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A New Model for Well Test Data Analysis for Naturally Fractured Reservoirs

by C.H. Lai, G.S. Bodvarsson, C.F. Tsang, and P.A. Witherspoon, Lawrence Berkeley Laboratory

ABSTRACT

A new model for the analysis of constant rate well test data from naturally fractured reservoirs is presented. The model considers three sets of orthogonal fractures with fluid flow from the fractures to the well and from the rock matrix to the fractures. In comparison to other models for naturally fractured reservoirs the present model allows fully transient fluid flow from the rock matrix to the fractures and a cubic geometry of the matrix blocks.

The model has been used to develop type curves for the analysis of drawdown and build-up tests as well as pressure transient data from observation wells. The reservoir systems considered include an infinite and a finite (no-flow outer boundary) system and a system with a constant pressure outer boundary. The effects of wellbore storage and skin are illustrated. The model is applied to field data to illustrate the method of analysis and the applicability of the model.

INTRODUCTION

In the last two decades considerable work has been devoted to the analysis of well test data from naturally fractured reservoirs. The need for new analysis models arose because of the distinct differences in the pressure response at wells completed in homogeneous porous media reservoirs to that of wells penetrating naturally fractured reservoirs. The approach used in developing analysis methods for well test data of naturally fractured reservoirs is to treat the fractures and the rock matrix separately, but couple their response by means of interaction terms. Thus, the fractures represent high permeability for fluid transport into the well, whereas the rock matrix has a much lower permeability, and provides gradual fluid drainage to the fractures. On the other hand, the fraction of the total volume occupied by the fractures (fracture porosity) is very small, and consequently the bulk of the fluids is stored in the rock matrix. This approach is currently referred to as the double porosity approach, and was developed by Barenblatt et al., 1,2 and Warren and Root. They considered the model shown in Figure 1, in which each point in the system is assigned two pressures, one for the fractures, \( p_f \), and the other for the rock matrix \( p_m \). Thus, for a rigorous solution to the problem, one must solve diffusion equations for both media. However, Barenblatt et al., 1,2 and Warren and Root assumed a quasi-steady flow between the rock matrix and the fractures. This approximation simplifies the problem considerably so that solutions for the pressures in the fractures and rock matrix can easily be obtained in the Laplace domain.

Warren and Root 3 found that the pressure solution could be characterized by two parameters \( \lambda \) and \( \omega \). The parameter \( \lambda \) represents the ratio of the rock matrix permeability to that of the fractures; whereas \( \omega \) represents the ratio of the fracture compressibility to the compressibility of the total system (see nomenclature for definitions of \( \lambda \) and \( \omega \)). For naturally fractured reservoirs typical values of \( \lambda \) and \( \omega \) fall within the ranges of \( 10^{-3} \) to \( 10^{-9} \) and 0.1 to 0.001, respectively.

Subsequent to the studies of Barenblatt et al. 1,2 and Warren and Root 3, various studies have been published on the applicability and extension of their models. Odeh 4 used a model similar to that of Warren and Root, and concluded that the pressure behavior in a naturally fractured reservoir is identical to that of homogeneous porous media reservoirs. However, in his study Odeh 4 only considered cases where the interporosity flow factor \( \lambda \) was relatively large (\( >10^{-3} \)), in which case the differences in the transient pressure behavior are only apparent at very early times. Later, Mayor and Cinco-Ley 5 extended the solution by Warren and Root to include the effects of wellbore storage and skin.

Many workers have developed models that do not require the approximation of quasi-steady fluid flow between the rock matrix and the fractures. However, due to the three-dimensional nature of the model considered by Barenblatt et al. 1,2 and Warren and Root (Fig. 1) the treatment of transient interporosity flow is mathematically very difficult, and has been accomplished only by more or less drastic simplification of matrix block geometry. Deruyck 6, Kazemi 7, Streltsova 8 and Serra et al. 9 considered a slab model, whereas de Swaan 10, Najurieta 11, and Cinco-Ley et al. 12 consider models based on spherically shaped matrix blocks.

References and illustrations at end of paper.

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The slab model is applicable to layered reservoirs as well as reservoirs with predominantly horizontal fractures. However, in the slab model the fluid flow in the layers is assumed one-dimensional; an approximation that is only valid if the permeability contrast between layers is large.

In contrast to the above transient models we use the original model proposed by Barenblatt et al. and Warren and Root (Fig. 1). This model considers three orthogonal fracture sets separated by cubic rock matrix blocks and does not invoke further simplifications of block shapes. Fully transient fluid flow between the rock matrix and the fractures is evaluated by means of a novel approximation for spatial dependence of pressure change in the blocks. In the following discussion we will describe the mathematical model and verify the solutions obtained by using a numerical model. Type curves will be given for the cases of constant rate production in infinite and finite systems as well as a system with a constant pressure outer boundary. The effects of wellbore storage and skin will be illustrated. Finally, application of the model will be illustrated in the analysis of field data.

**BASIC MODEL**

In formulating the governing equations for the pressures in the fractures and rock matrices, we use the Warren and Root approach of lumping the fractures and the rock matrices into two different continua. Using this approach the governing equation for the pressure in the fractures can easily be derived, but the geometry of the rock matrix (cubic) causes some problems. For a rigorous treatment of the fluid flow in the rock matrix continuum, a three-dimensional representation is necessary. However, we have developed a one-dimensional representation of the fluid flow in the rock that is quite adequate for the present problem and actually gives practically identical results for the pressure transients as those obtained using a three-dimensional model for the rock matrix.

The approach employed is as follows. Due to the high permeability of the fractures, pressure changes will propagate rapidly in the fracture network while migrating slowly into the low-permeability rock matrix. Therefore, as pointed out by Pruess and Narasimhan, pressures at each point in the rock matrix will depend primarily on the distance to the nearest fracture. In light of this and by neglecting gravity effects, flow in the rock cubes can be approximated by a one-dimensional basic model, as shown in Figure 2. The basic model represents one-sixth of a rock matrix block, with the surface area for flow decreasing from $D^2$ (D is the fracture spacing) at the edges of the cube to zero in the middle. Thus, the total flow from the rock matrix block to the fractures will be six times that given by the one-dimensional model.

