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Publication Date
2008

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UNIVERSITY OF CALIFORNIA, SAN DIEGO

On Physical Carrier Sensing for Random Wireless Ad-hoc Networks

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy

in

Electrical Engineering

(Communication Theory and Systems)

by

Chiu Ngok Eric Wong

Committee in charge:
Professor Rene L. Cruz, Chair
Professor Patrick J. Fitzsimmons
Professor Massimo Franceschetti
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Professor Alex C. Snoeren

2008
The dissertation of Chiu Ngok Eric Wong is approved, and it is acceptable in quality and form for publication on microfilm:

Chair

University of California, San Diego

2008
To my parents and Ju-Pei

In memory of my grandmother
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ACKNOWLEDGMENTS

I am truly indebted to my advisor, Professor Rene Cruz for his patience, encouragement and insightful suggestions throughout my graduate studies. None of my dissertation research would have been possible without his wisdom and guidance. I would also like to thank Professor Massimo Franceschetti for his valuable discussions on parts of my work.

I am also grateful to Professor Patrick Fitzsimmons, Professor Ramesh Rao, and Professor Alex Snoeren for serving on my doctoral committee.

I must thank my good friend Adrian, for always being there for me, as well as Yoav for his constant encouragement. Thanks also go to those whom I am fortunate to be acquainted with along the way, in particular to Andy and Foo-Chung. I would also like to thank Lee Wee, Alan, Kim Yaw, Kitt Onn, Yeow Teck, Yong Soon and Andrew for many years of enduring friendship and support.

No amount of words can express my gratitude to my parents and my brother Edmund for their understanding, encouragement and unflinching support. Lastly, to my dear wife Ju-Wei, for her patience, kindness and unwavering love. I attributed everything I have achieved to them.

The text of Chapter Two in part, is a reprint of material as it appears “A Spatio-Temporal Model for Physical Carrier Sensing Wireless Ad-Hoc Networks’,” in the Proceedings of the IEEE Communication Society Conference on Sensor, Mesh and Ad-hoc Communications and Networks (SECON), 2006, E.C. Wong; R.
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The text of Chapter Four in part, is a reprint of material as it appears “On Physical Carrier Sensing for Cognitive Radio Networks,” in the Proceedings of the *Allerton Conference on Communication, Control and Computing*, 2007, E. C. Wong; R. L. Cruz. The dissertation author was the primary investigator and author of this paper.
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ABSTRACT OF THE DISSERTATION

On Physical Carrier Sensing for Random Wireless Ad-hoc Networks

by

Chiu Ngok Eric Wong

Doctor of Philosophy in Electrical Engineering

(Communication Theory and Systems)

University of California, San Diego, 2008

Professor Rene L. Cruz, Chair

The popularity of wireless access will cause wireless ad-hoc networks to operate with higher node densities and increased levels of interference. Interference mitigation is therefore crucial in ensuring these networks operate efficiently. Often the lack of network planning and regulations for such networks require the targeted access strategy to be adaptive to network conditions and distributed.

In this dissertation, we develop analytical models for a random ad-hoc wireless network using physical carrier sensing. The interference at any node aggregates contributions from all concurrent transmissions on the infinite plane, and
is modeled as a shot noise process. A fixed-point equation for finding the intensity of actual transmission is formulated. Subsequently, the interference at the intended destination is defined as a conditional shot noise process given that the source transmits a packet. This modeled the inhibitory effect of nearby neighbors. Subsequently, we show that our model matches reasonably well with simulations.

We show that network throughput is a function of the ratio of idle threshold to transmit power, as well as the ratio of packet detection threshold to transmit power. Transmit power is in fact a scaling factor for network throughput, i.e. we need not consider transmit power as an independent parameter to be optimized. Hence, we focus on the joint adaptation of idle and packet detection threshold in order to maximize network throughput. We found that the optimal target SINR is much smaller than what is currently used in practice. We also found the optimal success probability is smaller than one, which is in contrast to commonly made assumptions in the literature. Both conditions are dependent on the intensity of transmission attempts. We develop a heuristic to jointly adapt idle and packet detection threshold, and an algorithm that results in maximum network throughput.

Physical carrier sensing is shown to be an effective medium access strategy for a secondary network, in the context of a cognitive radio network scenario. With simple enhancements, the proposed two-threshold protocol accounts for the degradation on each primary device due to the aggregate interference from secondary network. Using this, we show spectral efficiency can be significantly improved.
Chapter 1

Introduction

The concept of carrier sensing multiple access (CSMA) was first proposed in 1975 by Kleinrock and Tobagi [1] as an enhancement to the simple ALOHA protocol. The key distinguishing feature of the CSMA protocol is to listen to the medium prior to each packet transmission to avoid unnecessary packet failures. CSMA with collision detection was adopted for the IEEE 802.3 medium access control (MAC) standards for wired local area networks (LAN). More recently, CSMA with collision avoidance is the de facto MAC protocol for the IEEE 802.11 wireless LAN standards [2], which is now in widespread use for broadband access.

A network of IEEE 802.11-enabled devices typically shares a common radio spectrum. Consequently, packet transmissions will overlap in time, and may fail due to excessive interference. To further aggravate the problem, the popularity of IEEE 802.11 requires the network to accommodate a larger number of devices within the same area. Managing the aggregate interference from concurrent trans-
missions is therefore critical to overall network performance. The CSMA protocol
prevents close proximity transmissions, as well as regulates the number of eligi-
ble concurrent transmissions. For these reasons, CSMA is well-suited for wireless
networks.

The choice of CSMA protocol parameters affects each device’s eligibility
for packet transmission, which in turn affects the overall network performance. For
instance, the clear channel assessment described in the IEEE 802.11 standards, is
a mechanism for transmission eligibility. It uses the energy detection threshold,
which is a CSMA protocol parameter, to decide the state of the medium. However,
this threshold is a fixed number specified in the standards [2], irregardless of net-
work conditions. Rather, should be this threshold be adapted relative to current
network conditions? If so, what effects does a change in this threshold have on
other protocol parameters?

1.1 Carrier Sensing in IEEE 802.11

IEEE 802.11 MAC uses both physical and virtual carrier sensing mech-
anisms at the physical and MAC layers respectively, to determine eligibility for
packet transmission. A device is eligible if both mechanisms report an idle medium
simultaneously. Otherwise, packet transmission is prohibited. A good description
of IEEE 802.11 MAC protocol can be found in [3].

With physical carrier sensing, a busy medium is reported if the measured
energy is above the energy detection threshold, and/or the presence of IEEE 802.11
packets are detected. Otherwise, an idle medium at the physical layer is reported.
The energy detection threshold, which we refer as the *idle* threshold, limits the number of concurrent transmissions in the network; in turn, this threshold also regulates the overall level of interference in the system. Since the interference at the source and destination devices may be uncorrelated, transmitted packets may still fail at its intended destination. This is analogous to the *hidden terminal* problem highlighted in [4].

Virtual carrier sensing, on the other hand, consists of a four-way handshake between the source and destination devices for each data packet transmitted. This is also known in the literature as MACAW, which was proposed by Bharghavan et al. [5]. A source device with a packet first sends a Request-To-Send (RTS). If the destination device is ready for packet reception, it returns a Clear-To-Send (CTS). This is followed subsequently by the data packet and acknowledgment. Embedded within the messages is the network allocation vector (NAV), which indicate the amount of time requested by the source device to “reserve” the medium spatially. All IEEE 802.11-enabled devices within reception vicinity are required to update their NAV, and devices with non-zero NAV will report a busy medium.

In addition to hidden terminals, the performance of CSMA is also affected by exposed terminals. First, hidden terminals are devices unable to hear the source’s transmission, but yet close enough to the destination to cause failures. The four-way handshake proposed in MACAW addresses this issue. But, it is highlighted in [6] that the RTS/CTS handshake could lead to false blocking and
congestion in the network. In an ad-hoc network, Xu et al. [7] concluded the RTS/CTS mechanism by itself is inadequate against interfering devices unable to hear the RTS or CTS, as these devices may use large transmission powers. Exposed terminals, on the other hand, are devices prevented from transmission despite the fact that they do not interfere with the source’s transmission. Mittal and Belding [8] proposed using Request-To-Send-Simultaneously (RTSS) and Clear-To-Send-Simultaneously (CTSS) messages to synchronize transmissions between the source and exposed terminals, which require the identification of the hidden terminals. An interesting protocol Dual Busy Tone Multiple Access (DBTMA) proposed by Deng and Haas, addresses the hidden and exposed terminal problems together using two out-of-band busy tones; one for transmission and reception. Prior to sending a RTS, the source ensures no nearby device is receiving a packet by listening to the busy tone. Likewise, before sending a CTS, the destination checks the busy tone to ensure no other nearby device is sending a packet. Simulation results were also reported showing a doubling of network utilization with the DBTMA against MACAW.

Most proposals in the literature addressing the hidden and exposed terminal problems in CSMA, typically require additional control messaging or signaling. For instance, in the case of DBTMA, additional bandwidth for the two busy tones, as well as hardware modifications to devices in the network are needed. In addition, the use of RTS/CTS messaging is needed. Consequently, performance gain is at the cost of spectral efficiency and delay.
Besides RTS/CTS, physical carrier sensing may also be used to avoid hidden terminals. This can be achieved by using a suitably small idle threshold, which will ensure that the interfering devices will be far enough so as not to interfere with the source’s transmission. Moreover, the outcome of the physical carrier sensing mechanism is based on the aggregate interference from all concurrent transmissions in the network. It can be adapted to regulate the level of spatial reuse desired in the network through the appropriate choice of idle thresholds. In addition, physical carrier sensing can operate in a decentralized manner, which makes it suitable for use in an ad-hoc network environment.

