STOCHASTIC CONJUNCTIVE MANAGEMENT OF WATER RESOURCES IN YOLO COUNTY

TECHNICAL COMPLETION REPORT

WATER RESOURCES CENTER PROJECT W-874

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Abstract

Two management models are developed to determine the optimal operating policies for hydraulically connected time-variant surface and ground water supplies in a hypothetical system. The system involves a multipurpose reservoir, a hydraulically connected stream and aquifer, agricultural plot, water supply and observation wells, and an artificial recharge zone so as to address various hydrologic components experienced in Yolo County in the management models. The first model minimizes deviations from a set of rule curves defined for storage in the reservoir and along stream course so as to consider possibilities for storage excess water in wet periods and its distribution in subsequent dry periods. The second model, in addition to the objective of the first model, minimizes total operational costs of surface and subsurface water storages (pricing) while meeting the target storage levels in the surface water supplies. The first model is formulated as a linear programming model whereas the second one is formulated as a nonlinear programming model and a $\delta$-form approximating model is employed in the solution phase.

The hypothetical system is divided into components which are subsequently integrated using numerically-generated response functions that relate the system’s behavior to various system excitations. The management model is formulated and solved for monthly time steps, which include both dry and wet conditions, to determine reservoir release, pumpage rate from supply wells, artificial recharge rate, water diversion from reservoir and stream to the demand area, and storage both in the reservoir and along the stream course at an optimal level in each time step over a six-month planning horizon. Sensitivity analysis of the first management plan with respect to potential use of ground-water supplies is performed to analyze the latter’s impact on the operating policies. Furthermore, the performance of the $\delta$-form approximating model for conjunctive management models applied to the hypothetical system is found to be promising.

Key Words: 0455, 1665, 1525
1. Problem and Research Objectives

Substantial increase in water demand for municipal, agricultural, and industrial uses created primarily by rapid population growth, makes attractive the optimal joint operation of limited surface and ground water supplies. Joint operation of water supplies could be developed on a short- or long-term basis and takes advantage of: (1) storage of excess water in surface and subsurface reservoirs in wet periods of the year for use in subsequent dry periods; (2) mitigation of potential flood hazard in wet periods with high-peak streamflows by reservoir storage of excess water; (3) use of ground-water supplies to supplement surface water shortages; and (4) artificial recharge of excess surface water to sustain ground-water levels within a predetermined range.

The main objective of this research is to develop coupled simulation-optimization models to find the optima operating policies for the joint use of surface and ground water supplies that would lead to the development of a model for Yolo Basin, California so as to meet various water demands at optimal levels while preventing serious hydrogeological and environmental problems. Problems such as overdrafting, pumpage-induced land subsidence, and associated water quality problems experienced in Lower Cache-Putah and East Yolo Subbasin that threaten Yolo County's future supplies are considered within the constraints of the models. The management models developed are intended to manage the overall surface and ground water resources including their temporal and spatial hydraulic interactions. The main objectives of the management models are to: (1) use numerically generated response equations to couple the surface and subsurface water components to be able to include the system's heterogeneities associated with distribution of aquifer parameters and properties of stream channels (this is the most significant objective of this research, because numerically generated response equations have not been used in previously developed conjunctive management models for water use plans); (2) extend the concept of the rule curve storage that has been mostly used in reservoir operation in earlier works into the conjunctive water use planning models; (3) include all operational costs of surface and subsurface water supplies in the management models to make the problem more realistic and applicable to real-world problems; and (4) identify, apply and test the performance of a handy approximation method/model to cope with nonlinearities in the management models for the sake of ease in the real world applications. The management models developed in this work are powerful tool to solve practical problems.

Two conjunctive management model is applied to a hypothetical river basin involving a reservoir and a hydraulically-connected stream and underground water storage. The first model is a linear programming model and minimizes all deviation from the rule curve storages specified for the surface reservoir and stream course. The second model is a nonlinear programming model that minimizes total operational costs in addition to deviations from the rule curve storages. Nonlinearities associated with the physical settings of the system have been removed by assuming that: (1) the water level in the stream does not fall below the river bottom over the entire planning horizon; and (2)
the response of the system to releases from the reservoir is approximately linear so that this relation can be expressed with the help of the Green Function approach. However, nonlinearities that have arisen from the pricing associated with the operation of surface and subsurface supplies have been eliminated, in part, by imposing some additional mild assumptions related to the hydrology and operational policy of the basin.
2. Literature Review

Simulation and optimization methods have been widely used to determine operating strategies for regional water supplies. In earlier studies, groundwater and surface water were integrated for basin simulation by analytical models (e.g., Spiegel, 1962) by considering surface waters as either boundary conditions or hydraulically-connected aquifer elements of finite size but with unit storativity and infinite transmissivity, in aquifer systems that were broadly defined to include surface water, water laws, administrative rules, etc. Young and Bredehoeft (1972) developed a management model that incorporated aquifer behavior through a simulation, to analyze problems associated with unrestricted development of groundwater hydraulically connected to a stream. Using this model, the spatial and temporal interaction of the stream-aquifer system was analyzed through various excitations such as river flows, stream diversion, and well pumping. This model, however, does not guarantee an optimal solution because all feasible management strategies cannot be identified using only simulation. To overcome this difficulty, Maddock (1972) developed algebraic technological functions (response equations or discrete-kernel approach) that allow the explicit coupling of simulation and optimization models. Maddock (1974) extended ground-water management models and policy evaluation models to include surface water allocation and stream-aquifer interaction in addition to well pumpage where stream-stage is assumed to be time-invariant, i.e., withdrawals or losses from a stream to an aquifer do not affect the stream stage through response functions. This assumption is usually valid only along large rivers.

Illangasekare and Morel-Seytoux (1982) presented an analytical procedure to derive response equations of the physical components of a stream-aquifer system to simulate the behavior of the system when subjected to different excitations. The stream-aquifer system was divided into three basic physical components: (1) Surface conveyance system; (2) unsaturated flow region; and (3) saturated flow region. Each component was considered an isolated system, with independently derived system response equations. Since the various excitations are not in an optimization scheme, as in the Young and Bredehoeft (1972) model, an optimum solution may not be achieved.

Willis et al. (1989) developed a conjunctive ground and surface water planning model to maximize the net revenue obtained from various agricultural products. They developed hydraulic response equations of the conjunctive system using the finite element method and matrix exponential, which became part of the constraint set of the model.

Maddock and Lacher (1991) developed response functions to represent the potential flow in a stream-aquifer system through a discrete number of river reaches due to pumping from well fields, when linearity is maintained. They also extended the concept to handle nonlinear capture from the stream by introducing an additional sink term to the surface layer.
Matsukawa et al. (1992) presented a conjunctive-use management model to develop planning and operational strategies for the Mad River Basin, California. The river basin consisted of a single multipurpose reservoir and an unconfined aquifer system that hydraulically interacts with the river in the basin. They used a ground-water model and a hydrologic routing model interactively to develop mass-balance equations for surface water that relate reservoir releases, ground-water heads, and river stage to account for the loss from or gain to the river from the aquifer.

Lall (1995) presented a linked simulation-optimization yield based formulation, in which stream-aquifer interaction is not of direct concern, for screening ground and surface water resource development alternatives so as to satisfy various system constraints. In the complete management model, groundwater is considered to supplement surface water shortage in dry periods to meet the target yield.

The conjunctive management model formulated herein involves a set of deterministic stream flow data, which is not a serious shortcoming at all in light of the explanation given by Loganathan and Bhattacharya (1990).

From a computational standpoint, the nonlinearity associated with pumpage costs is of main concern in this study and has been replaced by a polygonal approximation. A traditional approach to handling the nonlinearities associated with the well pumpage cost in the objective function is to express the drawdowns in terms of pumpage rates using the response coefficients so as to put the objective function in a quadratic form (Colarullo, 1984; Misirli and Yazicigil, 1997). Global optimality then can be verified by checking the positive-definiteness of the Hessian matrix (Bazaraa and Shetty, 1979) through, for example, the Cholesky decomposition (Pres et al., 1992). However, if the drawdowns are considered to be a function of not only well pumpage but also irrigation return flow, reservoir release, diversion from the stream, and artificial recharge, then the objective function cannot be expressed simply in a quadratic form, but requires more in-depth analysis for the proof of global optimality. In this paper, the nonlinear portion of the objective function, after transforming it into a linear combination of convex and concave functions, will be linearized using a δ-form (Hadley, 1964; Williams, 1978) followed by corrections on the selection of the true segments in the concave portion of the objective function, by introducing a set of integer variables. After that, both linear and nonlinear models will be applied to a hypothetical system, and a comparison of the solutions from both models will be made.

