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Many-Body Correlation Dynamics Within a Green’s Function Formalism*

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Abstract

We have generalized the method used in the correlation dynamics of density matrix and established a set of equations of motion for many-body correlation green’s functions in the non-relativistic case. These non-linear and coupled equations of motion describe the dynamical evolution of correlation green’s functions of different order, and reflect how interactions produce many-body correlations and how different kinds of correlations interweave. The advantage of the present formalism resides in that it is expanded not in terms of the interaction strength but in terms of the many-body correlations, and therefore it is a non-perturbative approach providing a natural truncation scheme with respect to the order of correlations. It is shown that the non-perturbative results of the conventional green’s function theory are included in the present formalism as two limiting cases (the so-called ladder diagram limit and ring diagram limit). The correlation dynamics of green’s function is a multi-time generalization of the correlation dynamics of density matrix and contains the latter as an equal time limit.

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1 Introduction

Green's function approach is a basic method in quantum many-body problems and quantum field theory. In fact, quantum electrodynamics and quantum field theory are formulated in terms of green's functions[1]. The usage of green's function method is also wide-spread in condensed matter physics and nuclear physics[2]. Since the temperature green's function was invented, quantum equilibrium statistical physics has been formulated largely based on the green's function technique[3]. Ever since the pioneering work of Schwinger et al.[4, 5] was published, the closed time path green's function and the non-equilibrium green's function theory have been developed rapidly[6, 7, 8, 9]. Nowadays it has become a powerful tool for describing non-equilibrium quantum phenomena[10, 11, 12, 13, 14, 15].

Nevertheless, the practically tractable green's function theory is, in fact, a perturbation theory expanded in a series of interaction strength [16]. Equipped with renormalization method, generating functional, and Feynman diagram technique, the calculation of perturbative green's functions becomes simple and transparent. However, for systems with strong interactions or repulsive hard cores, the naive perturbation theory does not work, and non-perturbative methods are needed. The most obvious solution is to modify the conventional perturbation theory and combine the series expansion with a certain summation rule. The current non-perturbative method based on green's function is doing just that. In order to derive a set of non-perturbative differo-integral equations based on some infinite summation rules, one has to resort to the intuition of Feynman diagrams[2]. Apparently, this kind of non-perturbative approaches of green's function depends strongly on Feynman diagrams and their selection rules, and hence the completeness, the unification, and the systematicity are lacking for such an approach. Therefore, it is desirable to develop a unified, systematic, and non-perturbative green's function theory whose expansion is not in a series of interaction strength, but according to the order of many-body correlations. Moreover, it should provide a reasonable truncation scheme and each truncation should lead to a non-perturbative approximation.

It is the purpose of this article to develop such a non-perturbative dynamics theory within the framework of green's functions, to describe the interacting many-body systems, with the accuracy increased as more correlations included in the truncation scheme. Each truncation will lead to a correlation dynamics of certain order, which is non-perturbative and nonlinear. This kind of truncation scheme is natural, since it reflects the intrinsic property of the nature which manifests order by order its correlation structures and dynamics behaviors.

The density matrix correlation dynamics and its application in heavy ion physics[17, 18, 19, 20, 21, 22, 23, 24] have motivated the research above and also strengthened our desire for developing correlation dynamics of green's function. The density matrix is, in fact, a special equal time limit of the green's function. The density matrix correlation dynamics has been derived from the BBGKY hierarchy[25], and a set of coupled nonlinear equations of motion for correlation density matrices has been established. The advantage of the correlation dynamics is that it provides a natural truncation scheme with respect to the order of correlations, and that each truncation leads to a non-perturbative approximation. It has been shown[17] that the existing nuclear many-body theories, such as HF, TDHF, ETDHF, HF-Brueckner-Faddeev, can be obtained from the quantum correlation dynamics within its lowest order of truncation. Moreover, the wide-spread used BUU equation can be derived within the two-body correlation dynamics in the semi-classical limit and the phase space representation[18, 19, 20]. Recently, Tohyama et al.[26] have pointed out that the two-body correlation dynamics in the small amplitude limit provides a more general framework than the second RPA. Realistic calculations based on the two-body correlation dynamics (the so called TDDM) have been reported recently for damping of nuclear giant resonances[27] and for low energy heavy ion collisions[28]: TDDM reproduces the damping widths of isoscalar quadrupole resonances and gives much larger mass fluctuations in deep inelastic heavy ion collisions than TDHF does.

The success of these results assures us that the correlation dynamics of green's function will reach a similar achievement. To establish a full relativistic theory of correlation dynamics, two steps should be taken. The first step is to formulate a non-relativistic theory of correlation dynamics within the framework of green's function. This is the goal of this paper. The second step is to establish a correlation dynamics for relativistic quantum fields, which is a task left for future.

The article is organized as follows. In sect.2, the many-body correlation dynamics of green's function for a non-relativistic quantum system is formulated in detail. In sect.3, the normal equal time limit of the obtained dynamical equations is taken and the density matrix correlation dynamics is recovered under this limit. Sect.4 devotes to truncation approximations. In one body truncation, Dyson equation and TDHF follow as expected. A better
truncation leads to the two-body correlation dynamics which incorporates ladder diagrams (short range correlations) and ring diagrams (long range correlations) into a unified formalism. Yet, the well-known Bethe-Salpeter equation and the equation for long range polarization effects are derived as two extreme limits respectively. It is thus shown that all the non-perturbative results of the conventional green's function theory are included in the present formalism, which provides a unified and systematic approach for a non-perturbative treatment of quantum many body systems. Finally in sect.5, discussion and outlook cover the topics, such as the relationship between the present approach and other related ones, the possibility for establishing a non-perturbative temperature green's function theory, the relativistic generalization of the present formalism, and so forth. Some technical details are given in appendices A and B concerning the derivation of equations of motion for correlation green's functions and the equal time limit.

2 Correlation Dynamics of Many-Body Green's Function

2.1 Hamiltonian of a Many-Body System and Equations of Motion for Field Operators

For simplicity, we consider an interacting fermion system. \( \psi(x,t) \) and \( \psi^\dagger(x,t) \) denote fermion field operator and its hermitian conjugate. The fermion field may contain several components representing different kinds of fermion particles, such as lepton, nucleon or baryon et al. For systems physically interesting, the Hamiltonian can be written as

\[
\hat{H} = \int \psi^\dagger(x,t)\hat{\tau}(x)\psi(x,t) d^3x + \int \psi^\dagger(x,t)\psi^\dagger(x',t)\hat{V}(x,x')\psi(x',t)\psi(x,t) d^3x d^3x',
\]

where \( \psi(x,t) \) and \( \psi^\dagger(x,t) \) satisfy anti-commutation relations,

\[
[\psi(x,t), \psi^\dagger(x',t)]_+ = \delta^{(3)}(x - x'),
\]

\[
[\psi(x,t), \psi(x',t)]_+ = [\psi^\dagger(x,t), \psi^\dagger(x',t)]_+ = 0.
\]

The time evolution of the field operators obeys Heisenberg equations,

\[
i\partial_t \psi(x,t) = [\psi(x,t), \hat{H}] = \hat{\tau}(x)\psi(x,t) + \int d^4x' \psi^\dagger(x')\hat{V}(x,x')\psi(x),
\]

\[
i\partial_t \psi^\dagger(x,t) = [\psi^\dagger(x,t), \hat{H}] = -\hat{\tau}(x)\psi^\dagger(x,t) - \int d^4x' \psi^\dagger(x')\hat{V}(x',x')\psi^\dagger(x').
\]

In eqs.(2.3a,b), the notation of interactions in 4-dimensional space-time has been introduced,

\[
\hat{V}(x,x') = \tilde{V}(x',x) = \delta(t - t')\hat{\tau}(x - x').
\]

2.2 Many-Body Green's Functions and Their Equations of Motion

All information of a many-body system can be obtained from its green's functions which are conventionally defined, for 1-, 2-,... and n-body green's functions as

\[
iG(1;1') = \langle \hat{T}\psi(1)\psi^\dagger(1') \rangle
\]

\[
i^2G(2)(1;2,1';2') = \langle \hat{T}\psi(1)\psi(2)\psi^\dagger(2')\psi^\dagger(1') \rangle
\]

......
where $\psi(x_1)$ and $\psi^\dagger(x_1)$ are field operators in Heisenberg picture, $\hat{T}$ is a time ordering operator. The average of a field operator product is defined as a quantum mechanical and ensemble average, namely,

$$i^n G^{(n)}(1, \ldots n; 1', \ldots n') = \langle \hat{T} \psi(1) \ldots \psi(n) \psi(1') \ldots \psi(n') \rangle.$$  

(2.7)

The equations of motion for green’s functions can easily be derived from Heisenberg equations (2.3a,b). They read

$$[i \partial_t - \hat{t}(1)]G(1; 1') = \delta^4(1, 1') - i \int d^42\hat{V}(1, 2)G^{(2)}(1, 2; 1'2'),$$  

(2.9a)

$$[i \partial_t - \hat{t}(1)]G^{(2)}(1, 2; 1', 2') = \delta^4(1, 1')G(2; 2') - \delta^4(1, 2')G(2; 1') - i \int d^42\hat{V}(1, 3)G^{(3)}(1, 2, 3; 1', 2', 3'),$$  

(2.9b)

$$[i \partial_t - \hat{t}(1)]G^{(n)}(1, \ldots n; 1', \ldots n') = \sum_{j=1}^{n} \delta^4(1, j')(\delta^4(1, j') - \delta^4(1, j'))G^{(n-1)}(2, \ldots n; 1', (j - 1)', (j + 1)', \ldots n')$$

$$- i \int d^4(n + 1)\hat{V}(1, n + 1)G^{(n+1)}(1, \ldots (n + 1); 1', \ldots (n + 1')).$$  

(2.11)

where $t_{(n+1)} = t_{(n+1)}^{\pm} = t_{(n+1)}^{\pm} + \epsilon$, and $\epsilon$ is positive infinitesimal. Similar equations of motion can be obtained for any 4-d coordinates $(x_i)$ and $(x_i')$. Two obvious features can be observed from eqs. (2.9-11): (i) They are coupled, linear equations. (ii) The equation of motion for $G^{(n)}$ is coupled to $G^{(n-1)}$ and $G^{(n+1)}$. Since all the many-body green’s functions are on equal footing, one can not set any of them vanishing. This means that eqs. (2.9-11) themselves do not provide a truncation scheme and thus lose their value of practical use. Yet, another shortcoming of eqs. (2.9-11) is related to the unnecessary repetition of information. From the diagrammatic perturbation theory one knows that the n-body green’s function contains a large number of disconnected diagrams, while the useful information is contained in linked irreducible diagrams. Eqs. (2.9-11) describe the time evolution of a large number of disconnected diagrams, hence a large amount of information is repeated unnecessarily. Similar problems exist in the BBGKY hierarchy. To get rid of these shortcomings, as in the case of density matrix, one needs to separate correlation green’s functions $G^{(n)}_c$ from $G^{(n)}$, and transform the equations of motion for $G^{(n)}$ to those for $G^{(n)}_c$.