In this dissertation, we consider a wireless ad-hoc network using physical carrier sensing with energy detection. We develop an analytic model and study how the protocol parameters should be set and their effects on overall network throughput. Next, we identify which of these parameters should be adapted independently or jointly. Subsequently, we propose a heuristic to perform suitable adaptation on these protocol parameters.

Past approaches [9] [10] [11] [12] [13] to modeling physical carrier sensing wireless networks involve translating the carrier sense threshold to ranges. No other transmission is allowed within the carrier range irregardless of its distance to the transmitting node. The goal then is to find the optimal carrier sense range. This approach does not account for the interference due to the other concurrent transmissions outside the carrier sense range, nor does it account for the dynamics in the topology. Similarly in the past approaches, to determine the success of
a packet transmission, authors either consider the closest interferer (based on an interference range) or the six closest interferers spaced according to a honeycombed topology, commonly used in cellular networks. It is however conceivable that the aggregate interference due to concurrent transmission in the rest of the network is adequate to cause the source device to report a busy medium, or even worse cause the packet failures instead at its intended destination. Because of these reasons, we consider other approaches which can account for all the concurrent transmissions in the network, as well as adopt a gradual inhibitory model.

In our approach [14], we consider a network with devices distributed randomly over an infinite plane. The eligibility of each device is based on the aggregate interference from all concurrent transmissions. In fact, we can define the observed interference as a shot noise process. With this, we can model the joint statistics between the source device and its intended destination. This is in contrast to the common approach of assuming the interference at these two points are the same. Subsequently, we are able to define the conditional interference at the intended destination given that a packet is transmitted at the source. In doing so, we account for the inhibitory effect on neighboring devices around the source.

With the increased popularity of IEEE 802.11, physical carrier sensing is a simple, yet effective mechanism to mitigate interference. It is also a suitable way to control the level of spatial reuse in the network, which is becoming increasingly important due to the higher node densities in today’s systems. More importantly, physical carrier sensing can be implemented in a distributed manner, therefore it
is scalable to larger and denser networks.

1.2 Thesis Organization

In Chapter Two, we develop spatio-temporal models for a wireless networks that uses physical carrier sensing. Specifically, we account for transmit power, packet duration, average distance to the intended destination, attempt intensity, idle threshold and packet transmission rate. We consider a network on an infinite plane, and propose an interference model that accounts for all the concurrent transmissions in the network at a given time. We develop a fixed-point equation whose solution yields the intensity of actual transmissions. When a device starts a packet transmission, we capture the inhibitory effect on the neighboring devices using a threshold based model. Subsequently, we account for this inhibitory effect when we are computing the success probability at the intended destination. We show that our model, even with many simplifying assumptions, matches the simulations well. Finally, we point out that with careful protocol adaptation, significant improvements in network throughput can be achieved.

In Chapter Three, we show that the network throughput is a function of the ratio of idle threshold to transmit power, as well as the ratio of packet detection threshold to transmit power. Transmit power plays the role of a scaling factor. Using this property, we study the joint adaptation of idle and packet detection threshold. Based on extensive simulations, we show that the success probability when network throughput is maximized is high, but not necessarily
one. In fact, the success probability is dependent on the intensity of transmission attempts. Building on this insight, we develop a heuristic to relate idle and packet detection threshold, which is used for our joint adaptation. We also show that the optimal SINR is much smaller that what is currently used in practice. Finally, we propose an algorithm to achieve optimal network throughput operation.

In Chapter Four, we propose an architecture for a secondary network whose devices use physical carrier sensing and share the spectrum with a passive primary network. We show with a simple modification to the physical carrier sensing mechanism, the secondary network can successful exploit the excess bandwidth resources without causing significant degradation to the devices in the primary network. We also account for the aggregate effect from all concurrent transmitting secondary users on each primary device. The resulting solution is a two-threshold based access mechanism, which can be implemented distributedly and does not require any modifications to the primary devices.

At last, in Chapter Five, we provide a summary of our results, as well as mention some interesting directions for future work.
Chapter 2

Modeling Physical Carrier Sensing Networks

2.1 Introduction

Despite the popularity of CSMA in radio networks, there is only limited understanding of how protocol parameters should be set to optimize performance. In particular, the CSMA protocol for radio networks was based on the CSMA protocol for local wired networks where spatial reuse need not be aggressively sought. Consequently, the CSMA mechanism was originally designed so that with high probability concurrent transmissions in close proximity are avoided. In situations with aggressive spatial reuse, a high degree of concurrent transmissions may be desirable, so the question of an appropriate definition of an idle becomes an important one, in order to maximize network throughput.
2.2 Related Work

In order to address this problem, several models with simplifying assumptions have been used in the literature. In [9] [10] [11] [12] [13], the authors adopt heavy use of carrier sensing range in their analysis. The carrier sensing range is the farthest distance a node will detect an existing transmission and is related to the carrier sense threshold. Implicitly, this translates to a total block out and no other sources are allowed to transmit within the carrier sensing region. In general, the amount of the inhibitory effect depends on how far a node is from an existing transmission. A nearby neighbor is less likely to transmit than one farther away. Thus this motivates the need for an inhibiting model based on a threshold on the interference level rather than on a carrier sense region.

In [9] [10] [13], the authors assume the interference is only coming from a single source. Yang et al. [11] assume a fixed regular topology and consider only the six closest interfering sources. Zhai et al. [12] consider the six closest sources for a fixed regular topology and only the nearest source for a random topology. In general, the interference at any node could be from an infinite number of sources in a network with spatial reuse on an infinite plane. This motivates our Gaussian approximation on the interference process, which we will use heavily in this dissertation.

The authors in [15] based their analysis on carrier sense regions for a random topology. They use the Matern hard core process to compute the carrier sense regions while ensuring the sources are at least the carrier sense range apart.
Likewise, a total block out is assumed within each carrier sense region.

In [16], Fuemmeler et al. adopt the carrier sense model based on a threshold and assume the interference is coming from the $k$ worst case sources. The authors also assume the statistics of the interference at the transmitting node and its intended destination is similar, which they refer as the colocation approximation. A similar approximation is also made in [9]. We conjecture the correlation between the statistics of the interference at two different nodes will only be small for large transmission ranges. In other words, the interference statistics at the transmitter and receiver are only almost independent for large transmission ranges.

In general, there is a need for greater insight into the performance of CSMA network, so that protocol parameters, or even the protocol itself, can be adjusted in order to optimize network throughput. We discuss some of these parameters below.

The IEEE 802.11 wireless LAN standard uses both physical and virtual carrier sensing to determine the state of the medium. The latter, implemented in the form of an RTS/CTS handshake, addresses the hidden terminal problem. The focus of this dissertation lies only on physical carrier sensing and we define an idle threshold. If the signal power measured at a node falls below this threshold, the state of the medium is \textit{idle}. Otherwise, the state of the medium is \textit{busy}. This is analogous to the carrier sensing threshold used in the literature. The idle threshold is a protocol parameter of interest in optimizing the network throughput. If a high idle threshold is set, nodes access the medium liberally and the number
of transmissions increase. This, in turn, increases the number of collisions and can impact network throughput negatively.

Node density and retransmission rate are parameters that have a direct influence on network throughput. Consider a homogeneous network, where every node uses the same idle threshold. If node density and retransmission rate are low, the number of attempts to access the medium will be small. A smaller number of transmissions is expected in this case. If physical carrier sensing is available, nodes access the medium more conservatively and results in the network operating suboptimally.

Packet duration and transmission power impact the extent a transmitted packet has on the rest of the network. A packet transmitted using a larger power will have an impact on nodes farther away. The same analogy can be made for packet duration. In this dissertation, the optimization of packet duration and transmission power in order to maximize network throughput is not considered. However, they can be easily incorporated in future work.

We develop a model for CSMA networks with spatial reuse that accounts for physical layer parameters in the next section. In section III, we use the Poisson approximation to develop the model further to obtain estimates of performance. In the subsequent sections after that, we discuss results of Monte Carlo simulations, which suggest our simple model is reasonably accurate, and can be used for parameter optimization. In the last section of this chapter, we discuss the optimization of the protocol parameters.
2.3 Model

We consider a stationary three dimensional Poisson point process $N_a$ on $\mathbb{R}^3$ with mean intensity $\lambda_a$ per unit area and unit time. The points on this process denote the location of a node in $\mathbb{R}^2$ and the time it attempts to access the medium in $\mathbb{R}$. We assume the transmission attempts have the statistics of a Poisson point process. These attempts include new and retransmitted packets (packets which were transmitted but not successfully received by the intended destination), and also packets from previous unsuccessful attempts. An attempt is unsuccessful if the transmitting node senses the medium as busy. The node then backs off exponentially, moves to another random location and reattempts to access the medium later. Each node is assumed to adopt a random mobility model. The node moves randomly to another position once it completes sending the packet, irregardless of its transmission success or failure. We assume both transmitter and receiver are fixed in space throughout the duration of packet transmission. We can represent the transmission attempts by a random measure $N_a$. Specifically, define $S_a(B)$ as the set of points (transmissions attempts) in the Borel set $B \subset \mathbb{R}^3$. Then $N_a(B)$ counts the number of points (transmission attempts) in the set $B \subset \mathbb{R}^3$, i.e.,

$$N_a(B) = |S_a(B)|. \tag{2.1}$$

By definition, then, for every set $B$, $N_a(B)$ is a Poisson random variable with mean equal to $\lambda_a$ times the Borel measure of the set $B$. For disjoint sets $A$ and $B$, then $N_a(A)$ and $N_a(B)$ are independent Poisson random variables.
We assume every node uses physical carrier sensing. Physical carrier sensing involves measuring the total instantaneous power incident at each node. We assume every node has this capability. The instantaneous incident power measured at each node is the sum of the contributions from all concurrent transmissions on the infinite plane and background noise, which are denoted by $I$ and $\eta$ respectively. The output of this measurement is then compared against an idle threshold, $T_i$. If the measured instantaneous incident power falls below $T_i$, the state of the medium is *idle*. Otherwise, the state of the medium is *busy* and transmission is prohibited. A node with a packet to send first checks the state of the medium by sensing the carrier. If the state of the medium is idle, the packet is transmitted immediately with probability one. Otherwise, the transmission attempt fails and the node reattempts to send the packet later. This process is repeated until an idle medium is observed and the packet is finally sent. All packets are transmitted with constant power, $P_t$ and duration $L$ seconds.

