drag during a CE episode. The construction of our domain is best suited to study the flow within the accretion radius. At larger distances from the accretor, the approximations made here are less valid. A primary caveat is the missing gravitational vector of the other stellar core $\vec{g}$. Another consideration is that our simulation geometry is planar. Especially when $R_a \sim R_*$, the curved geometry of the CE may depart substantially from the planar approximation. Both of these effects will likely play a role in shaping the shock morphology at distances $\sim R_a$ from the accretor. Since this is where much of the thermalization occurs, we expect that some differences would arise in the full CE geometry. Further, Ricker and Taam (2008, 2012) have noted that at late times in relatively equal mass CE interactions, the envelope becomes distorted well out of its hydrostatic configuration and the drag force cannot be easily related to the initial envelope properties at a given radius. We intend to devote future simulations to study this question.

It is striking that although the accretion rate changes by nearly two orders of magnitude as the density gradient steepens, the drag changes very little. While this finding is initially surprising, its interpretation can be traced to Dodd and McCrea (1952)’s insightful analysis of gravitational capture from a medium containing a density gradient. While the functional form and normalization of drag and accretion rates differ substantially from their derived values, their analysis did point out that drag occurs when material is focussed within the vicinity of the accretor, for example, with impact parameter $\lesssim R_a$. To accrete, the gas must also be liberated of its angular momentum. With a stiff equation of state like $\gamma = 5/3$, disks do not form and most material is swept away from the vicinity of the accretor before it has the chance to redistribute its angular
Figure 7.13: Drag force along the direction of motion, equation (7.13) in simulations A-H and I-P. Drag is most in the steepest-gradient simulated, $\epsilon_\rho = 3$. In the mild-gradient simulations, $\epsilon_\rho = 0.1 - 1$, there is less drag than in the no-gradient, $\epsilon_\rho = 0$ case. The reasons for this can be seen visually in Figure 7.5, where a trade-off can be seen in the degree of thermalization realized in the orbital plane ($x$-$y$) and in the perpendicular plane ($x$-$z$) as a gradient is introduced. The drag realized is consistent to within an order of magnitude for each of the models, despite the accretion rate found in the simulations varying by a factor of 100.

7.6 Discussion

We have shown that the morphology of flows surrounding objects embedded within a CE can be characterized by a few key parameters. Among these are the flow Mach number $M_\infty$, and the relative sizes of the object radius, accretion radius, and stellar envelope radius. For the cases we consider here, $R_{\text{obj}} < R_a < R_*$, so gravitational focusing is important, and the whole accretion structure is embedded
inside the CE. We’ve emphasized that another key flow parameter is the upstream density inhomogeneity, which can be characterized by the local ratio of accretion radius to density scale height, $\epsilon_\rho$. The hydrodynamical implications of these density gradients have been explored in the previous sections.

Under the simplifying assumption that local flows inside CE can be described by a few key parameters, the lessons from our dimensionless calculations can be applied to any relevant CE system. In this section, we highlight some considerations in extending the results of our calculations to better understand flow properties across the range of typical encounters described in Section 7.3.2.2.

### 7.6.1 Cooling, Accretion, and Feedback from Embedded Objects

Accretion flows toward an embedded object can only be integrated into the object if there is an effective cooling channel or if the embedded object is a BH. In the case of objects with a surface, accretion liberates gravitational potential energy and generates feedback – which, if sufficiently large, can impinge on the flow field. In the very optically thick environment of the CE photons cannot easily propagate, and feedback from accretion will be primarily mechanical rather than photoionization-driven (for example, as modeled in HLA flows by Park and Ricotti, 2013). We have adopted a completely absorbing central boundary condition with radius in $R_s$ in our simulations. This is useful, of course, in evaluating accretion rates for cases which can accrete, but may not be appropriate in all scenarios. To parameterize the effect that a hard, reflecting, central boundary would have on these flow morphologies we compare the drag luminosity, $\dot{E}_d \approx F_{d,x}v_\infty$, to the accretion luminosity, $\dot{E}_a = GM\dot{M}/R_s$. Whether the drag luminosity or accretion luminosity is energetically dominant thus depends on both the compactness of the accretor, $R_s$, and the accretion rate, $\dot{M}$.

Figure 7.14 illustrates how the ratio between accreted energy and drag-generated heat change with density gradient and sink boundary size. For regimes in which
When \( \dot{E}_a / \dot{E}_d > 1 \), we would expect that replacing the absorbing central boundary with a hard boundary would make a significant energetic contribution to the post-shock region. The central sink boundary acts as a cooling term in absorbing \( \dot{E}_a \) from the central regions. If this energy were not deleted, we might expect it to contribute to overturning the flow or modifying either the bow shock configuration or stability. On the other hand, when \( \dot{E}_a / \dot{E}_d < 1 \) the accretion energy is small compared to the drag-generated energy in the post shock region. In this regime, the nature of the central boundary should not drastically affect the flow morphology. Figure 7.14 shows a transition between these regimes as the density gradient steepens. Steeper density gradients imply lower accretion rates into the central boundary. Because only a small fraction of material reaches the accretor, the accretion energy becomes a small contribution to the total energy budget.

Authors who have modeled the effects of a hard central boundary condition in flow have found that feedback from the central object impinges to create a broader, more unstable bow shock (Fryxell et al., 1987; Zarinelli et al., 1995). It may contribute to a higher rate of vortex-shedding, and thus hydrodynamic drag (Zarinelli et al., 1995), and almost certainly gives rise to a higher level of pressure support if there is angular momentum (Armitage and Livio, 2000). Much of this work has been performed under the assumption of 3D axisymmetry (Fryxell et al., 1987) or in 2D planar geometry (Zarinelli et al., 1995; Armitage and Livio, 2000), leaving open for future investigation the details of how non-axisymmetric 3D flows respond to feedback from a hard central boundary condition and how this affects critical properties like the drag coefficient.

Extending the lessons learned in these simulations to the reality of a CE episode as it plays out in nature is admittedly somewhat more complicated. The first fact to acknowledge is that, for objects with a surface, accretion is only possible when an adequate cooling channel exists. Comparing to the examples of Section 7.3.2.2, the microphysics of accretion flows onto NSs is such that neutrinos can carry away the
Figure 7.14: The ratio of accretion luminosity to drag luminosity as a function of density gradient in simulations A-H and I-P. The accretion and drag luminosity are approximated as $\dot{E} = GM\dot{M}/R_s$ and $\dot{E}_d = F_{d,x}i_{\infty}$, respectively. For shallow density gradients, $\epsilon_\rho \lesssim 1$, feedback of accretion energy into the flow would be energetically important were one to exchange the absorbing central boundary condition for a reflecting one. For steeper gradients, $\epsilon_\rho \gtrsim 1$, $\dot{M}$ is sufficiently small that accretion luminosity should represent only a perturbation to the energetics of the post-bow shock region.
accretion energy without interacting significantly with the surrounding gas (Houck and Chevalier, 1991; Chevalier, 1993, 1996; Fryer et al., 1996). Other stellar objects like main sequence stars and WDs are not sufficiently compact to promote neutrino emission, and yet the surrounding flow is extremely optically thick preventing the escape of heat through photon diffusion. These objects, therefore, are most appropriately modeled by a hard-surface boundary condition, while NSs and BHs are more appropriately modeled with an absorbing boundary.

Embedded objects like main sequence stars have radii of order $R_{\text{obj}} \approx 0.01 R_a$ (see Table 7.1), their inability to accrete, therefore, should affect flows with density gradients more shallow than $\epsilon_\rho \lesssim 1$. Embedded WDs, by contrast, have much smaller radii, $R_{\text{obj}} \approx 10^{-4} R_a$. Material that reaches these small scales will pile up rather than accreting and may contribute a substantial heating effect on the surrounding material. We cannot expect, though, to directly extrapolate our simulation $\dot{M}$ for $R_a = 0.01 R_a$ to $R_a = 10^{-4} R_a$ because the flow’s angular momentum as it passes the $R_a = 0.01 R_a$ indicates that only a fraction will reach the WD surface at $10^{-4} R_a$.

### 7.6.2 Mass Accumulation during CE Inspiral

NSs and BHs can gain mass by accretion during a CE episode. While the drag force, and resulting orbital energy dissipation rate $\dot{E}_d$ sets the rate of inspiral, $\dot{M}$ sets the rate of mass growth. We can therefore compare the inspiral timescale, $E/\dot{E}_d$, to the mass growth timescale $M/\dot{M}$. Several authors have noted that in HLA theory, the similarity in the function form of $\dot{M}_{\text{HL}}$ and $\dot{E}_{\text{HL}}$ imply a direct correlation between the energy required to liberate the CE and the mass accreted during the inspiral (Chevalier, 1993; Brown, 1995; Bethe and Brown, 1998). We can write $\dot{E}_{\text{HL}} = \dot{M}_{\text{HL}} v_\infty^2$. Within this framework, an integrated energy injection into the CE implies an accumulated mass. In the case of a NS inspiralling through a massive companion’s envelope, the implied accumulated mass would be enough to force the an accretion induced collapse to a BH.
Recent work by Ricker and Taam (2012) has indicated that accretion rates from the CE might be substantially lower than the HLA value, $\dot{M}_{HL}$. Our local simulations are complimentary to Ricker and Taam (2012)'s global models as we are able to allocate high resolution at scales smaller than $R_a$ and use an absorbing sink boundary condition. We confirm that in the presence of a density gradient the accretion rate may be severely limited. We show that the accretion rate drops off drastically with increasingly steep density gradients, reaching values $\sim 10^{-2}\dot{M}_{HL}$ for $\epsilon_\rho = 3$. Despite this, the drag force only changes mildly in response to the density gradient. Our results therefore indicate that the mass growth timescale may become substantially longer than expected in HLA theory and that embedded objects may grow significantly less than previously expected during CE evolution. We use the local calculations of accretion and drag rates presented in this paper to expand on the NS case study further in a companion paper (MacLeod and Ramirez-Ruiz, 2015b).

Accretion of material carrying angular momentum will impart net spin to the accreting object. If the object has radius equal to $R_a$ used in our simulations, then $\dot{L}_z$ (Figure 7.10) can be appropriately applied. However, if $R_{\text{obj}} < R_a$, as is the case for NSs and BHs, the a more realistic approximation of $\dot{L}$ may be to multiply $\dot{M}$ by the Keplerian specific angular momentum at the object’s surface: $\sqrt{GM R_{\text{obj}}}$. As mentioned earlier, the extrapolation of $\dot{M}$ to smaller radii is not trivial. However, the fact that we find $\dot{M} \ll \dot{M}_{HL}$ implies that both $\Delta M/M$ and $\Delta L/L$ are reduced proportionately with $\dot{M}/\dot{M}_{HL}$.

7.6.3 Loss of CE Symmetry during Inspiral

In computing local simulations of CE flows, we implicitly assume minimal disturbance of the envelope during the dynamical inspiral. While this is an extremely useful simplifying assumption, it may not be always justified in CE events as they occur in nature. The loss of spherical symmetry of the envelope material surrounding the
embedded object can be crudely estimated by comparing the ratio of inspiral timescale to orbital period. Livio and Soker (1988) define this ratio as $\beta_{\text{CE}} = (E_{\text{orb}}/\dot{E}_d)/P_{\text{orb}}$. When $\beta_{\text{CE}}$ is small, local, rather than orbit-averaged effects are important. As such, large departures from the initial hydrostatic structure can be expected. Making use of the Keplerian orbital energy and period, one derives the expression of Livio and Soker (1988, Equation 5),

$$\beta_{\text{CE}} \approx \frac{1}{12\pi} \left( \frac{m_*(a) + M}{M} \right) \left( \frac{\bar{\rho}}{\rho} \right),$$

(7.14)

where we have simplified the original author’s expression by neglecting any dependence on the sound speed in the flow accretion radius $R_a$. In the above expression the mean density is the that enclosed by the orbit, $\bar{\rho} = m_*(a)/(4/3\pi a^3)$.

Our local approximation of envelope properties as maintaining a quasi-hydrostatic structure similar to that of the original star is thus most justified when the embedded mass $M$ is small compared to $m_*(a)$ or when the local density $\rho \ll \bar{\rho}$, as is the case for highly evolved stars that develop tenuous convective envelopes. In Figure 7.15 we compare $\beta_{\text{CE}}$ to the ratio of $R_a/R_*$ for the binary systems described in Section 7.3. The ratio $R_a/R_*$ is representative of the fraction of stellar material that is being shocked in a given passage of the embedded body. The least disturbed envelopes lie at high $\beta_{\text{CE}}$ and low $R_a/R_*$ (the upper left of the diagram). An embedded planet is least disturbing of the examples shown, however, a NS embedded in a supergiant companion also appears to lie in the phase space best described by local approximation. Internal structure of the CE also plays a role. Near the stellar limb (diamond-shape points at 0.8$R_*$) where the local density is low, the local approximation is better justified than deeper in the stellar interior.

These scalings provide some guidance in comparing our local calculations to the global calculations of Ricker and Taam (2008, 2012) and Passy et al. (2012b). In particular, Ricker and Taam (2008, 2012) consider a $1.05M_\odot$ red giant companion to
a $0.6M_\odot$ star. Passy et al. (2012b) consider a $0.88M_\odot$ giant companion paired with point masses ranging from 0.1-0.9$M_\odot$, however, slices of the fluid conditions are only shown for the case with a $0.6M_\odot$ point mass. Thus, these simulations, with mass ratios of order unity, lie strongly in the regime where local effects (as parameterized by $\beta_{\text{CE}}$) should be important and the CE should quickly lose spherical symmetry and depart from its original structure, resulting in very rapid initial inspiral. However, during this short phase, flow properties compare favorably between local and global approaches. In particular, fluid is seen to trace out elliptical orbits around the embedded star in our local calculations, this is also observed in Figure 1 of Ricker and Taam (2008). The bow shock structures that originate near the embedded bodies in both local and global calculations sweep throughout the envelope to seed the the dominant spiral shock features seen in the orbital plane of global calculations (e.g. Ricker and Taam, 2012, Figure 10).

### 7.6.4 The End of Dynamical Inspiral: Envelope Spin-Up and Heating

In CE evolution, the rapid inspiral phase precedes a gradual stabilizing of the orbital separation as orbital evolution slows to the thermal rather than dynamical timescale of the envelope (Podsiadlowski, 2001). The rapid inspiral phase is hydrodynamic in nature, and is the phase principally addressed in simulations here as well as most recently by Passy et al. (2012b) and Ricker and Taam (2012). During the rapid inspiral, drag luminosity, $\dot{E}_d$, transforms the orbital energy into envelope thermal energy. Drag torques due to asymmetries in the flow transfer orbital angular momentum to CE angular momentum and orbital angular momentum is also carried away by any ejected material. Over time, the remaining CE spins closer to corotation with the embedded object (Iben and Livio, 1993). The combination of these effects lead to lower densities, higher local sound speeds, and a progressive reduction in the flow Mach number. These effects, in turn, bring a reduction in drag, gradually ending the rapid inspiral
Figure 7.15: Characteristic scales that describe the disturbance of the CE by the embedded body. $\beta_{\text{CE}}$ describes the rapidity of inspiral as compared to orbital period. Local effects should be very strong for low $\beta_{\text{CE}}$. The ratio of $R_a/R_*$ indicates the fraction of stellar material focused to interact with the embedded body each orbit, and perhaps an indication of the shock heating. Points are evaluated at $a = 0.2, 0.5,$ and $0.8 R_*$ (circles, squares, and diamonds, respectively) for the same binary combinations plotted in Figures 7.3 and 7.4. Our localized assumptions for CE flow properties are most valid in the upper left of this diagram, which arises when the embedded body's mass is small compared the giant-star's mass. Among the stellar cases, a NS embedded in a supergiant companion is particularly well described by local approximation. In other cases, our local assumptions are best justified early in the CE episode where the embedded object interacts with low-density material near the stellar limb.
phase. This phase of stabilization of orbital separation is observed clearly in the global simulations of Passy et al. (2012b) and Ricker and Taam (2012).