### 2.3 Correlation Green’s Functions and Their Equations of Motion

There are two major steps to derive the correlation dynamics: the separation of many-body correlation green’s functions and the transformation of equations of motion. The first step can be accomplished by a generalization of the method used in Ref.[17]. It is simply to consider the particle coordinates as ones containing 4-dimensional space-time variables, $x_i = (x_i, t_i)$ in the separation equations, namely,

$$G^{(2)}(1, 2; 1', 2') = G^{(2)}_c(1, 2; 1', 2') + \hat{A}P_{(1', 2')}G^{(2)}(1; 1')G(2; 2'),$$  

(2.12a)

$$G^{(3)}(1, 2, 3; 1', 2', 3') = G^{(3)}_c(1, 2, 3; 1', 2', 3')$$

$$+ \hat{A}P_{(1', 2', 3')}G^{(2)}(1, 1')G^{(2)}(2, 3; 2', 3') + G^{(2)}_c(1, 2, 1'2')G(3; 3'),$$  

(2.12b)

$$G^{(n)}(1, \ldots n; 1', \ldots n') = G^{(n)}_c(1, \ldots n; 1', \ldots n')$$

$$+ \hat{A}P_{(1'\ldots n')}G^{(n-1)}(1, \ldots k; 1'\ldots k')G^{(n-k)}((k + 1), \ldots n; (k + 1)', \ldots n'),$$  

(2.14a)
where \( G(1; 1') = G^{(1)}(1; 1') \). \( \hat{S}p(\ldots) \) denotes symmetrizing operation among particle variable pairs \((x_i, x'_i)\) and \((x_j, x'_j)\). \( \hat{A}p(\ldots) \) denotes anti-symmetrizing operation among variables \(x'_i\) and \(x'_j\). In combination of the operations \( \hat{S}p \) and \( \hat{A}p \), the repeated terms should be omitted[17]. The separation equations (2.12-14) are highly nonlinear. It is evident that \( G^{(n)} \) contains all possible correlations among \( n \) particles. The properties of eqs.(2.12-14) are as follows:

1. \( G^{(n)} \) can be expanded according to any variable pair \((x_i, x'_i)\). Due to symmetrization and anti-symmetrization operations, all the expansions are equivalent.

2. \( G^{(n)} \) and \( \hat{A}p \hat{S}p G^{(n-k)} \) possess the same symmetry as \( G^{(n)} \) does. Yet, \( \hat{A}p \) and \( \hat{S}p \) have following properties,

\[
\hat{A}p(\ldots), \hat{S}p(\ldots), \hat{A}p(\ldots)\hat{S}p(\ldots) = \hat{A}p(\ldots)\hat{S}p(\ldots)
\]

if \((\ldots)_1 \supset (\ldots)_2\) and \((\ldots)_1 \supset (\ldots)_2\).

3. \( G^{(n)} \) contains all possible correlations among \( n \) particles.

The advantage of eq.(2.14) is that the particle variable \( z_1 \) is fixed in \( G^{(k)} \). This will facilitate the derivation of equations of motion. It is checked that eqs.(2.12-14) are consistent with the corresponding relations between \( G^{(n)} \) and \( G^{(n)} \) obtained from the generating functional technique and the correlation green's functions \( G^{(n)} \) are simply the connected green's functions.

In order to give readers a specific feeling, we write down the lowest order separation equations,

\[
G^{(2)}(1, 2; 1', 2') = G(1; 1')G(2; 2') - G(1; 2')G(2; 1') + G^{(2)}(1, 2; 1', 2'),
\]

(2.16)

\[
G^{(3)}(1, 2, 3; 1', 2', 3') = G^{(3)}(1, 2, 3; 1', 2', 3')
+ G(1; 1')G(2; 2')G(3; 3') - G(1, 2')G(2; 1')G(3; 3') - G(1; 3')G(2; 2')G(3; 1')
- G(1; 1')G(2; 3')G(3; 2') + G(1; 2')G(2; 3')G(3; 1') + G(1; 3')G(2; 1')G(3; 2')
+ G(1; 1')G^{(2)}(2, 3, 2', 3') - G(1, 2')G^{(2)}(2, 3, 1', 3') - G(1; 3')G^{(2)}(2, 3, 2')
+ G^{(2)}(1, 2; 1', 2')G(3; 3') - G^{(2)}(1, 2, 3', 2')G(3; 1') - G^{(2)}(1, 2, 1', 3')G(3; 2')
+ G^{(2)}(1; 3, 1', 3')G(2; 2') - G^{(2)}(1, 3, 2', 3')G(2; 1') - G^{(2)}(1, 3, 1', 2')G(2; 3').
\]

(2.17)

Having accomplished the separation of \( G^{(n)} \), the next step is to derive equations of motion for \( G^{(n)} \). This is a complex task and the lengthy mathematical manipulation is put in Appendix A. While for the lowest order equations, the derivation is straightforward. Inserting eqs.(2.12-13) into eqs.(2.9-11), we obtain the equations of motion for \( G, G^{(2)} \) and \( G^{(3)} \),

\[
[i\partial_t - \hat{U}(1)]G(1; 1') = \delta^{(4)}(1; 1') - i \int d2\hat{V}(1, 2)[G^{(2)}(2; 1', 2') + G(1; 1')G(2; 2') - G(1; 2')G(2; 1')],
\]

(2.18)

\[
[i\partial_t - \hat{U}(1)]G^{(2)}(1, 2; 1', 2') = \int d3\hat{V}(1, 3)[G^{(2)}(2, 3; 1', 2') + G^{(2)}(2, 3; 1', 2') - G^{(2)}(2, 3; 1', 2')G^{(3)}(1, 2, 3; 1', 2', 3')
+ G^{(2)}(2, 1', 2')G(3; 3') - G^{(2)}(2, 1', 2')G(3; 3') + G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1') + G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1')
- G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1') - G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1')
+ G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1') - G^{(2)}(2, 3; 1', 2')G^{(2)}(3; 1')],
\]

(2.19)

\[
[i\partial_t - \hat{U}(1)]G^{(3)}(1, 2, 3; 1', 2', 3') = \int d4\hat{V}(1, 4)[G^{(2)}(2, 3; 4, 1')G^{(3)}(1, 4; 3', 2')
- G^{(3)}(2, 3; 4, 3')G^{(2)}(1, 3; 4', 2') - G^{(2)}(2, 3; 4, 1')G^{(2)}(1, 4; 3', 2')
- G^{(2)}(1, 2, 1', 2')G(3; 3') - G^{(2)}(1, 2, 1', 2')G(3; 3') + G^{(2)}(1, 2, 1', 2')G(3; 3') + G^{(3)}(1, 2, 3', 2')G(3; 4)G(4; 1')
- G^{(3)}(1, 3, 1', 2')G(2; 4)G(4; 2') + G^{(3)}(1, 2, 1', 2')G(2; 4)G(4; 2') + G^{(3)}(1, 3, 2', 3')G(2; 4)G(4; 1')],
\]

(2.20)
To obtain the equations of motion for $G^{(n)}$ in general, it is helpful to examine the structure of the r.h.s. of eqs. (2.18-20). The concepts of linked and unlinked terms introduced in Ref. [17] are very useful for describing the structure of eqs. (2.18-20). A term consisting of a product of several factors, is called unlinked, if one factor contains variables which do not appear in the others. Otherwise, it is called linked. In a linked term, each factor contains at least one variable which appears also in the other factors. In the language of Feynman diagram, the diagrams of linked terms are connected topologically. While the diagrams of unlinked terms are disconnected. In the above terminology, the structure of eqs. (2.18-20) can be described as follows: (i) All the terms in the r.h.s. of eqs. (2.18-20) are linked either due to interactions (called interaction correlations), or due to Pauli anti-symmetrization (called Pauli correlations). (ii) The two body interactions and Pauli anti-symmetrization can connect at most three factors. For $n$-body correlations, there are only three types of terms like $Tr(n+l)V(1,n+1)G^{(n)}$, $Tr(n+l)V(1,n+1)G^{(n)}G^{(m)}\delta_{k+l+m,n+1}$, and $Tr(n+l)V(1,n+1)G^{(k)}G^{(l)}G^{(m)}\delta_{k+l+m,n+1}$. (iii) The r.h.s. and l.h.s. of eqs. (2.18-20) possess identical symmetry with respect to permutation of particle variables. (iv) In eqs. (2.18-20), integration over continuous variables and summation over discontinuous variables are understood (see the definition of the trace operation, eq. (2.23)). From the above observations, the equation of motion for $G^{(n)}$ can be written as

\[ [i\partial_t - i(\lambda))G^{(n)}(1,...,n;1',...n') = \]

\[ -i \int d(n + 1)\tilde{V}(1,n+1)\{G^{(n+1)}(1,...,n,n+1;1',...n',n+1) \]

\[ + \sum_{k=1}^{n} G^{(k)}(1,...,k;1',...k')G^{(n+1-k)}((k+1),n,n+1;1',...n',n+1) \]

\[ - \sum_{k=2}^{n-1-k} G^{(k)}(1,...,k-1,n+1;1',...k')G^{(n+1-k)}((k+1),n,n+1;1',...n',n+1) \]

\[ - \sum_{k=p=1}^{n-1-k-p} G^{(k)}(1,...,k;1',...k')G^{(p)}((n+1),(k+2),...,(k+p);1',...,(k+p)) \]

\[ \times G^{(n+1-k-p)}((k+p+1),n,(k+1),(k+p+1);1',...n',n+1)) \}, (n \geq 2) \]

which has been proved rigorously in Appendix A. The above equation can be rewritten in a more compact form, namely

\[ [i\partial_t - i(\lambda))G^{(n)} = -i[Tr(n+1)y=(n+1)* \tilde{V}(1,n+1)AS_{(n+1)} \sum \sum \sum G^{(k)}G^{(l)}G^{(m)}\delta_{k+l+m,n+1}]_{L}, (n \geq 2) \]
where the notation \( AS(n+1) \) is an abbreviation of \( Ap\bar{Sp} \) and \([...]_L \) denotes linked terms as used in Ref.[17]. Here it is assumed that \( G_e^{(0)} = 1, G_e^{(1)} = G_e^{(1)} = G \). The trace operation is defined as first taking equal time limit, then integrating over space-time variables \( x \) and summing over spin-isospin variables \( \alpha \), i.e.