In order to verify the one-dimensional approximation, we consider flow to a cube with a uniform initial pressure of zero and a constant boundary pressure of unity. The solution for the transient flow rate in this three-dimensional problem is given by Carslaw and Jaeger:

$$ q_D = 24 \times \frac{1}{n^4} \sum_{k=1}^{n-1} \sum_{m=1}^{n-1} \frac{1}{(2k-1)^2(2m-1)^2} $$

The dimensionless flow rates for the two models (Eqs. 1 and 2) are plotted versus dimensionless time in Figure 3. The figure shows that at all times the dimensionless flow rate is practically identical for the two models. This indicates that the flow from the rock matrix to the fractures in naturally fractured reservoirs is quite well approximated by the one-dimensional model. However, it should be pointed out that the pressure distribution in the rock matrix blocks are not the same in the one- and three-dimensional models. This does not, however, have significant impact on interporosity flow and the pressure response at wells completed in naturally fractured reservoirs.

**MATHEMATICAL MODEL**

In addition to the approximation discussed above, the following assumptions are made:
1. The reservoir is of uniform thickness, with impermeable lower and upper boundaries.
2. The fluid flow from the system into the wellbore is radial and only the fractures feed the well.
3. The initial pressure $P_i$ is uniform throughout the system, but at time $t > 0$, a constant flow rate $q$ at the wellbore is imposed.
4. The pressure in the fractures is assumed to be equal to the pressure in the rock matrix at the contact region ($Z = D/2$).
5. All properties such as permeability, porosity, and compressibility, are constants in each continuum.
6. The fluid flow is isothermal and single-phase. The fluid is slightly compressible, with constant properties (viscosity and density).

The governing equation describing the fluid flow in the fracture system can be derived by performing a mass balance on a control volume in the fractures (see Appendix A):

$$ \frac{\partial^2 P_0}{\partial r^2} + \frac{1}{r} \frac{\partial P_0}{\partial r} - \frac{6 \kappa_1}{k_2 D^2} \frac{P_0}{Z = D/2} = \frac{q c \mu}{k_2 \delta t} \frac{\partial P_1}{\partial r} $$

where $P_0$ is the pressure in the fracture, and $P_1$ is the pressure in the rock matrix. Other symbols are defined in the nomenclature. The governing equation for fluid flow in the rock matrix can be expressed as:

$$ \frac{\partial^2 P_1}{\partial Z^2} + \frac{3}{2} \frac{\partial P_1}{\partial Z} = \frac{q c \mu}{k_1 \delta t} \frac{\partial P_1}{\partial Z} $$

The initial conditions are:

$$ P_2(r,0) = P_1(r,2,0) = P_1 $$

The dimensionless flow rates for the two models (Eqs. 1 and 2) are plotted versus dimensionless time in Figure 3. The figure shows that at all times the dimensionless flow rate is practically identical for the two models. This indicates that the flow from the rock matrix to the fractures in naturally fractured reservoirs is quite well approximated by the one-dimensional model. However, it should be pointed out that the pressure distribution in the rock matrix blocks are not the same in the one- and three-dimensional models. This does not, however, have significant impact on interporosity flow and the pressure response at wells completed in naturally fractured reservoirs.
The boundary conditions at the well are controlled by the constant flow rate, \( q \), and the effects of wellbore storage:

\[
\frac{\partial p}{\partial t} + 2\pi H \frac{1}{\mu} \frac{\partial p}{\partial r} \bigg|_{r=r_w} = qB
\]  
(6)

The effects of infinitesimal skin region around the wellbore can be expressed by:

\[
P_{wf} = \left[ P_2 - Sr_w \frac{\partial p}{\partial r} \right]_{r=r_w}
\]  
(7)

The boundary conditions for the rock matrix are:

\[
\frac{\partial p_1}{\partial z} \bigg|_{z=0} = 0
\]  
(8)

\[
P_1(r,z,t) \bigg|_{z=D/2} = P_1(r,t)
\]  
(9)

Three different cases are considered for outer boundary conditions; the reservoir is infinite in radial direction, a finite reservoir with a no-flow boundary, and a constant pressure boundary.

Infinite reservoir:

\[
\lim_{r \to \infty} P_2(r,t) = P_1
\]  
(10)

Finite reservoir:

\[
\frac{\partial p_2}{\partial r} \bigg|_{r=r_e} = 0
\]  
(11)

Constant pressure:

\[
P_2(r,t) \bigg|_{r=r_e} = P_1
\]  
(12)

In terms of dimensionless parameters, the governing equations (3)-(4), the initial conditions (5), and the boundary conditions (6)-(12), can be written as:

\[
\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \frac{\partial p_2}{\partial r} - 3\frac{\partial p_1}{\partial n} \bigg|_{n=1} = \omega \frac{\partial p_2}{\partial r} \bigg|_{r=r_e}
\]  
(13)

\[
\frac{\partial^2 p_1}{\partial n^2} + \frac{2}{n} \frac{\partial p_1}{\partial n} \bigg|_{n=0} = \frac{(1-\omega) \partial p_1}{\partial t} \bigg|_{t=1}
\]  
(14)

\[
P_2(r_D,0) = P_1(r_D,0) = 0
\]  
(15)

\[
\left. \frac{\partial p_2}{\partial r} \right|_{r=r_D} = 1
\]  
(16)

\[
P_2 = \left[ P_2 - S \frac{\partial p_2}{\partial r} \right] \bigg|_{r=r_D} = 1
\]  
(17)

\[
\left. \frac{\partial p_2}{\partial n} \right|_{n=0} = 0
\]  
(18)

The mathematical model is fully defined through equations (13) to (22). The simultaneous solution of the equations using the Laplace transformation is derived in Appendix B. In the Laplace domain the solutions for the pressure in the flowing well and the fractures are:

Infinite reservoir

\[
\tilde{P}_D = \frac{K_0(\sqrt{\lambda}) + S\sqrt{\lambda}X_1(\sqrt{\lambda})}{\frac{2\pi}{k_2H} + C_D p_1 K_0(\sqrt{\lambda}) + S\sqrt{\lambda}X_1(\sqrt{\lambda})}
\]  
(30)

\[
\tilde{P}_D = \frac{K_0(\sqrt{\lambda}) + S\sqrt{\lambda}X_1(\sqrt{\lambda})}{\frac{2\pi}{k_2H} + C_D p_1 K_0(\sqrt{\lambda}) + S\sqrt{\lambda}X_1(\sqrt{\lambda})}
\]  
(31)

\[
X_2 = 3\lambda X_1 \coth X_1 - 3\lambda + wp
\]  
(32)

\[
X_1 = \sqrt{\frac{(1-\omega)H}{\lambda}} = \sqrt{15(1-\omega)H}
\]  
(33)
and \( p \) is the Laplace parameter. It should be pointed out that this result is identical to de Swaan's result, which was obtained by approximating the behavior of cubical matrix blocks with that of spheres, provided the diameter of the spheres is equal to the side length of the cubes.\(^{10}\)

Finite reservoir

\[
\bar{P}_{Df} = \frac{\{ I_0(\sqrt{x_2})K_1(\sqrt{x_2}e_D) + I_1(\sqrt{x_2}e_D)K_0(\sqrt{x_2}) \}}{C_D p^2(1 - S + x_2 X) - p\sqrt{x_2} X}
\]

\[
\bar{S} x_2 \{ I_1(\sqrt{x_2}x_1(\sqrt{x_2}e_D) - I_1(\sqrt{x_2}e_D)K_1(\sqrt{x_2}) \}
\]

\[
C_D p^2(1 - S + x_2 X) - p\sqrt{x_2} X
\]

\[
\bar{P}_{D2} = \frac{I_0(\sqrt{x_2}D)K_1(\sqrt{x_2}e_D) + I_1(\sqrt{x_2}e_D)K_0(\sqrt{x_2}r) }{C_D p^2(1 - S + x_2 X) - p\sqrt{x_2} X}
\]

where

\[X = I_1(\sqrt{x_2})K_1(\sqrt{x_2}e_D) - I_1(\sqrt{x_2}e_D)K_1(\sqrt{x_2})\]

\[Y = I_0(\sqrt{x_2})K_1(\sqrt{x_2}e_D) + I_1(\sqrt{x_2}e_D)K_0(\sqrt{x_2})\]

Constant-pressure outer boundary

\[
\bar{P}_{Df} = \frac{K_0(\sqrt{x_2})I_0(\sqrt{x_2}e_D) - I_0(\sqrt{x_2}e_D)K_0(\sqrt{x_2})}{p\sqrt{x_2}[I_0(\sqrt{x_2}e_D)K_1(\sqrt{x_2}) + I_1(\sqrt{x_2}e_D)K_0(\sqrt{x_2}e_D)]}
\]

Wellbore storage and skin effects are not considered in the case of a constant-pressure outer boundary. The complex nature of the solutions prohibits analytical inversion from the Laplace domain into real space. Therefore, we employ a numerical inverter by Stehfest\(^{15}\) (1979) to obtain the solution in real space.

ASYMPTOTIC SOLUTIONS

In the following discussion we consider the case of the infinite reservoir without wellbore storage and skin effects and develop asymptotic solutions for the early- and late-time behavior.

Early time behavior

At early times the pressure response at the well is only governed by the characteristics of the fracture system:

\[
\bar{P}_{Df} = \frac{2}{\sqrt{\pi}} \sqrt{\frac{r_D}{\omega}}
\]

The period for which Equation (39) holds depends on the hydraulic properties of the fractures and the rock matrix. If the fracture storativity \( \omega \) is large and the interporosity flow factor \( \lambda \) small, the early-time behavior will last for a long time, and a semilog straight line can be observed. The flowing well pressure is given by:

\[
\bar{P}_{Df} = \frac{1}{2} \left[ \ln\left( \frac{r_D}{\omega} \right) + 0.80909 \right]
\]

Late-time behavior

At late times the flow between the rock matrix and the fractures becomes quasi-steady and the pressure response at the well is identical to that of a homogeneous reservoir with a storativity \( (\omega c)_2 \) = \((\omega c_1 + \omega c_2)\):

\[
\bar{P}_{Df} = \frac{1}{2} \left[ \ln(\omega) + 0.80909 \right]
\]

Comparison of Eqs. (40) and (41) shows that the early-time and late-time semilog straight lines will be parallel and offset by \( \ln(\omega) \).

The early- and late-time behavior described here is identical to that obtained by earlier models, e.g., the Warren and Root model\(^2\) and layered reservoirs\(^8,9\). The present model and the earlier models differ only in the transient pressure response at intermediate times.

COMPARISON BETWEEN MODELS FOR NATURALLY FRACTURED RESERVOIRS

The main difference between the present model and that of Warren and Root\(^2\) is that we assume transient fluid flow between the rock matrix and the fractures instead of the quasi-steady state fluid flow assumption. Results from these models for several values of \( \lambda \) are shown in Figure 4. As mentioned earlier, the early- and late-time semilog straight lines are identical for both models. However, significant differences are evident in the transient region at intermediate times. In the present model, significant flow from the rock matrix to the fractures occurs much earlier than in the Warren and Root model; consequently, the pressure deviates earlier from the first semilog straight line. Also, the pressure transients in the intermediate-time region last considerably longer in the present model than is predicted by the Warren and Root model. As will be shown later, the pressure transient data in the intermediate-time region are essential for the determination of the reservoir parameters, since in most cases the early-time data (first semilog slope) are masked by wellbore storage effects.