To explore qualitatively how flow morphologies might respond to reducing Mach numbers, we present two simulations in Figure 7.16. At lower Mach numbers, bow-shock features are broader, with larger opening-angles. While Figure 7.16 can be compared directly to Figure 7.5, the panel width is enlarged to accommodate the larger shock-features and greater upstream standoff distance. Additionally, the colorbar shows density only up to 10 $\rho_\infty$ – representative of the fact that shocks are weaker and compression is not as severe in the $M_\infty = 1.1$ simulations as with $M_\infty = 2$. We also find a milder tilt-angle in the bow shock structures than in the higher Mach number simulations. As expected, the weaker shocks in this scenario lead to somewhat lower drag rates with respect to the nominal value of $\pi R^2 \rho v^2_\infty$ (see Table 7.2). Despite these morphological differences, we find very similar median $\dot{M}$ for these simulations and their $M_\infty = 2$ counterparts.

These facts indicate that a dramatic change in flow morphology, accretion rate, and inspiral rate, probably occurs only when the the flow becomes subsonic (Ricker and Taam, 2012). A final consideration on flow properties toward the end of the dynamical phase is a the decoupling of the companion’s core from the envelope. In particular, when the enclosed envelope mass becomes small as compared to the binary mass that is formed by the embedded object and the core of the companion, both dense cores develop differential motion relative to the envelope. Ricker and Taam (2012) show that a low density, high pressure, region starts to open up surrounding the binary at this stage. Our guiding assumption of undisturbed envelope properties is clearly not realized here. Instead, typical relative motion is subsonic and local density gradients are mild.

In this case, the binary at the center of the extended envelope provides a heat source for all of the enshrouding CE gas. Modeling the subsequent thermal evolution that defines of the remainder of the CE event is certainly important in determining
Figure 7.16: $M_\infty = 1.1$ simulations T and U. These are representative of the flow at later times in the CE evolution after the envelope has been heated spun closer to corotation with the inspiralling object, and may otherwise be compared to Figure 7.5. These panels show the broader opening angle, smaller degree of tilt, and weaker shocks that develop in lower-Mach number flows. Note that the density scale here extends to $10^1$ while in Figure 7.5, it extends to $10^2$, and that the width of the panels is $3.2R_a$ rather than $2.2R_a$ to accommodate the larger bow-shock structures.
the outcome of the CE episode but is beyond the scope of this study. The stabilization phase may be particularly important as the envelope may not be fully ejected at the end of the dynamical phase (see Passy et al., 2012b, for a comparison between separations at the end of the dynamical inspiral phase and observed systems). Additional terms other than orbital energy, like recombination energy, may be important to finalize the envelope ejection after the dynamical inspiral (Ivanova et al., 2015).

7.7 Conclusions

This paper has examined flow structures in the immediate vicinity of objects during the dynamical inspiral phase of CE. We begin by exploring the typical scales during the dynamical inspiral using MESA calculations of stellar structure. We then use these conditions to motivate FLASH models of accretion flows with an upstream density gradient. We find that substantial asymmetry develops in these flows, which carry net angular momentum with respect to the embedded object. The main effects may be summarized as follows:

- Typical accretion radii, $R_a$, are much larger than the size of embedded objects, and may in fact span a large fraction of the stellar radius. As a result, material that is focused toward the embedded object can span a large range of density as $R_a$ sweeps across the radial density gradient within the giant star’s envelope. We parameterize the density gradient according to $\epsilon_{\rho} = R_a/H_{\rho}$. Typical values of $\epsilon_{\rho}$ are of order unity, so the density inhomogeneity represents a substantial perturbation to the flow. This effect is illustrated in Figure 7.1 and elaborated on in Section 7.3.

- We introduce this upstream density profile into simulations of HLA. We find that the upstream gradient breaks the symmetry of the flows, because momenta no longer cancel to form an accretion column in the wake of the accretor. With an
upstream gradient, the flow carries angular momentum about the accretor. Figure 7.5 illustrates the changing flow morphology with varying $\epsilon/\rho$.

- We find that drag forces are only mildly affected by the upstream density gradient, but mass accretion rates are sensitively dependent on $\epsilon/\rho$. For relatively steep gradients, accretion rates can be reduced by a factor of $\sim 10^{-2}$ compared to HLA theory, as shown in Figure 7.10. Because angular momentum plays a role in limiting accretion rates, we find that the accretion rates of mass and angular momentum depend on the size of the accretor, $R_s$. We have presented simulations with $R_s = 0.01R_a$, which is a realistic scale for a main sequence star embedded in its giant branch companion. Drag forces, and the corresponding energy dissipation rates, on the other hand, are only modified by a factor of a few as shown in Figure 7.13.

- Despite the presence of angular momentum in the flow, persistent disks do not form around the embedded object in our 3D simulations with a $\gamma = 5/3$ equation of state. This differs from results seen in 2D (for example, by Armitage and Livio, 2000). Figure 7.11 shows the lack of disk formation in our flow morphology, while Figure 7.12 shows that a more compressible equation of state, like $\gamma = 1.1$, allows the formation of a persistent disk within the bow shock. The lack of disks on large scales in CE flows may limit the degree to which viscous effects and magnetorotational instability (Balbus and Hawley, 1991) can contribute to the dissipation of angular momentum and, in turn, to accretion. This lack of accretion and disk formation is consistent with the observation of Tocknell et al. (2014) that jets seen in post-CE planetary nebulae might most naturally arise from interactions immediately before or after the dynamical phase.

- In a companion paper (MacLeod and Ramirez-Ruiz, 2015b) we apply the coefficients of drag and accretion derived here to the case of NS inspiralling through
the envelope of its supergiant companion. As argued in Section 7.6.3, this is one of the cases best described by local approximation of the CE flow properties. Due to the reduced efficiency of accretion relative to drag in the presence of a density gradient, we find that NSs undergoing typical CE episodes should accrete only a moderate mass, of order a few percent their own mass or less.

The local calculations of flow around an embedded object during CE inspiral presented in this paper are not intended to replace global calculations. Instead, in these complementary calculations we have adopted a highly simplified description of the complex physics of a CE interaction. Our calculations extend a tradition of numerical calculations of HLA, and treat the role of only one additional physical effect: density gradients within the CE. We find that the density gradient alone drastically modifies the flow around an embedded object as compared to homogenous HLA. This density gradient and ensuing loss of symmetry is likely responsible for the low mass accumulation rate observed by Ricker and Taam (2012) in their global calculation. Effects that may be particularly important to consider in future work include the full geometry and gravitational potential of the CE binary system as well as differential rotation and the disturbed background flow present in realistic CE events.

7.8 Numerical Details

7.8.1 Resolution Study

In Figure 7.17 we confirm the robustness of our results with respect to changing spatial resolution in the $\epsilon_\rho = 0.3$, $R_a = 0.05R_a$ case. This comparison draws on simulations D, Q, R, and S. Because the flow is chaotic, we should not expect formal convergence, but instead aim to demonstrate that the measurement of the mass (left panel) and angular momentum (right panel) accretion rates is not impacted by our choice of numerical resolution around the accretor. We show simulations with four dif-

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Figure 7.17: Study of the $\epsilon_{\rho} = 0.3$, $R_a = 0.05R_a$ case with four different levels of spatial resolution (simulations D, Q, R, and S). The two lower level simulations were run from $t = 0$, while the two higher resolution simulations were started from a checkpoint of the $R_a/\delta_{\text{min}} = 12.8$ simulation at $t = 20$. There appears to be good consistency between the three highest resolution levels for $10R_a/v_\infty$, the crossing time of material from the box boundary. The lowest-resolution simulation exhibits larger variability than its higher-resolution counterparts.

The three higher-resolution simulations all exhibit relatively uniform behavior in terms of mean accretion rate and amplitude of variability. This demonstrates that numerical resolution is not significantly impacting our conclusions in terms of the total rate of accreted mass and angular momentum.

### 7.8.2 Fitting Formulae

We derive fitting formulae for the accretion of mass, angular momentum, and for drag as a function of density gradient in our simulations. As shown in Table 7.1, the $R_a = 0.01R_a$ case is directly relevant to a main sequence star inside CE. Embedded compact objects like WDs, NSs, and BHs have much smaller radii and thus caution should be taken in the extrapolation of these results to those scales.
To fit the mass accretion rate, we use a function of the form

$$\log(\dot{M}/\dot{M}_{HL}) \approx a_1 + \frac{a_2}{1 + a_3 \epsilon_\rho + a_4 \epsilon_\rho^2}.$$  \hspace{1cm} (7.15)

We find $a_i = (-2.14034214, 1.94694764, 1.19007536, 1.05762477)$ for $R_s = 0.01R_a$ and $a_i = (-1.65171739, 1.49979486, 0.10226072, 3.93190671)$ for $R_s = 0.05R_a$ via a least-squares minimization in which weights for individual points are inversely proportional to the variability defined by the 5% and 95% bounds of the data. The accretion of angular momentum is approximated by

$$\frac{\dot{L}_z}{\dot{M}_{HL} R_a v_\infty} \approx \frac{b_1 \epsilon_\rho}{1 + b_2 \epsilon_\rho + b_3 \epsilon_\rho^2}.$$  \hspace{1cm} (7.16)

Using the same approach, we find fitting parameters of $b_i = (1.86818916 \times 10^{-2}, -6.42396570, 3.40135578 \times 10^1)$ for $R_s = 0.01R_a$ and $b_i = (0.06549409, -6.87212261, 27.02371844)$ for $R_s = 0.05R_a$. We fit the function

$$\frac{F_{d,x}}{\pi R_a v_\infty^2} \approx c_1 + c_2 \epsilon_\rho + c_3 \epsilon_\rho^2,$$  \hspace{1cm} (7.19)

to the drag realized in the simulations. Fitting parameters are

$$c_i = (1.91791946, -1.52814698, 0.75992092)$$  \hspace{1cm} (7.20)
for $R_s = 0.01R_a$ and

$$c_i = (1.98255197, -1.33691133, 0.62963326)$$

(7.21)

for $R_s = 0.05R_a$. 
Chapter 8

On the Accretion-Fed Growth of Neutron Stars During Common Envelope

8.1 Chapter Abstract

This paper models the orbital inspiral of a neutron star (NS) through the envelope of its giant-branch companion during a common envelope (CE) episode. These CE episodes are necessary to produce close pairs of NSs that can inspiral and merge due to gravitational wave losses in less than a Hubble time. Because cooling by neutrinos can be very efficient, NSs have been predicted to accumulate significant mass during CE events, perhaps enough to lead them to collapse to black holes. We revisit this conclusion with the additional consideration of CE structure, particularly density gradients across the embedded NS’s accretion radius. This work is informed by our recent numerical simulations that find that the presence of a density gradient strongly limits accretion by imposing a net angular momentum to the flow around the NS. Our calculations suggest that NSs should survive CE encounters. They accrete only modest amounts of envelope material, \( \lesssim 0.1M_\odot \), which is broadly consistent with mass determinations of double NS
binaries. With less mass gain, NSs must spiral deeper to eject their CE, leading to a potential increase in mergers. The survival of NSs in CE events has implications for the formation mechanism of observed double NS binaries, as well as for predicted rates of NS binary gravitational wave inspirals and their electromagnetic counterparts.

8.2 Introduction

The existence of a population of compact neutron star (NS) binaries (Hulse and Taylor, 1975) serves as a unique probe of general relativity (Stairs, 2004) and of binary stellar evolution (Bethe and Brown, 1998; Kalogera et al., 2007; Postnov and Yungelson, 2014). Mergers of NS binaries are promising sources for the detection of gravitational radiation (Phinney, 1991; Belczynski et al., 2002), and are the progenitors of short gamma ray bursts (Narayan et al., 1992; Behroozi et al., 2014). Yet, to inspiral and merge under the influence of gravitational radiation in less than a Hubble time, a compact binary must be separated by less than the radii of its main sequence progenitors (e.g., Peters, 1964). To reach their current small separations, these binaries must have passed through one or more common envelope (CE) phases (Paczynski, 1976).

In a standard evolutionary scenario to produce NS binaries, the companion to a NS evolves and engulfs the NS inside its growing envelope (Taam et al., 1978; Terman et al., 1995; Tauris and van den Heuvel, 2006). Within the shared envelope, the NS focusses envelope gas toward itself. Flow convergence leads to dissipation of orbital energy in shocks and to accretion (Hoyle and Lyttleton, 1939; Iben and Livio, 1993; Ivanova et al., 2013b). Neutrinos serve as an effective cooling channel for this convergent flow, allowing material to be incorporated into the NS at a hypercritical accretion rate well above the classical Eddington limit (Houck and Chevalier, 1991; Fryer et al., 1996; Popham et al., 1999; Brown et al., 2000; Narayan et al., 2001; Lee et al., 2005; Lee and Ramirez-Ruiz, 2006). The relative rates of drag and accretion implied
by Hoyle and Lyttleton (1939) accretion (HLA) theory suggest that an inspiralling NS is likely to grow to collapse to a black hole (BH) before the CE is ejected (Chevalier, 1993; Armitage and Livio, 2000; Bethe et al., 2007), leaving behind a tightened remnant binary (Webbink, 1984).

Despite this apparently clear prediction, reconciling the observed distribution of NS masses (e.g., Schwab et al., 2010; Özel et al., 2012; Kiziltan et al., 2013) with theories of hypercritical accretion in CE has posed a long-standing problem. In particular, double NSs exhibit a narrow range of inferred masses close to the suspected NS birth mass, centered at $1.33M_\odot$ with dispersion of $0.05M_\odot$ (Özel et al., 2012). Alternative evolutionary scenarios have been proposed where the NS can avoid CE, and accretion, entirely. For example, if the binary is sufficiently equal in mass, it could undergo a simultaneous, or double core, CE (Brown, 1995). The issue is that for each binary that passed through a preferred channel one would expect numerous massive NS and BH-NS binaries assembled through the more standard channels (Fryer and Woosley, 1998; Belczynski et al., 2002; Kalogera et al., 2007; Belczynski et al., 2010; Fryer et al., 2013). This picture remains at odds with the apparent paucity of BHs just above the maximum NS mass (Özel et al., 2010, 2012).

In this Letter we re-evaluate claims that BHs should necessarily form via accretion-induced collapse during CE events involving a NS and its massive companion. We draw on results of our recent simulations of accretion flows within a stellar envelope to demonstrate that it is critical to consider not just the binding energy, but also the structural properties of the whole envelope (MacLeod and Ramirez-Ruiz, 2015b). To this end, we follow the inspiral and accretion of a NS during its dynamical inspiral and show that all NSs should be expected to survive CE evolution, accreting only a small fraction of their own mass.
8.3 Characteristic Conditions in NS Accretion

When a NS becomes embedded within a CE, it exerts a gravitational influence on its surroundings and can accrete envelope material. In this section, we explore some characteristic scales for that accretion flow, focusing on how they depend on the supply of material and the microphysics of the gas.