\[
\operatorname{Tr}_{(n+1)'} = \sum_{a_{n+1}} \int_{t_{n+1}'}^{t_{n+1}} d^4x_{n+1}.
\]  

(2.23)

Because of the symmetry of \( G(n) \) and \( G_e(n) \), it is easy to write down equations of motion for any space-time variables \( x \) and \( x' \),

\[
[i\partial_t - i(j)]G_e^{(n)} = -i[\operatorname{Tr}_{(n+1)'}= (n+1) + \hat{V}(j, n+1)AS(n+1) \sum_{k \geq l \geq m=0} G_e^{(k)}G_e^{(l)}G_e^{(m)}\delta_{k+l+m,n+1}]_L, \quad (n \geq 2) \]  

(2.24)

and

\[
[i\partial_t' + i(j')]G_e^{(n)} = i[\operatorname{Tr}_{(n+1)'}= (n+1) - \hat{V}(j', n+1)']
\times AS(n+1) \sum_{k \geq l \geq m=0} G_e^{(k)}G_e^{(l)}G_e^{(m)}\delta_{k+l+m,n+1}]_L, \quad (n \geq 2).
\]  

(2.26)

Eqs.(2.18) and (2.21) constitute a set of basic equations of motion for many-body correlation green's functions. The advantage of the above equations are as follows. (i) The basic physical quantities are many-body correlation green's functions which contain sufficient and necessary information for description of a many-body system, there is no unnecessary repetition of information. (ii) They are nonlinear, non-perturbative and coupled, reflecting dynamical interweave and feedback processes among correlations of different orders. (iii) If the many-body correlations become weaker as their orders increase (this is the case for most physical systems), the above hierarchy of equations provides a natural truncation scheme and each truncation leads to a non-perturbative approximation. (iv) As to be proved, the equal time limit of the above equations leads to its density matrix correspondence—the correlation dynamics of density matrix.

3 Equal Time Limit of Many-Body Green's Functions-Correlation Dynamics of Density Matrix

A generalized theory should include its special theory as a limit case. The green's function theory is more general than the density matrix theory. Therefore the correlation dynamics of green's function should include the correlation dynamics of density matrix as a special case. This section devotes to prove that it is really the case.

According to the usual notation, the n-body green's function, under the so-called normal equal time limit (\( NETL \)):

\[
\text{equal time limit under the time order: } t_1 > t_2 > ... > t_n > ... > t_2 > t_1
\]

leads to the n-body density matrix,

\[
[r^nG(n)(1, ... n; 1', ... n')]_{NETL} = (-)^n\rho_n(1, ... n; 1'... n'; t).
\]  

(3.1)

Correspondingly, the n-body correlation green's function reduces to the n-body correlation density matrix,

\[
[r^nG_e(n)(1, ... n; 1', ... n')]_{NETL} = (-)^nC_n(1, ... n; 1', ... n'; t)
\]  

(3.2)
Rather than the NETL, other equal time limits of $G^{(n)}$ will result in extra terms which consist of unlinked products of $\delta$-functions and lower order density matrices. To get a more clear insight, we write down the one-body green's function under two different equal time limits, namely the normal and the anti-normal equal time limits,

\begin{align}
[iG(1; 1')]_{t'=t} &= -\rho(1; 1'; t), \\
[iG(t; t')]_{t'=t} &= \delta^{(3)}(1; 1') - \rho(1; 1'; t).
\end{align}

(3.3a) (3.3b)

In Appendix B, the following statement is proved to be true: The many-body correlation green's functions possess the identical equal time limit irrespective of the time order being taken, namely

\begin{align}
[i^n G^{(n)}(1, \ldots, n; 1', \ldots, n')]_{any \ equal \ time \ limit} &= [i^n G^{(n)}(1, \ldots, n; 1', \ldots, n')]_{NETL} = (-)^n C_n(1, \ldots, n; 1', \ldots, n'; t), (n \geq 2).
\end{align}

(3.4)

The above results can be understood intuitively as follows. Since different equal time limits only make difference which is unlinked terms and thus do not affect linked terms, the equal time limit of $G^{(n)}$ is the same and independent of time order. Graphically speaking, linked diagrams are connected in 4-dimensional space-time, equal time limit just deforms the connected diagrams continuously and does not change their topology of connectedness.

Taking the normal equal time limit, from eqs.(2.18) and (2.25) we have

\begin{align}
i\partial_t \rho(1; 1'; t) &= [\hat{t}(1) - \hat{t}(1')] \rho(1; 1'; t) \\
+ tr_{(2'=2)} [\hat{t}(1; 2) - \hat{t}(1', 2')][\rho(1; 1'; t) \rho(2; 2'; t)] \\
-\rho(1; 2'; t) \rho(2; 1'; t) + C_2(1, 2, 1', 2'; t).
\end{align}

(3.5)

Where the trace $tr_{(n+1)'=(n+1)}$ does not include integration over time $t$. For $n \geq 2$, under the NETL we have from eqs.(2.24) and (2.26),

\begin{align}
i\partial_t C_n(1, \ldots, n; 1', \ldots, n'; t) &= \sum_{j=1}^{n} [I(j) - I'(j')],
\end{align}

(3.6)

where

\begin{align}
I(j) &= \hat{t}(j) C_n(1, \ldots, n; 1', \ldots, n'; t) \\
+ [tr_{(n+1)'=(n+1)}] \delta(j; n + 1) A S_{(n+1)} \sum \sum_{k \geq l \geq m = 0} (-i)^{k+l+m} G^{(k)} G^{(l)} G^{(m)} \\
x \delta_{k+l+m,n+1} \mid (t'_1, \ldots, t'_n, t_{(j+1)}', t_{(n+1)}', t_{(j+1)}', t_{(n+1)}') = t.
\end{align}

(3.7a)

and

\begin{align}
I'(j') &= \hat{t}(j') C_n(1, \ldots, n; 1', \ldots, n'; t) \\
+ [tr_{(n+1)'=(n+1)}] \delta(j', (n + 1)) A S_{(n+1)} \sum \sum_{k \geq l \geq m = 0} (-i)^{k+l+m} G^{(k)} G^{(l)} G^{(m)} \\
x \delta_{k+l+m,n+1} \mid (t'_1, \ldots, t'_n, t_{(j+1)}', t_{(n+1)}', t_{(j+1)}', t_{(n+1)}') = t.
\end{align}

(3.7b)

Where $(t'_1, \ldots, t'_n, t_{(j+1)}', t_{(n+1)}', t_{(j+1)}', t_{(n+1)}') = t$ denotes a special equal time limit with the time order indicated by the arguments in the parenthesis. In eq.(3.7a), by using eq.(3.4), $(-i)_k G^{(k)}(k \geq 2)$ can be replaced by $C_k$. For $(-i) G^{(1)}(p; q)$, there are two cases: (i) For $-i G^{(1)}(p; q' \neq (n + 1)^+)$ and $-i G^{(1)}(p \leq j, or, p = n + 1; q' = (n + 1)^+)$, their equal time limit is normal as the normal $n$-particle equal time limit is taken. Thus these terms can be replaced by $C_1(p; q) = \rho(p; q; t)$ according to eq.(3.3a). (ii) For $-i G^{(1)}(n \geq p > j; q' = (n + 1)^+)$, their equal time limit is anti-normal as the normal $n$-particle equal time limit is taken. According to eq.(3.3b) we have

\begin{align}
-i G^{(1)}(n \geq p > j; q' = (n + 1)^+) &= -\delta^{(3)}(p; q' = (n + 1)) + \rho(p; (n + 1); t).
\end{align}

(3.8)
For eq.(3.7b), the calculation is similar.