Other models that consider transient fluid flow between the rock matrix and the fractures\(^8,9,11\) show similar transient pressure behavior at intermediate times. Therefore, depending upon the geologic conditions that prevail at a given site, the present model for naturally fractured reservoirs may be utilized or, in the case of layered reservoirs, models developed by Streltsova\(^8\) or Serra et al\(^9\) are applicable.

VERIFICATION OF THE PRESENT MODEL

In order to verify the mathematical model and the accuracy of the numerical inverter, we conducted independent numerical simulation studies using the three-dimensional simulator PT (Pressure and Temperature), recently developed at Lawrence Berkeley Laboratory\(^16\). Numerical analysis of pressure transients of wells completed in naturally fractured reservoirs were carried out using the multiple interacting continua (MINC) method\(^13\). The comparison between the numerical results and the results predicted by the present semi-analytic model is shown in Figure 5. The excellent agreement did not warrant additional simulation studies.
METHODOLOGY

Type curves for drawdown tests in naturally fractured reservoirs of infinite areal extent are shown in Figure 6 for three different values of \( \lambda \) \((10^{-3}, 10^{-6}, 10^{-9})\) and \( \omega \) \((10^{-1}, 10^{-2}, 10^{-3})\); these values cover the range of probable values for naturally fractured reservoirs. Not only \( \lambda \) but also \( \omega \) determines the time of pressure deviation. Figure 6 shows that \( \omega \) controls the shift of the early- and late-time semilog straight lines, whereas \( \lambda \) determines the time of pressure deviation from the first slope and the time of convergence to the late-time curve.

In order to develop methods for analysis of data from naturally fractured reservoirs, an approximate analytical solution is helpful. Applying the improved Schapery technique\(^{11,17}\) to Eq. (30) (without wellbore storage and skin effects), the following solution was obtained (Appendix C):

\[
P_{Df} = -\gamma + \ln 2 - \frac{1}{2} \ln \left[ \frac{\lambda}{5} \sqrt{\frac{15(1 - \omega)}{e^{\lambda t_D}}} \right] \coth \left( \frac{\sqrt{15(1 - \omega)}}{e^{\lambda t_D}} - \frac{\lambda}{5} \right) + \frac{\omega}{5} e^{\lambda t_D}
\]  
(42)

Equation (42) is valid for dimensionless times greater than \( t_D = 10 \), which covers times of most practical interest. For this time range Eq. (42) is generally accurate within 1%; the maximum deviation from values calculated using the numerical inverter is 2%. At late times the solution is exact. Equation (42) is used as a basis in the following discussion.

As mentioned earlier, the pressure response of naturally fractured reservoirs is characterized by three segments, a semilog straight line at early times, a transition period, and a late-time semilog straight line. In many cases, regardless of wellbore storage effects, the initial straight line is not present. Only in cases where \( \frac{\lambda}{\omega} \leq 7 \times 10^{-7} \) can the first linear segment be observed. By correlation, the initial semilog straight line ends at a dimensionless time of:

\[
t_D = \frac{\omega^2}{10 \lambda}
\]  
(43)

During the transition period the pressure changes are much less than at early and late times because of the large fluid flow from the rock matrix feeding the fractures. This period lasts for about 7 log cycles of dimensionless time. During the transition period two linear segments on the pressure-log time plot (Fig. 6) may be identified. The first segment has a slope half that of the initial and final slopes. This half-slope has also been identified by Streltsova\(^8\) and Serra et al.\(^9\) for the case of stratified reservoirs.

The half-slope occurs around the dimensionless time when the two last terms in Eq. (42) cancel each other:

\[
(t_D)^*_{IH} = \frac{\frac{5\omega}{e^{\lambda}}}{}
\]  
(44)

At that time the pressure declines according to the expression:

\[
P_{Df} = \frac{1}{4} \left[ \frac{\ln \left( \frac{\ln (1 - \omega) - \ln \frac{3}{80} - 3\gamma}{} \right)}{e^{\lambda t_D}} \right]
\]  
(45)

The time period over which a half-slope can be observed depends on \( \omega \). For \( \omega = 0.001 \), the half-slope occurs for over a log cycle whereas for \( \omega = 0.01 \) it lasts only a half-log cycle. Where \( \omega \) is larger than 0.1, the half-slope segment cannot be easily identified. The intersection between the initial semilog straight line and the half-slope straight line occurs at a dimensionless time of:

\[
(t_D)^*_{IH} = \frac{5w^2}{3e^{\lambda(1 - \omega)}}
\]  
(46)

Similarly, the intersection of the half-slope line with the final linear segment occurs at a dimensionless time of:

\[
(t_D)^*_{FH} = \frac{5}{3e^{\lambda(1 - \omega)}}
\]  
(47)

At a slightly later time in the transition period, a brief linear segment with a slope two-thirds that of the final slope is apparent. Due to the complex nature of the analytical approximation (Eq. (42)), it is not possible to mathematically derive the exact time of deviation of this linear segment. It is also of questionable value because of its short duration. However, as is the case with the half-slope, the 2/3-slope increases in duration with decreasing value of \( \omega \). The pressure transients converge on the final slope at a dimensionless time of:

\[
t_D = \frac{3(1 - \omega)}{\lambda}
\]  
(48)

However, for accurate determination of the final slope, one should only consider data points at dimensionless times exceeding:

\[
t_D > \frac{5(1 - \omega)}{\lambda}
\]  
(49)

PROCEDURES FOR ANALYSIS

In the above analysis we offer some insight into the pressure transients in naturally fractured reservoirs by using the approximate analytical solution (Eq. (42)). However, well test data rarely exhibit all of the theoretical characteristics displayed above. In most cases early data are masked by wellbore storage effects and in some cases the duration of the well test is too short for late-time behavior to be observed. Also, boundary effects may affect the well test data to the extent that the late time behavior predicted by the infinite reservoir model is never observed. The effects of wellbore storage and skin as well as the effects of different outer boundary conditions are discussed in a later section. Analysis procedures are given below for cases when the data are incomplete as well as for the case of a complete data set.