8.3.1 Hoyle-Lyttleton Accretion within a CE

The flow around the NS can be described in the context of the NS’s gravitational interaction with the surrounding medium in HLA theory (Hoyle and Lyttleton, 1939). The NS’s velocity relative to the envelope gas, $v_\infty$, is typically mildly supersonic, $\mathcal{M}_\infty = v_\infty / c_{s,\infty} \gtrsim 1$, where $\mathcal{M}_\infty$ is the flow Mach number and $c_{s,\infty}$ is the local sound speed. Material with an impact parameter less than an accretion radius,

$$R_a = \frac{2GM_{NS}}{v_\infty^2},$$

is focused toward the NS. The resulting accretion rate is

$$\dot{M}_{HL} = \pi R_a^2 \rho_\infty v_\infty,$$

where $\rho_\infty$ is the upstream density (Hoyle and Lyttleton, 1939). Flow convergence leads to shocks that imply a rate of dissipation of kinetic energy, or drag luminosity,

$$\dot{E}_{HL} = \pi R_a^2 \rho_\infty v_\infty^3 = \dot{M}_{HL} v_\infty^2,$$

or a drag force of $F_{d,HL} = \dot{E}_{HL} / v_\infty$ (e.g., Iben and Livio, 1993).

To estimate the growth of the NS during the inspiral, we first approximate the
inspiral timescale as
\[ t_{\text{insp}} \approx \frac{E_{\text{orb}}}{E_{\text{HL}}}, \]
where \( E_{\text{orb}} = GM_{\text{NS}}m/2a \) and \( m \) is the enclosed companion mass at a given orbital separation, \( a \). The accreted mass is thus \( \dot{M}_{\text{HL}} t_{\text{insp}} \), or

\[ \Delta M_{\text{NS}} \sim \dot{M}_{\text{HL}} \frac{E_{\text{orb}}}{E_{\text{HL}}} \sim \frac{M_{\text{NS}} m}{2(M_{\text{NS}} + m)} \]  

where we further assume that \( v_{\infty}^2 = G(M_{\text{NS}} + m)/a \) in the second equality. This estimate reproduces, at the simplest level, the arguments of Chevalier (1993) and later Brown (1995) and Bethe and Brown (1998) who argue that the NS should grow substantially during its inspiral.

### 8.3.2 Microphysics and Hypercritical Accretion

The microphysics of accreting gas imposes several further scales on the accretion rate. The first of these is the Eddington limit. When the accretion luminosity reaches \( L_{\text{Edd}} = 4\pi GM_{\text{NS}}c/\kappa \), where \( \kappa \) is the opacity, radiation pressure counteracts gravity and halts the accretion flow. This limit on the accretion luminosity implies a limit on the accretion rate,

\[ \dot{M}_{\text{Edd}} \approx 2 \times 10^{-8} \left( \frac{R_{\text{NS}}}{12\text{km}} \right) \left( \frac{\kappa}{0.34\text{cm}^2\text{g}^{-1}} \right)^{-1} M_\odot \text{yr}^{-1}. \]  

The Eddington limit may be exceeded under certain conditions if photons are trapped in the accreting flow and carried inward. Photons are trapped within the flow within a trapping radius of approximately (Houck and Chevalier, 1991),

\[ R_{\text{tr}} = \frac{\dot{M} \kappa}{4\pi c} \approx 5.8 \times 10^{13} \left( \frac{\dot{M}}{M_\odot\text{yr}^{-1}} \right) \left( \frac{\kappa}{0.34\text{cm}^2\text{g}^{-1}} \right) \text{cm}. \]  

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Because the NS has a surface, at small radii an accretion shock forms to stall the infalling gas. Within this shock neutrinos are the dominant cooling channel. The shock radius is

$$R_{\text{sh}} \approx 1.6 \times 10^8 \left( \frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right)^{-0.37} \text{ cm},$$  \tag{8.8}$$

where the scaling with accretion rate arises from the neutrino cooling function (Houck and Chevalier, 1991). When $R_{\text{tr}} > R_{\text{sh}}$, accretion energy is advected into the neutrino-cooling layer and super-Eddington, or hypercritical, accretion can proceed (Houck and Chevalier, 1991). This implies a lower-limit accretion rate of

$$\dot{M}_{\text{hyper}} \approx 1.9 \times 10^{-4} \left( \frac{\kappa}{0.34 \text{cm}^2 \text{ g}^{-1}} \right)^{-0.73} M_\odot \text{ yr}^{-1},$$  \tag{8.9}$$

where if $\dot{M} \gtrsim \dot{M}_{\text{hyper}} \sim 10^4 \dot{M}_{\text{Edd}}$ accretion can proceed despite the violation of the photon Eddington limit. No cooling solutions exist for $\dot{M}_{\text{Edd}} < \dot{M} < \dot{M}_{\text{hyper}}$, so mass supplied at these rates can only be incorporated at $\dot{M}_{\text{Edd}}$. Further investigation of these basic claims came in the form of multidimensional simulations, that confirmed that hypercritical accretion can reach a steady-state for some range of accretion rates while at others high entropy plumes intermittently overturn the flow (Fryer et al., 1996). For a flow with some rotational support, the critical $\dot{M}_{\text{hyper}}$ may be somewhat higher than for the spherical case described above (Chevalier, 1996; Brown et al., 2000).

### 8.3.3 Limits on the Accretion Rate due to Flow Asymmetry

In order to track accretion onto a NS during CE, we need a clear prediction of the accretion rate as a function of CE structure. In MacLeod and Ramirez-Ruiz (2015b), we use the FLASH adaptive mesh hydrodynamics code (Fryxell et al., 2000) to extend three-dimensional (3D) simulations of HLA to consider the role of an inhomogeneous
Figure 8.1: Flow morphologies, drag, and accretion in 3D simulations of HLA with an upstream density gradient. The introduction of upstream density gradients, as found in CE evolution, breaks the symmetry of HLA and gives rise to the tilted bow shock structures seen here. The coordinates in the flow panel are in units of the accretion radius, \( R_a \), and the accretor is defined as an absorbing sink condition with \( R_s = 0.01R_a \) surrounding a central point mass. Flow momenta do not cancel in the wake of the accretor with upstream inhomogeniety and the post-shock region is defined by rotation. In the right panel, we compare the resultant drag and accretion rate normalized to values anticipated by HLA. We find that the drag force depends only mildly on density gradient, but the accretion rate decreases drastically as the density gradient, \( \epsilon_\rho \), steepens. These calculations use a gamma-law equation of state with \( \gamma = 5/3 \).
upstream medium. We characterize the density gradient across the accretion radius as

\[ \epsilon_\rho = \frac{R_a}{H_\rho} \quad (8.10) \]

where \( H_\rho = -pdr/d\rho \), the density scale height. A planar density gradient is then applied along the simulation \( y \)-axis, perpendicular to the direction of motion, with \( \rho = \rho_\infty \exp(\epsilon_\rho y) \), where \( \rho_\infty \) is the density at zero impact parameter, \( y = 0 \). We find that typical values for \( \epsilon_\rho \) in CE range from \( \epsilon_\rho \approx 0.3 - 3 \).

Strong density gradients break the symmetry that defines HLA, severely limiting accretion. The momenta of opposing streamlines do not fully cancel with the introduction of inhomogeneity, and the resulting flow carries angular momentum with respect to the accretor. Thus, even if material is gravitationally captured it may not be accreted because of this angular momentum barrier. The HLA formula, Equation (8.2), drastically overestimates the resultant accretion rate (see also Ricker and Taam, 2008, 2012). In Figure 8.1, we show how flow morphology, drag force, and accretion rate change with steepening density gradients.

### 8.4 Inspiral and Accretion

In order to trace the NS inspiral through the dynamical phase of CE evolution, we integrate coupled equations for the evolution of the orbit and accretion onto the NS. We discuss our initial models, evolution equations, and findings below.

#### 8.4.1 Methods

To create approximate CE conditions, we evolve single stars in the MESA stellar evolution code (version 5527: Paxton et al., 2011, 2013). During the giant-branch phase, a CE event may be initiated when the radius of the stellar envelope grows to be similar to the binary separation, \( R_\ast \sim a \). We make the simplifying approximation of a
static CE profile. This is most valid when the companion mass is much greater than the NS mass, $M_{\text{comp}} \gg M_{\text{NS}}$. The progenitors of NSs in binaries are massive stars, so we calculate the structure of giant-branch envelopes of with initial masses of $12 - 20M_\odot$. A comparison of these envelope structures, and the typical flow Mach numbers and density gradients they give rise to, is shown in Figure 8.2.

Orbital energy is dissipated at a rate

$$\dot{E}_{\text{orb}} = -F_d(\epsilon, \rho)v_\infty,$$  

(8.11)

where $F_d(\epsilon, \rho)$ is approximated using a fit to our simulation results described in Section 8.3.3,

$$\frac{F_d(\epsilon, \rho)}{F_{d,\text{HL}}} \approx f_1 + f_2\epsilon + f_3\epsilon^2,$$  

(8.12)

with $f_i = (1.91791946, -1.52814698, 0.75992092)$. As a result of this drag, the orbital separation evolves at a rate $\dot{a} = \dot{E}_{\text{orb}}(da/dE_{\text{orb}})$. We terminate our integration of the dynamical inspiral when the integrated change in orbital energy equals the envelope binding energy at a given CE radius $\Delta E_{\text{orb}}(a) = E_{\text{env}}(a)$, equivalent to $\alpha_{\text{CE}} = 1$, (Webbink, 1984). The envelope binding energy is computed as

$$E_{\text{env}}(a) = \int_{M(a)}^M u - \frac{Gm}{r}dm,$$  

(8.13)

where we have included both the gravitational binding energy and internal energy of the stellar fluid.

The expression regulating accretion onto the NS is

$$\dot{M}_{\text{NS}} = \dot{M}(\epsilon, \rho),$$  

(8.14)
Figure 8.2: Envelope structure of a range of giant star models that share 600$R_\odot$. The top panel shows the density profile of the envelopes, while the center and lower panels show flow the Mach number and the density gradient that would be experienced by an inspiralling 1.33$M_\odot$ NS. For much of the stellar interior, mach numbers are moderate $M_\infty \approx 1.5 - 3$ with density gradients of $\epsilon_{\rho} \approx 1 - 2.5$, representing substantial density inhomogeneity across the accretion radius. Spikes in the density gradient are seen in the deep interior at transitions in chemical composition.
where, like the drag, $\dot{M}(\epsilon\rho)$ is a fit to our numerical results,

$$\log \left( \frac{\dot{M}(\epsilon\rho)}{\dot{M}_{\text{HL}}} \right) \approx m_1 + \frac{m_2}{1 + m_3 \epsilon\rho + m_4 \epsilon^2\rho}$$

with $m_i = (-2.14034214, 1.94694764, 1.19007536, 1.05762477)$. To give a baseline for comparison, we also compute orbital inspiral sequences using HLA theory, with $\dot{E}_{\text{orb}} = \dot{E}_{\text{HL}}$ and $\dot{M}_{\text{NS}} = \dot{M}_{\text{HL}}$.

We make several approximations that likely lead our calculation of the accreted mass in CE to be an overestimate. First, in assuming a static structure for the CE, we may overestimate the local density of the dispersing envelope (see, e.g., Ricker and Taam, 2012). Second, we assume that hypercritical accretion and cooling by neutrinos are effective above $\dot{M}_{\text{hyper}}$, despite the fact that this may not apply at all values of $\dot{M} > \dot{M}_{\text{hyper}}$, in particular with varying amounts of angular momentum (Fryer et al., 1996; Chevalier, 1996; Brown et al., 2000). Finally, we compute the mass accretion rate, Equation (8.15), assuming that all mass passing through $R_s = 0.01R_a$ is able to propagate the additional 2-3 orders of magnitude in radial scale to $R_{sh}$, where cooling can occur. Thus, our integration represents an upper limit for the potential accreted mass onto an embedded NS.

### 8.4.2 Results

We begin by comparing orbital inspirals based on HLA theory with simulation coefficients for drag and accretion in Figure 8.3. This comparison highlights the need to consider the role of the structure of the CE around the embedded NS. In the HLA case, the NS gains more than $1M_\odot$, enough mass to push it above the $\sim 2M_\odot$ maximum NS mass, and in agreement with our analytic prediction of Section 8.3.1. However, in the simulation case, we see that $\dot{M}_{\text{NS}}$, and in turn $\Delta M_{\text{NS}}$ are both severely limited by flow asymmetry. $\dot{M}_{\text{NS}}$ is still sufficiently high to allow hypercritical accretion to proceed
Figure 8.3: Orbital inspiral of an originally $1.33M_\odot$ NS through the envelope of its $12M_\odot$, $500R_\odot$ companion. The left panels show evolution of the orbital separation (top), and energies (bottom). The right hand panels show the mass accretion of the NS in terms of the separation. Blue lines show the results assuming HLA, while yellow lines take into account the effect of asymmetry on the accretion rate. In HLA, the NS grows well beyond the maximum NS mass, acquiring more than a solar mass during its inspiral. However, the loss of symmetry in the accretion flow limits $\dot{M}_{\text{NS}}$, such that the NS gains less than $0.1M_\odot$ and survives the CE.
(Chevalier, 1993), but the NS gains less than $0.1M_\odot$ during its inspiral. This accreted mass represents a few percent of the NS’s mass. Thus, the final compact object is a slightly more massive NS, rather than a BH.

We now extend our calculation to consider a diversity of pre-CE structures. In Figure 8.4, we plot only those structures for which the CE ejection is successful, where $\Delta E_{\text{orb}}(a) = E_{\text{env}}(a)$ at some radius in the stellar interior. In general, this criteria is satisfied when a distinct helium core and convective envelope structure forms. Minimum orbital periods are in the range 0.1-2 yr as determined by the masses and radii of the companions at the onset of CE. This is in agreement with the analysis of Terman et al. (1995) who found a dividing period of 0.8-2 yr for companions of 12-24 $M_\odot$. CE events involving less-evolved giants (smaller $R_{\text{comp}}$ than those plotted) likely lead to complete mergers because the NS is unable to eject its companion’s envelope. The merger could result in the formation of either stably burning Thorne-Zytkow objects (Thorne and Zytkow, 1977; Levesque et al., 2014) or explosive transients (Fryer et al., 2013).

In each CE structure considered, the NS survives CE. Perhaps more strikingly, it gains only a few percent of its own mass across a broad array of different envelope structures. In general, NSs gain the most mass in interactions with more massive companions. There are two reasons for this effect. First, the NS must spiral deeper to eject its companion’s envelope when the mass ratio is larger (Webbink, 1984). Second, large mass ratios imply that $R_a$ is a smaller fraction of $R_{\text{comp}}$, and as a result, the effective density gradient, $\epsilon_\rho$, is reduced (Figure 8.2), allowing for more efficient accretion (Figure 8.1). The NS gains less mass in interactions with more extended companions (for a given mass) because these envelopes are comparatively easier to unbind.