From the above discussion we know that, in the calculation of $I(j)$, only those terms containing $G_c^{(1)}(n \geq p > j; q' = (n + 1)^+)$ need special attention. They are

\[
\{ tr(n+1)\hat{v}(j, n+1)[ - \sum_{i=j+1}^{n} (-i)^n G_c^{(n)}(1..(i-1),(n+1),(i+1)..<n'(i-n)'\cdots n') \} \}
\]

\[
- \sum_{i=j+1}^{n} \hat{A} p_{1..n'} \hat{S} p_{1..(j-1)(j+1)..<(i-1)(i+1)..<n'(j-1)(j+1)..<(i-1)(i+1)..<n'} \sum_{k=1}^{n-1} (-i)^k \times
\]

\[
G_c^{(k)}(j, 1..(k-1); j', 1'..(k-1)')(i-1)(n+1),(i+1)..<n; k', ..(i-1)', i', (i+1)..<n')
\]

\[
\times (-i)^{n-k} G_c^{(n-k)}(j, 1..(n+1)^+)(i; n+1^+)(i; n+1^+) \}
\]

\[
= \{ \sum_{i=j+1}^{n} \hat{v}(j, i) C_n(1..n^+; i', t), (I)
\]

\[
+ \sum_{i=j+1}^{n} \hat{v}(j, i) \hat{A} p_{1..n'} \hat{S} p_{1..(j-1)(j+1)..<(i-1)(i+1)..<n'(j-1)(j+1)..<(i-1)(i+1)..<n'} \times \sum_{k=1}^{n-1} C_k(j, 1..(k-1); j', 1'..(k-1)'; t) C_{n-k}(k..i..n; k', ..(i-1)', i', (i+1)..<n'); (II)
\]

\[
- tr(n+1)\hat{v}(j, n+1) \hat{A} p_{1..n} \hat{S} p_{1..n} \}
\]

\[
\sum_{i=j+1}^{n} \sum_{k=1}^{n-1} C_k C_{n-k} C_1(i; n+1^+; t), (III)
\]

\[
+ \sum_{k=1}^{n-1} C_k C_{n-k} C_1(i; n+1^+; t)), (IV)
\]

(3.9)

where the second term in eq.(3.9) can be written as

\[
\{ [ \sum_{i=j+1}^{n} \hat{v}(j, i) A S(n) \sum_{k=1}^{n-1} C_k C_{n-k} ] L \}.
\]

(3.10)

Where $A$ and $S$ are anti-symmetrizing and symmetrizing operators used in Ref.[17]. The third and fourth terms of eq.(3.9) are in the same form as the other terms of eq.(3.7a). By using eqs.(3.9-10), eq.(3.7a) turns to be

\[
I(j) = [ \hat{t}(j) + \sum_{i=j+1}^{n} \hat{v}(j, i)] C_n + [ \sum_{i=j+1}^{n} \hat{v}(j, i) A S(n) \sum_{k=1}^{n-1} C_k C_{n-k} ] L
\]

\[
+ tr(n+1) [ \hat{v}(j, n+1) A S(n+1) \sum_{k=1}^{n-1} \sum_{m=0}^{k-1} C_k C_l C_m \delta_{k+l+m, n+1} ] L \]

(3.11a)

Similarly

\[
I'(j') = [ \hat{t}(j') + \sum_{i=j+1}^{n} \hat{v}(j', i') ] C_n + [ \sum_{i=j+1}^{n} \hat{v}(j', i') A S(n) \sum_{k=1}^{n-1} C_k C_{n-k} ] L
\]

\[
+ tr(n+1) [ \hat{v}(j', n+1) A S(n+1) \sum_{k=1}^{n-1} \sum_{m=0}^{k-1} C_k C_l C_m \delta_{k+l+m, n+1} ] L \]

(3.11b)

Inserting eqs.(3.11a,b) into eq.(3.6), we obtain the equation of motion for $C_n$

\[
\hat{i} \partial_t C_n = [ \sum_{j=1}^{n} (\hat{t}(j) - \hat{t}(j')) + \sum_{i>j=1}^{n-1} (\hat{v}(j, i) - \hat{v}(j', i')) ] C_n
\]

\[
+ \sum_{k=1}^{n-1} C_k C_{n-k} C_1(i; n+1^+; t), (IV)
\]

(3.9)
\[ 
\sum_{i>j=1}^{n-1} (\bar{v}(j, i) - \bar{v}(j', i'))AS_{(n)} \sum_{k=1}^{n-1} C_k C_{n-k} \] 
\[ + tr_{(n+1)} \left[ \sum_{j=1}^{n} (\bar{v}(j, n + 1) - \bar{v}(j', n + 1))AS_{(n+1)} \sum_{k \geq 2} \sum_{m=0} C_k C_l \delta_{k+l+m, n+1}^{\pm} \right] 
\] 

Eqs. (3.5) and (3.12) are exactly the equations of motion for many-body correlation density matrices given in Ref. [17]. So far we have proved that the correlation dynamics of density matrix is the equal time limit of that of green's function. Since in the green's function formalism, space and time variables are on equal footing, the present formalism is thus a necessary step to a relativistic correlation dynamics of quantum fields.

Before concluding this section, we would like to make a brief remark on conservation law. Since the average of physical observables can be calculated from the many-body green's functions under the normal equal time limit [2, 4], the conclusion concerning conservation law drawn in Ref. [17] is also applicable to the present formalism: the conservation of one-body quantities is independent of the approximation made for \( G_x^{(2)} \), while the conservation of two body quantities is irrespective of how the approximation for \( G_x^{(3)} \) is taken.

### 4 Lowest Order Truncations—Mean Field Theory and Two-Body Dynamics

As pointed out in the last section, one of the merits of correlation dynamics is that it provides a natural truncation scheme with respect to the order of correlations. As is in the case of density matrix formalism, the lowest order truncations of the correlation dynamics of green's function lead to mean field theory (Dyson equation and TDHF), two-body dynamics (including Bethe-Salpeter equation and the equation for polarization effect as two limits), and three-body dynamics (Faddeev-like equations in the form of green's function).

#### 4.1 Mean Field Approximation. Dyson Equation and TDHF

The lowest order truncation is simply to neglect all many-body correlations, i.e., to assume

\[ G_x^{(n)} = 0, (n \geq 2). \]  

(4.1)

This leads to the equation of motion for \( G(1; 1') \) in mean field approximation,

\[ [i\partial_t - \hat{\mathcal{H}}(1)]G(1; 1') = \delta^{(4)}(1; 1') - i \int d^4x \mathcal{V}(1, 2)[G(1; 1')G(2; 2') - G(1; 2)G(2; 1')]. \]  

(4.2a)

and

\[ [i\partial_t + \hat{\mathcal{H}}(1')]G(1; 1') = -\delta^{(4)}(1; 1') + i \int d^4x \mathcal{V}(1', 2)[G(1; 1')G(2'; 1) - G(1; 2)G(2; 1')]. \]  

(4.2b)

Introduce the green's function of a free particle, \( G^0(1; 1') \),

\[ [i\partial_t - \hat{\mathcal{H}}(1)]G^0(1; 1') = \delta^{(4)}(1, 1'), \]  

(4.3a)

\[ G^0(1; 1') = [i\partial_t - \hat{\mathcal{H}}(1)]^{-1}\delta^{(4)}(1, 1'). \]  

(4.3b)

Equation (4.2) can be rewritten as

\[ G(1; 1') = G^0(1; 1') + \int d^4x d^4y G^0(1; 3) \sum (3, 2) G(2; 1'), \]  

(4.4a)

\[ \sum (3, 2) = -i \int d^4x d^4y \mathcal{V}(3, 4) G(4; 4') - \mathcal{V}(3, 2) G(3; 2)). \]  

(4.4b)
Eqs.(4.4a,b) are Dyson equations for the single particle green’s function and its self-energy in mean field approximation.

In literatures, only Hartree-Fock equation was derived from Dyson equation in a special case. In the following, we will derive TDHF from Dyson equation in general case. Because of Hermiticity of $iG(1;1')$, namely,

\begin{align}
[iG^>(1;1')]^* &= (\langle \psi(1)\psi^\dagger(1') \rangle) = \langle \psi(1')\psi^\dagger(1) \rangle = iG^<(1';1), \\
[iG^<(1;1')]^* &= (-\langle \psi^\dagger(1')\psi(1) \rangle)^* = -\langle \psi^\dagger(1)\psi(1') \rangle = iG^>(1';1),
\end{align}

$iG(1;1')$ can be written in a diagonal form. From its definition we know that as $t > t'$, $G(1;1')$ represents propagation of a particle from $(x',t')$ to $(x,t)$; while as $t < t'$, it represents propagation of a hole from $(x',t')$ to $(x,t)$. From the above mathematical-physical consideration, $iG(1;1')$ can be written as

\begin{equation}
\begin{aligned}
iG(1;1') &= \theta(t_1 - t'_1) \sum_\alpha \psi_\alpha(x_1,t_1)\bar{\psi}_\alpha(x'_1,t'_1)(1 - n_\alpha) \\
&\quad - \theta(t'_1 - t_1) \sum_\alpha \psi_\alpha(x_1,t_1)\bar{\psi}_\alpha(x'_1,t'_1)n_\alpha,
\end{aligned}
\end{equation}

where $n_\alpha$ are occupation probabilities of the single particle state $\psi_\alpha$, which are independent of time and should be given by initial conditions. The single particle states $\psi_\alpha(x,t)$ are to be determined and satisfy the following orthonormal and completeness conditions,

\begin{align}
\langle \psi_\alpha(x,t) | \psi_\beta(x,t) \rangle &= \delta_{\alpha\beta} \\
\sum_\alpha \psi_\alpha(x,t)\bar{\psi}_\alpha(x',t) &= \delta^{(3)}(x - x').
\end{align}

The equation of motion for $\psi_\alpha(x,t)$ can be obtained from eqs.(4.2a,b). Inserting eq.(6) into eq.(4.2a) and using eqs.(4.7a,b), we have the TDHF equation,

\begin{align}
i\partial_t \psi_\alpha(x,t) &= \hat{h}(x,t)\psi_\alpha(x,t) \\
\hat{h}(x,t) &= \hat{i}(x) + U_{HF}(x,t)
\end{align}

\begin{equation}
\begin{aligned}
\hat{U}_{HF}(x,t)G(x,t;x'\prime) &= \int d^3x_1 \psi(x-x')[G(x,t;x'\prime)G^<(x_1t;x_1t_1) \\
&\quad - G^<(x_1t;x_1t)G(x_1t;x'\prime)].
\end{aligned}
\end{equation}

By virtue of eq.(4.6), eq.(4.9b) can be written down explicitly,

\begin{equation}
\begin{aligned}
\hat{U}_{HF}(x,t)\psi_\alpha(x,t) &= \int d^3x_1 \bar{\psi}(x_1 - x) \sum_\beta n_\beta [\psi_\beta(x_1,t)\bar{\psi}_\beta(x_1,t)\psi_\alpha(x,t) \\
&\quad - \psi_\beta(x,t)\bar{\psi}_\beta(x_1,t)\psi_\alpha(x_1,t)].
\end{aligned}
\end{equation}

which is exactly the TDHF mean field. If $\hat{h}$ is independent of time, eq.(4.8) has stationary solutions, namely the HF solutions,

\begin{align}
\psi_\alpha(x,t) &= \exp(-i\epsilon_\alpha t)\psi_\alpha(x), \\
\hat{h}(x)\psi_\alpha(x) &= \epsilon_\alpha \psi_\alpha(x).
\end{align}

For TDHF and HF solutions, $n_\alpha = 1$ or 0. Inclusion of two-body correlations will destroy the independent particle picture and lead to $0 < n_\alpha < 1$. 