Complete data set

In this case the transmissivity \( kH \) and total storativity \( \{\gamma\} \) of the reservoir can be determined from the early-time or late-time slopes using conventional methods. \( \omega \) can be determined from the pressure transients using the above analysis. The deviations from the early-time and late-time slopes are used as a basis in the following discussion.

For a given set of early-time or late-time slopes, the time of deviation from each slope is calculated using Eq. (42). The intersection of the early-time or late-time semilogs and the half-slope straight line occurs at a dimensionless time of:

\[
t_D^{*}_{E} = \frac{\frac{5\omega}{e^{\lambda}}}{}
\]  
(50)

Similarly, the intersection of the late-time semilog and the half-slope straight line occurs at a dimensionless time of:

\[
t_D^{*}_{L} = \frac{5}{3e^{\lambda(1 - \omega)}}
\]  
(51)

At a slightly later time than the time of deviation from the half-slope, the 2/3-slope occurs. Due to the complex nature of the analytical approximation (Eq. (42)), it is not possible to mathematically derive the exact time of deviation of this linear segment. It is also of questionable value because of its short duration. However, as is the case with the half-slope, the 2/3-slope increases in duration with decreasing value of \( \omega \). The pressure transients converge on the final slope at a dimensionless time of:

\[
t_D = \frac{3(1 - \omega)}{\lambda}
\]  
(52)

However, for accurate determination of the final slope, one should only consider data points at dimensionless times exceeding:

\[
t_D > \frac{5(1 - \omega)}{\lambda}
\]  
(53)

The pressure transients converge on the final slope at a dimensionless time of:

\[
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In this case the transmissivity \( kH \) and total storativity \( \{\gamma\} \) of the reservoir can be determined from the early-time or late-time slopes using conventional methods. \( \omega \) can be determined from the pressure transients using the above analysis. The deviations from the early-time and late-time slopes are used as a basis in the following discussion.

For a given set of early-time or late-time slopes, the time of deviation from each slope is calculated using Eq. (42). The intersection of the early-time or late-time semilogs and the half-slope straight line occurs at a dimensionless time of:

\[
t_D^{*}_{E} = \frac{\frac{5\omega}{e^{\lambda}}}{}
\]  
(56)

Similarly, the intersection of the late-time semilog and the half-slope straight line occurs at a dimensionless time of:

\[
t_D^{*}_{L} = \frac{5}{3e^{\lambda(1 - \omega)}}
\]  
(57)

At a slightly later time than the time of deviation from the half-slope, the 2/3-slope occurs. Due to the complex nature of the analytical approximation (Eq. (42)), it is not possible to mathematically derive the exact time of deviation of this linear segment. It is also of questionable value because of its short duration. However, as is the case with the half-slope, the 2/3-slope increases in duration with decreasing value of \( \omega \). The pressure transients converge on the final slope at a dimensionless time of:

\[
t_D = \frac{3(1 - \omega)}{\lambda}
\]  
(58)

However, for accurate determination of the final slope, one should only consider data points at dimensionless times exceeding:

\[
t_D > \frac{5(1 - \omega)}{\lambda}
\]  
(59)
It is obvious that the wellbore storage effects become more critical when the value of $\lambda$ is higher. In many cases wellbore storage effects will mask all of the data during the transition period so that only the final semilog straight line can be observed. In this case neither $\lambda$ nor $\omega$ can be determined, but only the overall integrated reservoir parameters $k_2H$ and $S$. Our analysis shows that if all the parameter values of Eqs. (44), (46)–(48) are simultaneously determined then $\lambda$ and $\omega$ can only be determined if the following constraint holds:

$$C_0 \leq \frac{5\omega}{4e^{\frac{1}{\lambda}}/(60 + 3.55)}$$

The combined effects of wellbore storage and skin are shown in Figure 8 for $\lambda = 10^{-6}$ and $\omega = 0.01$. The skin factor $S$ represents permeability reduction in the near-wellbore region as a result of formation damage (positive skin) or permeability enhancement due to the presence of natural or man-made (hydraulic) fractures. The figure shows the characteristic unit-slope due to wellbore storage at early times, and a steady-state pressure drop associated with positive skin. The wellbore storage factor $C_0$ can be determined from type curves such as the ones shown in Figure 8. The skin factor $S$ is determined by conventional methods by assuming a value for the total storativity.

Mavor and Cinco-Ley extended the Warren and Root model to include the effects of wellbore storage and skin. Their results differ considerably from those presented here, mainly because of the quasi-steady flow assumption employed by Mavor and Cinco-Ley. For example, these authors develop criteria to determine at what values of the wellbore storage factor the initial straight line will appear. Our studies show, however, that if wellbore storage is present, the initial straight line will never appear for realistic values of $\lambda$ and $\omega$. The reason for this discrepancy is that the initial straight line lasts much longer in the Warren and Root model due to the assumption of quasi-steady interporosity flow.

**FINITE RADIUS RESERVOIR**

In this section we consider the cases of a closed and a reservoir with a constant pressure boundary. The mathematical solutions for these cases are given in an earlier section.

Figure 9 shows the pressure transient behavior in a closed reservoir $(r_{ap} = 100)$ for $\lambda = 10^{-6}$ and various values of $\omega$. We show also for comparison, the solutions for the same parameters based on the Warren and Root quasi-steady flow model. In the case considered here, the no-flow outer boundary effects are felt before the rock matrix significantly contributes to the flow. Consequently, the boundary effects are felt a factor of $(1/\omega)$ times earlier than they would be in the case of a homogeneous reservoir $(\omega = 1)$. Thus, if conventional methods for homogeneous reservoirs were used to analyze such data, the drainage radius may be underestimated by orders of magnitude. In general, the boundary effects will be felt before significant fluid flow from the matrix occurs if $(1/\omega^2) < (\pi/10r_{ap}^2)$. It is of interest to compare the solutions by the present model and the Warren and Root quasi-steady type model. In the case of the Warren and Root model, a plateau can be observed in the pressure transients (Fig. 9). The plateau appears only because of the quasi-steady state assumption. When transient fluid flow between the rock matrix and the fractures is...
considered, as in the present model, the pressure decline in the reservoir is more monotonic and a smooth transition to the final straight line is observed. As a result of the above discussions, methods that have been developed to determine the drainage radius of finite naturally-fractured reservoirs using the Warren and Root model may also significantly underestimate the drainage radius.