Mass accretion implies a spin-up of the NS based on the specific angular momentum of accreting material. In cases where the NS mass is not well determined, the pulsar spin period can still offer constraints on the accreted mass. The spin-up is $\Delta \Omega = \Delta L/I_{\text{NS}}$, where $\Delta L$ is the accreted angular momentum and $I_{\text{NS}}$ is the NS’s
Figure 8.4: Post-CE states for originally $1.33M_{\odot}$ NSs involved in interactions with wide variety of companions. Companions range in pre-CE mass from $12 - 20M_{\odot}$, in radius from $100 - 1100 R_{\odot}$, and have convective envelopes. CE events initiated with smaller-radius companions than those plotted (for a given mass) result in merger rather than envelope ejection. All evolutions computed result in NSs surviving CE, with none expected to undergo accretion-induced collapse to a BH. NSs generally gain more mass in interactions with more massive companions, but very extended radius at the onset of CE can lead to less mass accumulation. To compute the right-hand axis we assume material is accreted from a Keplerian disk, and that adopt median NS properties of $M_{\text{NS}} \approx 1.39M_{\odot}$, $R_{\text{NS}} = 12\text{km}$. NSs undergoing these CE episodes survive to find themselves in close partnerships with helium-rich companions and orbital periods ranging from hours to days.
moment of inertia. We estimate $\Delta L$ assuming that material is accreted from a neutrino cooled disk surrounding the NS, $\Delta L \approx \Delta M \sqrt{GM_{\text{NS}}R_{\text{NS}}}$, and that $I_{\text{NS}} = 2/5M_{\text{NS}}R_{\text{NS}}^2$. The post-CE spin period of the recycled NS can then be estimated as $2\pi/\Delta \Omega$. The calculated spin periods are shorter than those observed presently for first-born pulsars in double NS systems ($P \sim 20 - 100$ ms, Osłowski et al., 2011). However, NSs with magnetic fields $\gtrsim 10^{10}$G will quickly spin down from these initial periods, so spin constraints are most valuable where the NS magnetic field is small (and thus the spin-down timescale is long). Two of the ten pulsars listed by Osłowski et al. (2011) meet this criterion, pulsars J1518+4904 and J1829+2456. Neither of these object has a well-determined mass, but their measured pulse periods (both $\approx 41$ms) and period derivatives $\lesssim 10^{-19}$ss$^{-1}$ imply spin down timescales $> 10$ Gyr, allowing direct comparison to Figure 8.4. If spun-up by accretion during CE, the spin of these objects is consistent with having gained of order $0.01M_{\odot}$.

8.5 Discussion

We have self-consistently evaluated the mass growth of a NS embedded within a CE, taking into account that the accretion rate depends sensitively on the structure of the CE (MacLeod and Ramirez-Ruiz, 2015b). Although our integration likely represents an upper limit (as discussed in Section 8.4.1), we observe that NSs gain only a few percent of their own mass during CE episodes. These objects emerge from CE only mildly heavier and more rapidly spinning, rather than undergoing accretion-induced collapse to BHs (Chevalier, 1993; Armitage and Livio, 2000). It appears that density gradients in typical CE structures are the missing link needed to reconcile theories of hypercritical accretion onto NSs (Houck and Chevalier, 1991; Chevalier, 1993; Fryer et al., 1996) with the narrow observed mass distribution of NS masses (Schwab et al., 2010; Özel et al., 2012; Kiziltan et al., 2013). This result hints that forming double
NS binaries may not require a finely-tuned evolutionary channel (Brown, 1995; Bethe and Brown, 1998), but they could instead emerge from within the standard CE binary evolution framework (e.g., Stairs, 2004; Tauris and van den Heuvel, 2006).

Further investigation is certainly needed to probe the efficiency of CE ejection by embedded NSs, as well as the dynamical timescale effects of envelope dispersal (for example, as studied by Terman et al., 1995). We note that a reduction in $\dot{M}$, as compared to $\dot{M}_{\text{HL}}$, may hinder envelope ejection in that any potential accretion disk feedback (Armitage and Livio, 2000; Papish et al., 2013) would be weakened relative to the envelope’s binding energy. This hints that other forms of feedback that may be less dependent on $\dot{M}$, like nuclear burning or recombination, may be of more assistance in CE ejection (Iben and Livio, 1993; Ivanova et al., 2015). Studies that consider these energy sources can best determine the critical separation (or orbital period) that divides binaries that merge from those that successfully eject their envelopes.

The CE stage described here is not the full story of the evolution of a binary. In many binaries, the first-born pulsars interact with their helium-star companions following the CE (Tauris and van den Heuvel, 2006). If an additional CE were to occur, the low mass and steep density gradients of the He star’s typically radiative envelope (Dewi et al., 2002) suggest a relative ease of envelope ejection and a low accretion efficiency. However, as the most-recent interaction, this phase of mass transfer or CE would be responsible for the current spin of the first-born pulsar. These more complex interaction histories are best traced with population synthesis calculations, where the ramifications of observed masses, spins, and orbital eccentricities offer a window to the outcome of the CE phase (Kalogera et al., 2007; Dominik et al., 2012).

We anticipate that moving beyond the energy formalism of CE (Webbink, 1984) to also consider CE structure, as parameterized by density gradient $\epsilon_\rho$, will shape the channels through which double compact binaries can be expected to form. As a result of the structures of their companions, few to none of the NSs entering CE
episodes should be expected to collapse to BHs.
Chapter 9

The Onset of a Common Envelope Episode: Lessons from the Remarkable M31 2015 Luminous Red Nova Outburst

9.1 Chapter Abstract

This paper investigates the recent transient M31LRN 2015 in the Andromeda galaxy. We analyze published optical photometry and spectroscopy along with a Hubble Space Telescope detection of the color and magnitude of the pre-outburst source. Using these data, we determine that the transient outburst is caused by dynamically driven ejecta at the onset of a common envelope episode, which eventually leads to the complete merger of a binary system. Just prior to merger, we find that the primary star is a $3 - 5.5M_\odot$ sub-giant branch star of $30 - 40R_\odot$. Its position in the color-magnitude diagram shows that it is growing in radius to eventually engulf its companion. By matching the properties of the binary system to the transient outburst, we show that the light curve contains two components: first $10^{-2}M_\odot$ of fast ejecta driven by shocks at contact in between the primary and secondary, and later, $0.2M_\odot$ of slower ejecta as the secondary becomes more deeply engulfed within the primary. The optical transient lasts
about ten orbits of the original binary. We also use the orbital dynamics leading up to merger to place constraints on the unseen secondary star, we find that it is likely to be a lower mass main sequence star of $0.1 - 0.6 M_\odot$. This analysis represents a promising step toward a more detailed understanding of flows in common envelope episodes through direct observational constraints.

9.2 Introduction

The majority of stars live in binary and higher order multiple systems (Duchêne and Kraus, 2013). As stars evolve off the main sequence and their radii grow significantly, binary systems that were non-interacting can evolve into contact. Among triple and higher-order multiple systems, secular interactions can drive pairs of objects into eccentric orbits that lead to their direct impact (e.g. Kozai, 1962; Thompson, 2011). Through these interaction channels, stellar multiplicity plays a key role in shaping the evolution of stellar populations (e.g. Sana et al., 2012; de Mink et al., 2013).

The outcome of a stellar interaction is shaped by the orbital dynamics, relative masses and evolutionary states of the component stars. Tidally synchronized close binaries which come into contact on the main sequence can form peanut-shaped, yet stable, overcontact systems known as W UMa stars (e.g. Shu et al., 1976; Webbink, 1976; Robertson and Eggleton, 1977; Rasio, 1995). Other interacting pairs, especially those with a more massive evolved-star donor, become unstable at or before the onset of mass transfer. This instability can arise from the transfer of mass from a more massive component to a less massive one, narrowing the orbital separation (e.g. Pejcha, 2014) or from orbital instability prior to mass exchange (as in the Darwin (1879) tidal instability). In either of these cases, or in the more-violent case of eccentric interaction, the component objects are driven toward runaway merger.

The runaway merger of two stars can lead to either a remnant composed of the
bulk of the mass of the pair, or to the formation of a new, tighter binary system. In the case of a remnant binary, the new pair formed by the secondary star along with the core of the primary must inject enough energy into its surrounding gaseous common envelope to clear its surroundings and stabilize as a system transformed by a phase of orbital inspiral (Paczynski, 1976; Meyer and Meyer-Hofmeister, 1979; Iben and Livio, 1993; Taam and Sandquist, 2000; Taam and Ricker, 2010; Ivanova et al., 2013b). These phases of orbital transformation are key in shaping populations of compact binaries which interact or merge (Kalogera et al., 2007; Postnov and Yungelson, 2014) – producing some of the most dramatic electromagnetic and gravitational transients when they do (Abbott et al., 2016). Yet the hydrodynamics, overall efficiency, and division of systems that merge and those that eject their envelopes following these phases of unstable binary interaction remain poorly understood subjects of intense scrutiny (Ivanova et al., 2013b).

In order to improve our understanding of common envelope episodes, we can rely on before-and-after comparisons of stellar populations or attempt to catch common envelope events and stellar mergers in action (e.g. Ivanova et al., 2013a,b). We should expect the onset of common envelope episodes and binary mergers to proceed similarly – particularly if the system is composed of one evolved star and a more compact companion. A possible distinction arises later, following a phase of orbital inspiral, when the bulk of the envelope material either is (or is not) driven off (e.g. Ricker and Taam, 2008, 2012; Passy et al., 2012a). At the onset of interaction between two stars, a small portion of mass will be driven off, powering an optical or infrared transient as it expands and becomes transparent (Soker and Tylenda, 2006; Metzger et al., 2012b). The detection and detailed study of this category of stellar merger and common envelope transients offers direct constraints on the conditions and flow properties at the onset of this highly uncertain phase of binary evolution.

An emergent class of transients – luminous red novae (LRN) – have come to be associated with stellar mergers through detailed study of a few key events. M31
RV was one of the first red transients to be identified, in 1988, but its lightcurve is only captured during the decline (Mould et al., 1990; Bryan and Royer, 1992; Boschi and Munari, 2004; Bond and Siegel, 2006; Bond, 2011). On the other hand, V838 Mon illuminated its surroundings with a spectacular light echo imaged by the Hubble Space Telescope (HST) from 2002-2008 following its outburst (Bond et al., 2002; Munari et al., 2002; Kimeswenger et al., 2002; Bond et al., 2003; Munari et al., 2005; Sparks et al., 2008; Antonini et al., 2010). The object V1309 Sco, another galactic transient, proved critical in establishing the origins of this class of transients (Mason et al., 2010; Nicholls et al., 2013). A multi-year time series of photometric data taken by OGLE (Rucinski, 1998; Udalski et al., 2008) revealed an eclipsing binary with decreasing orbital period prior to the outburst (Tylenda et al., 2011; Stepien, 2011). Following the event, a single object remained (Tylenda et al., 2011; Nicholls et al., 2013; Kamiński et al., 2015).

Other recently-discovered transients populate a similar (or slightly longer duration and more luminous) phase space in a luminosity-timescale diagram (Kasliwal, 2012). These include objects like NGC 300-OT (Bond et al., 2009; Berger et al., 2009), PTF 10fqs (Kasliwal et al., 2011), M85-OT (Kulkarni et al., 2007; Rau et al., 2007; Ofek et al., 2008), and supernova 2008S (Smith et al., 2009), and are generally categorized as intermediate-luminosity optical transients with a massive star-outburst (rather than binary) origin (e.g. Thompson et al., 2009; Kochanek, 2011; Smith et al., 2016).

This paper focuses on a new, and particularly remarkable, addition to the category of stellar merger transients: M31LRN 2015. Discovered in January 2015 by the MASTER network\(^1\), the transient resides in the Andromeda galaxy. The transient was discovered early, about 8 days prior to peak brightness, and both Williams et al. (2015) and Kurtenkov et al. (2015) have published multicolor photometry of the outburst light curve along with spectra. What makes this event unique is the existence of multi-band pre-outburst imaging of the source ten years prior to the transient. Since the association

\(^1\)http://observ.pereplet.ru/
with M31 (and thus the distance) is known, confirmation of the pre-outburst color and magnitude allow us to compare the detailed physical properties of the binary system to the merger-driven outburst it produces. Uncertainties in the progenitor mass and color (in the case of V838 Mon Munari et al., 2005; Tylenda et al., 2005) or in its distance (in the case of V1309 Sco Tylenda et al., 2011) have hindered such an analysis of previous well-studied transients.

This comparison of the M31LRN 2015 binary system to its transient outburst reveals a binary in which the primary star is evolving until it engulfs its companion. The presence of pre-outburst data allows us to confirm that the M31LRN 2015 transient thus marks the onset of a common envelope episode. This connection adds significant value to the detailed data presented by Williams et al. (2015) and Kurtenkov et al. (2015) focusing on the outburst itself. We are able to compare ejecta velocities and estimated masses to the masses of the binary components, and use this information to attempt to constrain the pathway through which the binary merged and the mass of the unseen secondary star. The observational constraints that this system imposes on the hydrodynamics of the onset of common envelope are very useful given the enormous theoretical uncertainty surrounding this phase of common envelope events.

Our analysis of this source proceeds as follows. In Section 9.3, we focus on the transient outburst, and derive some physical properties of the ejecta from the photometric and spectroscopic data published by Williams et al. (2015) and Kurtenkov et al. (2015). In Section 9.4, we consider the pre-outburst detection by HST, and fit stellar tracks to find the primary-star’s mass, radius, and internal structure from its position in the color-magnitude diagram. In Section 9.5, we consider the properties of the transient that emerge when we consider both the outburst and the pre-merger source together. Section 9.6 extends our discussion to consider the pathways that a binary may take to the onset of a common envelope phase. In the context of these arguments, we are able to place constraints on the mass of the unseen secondary star that gave rise to the
M31LRN 2015 outburst. In Section 9.7, we discuss M31LRN 2015 in the context of other similar transients and conclude.

9.3 Outburst

9.3.1 Summary of Observations

M31LRN 2015 (MASTER OT J004207.99+405501.1) was discovered in January 2015 (Williams et al., 2015), and both Williams et al. (2015) and Kurtenkov et al. (2015) have since published photometry and spectroscopy of the outburst. The source is located in M31 (Williams et al., 2015). While the distance is therefore well known, the reddening to the source has been estimated at $E(B-V) = 0.12 \pm 0.06$ mag (Williams et al., 2015) or $E(B-V) = 0.35 \pm 0.1$ mag (Kurtenkov et al., 2015). In Figure 9.1, we show the outburst absolute light curve using photometric data from both Williams et al. (2015) and Kurtenkov et al. (2015) under the assumption of $E(B-V) = 0.15$ mag. The transient peaks at $M_V \approx -9.5$, and reddens progressively during its decline. The elapsed time from discovery to peak is approximately 8 d, and the optically-bright portion of the light curve persists for $\sim 50$ d more.

These photometric data have been supplemented by spectroscopic observations reported both by Williams et al. (2015) and Kurtenkov et al. (2015). Williams et al. (2015)’s data has a spectral resolution of 18 Å, or $R \approx 360$ at 6500 Å (821 km s$^{-1}$). A spectrum taken 3.2 days after the discovery of the transient and 4.7 days prior to peak shows a strong Hα emission feature with an uncorrected FWHM of $900 \pm 100$ km s$^{-1}$ (Williams et al., 2015). When corrected to instrumental resolution (by subtracting the instrumental resolution in quadrature) this suggests an intrinsic FWHM that may be associated with the expanding ejecta of

$$v_{ej} = 370 \pm 240 \text{ km s}^{-1}. \quad (9.1)$$
Figure 9.1: Absolute light curve of the optical outburst of M31LRN 2015. Data from Williams et al. (2015) (open points) and Kurtenkov et al. (2015) (closed points). This figure is constructed assuming $E(B-V) = 0.15$ mag and error bars show measurement and distance errors while ignoring reddening error. The similarity of this lightcurve to other LRN transients like V838 Mon initially marked M31LRN 2015 as a potential stellar merger.

Over time, the H\textalpha emission fades and Na I D and Ba II absorption features appear. Kurtenkov et al. (2015) also report H\textalpha emission in early spectra, but they do not report an emission line FWHM. In an $R = 1000$ spectrum covering 4000-5000 Å taken 6 days before peak, Kurtenkov et al. (2015) report a host of absorption lines and a similarity to a stellar F5I spectrum.

9.3.2 Modeled Properties

In this section we use the photometry from Williams et al. (2015) and Kurtenkov et al. (2015) shown in Figure 9.1 to derive some physical properties of the transient outburst under the assumption of a blackbody spectral energy distribution.

