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Authors
Ahmadzadeh, Akbar
Kaufmann, William B.

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EXCHANGE DEGENERATE REGGE TRAJECTORIES AND
MESON NUCLEON CHARGE EXCHANGE SCATTERING

Akbar Ahmadzadeh and William B. Kaufmann

Lawrence Radiation Laboratory
University of California
Berkeley, California

and

Department of Physics
Arizona State University
Tempe, Arizona 85281

August 12, 1969

ABSTRACT

Based on exchange degeneracy and SU(3) symmetry, a Regge pole model fitting the differential cross sections for pion-nucleon and kaon-nucleon charge exchange and \( \eta \) production as well as polarization in \( \pi^- p \rightarrow \pi^0 n \) is given. Besides the \( \rho - A_2 \) trajectory, a conspiring and exchange degenerate \( \rho' - A_2 \) trajectory with the same slope as \( \rho - A_2 \) but with intercept near zero is taken into account. The residues are assumed to have \( 1/\Gamma(\delta) \) behavior. A four parameter fit gives good agreement with the experimental data.

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†Permanent address.
INTRODUCTION

A possible classification of Regge trajectories according to SU(3) symmetry, exchange degeneracy and the Lorentz pole quantum number has been given previously.\(^1,2\) Our aim here is to combine these considerations with the Veneziano-type representation\(^3\) in a phenomenological study of pion-nucleon and kaon-nucleon charge exchange reactions and \(\eta\) production. The model presented here has numerous other phenomenological implications which are subject to future investigations.

The basic ideas of Refs. 1 and 2 utilized here are the following: The vector and tensor octets of trajectories form an octet of exchange degenerate trajectories with SU(3) splittings of trajectories determined by the masses of resonances. These trajectories are coupled to mesons and baryons SU(3) symmetrically. Similarly, the octet of pseudoscalar trajectories (to which the pion belongs) together with the octet of axial vector trajectories (to which the B meson belongs) form an exchange degenerate octet of conspiring trajectories. The co-conspirator of this octet is an exchange degenerate octet of vector-tensor trajectories. For example, the exchange degenerate \(\pi\)-B trajectory conspires with an exchange degenerate \(\rho'-A_2\) trajectory of the same intercept.

In addition, we shall assume that the trajectories have a universal slope. In the spirit of the Veneziano formula this simplifying assumption makes extra baryon trajectories in the \(s\) and \(u\) channels unnecessary. Furthermore, the possible resonances
belonging to the co-conspirators would be hidden underneath the other (known) resonances. This would explain why they have not been observed experimentally.

For the reactions we consider in this article only the \( \rho \) and \( \Lambda_2 \) quantum numbers can be exchanged in the \( t \) channel. Our model consists of exchange degenerate \( \rho-\Lambda_2 \) plus exchange degenerate and conspiring \( \rho'-\Lambda'_2 \) trajectories. In the next section the formalism and the results of fitting the parameters are given.
2. FORMULATION AND RESULTS

A phenomenological study of pion nucleon charge exchange differential cross section and polarization based on $\rho$ and $\rho'$ trajectories has been given in a separate article. Here we include the $\eta$ production and kaon-nucleon charge exchange as well. Because we assume exchange degeneracy and $SU(3)$ symmetry of the couplings, no new parameter is needed.

Let us define the helicity nonflip and helicity flip parts of the $\rho$ trajectory contribution in meson nucleon scattering to be of the form