11
4.2 Two-Body Dynamics. Bethe-Salpeter Equation and Equation for Polarization Effect

A better approximation is to keep two-body correlations and neglect more than three-body correlations, namely

\[ G^{(n)} = 0, \quad (n \geq 3). \tag{4.11} \]

This truncation leads to the two-body dynamics which consist of two coupled equations of motion, one for \( G(1; 1') \),

\[
[i\partial_t - i(1)]G(1; 1') = \delta^{(4)}(1, 1') - i \int d2\hat{V}(1, 2)[G(1; 1')G(2; 2^+)] - G(1; 2)G(2; 1') - i \int d2\hat{V}(1, 2)G_z^{(2)}(1, 2; 1', 2^+), \tag{4.12}
\]

and the other for \( G_z^{(2)} \),

\[
[i\partial_t - i(1)]G_z^{(2)}(1, 2; 1', 2') = -i \int d3\hat{V}(1, 3)[G_z^{(2)}(1, 2; 1', 2')G(3; 3^+)] + G_z^{(2)}(2, 3; 1', 2')G(3; 3^+) - G_z^{(2)}(2, 3; 1', 2')G(3; 3^+) - G_z^{(2)}(2, 3; 1', 2')G(3; 3^+) - G_z^{(2)}(2, 3; 1', 2')G(3; 3^+). \tag{4.13}
\]

Equations (4.12) and (4.13) are an extension of Bethe-Salpeter (B-S) equation [2], since they contain both ladder diagram series and ring diagram series in a unified way.

We first proceed to prove that under the ladder diagram approximation, eqs. (4.12) and (4.13) lead to the B-S equation. Define a mean field \( \hat{U} \) as follows,

\[
\hat{U}(1)G(1; 1') = -i \int d2\hat{V}(1, 2)[1 - \hat{P}_{12}]G(1; 1')G(2; 2^+), \tag{4.14a}
\]

or

\[
\hat{U}(1) = -i \int d2\hat{V}(1, 2)[1 - \hat{P}_{12}]G(2; 2^+). \tag{4.14b}
\]

Where \( \hat{P}_{12} \) is a permutation operator of variables 1 and 2. Under mean field approximation, eq.(4.12) becomes

\[
[i\partial_t - i(1) - \hat{U}(1)]G(1; 1') = \delta^{(4)}(1, 1'), \tag{4.15a}
\]

or

\[
G(1; 1') = [i\partial_t - i(1) - \hat{U}(1)]^{-1}\delta^{(4)}(1, 1'). \tag{4.15b}
\]

The equation of motion for \( G_z^{(2)} \) can be rewritten as

\[
[i\partial_t - i(1)]G_z^{(2)}(1, 2; 1', 2') = -i \int d3\hat{V}(1, 3)[G_z^{(2)}(1, 2; 1', 2')G(3; 3^+)] + (1 - \hat{P}_{13})G(3; 3^+)G_z^{(2)}(1, 2; 1', 2') \tag{4.16a}
\]

\[
- i \int d3\hat{V}(1, 3)[(1 - \hat{P}_{13})(1 - \hat{P}_{12'})]G(1; 1')G_z^{(2)}(2, 3; 2', 3^+). \tag{4.16b}
\]

A truncation up to eqs.(4.16a,b) corresponds to the ladder diagram approximation and leads to Bethe-Salpeter equation, while the terms indicated by eq.(4.16c) are contributions from ring diagrams. By virtue of eq.(4.14b), eqs.(4.16a,b) turn to be,

\[
[i\partial_t - i(1) - \hat{U}(1)]G_z^{(2)}(1, 2; 1', 2') = i \int d3\hat{V}(1, 3)G_z^{(2)}(1, 3; 1', 2')G(2; 3). \tag{4.17}
\]
Operating \([i\partial_t - i(2) - \hat{U}(2)]\) on the above equation from left and using eq.(4.15a), we have
\[
[i\partial_t - i(2) - \hat{U}(2)][i\partial_t - i(1) - \hat{U}(1)]G(2)(1, 2; 1', 2') = i\tilde{V}(1, 2)G(2)(1, 2; 1', 2').
\] (4.18)
By virtue of eq.(4.15b), the solution of eq.(4.18) reads
\[
G(2)(1, 2; 1', 2') = i\int d3d4G(1; 3)G(2; 4)\tilde{V}(3, 4)G(3; 1')G(4; 2'),
\] (4.19a)
or
\[
G(2) = GGi\tilde{V}G(2).
\] (4.19b)
Defining an effective interaction \(\hat{\Gamma}\),
\[
\tilde{V}G(2) = \hat{\Gamma}\hat{\Delta}\hat{S}(2)GG,
\] (4.20)
and noticing
\[
G(2) = \hat{\Delta}\hat{S}(2)GG + G(2),
\] (4.21)
after a simple algebraic manipulation we finally obtain the Bethe-Salpeter equation [2],
\[
\hat{\Gamma} = \tilde{V} + i\tilde{V}GG\hat{\Gamma},
\] (4.22)
where the single particle green's function is not for a free particle, but for a particle moving in a mean field.

Now we consider the terms of eq.(4.16c)-contributions from ring diagrams. For simplicity, we investigate only the direct terms of ring diagrams (i.e. neglect the terms containing \(\hat{P}_{13}\) and \(\hat{P}_{1;2'}\) in eq.(4.16c)). In eq.(4.16a) we drop the term \(G(2)\) responsible for ladder diagrams and consider also direct term for the rest, i.e. assume \(G(2) = GG\). Under the above considerations, from eqs.(4.16a,b,c) and (4.14b) we have
\[
[i\partial_t - i(1) - \hat{U}(1)]G(2)(1, 2; 1', 2')
= -i\int d3\tilde{V}(1, 3)[-G(1; 1')G(3; 2')G(2; 3) + G(1; 1')G(2; 3; 2', 3+)].
\] (4.23)
The solution of the above equation reads
\[
G(2)(1, 2; 1', 2') = i\int d3d4G(1; 4)G(4; 1')\tilde{V}(4, 3)[G(3; 2')G(2; 3) - G(2; 3; 2', 3+)].
\] (4.24)
Consider the relation between the time-ordering correlation function and the two body correlation green's function,
\[
iD(1, 2) = iD(2, 1) = \langle \hat{T}[\hat{n}(1) - \hat{n}(2)]\rangle = \langle \hat{T}[\psi(1)\psi(2)]\rangle
= -G(2)(1, 2; 1^+, 2^+) + G(1; 1^+)G(2; 2^+) = -G(2)(1, 2; 1^+, 2^+) + G(1; 2)G(2; 1).
\] (4.25)
From eqs.(4.24) and (4.25) we have
\[
G(2)(1, 2; 1^+, 2^+) = i\int d3d4G(1; 4)G(4; 1')\tilde{V}(4, 3)[-G(2)(2; 3; 2^+, 3^+) + G(2; 3)G(3; 2)].
\] (4.26)
Introducing \(D^\theta(1, 2)\) such that
\[
D^\theta(1, 2) = -iG(1; 2)G(2; 1),
\] (4.27)
from eqs.(4.26) and (4.27) we obtain

\[ D(1, 2) = D^0(1, 2) + \int d3d4D^0(1, 3)\tilde{V}(3, 4)D(4, 2), \]

or

\[ D = D^0 + D^0\tilde{V}D, \quad D = (1 - D^0\tilde{V})^{-1}D^0. \]

Now consider the effect of \( D \) on \( G^{(2)}_\varepsilon \). Operating \([i\partial_2 - i(2) - \tilde{U}(2)]\) from left on eq.(4.23), we have

\[ [i\partial_2 - i(2) - \tilde{U}(2)][i\partial_1 - i(1) - \tilde{U}(1)]G^{(2)}_\varepsilon(1, 2; 1', 2') \]
\[ = i\tilde{V}(1, 2)G(1; 1')G(2; 2') + \int d3d4\tilde{V}(1, 3)\tilde{V}(2, 4)G(1; 1')G(2; 2')[-G^{(2)}_\varepsilon(3, 4; 3', 4') + G(4; 3)G(3; 4)]. \]

The solution is

\[ G^{(2)}_\varepsilon(1, 2; 1', 2') = i\int d3d4G(1; 3)G(2; 4)\tilde{V}(3, 4)G(3; 1')G(4; 2') \]
\[ + i\int d3d4d5d6G(1; 3)G(2; 4)\tilde{V}(3, 5)D(5, 6)\tilde{V}(4, 6)G(3; 1')G(4; 2'). \]

In brevity

\[ G^{(2)}_\varepsilon = iGG\tilde{U}_rGG, \]

where the effective interaction \( \tilde{U}_r \) is defined as

\[ \tilde{U}_r = \tilde{V} + \tilde{V}D\tilde{V} = \tilde{V}(1 - D^0\tilde{V})^{-1}. \]

Eqs.(4.28,4.30,4.31) are used to describe the polarization effect of ring diagrams[2]. In terms of Fyenman diagrams, they correspond to

\[ D^0 = \]

\[ D = \]

\[ \tilde{U}_r = \]

\[ G^{(2)}_\varepsilon = \]

The above Fyenman diagrams include only direct terms. The exchange terms correspond to diagrams with more complicated structures.

Up to now, we have proved that the two-body dynamics, eqs.(4.12,4.13), as a whole, contains both ladder diagram series and ring diagram series, and the non-perturbative results of the conventional green's function theory can be
obtained as two limiting cases. This indicates that the correlation dynamics is a complete and unified theory which contains all non-perturbative effect up to certain truncated orders.

Before concluding this section, we should mention that a green's function correspondence of the time-dependent G-matrix (TDGM) in density matrix correlation dynamics[18] has been obtained also in the present formalism[29] and the two-body green's function dynamics can be solved in a parallel way as in Refs.[18,19,20].