Because of the uncertain source reddening, we apply corrections for $E(B-V)$
Figure 9.2: Photometric properties of the outburst modeled as blackbody emission. Overset lines in each panel represent different assumed reddening, \( E(B - V) \). Points and error bars are plotted for \( E(B - V) = 0.15 \) mag with the assumption of no error in the reddening vector itself. The transient first becomes bluer with peak temperature of \( 10^4 \) K at the time of lightcurve peak, then progressively redder. The bolometric lightcurve shows two components 1) a rapid rise to peak lasting \( \sim 8 \) d, followed by a brief fading of similar duration and 2) a longer lived plateau extending from 10 to 50 d after the time of peak. During this time the photosphere radius increases from \( \sim 2 \) AU around peak to a maximum of \( \sim 10 \) AU approximately 35 d after peak before beginning to recede. In the background of this panel we mark lines of constant expansion velocity of 100-500 km s\(^{-1}\).
between 0.05 mag and 0.45 mag (in increments of 0.1 mag), spanning the full range of estimates by Williams et al. (2015) and Kurtenkov et al. (2015). Figure 9.2 shows different values of the reddening to the source as different line colors. We plot the data points (in black) only for one value of the reddening, $E(B - V) = 0.15$ mag. The plotted error bars represent the propagation of photometric measurement errors with the assumption of no reddening uncertainty.

We use the $B - R$ color (upper left panel) to estimate the effective temperature assuming that the spectral energy distribution is a blackbody (upper right panel). With the effective temperature, we derive the $R$-band bolometric correction, and apply that to find $M_{bol}$ and $L_{bol}$ (lower left panel). Finally, assuming $L_{bol} = 4\pi R_{bb}^2 \sigma_b T_{eff}^4$, we derive the radius of the blackbody photosphere (lower right panel). The effective temperature starts around 6000 K at the time of the first observations, and peaks near the time of maximum brightness of the transient, with maximum temperatures of $\sim 10^4$ K (dependent on reddening). Over the following $\sim 40$ d, the transient’s photosphere becomes cooler, with inferred temperatures falling to the 3000-4000 K range.

During the time mapped by the observations, the inferred photosphere radius, $R_{bb}$, first expands, then recedes. Near the time of peak the photosphere radius is approximately 2 AU. It reaches a maximum of approximately 10 AU $\sim 35$ days after the lightcurve peak. The velocity of this expansion and the maximum photosphere radius reached are a weak function of the assumed reddening. Lines of constant expansion velocity are shown in the background of Figure 9.2, and the typical expansion velocity inferred is $\sim 400$ km s$^{-1}$.

The transient’s bolometric light curve exhibits a two-part structure consisting of an early rise to peak and subsequent decay (times of approximately -10 to +10 days relative to the peak time in Figure 9.2) followed by a longer plateau of approximately constant bolometric luminosity. On the basis of this observation, we will refer to two
portions of the lightcurve as the peak (times of ±10 d relative to peak at JD 2457044.189) and plateau (times of 10 to 50 d after peak).

Figure 9.3 explores the radiated energy during the outburst. The upper panel shows the cumulative radiated energy. This rises steeply in the peak portion of the lightcurve, and more shallily after peak. The relative slopes are determined by the reddening. When the reddening is high, the peak is most accentuated, as is also the case in the lower-left panel of Figure 9.2. The relative contributions of the peak and plateau to the total integrated radiated energy (≈ 10^{46} erg) are also shown in Figure 9.3. The plateau always dominates the energy release, with the peak contributing 30-50% of the radiated energy, or an energy of 2 − 4 × 10^{45} erg.

9.3.3 Ejecta Mass and Outburst Energetics

Although the properties of the merger-driven outflow are likely to be complex, we can still draw some important conclusions from the physical properties inferred from the light curve, especially when paired with the spectroscopic data. In this section we divide our focus based on the two key phases of the bolometric light curve as visible in the lower left panel of Figure 9.2: the peak (±10 d) and the plateau (10-50 d).

To begin, we estimate the minimum mass needed to produce an opaque shell at the observed blackbody radius \( R_{bb} \) by assuming that the obscuring mass is spread over a sphere of radius \( R_{bb} \) with constant density, \( \rho \). We then estimate the gas mass \( \Delta m_{\text{obsc}} \) needed to produce an optical depth, \( \tau = \rho \kappa R_{bb} = \Delta m_{\text{obsc}} \kappa / (4/3 \pi R_{bb}^2) \), of order unity,

\[
\Delta m_{\text{obsc}} \approx 1.4 \times 10^{-6} M_\odot \left( \frac{R_{bb}}{\text{AU}} \right)^2,
\]

where we’ve assumed an electron scattering opacity in solar composition material of \( \kappa = 0.34 \text{ cm}^2 \text{ g}^{-1} \). This shows that even a tiny ejecta mass can be responsible for the growth in the emitting surface.
Figure 9.3: The radiated bolometric energy inferred from the optical light curve of M31LRN 2015. The upper panel shows the cumulative radiated energy throughout the outburst for the same reddening assumptions as in Figure 9.2. The lower panel plots the integral quantities as a function of reddening to the source. The contributions from the peak and plateau sub-components are computed from the ±10 d and the 10-50 d portions of the light curve shown in Figure 9.2. The radiated energy is dominated by the plateau in all cases, with the peak contributing roughly 30-50% of the total. Regardless of the reddening, the total radiated energy is within a factor of two of $10^{46}$ erg.
9.3.3.1 Early Peak

Here we focus on the early portion of the light curve in order to derive some additional constraints. The minimum obscuring mass (estimated above) is sufficient to explain the growth of $R_{bb}$. But it cannot explain the rise and fall timescales of the light curve, which require that heat does not diffuse out of the expanding gas instantaneously.

When the photon mean free path is of order the size of the object, the heat diffusion time is of order the light crossing time – just 8 minutes for 1 AU. Instead, we use the behavior of an expanding shell of gaseous ejecta with diffusion but no other heating or cooling, following Padmanabhan (2001, chapter 4.8). The first law of thermodynamics implies that the light curve will peak at $\tau_{\text{peak}} = \sqrt{2\tau_d \tau_h}$, where $\tau_d$ is the diffusion time of photons through the gas at the initial radius $\tau_d = f\kappa \Delta m_{\text{ej,peak}}/(cr_0)$. The constant is $f \approx 0.07$ for a spherical shell (Padmanabhan, 2001). $\tau_h = r_0/v_{\text{ej}}$ is the hydrodynamic timescale based on the initial radius $r_0$ and velocity $v$. The time of peak is

$$\tau_{\text{peak}} \approx 10.7 \text{ d} \left(\frac{\Delta m_{\text{ej,peak}}}{0.01 M_\odot}\right)^2 \left(\frac{v_{\text{ej}}}{370 \text{ km s}^{-1}}\right)^{-1/2},$$

(9.3)

with $\kappa = 0.34 \text{ cm}^2 \text{ g}^{-1}$. We’ve substituted a characteristic velocity of the outflowing material based on the corrected Hα FWHM velocity, $v_{\text{ej}} = 370 \text{ km s}^{-1}$, equation (9.1), measured by Williams et al. (2015) approximately 4.7 d before the lightcurve peak.

We can re-express the diffusion argument above to solve for the ejected mass in terms of the observed properties of the early peak of the light curve,

$$\Delta m_{\text{ej,peak}} \approx 5.6 \times 10^{-3} M_\odot \left(\frac{\tau_{\text{peak}}}{8 \text{ d}}\right)^2 \left(\frac{v_{\text{ej}}}{370 \text{ km s}^{-1}}\right).$$

(9.4)

This ejected mass is large compared to the minimum obscuring mass estimated above (as is evident from the fact that the light curve’s rise to initial peak is much longer than the light crossing time). The total kinetic energy carried by the outflowing shell
of material, \( E_K = \frac{1}{2}mv^2 \), is

\[
E_{K,\text{peak}} \approx 7.6 \times 10^{45} \text{ erg} \left( \frac{\tau_{\text{peak}}}{8 \text{ d}} \right)^2 \left( \frac{v_{\text{ej}}}{370 \text{ km s}^{-1}} \right)^3. 
\]

(9.5)

We note that because of the velocity-cubed behavior of this expression, the kinetic energy estimate is relatively sensitive to the ejecta velocity uncertainty. To illustrate this more quantitatively, the ±240 km s\(^{-1}\) error bars of equation (9.1) allow for an ejecta velocity of up to 610 km s\(^{-1}\) within one sigma, which gives \( E_{K,\text{peak}} \approx 3.2 \times 10^{46} \text{ erg} \) in equation (9.5).

The kinetic energy estimate of equation (9.5) can be compared to the radiated energy in the peak portion of the light curve, as plotted with a dot-dash line in the lower panel of Figure 9.3. For a reddening of \( E(B-V) = 0.12 \text{ mag} \), the radiated energy during the early-peak portion of the lightcurve is \( \approx 2.0 \times 10^{45} \text{ erg} \). The kinetic energy estimated in equation (9.5) is thus a factor of \( \sim 4 \) larger than the radiated energy. If the velocity of the ejecta is higher than the nominal value, then this ratio increases. For \( v_{\text{ej}} = 610 \text{ km s}^{-1} \), the kinetic energy exceeds the radiated by a factor of \( \sim 16 \). In the absence of additional heating or cooling, adiabatic expansion decreases the internal energy – and thus also the radiated energy – relative to the kinetic by \( (R_{bb}/r_0)^{-1} \) for radiation pressure dominated ejecta. Because the material must expand before it becomes transparent, we expect the radiated energy to be lower than the kinetic.

9.3.3.2 Plateau

The second phase of the light curve morphology, the plateau, exhibits relatively constant bolometric luminosity for \( \sim 40 \text{ d} \) and spans from 10 to 50 d after peak (as seen in the lower-left panel of Figure 9.2). The color evolution slows during this phase, and the effective temperature stabilizes in the \( 3 - 4 \times 10^3 \text{ K} \) range (depending on reddening). These properties suggest that hydrogen recombination might be stabilizing
the effective temperature as we observe a recombination wave propagating through the ejecta, lowering the opacity of material that recombines, as described by Popov (1993) for type IIP supernovae. This idea was introduced in the context of LRN by Ivanova et al. (2013a) where it was invoked as a potential explanation of the light curve evolution in various LRN outbursts.

During the peak phase of the light curve, recombination energy is not an important contribution to the energetics. The photosphere is too hot, and the total energy,

$$E_{\text{recomb}} \approx 2.9 \times 10^{44} \text{erg} \left( \frac{\Delta m}{0.01 M_\odot} \right),$$

(9.6)

is too low by 1-2 orders of magnitude to explain the radiated energy given our estimates of the ejecta mass. The expression above describes the recombination potential energy per unit mass $\Delta m$ in a fully ionized plasma with solar composition abundances of H and He (Ivanova et al., 2013a, 2015). In short, a recombination powered transient cannot simultaneously be of such short duration and as energetic as the early peak of M31LRN 2015 given the characteristic velocities observed. Thus, it appears that simple heat diffusion, rather than recombination power, serves as the primary regulator of the early light curve’s temporal evolution.

However, during the plateau phase of the light curve, recombination energy could well play an important role in increasing the radiated energy (and modulating the photosphere location), as outlined above. We use the recombination potential energy per unit mass, equation (9.6), to estimate the ejecta mass contributing to the plateau phase of the light curve. Given the energetics of this phase, approximately $0.4 - 0.5 \times 10^{46}$ erg, we can estimate the mass as

$$\Delta m_{\text{ej,plateau}} \approx 0.17 M_\odot \left( \frac{E_{\text{rad,plateau}}}{5 \times 10^{45} \text{erg}} \right).$$

(9.7)

The implicit assumptions in this expression are that the ejected plasma is fully ionized.
as it is ejected, and that recombination happens at or near the photosphere and thus efficiently supplies the power source for the plateau of the light curve.

Even taken as order of magnitude estimates, a comparison of equations (9.4) and (9.7) suggest that a small amount of mass ejected at somewhat higher velocity forms the early light curve peak, while later a larger amount of ejecta contributes to the plateau phase.

9.4 Pre-Outburst Source

The detection of a progenitor source with known distance in the years prior to the transient outburst makes the lessons of M31LRN 2015 uniquely useful. In this section we analyze the properties of that pre-outburst source, and in Section 9.5 we return to the outburst and examine its properties given the pre-outburst detection. This effort to examine both the pre-outburst source along with the transient is useful because it allows us to link the binary system to the properties of transient outburst it produced.

Williams et al. (2015) report a source at the outburst location in Hubble Space Telescope (HST) imaging from August 2004, ~10.5 yr before the outburst took place. F555W and F814W magnitudes were converted into $M_V$ and $V - I$ colors assuming a reddening of $E(B - V) = 0.12 \pm 0.06$ mag to be $M_V = -1.50 \pm 0.23$ mag, and $(V - I) = 1.05 \pm 0.15$ mag (Williams et al., 2015). We transform these observations to a broader range of possible reddening values in what follows. This detection is extremely powerful in aligning the properties of the pre-outburst system with those of the transient outburst itself. In this section, we compare this detection to detailed stellar models to infer the properties of the source ten years prior to outburst.
9.4.1 Stellar Evolution Models

To map the pre-outburst HST color and magnitude to the properties of a progenitor star we compute a grid of stellar models with the MESA stellar evolution code\(^2\) (Paxton et al., 2011, 2013, 2015). We compute stellar tracks for stars of solar metallicity with masses between 2 and 8 \(M_\odot\) from the pre main sequence until core helium ignition.

We use a modified version of the input lists provided to recreate Figure 16 of (Paxton et al., 2013) which are available on mesastar.org. The initial models have a composition of \(Y = 0.272\) and \(Z = 0.02\) and convection is determined by the Schwarzschild criterion using \(\alpha_{\text{MLT}} = 2\) and allows for overshoot. We include mass loss along the RGB through a Reimers (1975) wind prescription with \(\eta_R = 0.5\) and do not include the effects of rotation. We calculate nuclear reactions using the JINA reaclib rates (Cyburt et al., 2010). We use standard OPAL opacities (Iglesias and Rogers, 1996) and we calculate surface properties using a grey atmosphere approximation.

To map the MESA variables to a color and magnitude we use the MESA colors package, which follows the method outlined in Lejeune et al. (1997, 1998) to yield \(UBVRI\) colors given surface temperature, surface gravity, and metallicity.

9.4.2 Source Properties

In Figure 9.4, we compare the HST detection to single-star evolutionary tracks in the \(M_V\) versus \(V - I\) color magnitude diagram (CMD). The tracks plotted are the post main sequence evolution of stars between 2\(M_\odot\) and 8\(M_\odot\). We make the initial assumption that we can approximate the net emission as being solely due to the primary star. The tracks are masked to only show portions of the evolution during which the stellar radius has not previously been larger. This selection is important in our present

\(^2\)version 7624
Figure 9.4: Color-magnitude diagram comparing the HST-detected source to post-main sequence stellar evolutionary tracks computed in MESA. The tracks are masked to exclude portions of the evolution where the radius decreases (because an encounter could only occur as the primary star grows). The pre-outburst color and magnitude are re-computed from the Williams et al. (2015) data to take $E(B-V)$ = 0.05, 0.15, 0.25, 0.35, 0.45 mag. Error bars include measurement and distance error, but not reddening error. The left panel shows tracks of stars between 2 and $8M_\odot$, in intervals of $1M_\odot$. The right panel zooms in and shows tracks with intervals of $0.1M_\odot$ between 3 and $6M_\odot$ (integer values are plotted with dashed lines). From these color magnitude diagrams, the pre-outburst source is clearly identified as subgiant star of several solar masses.
binary merger context because a binary interaction can only occur while the evolving primary star is growing to larger radial extent.