We now define another stationary three dimensional point process, and we let $N$ be the associated random measure on $\mathbb{R}^3$ with mean intensity $\lambda$ per unit area and unit time. The points of this process denote the location and time at which each transmission starts. An attempt to transmit on the medium is allowed when the state of the medium is perceived as *idle*, which is defined in accordance with the total instantaneous power received being less than the idle threshold, $T_i$. The total received power is the sum of the received power from all other concurrent transmissions, so that in fact the point process of transmissions is
defined recursively in time. We see that \( N(B) \leq N_a(B) \) for all \( B \). The interference model takes into account infinitely many concurrent transmissions over an infinite space.

### 2.3.1 Interference from Concurrent Transmissions

To define \( N \) precisely, we need to define the interference model precisely. To this end, suppose at the origin a node starts sending a packet at time zero. This transmission will have a non zero contribution \( h(\vec{u}, t) \) to the received power at another node located at \( \vec{u} \) and time \( t \). We assume the contributions from infinitely many concurrent transmissions at the point \( (\vec{u}, t) \) add linearly and propagation delay is small compared to the duration of the packet. We can view the interference process as the output of a linear time-invariant filter with transfer function \( h \), i.e. as a filtered shot noise process. In particular, the total interference power received at location \( \vec{u} \) and time \( t \) is denoted by \( I(\vec{u}, t) \), where

\[
I(\vec{u}, t) = \int_{\mathbb{R}^2} \int_{-\infty}^{t} h(\vec{u} - \vec{v}, t - \tau) \, dN(\vec{v}, \tau).
\]

We assume that \( h \) is separable into its space and time components, i.e.

\[
h(\vec{u}, t) = P_t f(\vec{u}) g(t).
\]

Recall that \( P_t \) denotes the transmission power. The function \( g \) is defined as \( g(t) = 1 \) if \( 0 \leq t \leq L \) and \( g(t) = 0 \) otherwise, where we recall \( L \) is the duration of each packet transmission. The function \( f \) incorporates signal power attenuation. We assume, for simplicity, a flat fading channel model where

\[
f(\vec{u}) = \left(1 + ||\vec{u}||\right)^{-\alpha}, \quad || \cdot || \text{ is the Euclidean norm and } \alpha \text{ is the path loss exponent.}
\]

We can interpret \( h \) as specifying the impact a transmission has on the rest of the
network in space and time. Since \( g(t) = 0 \) for \( t < 0 \), it is clear that \( I(\vec{u}, t) \) depends only on transmissions which commenced before time \( t \).

For an attempt to access the medium occurring at a node at location \( \vec{u} \) and time \( t \), the channel will be sensed idle if the observed interference \( I(\vec{u}, t^-) \) is less than the idle threshold \( T_i \). Therefore, recalling that \( N \) is the random measure that counts the number of transmissions in a set, we have

\[
N(\mathcal{B}) = \left| \left\{ i : (\vec{u}_i, t_i) \in \mathcal{S}_a(\mathcal{B}), I(\vec{u}_i, t_i^-) \leq T_i \right\} \right|, \tag{2.3}
\]

where \( t_i^- := t_i - \epsilon \) for small \( \epsilon > 0 \).

Here, we adopt the notation introduced in [17]. We assume the probability of two concurrent transmissions falling in \( \Delta \) is \( o(\Delta) \) as \( \Delta \to 0 \). In other words, \( dN(\vec{u}, t) \) takes only 0 or 1, and note that

\[
\mathbb{E}[dN(\vec{u}, t)^2] = \mathbb{E}[dN(\vec{u}, t)]. \tag{2.4}
\]

**Lemma 2.3.1.** Let \( \lambda \) be the intensity of transmissions per unit area and unit time, i.e.,

\[
\lambda = \frac{\mathbb{E}[dN(\vec{u}, t)]}{d\vec{u}dt}. \tag{2.5}
\]

Then

\[
\lambda = \lambda_a \times \Pr(I(\vec{u}, t) \leq T_i), \tag{2.6}
\]

for any location \( \vec{u} \) and time \( t \).
Proof.

\[
\lambda = \mathbb{E}[dN(\bar{u}, t)]/d\bar{u}dt \\
= \Pr \left( dN_a(\bar{u}, t) = 1, I(\bar{u}, t^-) \leq T_i \right)/d\bar{u}dt \\
= \Pr \left( I(\bar{u}, t^-) \leq T_i \right) \Pr \left( dN_a(\bar{u}, t) = 1 \right)/d\bar{u}dt \\
= \lambda_a \Pr \left( I(\bar{u}, t^-) \leq T_i \right)
\]

A transmission on the medium affects the interference received at all nodes, which in turn affects whether or not transmission attempts at other locations in space and time will be successful or not. It is clear that the point process associated with \( N \) is not Poisson, because of this dependency. In order to evaluate (2.6), we need to look at the statistics of \( I(\bar{u}, t) \). Figure 2.1 plots the probability density function of \( I \), obtained through simulation for a node located at the origin for different values of \( T_i \).

Since \( I(\bar{u}, t) \) is the sum of a large number of random and largely independent terms, we appeal to the central limit theorem and model \( I(\bar{u}, t) \) as a Gaussian random process. The statistics are then determined by its mean and covariance function. For the mean, we take the expectation of (2.2) and apply Campbell’s theorem. We have

\[
m_I = \mathbb{E}[I(\bar{u}, t)] = \lambda \times H_1,
\]
where

\[ H_1 = \int_{\mathbb{R}^2} \int_{-\infty}^{t} h(\vec{u} - \vec{v}, t - \tau) d\vec{v} d\tau 
= \int_{\mathbb{R}^2} \int_{-\infty}^{t} h(\vec{v}, \tau) d\vec{v} d\tau 
= P_t L \int_{\mathbb{R}^2} f(\vec{v}) d\vec{v} 
= 2\pi P_t L \int_{0}^{\infty} \frac{r}{(1 + r)\alpha} dr 
= \frac{2\pi P_t L}{(\alpha - 1)(\alpha - 2)}. \]

(2.8)

For the variance of the interference power \( I(\vec{u}, t) \), we have

\[ \sigma_I^2 = \text{var}[I(\vec{u}, t)] = (h_2 * K_N)(\vec{u}, \vec{u}, t, t), \]

(2.9)

where \( h_2 * K_N \) denotes convolution of \( h_2 \) and \( K_N \), and \( K_N \) is the covariance function
of the transmissions

\[
\text{cov}(dN(\vec{u}_1, t_1), dN(\vec{u}_2, t_2)) \\
= K_N(\vec{u}_1, \vec{u}_2, t_1, t_2) \, d\vec{u}_1 d\vec{u}_2 dt_1 dt_2,
\]

\hspace{1cm} (2.10)

and

\[
h_2(\vec{u}_1, \vec{u}_2, t_1, t_2) = h(\vec{u}_1, t_1) \times h(\vec{u}_2, t_2).
\]

\hspace{1cm} (2.11)

In order to evaluate \( K_N \), we need the conditional intensity of a packet transmission at the point \((\vec{u}_2, t_2)\) given a packet transmission started at another point \((\vec{u}_1, t_1)\). Below, we will formally define and present an analysis based on the model.

### 2.3.2 Conditional Transmission Intensity

A key mechanism of physical carrier sensing is the inhibitory effect the transmitting node has on its neighbors and to prevent them from sending. Recall many authors have associated a carrier sense region with every transmission, within which no other node is allowed to transmit. This is analogous to a total block out within the carrier sense region. As one would expect, a node closer to the transmitting node will be less likely to transmit than another node located farther away. However, transmissions are still possible at all distances away from the transmitting node.

We propose an inhibiting model that does not require the use of carrier sense regions. The probability that another node transmits conditioned on an ongoing packet transmission depends on the distance in time and space between...
them. We model this by the conditional intensity of transmissions given a packet transmission. Let $\mathcal{N}_0$ be the event that a node at the origin starts sending a packet at time zero. Formally, we define the conditional intensity of transmissions given $\mathcal{N}_0$ as

$$\phi(\vec{u}, t) = \mathbb{E}[dN(\vec{u}, t)|\mathcal{N}_0]/d\vec{u}dt,$$  

(2.12)

and the complete conditional intensity of transmissions for all $\vec{u}$ and $t$ is

$$\phi_c(\vec{u}, t) = \delta(\vec{u}, t) + \phi(\vec{u}, t),$$  

(2.13)

where $\delta$ is the Dirac delta-function, and $\phi$ is continuous at the origin.