5 Discussion and Outlook

So far we have generalized the correlation dynamics of density matrix and established a complete set of dynamical equations for many-body correlation green's functions. The resulting formalism is non-perturbative and nonlinear. It provides a natural truncation scheme with respect to the order of many-body correlations. In the lowest order truncations, the main results of the conventional green's function theory have been obtained. Besides, the two-body correlation dynamics contains ladder diagrams and ring diagrams in a compact way. In general, the correlation dynamics of green's function provides a unified and systematic method to treat quantum many-body problems in a non-perturbative manner.

It is desirable to compare present approach with other ones. The conventional green's function approach aimed at a perturbative calculation of green's functions, while our approach aimed at establishing a set of dynamical equations. By virtue of Feynman diagram technique and resummation rules, the conventional green's function theory is able to obtain some non-perturbative results. However it is difficult to establish a complete non-perturbative theory within a perturbative approach. On the other hand, the advantage of the present approach resides in that it provides a complete set of non-perturbative equations and that its truncation scheme is natural and systematic.

In comparison with density matrix theory, the present approach is of a multi-time form. In the non-relativistic case, the difference is not important. While in the relativistic case, since propagation of interactions needs time, multi-time green's functions are essential for a proper description of retardation effect and causality. Therefore, the correlation dynamics in a multi-time form is a necessary step to reach a full relativistic theory of correlation dynamics.

The temperature green's function is an analytical continuation of the real time green's function. This observation has motivated an investigation of a non-perturbative method for the temperature green's function theory. A project in this respect is now in progress.

The problem of formulating a non-perturbative green's function dynamics for quantum field theory is very attractive and interesting. The method developed in the present formalism is of course applicable to the relativistic case. The complex of the problem is such that the physical quantum gauge field theory contains both fermion and boson fields and the gauge fields include nonlinear self-interactions. To describe the whole dynamical process by correlation functions, one needs to define different kinds of correlation green's functions, vertex functions, and to establish their coupled, nonlinear equations of motion. Besides, the gauge condition makes the problem more difficult. Therefore this is a rather complicated task.

APPENDIX

A Proof of Equation (2.21)

We proceed to prove equation (2.21) by induction.

1. Since for \( n = 2 \) and 3 it reduces to eqs.(2.19) and (2.20) respectively, equation (2.21) is correct for \( 2 \leq n \leq 3 \).

2. Next we shall prove that if for \( 2 \leq n \leq m \) eq.(2.21) is valid, then for \( 2 \leq n \leq m + 1 \) it is also valid.

According to the separation equation (2.14), we have

\[
G^{(m+1)}(1\ldots(m+1);1'\ldots(m+1)') = G^{(m+1)}(1\ldots(m+1);1'\ldots(m+1)')
\]

15
\[-\hat{A}p(1'..(m+1)') \hat{S}p(2'..(m+1)') \sum_{k=1}^{m} G^{(k)}(1..k; 1'..k') \]
\[ \times G^{(m+1-k)}((k+1)..'..(m+1)); (k+1)'..(m+1)'). \quad (A.1) \]

Operating \([i\partial_{1} - i(1)]\) from left on eq.(A.1), we obtain
\[ [i\partial_{1} - i(1)]G^{(m+1)}(1..(m+1); 1'..(m+1)') = [i\partial_{1} - i(1)]G^{(m+1)}(1..(m+1); 1'..(m+1)') \]
\[ -\hat{A}p(1'..(m+1)') \hat{S}p(2'..(m+1)') \sum_{k=1}^{m} [(i\partial_{1} - i(1))G^{(k)}(1..k; 1'..k')] \]
\[ \times G^{(m+1-k)}((k+1)..'..(m+1)); (k+1)'..(m+1)'). \quad (A.2) \]

Inserting eq.(2.11) into eq.(A.2), using eq.(2.9) for \(G(1) = G\) and eq.(2.21) for \(\sim k \sim m\), and noticing eq.(2.15), we have
\[ [i\partial_{1} - i(1)]G^{(m+1)}(1..(m+1); 1'..(m+1)') = -i \int d(m+2)\tilde{V}(1, m+2) \]
\[ \left\{ G^{(m+2)}(1..(m+1), (m+2); 1'..(m+1)', (m+2)^+), (I) \right\} \]
\[ -\hat{A}p(1'..(m+1)') \hat{S}p(2'..(m+1)') \left( \sum_{k=1}^{m} G^{(k+1)}(1..k, (m+2); 1'..k', (m+2)^+), (II) \right) \]
\[ + \sum_{p=1}^{k} G^{(p)}(1..p; 1'..p')G^{(k+1-p)}((p+1)..'..k, (m+2); (p+1)'..k', (m+2)^+), (III) \]
\[ - \sum_{p=1}^{k} G^{(p)}(1..p; 1'..(p-1)', (m+2)^+)G^{(k+1-p)}((p+1)..'..k, (m+2); (p+1)'..k', (m+2)^+), (IV) \]
\[ \times G^{(m+1-k)}((k+1)..'..(m+1)); (k+1)'..(m+1)') \]
\[ - \sum_{k=2}^{m} \sum_{p=2}^{k-1} G^{(p)}(1..(p-1), (m+2); 1'..(p-1)', p')G^{(k+1-p)}((p+1)..'..k, (m+2); (p+1)'..k', (m+2)^+), (V) \]
\[ + \sum_{p=1}^{k-1} \sum_{l=1}^{k-p} G^{(p)}(1..p; 1'..p')G^{(l)}((p+1)..'..(p+l-1), (m+2); (p+1)'..(p+l)', (m+2)^+), (VI) \]
\[ \times G^{(k+1-p-l)}((p+l+1)..'..k, (p+l); (p+l+1)'..k', (m+2)^+) \]
\[ \times G^{(m+1-k)}((k+1)..'..(m+1)); (k+1)'..(m+1)'). \right\} \quad (A.3) \]

The following multiple summation relations are needed for further calculation,
\[ \sum_{k=1}^{m} \sum_{p=1}^{k} = \sum_{p=1}^{m} \sum_{k=p}^{m}, \quad (A.4a) \]
\[ \sum_{k=2}^{m} \sum_{p=2}^{k} = \sum_{p=2}^{m} \sum_{k=p}^{m}, \quad (A.4b) \]
\[ \sum_{k=2}^{m} \sum_{p=1}^{k-1} \sum_{l=1}^{k-p} = \sum_{p=1}^{m-p} \sum_{l=1}^{m-p} \sum_{k=p+l}^{m}. \quad (A.4c) \]
The above relations can be expressed by Fig. A.a,b,c. After having changed the order of summations, eq. (A.3) is simplified and the third to sixth terms of the r.h.s. are as follows:

\[ (VI) = -i \int d(m + 2) \hat{V}(1, m + 2) \hat{A} \hat{P}(1', \ldots, (m + 1)') \hat{S} \hat{P}(2, \ldots, (m + 1)') \sum_{p=1}^{m} \sum_{l=1}^{m+1-p} \]

\[ G^{(p)}(1, \ldots, (p-1), (m + 2)) \frac{1}{(p + 1) \ldots (p + l - 1), (m + 2); (p + 1)' \ldots (p + l)'} \times \]

\[ G^{(m+2-p-l)} - G^{(m+2-p-l)'} [p + l + 1 \ldots m + 1, p + l; (p + l + 1)' \ldots (m + 1)', (m + 2)^+] \]

\[ (A.5a) \]

\[ (III) + (VI) = i \int d(m + 2) \hat{V}(1, m + 2) \hat{A} \hat{P}(1', \ldots, (m + 1)') \hat{S} \hat{P}(2, \ldots, (m + 1)') \sum_{p=1}^{m} \sum_{l=1}^{m+1-p} \]

\[ G^{(p)}(1, \ldots, (p-1), (m + 2)) \frac{1}{(p + 1) \ldots (p + l - 1), (m + 2); (p + 1)' \ldots (p + l)'} \times \]

\[ G^{(m+2-p-l)} - G^{(m+2-p-l)'} [p + l + 1 \ldots m + 1, p + l; (p + l + 1)' \ldots (m + 1)', (m + 2)^+] \]

\[ (A.5b) \]

\[ (IV) = -i \int d(m + 2) \hat{V}(1, m + 2) \hat{A} \hat{P}(1', \ldots, (m + 1)') \hat{S} \hat{P}(2, \ldots, (m + 1)') \sum_{p=1}^{m+1} \]

\[ G^{(p)}(1, \ldots, (p-1), (m + 2)) \frac{1}{(p + 1) \ldots (p + l - 1), (m + 2); (p + 1)' \ldots (p + l)'} \times \]

\[ G^{(m+2-p-l)} - G^{(m+2-p-l)'} [p + l + 1 \ldots m + 1, p; (p + l + 1)' \ldots (m + 1)', (m + 2)^+] \]

\[ (A.5c) \]

\[ (V) = -i \int d(m + 2) \hat{V}(1, m + 2) \hat{A} \hat{P}(1', \ldots, (m + 1)') \hat{S} \hat{P}(2, \ldots, (m + 1)') \sum_{p=2}^{m+1} \]

\[ G^{(p)}(1, \ldots, (p-1), (m + 2); 1', \ldots, (p-1)', (m + 2)) \frac{1}{(p + 1) \ldots (p + l - 1), (m + 2); (p + 1)' \ldots (p + l)'} \times \]

\[ G^{(m+2-p-l)} - G^{(m+2-p-l)'} [p + l + 1 \ldots m + 1, p; (p + l + 1)' \ldots (m + 1)', (m + 2)^+] \]

\[ (A.5d) \]