3. Review of Methodology and Assumptions

3.1 Response Function

The response function approach is used to: (1) couple the subsurface and surface water components; (2) relate the state variables (drawdown and water exchange rate between stream and aquifer) to the decision variables (reservoir release, pumpage from well fields, irrigation return flow, surface water diversion from the stream, and artificial
recharge); and (3) incorporate transient hydraulic interaction between stream and aquifer into the management model. In derivation of the response equations, it is assumed that the response of the system to the reservoir releases can be approximated linearly. To generate response coefficients, the hypothetical system is subjected to a unit impulse of system excitations (e.g., reservoir release, pumping) in the first period with no disturbance thereafter, and then the system response to each excitation is monitored over the entire planning horizon (Gorelick, 1983; Louie et al., 1984). On the basis of the Gorelick (1982) interpretation of the unit impulse, unit reservoir release and stream diversion are considered to be 500 m$^3$/s, whereas unit pumpage, artificial recharge, and irrigation return flow of 1 m$^3$/s are used as system excitations. For this particular problem setting, with numerical development of response coefficients, the system reached a steady-state condition (i.e., state variables did not change) after eight periods. Thus, the model was run semi-annually (6 months), rather than yearly.

In deriving the response equations, stream stage is assumed to be linearly related to the stream flow. However, generally speaking, stream stage is related nonlinearly to the stream flow (e.g., Chezy or Manning’s equations) and thus the assumption of linearity definitely carries some error to the model. Thus, an error introduced by this assumption needs to be identified first. In all management models, MODFLOW was used to generate the response equations. In MODFLOW, stream flow is related to the stream stage through Manning’s equation, which is based on the assumption that the width of a stream channel is much greater than the stream’s depth. Based on the channel properties and the associated parameters described in section 5 in-depth, the validity of the assumption of linearity can be checked as follows:

Manning’s equation that relates the stream flow to stream stage can be written as:

\[
Q = \frac{C_n}{n} A \cdot R^{3/2} \cdot S_0^{1/2}
\]  
(1)

in which \(Q\) = stream flow; \(A\) = cross-section of the stream channel; \(R\) = hydraulic radius; and \(S_0\) = bottom slope (Manning’s equation above is based on the assumption of uniform flow along the stream channel). A rectangular stream channel was assumed, then \(A\) and \(R\) can be expressed as:

\[
A = B_0 \cdot y
\]

\[
R = \frac{B_0 \cdot y}{B_0 + 2y}
\]  
(2)

in which \(B_0\) = stream width (is constant along the stream depth, since a rectangular stream channel was assumed); and \(y\) = stream stage elevation above the stream bed.

For the geometry of the stream channel chosen for this study;

\[
B_0 \gg 2y \gg y
\]  
(3)

Then the denominator in equation (2) can be approximated as:
Substitution of equation (4) into equation (1), and using the values of parameters given in section 5, allows equation (1) to be written as:

\[ Q = 27.2727 \cdot y^{5/3} \]  

Using this simple equation (equation 5), one may observe that the linear and nonlinear relationships between the stream stage and flow give similar results with not more than a 10% difference, when the reservoir releases are in the range of 23.5 to 31.5 m³/s. However, for reservoir releases outside this range, some coefficients can be used to reduce the error. Referring to Figure 10(a), for example, for reservoir releases in the range of 5 to 10 m³/s (with and without that coefficient a maximum reservoir release of 10 m³/s was calculated in the management model) the following stage values were calculated for a set of stream flow values when they are related linearly and nonlinearly.

Table 1. Verification of Linear Assumption Between the Stream Stage and Discharge

<table>
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<tr>
<th>Stream Flow (L³)</th>
<th>Stream Stage (L)</th>
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<tr>
<td></td>
<td>Linear/(2.3)</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>5.5</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
</tr>
<tr>
<td>6.5</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.11</td>
</tr>
<tr>
<td>7.5</td>
<td>0.12</td>
</tr>
<tr>
<td>8</td>
<td>0.13</td>
</tr>
<tr>
<td>8.5</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>0.14</td>
</tr>
<tr>
<td>9.5</td>
<td>0.15</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Since the variations in the stream stage in both approximated and nonlinear forms (the second and third columns in Table 1) are similar for all practical purposes and for the sake of simplicity, equation (5) can be linearly approximated as:

\[ Q \approx 27.2727 \cdot (y/2.3) \approx 11.86 \cdot y \]  

However, for this particular problem the response equations are described in terms of water exchange rate between the stream and aquifer. The appropriateness of the linear approximation in this case can be verified as follows. The leakage to or from the stream reach, \( Q_{RR} \), can be calculated as follows:

\[ Q_{RR} = SCTR \cdot (H_s - H_{Aq}) \]  

in which \( CSTR = \) conductance of stream bed; \( H_s = \) head in the stream; and \( H_{Aq} = \) head in the aquifer side of the streambed. The conductance term can be calculated as:
in which \( k \) = hydraulic conductivity of the streambed; \( L \) = length of the streambed; \( w \) = width of streambed; and \( t \) = thickness of the streambed. Using the values of parameters given in the section 5, CSRT is calculated as 0.03 m\(^3\)/s.

It should be noted that equation (7) remains linear, since ground water levels can not drop below the streambed, which is enforced by equation (18) given section 4.1. Thus, a nonlinear flux through the stream bed is not possible as long as the stream flow and stream stage are approximated linearly as in equation (6).

The approximation suggested in equation (6) was incorporated into the existing numerical simulation model (MODFLOW) for the derivation of the response coefficients, so that the stream stages calculated on the basis of a nonlinear relation between the stream stage and flow can then be replaced by its linear approximation (see equation 6) before calculating the leakage through the stream bed. In this way, the overall system is approximated linearly.

### 3.2 Rule Curve Storage

In general, planners prefer not to violate target levels for critical parameters such as storage in, or release from surface and/or subsurface reservoirs. To develop operating strategies that rely on rule curves, planners must specify target levels, and acceptable ranges for those critical parameters. The feasible range of values above and below the target level is divided into subzones, each representing a deviation from the target level (Figure 1). The goal is to minimize deviations from the target levels to maintain the critical parameter at its target level over a planning horizon and has been applied mostly to reservoir operations (Yeh, 1985; Vasiliadis and Karamouz, 1994). In this paper, rule curve storage for both reservoir (RCS) and along the entire stream course (TSSTR) is defined, and deviations from those target levels are penalized in the objective function of the management model.

This is the first work in which the concept of rule curve storage has been applied to water storage along the stream course in a conjunctive management model. This is motivated by the following fact. Application of agrochemical (pesticides and fertilizers) on large agricultural fields is a common practice to improve the quality and quantity of crop yields. In addition to non-point sources, point sources are also a major threat on the quality of the stream water and thus on the stream habitat. The upper and lower storage levels along the stream course were defined not to flood the riverbank but to achieve the minimum water level for recreation and for river biota, respectively. In this case, storage limits within that range, for example, can be defined in terms of tolerance limits of different biota of the river to incorporate the various trade-offs into the model. Temporal variations in storage zones and target levels within the stream course can be associated with the temporal variations in pesticide, fertilizer, and non-point flux into the river. This flux rate depends on when the chemicals are put on the field and on meteorological conditions in the current and subsequent periods. Thus, using the concept of rule curves for the stream, decision-makers can easily incorporate their preferences and various trade-offs between quantity, quality, and stream habitat into the management model.
In any water resources management plan, the parts that surface and ground water resources take need to be analyzed first. In this research, we considered the situation in central California in determining the role of surface and ground water supplies in an overall management plan. Ground water in central California (San Joaquin Valley, one of the largest agricultural production valleys in the world) is more saline and less agriculturally productive than the surface water; thus, it is used as a supplementary or back up water storage (Dinar et al., 1993). Thus, the surface water supplies are more significant sources for central California (however, this is not necessarily true for other basins). Based on this observation, we specified the storage zones for all surface water supplies to meet the target levels at the optimal levels. However, this does not mean that ground water supplies are left outside the analysis. As mentioned earlier, ground water supplies are treated as back up water storage in this research, according to their allowable use capacity defined in equation (10). Thus, they will be operated: (1) when all surplus surface water supplies above the target levels are used to meet the demands and there is still a deficiency in supply or (2) when the storage in the reservoir happens to be far above the target levels and the ground water reservoir has enough room for artificial recharging practices.
For the reservoir and stream course, selection of the number of deviation (storage) zones and whether or not they are equally distant undoubtedly depends on the physics of the problem under consideration in the application phase. For example, if all consecutive planning periods happen to be dry, then it is not wise to penalize the surplus storage above the target levels (in contrast to the target storage equations shown in this report) as long as they do not violate the upper safety limits of the storage in the surface reservoirs. On the other hand, if snowmelt, for example, is a significant process in water mass balance equations for the surface water supplies, then the rule curve storages might be set just the opposite to what is considered herein. Since the models are applied to a hypothetical basin, we do not go through such a discussion in depth in this report. We looked at the problem from a mathematical point of view and treated it as a goal programming problem so as to satisfy the target levels strictly at the target levels and all deviations above and below the target levels are penalized (see Loganathan and Bhattacharya, 1990). Five storage zones above and below the target levels were considered to be sufficient for this application, in essence so as not to increase the number of integer variables excessively for the sake of ease of model solution with less CPU time, but to provide a clear interpretation of the model results.