**Lemma 2.3.2.** Using (2.12). Then,

$$\phi(\vec{u}, t) = \lambda_a \times \Pr (I(\vec{u}, t^-) \leq T_i|\mathcal{N}_0).$$  

(2.14)

for $\vec{u} \neq 0$ and $t \neq 0$.

**Proof.**

\[
\begin{align*}
\phi(\vec{u}, t) & = \Pr (dN(\vec{u}, t) = 1|\mathcal{N}_0) /d\vec{u}dt \\
& = \Pr (dN_a(\vec{u}, t) = 1, I(\vec{u}, t^-) \leq T_i|\mathcal{N}_0) /d\vec{u}dt \\
& = \Pr (I(\vec{u}, t^-) \leq T_i|\mathcal{N}_0) \Pr (N_a(\vec{u}, t) = 1) /d\vec{u}dt \\
& = \lambda_a \Pr (I(\vec{u}, t^-) \leq T_i|\mathcal{N}_0),
\end{align*}
\]

since $\mathbb{E}[dN_a(\vec{u}, t)] = \lambda_a d\vec{u}dt$ by assumption. \qed

Recall to evaluate the variance of $I(\vec{u}, t)$, we need to find the covariance function $K_N$ of the transmissions and it is related to $\phi_c$ by the following lemma
Lemma 2.3.3.

\[ K_N(\vec{u}, t) = \lambda \times (\phi_c(\vec{u}, t) - \lambda) \]  

(2.15)

Proof. Using (2.10),

\[
K_N(\vec{u}_1, \vec{u}_1 + \vec{u}, t_1, t_1 + t)
\]

\[
= \text{cov}(dN(\vec{u}_1, t_1), dN(\vec{u}_1 + \vec{u}, t_1 + t))/(d\vec{u}_1 dt_1)^2
\]

\[
= (\mathbb{E}[dN(\vec{u}_1, t_1)dN(\vec{u}_1 + \vec{u}, t_1 + t)]
\]

\[
- \mathbb{E}[dN(\vec{u}_1, t_1)]\mathbb{E}[dN(\vec{u}_1 + \vec{u}, t_1 + t)]/(d\vec{u}_1 dt_1)^2
\]

\[
= \lambda\mathbb{E}[dN(\vec{u}_1 + \vec{u}, t_1 + t)]dN(\vec{u}_1, t_1) = 1]/d\vec{u}_1 dt_1 - \lambda^2
\]

\[
= \lambda(\phi_c(\vec{u}_1, \vec{u}_1 + \vec{u}, t_1, t_1 + t) - \lambda).
\]

Since \( N \) is translation and rotation invariant, we adopt the notation \( \phi(\vec{u}, t) = \phi(\vec{u}_1, \vec{u}_1 + \vec{u}, t_1, t_1 + t) \) and (2.15) follows.

Since the transmissions have an inhibitory effect, one would expect \( \phi(\vec{u}, t) \) less than or equal to \( \lambda \) for all \((\vec{u}, t)\). From (2.15), it is easy to show that \( K_N(\vec{u}, t) \) less than or equal to zero for \((\vec{u}, t) \neq (0, 0, 0)\). In the literature, the point process for transmissions \( N \) belongs to a class of self-correcting point processes [18].

In order to evaluate (2.14), we require the conditional statistics of \( I(\vec{u}, t) \) received at another potential transmitter given a node at the origin started packet transmission at time zero. We call this conditional interference power as \( X(\vec{u}, t) \). Similarly, \( X(\vec{u}, t) \) is the sum of a large number of random and largely independent terms. We appeal again to the central limit theorem and model \( X(\vec{u}, t) \) as a Gaussian random process, whose statistics are determined by its mean and covariance.
function. For the mean, apply Campbell’s theorem and we have

\[ m_X(\vec{u}, t) = h(\vec{u}, t) + (h * \phi)(\vec{u}, t), \]  

(2.16)

where \( h(\vec{u}, t) \) is the received power at \((\vec{u}, t)\) from the packet transmission that started at the time zero at the origin and \((h * \phi)(\vec{u}, t)\) is the sum of received power from all other transmissions at \((\vec{u}, t)\) including the inhibited nodes around the transmitting node.

For the variance of the conditional interference power \( X(\vec{u}, t) \), we have

\[ \sigma^2_X(\vec{u}, t) = (h_2 * K_{N_0})(\vec{u}, \vec{u}, t, t), \]  

(2.17)

where \( K_{N_0} \) is the conditional covariance function of the transmissions

\[
\text{cov}(dN(\vec{u}_1, t_1), dN(\vec{u}_2, t_2); N_0) = K_{N_0}(\vec{u}_1, \vec{u}_2, t_1, t_2) \, d\vec{u}_1d\vec{u}_2dt_1dt_2. \]  

(2.18)

Appealing to the central limit theorem, we can write (2.14) as

\[
\phi(\vec{u}, t) = \lambda_o \times \left[ 1 - Q \left( \frac{T_i - h(\vec{u}, t) - (h * \phi)(\vec{u}, t)}{\sqrt{(h_2 * K_{N_0})(\vec{u}, \vec{u}, t, t)}} \right) \right], \]  

(2.19)

where \( Q(x) = (2\pi)^{-1} \int_x^\infty e^{-y^2/2}dy \) is the complementary distribution function of a standard normal random variable. In (2.19), we have expressed \( \phi \) as a function of itself and \( K_{N_0} \).

In order to evaluate \( K_{N_0} \), we need to find the probability of two transmissions at \((\vec{u}_1, t_1)\) and \((\vec{u}_2, t_2)\) given a packet transmission at the origin started at time zero. In order words, the third order statistics of the transmissions are needed.
Lemma 2.3.4. Using (2.18).

For \((\vec{u}_1, t_1) \neq (\vec{u}_2, t_2)\),

\[
K_{N_0}(\vec{u}_1, \vec{u}_2, t_1, t_2) = \lambda_a^2 \times \Pr \left( I(\vec{u}_1, t_1^{-1}) \leq T_i, I(\vec{u}_2, t_2^{-1}) \leq T_i \mid N_0 \right) - \phi_c(\vec{u}_1, t_1) \times \phi_c(\vec{u}_2, t_2). \tag{2.20}
\]

For \((\vec{u}_1, t_1) = (\vec{u}_2, t_2) = (\vec{u}, t)\), we have

\[
K_{N_0}(\vec{u}, \vec{u}, t, t) = \phi_c(\vec{u}, t). \tag{2.21}
\]

We can evaluate (2.20) using the formula for a bivariate normal integral for \((X(\vec{u}_1, t_1^{-1}), X(\vec{u}_2, t_2^{-1}))\) whose statistics are determined by its mean vector, and its elements can be computed (2.16). For the covariance matrix, its elements are given by

\[
K_X(\vec{u}_i, \vec{u}_j, t_i, t_j) = (h_2 \ast K_{N_0})(\vec{u}_i, \vec{u}_j, t_i, t_j), \tag{2.22}
\]

for \(i, j = \{1, 2\}\). A simple approximation for the bivariate normal integral is described in [19], while a more efficient and accurate algorithm is presented in [20].

In (2.20) and (2.21), we have expressed \(K_{N_0}\) as a function of itself and \(\phi\). We now have a system of two fixed point equations in terms of \(\phi\) and \(K_{N_0}\), which are given by (2.19), (2.20) and (2.21). Recall the analysis in this section is to discuss the finding of \(\phi\), which in turn is required to evaluate the variance of the interference \(I(\vec{u}, t)\).
2.3.3 Probability of Successful Transmission

In this section, we discuss the impact of inhibiting neighbors around the transmitting node on its intended destination. Let $p_{suc}$ be the probability that a given transmission was successfully received by the destination, conditioned on the transmission occurring. We develop further from the analysis made in the previous section to evaluate $p_{suc}$, which a similar methodology for finding $\Pr (I(\vec{u}, t) \leq T_i|N_0)$ is used. We let $Y(\vec{u}, t)$ be the conditional statistics of $I(\vec{u}, t)$ received at the intended receiver given a node at the origin started packet transmission at time zero. This received interference power does not include the signal power from the transmitting node at the origin. Recall, since the conditional statistics of $I(\vec{u}, t)$ given that a packet transmission at the origin started at time zero is Gaussian, it is easy to see that the statistics of $Y(\vec{u}, t)$ is also Gaussian.

We assume that the intended destination is a fixed distance $||\vec{u}||$ away from the transmitting node. Thus, the signal power at the intended destination is $P_t(1 + ||\vec{u}||)^{-\alpha}$. If the total power of the thermal noise and interference is equal to $T_d$, then the signal to interference plus noise ratio (SINR) is $\gamma = P_t(1 + ||\vec{u}||)^{-\alpha}/T_d$. We call $T_d$ the packet detection threshold. To approximate $p_{suc}$ formally, several models are possible. One model for a successful packet transmission is the interference plus noise must not be greater than $T_d$ throughout a substantial part of the packet duration. Exact solutions are difficult and an upper bound is presented in [21]. A second, less complicated model is to sample the received interference plus noise at the intended destination at a finite set of time instants throughout
the duration of the packet. Formally, we can define $p_{\text{suc}}$ as

$$p_{\text{suc}} = \Pr \left( \bigcap_i \{ Y_i \leq T_d \} \right),$$

(2.23)

where $Y_i = Y(\bar{u}, t_i)$. Recall since $Y(\bar{u}, t)$ is Gaussian random process, then $\{Y_i\}$ necessarily form a Gaussian random vector. To evaluate (2.23), the elements of the mean vector are given by

$$m_{Y_i} = (h \ast \phi)(\bar{u}, t_i),$$

(2.24)

where $\phi$ accounts for the clearing out around the transmitting node. The covariance matrix $\Lambda$ is assumed to be nonsingular and its elements are given by

$$\Lambda_{ij} = K_Y(\bar{u}, \bar{u}, t_i, t_j) = K_X(\bar{u}, \bar{u}, t_i, t_j),$$

(2.25)

where $K_Y$ is the covariance function of $Y$. In order to evaluate (2.24) and (2.25), recall $\phi$ and $K_{N_0}$ are found using the system of two fixed point equations, (2.19), (2.20) and (2.21) given in the previous section.