When the boundary effects are felt during the transition period similar results as discussed above can be observed, but the time shift will be less. Obviously, a pressure response identical to that of a homogeneous reservoir will result if boundary effects are felt during the final semilog straight line. This will be the case if $\lambda > 1/11r_{D0}$.

Figure 10 shows the effects of no-flow and constant pressure boundaries for $\lambda = 10^{-6}$ and various values of $w$ and $r_{D0}$. The figure shows there is a much shorter transient region for a constant pressure boundary than for a closed boundary. The boundary effect on the pressure behavior in the constant pressure boundary case is similar to that in the no-flow boundary case.

PRESSURE BUILD-UP ANALYSIS

The analysis of pressure buildup tests is very similar to that of drawdown tests described earlier. Using rules of superposition, the dimensionless shut-in pressure, $P_{DS}$, is given by:

$$P_{DS} = P_{DF} \left( \frac{t + \Delta t}{D} \right) - P_{DF} \left( \frac{\Delta t}{D} \right)$$

(52)

If we assume that $t_p$ is large enough that the pressure transients follow the final slope before shut-in, the build-up data will also exhibit a half-slope at a dimensionless shut-in time given by Eq. (44). At that time the shut-in pressure is given by:

$$P_{DS} = \frac{1}{4} \left( \ln \left( \frac{t + \Delta t}{D} \right) - \ln \left( \frac{\Delta t}{D} \right) \right)$$

(53)

Assuming that $\ln (t_p + \Delta t) = \ln (\frac{t_p}{D})$, Eq. (53) simplifies to:

$$P_{DS} = \frac{1}{4} \left( \ln \left( \frac{t + \Delta t}{D} \right) - \ln \left( \frac{\Delta t}{D} \right) \right)$$

(54)

The late-time behavior of the build-up test is given by the expression:

$$P_{DS} = \frac{1}{2} \ln \left( \frac{t + \Delta t}{D} \right) - \ln \left( \frac{\Delta t}{D} \right)$$

(55)

The dimensionless times for the intersections of the half-slope straight line with the initial and final slope straight lines, respectively, are identical to those presented in Eqs. (46) and (47).

FIELD EXAMPLE

Bourdet and Gringarten presented the build-up data from a naturally fractured reservoir shown in Table 1. We use this data to illustrate how the present model can be used to determine important reservoir parameters. The best match obtained between the observed data and the calculated values using the present model is shown in Figure 11. An excellent match was obtained. The analysis proceeds as follows: Using the final slope of 141.8 psi/cycle the transmissivity of the reservoir can be calculated as:

$$k_r = \frac{162.6 \times 4500 \times 1.2 \times 0.5}{141.8} = 3088 \, \text{md-ft}$$

For a reservoir thickness of 100 feet, the average reservoir permeability is $k_r = 30.88 \, \text{md}$.  

We can now proceed to calculate $w$. As is evident from the data shown in Figure 11, wellbore storage effects mask the initial fracture controlled linear segment. One must therefore use the methodology developed earlier to determine $\lambda$. In the data shown in Figure 11 a half-slope segment can be observed at Horner time of about $t_p + \Delta t/\Delta t = 35$, or $\Delta t = 0.617 \, \text{hrs}$. It equals 21 hrs.

The intersection of the half-slope line with the final straight line occurs at $t_p + \Delta t/\Delta t = 9$, or $\Delta t = 2.625 \, \text{hrs}$. Dividing Eq. (47) by Eq. (44):

$$\frac{\Delta t}{\Delta t_H} = \frac{1}{3w(1 - w)}$$

since $\Delta t_H/\Delta t_H = (\Delta t)/\Delta t_H$, one can determine $w = 0.078$. To determine $k_1/\Delta r^2$ one can use Eq. (47), after substituting for $\lambda$ and $w$ in Eqs. (27) and (28):

$$\frac{1.691 \times 10^{-2} k_1 (\Delta t)}{\mu (\phi c) r^2} = 1$$

Solving for $k_1/\Delta r^2$ yields $k_1/\Delta r^2 = 1.03 \times 10^{-5} \, \text{md/ft}^2$.

Unfortunately, there are no core data available on the matrix permeability, $k_1$. However, if one assumes a reasonable value for the matrix permeability, say, $k_1 = 0.001 \, \text{md}$, the average fracture spacing $\Delta r$ equals 10 ft.

One can now proceed to calculate $\lambda$ based on its definition:

$$\lambda = \frac{60 k_1 \Delta r^2}{k_2 \Delta r^2} = 1.82 \times 10^{-6}$$

The skin factor $S$ can be calculated from:

$$S = 1.151 \left[ \frac{P_{HR} - P_{WR} (\Delta t = 0)}{m} \right] - \log \left( \frac{k_2}{\phi_1 c_1 + \phi_2 c_2} \right) + 3.23 = -0.7$$
The above analysis is based on the approximate solution given by Eq. (42). However, if a more accurate analysis is needed, Eq. (30) can be employed. The match shown in Figure 11 was obtained using Eq. (30) and the following parameters were determined:

\[ \lambda = 2.63 \times 10^{-6}, \quad \omega = 0.085, \quad \text{and} \quad S = -0.7, \quad C_p = 1200 \]

and \( C = 0.012 \text{ bbl/psi} \).