4. Formulation of Management Models

4.1 Conjunctive Management Model–I (CMM-I)

The objective function of the management model minimizes penalties associated with deviations from the rule curves for storage in the reservoir and along the stream course:

$$
\text{Minimize } Z = w_1 \left( \sum_{n=1}^{NTS} \sum_{i=NSSA}^{NSA} PU(i,n) \cdot US(i,n) + \sum_{n=1}^{NTS} \sum_{i=NSSB}^{NSB} PL(i,n) \cdot LS(i,n) \right) \\
+ w_2 \cdot \left( \sum_{n=1}^{NTS} \sum_{i=NSSA}^{NSSA} PUSTR(i,n) \cdot USTR(i,n) + \sum_{n=1}^{NTS} \sum_{i=NSSB}^{NSSB} PLSTR(i,n) \cdot LSTR(i,n) \right)
$$

(9)

in which $Z = \text{value of the objective function (L}^3)$; NSA, NSB = number of storage zones above and below the storage rule curve for the reservoir, respectively; NSSA, NSSB = number of storage zones above and below the target storage along the stream course, respectively; PU(i,n), PL(i,n) = storage penalty functions for the reservoir; PUSTR(i,n), PLSTR(i,n) = storage penalty functions for the stream; US(i,n), LS(i,n) = excess and deficit storage in the reservoir, respectively (L$^3$); USTR(i,n), LSTR(i,n) = excess and deficit storage along the stream (L$^3$); $w_1, w_2 = \text{weighting factors representing relative weights of different objectives}$; NTS = total number of time steps; n = time index; and i = deviation zone index.

The constraint set of the management model is comprised of the following conditions:
1. Reservoir mass-balance equations for the surface reservoir in each time step must be satisfied to ensure continuity:

\[ SRES(n) = SRES(n-1) + (P(n) \times RSA + I(n) - Q_{RR}(n) - Q_D(n) - \text{SPILL}(n)) \times \Delta n \]

in which \( SRES(n-1) \), \( SRES(n) \) = reservoir storage volume at beginning and end of the time step \((L^3)\); \( P(n) \) = monthly net average precipitation rate \((L/T)\); \( RSA \) = surface area of the reservoir \((L^2)\); \( I(n) \) = net inflow to the reservoir \((L^3/T)\); \( Q_{RR}(n) \) = reservoir release to the first reach of the stream \((L^3/T)\); \( Q_D(n) \) = reservoir release directly to the demand area \((L^3/T)\); \( \text{SPILL}(n) \) = spilled water from the reservoir \((L^3/T)\); and \( \Delta n \) = time step.

2. Storage in the reservoir can be represented as the sum of the rule curve storage (RCS) plus deviations from the RCS:

\[ SRES(n) = RCS(n) + \sum_{i=1}^{NSA} US(i,n) - \sum_{i=1}^{NSB} LS(i,n) \]

in which \( RCS(n) \) = rule curve storage for the reservoir \((L^3)\).

3. Upper and lower specific storage safe levels are required to prevent overtopping of the structure in case of high inflow to the reservoir, and to provide sufficient releases from the reservoir to the stream at all times:

\[ 0.1 \times \text{CAP} \leq SRES(n) \leq 0.9 \times \text{CAP} \]

in which \( \text{CAP} \) = capacity of the reservoir \((L^3)\).

4. Agricultural, municipal, and industrial water demands must be met. Water diverted from the stream is used for agricultural demand exclusively:

\[ \eta \sum_{k=1}^{\text{NWELL}} Q_W(k,n) + \kappa Q_D(n) + \text{SDIV}(n) \geq WDA(n) \]

\[ (1 - \eta) \sum_{k=1}^{\text{NWELL}} Q_W(k,n) + (1 - \kappa) Q_D(n) \geq WDM(n) + WDI(n) \]

in which \( Q_W(k,n) \) = volume of water withdrawn from the \( k^{th} \) well field \((L^3/T)\); \( \text{SDIV}(n) \) = diverted water from the stream \((L^3/T)\); \( \kappa \), \( \eta \) = weighting factors indicating the allocation of water supplies for various water demands; and \( WDI \) (n), \( WDA(n) \), \( WDM(n) \) = industrial, agricultural, and municipal water demand requirements, respectively \((L^3/T)\).

5. Excess water not made available to crops or to other uses would be a potential source of deep percolation to the aquifer:
\[ Q_{Ex}(n) = (1 - E(n)) \left( \sum_{k=1}^{N\text{WELL}} Q_w(k,n) + Q_D(n) + SDIV(n) \right) \quad \forall n \] (14)

in which \( Q_{Ex}(n) \) = volume of water which is not made available to users and crops (L^3/T) and \( E(n) \) = water use efficiency.

6. Potential deep percolation from the unsaturated zone underlying an irrigated field to the water table can be calculated using the concept of field capacity:

\[
\begin{align*}
SUNST(n) &= SUNST(n-1) + \left( (1 - \xi)Q_{Ex}(n) + (1 - \Phi_{ef\ell})P(n)A(n) \right) \cdot \Delta n - PERC(n) \\
SUNST(n) &= PERC(n) + FC 
\end{align*}
\] (15a, 15b)

in which \( SUNST(n-1), SUNST(n) \) = water content in the soil above the water table at the beginning and end of the time step (L^3); \( \xi \) = percent of consumed water for various uses that is returned to the stream; \( \Phi_{ef\ell} \) = effective precipitation index (percent contribution of precipitation to the stream flow); \( A(n) \) = surface area of the agricultural zone (L^2); \( PERC(n) \) = deep percolation to the water table at the end of the n\textsuperscript{th} period (L^3); and \( FC \) = field capacity (L^3).

7. Spilled water from the reservoir is a source of artificial recharge to the aquifer. Spilled water exceeding the artificially rechargeable volume will be exported to outside the basin:

\[ SPILL(n) \geq \sum_{ar=1}^{TARW} Q_{AR}(ar,n) \quad \forall n \] (16)

in which \( Q_{AR}(ar,n) \) = artificial recharge at the ar\textsuperscript{th} plots (L^3/T) and \( TARW \) = total number of artificial recharge plots.

8. Response equations are used to incorporate spatial and temporal stream-aquifer interaction, as functions of decision and state variables, into the model to ensure continuity:

\[
\begin{align*}
Q_r(r,n) &= f(Q_{RR}(n), Q_w(k,n), PERC(n), SDIV(n), Q_{AR}(ar,n)) \quad \forall r, \forall n \\
s(l,n) &= f(Q_{RR}(n), Q_w(k,n), PERC(n), SDIV(n), Q_{AR}(ar,n)) \quad \forall l, \forall n 
\end{align*}
\] (17a, 17b)

in which \( Q_r(r,n) \) = flow between the r\textsuperscript{th} reach of the stream and the aquifer (L^3/T); \( s(l,n) \) = drawdown at the l\textsuperscript{th} observation point (L); \( r \) = reach index; \( l \) = observation well index; and \( k \) = pumping well field index.
9. Upper ($s_{\text{max}}(l,n)$) and lower ($s_{\text{min}}(l,n)$) bounds on drawdown must be specified to prevent water logging at the surface or excessive depletion of the aquifer:

$$s_{\text{min}}(l,n) \leq s(l,n) \leq s_{\text{max}}(l,n) \quad \forall n, \forall l \tag{18}$$

Ground-water storage in those limits is considered to be allowable ground-water use capacity.

10. Continuity equations for the surface water system must be met:

$$S_{\text{STR}}(n) = S_{\text{STR}}(n-1) + \left( Q_{RR}(n) - \sum_{r=1}^{N_{\text{REACH}}} Q_{RR}(r,n) - Q(n) + \xi Q_{\text{Ex}}(n) + P(n) A_{\text{str}} - S_{\text{DIV}}(n) \right) \Delta n \quad \forall n \tag{19}$$

in which and $S_{\text{STR}}(n-1)$, $S_{\text{STR}}(n) =$ stream storage volume at the beginning and end of the time step; $A_{\text{str}} =$ surface area of the stream course ($L^2$); and $Q(n) =$ downstream flow from the last reach ($L^3/T$).

11. Rule curve for storage for the entire stream course ($TS_{\text{STR}}$) is defined as:

$$S_{\text{STR}}(n) = TS_{\text{STR}}(n) + \sum_{i=1}^{N_{\text{SSA}}} USST(i,n) - \sum_{i=1}^{N_{\text{SSB}}} LSST(i,n) \quad \forall n \tag{20}$$

in which $TS_{\text{STR}}(n) =$ rule curve storage for the stream.

12. Upper and lower bounds are specified for state and decision variables (they have already been defined for drawdown and storage in the reservoir, in the preceding constraints).

4.2 Conjunctive Management Model–II (CMM-II)

Ground water usually costs less than surface water for small and moderate-sized municipal and industrial (M&E) applications, while in agriculture the comparative costs of surface and ground water depend on local conditions and water subsidies (Jenkins, 1992). In general, surface water supplies are used to satisfy water demands in wet periods with excess water stored either in a surface reservoir or underground through artificial recharge to meet the water deficiency in subsequent dry periods. Although legal constraints to conjunctive use management may be difficult to overcome, appropriately adjusted economic prices and incentives may help self-regulate the ground water and surface water use. The comparative costs of using surface and ground water can be adjusted so that water users should pay lower electricity rates for ground water pumping in dry periods and higher rates in wet periods (Boyd, 1991). Thus, higher unit pumpage cost and lower operation, maintenance, and replacement (OMR) costs will be assigned in the wet periods and the reverse in dry periods. In calculating unit pumpage costs, it is assumed that the energy needed to lift $1 \text{ m}^3/\text{s}$ of water vertically to a height of $1 \text{ m}$ in one second is $2.80$
watt-h. Unit pumpage costs given in Table 2 were estimated using a wire-to-wire efficiency of 0.55 and electrical costs of 0.30 $/KWH in dry and 0.90 $/KWH in wet periods.