In short, we presented a model for conditional probability of successful packet transmission, which accounts for the inhibitory effect of the transmitting node, the joint statistics of the interference received at the transmitter and its intended destination and the received interference based on an infinitely many concurrent transmissions. This heuristic for $p_{\text{suc}}$ can be extended to incorporate error correcting codes by sampling the received interference $Y(\bar{u}, t)$ at bit rate.

Recall the analysis developed up to now, depends heavily on the Gaussian assumption on unconditional and conditional received interference power at any
node. However, finding of the variances of both the unconditional and conditional interference power are computationally intensive. In order to find a simpler analytical model that can be used as a guide to setting the protocol parameters, a further approximation is made below.

2.4 Poisson Approximation

In order to evaluate the variance of the interference power $I(\vec{u}, t)$, we make the approximation that the point process associated with $N$ is Poisson. This is despite the fact that we argued it is clearly not Poisson. The transmissions are now assumed uncorrelated and $\phi(\vec{u}, t) = \lambda$. Equation (2.15) then reduces to $K_N(\vec{u}, t) = \lambda \delta(\vec{u}, t)$, which is a well-known result for Poisson point process. We write the variance of the interference power $I(\vec{u}, t)$ as

$$\sigma_I^2 = \lambda \times H_2,$$  \hspace{1cm} (2.26)

where

$$H_2 = (h_2 \ast \delta)(\vec{u}, \vec{u}, t, t)$$

$$= \int_{\mathbb{R}^2} \int_{-\infty}^{t} h^2(\vec{u} - \vec{v}, t - \tau)d\vec{v}d\tau$$

$$= \int_{\mathbb{R}^2} \int_{-\infty}^{t} h^2(\vec{v}, \tau)d\vec{v}d\tau$$

$$= (Pt)^2 L \int_{\mathbb{R}^2} f^2(\vec{v})d\vec{v}$$

$$= 2\pi(Pt)^2 L \int_0^{\infty} \frac{r}{(1 + r)^{2\alpha}}dr$$

$$= \frac{2\pi(Pt)^2 L}{(2\alpha - 1)(2\alpha - 2)}. \hspace{1cm} (2.27)$$
Recall the mean of the unconditional interference power \( I(\vec{u}, t) \) is given by \( \lambda H_1 \).

Furthermore, we approximate (2.6) by

\[
\lambda = \lambda_a \times \left[ 1 - Q \left( \frac{T_i - \lambda H_1}{\sqrt{\lambda H_2}} \right) \right]. \tag{2.28}
\]

It is easy to show that the right side of (2.28) is a monotone decreasing function of \( \lambda \), and thus the fixed point equation (2.28) has a unique solution in \([0, \lambda_a]\) which can easily be solved numerically.

Equation (2.28) relates the intensity of actual transmissions \( \lambda \) to intensity of transmission attempts \( \lambda_a \), idle threshold \( T_i \), transmission power \( P_t \) and packet duration \( L \). As the \( T_i \) increases, \( \lambda \) increases to \( \lambda_a \) monotonically, which is to be expected since it becomes easier for a transmission attempt to see an idle medium.

For the case of \( T_i \) infinitely large, the model approaches to that of ALOHA.

The intensity of transmissions is also a measure of the aggressiveness of spatial reuse. A large idle threshold means a large transmission intensity and high spatial reuse. In periods of high load, i.e. large \( \lambda_a \), it may be desirable to exercise control and decrease the value of \( \lambda \), which can be accomplished by an appropriate value of \( T_i \). We have discussed the impact that \( T_i \) has on the network performance and its usefulness as a protocol parameter.

Below, we will present results from Monte Carlo simulations in order to check the accuracy of the above approximations. As we will see, the solution to the fixed point equation (2.28) yields a transmission intensity \( \lambda \) that is reasonably close to that observed in simulations.
2.4.1 Successful Transmission Intensity

Let \( \lambda_s \) be the intensity of successful transmissions per unit area and unit time, i.e. transmissions attempts that completed because the medium was sensed to be idle, and were successfully received by the destination. We have

\[
\lambda_s = \lambda \times p_{\text{suc}}. \tag{2.29}
\]

We assume the intended destination is a fixed distance \( ||\vec{u}|| = d \) away from the transmitter. Thus the signal power at the intended destination is \( P_t (1 + d)^{-\alpha} \). We adopt the second model mentioned in Section II.B for the conditional probability of successful transmission by sampling the received interference plus noise. We make the approximation that \( \gamma \) must be at least \( P_t (1 + d)^{-\alpha} / T_d \) midway through the packet transmission. In other words, for example, a packet transmission will be received correctly if \( \eta + I(d, 0, L/2) \leq T_d \), otherwise the packet must be retransmitted. Let \( \eta \) be additive white Gaussian noise, whose statistics are given by zero mean and variance \( \sigma^2_\eta \). Formally, we have

\[
p_{\text{suc}} = \Pr ( Y(d, 0, L/2) + \eta \leq T_d ). \tag{2.30}
\]

Consider a packet transmission at the origin that started from time \( t = 0 \). The statistics of the conditional interference \( I(d, 0, L/2) \) will depend on a number of factors. First, since the packet transmission commences at the origin at time \( t = 0 \), then at time \( t = 0^- \), \( I(0, 0, 0^-) \leq T_i \). It is still not clear how this will influence the conditional statistics of \( I(d, 0, L/2) \). On one hand, it appears that \( I(0, 0, 0^-) \) being small suggests that \( I(d, 0, L/2) \) will also be small, since they may be determined
by the same packet transmissions. This suggest a positive correlation between 
$I(0, 0, 0^-)$ and $I(d, 0, L/2)$. On the other hand, $I(0, 0, 0^-)$ being small suggests 
that other nodes nearby to the origin are more likely to transmit a packet, which 
suggests a negative correlation between $I(0, 0, 0^-)$ and $I(d, 0, L/2)$. In fact, though, 
in the model we suggest in this dissertation, we ignore this effect.

Instead, we focus on the dominant mechanism of CSMA which appears 
to largely determine the performance. Specifically, a transmission of a packet has 
the effect of inhibiting other nearby packet transmissions in space and time, due to 
the carrier sensing that takes place before each packet transmission. In particular, 
suppose a packet transmission commences at the origin at time $t = 0$. The 
interference power from this transmission alone at the point $(\bar{u}, t)$ is $h(\bar{u}, t)$, recall 
the conditional mean of the interference is given by (2.16). For simplicity, we shall 
assume the conditional mean of the interference power is $h(\bar{u}, t) + \lambda H_1$. We shall 
assume the conditional variance of $I(\bar{u}, t)$ is equal to the unconditional variance of 
$I(\bar{u}, t)$, namely $\lambda H_2$.

We shall evaluate (2.14) using the following approximation, which motivated the discussion above:

$$\phi(\bar{u}, t) \approx \lambda_0 \times \left[1 - Q \left( \frac{T_i - h(\bar{u}, t) - \lambda H_1}{\sqrt{\lambda H_2}} \right) \right].$$  \hspace{1cm} (2.31)

In order to evaluate (2.30), the mean of the received interference power
\[ Y(d, 0, L/2) \text{ at the intended destination is approximated by} \]

\[
m_Y = (h * \phi)(d, 0, L/2) \\
= \int_{\mathbb{R}^2} \int_{-\infty}^{L/2} h(d - u_1, -u_2, L/2 - \tau) \phi(u_1, u_2, \tau) \, du_1 du_2 d\tau. \quad (2.32)
\]

For variance of the received interference power \( Y(d, 0, L/2) \), we approximate by a conditionally inhomogeneous Poisson process and

\[
\sigma_Y^2 = (h^2 * \phi)(d, 0, L/2) \\
= \int_{\mathbb{R}^2} \int_{-\infty}^{L/2} h^2(d - u_1, -u_2, L/2 - \tau) \times \phi(u_1, u_2, \tau) \, du_1 du_2 d\tau. \quad (2.33)
\]

We can approximate (2.30) by

\[
p_{\text{suc}} = 1 - Q\left( \frac{T_d - (h * \phi)(d, 0, L/2)}{\sqrt{h^2 * \phi(d, 0, L/2)}} \right). \quad (2.34)
\]

In summary, in order to evaluate the intensity of packet transmissions that are successfully received by the destination \( \lambda_s \), we use Equation (2.29). The intensity of transmissions \( \lambda \) is found by solving the fixed point equation (2.28). The packet success probability \( p_{\text{suc}} \) is determined by first calculating \( \phi(\vec{u}, t) \) for all \((\vec{u}, t)\) from (2.31). Once this is done, we can then calculate \( p_{\text{suc}} \) from (2.34).