Bourdet and Gringarten analyzed the same data (Fig. 11) using the Warren and Root model. They do not show the comparison between the calculated and observed pressures, but give values for \( \lambda \) and \( \omega \) of \( 2 \times 10^{-6} \) and 0.25, respectively. Using these parameters and the Warren and Root model, we compared the calculated and observed data and found a very unsatisfactory match. A much more reasonable match is obtained using \( \lambda = 3.5 \times 10^{-5} \) and \( \omega = 0.25 \). This value of \( \lambda \) is more than an order of magnitude higher than the value obtained using the present model. The value of \( \lambda \) is also considerably greater than our value, and is unrealistically high.

**SUMMARY AND CONCLUSIONS**

A new model for the analysis of pressure transient data from naturally fractured reservoirs has been developed. The model is considered to be that of Barenblatt et al.\(^1\) and Warren and Root\(^3\), consisting of the sets of orthogonal fractures separated by cubic matrix blocks. The model considers fully transient flow between the rock matrix and the fractures. Pressure transient solutions are presented for drawdown/build-up tests in naturally fractured reservoirs that are infinite, finite or with a constant pressure outer boundary. Wellbore storage and skin effects are also included. Some of the results obtained are:

1. Wellbore storage effects will mask the early-time semilog straight line for realistic values of \( \lambda \) and \( \omega \).

2. The pressure transient data during the transition period for values of \( \omega \) smaller than 0.1 exhibit a "half-slope" similar to that observed for the layered reservoir case. The half-slope is followed by a brief segment with a slope of 2/3.

3. All reservoir parameters can be determined if the half-slope segment is observed, even if the early-time straight line is not present. The appropriate procedure for analysis is given.

4. In the case of a finite reservoir, the drainage radius cannot be properly determined using a Warren and Root type model.

5. The model presented here is similar to other transient models, e.g., layered reservoir models. Geologic information must be used to determine which model is appropriate.

6. The field example given illustrates the applicability of the present model for naturally fractured reservoirs.

**NOMENCLATURE**

- \( A \) = interporosity flow parameter defined by Warren and Root, \( 60 k_1 r_p/2D^2 \)
- \( \alpha \) = porosity
- \( \omega \) = ratio of storativity of the fracture to total storativity, \( \frac{2c_2}{(4c_l + 2c_2)} \)
- \( \gamma \) = Euler constant, 0.5772
- \( \phi \) = dimensionless distance in the rock matrix block, \( 2z/D \)
- \( t_o \) = dimensionless time, \( k_1t/(41c_1)(D/2)^2 \)
- \( r_o \) = external radius, ft
- \( p_{wf} \) = flowing bottom-hole pressure, psi
- \( p_b \) = boundary pressure on the surface of rock matrix block
- \( P_D \) = dimensionless pressure difference, \( PD = \frac{(27k_2H)/qUb}{P_l - p(r,t)} \)
- \( Q \) = flow rate, STB/D
- \( e \) = exponential function
- \( f \) = storativity, \( p_2c_2/0_1c_1 + 0_2c_2) \)
- \( I_0 \) = modified Bessel function of the first kind, zero order
- \( I_1 \) = modified Bessel function of the first kind, first order
- \( K_0 \) = modified Bessel function of the second kind, zero order
- \( K_1 \) = modified Bessel function of the second kind, first order
- \( K \) = permeability, md
- \( C_D \) = dimensionless wellbore storage, \( C/(2\pi(\rho c_1 + 2c_2)Hr_p^2) \)
- \( D \) = fracture spacing, ft.
- \( c \) = compressibility, psi\(^{-1}\)
- \( B \) = formation volume factor, RB/STB
- \( C \) = wellbore storage coefficient, RB/psi
- \( \beta \) = dimensionless wellbore storage
- \( q \) = flow rate, STB/D
- \( p \) = Laplace parameter
- \( r_w \) = wellbore radius, ft.
- \( r_e \) = external radius, ft.
- \( r_e/D \) = dimensionless external boundary radius, \( r_e/r_w \)
- \( r/D \) = dimensionless radius, \( r/r_w \)
- \( r_1 \) = the radius of the outer boundary.
- \( u \) = exponential function
- \( w \) = viscosity, cp
- \( P_D \) = dimensionless vertical semilog pressure difference
- \( P_D \) = dimensionless flow rate, \( Q/\rho c_1 \)
- \( r_f \) = dimensionless flow rate, \( Q/r_w \)
- \( t \) = time, hours
- \( t_D \) = dimensionless time, \( k_1t/(4k_1c_1 + 2c_2)^{1/2} \)
- \( t_T \) = dimensionless time, \( k_1t/(4k_1c_1 + 2c_2)^{1/2} \)
- \( X \) = Interporosity flow parameter, \( 4k_1r_p^2/2D^2 \)
- \( \theta \) = flow rate, STB/D
- \( s \) = van Everdingen-Hurst skin factor
- \( \eta \) = dimensionless pressure, \( PD = \frac{(27k_2H)/qUb}{P_l - p(r,t)} \)
- \( \lambda \) = interporosity flow parameter defined by Warren and Root, \( 60k_1r_p/2D^2 \)
- \( X \) = Interporosity flow parameter, \( 4k_1r_p^2/2D^2 \)
- \( \phi \) = storativity, \( p_2c_2/0_1c_1 + 0_2c_2) \)
- \( \psi \) = semi-infinite wellbore storage coefficient
- \( \psi \) = semi-infinite wellbore storage coefficient

**REFERENCES**


In the radial flow system, one can define a control volume $V_n$ as:

$$V_n = \pi \left[ (r + dr)^2 - r^2 \right] H = 2\pi r dr H \quad (A-1)$$

The interface area $A_c$ between the rock matrix and the fractures in the control volume can be expressed:

$$A_c = 6D^2 \left( \frac{V_n}{D^3} \right) = \frac{12\pi r dr H}{D} \quad (A-2)$$

We can now write a mass balance equation for the control volume in the fractures:

$$q_A - \left( q_A + \frac{3}{\partial t} (q_A) dr \right) + \left( \frac{3}{\partial t} q_A \right)_{z=D/2} = \frac{\partial^2 q}{\partial z^2} \quad (A-3)$$

If the fracture and fluid properties are constant, the pressure gradients in the system are small and the fluid is slightly compressible then after introducing Darcy's law, Eq. (A-3) becomes:

$$q_A(z) = \frac{6k_1}{k_2} \left( \frac{1}{\partial r} \frac{\partial p}{\partial r} - \frac{6k_1}{k_2} \frac{1}{D} \frac{\partial p}{\partial z} \right) \frac{1}{k_2} \frac{3p_2}{\partial t} \quad (A-4)$$

The governing equation describing the mass conservation in the rock matrix can be expressed as:

$$q_{A_z} - \left( q_{A_z} + \frac{3}{\partial z} (q_{A_z}) \right)_{z=D/2} = \frac{3(A dz \phi \rho)}{\partial t} \quad (A-5)$$

where

$$A_z = k z^2 \quad (A-6)$$

and $k$ is a constant.