We transform the HST detection to a range of $E(B - V)$ values between 0.05 mag and 0.45 mag to span the range of uncertainty reported by Williams et al. (2015) and Kurtenkov et al. (2015). We plot error bars that include the distance modulus error and measurement error only (reddening error is not included). The reddening uncertainty thus implies a diagonal swath of possible color-magnitude pairs (not an uncorrelated error space).

Figure 9.4 shows that the allowed region of CMD space selects portions of stellar tracks which range from $3M_\odot$ to about $5.5M_\odot$. The selected regions along these tracks always correspond with phases in which the primary star is growing in radius as it evolves. This presents a picture very consistent with a star evolving to engulf its companion. Further, it suggests that, unless the companion is in the very unlikely configuration of also being a giant, the light will be dominated by the much-more luminous evolved primary, which justifies our comparison to single star tracks.

In Figure 9.5 we map the CMD space to the physical properties of the progenitor primary star, in particular radius, specific moment of inertia, and escape velocity. The panels of Figure 9.5 use colored dots along evolutionary tracks to show how the evolution of stars in the $3M_\odot$ to $6M_\odot$ range compare to the vector of progenitor source colors and magnitudes allowed by the HST data. The growth in radius along the post-main-sequence stellar tracks can be observed in the top panel of this diagram. Interestingly, the CMD swath allowed by the data selects objects of relatively similar radius regardless of reddening. The stellar escape velocity $v_{\text{esc}}$ is shown in the center panel of Figure 9.5. The escape velocity decreases along the evolutionary tracks as stars go from their compact main sequence radii to extended giant-branch radii.

The stellar moment of inertia is important in the context of binary star evolution because it relates to how much energy and momentum are needed to lock the
Figure 9.5: Properties of pre-outburst progenitor models plotted along stellar evolutionary tracks. The source data are again plotted for $E(B-V) = 0.05, 0.15, 0.25, 0.35, 0.45$ mag. Shortly before outburst, the primary star in the binary system was ascending the subgiant branch, during which its radius grows (top panel) and its escape velocity decreases (center panel). The specific stellar moment of inertia increases (lower panel) as the envelope’s internal structure transitions from radiative to convective.
primary star into corotation with its companion. We compute the moment of inertia from the radial profile of the stars’ interior density profiles, \(\rho_1(r)\), as

\[
I_1 = \frac{8}{3\pi} \int_0^{R_1} \rho_1(r) r^4 dr,
\]

and define a specific moment of inertia \(\eta_1 = I_1/M_1 R_1^2\) (e.g. Soker and Tylenda, 2006). This specific moment of inertia is plotted in the lower panel of Figure 9.5. It changes dramatically in the CMD space selected by our HST measurements. In this transition we are seeing the sub-giant branch transition from models with primarily radiative envelopes (and low specific moments of inertia) to those with convective envelopes (with more mass at larger radii and thus higher specific moments of inertia).

In Figure 9.6, we derive numerical values for a few key properties of the progenitor primary star as a function of reddening. We plot derived mass, radius, specific moment of inertia, and escape velocity along with their \(\pm 1\sigma\) error regions. As in the previous figures, we assume a fixed reddening value and consider the contribution of other sources to the error budget (no reddening error). We find that the pre-outburst star is a \(\sim 3 - 5.5 M_\odot\) star, with radius of \(\sim 25 - 40 R_\odot\). These masses and radii imply typical escape velocities from the primary of \(\sim 180 - 280\) km s\(^{-1}\). As a function of the assumed reddening, the interior structure of the fitted progenitor star changes: if the reddening is \(E(B-V) \lesssim 0.25\) mag, then the primary star has a convective envelope, and a relatively high specific moment of inertia, \(\eta_1\). If, however, the reddening is higher, the primary envelope is radiative and has lower specific moment of inertia.
Figure 9.6: Properties of pre-outburst progenitor models as a function of assumed reddening. These properties are measured from MESA models in the $M_V$, $V - I$ color-magnitude space allowed for a given reddening. Shaded regions show the range of properties within the ±1σ photometric measurement error bars of Figures 9.4 and 9.5. The pre-outburst source is a 3 to 5.5 $M_\odot$ star of approximately 35 $R_\odot$. It has an escape velocity in the range of $\sim 180 - 280$ km s$^{-1}$. The object’s internal structure and thus specific moment of inertia spans a wide range depending on reddening, at low reddening the star has a convective envelope and $\eta_1 \approx 0.14$, while at high reddening the star has a radiative envelope and lower specific moment of inertia of $\eta_1 \approx 0.02$. 
9.5 System and Transient Together

9.5.1 Combined Requirements

Taken together, the outburst and progenitor properties of M31LRN 2015 are strongly consistent with a stellar merger originating in a binary system. This identification was initially made by Williams et al. (2015) and Kurtenkov et al. (2015) through comparison of the outburst properties to similar optical transients like V838 Mon and V1309 Sco, which have been associated with a stellar merger origin (Tylenda and Soker, 2006; Soker and Tylenda, 2006; Tylenda et al., 2011; Pejcha, 2014; Nandez et al., 2014). The pre-outburst progenitor source supports this possibility. We find that the HST detection places the sub-giant primary star in a portion of the CMD in which it is evolving and growing in radius to engulf its companion. We are thus capturing the signature of the onset of a common envelope episode through this transient outburst.

A comparison of the progenitor source and the outburst itself allows us to draw several conclusions about this process. The optical transient rises from discovery to peak in a timescale of $\sim 8$ d. Depending on the reddening, the orbital period of a test mass at the surface of the primary varies from $\sim 13$ to 7 d (for low to high $E(B-V)$), the primary’s dynamical timescale, $(R_1^3/GM_1)^{1/2}$, is 2 to 1.2 d, respectively. Thus the initial transient outburst is quite rapid relative to the binary orbital period (of the same order) and the entire optical transient transpires over only tens of orbits of the binary.

Although the ejecta giving rise to the transient appear to be liberated on a dynamical timescale in the merger, the mass ejected is only a small fraction of the total system mass. In Section 9.3 we estimated that of order $10^{-2} M_\odot$ is ejected in the early peak of the outburst light curve (the portion with timescale similar to the orbital period). Of order $0.2 M_\odot$ may be ejected in the plateau portion. These ejecta masses are small by comparison to our estimated primary-star masses from the progenitor imaging.

A final useful point of comparison lies in the characteristic velocities of the
ejecta as compared to the progenitor system escape velocity. We have two observational handles on the velocity of the ejecta. The first is the Hα line FWHM from the Williams et al. (2015) data near the peak of the optical transient, equation (9.1). A second velocity measure is the expansion velocity of the photosphere as computed in Figure 9.2.

In Figure 9.7 we plot these characteristic velocities describing the progenitor system and outburst along with their ratio to the primary’s escape velocity. Across the range of possible source reddening values, the ejecta outflow at velocities similar to or greater than the primary star’s escape velocity. From this analysis we therefore can glean that in the M31LRN transient, we are observing the rapid, dynamical ejection of a small portion of the system mass. The characteristic masses are small relative to the system mass, the velocities are of the same order as the system escape velocity, and the timescale is similar to the primary’s dynamical time.

9.5.2 Interpretation

9.5.2.1 Early Peak: Shocked Ejecta from Contact

The M31LRN 2015 light curve shows an early peak, analyzed in Section 9.3.3.1. We found a small amount of mass, of order $10^{-2} M_\odot$, ejected at velocities similar to or exceeding the escape velocity, best explains this signature. Due to the small mass and rapid timescale, we associate these ejecta with the first phase of contact of the merging binary, where the relative velocity of the secondary across the primary’s atmosphere drives shocks through the primary’s outer envelope (e.g. Soker and Tylenda, 2006; Metzger et al., 2012b). In this scenario shocks are generated by the relative motion of the secondary through the primary’s envelope.

Material from the stellar envelope is gravitationally focused toward the secondary as it skims through the stellar atmosphere. This gravitational focussing effect
from the strong density gradient of the stellar limb creates a flow morphology in which material is shock-heated and a fraction is ejected radially outward (see, for example, the flow morphology in simulations presented by MacLeod and Ramirez-Ruiz, 2015a,b). These shocks accelerate a small portion of the mass to high velocities as they run down the density gradient of the primary’s disturbed envelope. The eventual ejecta in such a scenario will have a distribution of velocities of the same order, or faster than, the escape velocity from the primary star’s envelope.

As a check on this scenario, we can use the scalings of Section 9.3.3.1 and the size of the primary, $R_1$, as the ejecta’s origin to estimate a peak luminosity of

$$
L_{\text{peak}} \approx 1.0 \times 10^{39} \text{erg s}^{-1} \left( \frac{v_{\text{ej}}}{370 \text{ km s}^{-1}} \right)^2 \left( \frac{R_1}{35R_\odot} \right),
$$

that results from the ejection of a shell of mass of $\Delta m_{\text{ej,peak}}$ with velocity $v_{\text{ej}}$. We have assumed $L_{\text{peak}} = E_{\text{int}}(t_{\text{peak}})/t_{\text{peak}}$, that the original internal energy is similar to the ejecta kinetic energy ($E_{\text{int}}(0) = E_{K,\text{peak}}$, as would be the case if the material were strongly shocked) and that the internal energy declines as the gas expands with $(R_{\text{bb}}/r_0)^{-1}$ where $r_0 = R_1$ (e.g. Padmanabhan, 2001; Kasen and Woosley, 2009). Despite the many crude ingredients that form the basis of this calculation, this estimate agrees with the bolometric light curve of Figure 9.2. This agreement offers confirmation that the ejection of a small amount of mass from the stellar surface can produce the early transient light curve.

### 9.5.2.2 Later Outflow: Embedded Phase and Merger

When the secondary star is deeply embedded within the envelope of the primary, gravitational interaction drives high density material from the stellar interior outward (MacLeod and Ramirez-Ruiz, 2015a). In contrast to the phase of early contact, this material is not free to expand into the low-density stellar atmosphere but
instead thermalizes on its surroundings, effectively sharing the orbital-energy (drained from the secondary’s orbit) with a large amount of surrounding primary-star envelope material (see, for example, the simulations of Nandez et al., 2014). The continuous stirring of the envelope by the inspiralling-secondary is a violent process, but this phase drives a slower, albeit more massive outflow (Taam and Bodenheimer, 1989; Sandquist et al., 1998). Because the energy deposited by the secondary is shared with more mass in the embedded phase, the specific energy of the ejecta is lower and the imprint of these material on the light curve is longer, lower-temperature, and of lower peak luminosity.

In this case, the photosphere is at larger radii (and lower temperatures) implying that the material is able to recombine prior to becoming transparent. The Hα emission lines in the spectra fade, and the photosphere temperature stabilizes as hydrogen recombination controls the opacity. In this phase, we associate much of the radiated energy with local heating near the photosphere radius due to recombination, and we estimate the mass of the outflow from its energetics using this connection in Section 9.3.3.2.

Finally, the photosphere begins to recede after day 40 in the lower-right panel of Figure 9.2 suggesting that mass ejection has slowed or shut off at this time. We interpret this transition as the end of the binary merger: following a phase of orbital inspiral, the secondary tidally disrupts inside the primary envelope, leaving a merged remnant. Ongoing observations of the later phases of the transient (and its subsequent evolution in later stages) will allow us to constrain the total heat deposited into the primary’s envelope and to follow its resultant relaxation as it cools.

9.6 From Binary System to Transient Outburst

We have been able to make direct comparisons between the observed transient in M31LRN and the pre-outburst source. Here we discuss possibilities surrounding the
Figure 9.7: Taking the outburst and pre-outburst system together, this figure compares characteristic velocities. The blue, solid line compares the ejecta velocity inferred from spectral emission-line FWHM to the escape velocity, while the dashed yellow line compares the average photosphere expansion velocity to the escape velocity. The ejecta velocities are similar to or slightly larger than the escape velocity.
orbital dynamics, the hydrodynamics of mass ejection, and the component stars of the merging pair that drove the M31LRN transient.

### 9.6.1 Pathways to the Onset of Common Envelope

Two channels can drive a binary from its long-term stable orbital state toward the sort of dynamical merger observed in M31LRN 2015. The distinction between these channels depends on the mass ratio of the binary and on the internal structure of the primary star. These channels are Roche lobe overflow through the outer Lagrange point, which carries mass and angular momentum away from the system, and the Darwin tidal instability, which deposits orbital angular momentum into the reservoir of the primary’s envelope (Darwin, 1879).

Both channels destabilize and desynchronize the orbital motion of the secondary from the rotation of the primary envelope leading the two objects to plunge together with significant velocity shear at the moment of contact. They differ in the hydrodynamics of the initial mass ejection and, as a result, in the transients they should give rise to. In the following subsections, we explore these two pathways and discuss their observable consequences.

#### 9.6.1.1 Roche Lobe Overflow Leading to $L_2$ Mass Loss

If the primary star is locked into corotation with the secondary, it can overflow its Roche Lobe at the $L_1$ (inner) Lagrange point as it evolves toward contact. If the ensuing mass transfer is unstable (e.g. Hjellming and Webbink, 1987; Soberman et al., 1997; D’Souza et al., 2006; Woods and Ivanova, 2011; Passy et al., 2012b; Pavlovskii and Ivanova, 2015), the mass transfer rate can run away until the system also overflows the $L_2$ (outer) Lagrange point (Shu et al., 1979; Pejcha et al., 2016). Mass lost from the $L_2$ point carries angular momentum away from the bound pair, driving them toward merger.
Figure 9.8: Pathways to the onset of runaway interaction in a binary system. The left panel shows characteristic separations of orbital de-synchronization. Two possible processes, the Darwin tidal instability and non-conservative $L_2$ Roche lobe overflow can drive a binary pair toward merger. Points mark the transition where $a_{L_2} = a_c(\eta_1)$. The lower-right panel shows this transitional mass ratio for a range of primary-star specific moments of inertia, $\eta_1$. Whichever process occurs at larger separation mediates the merger of the two objects. Systems with $q \ll 1$ are likely to merge via the Darwin instability, while systems with $q \sim 1$ maintain corotation until mass transfer commences and merge by Roche lobe overflow. The pathway a merging system takes has implications for the observable properties its transient, as outlined in the upper-right hand side panels.
In Figure 9.8, we plot the separation, $a$, at which the primary undergoes Roche lobe overflow and begins to transfer mass onto the secondary (assuming corotation), this is approximated by Eggleton (1983)’s formula,

$$\frac{a_{L_1}}{R_1} = \frac{0.6q^{-2/3} + \ln(1 + q^{-1/3})}{0.49q^{-2/3}},$$

(9.10)

where $q = M_2/M_1$ is the binary mass ratio. Eggleton (1983)’s formula is an approximation of the separation at which the volume of the primary’s Roche lobe is equal to the volume of the unperturbed star, $4/3\pi R_1^3$. We perform a conceptually similar calculation to determine the approximate separation at which the system will overflow the $L_2$ Lagrange point, $a_{L_2}$. To compute this value, we compare the volume enclosed by the equipotential surface passing through $L_2$ (limiting our integration to the primary’s lobe) to the primary’s effective volume. We plot $a_{L_2}$ along with $a_{L_1}$ in the left panel of Figure 9.8. We find that $a_{L_2}$ is always smaller than $a_{L_1}$, with the difference most substantial for $q \sim 1$.

Mass carried away from the binary system at the $L_2$ Lagrange point is lost gently – the ejecta are trailed off from the inner binary and expand in a spiral wave. This wave of material has a finite width (based on its sound speed) and eventually overlaps, piling up into a wedge-like outflow with relatively constant opening angle (Shu et al., 1979; Pejcha et al., 2016). In Appendix 9.8, we compute the mass loss needed to bring the binary from the point where it begins to shed material to contact, and find that this mass is 10-15% of $M_2$ across a range of mass ratios $q$, as shown in Figure 9.11.