### 2.4.2 Network Throughput

Let \( C \) be the average number of successfully transmitted bits per unit area and time and per Hertz. We call \( C \) the network throughput, which is defined as

\[
C = \lambda_s \times R \quad (2.35)
\]
bps/sec/m²/Hz, where \( R \) is the average number of bits per packet transmission and Hertz. Recall a packet transmission is received successfully if \( \gamma \) is at least \( P_t (1 + d)^{-\alpha} / T_d \). To evaluate \( R \), we adopt Shannon’s capacity formula and write

\[
R = \log_2 (1 + \gamma)
\]

bps/Hz, where signal to interference plus noise ratio is

\[
\gamma = \frac{P_t (1 + d)^{-\alpha}}{T_d}.
\]

We see from (2.30), the optimal success probability for a packet transmission is achieved when \( T_d \) is infinitely large. A higher success probability requires fewer retransmissions before the packet is finally successfully received. Since the average number of retransmitted packets due to transmission failures is approximately \( 1 / p_{suc} \), this relates to a smaller latency in packet delivery. However in (2.36), a large \( T_d \) implies a small \( R \), which suggests low spectral efficiency (measured by the number of bits per Hertz). If the performance metric is network throughput, then the trade off discussed above suggests an optimal \( T_d \) and the effect of using \( T_d \) as a protocol parameter.

2.5 Model Validation

2.5.1 Numerical Example

We present here some numerical results obtained with the Poisson approximated model and compare with simulation. We take \( P_t = 1, L = 1 \) and
\( \alpha = 4 \). For a packet transmission that has occurred at \( \vec{u} \), we let the intended destination be located at \( \vec{u} + (0, L/2) \). We also adopt the sampling of the interference at the receiver midway through the packet transmission. We call this sampled value \( Y \). The packet transmission is a success if \( Y \) falls below \( T_d \). We use \( T_d = 0.9 \). For a given \( \lambda_a, T_i, L \) and \( P_t \), we use (2.28) to find \( \lambda \). Thus, we can write the intensity of successful transmissions as

\[
\lambda_s = \lambda \times \left[ 1 - Q \left( \frac{T_d - (h \ast \phi)(0, L/2, L/2)}{\sqrt{(h^2 \ast \phi)(0, L/2, L/2)}} \right) \right].
\] (2.38)

### 2.5.2 Simulation Setup

We simulate a wireless network using a discrete time system. The network consist of nodes distributed randomly on the plane of size \( K \times K \) according to a Poisson point process with mean \( \lambda_a \) attempts/sec/m\(^2\). To address the edge users, we consider a two-dimensional torus structure, where the points in the center square of size \( K \times K \) is repeated 8 times around it. The total interference power incident on each node is due to any points within an area of \( K \times K \) with that node lying in the middle. We also divide time into slots of \( L/10 \) seconds long. Each receiver associated with the transmitting node is located at a distance \( d \), with uniform angular distribution. Therefore, in each new time slot, there are \( \lambda_a K^2/10 \) new attempts. Simulations are ran for 2,000 seconds, including a 200 seconds warm-up. We set thermal noise \( \eta = 0 \).
2.5.3 Results

In Figure 2.2, we compute $\lambda$ numerically using the fixed point equation (2.28) for values of $\lambda_a = 0.5, 1.0$ and $2.0$. The full line represent the plots for $\lambda_a = 2.0$, dotted line represent plots for $\lambda_a = 1.0$ and dashed line represent plots for $\lambda_a = 0.5$. The choice of $\lambda_a$ is to show the impact of different levels of offered load on the network, with the highest at $\lambda_a = 2.0$. We see that the simulated (sim, with triangle markers) and numerical (num, with no markers) results match fairly well, especially for $\lambda_a$ small. For $\lambda_a$ large, the Poisson approximation on $N$ is inaccurate because the points are more correlated with each other. In fact, for small values of $T_i$, a transmission attempt becomes more sensitive to other transmissions. Only a small number of transmissions is necessary to cause the transmission attempt to fail. This discrepancy is a result of the Poisson assumption, which does not account for the correlation between the points. For $\lambda_a$ small, the average distance between the nodes is large, which relates to a smaller correlation between the points. Thus for $T_i$ large, the numerical and simulated results match well.

In the same figure, we plotted (2.6) (denoted by (gaussian, with circle markers)) with the statistics for the interference power obtained through the simulation. The purpose of this is to verify the Gaussian assumption on the interference power. For most values of $T_i$, we see this assumption Gaussian is fairly accurate. From the literature, we know the statistics of the filtered Poisson shot noise is not Gaussian in general. However in [22] and [23], it is shown the Gaussian assumption is good in regime of large intensities. For $T_i$ small, we expect $\lambda$ to be small, but
Figure 2.2 Transmission Intensity vs Idle Threshold under different Offered Loads despite this weakness in the Gaussian assumption, our model fits relatively well from the results obtained through simulation. In Figure 2.3 and Figure 2.4, we plot the mean and the variance of the unconditional interference power received using (2.7) and (2.26) for one particular realization of the node topology. We see the numerical and simulated results maps well in Figure 2.3. As for the discrepancy in the variance, it occurs is two-fold. First, due to the inaccuracy of Gaussian assumption for the interference power. Second, not accounting for the negative correlation between the transmissions by making the Poisson assumption.

Figure 2.5 plots the probability density of the interference process $I$ and the Gaussian random variable whose mean and variance are obtained through simulation. We see our Gaussian approximation on the power of the interference
process is reasonable especially for larger $T_i$.

In Figure 2.6, we look at the performance of CSMA under different levels of offered loads and compare them against the ALOHA protocol. The performance metric we shall use is success intensity. We recall for $T_i$ large, our model approaches ALOHA. In other words, we have $\lambda_a = \lambda$. Thus, the results obtained for $T_i = 4.0$ can be used to represent the performance of ALOHA. At low offered load, i.e. $\lambda_a = 0.5$, CSMA performs poorly relative to ALOHA. Especially for $T_i$ small, the network is operating suboptimally because physical carrier sensing causes the nodes to access the medium conservatively. At high offered load, i.e. $\lambda_a = 2.0$, the range of $T_i$ for optimal network throughput is small and sensitive to any changes in $T_i$. If $T_i$ is large, a packet transmission is more likely to fail because of the
Figure 2.4 Interference Power Variance vs Idle Threshold under different Offered Loads
Figure 2.5 Probability Density of Interference: Simulation vs Gaussian

increased number of transmissions and interference level. This impacts network performance negatively.

Figure 2.6 also shows that the simulation and analysis agree reasonably well and support our intuition of having a suitable $T_i$ to regulate the amount load carried on the network. We know that making the Poisson assumption reduces some fidelity in the analysis, nonetheless the result is a simple and tractable model. Below, we will use this model to gain insights on the impact the protocol parameters has on the performance of CSMA.

Refer to Figure 2.7. We consider the joint optimization of the intensity of transmission attempts $\lambda_a$ and idle threshold $T_i$ in order to maximize the intensity of successful transmissions $\lambda_s$. The numerical results suggest the success intensity
Figure 2.6 Success Intensity vs Idle Threshold under different Offered Loads is optimized for large $T_i$ and small $\lambda_a$. In other words, the optimal policy is to use ALOHA with small offered load. Recall the intensity of transmission attempts consists of new and retransmitted packets due to previous transmission failures or unsuccessful attempts. It is reasonable to assume that $\lambda_a$ is proportional to the product of spatial density of the nodes and the transmission rate in time. In most cases, we assume the spatial density of the nodes is given. We can make $\lambda_a$ small by waiting long before making another attempt to transmit.

As we see in Figure 2.7, there are many local maximums. This suggests either parameter alone can be adjusted to optimize performance. It is also suggested in Figure 2.7, the optimal policy for success intensity is to increase $\lambda_a$ and decrease $T_i$. With an increased $\lambda_a$, this results in more attempts therefore improv-
Figure 2.7 Success Intensity vs Idle Threshold and Transmission Attempts Intensity showing the chances of finding regions of low interference. However, in order to control the level of interference in the system, a smaller $T_i$ is desired.

For a more realistic measure of performance, we adopt network throughput as our metric in subsequent discussions below. Figure 2.8 contains the numerical results for network throughput $C$ evaluated as a function of $T_i$ and $T_d$. We set $\lambda_a = 1.0$. From the plot, the optimal $C = 0.1680$ bps/sec/m²/Hz is achieved at approximately $T_i = 1.0$ and $T_d = 1.0$. This success probability at which this maximum is achieved is 0.8657. This results suggests that CSMA outperforms ALOHA.
Figure 2.8 Network Throughput vs Idle and Packet Detection Threshold

\[ \lambda_a = 1, P_t = 1, L = 1, D = 0.5, \alpha = 4, \eta = 0 \]
Figure 2.9 Network Throughput vs Transmission Attempts Intensity and Packet Detection Threshold

Figure 2.9 shows the numerical results for network throughput evaluated as a function of $T_d$ and $\lambda_a$. From the plot, the optimal $C = 0.1925$ bps/sec/m$^2$/Hz with $\lambda_a = 4.0$ and $T_d = 1.2$. And the success probability is 0.8656. This results suggest for a fixed $T_i$, the best point is to operate with high $\lambda_a$ and lower rate. With a high $\lambda_a$, more quiet regions in space and time are found; therefore making full use of these opportunities.
2.6 Conclusion

The purpose of a node sensing the medium is to make an estimate of the interference environment at its intended destination prior to transmitting. From (2.28) and (2.31), it is easy to see that as $||\vec{u}||$ increases, $\phi(\vec{u}, t)$ increases monotonically to $\lambda$ and $K_N(\vec{u}, t)$ approaches to zero. In other words, the interference statistics at a node and its intended destination are uncorrelated if they are separated far enough. On the other hand, once a node starts sending a packet, its transmission will have an inhibitory effect on its neighbors. The set of inhibited nodes will be closer and more likely to have damaging effect on the intended destination located nearby than one located further away. Thus, this discussion motivates the use of small transmission ranges for CSMA.