Now we proceed to sum up all the unlinked terms. They are the second term of eq. (A.3), the second term of eq. (A.5b), and the first term of eq. (A.5c,d). The sum of the four terms amounts to

\[ i \int d(m + 2) \hat{V}(1, m + 2) [G^{(m+2)} - G^{(m+2)'}] [1, \ldots, (m + 1), (m + 2)^+] \]

which exactly cancels the unlinked terms containing in \( G^{(m+2)} \) of the first term of eq. (A.3). Thus only the linked terms are left in the r.h.s. of eq. (A.3). We finally obtain

\[ [i \hat{t}_1 - i(1)G^{(m+1)}] [1, \ldots, (m + 1), 1', \ldots, (m + 1)'] = -i \int d(m + 2) \hat{V}(1, m + 2) \times \]

\[ \left\{ G^{(m+2)}(1, \ldots, (m + 1), (m + 2); 1', \ldots, (m + 1)', (m + 2)^+) \right\} \]

\[ + \hat{A} \hat{P}(1', \ldots, (m + 1)') \hat{S} \hat{P}(2, \ldots, (m + 1)') \hat{P}(1', \ldots, (m + 1)') \]

\[ \left[ \sum_{p=1}^{m+1} G^{(p)}(1, \ldots, (p-1), (m + 2)'; 1', \ldots, (p-1)', (m + 2)^+) \right] \]

\[ - \sum_{p=1}^{m+1} G^{(p)}(1, \ldots, (p-1)', (m + 2)^+) G^{(m+2-p)}(1, \ldots, (p+1)', (m + 2)^+) \]

\[ (A.6) \]
\[ - \sum_{p=2}^{m+1} G_{c}^{(p)}(1..(p-1),(m+2);1'..p')G_{c}^{(m+2-p)}((p+1)..(m+1),p;(p+1)'..(m+2)^+) \]
\[ - \sum_{p=1}^{m} \sum_{l=1}^{m-1} G_{c}^{(p)}(1..p;1'..p')G_{c}^{(l)}((p+1)..(p+l-1),(m+2);(p+1)'..(p+l)^+) \]
\[ \times G_{c}^{(m+2-p-l)}((p+l+1)..(m+1),(p+l);(p+l+1)'..(m+1)^+,(m+2)^+) \} \]

(A.7)

This is exactly the equation (2.21) for \( n = m + 1 \) and means that it is valid for \( 2 \leq n \leq m + 1 \).

3. From (1) and (2) we have succeeded in proof by induction.

The key point of the above proof is to confirm a complete cancellation of unlinked terms in eq.(2.21).

### B Proof of Equation (3.4)

Equation (3.4) can be proved also by induction. Let \( \hat{T}(\alpha)(\alpha = (pq), (p'q'), (pq')) \) be a time-ordering operator such that

\[
\begin{align*}
\hat{T}(p'q') &< \ldots \psi^\dagger(p') \ldots \psi^\dagger(q') \ldots > = - \ldots \psi^\dagger(q') \ldots \psi^\dagger(p') \ldots >, \quad \text{(B.1a)} \\
\hat{T}(pq) &< \ldots \psi(p) \ldots \psi(q) \ldots > = - \ldots \psi(q) \ldots \psi(p) \ldots >, \quad \text{(B.1b)} \\
\hat{T}(pq') &< \ldots \psi^\dagger(q') \ldots \psi(p) \ldots > = - \ldots \psi(p) \ldots \psi^\dagger(q') \ldots >. \quad \text{(B.1c)}
\end{align*}
\]

In what follows we shall refer \( p \) and \( q \) as \( \psi(p) \) and \( \psi(q) \), and \( p' \) and \( q' \) as \( \psi^\dagger(p') \) and \( \psi^\dagger(q') \). Now introduce the normal time ordering green's function \( G^{(m)}_{c,N} \) which is defined as the average of normal product of field operators,

\[( -i )^m G^{(m)}_{c,N}(1..m;1'..m') = < \psi^\dagger(1') \ldots \psi^\dagger(m') \psi(m) \ldots \psi(1) >. \quad \text{(B.2)} \]

Any time ordering green's function \( G^{(m)}_{c,aN} \) can be obtained by operating \( T(\alpha)T(\beta) \ldots T(\gamma) \) on \( G^{(m)}_{c,N} \),

\[ G^{(m)}_{c,aN}(1..m;1'..m') = T(\alpha)T(\beta) \ldots T(\gamma)G^{(m)}_{c,N}(1..m;1'..m'). \quad \text{(B.3)} \]

Therefore one needs only to study the effect of \( \hat{T}(\alpha) \). Since

\[ \hat{T}(pq)G^{(m)}_{c,N} = \hat{T}(p'q')G^{(m)}_{c,N} = G^{(m)}_{c,N}, \quad \text{(B.4)} \]

one concludes that

\[ \hat{T}(pq)G^{(m)}_{c,N} = \hat{T}(p'q')G^{(m)}_{c,N} = G^{(m)}_{c,N}. \quad \text{(B.5)} \]

Thus only \( \hat{T}(pq') \) needs to be investigated.

1. Firstly we examine the effect of \( \hat{T}(pq') \) on \( G^{(1)}_{c,N} = G^{N} \) and \( G^{(2)}_{c,N} \). Let us use \( ETL(\text{Equal Time Limit}) \), \( NETL(\text{Normal Equal Time Limit}) \), \( ANETL(\text{Anti-Equal Time Limit}) \), and \( any-ETL(\text{any Equal Time Limit}) \) to denote different equal time limits. For \( n = 1 \), from eqs.(3.3a,b) we have

\[ \hat{T}(pq')G^{N}(p;q')_{ETL} = G(p;q')_{ANETL} = G(p;q')_{NETL} - i\delta^{(3)}(p,q'). \quad \text{(B.6)} \]

For \( n = 2 \), we calculate

\[ \hat{T}(pp')( -i)^2 G^{(2)}_{c,N}(pq;p'q')_{ETL} = \hat{T}(pp') < \psi^\dagger(p') \psi^\dagger(q') \psi(q) \psi(p) >_{ETL} \]
\[ = - < \psi(p) \psi^\dagger(q') \psi^\dagger(q) \psi(p') >_{ETL} \]
\[ = ( -i )^2 G^{(2)}_{c,N}(pq;p'q')_{ETL} - \delta^{(3)}(p,q')\delta^{(3)}(q,p') - i\delta^{(3)}(p,q')G(q;p')_{NETL} \]
\[ - i\delta^{(3)}(q,p')G(p;q')_{NETL} + i\delta^{(3)}(p,p')G(q;q')_{NETL}. \quad \text{(B.7)} \]

18
From eq. (2.14) we have, for $n = 2$,

$$G^{(2)}(pq; p'q') = G^{(2)}(pq; p'q') + G^{(2)}(p; q')G^{(2)}(q; q') - G^{(2)}(p; q')G^{(2)}(q; q'),$$  \hspace{1cm} (B.8)

and

$$G^{(2)}(pq; p'q')NETL = G^{(2)}(pq; p'q')NETL + G(p; p')NETL G(q; q')NETL - G(p; q')NETL G(q; q')NETL.$$  \hspace{1cm} (B.9)

Applying $\hat{T}(pp')$ on eq. (B.8), we have

$$\hat{T}(pp')G^{(2)}(pq; p'q')ETL = \hat{T}(pp')G^{(2)}(pq; p'q')ETL + \hat{T}(pp')G^{(2)}(p; q')ETL G(q; q')ETL - \hat{T}(pp')G^{(2)}(p; q')ETL G(q; q')ETL.$$  \hspace{1cm} (B.10)

Noticing that $\hat{T}(pp')G^{(2)}(p; p')ETL, \hat{T}(pp')G^{(2)}(p; p')ETL$ and $\hat{T}(pp')G^{(2)}(p; p')ETL$ are in anti-equal time limit, from eq. (B.6) we thus have,

$$\hat{T}(pp')G^{(2)}(pq; p'q')ETL = \hat{T}(pp')G^{(2)}(pq; p'q')ETL + \delta^{(3)}(p, p')G(q; q')NETL + \delta^{(3)}(p, q')G(p; q')NETL + \delta^{(3)}(q, q')\delta^{(3)}(q, q').$$  \hspace{1cm} (B.11)

Comparing eqs. (B.7) and (B.11) we obtain

$$\hat{T}(pp')G^{(2)}(pq; p'q')ETL = G^{(2)}(pq; p'q')NETL.$$  \hspace{1cm} (B.12a)

Similarly one has

$$\hat{T}(pp')G^{(2)}(pq; p'q')ETL = \hat{T}(pq')G^{(2)}(pq; p'q')ETL = \hat{T}(pp')G^{(2)}(pq; p'q')ETL = G^{(2)}(pq; p'q')NETL.$$  \hspace{1cm} (B.12b)

From eqs. (B.3), (B.5) and (B.12a,b) we have proved that

$$[G^{(2)}_e]_{\text{anti-ETL}} = [\hat{T}(\alpha)\hat{T}(\beta)\cdots\hat{T}(\gamma)G^{(2)}_e]_{\text{ETL}} = [G^{(2)}_e]_{\text{NETL}} = i^2C_2.$$  \hspace{1cm} (B.13)

2. Now we turn to the general case and proceed to prove by induction. Assume eq. (3.4) is valid for $2 \leq n \leq m-1$, namely

$$[i^nG^{(n)}_e]_{\text{anti-ETL}} = [i^nG^{(n)}_e]_{\text{NETL}} = (-1)^nC_n, (2 \leq n \leq m-1).$$  \hspace{1cm} (B.14)

We shall prove that it is valid for $2 \leq n \leq m$. Eq. (B.14) means that

$$[\hat{T}(pq')]i^nG^{(n)}_e]_{\text{ETL}} = [i^nG^{(n)}_e]_{\text{NETL}} = (-1)^nC_n, (2 \leq n \leq m-1).$$  \hspace{1cm} (B.15)

Since the separation equation (2.14) is a recursive relation, we can rewrite it as

$$G^{(m)}_e = G^{(m)} - \hat{A}p\hat{S}p \sum_{k+l+\cdots+r+t=m} [G^{(k)}_eG^{(i)}G^{(r)}G^{(s)}G^{(t)}].$$  \hspace{1cm} (B.16)