### 9.6.1.2 Darwin Instability

Binary systems with low mass secondaries are subject to an orbital instability known as the Darwin instability (Darwin, 1879). As the primary’s envelope grows,
a situation can arise where the angular momentum budget of the secondary’s orbital motion is too small to lock the primary’s envelope into corotation. As desynchronization ensues, angular momentum is drained from the secondary object’s orbit and the orbit decays on a tidal-dissipation timescale. The condition for this instability can be written in terms of the moments of inertia of the orbit $I_{\text{orb}}$ and the moment of inertia of the primary, $I_1$, as (Eggleton and Kiseleva-Eggleton, 2001),

$$I_{\text{orb}} \lesssim 3I_1. \quad (9.11)$$

In the above expression, $I_{\text{orb}} = \mu a^2$, where $\mu = \frac{M_1 M_2}{M}$ is the reduced mass, $M = M_1 + M_2$ is the total mass, and $a$ is the orbital separation. The primary’s moment of inertia can be written as $I_1 = \eta_1 M_1 R_1^2$; it is computed from the interior structure of the primary using equation (9.8). Though the moment of inertia of the secondary could enter into the above expression (Hut, 1980), we ignore it here because the evolved primary star in the M31LRN 2015 has a much larger moment of inertia than its lower mass and more compact companion.

Whether or not a binary system is Darwin unstable is thus a property of both the radius of the primary star and its interior structure. The critical separation which leads to instability can thus be written,

$$a_c = 3\eta_1 \left(1 + q^{-1}\right) R_1. \quad (9.12)$$

In the left-hand panel of Figure 9.8, we compare critical separations to the radius of the primary and the separations on Roche lobe overflow for synchronized systems, $a_{L1}$ and $a_{L2}$. We use three different specific moments of inertia, $\eta_1$, which span the range of typical stellar values shown in Figure 9.6 (low $\eta_1$ is relevant for main-sequence stars, while evolved stars with convective envelopes have higher $\eta_1$). The lower-right
panel of Figure 9.8 delineates the transition in mass ratio (as a function of primary-star specific moment of inertia, $\eta_1$) between Darwin unstable systems and Roche lobe overflow systems as divided by the line $a_c = a_{L_2}$.

9.6.1.3 Consequences of Pathway for Merger Transients

A comparison of the desynchronization radii plotted in Figure 9.8 shows that in some cases the system becomes Darwin unstable prior to mass transfer, while in other cases unstable Roche lobe overflow proceeds to $L_2$ mass loss. Both of these channels lead to the merger of the pair of objects, but they may generate very different observable consequences – particularly in the early phases of a merger transient. In general, low-mass-ratio systems come to contact through the Darwin instability, while systems that are closer to equal mass tend to remain synchronized and merge through Roche lobe overflow. We explore these differences here and in the cartoon panels in Figure 9.8.

In systems that merge through Roche lobe overflow, the initial ejecta are trailed off in a spiral wave from the $L_2$ point. Beyond a few times the orbital semi-major axis, these material pile up into a wind-like outflow with $\rho \propto r^{-2}$ (Pejcha et al., 2016). In general, the material is cool, and the radial component of the ejected material’s velocity is small. Depending on the mass ratio, the majority of the material may be either bound or unbound, with the maximum velocity at infinity being about 25% of the system escape velocity, $v_{\infty} \lesssim 0.25v_{\text{esc}}$ (Shu et al., 1979). This result has recently been confirmed in a detailed numerical study by Pejcha et al. (2016).

Mass loss from $L_2$ begins slowly but eventually exponentiates leading to the runaway merger. As the mass loss rate increases, the photosphere moves outward in the “wind”, and the luminosity of the system rises (Pejcha et al., 2016). This transition takes place over many orbital periods, leading to a gradual brightening of the system toward peak. This is exactly what was observed in the progressively-steepening six-month rise of V1309 Sco (Pejcha, 2014). The characteristic features of transients generated by
systems brought to merger through Roche lobe overflow will thus be a long rise time (relative to the orbital period) and low ejecta velocities (relative to the system escape velocity).

Systems that are driven to merger through the Darwin instability desynchronize when there is no longer sufficient angular momentum in the secondary’s orbit to maintain the corotation of the primary. Thus, they proceed toward merger (on a tidal-dissipation timescale) without significant mass loss (Eggleton and Kiseleva-Eggleton, 2001; Nandez et al., 2014). Even when the separation reaches the mass-transfer separation for synchronized systems, $a_{L_1}$, significant mass loss is unlikely. Flow through the $L_1$ and $L_2$ Lagrange points will be suppressed by the lack of corotation, creating an effective potential with much higher barriers for material to cross at these points (Sepinsky et al., 2007).

Initial mass loss in systems merging by the Darwin instability comes as pair of objects comes into contact. Gravitational focussing of material from the primary envelope shock heats and ejects gas. Without the dense circumbinary outflow that characterizes Roche lobe overflow systems, this initial ejecta expands into the uninhibited with characteristic velocity of order, or larger than, the system escape velocity. The characteristic timescale for this initial ejecta is short, because these ejecta mark the onset of the dynamical merger. Thus, rather than a slow rise, systems merging by the Darwin instability will rise toward peak on a timescale of order the binary orbital period.

The timescale and velocity of the initial ejecta in M31LRN 2015 are strongly consistent with a system that is driven toward merger through the Darwin instability. Although our comments on merger pathways have been, up to this point, left general, a comparison to Section 9.5 reveals that the properties of the outburst – a timescale comparable to the stellar dynamical time, an outflow with $v_{ej} \gtrsim v_{esc}$, and the ejection of a small quantity of mass – all point to the initial ejecta being driven by shocks through
the atmosphere of the primary star. In the following section, we rely on this observation (along with the outburst energetics) to constrain the mass ratio of the M31LRN 2015 binary system.

9.6.2 Constraining $M_2$ in M31LRN 2015

In this section, we consider what limits may be placed on the mass of the unseen secondary star that was engulfed to create M31LRN 2015. We use the properties and energetics of the transient outburst along with our discussion above of the possible pathways to merger in order to consider the constraints on the properties of the merging binary.

9.6.2.1 An Upper Limit: Merger Pathway

Based on our analysis of the observational implications of the two pathways to merger in a binary system in Section 9.6.1.3, we require that the M31LRN 2015 desynchronize via the Darwin instability. This pathway to merger is consistent with the high velocity initial ejecta and the rapid rise to outburst peak. The requirement that the system merge via the Darwin instability has the effect of placing an upper limit on the mass ratio, $q$. Because the pre-outburst HST imaging constrains the primary mass and structure, it is also possible to place an upper limit on the mass of the secondary, $M_2$.

To apply this constraint, we specify that the system must destabilize due to the Darwin instability at greater separation that it begins to lose mass from $L_2$, $a_c \geq a_{L_2}$. This implies a maximum allowed $q$ for a given $E(B-V)$ reddening, which gives $a_c = a_{L_2}$. As can be seen in equation (9.12) and Figure 9.8, this constraint depends on the specific moment of inertia of the primary star’s envelope, $\eta_1$. We therefore apply the constraints derived in Section 9.4 and plotted in Figure 9.6. For lower $q$, the pair becomes Darwin unstable at larger separations ($a_c > a_{L_2}$). However, for higher masses, the system
remains synchronous until mass loss from $L_2$ begins.

As a more general comment, the pathway to merger is an upper limit on the mass ratio only for Darwin-unstable systems. Were the transient’s observational properties consistent with a Roche lobe overflow pathway to merger, this would instead imply a lower, rather than upper, limit on the binary mass ratio.

9.6.2.2 A Lower Limit: Energetics

We can place a lower limit on the mass of the secondary object based on the outburst’s energetics. In particular, if we consider the primary source of energy in the transient to be the dissipation of the orbital energy of the secondary into the ejected envelope gas, then we can place a lower limit on the change in orbital energy during
this phase (and in turn on $M_2$).\footnote{We note that orbital energy isn’t the only possible energy source in a common envelope episode (Iben and Livio, 1993). The possibility of accretion energy mediated by jets has also been proposed to explain red transients by Kashi and Soker (2015).} The first peak of the transient outburst is particularly useful in this regard because we have argued that it originates from the phase when the secondary is grazing the surface of the primary. We also know that the radiated energy during this peak originates primarily from the original ejection, not via hydrogen recombination, as argued in Section 9.3.

The rapid velocities, $v_{\text{ej}} \gtrsim v_{\text{esc}}$, measured during the outburst peak imply that shocked ejecta are free to expand uninhibited and without thermalizing on the surrounding envelope gas. This suggests that we are observing the early phase of interaction before the secondary object has become ‘buried’ within the envelope of the primary. In Figure 9.9 we plot a quantity $\Delta E_{\text{bury}}$, which we define as the magnitude of the change in orbital energy liberated before the secondary is subsumed within the envelope of the primary. Because the orbital energy of the binary is the energy reservoir which drives the ejecta, $\Delta E_{\text{bury}}$ must be sufficiently large to explain the early light curve energetics. Below, we explore the limits this places on the range of possible mass ratios.

To approximate $\Delta E_{\text{bury}}$, we assume that the secondary can eject material to infinity when it is within one (secondary) Roche lobe radius of the surface of the primary. We therefore compute the liberated energy as (minus) the difference in orbital energy between $a = R_1$ and $a = R_1 - R_{R_2}$,

$$\Delta E_{\text{bury}} = -[E_{\text{orb}}(R_1 - R_{R_2}) - E_{\text{orb}}(R_1)], \quad (9.13)$$

where $E_{\text{orb}} = -GM_1M_2/(2a)$, and $R_{R_2}$ is the Roche lobe of the secondary, approximated by the inversion of Eggleton’s formula, equation (9.10) (thus $\Delta E_{\text{bury}} > 0$ because the
orbital energy becomes more negative). For our definition of $q = M_2/M_1$ we have,

$$\frac{R_{R_2}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}. \quad (9.14)$$

In Figure 9.9, we don’t include the (few percent) change in enclosed primary mass as the secondary plunges into the outer layers of the primary star’s envelope. This approximation of $\Delta E_{bury}$ has desirable qualitative features. When the secondary mass is small, it need not plunge deep within the envelope of the primary before the material in its vicinity is thermalized rather than directly ejected. When the secondary mass is large, it instead causes a more global disturbance to the primary envelope and can eject material to infinity from deeper depths within the envelope (smaller $a$).

Returning now to Figure 9.9, $\Delta E_{bury}$ is plotted normalized to the orbital energy at contact ($E_{orb}$ at $a = R_1$) and to the approximate binding energy of the primary, $GM_1^2/R_1$. The orbital energy released in the initial contact is a steep function of the secondary mass (roughly $\propto q^{1.36}$) because of the mass dependence not only of the orbital energy at contact, but also of the depth to which the secondary can penetrate before we assume it to be buried. We use this information to place a limit on the binary mass ratio by requiring that the liberated energy is larger than the radiated energy of the peak: $\Delta E_{bury}(q) > E_{rad,peak}$, where the radiated energy is plotted as a function of reddening in Figure 9.3. We note, that is likely that the kinetic energy of the outflow is larger than the radiated (see equation (9.5) and the following discussion). But, because the radiated energy is the observed quantity, this is a strictest constraint that we must ensure is satisfied by any potential system.

9.6.2.3 Range of Possible Secondary Masses

Figure 9.10 shows the range of mass ratios (upper panel) and masses (lower panel) permitted by the upper and lower limits described above. Since our knowledge
Figure 9.10: The range of allowed mass ratios ($q$, upper panel) and masses ($M_1, M_2$, lower panel) for M31LRN 2015 as constrained by the radiated energy on the lower boundary and the Darwin instability on the upper boundary. The lower bound requires that there is sufficient energy to drive the early peak of the observed light curve. The upper bound requires that the binary desynchronize through the Darwin instability because this implies an unpolluted circumbinary environment in which the small mass but high velocity early shocked ejecta (from contact) can propagate uninhibited. In the lower panel, the purple dashed line and shaded region show the inferred constraints on $M_1$, while the blue region uses the constraint on $q$ to deduce $M_2$. 

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surrounding the processes regulating stellar mergers remains quite uncertain, these limits are most appropriately interpreted in terms of the qualitative guidance they offer, rather than an exact numerical value.

The range of allowed $q$, like the other properties of the binary system, is a function of the reddening along the line of sight to the source. The upper mass limit, derived on the basis of the Darwin instability, contains the imprint of the internal structure of the primary star, in particular its specific moment of inertia, $\eta_1$. At low reddening, the primary star properties we derive point to an object with a convective envelope, while at higher reddening, the primary star would be more massive and have a radiative envelope.

Across all reddening values, the secondary is required to be much less massive than the primary, and $q \ll 1$. Taking a reddening value of $E(B - V) = 0.12$ mag, as assumed by Williams et al. (2015), this implies that we are observing the merger of a primary star with mass $M_1 \sim 4M_\odot$, radius of $R_1 \sim 35R_\odot$ with a low-mass secondary star with mass $M_2 \sim 0.1 - 0.6M_\odot$. This suggested mass ratio compliments the recent discovery of eclipsing binaries with B-type main sequence stars and extreme mass ratios by Moe and Di Stefano (2015a,b).

9.6.2.4 A Caveat: The Possible Role of Secular Dynamics

The calculations presented in this section assumed that the source was a binary, and thus its desynchronization and merger dynamics were those of a two-body system. Another possible channel to create a violent initial interaction with an unpolluted environment is if the primary and secondary were to collide in a parabolic or eccentric orbit. Even in dense star clusters, direct stellar collisions are rare and further a cluster would be obvious in HST pre-outburst imaging. Perhaps more common are eccentric interactions driven by secular torques in hierarchical triple systems (e.g. Kozai, 1962; Shappee and Thompson, 2013; Naoz et al., 2013; Pejcha et al., 2013; Antognini
et al., 2014; Naoz and Fabrycky, 2014; Michaely and Perets, 2014; Hamers et al., 2015; Antognini, 2015).

If the colliding pair were driven to a collision in an eccentric orbit, the impulsive nature of the outburst could be readily explained (e.g. Smith, 2011), but the constraints derived above on the secondary mass would not apply. More study is needed to confirm which channels can produce ejecta similar to this event and other luminous red novae transients. In particular, hydrodynamic simulations of merger events can assess the velocity distribution of ejecta in different cases of the orbital synchronization and eccentricity. The generation of transients we are most likely to observe relies on a convolution of the relative frequency with which different systems occur in nature and the luminosity and characteristic wavelength of the transients they produce (e.g. Kochanek et al., 2014). Thus, even if hierarchical triple systems were rare, if they generated particularly luminous transients, they could conceivably make up a disproportionate fraction of the transient population.

9.7 Discussion and Conclusion

9.7.1 Summary

This paper has studied initial photometric and spectroscopic data published by Williams et al. (2015) and Kurtenkov et al. (2015) on a particularly remarkable addition to the growing class of stellar merger-transients. This event is especially useful in teaching us about the physics of this process because of the additional detection of a pre-outburst source (with both color, magnitude, and known distance). In this paper we’ve linked the observational data to modeling of the light curve and pre-outburst source to map the properties of the binary before its merger to the transient it created.

Our analysis reveals that the M31LRN 2015 transient marked the onset of a common envelope episode. A several solar mass star evolved away from the main
sequence and grew to a radius of $\sim 35R_{\odot}$ before engulfing its companion. The light curve and spectra reveal multiple components: 1) a fast early outflow that becomes transparent at $\sim 2$ AU with characteristic temperature of around $10^4$ K, and 2) a cooler, longer lived outflow which drives the expansion of the photosphere to $\sim 10$ AU, where the temperature is low enough for hydrogen to recombine (and dramatically lower the opacity). We infer that this early component (with characteristic length similar to one binary orbital period) is driven by shocks that race through the stellar atmosphere during the violent onset of the merger, driving off $\sim 1\%$ of the system mass with characteristic velocity similar to the system escape velocity. The later, more mass-rich ejecta arise from the continued insprial of the secondary after it has plunged deeper and become embedded in the common envelope.