We have discussed the effectiveness of adapting the idle threshold as a protocol parameter, which has the potential of leading to a distributed protocol. In other words, a node can adapt to the dynamics of the network traffic in a distributed, scalable manner. Related works have focus on the analysis and simulation of CSMA. The authors in [24] have deployed an experimental testbed to investigate the effectiveness of carrier sensing in a practical system for improving network throughput. In general, there is a need for a comprehensive analytical model that captures the dynamics of the physical layer while modeling the behavior of a real CSMA network.

We have developed a spatio-temporal model for a physical carrier sensing network on an infinite plane that factors in all physical layer parameters, i.e.
intensity of transmission attempts, mean distance between the transmitting node and its intended destination, transmission power, packet duration, idle and packet detection threshold. We have provided a simple approximate model for computing the network throughput in terms of these physical layer parameters. We have demonstrate our model performs reasonably close to simulation and can be used to perform a comprehensive optimization network throughput factoring in all of the above protocol parameters. The intensity of transmission attempts $\lambda_a$ is a measure of offered load on the network. In periods of high load, it maye desirable to exercise control and decrease the value of $\lambda_a$. Our results indicate that adjusting the idle threshold also appears to be a viable way of dealing with temporary high loads. Another interesting approach for further investigation would be to consider adjusting the data rate of transmissions in response to variations in node density and traffic load.

The text of this Chapter in part, is a reprint of material as it appears “A Spatio-Temporal Model for Physical Carrier Sensing Wireless Ad-Hoc Networks’,” in the Proceedings of the IEEE Communication Society Conference on Sensor, Mesh and Ad-hoc Communications and Networks (SECON), 2006, E.C. Wong; R. L. Cruz. The dissertation author was the primary investigator and author of this paper.
Chapter 3

On Achieving Optimal Network Operation

3.1 Introduction

In the previous chapter, we presented a simple analytical model for a wireless network that adopts the physical carrier sensing protocol. We accounted for many protocol parameters with that model, and compared it with extensive simulation results. In this chapter, we use that model to study the inter-dependence of these parameters, as well as investigate regions of optimal network throughput operation. We express network throughput as the product of successful transmission intensity and per packet transmission rate. More specifically, we consider transmission attempts intensity, transmission power, idle and packet detection threshold as the parameters of interest. This is elaborated in Sections 3.3.1, 3.3.2 and 3.4.
Network throughput of a wireless network is generally affected by two factors. Namely, the number of concurrent transmissions and the transmission rate used for each packet. The number of transmissions allowed at any given time is influenced by the idle threshold. If this threshold is small, the network is operating too conservatively. In contrast, if this threshold is too high, the increased level of interference from concurrent transmissions is likely to cause transmissions to fail. Meanwhile, the transmission rate used in each packet is directly related to the packet detection threshold. A small packet detection threshold corresponds to using a higher packet transmission rate. This packet, however, is required to meet a higher target SINR at its intended destination.

It is evident that the network can operate more efficiently by adapting either the idle or packet detection threshold suitably. In situations where the network is operating at regions of high idle threshold, devices may improve their throughput by sending at a lower rate. Likewise, when a device is sending a packet at a high transmission rate, the probability of success may be improved by using a smaller idle threshold. Therefore, this motivates the joint adaptation of idle and packet detection thresholds to improve network throughput. This is elaborated in Section 3.4.

Finally, in Section 3.5.1, we study the performance CSMA with joint adaptation under noise-limited and interference-limited conditions. Similarly, we compare the performance of CSMA against a wireless network using the ALOHA protocol. In fact, this is equivalent to the CSMA protocol with the idle threshold
set suitably large. We show that there is substantial gain in network throughput for CSMA over ALOHA, especially when the intensity of transmission attempts is small.

3.2 Related Work

Numerous studies on physical carrier sensing to improve network throughput and reliability have been conducted in recent years. For instance, the authors in [25] considered the impact of transmit power and carrier sense threshold on network throughput. They suggested that to achieve high network throughput, these two parameters should be adapted such that the carrier sense region completely overlaps the interference region. That is, the intended destination does not suffers from hidden terminals. On the contrary, network throughput is suboptimal in such a scenario because of an increased number of exposed terminals. This is also highlighted in [26], where the authors found that the optimal carrier sense range is smaller than what is required to cover the interference range. They suggested adapting the ratio of transmit power and carrier sense threshold. Similarly, Zhu et al. in [27] and [28] also identified the necessity of balancing the number of hidden and exposed terminals in order to maximize per user throughput. Subsequently, a heuristic based on estimating the periods of success, capture, busy and idle is used to adapt the carrier sense threshold.

In addition, there are other studies involving jointly adapting carrier sensing threshold and transmission power. For example, the authors in [16] found that
they should be adapted so that product of transmission power and carrier sense threshold should be kept constant for maximal spatial reuse. They advocate using step-wise increase and decrease or gradient-descent to adapt the carrier sense threshold.

These exist also other proposals that involves tuning only the carrier sense thresholds. Zhai et al. in [12] suggested using the same idle threshold for different channel rates. However, this is only possible if the carrier sense threshold is small enough to ensure that the target SINR for the highest possible channel rates is met. Also, in [29], the authors found the optimal carrier sense threshold is linearly proportional to node density. Finally, Ma et al. in [30] and [31] proposed adapting the carrier sense threshold subjecting to a target packet error rate. This suggest that optimal network throughput is achieved when the probability of successful transmission is smaller than one.

Xue et al. in [32] are probably the first to adapt jointly the carrier sense threshold and transmission rate. They first make the assumption that the carrier sense and packet detection threshold should be set the same, and proposed a two-dimensional search over finite values of carrier sense thresholds and rates. Each device uses its past transmission successes and failures as inputs to their algorithm seek to find its own optimal operating point, and targeting a success probability close to one. Rather, we show later that the probability of success for optimal network throughput depends on the intensity of transmission attempts. For example, when $\lambda_a = 1.0$, the optimal success probability is approximately $0.83$. 
Finally, in [33], they proposed two simple rules to jointly tune the physical carrier sensing threshold, transmission power and date rate. In particular, the two rules are the product of carrier sense threshold and transmission power, as well as, the product of carrier sense threshold and the maximum achievable rate are both constants. Despite the fact that their proposal achieve a higher aggregate throughput for the simulated scenarios, they fail to mention if it achieves the optimal network throughput.

### 3.3 Network Model

We consider the simpler analytical model developed earlier for a physical carrier sensing wireless network. Devices in this network share a common channel of bandwidth $W$ Hertz, and transmission attempts are made according to a three-dimensional Poisson point process with mean intensity $\lambda_a$ per unit area and unit time. Thus, $\lambda_a$ is used as a measure of offered load to the network. An attempt is *eligible* for transmission if the incident interference plus thermal noise, which we denote by $I$ and $\eta$ respectively, falls below the idle threshold $T_i$. Otherwise, this attempt is prohibited from making transmission. Eligible packets are transmitted with power $P_t$ and duration $L$ seconds. Thus, the intensity of *actual* transmissions is

$$\lambda = \lambda_a \times \Pr(I + \eta \leq T_i)$$  \hspace{1cm} (3.1)
per unit area and per unit time. Refer to Section 2.3.1 for a formal definition of $I$. In order to evaluate further, we make the Gaussian assumption on $I$ and certain Poisson approximation on the actual transmissions. As a result, the complete statistics of $I$ can be described by its mean and variance, which can be computed by

$$m_I = \lambda \times H_1$$

$$\sigma_I^2 = \lambda \times H_2,$$

where

$$H_1 = \frac{2\pi P_t L}{(\alpha - 1)(\alpha - 2)},$$

$$H_2 = \frac{2\pi P_t^2 L}{(2\alpha - 1)(2\alpha - 2)},$$

and $\alpha$ denote path loss exponent. Consequently, $\lambda$ is a solution of a fixed-point equation (3.1), which can be easily solved numerically.

Eligible packets are then transmitted using a rate of $R = \log_2(1 + \text{SINR}_{\text{target}})$ bps/Hz. The outcome of each packet transmission depends if the target SINR is met at its intended destination, which we can write as

$$\text{SINR}_{\text{target}} = \frac{P_t f(d)}{T_d},$$

(3.2)

where function $f$ models signal attenuation due to path loss, $d$ is the distance to the intended destination, and $T_d$ is the packet detection threshold. Therefore, we write

$$p_{\text{suc}} = \Pr(I + \eta \leq T_d | \mathcal{N}_0)$$

(3.3)
where $\mathcal{N}_0$ denote an actual transmission event by the source device. Likewise, we make the Gaussian assumption for the aggregate interference at the intended destination, and its statistics are

$$m_{I|\mathcal{N}_0}(\vec{u}, t) = (h \ast \phi)(\vec{u}, t)$$

$$\sigma^2_{I|\mathcal{N}_0}(\vec{u}, t) \approx (h^2 \ast \phi)(\vec{u}, t)$$

where $h(\vec{u}, t) = P_t f(\vec{u}) g(t)$ represent the impact of one packet transmission and $\phi$ is the conditional transmission intensity as described in Section 2.3.2.