From eq. (B.16) we have

$$[(-i)^m\hat{T}(pq')G^{(m)}]_{\text{ETL}} = [(\hat{T}(pq')G^{(m)}N]_{\text{ETL}}, (I)$$

$$-(-i)^m [\hat{T}(pq')\hat{A}p\hat{S}p \sum_{k+l+\cdots+r+t=m} (G^{(k)}_eG^{(i)}N\cdotsG^{(r)}N\cdotsG^{(s)}N\cdotsG^{(t)}N)]_{\text{ETL}}, (II).$$  \hspace{1cm} (B.17)
The first term of the r.h.s. of eq.(B.17) can be calculated directly,

\[ (I) = [-i]^m \hat{T}(pq') G^{(m)} (l..m; 1'..m') \text{ETL} \]
\[ = [\hat{T}(pq') < \psi^i (1')..\psi^i(q')..\psi^i(m')\psi(m)\psi(p) \psi(1) > \text{ETL} \]
\[ = - < \psi^i (1')..\psi(p)\psi(m')\psi(m)\psi(q')..\psi(1) > \text{ETL} \]
\[ = < \psi^i (1')..\psi(q')\psi^i(m')\psi(m)\psi(p) \psi(1) > \text{ETL} \]
\[ - \delta^{(3)}(p, q')( -1)^{p+q'} < \psi^i (1')..\tilde{q}'..\psi^i(m')\psi(m)\tilde{p}..\psi(1) > \text{ETL} \]
\[ - \sum_{j=p+1}^{m} \delta^{(3)}(j, q')( -1)^{-j} < \psi^i (1')..\tilde{q}'..\psi^i(m')\psi(m)\tilde{j}..\psi(1) > \text{ETL} \]
\[ - \sum_{i'=q'+1}^{m'} \delta^{(3)}(p, i')( -1)^{-i'} < \psi^i (1')..\tilde{i}'..\psi^i(m')\psi(m)\tilde{p}..\psi(1) > \text{ETL} \]
\[ - \sum_{i'=q'+1}^{m'} \sum_{j=p+1}^{m} ( -1)^{j+i'-p} \delta^{(3)}(p, i') \delta^{(3)}(j, q') < \psi^i (1')..\tilde{i}'..\tilde{j}..\psi^i(m')\psi(m)\tilde{p}..\psi(1) > \text{ETL} \]
\[ = (-i)^m G^{(m)} (1..m; 1'..m') \text{NETL} + (-i)^m \Delta G^{(m)} \text{ETL}. \]  

where \( \tilde{p}, \tilde{q}', \tilde{j} \) and \( \tilde{i}' \) indicate that they do not appear in the related green's functions and \( \Delta G^{(m)} \text{ETL} \) is

\[ \Delta G^{(m)}_{\text{NETL}} = -i \delta^{(3)}(p, q')( -1)^{p+q'} G^{(m-1)} (1..\tilde{p}..m; 1'..\tilde{q}'..m') \text{NETL} \]
\[ - i \sum_{j=p+1}^{m} \delta^{(3)}(j, q')( -1)^{-j} G^{(m-1)} (1..\tilde{j}..m; 1'..\tilde{q}'..m') \text{NETL} \]
\[ - i \sum_{i'=q'+1}^{m'} \delta^{(3)}(p, i')( -1)^{-i'} G^{(m-1)} (1..\tilde{i}'..m; 1'..\tilde{q}'..m') \text{NETL} \]
\[ + \sum_{i'=q'+1}^{m'} \sum_{j=p+1}^{m} \delta^{(3)}(p, i') \delta^{(3)}(j, q')( -1)^{j+i'-p-q'} G^{(m-2)} (1..\tilde{i}'..\tilde{j}..m; 1'..\tilde{i}'..\tilde{q}'..m') \text{NETL}. \]  

The calculation of the second term of eq.(B.17) is cumbersome. It reads

\[ (II) = (-i)^m \hat{T}(pq')(( -1)^{p+q'} G^N (p; q') \Delta p \Delta \tilde{S} \tilde{p} \sum_{l_1+..+r_1+s_1+t_1=m-1} [G^{(l_1)} e_G^{(r_1)} e_G^{(s_1)} G^{(t_1)}](1..\tilde{p}..m; 1'..\tilde{q}'..m') \]
\[ + \sum_{j=1}^{m} ( -1)^{-j} G^N (j; q') \Delta p \Delta \tilde{S} \tilde{p} \sum_{l_1+..+r_1+s_1+t_1=m-1} [\text{terms where } p \text{ is not contained in } G^N ](1..\tilde{p}..j..m; 1'..\tilde{q}'..m') \]
\[ + \sum_{i'=1'}^{m'} ( -1)^{-i'} G^N (p; i') \Delta p \Delta \tilde{S} \tilde{p} \sum_{l_1+..+r_1+s_1+t_1=m-1} [\text{terms where } q' \text{ is not contained in } G^N ](1..\tilde{p}..m; 1'.q'.\tilde{q}'.m') \]
\[ - \sum_{i'=1'}^{m'} \sum_{j=1}^{m} ( -1)^{j+i'-p-q'} G^N (j; q') G^N (p; i') \Delta p \Delta \tilde{S} \tilde{p} \sum_{r_1+..+s_1+t_1=m-2} [..G^{(r_1)} e_G^{(s_1)} G^{(t_1)}](1..\tilde{p}..j..m; 1'.\tilde{q}'..\tilde{i}'..m') \]
\[ + \text{other terms where } p, q' \text{ are not contained in } G^N \text{ETL}. \]  

Since \( \hat{T}(pq') \) causes anti-equal time limit, one should consider its effect term by term. By using eqs.(B.6) and (B.15), we have

\[ \hat{T}(pq') G^N (p; q') \text{ETL} = G(p; q') \text{NETL} - i \delta^{(3)}(p, q'), \]  

(B.21a)
\[ \hat{T}(pq')G^N(j; q') = G(j; q')NETL, (j \leq p), \]
\[ = G(j; q')NETL - i\delta^{(3)}(j, q'), (j > p), \]  
\hspace{1cm} (B.21b) 

\[ \hat{T}(pq')G^N(p; i') = G(p; i')NETL, (i' \leq q'), \]
\[ = G(p; i') - i\delta^{(3)}(p; i'), (i' > q'), \]  
\hspace{1cm} (B.21c) 

\[ \hat{T}(pq')G^{(l)}_{\varepsilon}(...p...; ...q') = [G^{(l)}_{\varepsilon}]_{NETL}, (2 \leq l \leq m - 1). \]
\hspace{1cm} (B.21d) 

Inserting the above results in eq. (B.20), we have

\[ (II) = \sum (-1)^{\mu'} \hat{T}(j; q') \hat{A}_p \hat{S}_p \sum [G^{(l)}_{\varepsilon}...G^{(r)}_{\varepsilon}...G^{(s)}_{\varepsilon}...G^{(t)}_{\varepsilon}]^{(m-1)}(1...p...m...; 1...q...m') \]
\[ + \sum (-1)^{\mu'} G(j; q') \hat{A}_p \hat{S}_p \sum [G^{(l)}_{\varepsilon}...G^{(r)}_{\varepsilon}...G^{(s)}_{\varepsilon}...G^{(t)}_{\varepsilon}]^{(m-1)}(1...j...m...; 1...q...m') \]
\[ + \sum (-1)^{\mu'} \hat{A}_p \hat{S}_p \sum [G^{(l)}_{\varepsilon}...G^{(r)}_{\varepsilon}...G^{(s)}_{\varepsilon}...G^{(t)}_{\varepsilon}]^{(m-1)}(1...\tilde{p}...m...; 1...\tilde{q}...m') \]
\[ + \sum \sum (-1)^{\mu'} \delta^{(3)}(j, q') G(j; q') \hat{A}_p \hat{S}_p \sum [G^{(l)}_{\varepsilon}...G^{(r)}_{\varepsilon}...G^{(s)}_{\varepsilon}...G^{(t)}_{\varepsilon}]^{(m-2)}(1...\tilde{p}...j...m...; 1...\tilde{q}...\tilde{p}...m') \]
\[ + \text{other terms where } p, q' \text{ are not contained in } G \]
\[ \text{(the above is unlinked terms of } G^{(m)}_{NETL} \text{)} \]
\[ -i \sum \sum \sum (-1)^{\mu'} \delta^{(3)}(p, i') \hat{A}_p \hat{S}_p \sum [G^{(l)}_{\varepsilon}...G^{(r)}_{\varepsilon}...G^{(s)}_{\varepsilon}...G^{(t)}_{\varepsilon}]^{(m-2)}(1...\tilde{p}...\tilde{p}...m...; 1...\tilde{q}...\tilde{p}...m') \]
\[ \text{(the above terms amount to } \Delta G^{(m)}_{NETL}) \]
\[ = (-i)^m [G^{(m)}_{\varepsilon}(1...m...; 1...m') - G^{(m)}_{\varepsilon}(1...m; 1...m')]_{NETL} + (-i)^m \Delta G^{(m)}_{NETL}. \]
\hspace{1cm} (B.22) 

Comparing eqs. (B.18) and (B.22), we obtain

\[ [\hat{T}(pq')G^{(m)}_{\varepsilon}]_{NETL} = [G^{(m)}_{\varepsilon}]_{NETL} = (i)^m C_m \]  
\hspace{1cm} (B.23) 

Considering eqs. (B.3), (B.5) and (B.23), we succeed in proving

\[ [G^{(m)}_{\varepsilon}]_{any-ETL} = [\hat{T}(\alpha)\hat{T}(\beta...\hat{T}(\gamma)G^{(m)}_{\varepsilon}]_{NETL} = [G^{(m)}_{\varepsilon}]_{NETL} = (i)^m C_m. \]  
\hspace{1cm} (B.24) 

3. From (1) and (2) we have proved eq. (3.4).
It is worth to mention that the above proof is equivalent to prove the consistence of two kinds of definition of $G^{(n)}$. The first definition is given by the first kind of separation equation (2.14). The second one can be given as follows: (a) First transform the T-product to the N-product by Wick's theorem. (b) Then apply the separation equation to the averages of N-products (the normal green's function $G(N)^{(n)}$) and give the second definition of $G^{(n)}_e$. This appendix tells that all the contractions in Wick's expansion can be absorbed by the one-body normal green's function $G^N$. After absorption of the contraction, $G^N$ turns to be $G$, and the second kind of separation equation becomes the first kind. Thus $G^{(n)}_e$ and $G^{(n)}_e$ are equivalent.
References


Caption:

Fig. A.a  Illustration of eq. (A.4a)

Fig. A.b  Illustration of eq. (A.4b)

Fig. A.c  Illustration of eq. (A.4c)
(Fig A. a.)

(Fig A. b.)

(Fig A. c.)