Table 2. Unit Costs for the System Operation and Target levels

<table>
<thead>
<tr>
<th>Period</th>
<th>$/m^3/s</th>
<th>$/m^3/s/m</th>
<th>$/m^3</th>
<th>m^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet</td>
<td>2.20E-04</td>
<td>1.35E-04</td>
<td>4.58E-04</td>
<td>3.0E+07</td>
</tr>
<tr>
<td>Dry</td>
<td>4.22E-04</td>
<td>3.48E-04</td>
<td>1.53E-04</td>
<td>2.0E+07</td>
</tr>
</tbody>
</table>

The first and fourth periods are considered to be wet periods.

The objective function of the management model is formulated so as to minimize the total operation costs of surface and subsurface water storages (pricing) and meet the target storage levels in the surface water supplies so as to meet the system constraints given in section 2.3.1:

$$
\min Z = \sum_{n=1}^{N} \frac{C_{STR}(n)}{(1+r)^n} SDIV(n) + \sum_{n=1}^{N} \frac{C_{D}(n)}{(1+r)^n} Q_{D}(n) + \sum_{n=1}^{N} \sum_{l=1}^{N_{RW}} \frac{C_{AR}(l,n)}{(1+r)^n} [L(l) + s(l,n)] \cdot Q_{AR}(l,n)
$$

$$
+ \sum_{n=1}^{N} \sum_{k=1}^{N_{W}} \frac{C_{W}(k,n)}{(1+r)^n} [L(k) + s(k,n)] \cdot Q_{W}(k,n)
$$

$$
+ W1 \cdot \left( \sum_{n=1}^{NTS} \sum_{i=1}^{NSA} PU(i,n)US(i,n) + \sum_{n=1}^{NTS} \sum_{i=1}^{NSB} PL(i,n)LS(i,n) \right)
$$

$$
+ W2 \cdot \left( \sum_{n=1}^{NTS} \sum_{i=1}^{NSSA} PUSTR(i,n)USSTR(i,n) + \sum_{n=1}^{NTS} \sum_{i=1}^{NSSB} PLSTR(i,n)LSTR(i,n) \right)
$$

(21)

in which $C_{STR}(n)$, $C_{D}(n) =$ unit costs for diversion of water from the stream and reservoir ($$/m^3/s)$; $C_{AR}(l,n)$, $C_{W}(k,n) =$ unit costs for artificial recharge and pumpage ($$/m^3/s/m)$; $SDIV(n) =$ diverted water from the stream ($L^3/T$); $Q_{D}(n) =$ reservoir release directly to the demand area ($L^3/T$); $Q_{AR}(ar,n) =$ artificial recharge rate at the recharge basin ($L^3/T$); $Q_{W}(k,n) =$ pumpage rate at the supply wells ($L^3/T$); $L(l), L(k) =$ initial lifts at the
artificial recharge site and the supply wells, respectively; \( s(k,n), s(l,n) \) = change in ground water levels at the supply wells and recharge sites, respectively; \( NSA, NSB \) = number of storage zones above and below the storage rule curve for the reservoir, respectively; \( NSSA, NSSB \) = number of storage zones above and below the target storage along the stream course, respectively; \( PU(i,n), PL(i,n) \) = storage penalty functions for the reservoir; \( PUSTR(i,n), PLSTR(i,n) \) = storage penalty functions for the stream; \( US(i,n), LS(i,n) \) = excess and deficit storage in the reservoir, respectively; \( USST(i,n), LSST(i,n) \) = excess and deficit storage along the stream; \( w_1, w_2 \) = weighting factors representing relative weights of different objectives; \( NTS \) = number of total time steps; \( TARW \) = total number of artificial recharge plots; \( r \) = interest rate; \( n \) = time index; and \( i \) = deviation zone index.

The first two terms in equation (1) represent the costs of operation, maintenance, and replacement (OMR) for the diversion of water from the stream and reservoir. According to data from a Bureau of Reclamation project referred by Maass et al. (1964), a linear relation was found between the annual OMR costs and annual target output for water demand. In this study, the conveyance canal is presumed to be lined. It is also assumed that the irrigable area can be served by gravity without requiring pumping of diverted water, which is consistent with the assumptions of Maass et al. (1964). The third term in the objective function represents the artificial recharge costs through a well injection. This term is similar to the fourth term representing the pumpage costs at supply wells. However, it is different from the fourth term in that ‘\( s \)’ in the third term takes negative values as opposed to the term ‘\( s \)’ in the fourth term. The energy cost of injection can be assumed to be negligible for this problem, since upper and lower limits on drawdown are imposed to prevent pressurized injection (Lefkoff and Gorelick, 1986). Furthermore, all conveyance systems and supply wells are assumed to exist prior to the management model, thus no capital cost is involved in the objective function. The fourth term, representing the pumpage costs, causes nonlinearity since it includes the product of two decision variables, \( s(k,n) \) and \( Q_w(k,n) \). The rest of the terms in equation (21) minimize the deviations from the rule curve storages for the reservoir and stream.

In light of Maass et al. (1964) findings, a linear relation is assumed between the annual target water demand and annual OMR costs with a slope of 0.52 and 0.43 in dry and 0.27 and 0.17 in wet periods in the calculation of unit costs for water diversion from the reservoir and stream. Lower slopes in wet periods are used to account for the economic incentives and/or subsidies by the government or water agencies to encourage farmers to rely on surface water supplies in wet periods and to save the underground stored water for subsequent dry periods.
4.2.1 Nonlinearity and $\delta$-form Approximating Model for the Conjunctive Management Model–II (CMM–II)

4.2.1.1 Nonlinearity

As discussed in the preceding section, nonlinearity is caused by the product form of decision variables, $s(k,n)$ and $Q_w(k,n)$, in the fourth term of equation (21), representing the pumpage costs. It should also be noted that $s(k,n)$ in the pumpage cost is a function of a set of decision variables through a drawdown response equation given in equation (17b).

In subsequent sections, the product form will be converted first into a linear combination of nonlinear separable functions of a single variable. This modified model will then be converted into the $\delta$-form of the approximating problem that can be solved by standard linear programming techniques, after resolving the problems in selection of the correct segments in the concave function of the transformed objective function.

4.2.1.2 $\delta$-Form of the Approximating Problem

The aforementioned product form of the decision variables may be eliminated by casting the nonlinear term into a separable form through the introduction of a new set of variables as follows:

$$V(k,n) = \frac{Q_w(k,n) + s(k,n)}{2} \quad (22)$$

$$U(k,n) = \frac{Q_w(k,n) - s(k,n)}{2} \quad (23)$$

It should be noted that in equations (22) and (23) both $V(k,n)$ and $U(k,n)$ are unrestricted in sign, because rises in ground water levels are allowed in the management model. After performing the transformations indicated in equations (22) and (23), the product form in equation (21) may be written in terms of a linear combination of two nonlinear functions, convex and concave, in a separable form as follows:

$$Q_w(k,n) \cdot s(k,n) = f_1[V(k,n)] + f_2[U(k,n)] \quad (24)$$

where $f_1[V(k,n)] = [V(k,n)]^2$ and $f_2[U(k,n)] = -[U(k,n)]^2 \quad (25)$

Alternatively, equations (24) and (25) can also be interpreted as a difference of two convex functions, by which the global optimization problem is known as a multiextremal global optimization problem (Horst and Tuy, 1990).
In the separable programming technique, each separable function can be approximated by piecewise linear functions either in a \( \delta - \) or \( \lambda - \) form (Hadley, 1964; Williams, 1978). Due to its computational efficiency (Williams, 1978), the former is used in this report, in which each linear function in the polygonal approximation is represented by \( \delta_v(k,n,i) \) and \( \delta_u(k,n,i) \) (Figure 2) where the value of a special variable is the fraction of the segment covered by the variable \( V(k,n) \) and \( U(k,n) \). These special variables may be defined as:

\[
0 \leq \delta(k,n,i) \leq 1 \quad \text{and} \quad 0 \leq \delta(k,n,i) \leq 1 \quad \forall k, \forall n, \forall i \tag{26}
\]

\[
\text{if} \quad \delta_v(k,n,i) > 0 \quad \text{then} \quad \delta_v(k,n,i) = 1, \quad j = 1, \ldots, i - 1 \tag{27}
\]

\[
\text{if} \quad \delta_u(k,n,i) > 0 \quad \text{then} \quad \delta_u(k,n,i) = 1, \quad j = 1, \ldots, i - 1 \tag{28}
\]

\[
V(k,n) = V_{\min}(k,n) + \sum_{i=1}^{ND} \Delta V(k,n,i) \cdot \delta_v(k,n,i) \quad \forall k, \forall n \tag{29}
\]

\[
U(k,n) = U_{\min}(k,n) + \sum_{i=1}^{ND} \Delta U(k,n,i) \cdot \delta_u(k,n,i) \quad \forall k, \forall n \tag{30}
\]