Similar considerations yield:

$$q_{A_z} - \left( q_{A_z} + \frac{3}{\partial z} (q_{A_z}) \right)_{z=D/2} = \frac{3(A dz \phi \rho)}{\partial t} \quad (A-7)$$
Equations (A-4) and (A-7) describe the pressure transient behavior in the fractures and the rock matrix, respectively.

**APPENDIX B:** Simultaneous solution of fracture and rock matrix equations.

Applying Laplace transformation to Eqs. (13) through (22) yields:

\[ \frac{\partial^2 P_{D2}}{\partial r^2_D} + \frac{1}{r_D} \frac{\partial P_{D2}}{\partial r_D} + 3 \lambda \frac{\partial P_{D1}}{\partial n} \bigg|_{n=1} - \omega pD2 = 0 \]  
\[ (B-1) \]

\[ \frac{\partial^2 P_{D1}}{\partial n^2} + \frac{2}{n} \frac{\partial P_{D1}}{\partial n} - \frac{(1 - \omega)}{\lambda} pD1 = 0 \]  
\[ (B-2) \]

\[ \left. \frac{\partial P_{D2}}{\partial r_D} \right|_{r_D=1} = \frac{1}{p} \]  
\[ (B-3) \]

\[ \bar{P}_{Df} = \bar{P}_{D2} - \frac{1}{r_D} \frac{\partial P_{D2}}{\partial r_D} \bigg|_{r_D=1} \]  
\[ (B-4) \]

\[ \left. \frac{\partial P_{D1}(r_D, n)}{\partial n} \right|_{n=0} = 0 \]  
\[ (B-5) \]

\[ \left. \frac{\partial P_{D1}(r_D, n)}{\partial n} \right|_{n=1} = \bar{P}_{D2}(r_D) \]  
\[ (B-6) \]

After applying boundary conditions given by Eqs. (8-3), (8-4), and (3-7), the solution for the pressure at the wellbore and in the fractures can be expressed as Eqs. (30) and (31) for an infinite reservoir. Similar procedures can be used for a finite reservoir.

**APPENDIX C.** Approximate solution

Without skin and wellbore storage effects, the pressure at the wellbore can be expressed as:

\[ P_{Df} = \frac{K_0(\sqrt{X_2})}{\sqrt{X_2}K_1(\sqrt{X_2})} \]  
\[ (C-1) \]

Approximate inversion of equation (C-1) is possible using the improved Schapery method. As a first step, one approximates:

\[ P_{Df} \approx |pP_{Df}| \]  

Hence,

\[ P_{Df} \approx K_0(\sqrt{X_2}) \]  
\[ (C-2) \]

If \( X_2 \) is small, equation (C-2) becomes

\[ P_{Df} \approx K_0(\sqrt{X_2}) \]  
\[ (C-3) \]

In general, for \( t_D > 10 \), within 2% accuracy in comparison to the results of the Laplace numerical inversion, \( P_{Df} \) can be expressed as equation (C-4):

\[ P_{Df} = -\gamma + \ln \left( 2 - \frac{1}{2} \ln \left( \frac{\lambda}{5} X_3 \coth(X_3) - \frac{\lambda}{5} + \frac{\omega}{\epsilon t_D} \right) \right) \]  
\[ (C-4) \]

where

\[ X_3 = \sqrt{\frac{15(1 - \omega)}{\lambda}} e^{\lambda t_D} \]  
\[ (C-5) \]

The half-slope can be observed around the dimensionless time, \( t_D = 5w/\lambda e^\gamma \). In this region Eq. (C-4) can be further simplified to yield:

\[ P_{Df} = \frac{1}{4} \left\{ \ln t_D - \ln \left( 1 - \omega \right) - \ln \left( \frac{3}{80} - 3\gamma \right) \right\} \]  
\[ (C-5) \]
Table 1. Data for pressure build-up test.

\[ H = 100 \text{ ft}, \quad B = 1.2 \text{ RB/STB}, \quad u = 0.5 \text{ cp,} \]
\[ (\phi_1 + \phi_2) = 10^{-6} \text{ psi}^{-1}, \quad q = 4500 \text{ bbl/D}, \]
\[ r_w = 0.3 \text{ ft}, \quad P(\Delta t = 0) = 3420.8 \text{ psi,} \quad t_p = 21 \text{ hrs} \]

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Fig. 1—Idealized model of naturally fractured reservoirs.

Fig. 2—One-dimensional approximation of fluid flow in rock matrix block.
Fig. 3—Comparison between three-dimensional and one-dimensional models.

Fig. 4—Comparison between Warren and Root and the present solutions for an infinite reservoir.

Fig. 5—Comparison between numerical simulation and semi-analytic solutions.

Fig. 6—Pressure drawdown behavior without wellbore and skin effects.
Fig. 7—Effects of wellbore storage on pressure drawdown behavior.

Fig. 8—Effects of wellbore storage and skin on pressure drawdown behavior.
Fig. 9—Comparison between Warren and Root and the present solution for a closed reservoir.

Fig. 10—Effects of outer boundary conditions on pressure behavior.

Fig. 11—Pressure buildup analysis.