In Section 9.6, we extended our analysis to consider the possible pathways to merger for a binary system like that observed in M31LRN 2015. We show that a scenario in which the binary desychronized and was driven toward merger by the Darwin instability is highly favorable (Figure 9.8) in that it leaves the circumbinary environment relatively pristine – the mass ejected at contact would be free to race outward and produce the observed transient. This constraint suggests a binary with a low-mass secondary and a relatively asymmetric mass ratio, $q \sim 0.1$, as shown in Figure 9.10. Intriguingly, Moe and Di Stefano (2015a) find that 2% of B-type main sequence stars have low-mass companions in the 3-8.5 day period range similar to the M31LRN 2015 system’s binary period of between 7 d and 13 d (depending on reddening). This result suggests that very asymmetric binaries of the sort apparently giving rise to M31LRN 2015 might be relatively common in this stellar mass range.

### 9.7.2 Comparison to Other Stellar-Merger Transients

The similarities of the optical light curve and spectra of M31LRN 2015 to transients like V838 Mon enabled its early identification and the follow-up observations
published by Williams et al. (2015) and Kurtenkov et al. (2015). The class of binary merger transients has grown to contain objects with a range of properties. In this section we briefly compare of M31LRN 2015 to three recent and well-studied transients which may share similar origin: V1309 Sco, V838 Mon, and NGC4490-OT.

V1309 Sco was a particularly useful detection in defining the class of stellar merger transients because an eclipsing binary with a decreasing orbital period was observed by the OGLE experiment (Udalski et al., 2008) for nearly a decade prior to the event (Tylenda et al., 2011). Eventually, the modulation of the light curve disappeared and was replaced by a steady brightening over approximately 6 months. Pejcha (2014) have associated this phase with mass loss from the $L_2$ Lagrange point, which is carrying away orbital angular momentum and driving the binary toward merger as it also obscures the central objects and leads to a growing (and brightening) photosphere. This phase of steady brightening transitions smoothly to an abrupt peak. Nandez et al. (2014) and Pejcha (2014) both associate this abrupt peak with shocks driven through the outflow at the moment of contact – where one star plunges within its companion.

Many similarities between V1309 Sco’s evolution and the qualitative phases of the M31LRN 2015 outburst present themselves. In particular, a fast, shock-driven outflow seems likely to drive the light curve peak. However, no gradual rise is observed in M31LRN 2015 (and the transient was identified 5 magnitudes below peak). Were a feature similar to that seen in V1309 Sco present, it seems likely it would have been detected. The characteristic velocities measured from the Hα FWHM in V1309 Sco (150 km s$^{-1}$) are only $\sim$ 40% of the modeled primary-star’s ($M_1 \approx 1.5 M_\odot$, $R_1 \approx 3.5 R_\odot$) escape velocity ($\approx 400$ km s$^{-1}$) (Nandez et al., 2014). By contrast, for M31LRN 2015, velocities of $v_{ej} \gtrsim v_{esc}$ are observed; in Section 9.6, we argue that these high relative velocities suggest a comparatively unpolluted circumbinary environment without substantial $L_2$ mass loss to decelerate the early ejecta. These contrasting properties reveal that while V1309 Sco appears to have been driven to merger through Roche lobe over-
flow, M31LRN 2015 seems likely to have merged because the system became Darwin unstable.

V838 Mon’s light curve shows an even more dramatic multiple-component structure than either that of M31LRN 2015 or V1309 Sco, with at least three individual peaks (e.g. Bond et al., 2003). V838 Mon’s merger generated significantly faster outflows than V1309 Sco, with velocity \( \sim 500 \text{ km s}^{-1} \) (Munari et al., 2002). It does, however, reach a very similar peak luminosity to M31LRN 2015 (Williams et al., 2015). Light echos from the outburst allowed the determination of an accurate distance (Bond et al., 2003; Munari et al., 2005; Sparks et al., 2008) and archival data showed the progenitor system to consist of a young (main sequence or pre-main sequence) B-type (5 – 10 \( M_{\odot} \)) primary (Tylenda et al., 2005) along with a distant B-type companion (Munari et al., 2007; Antonini et al., 2010). However, there remains discussion on the nature of the progenitor, with Munari et al. (2005) suggesting that the pre-outburst source is better explained by a hotter star that was initially substantially more massive (\( \sim 65 M_{\odot} \)), while evidence from the surrounding stellar cluster points toward the lower-mass alternative (Afşar and Bond, 2007). Recently, Loebman et al. (2015) have found that a decade after its original outburst, V838 Mon exhibits a very cool, extended L-type supergiant remnant. The uncertainty surrounding the nature of the progenitor of V838 Mon makes it difficult to draw firm conclusions about this object’s evolutionary history. Even so, the light curves show distinctly similar color evolution and duration to that of M31LRN 2015 (as emphasized by Williams et al., 2015, in their original presentation of the light curve), perhaps suggesting similar ejecta masses or hydrodynamics.

The properties of NGC 4490-OT were recently presented by Smith et al. (2016), who emphasize the similarity of this event in color evolution and light curve structure to other stellar-merger transients – and its distinction from the supernova 2008S-class intermediate luminosity optical transients. The optical transient is significantly brighter, peaking at \( M_R \sim -14.2 \), and the duration is \( \sim 200 \text{ days} \). The radiated energy during
this time is $1.5 \times 10^{48}$ erg, or approximately 2.5 orders of magnitude larger than that of M31LRN 2015. The scalings of Section 9.3.3.1 suggest that the ejection of several solar masses would reproduce these timescales and energetics given the ejecta velocities of several hundred km s$^{-1}$. Similar considerations (along with spectroscopic similarities to V838 Mon) lead Smith et al. (2016) to suggest that NGC 4490-OT could be the massive star equivalent of the stellar-merger transients mentioned previously. Indeed, the detection of a progenitor (though single-color) point toward a massive, late B-type star of $\sim 30 M_\odot$. With this large of an ejecta mass, hydrogen recombination could play a major role in the radiated energetics, as indicated by equation (9.7) and Ivanova et al. (2013a, 2015), but the full transient luminosity and radiated energy remain difficult to explain (under simplistic assumptions) by $\sim 1$ order of magnitude.

Some diversity in these events should absolutely be expected. After all, with two (or more) stars in a stellar multiple system, there are innumerable combinations of stellar mass and type that could be achieved at the onset of merger. It is interesting to note that the range of masses estimated for progenitor stars involved in driving merger-transients spans a huge range – from sources with primaries near a solar mass to those with massive stars. Kochanek et al. (2014) have noted from this evidence that the peak luminosity of merger transients scales steeply with progenitor mass, perhaps exchanging with the stellar initial mass function to make transients similarly observable across a wide range in progenitor masses. If this trend in the data continues to hold, it will allow us to probe the binary evolution of a range of both low-mass and massive stars.

9.7.3 Future Prospects

The discovery of M31LRN 2015 marks a step forward in our understanding of flows at the onset of a common envelope episode; we can say with some confidence that we have observed a giant star swallowing its much-lower-mass companion. Although the details of flows in common envelope have remained a theoretical mystery for the past
forty years, with observations of common envelope events “in action” we are now well positioned to start to constrain this important, yet highly uncertain process in binary evolution (Ivanova et al., 2013a). Indeed, the stakes have never been higher: common envelope episodes are thought to be essential in tightening the orbits of compact binaries to the point that they can merge by gravitational radiation (e.g. Postnov and Yungelson, 2014) – as in the recent LIGO detection of merging black holes (Abbott et al., 2016). Whether a particular interaction leads to merger or to envelope ejection, is a critical question for forming gravitational wave sources, especially because the tightest binaries that form are those that are driven nearly to merger.

There remains much to be learned from other nearby extragalactic transients in which we are able to identify a pre-outburst source. In the case of this work, the identification and modeling of the pre-outburst detection allowed us to compare the properties of the outburst to the binary system and to constrain the mass of the subsumed secondary star. Recently-published work by Goranskij et al. (2016) shows that the transient PSNJ14021678+5426205 in M101 shares properties with many LRNe. The transient undergoes multiple outburst peaks (separated by about a year in 2014 and 2015) with a relatively small amplitude of ∼ 5 mag. There is a robust detection of a slowly-brightening pre-outburst source between 1993 and 2014. These data appear consistent with a massive-star merger origin, as Goranskij et al. (2016) point out, in particular one in which the evolution to contact is mediated by $L_2$ Roche lobe overflow, rather than the Darwin instability. Future modeling of this object might, therefore, provide an extremely valuable evolutionary counterpoint to M31LRN 2015. The event rates of these (and similar) events are also promising: Kochanek et al. (2014) predict galactic rates of ∼ 0.1 yr$^{-1}$ for LRNe, with a steeply-rising number of shallower-amplitude outbursts. It therefore appears reasonable to expect one or more transients such per year in local galaxies with extensive multi-color imaging to rely on for pre-outburst detections.

While this class of objects can teach us many lessons about dynamical phases
of binary evolution, there remains much to be learned from M31LRN 2015 itself. This paper has presented analysis and possible interpretations of the early optical light curve and pre-outburst source. Ongoing observations of this object at infrared wavelengths can map out the now-dusty ejecta’s mass and energetics. The contracting photosphere in the optical observations suggests that the binary merged completely and promptly, but future multi-wavelength observations of the remnant object can better constrain the fate of this common envelope episode. Will we observe the remnant’s thermal relaxation in future years? Or does a central binary remain enshrouded within the envelope that will instead continue to drive dusty ejecta? The answers to these questions will be especially interesting in comparison to the properties of the seemingly-similar M31 RV red transient, thirty years further into its post-outburst phase (Bond, 2011).

9.8 Mass Lost from $L_2$ Prior to Contact in Roche Lobe Overflow Mergers

One pathway to the onset of common envelope is mass loss from the outer, $L_2$, lagrange point. This material forms a cool, equatorial outflow with characteristic radial velocity significantly less than the system escape velocity (Shu et al., 1979; Pejcha et al., 2016).

We can calculate how much mass would need to be lost to carry away the angular momentum needed to bring the binary from $a = a_{L_2}$ into contact at $a = R_1$. The orbital angular momentum of the binary is

$$L_{\text{orb}} = \mu (GMa)^{1/2}. \quad (9.15)$$

Thus the change in orbital angular momentum between $a = a_{L_2}$ and $a = R_1$ (for the
moment neglecting the loss in mass) is

\[ \Delta L_{\text{orb}} \approx L_{\text{orb}} (a_{L_2}) - L_{\text{orb}} (R_1). \]  

(9.16)

Mass loss from the \( L_2 \) point carries angular momentum away from the binary at a rate of

\[ \frac{dL}{dm} = \left( GMa \right)^{1/2} r_{L_2}^2, \]  

(9.17)

where \( r_{L_2} = x_{L_2}/a \) is a dimensionless ratio, dependent on \( q \), which describes the distance to the \( L_2 \) point from the center of mass of the binary in units of the semi-major axis (Pribulla, 1998). Typical values for \( r_{L_2} \approx 1.2 \) and it varies by < 10% across a wide range of \( q \). We can then estimate a mass loss \( \Delta m_{L_2} \approx \Delta L_{\text{orb}} /(dL/dm) \), where, for simplicity, we evaluate \( dL/dm \) at \( a_{L_2} \). This gives,

\[ \frac{\Delta m_{L_2}}{M_1} \approx \frac{q}{1 + q r_{L_2}^{-2}} \left[ 1 - \left( \frac{R_1}{a_{L_2}} \right)^{1/2} \right]. \]  

(9.18)

We’ve plotted this quantity in the right-hand panels of Figure 9.8 labeled as ‘Simple’.

To obtain a more numerically accurate version of \( \Delta m_{L_2} \), we need to integrate the mass lost as the binary separation decreases from \( a = a_{L_2} \) to \( a = R_1 \). We use equations (9.17) and (9.15) to express \( dm/da = (dm/dL)(dL/da) \), to find

\[ \frac{dm}{da} = -\frac{\mu}{2a} r_{L_2}^{-2}. \]  

(9.19)

We integrate \( dm/da \) numerically from \( a = a_{L_2} \) to \( a = R_1 \) to find \( \Delta m_{L_2} \). We use the midpoint method and \( 10^3 \) evenly-spaced integration steps. We assume that mass lost comes from the envelope of the primary, thus decreasing \( M_1 \) and modifying \( \mu \) and \( q \) through the integration. This numerically derived version of \( \Delta m_{L_2} \) is slightly higher than the estimate of equation (9.18) and is plotted in the right-hand panels of Figure
Figure 9.11: Mass lost from the outer Lagrange point, $L_2$, prior to contact in systems merging by Roche lobe overflow. This is the mass loss needed to carry away the orbital angular momentum of the binary. Two estimates of this mass loss are shown, the first, labeled “Simple” is based on equation (9.18), while the second, which is derived from a numerical integration of equation (9.19), is labeled “Numerical”. Approximately 10-15% of $M_2$ is needed to drive systems to merger through Roche lobe overflow. This material is trailed off from the $L_2$ point and expands with radial velocity $\lesssim 0.25v_{\text{esc}}$ (Shu et al., 1979; Pejcha et al., 2016). This substantial mass loss shapes the circumbinary environment and leads to a steady increase in photosphere radius and a slow rise in system luminosity, as has been observed in V1309 Sco (Pejcha, 2014). This pile-up of slow-moving material can also decelerate less massive but faster shock-driven ejecta from the moment of contact.

9.8 labeled ‘Numerical’.

Figure 9.8 shows that the mass loss needed to bring the binary from the point where it begins to shed material to contact is of order 10% of $M_2$. This calculation makes the assumption that Roche lobe overflow occurs when the binary reaches $a = a_{L_1}$. The desynchronization of the orbital motion and the primary’s rotation can substantially suppress mass transfer in systems that first destabilize through the Darwin instability. Sepinsky et al. (2007) have shown that non-synchronous systems need to reach closer separations before the effective potential allows material to flow from one Roche lobe to another. In these systems the initial mass loss will come when material is ejected at contact, not through Roche Lobe overflow.
Chapter 10

Conclusion

This thesis has focused on a range of stellar encounters: the “social” moments in stellar lifetimes. We’ve used a combination of computational modeling techniques to reproduce these encounters and to try to predict how they might light up the transient night sky.

Observational detections of tidal disruption events and common envelope episodes remain rare, but these events are beginning to found. The transient night sky is being surveyed in an increasingly systematic manner at a wider range of wavelengths than ever before. That transition is partially represented in the arc of this thesis: Chapter 9 focuses on a probable detection of a common envelope episode and the lessons we can draw from it.

In the coming era of time domain astronomy, event detections will far outnumber the possibility of in-depth follow up. That implies a few things. One potential take away is the value of a few well-studied empirical “template” events per class of transient. By modeling these events in detail we can attempt, iteratively, to uncover the physical properties that underlie a given light curve. Another potential lesson is the need to derive more direct model-to-data comparisons from multiband photometry alone, without as extensive reliance on spectra and other forms of more-detailed follow
up observations. On the modeling side this implies a need to generate light curves in standard photometric bands that are as accurate as possible. With high-fidelity template light curves, we can attempt to distinguish between potential driving scenarios in samples of observed transients using photometry alone.

Though there are challenges to be faced in the coming era of high-volume time domain astronomy, the prospects are extremely exciting. We are likely to see the science of astronomical transients evolve from one of predictive modeling to one driven empirically and through extensive model-to-template comparisons.
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