in which \( ND \) is the total number of segments, and \( \Delta V(k,n,i) \) and \( \Delta U(k,n,i) \) correspond to a particular interval defined by the \( i^{th} \) segment of \( V(k,n) \) and \( U(k,n) \).
Figure 2. Piecewise Linearization of Convex and Concave Functions
In the model formulation, the approximation was made by subdividing the interval that each new variable can take into five equally-spaced segments. However, in order to simplify the presentation of the problem herein, \( V(k,n) \) and \( U(k,n) \) are assumed to be subdivided into two equal segments as shown in Figure 2, in which the polygonal approximations to the convex and concave functions are denoted by a dashed line and the approximations may be written as:

\[
\begin{align*}
    f_1[V(k,n)] &= \left[ V_{\text{min}}(k,n) \right]^2 + \left\{ \left[ V_{\text{max}}(k,n) + \Delta V(k,n) \right]^2 - \left[ V_{\text{min}}(k,n) \right]^2 \right\} \cdot \delta_1(k,n,1) \\
    &+ \left\{ \left[ V_{\text{min}}(k,n) + 2\Delta V(k,n) \right]^2 - \left[ V_{\text{max}}(k,n) + \Delta V(k,n) \right]^2 \right\} \cdot \delta_1(k,n,2) \\
\end{align*}
\]

and

\[
\begin{align*}
    f_2[U(k,n)] &= -\left[ U_{\text{min}}(k,n) \right]^2 + \left\{ -\left[ U_{\text{max}}(k,n) + \Delta U(k,n) \right]^2 - \left( -\left[ U_{\text{min}}(k,n) \right]^2 \right) \right\} \cdot \delta_2(k,n,1) \\
    &+ \left\{ -\left[ U_{\text{min}}(k,n) + 2\Delta U(k,n) \right]^2 - \left( -\left[ U_{\text{max}}(k,n) + \Delta U(k,n) \right]^2 \right) \right\} \cdot \delta_2(k,n,2) \\
\end{align*}
\]

In Figure 2, maximum and minimum values that each new variable can take are calculated separately for each well field and for each period, using equation (22) and (23). For the sake of simplicity, maximum pumpage rates and upper drawdown levels are assumed to be constant at all production wells. Furthermore, the coefficients of special variables in equation (31) and (32) may be expressed in terms of new constant terms as follows:

\[
\begin{align*}
    s_v(k,n,i) &= \left[ V_{\text{min}}(k,n) + i \cdot \Delta V(k,n) \right]^2 - \left[ V_{\text{min}}(k,n) + (i - 1) \cdot \Delta V(k,n) \right]^2 \\
    s_u(k,n,i) &= \left[ U_{\text{min}}(k,n) + (i - 1) \cdot \Delta U(k,n) \right]^2 - \left[ U_{\text{min}}(k,n) + i \cdot \Delta U(k,n) \right]^2 \\
\end{align*}
\]

Using these new constant terms, the nonlinear portion of the objective function (equation 21) can be rewritten by substituting the variables \( [V(k,n)]^2 \) and \( -[R(k,n)]^2 \) with their equivalents defined in equations (31) and (32):

\[
\begin{align*}
    \sum_{n=1}^{N} \sum_{k=1}^{NW} \frac{C_v(k,n)}{(1+r)^n} \left[ V_{\text{min}}(k,n) \right]^2 \\
\end{align*}
\]

\[
\begin{align*}
    + \sum_{n=1}^{N} \sum_{k=1}^{NW} \sum_{i=1}^{NB} \frac{C_v(k,n)}{(1+r)^n} \left[ s_v(k,n,i) \cdot \delta_v(k,n,i) \right] \\
\end{align*}
\]

\[
\begin{align*}
    + \sum_{n=1}^{N} \sum_{k=1}^{NW} \frac{C_u(k,n)}{(1+r)^n} \left[ U_{\text{min}}(k,n) \right]^2 + \sum_{n=1}^{N} \sum_{k=1}^{NW} \sum_{i=1}^{NB} \frac{C_u(k,n)}{(1+r)^n} \left[ s_u(k,n,i) \cdot \delta_u(k,n,i) \right] \\
\end{align*}
\]

In the above expression, the term represented by
\[ \sum_{n=1}^{N} \sum_{i=1}^{N_W} \frac{C_i(k,n)}{(1+r)^n} \left[ V_{min}(k,n) \right]^2 + -\left[ U_{min}(k,n) \right]^2 \] (35)

is constant since both \( V_{min}(k,n) \) and \( U_{min}(k,n) \) are constant. Consequently, this term can be represented by another constant term such as "A", and then the objective function of the model may further be simplified to:

\[
A + \text{Min} \left\{ USST, LSST, USST, LLST, \delta_u(k,n,i), \delta_u(k,n,i) \right\} 
\] (36)

Equation (36) becomes an approximating objective function for equation (21). As given in equations (31) and (32), the slope of each line in Figure 2 represents the rate of change of each separated nonlinear function with the increment chosen. Since the increments for both convex and concave functions in the objective function are assumed to be equally spaced, the slope of each line is then directly related to the cost coefficient of each special variable in the objective function. Furthermore, since the management model is a minimization model, variables with a lower cost coefficient will have the higher priorities in entering the basis, which is consistent with the conditions imposed for the convex function in equations (26), (27), and (29). In contrast, for the concave function those conditions are not met, thus the method results in a selection of a wrong segment. This is due to the fact that the first segment has a greater slope than the second segment, and equivalently the cost coefficient of the \( \delta_u(k,n,1) \) is greater than that of \( \delta_u(k,n,2) \). Thus, the model always gives the priority to the second segment and if it is entirely used up, then the first segment will be used. This is in conflict with equations (26), (28) and (30). In order to eliminate this conflict, a set of new integer variables \( G(k,n,i) \) are introduced as follows:

\[
\delta_u(k,n,i) - G(k,n,i) \geq 0 
\] (37)

\[
\delta_u(k,n,i+1) - G(k,n,i) \leq 0 
\] (38)

If \( G(k,n,i) = 1 \), then \( \delta_u(k,n,i) \geq 1 \) and \( \delta_u(k,n,i) + 1 \) \leq 1. Because the upper bound of these variables is "1", \( \delta_u(k,n,i) = 1 \) and no new restrictions are placed on \( \delta_u(k,n,i+1) \). However, if \( G(k,n,i) = 0 \), then inequalities (37) and (38) require that \( \delta_u(k,n,i) \geq 0 \) and \( \delta_u(k,n,i+1) \leq 0 \). Because the lower bound of these variables is "0", \( \delta_u(k,n,i+1) = 0 \). These new constraints ensure that the \((i+1)^{th}\) segment can be selected only if the \(i^{th}\) segment is covered entirely.
5. Hypothetical System

The hypothetical river basin taken up for this research is presented in Figure 3. Figure 4 shows the distribution of aquifer hydraulic conductivity and storativity. It is assumed that the basin has a square areal extent and is discretized into a uniform grid, comprising 15 rows and 15 columns with a grid spacing of 250 m in the x-direction and variable in the y-direction. The hypothetical basin consists of a single, multipurpose reservoir (water supply development and flood control) and an unconfined and laterally confined (except for four blocks at constant head) aquifer system that is connected hydraulically to a stream occupying the central portion of the basin. A rectangular channel with a stream width of 60 m, which is seven and a half times greater than its depth, is assumed. The channel is considered to be a natural channel with a Manning's roughness coefficient of 0.022 and a constant slope of 0.0001. The stream is hydraulically connected to the aquifer through a semipervious streambed having an average thickness of 0.5 m and a hydraulic conductivity of $1 \times 10^{-6}$ m/s. An agricultural field extends over a ten grid-cell along the stream in the upper portion of the basin. Eight cells in the upstream side are chosen for artificially recharging the aquifer, when excess water is spilled from the reservoir. Furthermore, five observation and three pumping well fields are located in the basin. Surface water diversions from the reservoir and along the stream course are considered as two options to transfer water to the demand area. Downstream flow requirements were set equal to $3.2 \ m^3/s$ in wet periods, $2.1 \ m^3/s$ for the second and third periods, and $2.4 \ m^3/s$ for the rest of the periods. The system is assumed to be initially at rest (ground-water level coincides with the water level in the stream), and the quasi-three-dimensional ground-water model of McDonald and Harbaugh (1992) is used to simulate the stream-aquifer system and to derive the response function coefficients.

6. Application

6.1 Numerical Application of CMM-I

The management model discussed in section 4.1 is applied to the hypothetical system depicted in Figure 3. Feasible regions above and below the RCS and TSSTR are divided into five equal deviation zones with associated unitless penalties in an ascending order of 1, 3, 15, 90, and 900, as deviation from the rule curve storage increases so as to fit a convex function (see Vasiliadis and Karamouz, 1994). So, as a rule of thumb, more deviations from the target levels are penalized more in the model.