$$A^\rho = \frac{\beta^\rho}{\Gamma(\alpha_1)} \xi^\rho (as)^{\alpha_1}$$

and

$$B^\rho = -\frac{\beta^f}{\Gamma(\alpha_1)} \xi^\rho (as)^{\alpha_1-1}$$

respectively; where $\xi^\rho = \frac{1-e^{-i\pi\alpha_1}}{\sin\pi\alpha_1}$ and $\beta^\rho(\beta^f)$ is the helicity nonflip (flip) residue constant. Eqs. (1) and (2) can be considered as the leading term (in energy) of a suitable Veneziano-type representation. In fact, Igi$^5$ has given such a representation for pion nucleon scattering and his formula reduce to Eqs. (1) and (2) in the high energy limit. Furthermore, even if one has to take an infinite number of Veneziano terms (corresponding, perhaps, to a single Lorentz pole), still the leading term in energy will be in the form of Eqs. (1) and (2). The exchange degenerate $\rho-A_2$ trajectory passing through $\rho(760)$ and $A_2(1310)$ is given by
\[ \alpha_1 = 0.5 + at; \alpha = 0.9 \ (GeV)^{-2} \]  

Similarly, the amplitudes for the \( A_2 \) trajectory contribution (denoted by the subscript \( R \)) are given by

\[ A_R^1 = \frac{\beta_R^n}{\Gamma(\alpha_1)} \xi_R \ (as)^{\alpha_1} \]  
\[ B_R^1 = \frac{\beta_R^f}{\Gamma(\alpha_1)} \xi_R \ (as)^{\alpha_1-1} \]  

where \( \xi_R = (1 + e^{-i\pi\alpha_1})/\sin\pi\alpha_1 \)

The exchange degenerate \( \rho-A_2 \) trajectory is assumed to have the same slope as the \( \rho'-A'_2 \) trajectory. This simplifying assumption is made in order to avoid introducing new baryon trajectories in the \( s \) and \( u \) channels. Namely, one can imagine a Veneziano-type formula in which the same baryon trajectories coexist with \( \rho-A_2 \) as with \( \rho'-A'_2 \). Furthermore, as pointed out earlier, this model would predict existence of a particle with the same mass as the \( B(1220) \) meson and with the \( \rho \) quantum numbers. This is because in our scheme \( \rho'-A'_2 \) trajectory coincides with the \( \pi-B \) trajectory. This coincidence would be a realization of chiral symmetry. The same coincidence should occur for higher recurrences. On the other hand, we assume that the \( A_2^1 \) trajectory chooses nonsense at \( \alpha = 0 \).
Therefore we do not expect a scalar particle at the pion mass. This scheme is consistent with the quark model; namely, if as in Ref. 2 we assume that all trajectories are coupled to a quark-antiquark system, then the point \( \alpha = 0 \) is a nonsense point for the \( A_2' \) trajectory. Note that the \( A_2' \) trajectory couples to the spin triplet of \( q\bar{q} \) system. The \( \rho' - A_2' \) trajectory, based on the masses of the pion and the \( B \) meson is given by

\[
\alpha_2 = 0.02 \pm 0.9 t
\]  

The \( \rho' \) contribution is given by

\[
A_{\rho'} = \frac{t \beta_{\rho'}^n}{\Gamma(\alpha_2)} \xi_{\rho'}(as)^{\alpha_2}
\]  

and

\[
B_{\rho'} = \frac{t \beta_{\rho'}^f}{\alpha_2 \Gamma(\alpha_2)} \xi_{\rho'}(as)^{\alpha_2-1}
\]

Similarly, for the \( A_2' \) trajectory we have

\[
A_{R'} = \frac{t \beta_{R'}^n}{\Gamma(\alpha_2)} \xi_{R'}(as)^{\alpha_2}
\]

\[
B_{R'} = \frac{t \beta_{R'}^f}{\alpha_2 \Gamma(\alpha_2)} \xi_{R'}(as)^{\alpha_2-1}
\]

The factor of \( t \) in Eqs. (7) and (9) is included because \( \rho' \) and \( A_2' \) are assumed to be conspiring trajectories (see Ref. 7, for example).
Note the factor $\alpha$ in the denominators of the right hand sides of Eqs. (8) and (10). Due to this factor there is a pole in the flip amplitude of the $A'_2$ contribution at $\alpha_2 = 0$. This is a nonsense pole and thus we are assuming the existence of a compensating trajectory. In Ref. 4, this factor of $\alpha$ was not included in the flip amplitude of the $\rho'$. It turns out that this extra factor improves our fit for polarization in $\pi^- p \to \pi^0 n$.