Moreover, because the storage in the river is about one-tenth of the storage in the reservoir, a weighting factor, $w_2$, for the former is chosen to be ten times more than the weighting factor, $w_1$, for the latter to assign equal priority for both storages. Table 3 shows additional parameters pertaining to the stream, reservoir, and aquifer.
Figure 3. Finite-Difference Grid with Boundary Conditions, Pumping and Observation Wells
(Q(n) = Downstream Flow Leaving the System; Q_{RR}(n) = Reservoir Release)
Figure 4. Distribution of Aquifer Parameters

Hydraulic Conductivity (L/T)

Storativity

- Stream course
- Constant head
- No flow
Table 3. Additional Parameters Associated with the Stream, Reservoir, and Aquifer

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Capacity ((10^6 \text{m}^3))</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Storage ((10^6 \text{m}^3))</td>
<td>7.33</td>
</tr>
<tr>
<td></td>
<td>Diversion Channel Capacities ((\text{SDIV}(n); \text{QD}(n); \text{RR}(r,n)) \text{ (m}^3/\text{s}))</td>
<td>12; 4.8; 20</td>
</tr>
<tr>
<td>Stream</td>
<td>Maximum and Minimum Required Storage ((10^6 \text{m}^3))</td>
<td>3.15; 0.7875</td>
</tr>
<tr>
<td></td>
<td>Initial Storage ((10^6 \text{m}^3))</td>
<td>0.6510</td>
</tr>
<tr>
<td></td>
<td>Diversion Channel Capacity ((\text{m}^3/\text{s}))</td>
<td>3.9</td>
</tr>
<tr>
<td>Aquifer</td>
<td>Maximum Allowable Rise and Decline ((\text{m}))</td>
<td>0.5; 1</td>
</tr>
<tr>
<td></td>
<td>Maximum and Minimum Required Pumpage Rate ((\text{m}^3/\text{s}))</td>
<td>0.1; 0.8</td>
</tr>
</tbody>
</table>

Deep percolation to the water table is assumed to occur when the total moisture content in the pore volume of the unsaturated zone above the maximum rise level, underlying the agricultural plot, exceeds 21%, given that porosity in the unsaturated zone is 0.2. An initial moisture content of approximately 6% is assumed for the unsaturated zone. Furthermore, water use efficiency is considered to be 95%, and the effective precipitation index is assumed to be 0.2.

The agricultural zone (Figure 3) is considered to be the sole demand area in the basin, with higher water demand for agriculture than for municipal and industrial uses (Figure 5), and all demands increasing in time during the planning period. It is also assumed that ground-water is mostly used to meet municipal and industrial water needs, \(\eta = 0.4\), and diversion from the reservoir is used equally to meet both municipal & industrial and agricultural water needs, \(\epsilon = 0.5\).

![Figure 5. Industrial, Agricultural, and Municipal Water Demand Requirements](image_url)
Figure 6. Optimal Operating Policies for Surface Water Components
Initial storages in the reservoir and stream are assumed to be 76% below the RCS, and 67% below the TSSTR, respectively. Target storage levels, inflow to reservoir and precipitation data are assumed to be deterministic as given in Figure 6. The management model is solved using GAMS (Brooke et al., 1992) for the aforementioned hypothetical system and the resultant optimal operating policies for the surface water components and ground-water are shown in Figures 6 and 7.

![Figure 7. Optimal Pumpage Pattern, $Q_{w(k,n)}$, from Pumping Wells](image)

As can be seen from Figure 6, a high inflow rate into the reservoir in the first period raises the initial low storages in the reservoir and along the stream course to the RCS and TSSTR, respectively. Surplus storage, above the RCS, is eliminated by (1) releasing more water into the stream and (2) diverting more water to the artificial recharge zone. A high release into the stream, after accomplishing the TSSTR, is balanced by releasing the excess volume from the last reach (Figure 6) and recharging the aquifer through the streambed along the seventh and eighth reaches (Figure 8). A high artificial recharge rate leads to an increase in pumpage rates at the supply wells to create extra room for the recharged water and not violate the maximum allowable rise limit for the ground-water levels. However, the higher pumpage rates are used to satisfy the water demands, thus offsetting the water transfer from reservoir to the demand area. This is due to the fact that the water supply in excess of the demand rate would be a potential source of deep percolation to the ground-water from the demand area (Equations 13 and 14). This would be undesirable because water levels at the observation wells have already reached their maximum rise as a result of the artificial recharge practices.
The second period is characterized by considerably lower inflow and precipitation rates. RCS and TSSTR are reduced by 33% and 45% as compared to the first period. Some of the surplus water in the reservoir, as a result of a decrease in RCS, is diverted to the artificial recharge zone to maintain the reliance on the ground-water supplies, yet some excess water remains in the reservoir in this period to be able to accomplish RCS and TSSTR for the next drier period. The artificial recharge rate is, however, reduced by 23.1% which can be attributed mostly to a 61% reduction in the inflow rate to the reservoir and high water levels resulting from the recharge-discharge practices during the preceding period. Continuation of the artificial recharge at a high rate increases the water gain from the aquifer to the stream course by 282% (Figure 8). As a result of artificial recharge, total pumpage at supply wells is increased by 37.4% to avoid excessively high ground-water levels. An increase in the total pumpage offsets water diversion from the reservoir to the demand area by 27.2% and allows retention of water in the reservoir for the next period. To meet the RCS in the reservoir, reservoir release to the stream is reduced by 39.3%; however, the release rate to the stream is still enough to match the TSSTR in the stream. Furthermore, a decrease in reservoir release rate leads to a substantial decrease in the downstream flow rate.

In the third period, a 14% decrease in inflow to the reservoir and a 80% decrease in precipitation rate prevent outflow to the artificial recharge zone to be able to accomplish the RCS for the reservoir. High reliance on ground-water sources in the preceding period, and no artificial recharge practices in this period, result in lowering the total pumpage by 52.3% in order to maintain acceptable drawdown levels. This water supply deficiency at the demand area is balanced by increasing the water diversion from the reservoir directly to the demand area by 120.6%, using the excess water stored in the preceding period.
Storage of surplus water in the preceding period and the aforementioned operational practices make the rule curve storage for both reservoir and the stream achievable for this period.

High precipitation and inflow rate in the fourth period (but 33% and 11% lower, respectively, than those rates observed in the first period) is mostly stored in the reservoir and along the stream course, above the rule curve storages, to be used for the next drier periods. Total pumpage is reduced by 79% so as to keep them at their lower limits, while using the surface water supplies, to a greater extent, to meet the water demands. Low pumpage rate, artificial recharge practices, and gain from the higher stream levels (22.5% more storage exceeding the TSSTR) reverses the decline in ground-water supplies of the previous periods. A 23.5% higher reservoir release in this period leads to a 22.5% increase in storage along the stream course and satisfies the 52% increases in the downstream flow requirement.

Low inflow, low precipitation in the fifth period and high downstream flow requirement, necessitates an increase in pumpage rates at supply wells by 345.3%. Stored surplus water in the reservoir (17.6% above the RCS) and along the stream course (23% above the TSSTR) in the preceding period, lead to attainment of the TSSTR in the stream in this period. However, some of the water in the reservoir from the previous periods is reserved for the next drier period, by exceeding the RCS by 39.7% in this period. Some portion of the stored water in the reservoir is transferred to the demand area to satisfy the water deficiency caused by low pumpage at supply wells. The latter results in lowering the reservoir release to the stream by 21.8% so as to satisfy the RCS and TSSTR.

Very low precipitation and inflow in the last period lowers the storage in the reservoir below the RCS by 30.4%. Excess storage in the previous period helped to maintain the reservoir storage in the feasible region, and not to allow further decline in reservoir storage below the second deviation zone, keeping the associated penalties at a minimum. Total pumpage is lowered by 5% so as not to violate the maximum allowable drawdown limits, which results in an 8.2% increase in the water diversion rate from the reservoir to the demand area. Insufficient reservoir release (although 7% higher than the preceding period) to the stream as a result of low storage in the reservoir, and a high downstream flow requirement result in missing the target storage levels in the stream by 28%.

The stream is the main source of water for irrigation and is used at its maximum capacity in all periods. The water exchange rate between the stream and aquifer through the streambed for a six-month planning period is shown in Figure 8 (negative sign indicates gaining stream conditions). Figure 8 indicates that the first (except seventh and eighth reaches), second (except eighth reach), and fourth periods represent stream-gaining conditions while the rest of the periods represent losing stream condition. The findings for the seventh and eighth reaches in the first and the eighth reach in second period result from a combination of: (1) high pumpage rates in third pumping well field; (2) a relatively long distance from the artificial-recharge well; and (3) lack of direct infiltration from the demand area at the eighth cell. The higher rate of water gain through the third
reach of the stream from the aquifer is due to the fact that only cells along the third column of the grid in Figure 3 have been artificially recharged whenever excess water is available from the reservoir.

6.2 Sensitivity Analysis of Ground-Water Use Potential

The impact of the potential use capacity of the ground-water supply on the conjunctive use plans is analyzed through a sensitivity analysis of variations in the upper bound of drawdown levels ($s_{\text{max}}$). Variations in the objective function value (total deviations from the rule curves) for a set of $s_{\text{max}}$ values are given in Figure 9, indicating that as the ground-water use capacity increases, the sum of the deviations from the rule curves decreases. Thus, higher ground-water use capacity makes the model more flexible in achieving the target storage levels for the surface water system, a usual benefit of using true conjunctive use management.

![Figure 9. Variation in Objective Function, $Z$, with Respect to Ground-Water Use Potential](image)

Impacts of the variations in the ground-water use potential on the operating policies for a set of $s_{\text{max}}$ values of 0.5, 1, 1.5, and 2 are given in Figure 10. The scenario with $s_{\text{max}}$ of 0.5 m shows a slightly different trend from the rest, thus will be discussed separately. For the rest of the scenarios, generally speaking, as ground-water use potential increases, total pumpage from the supply wells also increases, which subsequently offsets the water diversion rate from the reservoir to the demand area, because the stream water is being used to meet the agricultural water needs at its maximum allowable limit in all periods in all scenarios. It should be noted that the higher total pumpage rate creates more room underground for artificial recharging, thus requires more water to be diverted to the artificial recharge zone to counteract the high decline of ground-water levels, as long as surplus water is available in the reservoir. However, more water diversion from the reservoir to artificial recharge zone and the demand area requires a decrease in the reservoir release to the stream to keep the water storage at the RCS levels in the reservoir so as to balance the water losses. Although this conclusion is valid for all periods, the operation policy of each component is sensitive to the initial storage in the reservoir and stream, initial ground-water level, and inflow and precipitation rate in each period. If economic measures were used in this analysis, differences would be more obvious.
a-Variations in the Reservoir Release to the Stream, $Q_{RR}(n)$

b-Variations in the Reservoir Release to the Demand Area, $Q_{D}(n)$
c-Variations in the Total Pumpage from the Supply Wells, $Q_w(k,n)$

d-Variations in the Artificial Recharge Rate, $Q_{AR}(ar,n)$

Figure 10. Sensitivity of Model Results with Respect to Potential Capacity of Ground Water Use

In an extreme case when $s_{\text{max}} = 2$ m, all water demand is met by the stream and wells without diverting any water from the reservoir to the demand area. However, a high ground-water decline as a result of high pumpage is balanced by a high artificial recharge rate in the first two periods. The very high artificial recharge rate in the first period leads to a significant decrease in the reservoir release to the stream to balance the water outflow from the reservoir. Furthermore, when $s_{\text{max}} = 2$ m, very high pumpage rates in the first period could not be maintained in the second period as a result of the low artificial
recharge rate and need to keep some of the ground-water supplies for the next, drier periods.

When \( s_{\text{max}} \) is reduced to 0.5 m, the total pumpage rate is still high (higher than the total pumpage rate when \( s_{\text{max}} = 1 \)) in the first period, which is compensated by high artificial recharge rate, because less room is available underground for the recharging practices. On the other hand, a high spill rate to the artificial recharge zone and a high reservoir release to the stream in this period lead to satisfaction of the RCS for the reservoir. Furthermore, a low ground-water use potential, when \( s_{\text{max}} = 0.5 \), restricts the ground-water operation options a great deal. Thus, the total pumpage rate increases at a very small rate from the fourth period to the last period. However, water diversion from the reservoir to the demand area increases linearly to provide enough percolation to the ground-water (equations 13 and 14). The operation plans for the reservoir releases to the stream help to reduce penalties, however, total deviations from the target storage levels are still very high (Figure 9). Thus, total deviations from the target levels and operational policies for surface and subsurface water resources are highly sensitive to the potential ground-water use capacity, and to the initial condition of each water supply at the beginning of each period.

6.3 Numerical Application of CMM-II

The nonlinear and the approximating form of the conjunctive management model were solved for a hypothetical river basin, using GAMS (Brooke et al., 1992). The nonlinear form, without approximation, was solved using MINOS, and the approximating problem was formulated as a linear mixed integer programming problem and solved using XA. In the latter, for both convex and concave functions, five segments were used for polygonal approximation, and the new constraints given in equations (22), (23), (26), (27), (29), (30), (37), and (38) were added to the constraints listed in equations (10) to (20).

The global optimality analysis for the nonlinear form without approximation requires in-depth analysis to prove global optimality, if at all possible. Thus, alternatively, in the solution phase one hundred sets of (arbitrarily selected) initial guesses were provided for the important decision variables, \( Q_w(k,n), Q_{RR}(n), Q_{D}(n), SDIV(n) \) and \( SPILL(n) \). The initial guesses, within the allowable ranges, were generated through a random number generator using the Box-Muller method (Press et al. 1992). However, this analysis showed that the value of the objective function of the management model remained unchanged irrespective of the initial guesses chosen.

It should also be noted that the set of feasible solutions given in equations (10) to (20) forms a convex set. Thus, any feasible solution to the approximating problem will also be a feasible solution to the original problem. Moreover, because the stipulations in equations (29) and (30) need to hold simultaneously, they do not lead to disjunctive constraints that may turn the feasible solution set into a nonconvex set (Horst and Tuy, 1990). On the other hand, discontinuities in the solution set makes it nonconvex. Furthermore, every piecewise smooth function (each linear segment) in Figure 2 is
bounded and has a bounded derivative everywhere, except at the corners. In the solution, it was found that some of those linearized segments were used fully so as to include the corner points, without using the next segment at all. Thus, the solution to the approximated problem is a local optimal solution which is also a feasible solution to the original nonlinear model. In the next section, the optimal solution, obtained as a result of one hundred runs with different initial guesses, to the nonlinear form without approximation was considered to be a global optimal solution against which the performance of the approximating problem was evaluated.

Optimal operating policies obtained from the nonlinear model without approximation (Model I) and from the approximating problem (Model II) are shown in Figure 11. As can be seen, the operating policies from the two models have the same pattern and are considered to be close enough to each other in practical applications. However, the major discrepancy in the operating policies from the two models is in the well pumpage rates and rates of water diversion from the reservoir in dry periods. The former might be attributed to the fact that the piecewise linearization technique was carried out directly on the product of the pumpage rates and the drawdown levels at the production wells. The discrepancies in the water diversion rates from the reservoir in the dry periods are caused by the water demand requirement constraints given in equation (13). Water diversion along the stream channel was found by both models at its upper limit of 3.9 m$^3$/s. Thus, the only source to satisfy the deficiencies in water requirements, due to the low ground water withdrawal, is the diversion of water from the reservoir. Moreover, relatively lower spilled water rates by the approximating model in the second and fourth periods are a result of higher diversion from the reservoir given that any deviations from the rule curve storage defined by equation (11) will be penalized in the objective function (equation (21)).

As can be seen from Figure 11, the sums of all deviations from target storage levels for the reservoir and the stream from the two models are in good agreement. In the management model, all terms representing the operation costs and deviations from the target levels were put in the same order of magnitude in equation (1), using appropriate weighting factors.

![Figure 11](image.png)

**Figure 11. Optimal Operating Policies from the Two Models**
Figure 11. Optimal Operating Policies from the Two Models (continued)
Figure 11. Optimal Operating Policies from the Two Models (continued)

Table 4 shows the total cost of the operation of surface and ground water resources. On the basis of the discussion on the operating policies in the preceding paragraph, Model II yields about 5% less total pumpage cost but about 6% more water diversion cost from the reservoir than Model I. It should be noted that high drawdown values as a result of relatively higher pumpage rates calculated by Model I are partly balanced by relatively higher artificial recharge rates, thus the total pumpage costs from the two models were found to be close enough with a difference of 5%.
Table 4. Comparison of Total Costs of the System Operation from the Two Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Total Pumpage Costs</th>
<th>Total Surface Water Diversion Cost (from Stream)</th>
<th>Total Surface Water Diversion Costs (from Reservoir)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model I</td>
<td>4962.1</td>
<td>63943.3</td>
<td>29963.9</td>
</tr>
<tr>
<td>Model II</td>
<td>4708.2</td>
<td>63943.3</td>
<td>31878.3</td>
</tr>
<tr>
<td>% Difference</td>
<td>5.1</td>
<td>0.</td>
<td>6.05</td>
</tr>
</tbody>
</table>

7. Summary, Conclusion and Discussion

A deterministic linear programming model (CMM-I) minimizing the deviations from the target storage levels in the reservoir and stream has been developed for the joint use of surface and ground-water resources to determine optimal operating rules for regional water supplies over a six-month planning horizon. Interaction between surface and groundwater has been introduced into the management model using numerically generated response equations. Target storage levels were defined for the reservoir and stream, while only a feasible range was defined for the ground-water supplies. Deviations from target levels in the reservoir and stream were penalized while satisfying a set of system constraints in the management model.

The management model yielded optimal operation of both surface and subsurface resources in wet and dry periods, while considering the possibilities of storing the surplus water in the wet periods for use in subsequent dry periods. If the RCS for the reservoir is exceeded, then the excess water is released to the stream, diverted to the demand area, and stored in the subsurface. However, this may lead to high pumpage rates to create enough room for artificial recharge, if the artificial recharge area is very close to the supply area, and a high volume water is being used for the recharging operation. However, use of economic measures associated with each alternative in the management model may yield different scenarios.

The sensitivity of the management model to ground-water use capacity was evaluated. It was found that deviations from rule curve storages are reduced as ground-water use capacity increases. Thus, variations in ground-water use capacity has direct impacts on all operational elements of water use policies and is very sensitive to water storage in the stream, reservoir, and aquifer, and the inflow and precipitation rate in each period.