Now using the exchange degeneracy of the residues we have

$$\beta_1^n = \beta_1^f = \beta_R^n = \beta_R^f$$

(11)

and similarly

$$\beta_2^n = \beta_2^f = \beta_{R'}^n = \beta_{R'}^f$$

(12)

Note that, since the trajectories are fully determined by Eqs. (3) and (6), we are left with only four parameters $\beta_1^n, \beta_1^f, \beta_2^n$ and $\beta_2^f$.

Defining the new amplitudes $\mathcal{A}$ and $\mathcal{B}$ as

$$\mathcal{A}_\rho = A_\rho' + A_\rho' ; \quad \mathcal{A}_R = A_{R'} + A_{R'}$$

(13)

and

$$\mathcal{B}_\rho = B_\rho' + B_\rho' ; \quad \mathcal{B}_R = B_R + B_{R'}$$

from SU(3) symmetry of the couplings we have

$$A'(\pi^- p \to \pi^0 n) = \sqrt{2} \mathcal{A}_\rho$$

(14)

$$A'(K^- p \to K^0 n) = \mathcal{A}_R - \mathcal{A}_\rho$$

$$A'(\pi^- p \to \eta n) = -(2/3)^{1/2} \mathcal{A}_R$$

$$A'(K^+ n \to K^0 p) = -\mathcal{A}_\rho - \mathcal{A}_R$$

with similar expressions for the $B$ amplitudes. In what follows we
shall consider the first three reactions in Eqs. (14). We are not including the $K^+n$ charge exchange case because of the meager high energy experimental data and because of the complications due to the Glauber corrections for deuteron target. Note however that with the four parameter determined below our model makes unambiguous prediction of the $K^+n$ charge exchange reaction. The differentiated cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{64 \pi q^2 s} \left\{ (4m^2-t) |A'|^2 + \frac{t}{4m^2-t} \left[ 4\mu^2m^2-ts-(s-m^2-\mu^2)^2 \right] |B|^2 \right\}$$

where $q$ is the center of mass momentum, $m$ is the nucleon mass, $\mu$ is the meson mass and $s$ and $t$ are the usual invariant energy and momentum transfer. Note that in $\eta$ production we should use unequal mass kinematics. However, at the energies we are considering this extra complication is not necessary. The polarization is given by

$$P = \frac{-\sin \theta \operatorname{Im}(A' B^*)}{16 \pi \sqrt{s} \frac{d\sigma}{dt}}$$

where $\theta$ is the scattering angle in the center of mass system.

The experimental data are fitted using the four free parameters. The minimum Chi square is obtained for

$$\beta_1^n = 9.81 \pm 1, \beta_1^f = 119. \pm 9, \beta_2^n = -36 \pm 15, \beta_2^f = 38 \pm 15$$
For a total of 223 data points we have obtained $\chi^2 = 369$ which we consider to be rather reasonable especially in view of ignoring the other (non-leading) terms. One could of course break SU(3) and/or exchange degeneracy and obtain a smaller $\chi^2$. But as long as we do not know how to take all other possible terms into account, there is very little information to be gained.

The errors given in Eqs. (17) indicate the change in each parameter necessary to increase $\chi^2$ by 10 percent. The large uncertainty in $\beta_2^n$ and $\beta_2^f$ reflects the lack of extensive polarization data, to which these parameters are sensitive. We are looking forward to more polarization data in these reactions especially at higher momentum transfers to determine these parameters with better confidence. One feature of our model worth mentioning is that it predicts zero polarization in $\pi^-p$ charge-exchange at $t \approx -0.6\text{(GeV/c)}^2$ where $\alpha_\rho = 0$, and also predicts a zero in the $\pi^-p\to\eta n$ polarization at $t \approx -1.6\text{(GeV/c)}^2$ where $\alpha_R = -1$.

Note also that our formalism can be used to calculate the differential cross section and polarization in $\pi^+p\to K^+\Sigma^+$ reaction. In this case only the d/f ratios remain as free parameters; for the $K^* - K^{**}$ and $K^{*'} - K^{**'}$ trajectories are determined from the masses in the same way as our $\rho - A_2$ and $\rho' - A_2'$ trajectories. The residues in this case are of course related to those of Eqs. (17).

Figures 1. through 8. show the experimental data as well as our theoretical curves. The error bars shown on these curves are statistical only. In calculating the $\chi^2$, systematic errors in the
data are also taken into account when given in the experimental articles. In the $\eta$ production data we have assumed a branching ratio of $0.381$.

Based on our model and the parameters obtained here predictions are made for future data up to 100 GeV/c. Figures 9. through 11. show the results of our prediction.
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REFERENCES


FIGURE CAPTIONS

Fig. 1. $\frac{d\sigma}{dt}$ for $\pi^- p \to \pi^0 n$. Upper curve: $100 \times \frac{d\sigma}{dt}$ at 4.83 GeV/c laboratory momentum; middle curve: $10 \times \frac{d\sigma}{dt}$ at 5.85 GeV/c; lower curve $\frac{d\sigma}{dt}$ at 6.0 GeV/c.

Fig. 2. $\frac{d\sigma}{dt}$ for $\pi^- p \to \pi^0 n$: Upper curve: $100 \times \frac{d\sigma}{dt}$ at 10 GeV/c; middle curve $10 \times \frac{d\sigma}{dt}$ at 13.3 GeV/c; lower curve $\frac{d\sigma}{dt}$ at 18.2 GeV/c.

Fig. 3. Polarization in $\pi^- p \to \pi^0 n$ at 5.9 GeV/c.

Fig. 4. Polarization in $\pi^- p \to \pi^0 n$ at 11.2 GeV/c.

Fig. 5. $\frac{d\sigma}{dt}$ for $K^- p \to K^0 n$: Upper curve $10 \times \frac{d\sigma}{dt}$ at 5 GeV/c; lower curve $\frac{d\sigma}{dt}$ at 7.1 GeV/c.

Fig. 6. $\frac{d\sigma}{dt}$ for $K^- p \to K^0 n$: Upper curve $10 \times \frac{d\sigma}{dt}$ at 9.5 GeV/c; lower curve $\frac{d\sigma}{dt}$ at 12.3 GeV/c.

Fig. 7. $\frac{d\sigma}{dt}$ for $\pi^- p \to \eta n$. Upper curve $100 \times \frac{d\sigma}{dt}$ at 5.9 GeV/c; middle curve $10 \times \frac{d\sigma}{dt}$ at 10. GeV/c; lower curve $\frac{d\sigma}{dt}$ at 9.8 GeV/c.

Fig. 8. $\frac{d\sigma}{dt}$ for $\pi^- p \to \eta n$. Upper curve $10 \times \frac{d\sigma}{dt}$ at 13.3 GeV/c; lower curve $\frac{d\sigma}{dt}$ at 18.2 GeV/c.

Fig. 9. Predicted $\frac{d\sigma}{dt}$ for $\pi^- p \to \pi^0 n$. Laboratory momentum from top to bottom is 25, 50, 75, 100 GeV/c respectively.

Fig. 10. Predicted $\frac{d\sigma}{dt}$ for $K^- p \to K^0 n$. At same momenta as Fig. 8.

Fig. 11. Predicted $\frac{d\sigma}{dt}$ for $\pi^- p \to \eta n$. At same momenta as Fig. 8.
Fig. 1

\[ \frac{d\sigma}{dt} \left[ \frac{\mu b}{(GeV/c)^2} \right] \]

\(-t (GeV/c)^2\)
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 10
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