This management model may be used for developing and improving target levels, exploring the best strategies in case of droughts and wet periods, and testing out various
possibilities for changing the water elements, including reservoirs, ground-water wells, surface water streams and canals. Assumptions and limitations associated with both simulation and management models, however, should be considered.

It was also shown that the $\delta$-form approximating model can be used in practical applications for the solution of the nonlinear conjunctive water resource management model (second model) with an explicit economic objective function. Solutions to the original nonlinear management model, without approximation, was found to be insensitive to the initial guesses chosen for the decision variables (a hundred different sets of initial guesses were tested). Alternatively, nonlinearity in the objective function was eliminated by replacing the term causing the nonlinearity with its polygonal approximation. However, in this method, problems associated with the selection of the true segments in the concave function of the objective function had to be resolved by introducing a new set of integer variables. Although the model was linearized successfully and the optimal solution to the approximated form was achieved using the $\delta$-form approximating method, the size of the model is increased as a result of the introduction of new variables and constraints. However, the structure of the conjunctive management model was suitable for the use of the $\delta$-form approximating method and its solution was quite easy using GAMS. Furthermore, the optimal operating policies obtained from the two models were in good agreement and they displayed the same trends. Diversion rates from the stream course were found to be at the maximum capacity of the diversion channels, from both models. Furthermore, the approximated model ended up with a 5% less total pumpage cost, but 6% more total water diversion cost from the reservoir, which can be considered to be acceptable in practical applications.

Regarding the advantage of the approximation method presented herein, it is worthwhile to read the following quote from Hadley (1964);

"...It is interesting to note that for some classes of practical problems, in spite of large number of local maxima, the value of the objective function at these local maxima appears to be quite close to the global optimum, and hence the fact that only a local optimum is obtained is not a serious drawback..." (page 110; note that problem that Hadley considered was a maximization problem. Since we considered a minimization problem, replace the word 'maxima', with 'minima').

Furthermore, if the set of feasible solutions to the original problem is convex (which is the case for CMM-II under consideration), then any feasible solution to the approximating problem will also be a feasible solution to the original problem, and there does not exist the possibility that the solution obtained to the approximating problem will not be feasible solution to the original problem.

The following observations can be made on the advantages of the approximation method employed in this report:
1. The optimal solutions obtained from the approximating and nonlinear models with 100 different sets of initial conditions were found to be similar and thus can even be considered to be the same for all practical purposes (the local optima from the approximating problem can be treated as global optimal, since the local optimum in this case is not a serious drawback).

2. Since the set of feasible solutions to the original problem forms a convex set, any solution to the approximating problem is also a feasible solution to the original problem and does not require any in-depth analysis to prove it.

3. The δ-form is preferred over the λ-form approximating problem due to its computational efficiency (Williams, 1990).

The δ-form approximating problem is a convenient method of converting the nonlinear model into a separable model, when the nonlinearity is due to product of two or more decision variables. The alternative way of dealing with product terms in a nonlinear model is to use logarithms (Williams, 1990). However, this procedure fails when the drawdown happens to be zero in any period (although pumpage rates at the supply well fields are lower-bounded so as not to leave them idle in any period, artificial recharge practices may still cause zero drawdown in the observation wells). However, the δ-form approximating problem does not have any such restrictions.

8. Significance of the Conjunctive Management Model

The main innovative ideas and the advantages of our conjunctive management model over earlier models are as follows:

1) Unlike the conjunctive management models developed by Young and Bredehoeft (1972), Illangasekare and Morel-Seytoux (1982, 1986), Hantush and Marino (1986), Lall (1995), Reichard (1996), etc., the system's heterogeneity (spatial distribution of aquifer parameters and stream channel properties) can be handled better, since we used numerical models rather than analytical models to couple the surface and subsurface components of the hypothetical hydrologic system.

2) Unlike the conjunctive management models developed by Peralta et al. (1995) Reichard (1996), etc., our model explicitly consider costs associated with the operation of various components of surface and subsurface supplies.

3) Unlike the conjunctive management models developed by Maddock (1972), Hantush and Marino (1986), Peralta et al. (1995), etc., temporal variations in the stream stage are also incorporated into the management model.

4) Unlike the conjunctive management model developed by Danskin and Gorelick (1985), all the costs associated with both surface ad subsurface water supply operation are explicitly included in the objective function of the management model.

5) Unlike the management model formulated by Matsuwaka et al. (1992), flow through the unsturated zone is included in the management model.
6) Unlike previously developed management models, the concept of rule curve storage for both reservoir and stream is introduced in the conjunctive management models. This is the first time that such a concept has been considered in conjunctive management models.

7) As far as we know, this is the first time that the, ‘δ – form approximating problem’ has been used to solve a conjunctive management model. It is a convenient technique to use, because: (1) the nonlinearity arises in the model due to the product of two decision variables; and (2) one should not be overly concerned about the global optimality in practical applications when this technique is employed (Hadley, 1964). This study clearly indicates that its use in the solution of the conjunctive management model is quite promising.

9. References:


Jenkins, M. 1992. Yolo County, California's water supply system: conjunctive use without management. M.S. Report, Department of Civil and Environmental Engineering, University of California, Davis, CA.


SDIV(n) = diverted water from the stream (L^3/T);
s_{\text{max}}(l,n) = upper bound on drawdown (L);
s_{\text{min}}(l,n) = lower bound on drawdown (L);
SPILL(n) = spilled water from the reservoir (L^3/T);
SRES(n-1), SRES(n) = reservoir storage volume at beginning and end of the time step (L^3);
SSTR(n-1), SSTR(n) = stream storage volume at the beginning and end of the time step (L^3);
SUNST(n-1), SUNST(n) = water content in the soil above the water table at the beginning and end of the time step (L^3);
TARW = total number of artificial recharge plots;
TSSTR(n) = rule curve storage for the stream (L^3);
US(i,n), LS(i,n) = excess and deficit storage in the reservoir, respectively (L^3);
USTR(i,n), LSTR(i,n) = excess and deficit storage along the stream (L^3);
w_1, w_2 = weighting factors representing relative weights of different objectives;
WDI(n), WDA(n), WDM(n) = industrial, agricultural, and municipal water demands (L^3/T);
Z = value of the objective function (L^3);
Greek Symbols:
\Delta n = time step;
\kappa, \eta = weighting factors indicating the allocation of water supplies for various water demands;
\xi = percent of consumed water for various uses that is returned to the stream;
\Phi_{\text{eff}} = effective precipitation index.
10. Notation:

\[ A(n) = \text{surface area of the agricultural zone (L}^2) \];
\[ A_{sr} = \text{surface area of the stream course (L}^2) \];
\[ C_{AR}(l,n), C_{W}(k,n) = \text{unit costs for artificial recharging and pumpage ($/m^3/s.m$)} \];
\[ C_{SR}(n), C_{D}(n) = \text{unit costs for diversion of water from the stream and reservoir ($/m^3/s$)} \];
\[ \text{CAP} = \text{capacity of the reservoir (L}^3) \];
\[ \text{E}(n) = \text{water use efficiency} \];
\[ \text{FC} = \text{field capacity (L}^3) \];
\[ i = \text{deviation zone index} \];
\[ I(n) = \text{net inflow to the reservoir (L}^3/T) \];
\[ k = \text{pumping well index} \];
\[ l = \text{observation well index} \];
\[ L(l), L(k) = \text{initial lifts at the artificial recharge site and the supply wells, respectively} \];
\[ n = \text{time index} \];
\[ \text{NSA}, \text{NSB} = \text{number of storage zones above and below the storage rule curve for the reservoir} \];
\[ \text{NSSA}, \text{NSSB} = \text{number of storage zones above and below the target storage along the stream course} \];
\[ \text{NTS} = \text{total number of time steps} \];
\[ \text{P}(n) = \text{monthly net average precipitation rate (L/T)} \];
\[ \text{PERC}(n) = \text{deep percolation to the water table at the end of the n}\text{th period (L}^3) \];
\[ \text{PU}(i,n), \text{PL}(i,n) = \text{storage penalty functions for the reservoir} \];
\[ \text{PUSTR}(i,n), \text{PLSTR}(i,n) = \text{storage penalty functions for the stream} \];
\[ \text{Q}(n) = \text{downstream flow from the last reach (L}^3/T) \];
\[ \text{Q}_{AR}(ar,n) = \text{artificial recharge at the ar}^{th} \text{plots (L}^3/T) \];
\[ \text{Q}_{R}(n) = \text{reservoir release directly to the demand area (L}^3/T) \];
\[ \text{Q}_{ex}(n) = \text{volume of water which is not made available to users and crops (L}^3/T) \];
\[ \text{Q}_{R}(r,n) = \text{flow between the r}^{th} \text{reach of the stream and the aquifer (L}^3/T) \];
\[ \text{Q}_{RR}(n) = \text{reservoir release to the first reach of the stream (L}^3/T) \];
\[ \text{Q}_{w}(k,n) = \text{volume of water withdrawn from the k}^{th} \text{well (L}^3/T) \];
\[ r = \text{reach index} \];
\[ \text{RCS}(n) = \text{rule curve storage for the reservoir (L}^3) \];
\[ \text{RSA} = \text{surface area of the reservoir (L}^2) \];
\[ s(k,n), s(l,n) = \text{change in ground water levels at the supply wells and recharge sites, respectively} \];