At last, we adopt network throughput

$$C = \lambda \times p_{suc} \times \log_2(1 + \text{SINR}_{target})$$

bps/sec/m²/Hz, as the metric for performance.

### 3.3.1 Adapting Idle Threshold and Transmit Power

The choice of idle threshold controls the number of concurrent transmissions, while transmission power influences the extend each of these transmissions has on its neighboring devices. Therefore, it is possible to consider the joint adaptation of idle threshold and transmit power. For instance, if devices in the network use a large idle threshold, then the increased level of interference will degrade the success probability of packet transmission. Thus, it is desirable to use a higher transmit power to inhibit neighboring nodes from interfering with its packet reception.
Suppose that each device in the network uses the same rate for packet transmission, i.e., $\text{SINR}_{\text{target}}$ is fix. And, assuming no thermal noise, i.e. $\eta = 0$. Then the probability of successful transmission can be expressed as

$$p_{\text{suc}} = \Pr \left( I \leq \frac{P_t f(d) \text{SINR}_{\text{target}}}{\text{SINR}_{\text{target}}} \right| \mathcal{N}_0)$$

$$= 1 - Q \left( \frac{P_t f(d)}{\text{SINR}_{\text{target}}} - m I_{\mathcal{N}_0}(d, L/2) \right)$$

$$= 1 - Q \left( \sqrt{h^2 \phi(d, L/2)} \right)$$

$$= 1 - Q \left( \frac{f(d)}{\text{SINR}_{\text{target}}} - (h \phi(d, L/2)) \right)$$

where $Q$ is complementary distribution of a standard normal random variable. It is evident from (3.5) that the success of packet transmission depends directly on the intensity of actual transmissions, and indirectly on transmit power. In fact, the intensity of actual transmissions is a function of transmit power, and assuming no thermal noise, we have

$$\lambda = \lambda_a \times \Pr(I \leq T_i)$$

$$= \lambda_a \times \left( 1 - Q \left( \frac{T_i - m I}{\sigma_I} \right) \right)$$

$$= \lambda_a \times \left( 1 - Q \left( \frac{T_i - 2\pi \lambda P_t L}{(\alpha-1)(\alpha-2)} \right) \right)$$

$$= \lambda_a \times \left( 1 - Q \left( \frac{T_i P_t}{2\pi \lambda L} \right) \right)$$

(3.6)

From the analysis above, we conclude that the number of concurrent transmissions in the network depends on the ratio of idle threshold and transmit power. In turn, network throughput, which is a function of $\lambda$, is also a function of idle threshold.
and transmit power. A similar conclusion is made in [26], where the authors also conclude that the optimal network throughput depends on the ratio of carrier sense threshold and transmission power. As a result, any proposal that seek to jointly adapt idle threshold and transmit power reduce to searching over a single parameter, i.e. the ratio idle threshold to transmit power.

### 3.3.2 Scaling Network Throughput with Transmit Power

In addition to idle threshold, we focus on the controls of packet detection threshold and transmit power on network throughput. Assuming a fixed idle threshold, if the eligible devices want to send packets at a high transmission rate, they need to transmit at a higher power, and select a smaller packet detection threshold. However, there are more packets failures due to the higher SINR requirements, as well as increased levels of interference from other concurrent transmissions. On the other hand, if devices choose to operate with a smaller target SINR, they can then transmit at a smaller power and select a higher packet detection threshold. Consequently, this causes the network to operate with suboptimal spectral efficiency.

Suppose an eligible device transmit a packet with a target SINR of \( \frac{P_t f(d)}{I_d} \)
to its destination located at distance $d$. The per packet transmission rate is

$$\mathcal{R} = \log_2 (1 + \text{SINR}_{\text{target}})$$

$$= \log_2 \left( 1 + \frac{P_t f(d)}{T_d} \right)$$

$$= \log_2 \left( 1 + f(d) \frac{P_t}{T_d} \right)$$

(3.7)

bps/Hz. The rate used for each transmitted packet depends on the ratio of packet detection threshold and transmit power. Meanwhile, the success of each transmitted packet can be expressed as

$$p_{\text{suc}} = \text{Pr}(I \leq T_d|N_0)$$

$$= 1 - Q \left( \frac{T_d - m_{I|N_0}(d, L/2)}{\sigma_{I|N_0}(d, L/2)} \right)$$

$$= 1 - Q \left( \frac{T_d - m_{I|N_0}(d, L/2)}{\sigma_{I|N_0}(d, L/2)} \right)$$

$$= 1 - Q \left( \frac{T_d - (h \ast \phi(d, L/2))}{\sqrt{h^2 \ast \phi(d, L/2)}} \right)$$

$$= 1 - Q \left( \frac{T_d}{P_t} - \frac{(h \ast \phi(d, L/2))}{P_t} \right) \sqrt{\frac{h^2 \ast \phi(d, L/2)}{P_t^2}}.$$ (3.8)

Likewise, the success probability is a function of packet detection threshold and transmit power.

Refer to Equations (3.6) - (3.8). In addition to the ratio of idle threshold and transmit power, we conclude that the optimal network throughput also depends on the ratio of packet detection threshold and transmission power. Indeed, transmission power is a scaling factor for network throughput. With this insight, we normalize idle and packet detection threshold with transmit power, and reduce the search space among the list of parameters affecting network throughput. This
motivates the study of joint adaptation of idle and packet detection threshold.

3.4 Joint Rate and Idle Threshold Adaptation

We focus on the controls of idle and packet detection threshold on network throughput. Note that idle threshold affects the number of concurrent transmissions, as well as the success probability of each packet transmission. Likewise, packet detection threshold also affects the success probability of each packet transmission. Furthermore, it affects the transmission rate for each packet sent as well. These are all factors that directly impact system performance, i.e. network throughput.

How should one adapt idle and packet detection threshold for optimal network operation? For example, if the devices in the network use a small idle threshold, then the interference due to concurrent transmissions would be small. Packets are always successful received at their intended destinations. Therefore a higher spectral efficiency is achieved if a small packet threshold is adopted. Alternatively, in situations of aggressive spatial reuse, devices in the network use a high idle threshold. In this case, the interference due to concurrent transmissions would dominate, therefore a high packet detection threshold is desired to ensure transmission success.

To study this, we first conduct many experiments through Monte Carlo simulation, and present some result highlights here. We simulate a wireless network consisting of nodes distributed randomly on a $K \times K$ plane according to a Poisson
Figure 3.1 Network Throughput vs Idle and Packet Detection Threshold

point process with mean $\lambda_a$ attempts/sec/m$^2$. We consider a two-dimensional torus structure, where the points in the center of the $K \times K$ square is repeated 8 times around it. The total interference power incident on each node is caused by any nodes within a $K \times K$ area. We also discretized time into slots of $\frac{L}{10}$ seconds. In each new time slot, there are $\frac{\lambda_a K^2}{10}$ new attempts. The intended receiver is located at a distance $d$ away with uniform angular distribution. Simulations are ran for 2,000 seconds, including a warm up of 200 seconds. We let thermal noise be $\eta = 0.01$.

In Figure 3.1, we plot different values of $T_i$ and $T_d$ through simulations. We observe that for each fixed value of $T_i$, there is an corresponding optimal $T_d$
that achieves maximal network throughput. When $T_d$ is small, the target SINR is too high and excessive packet failures caused performance to degrade. Meanwhile, when $T_d$ is large, actual SINR is much higher than target SINR and the network is operating with poor spectral efficiency. Likewise, for each fixed value of $T_d$, there is a corresponding value of $T_i$ that achieves maximal network throughput. When $T_i$ is small, the spatial reuse in the network is too conservative and packets are received with high probability of success. On the other hand, when $T_i$ is large, the spatial reuse in the network is too aggressive and packet failures are common. The reason for these failures is the high level of interference from concurrent transmissions.

Note that when $T_i$ is higher than 12, the CSMA protocol approaches ALOHA and the optimal $T_d$ flattens off. This transition is captured in our analytical model.

In Figure 3.2, we plot $C$ versus $T_i$ with optimal $T_d$ for $\lambda_a = 10$. The maximum network throughput $C$ (with square markers) attained is 0.2427 bps/sec/m$^2$/Hz. This is achieved by setting $T_i = 8$ and $T_d = 8.6$, and the probability of idle and success are 0.7673 and 0.9654 respectively. The target SINR is $-16.39$ dB. This suggests that present day wireless networks, which operate at much higher SINR, are poorly optimized in terms of network throughput. It is also observed that the target SINR also depends on $\lambda_a$. For example, when $\lambda_a = 1$, the target SINR is $-1.20$ dB; and when $\lambda_a = 0.1$, the target SINR is $6.93$ dB.

From this result, further network throughput can be gained through careful joint adaptation of idle and packet detection threshold. Note also for $\lambda_a = 10$, the network throughput of CSMA over ALOHA is approximately 5.9%.
Figure 3.2 Network Throughput vs Idle Threshold with Optimal Packet Detection Threshold

\[ \lambda_a = 10, P_t = 1, L = 1, D = 0.5, \alpha = 4, \eta = 0.01, 20000 \text{ slots} \]
same figure, we also plot (with triangle markers) for each $T_i$ the network throughput where $p_{suc}$ is close to 1.0. This is the targeted success probability [32] for the joint idle threshold and rate control algorithm. We see from this plot that further network throughput can be achieved by operating at a smaller target success probability. Also, in this figure, we plot the network throughput where $T_i$ is equal to $T_d$. This is a common assumption used in many studies assuming the interference at the source and intended destination are the same. However, this results in very poor performance because the variation in the interference is not taken into consideration.

### 3.4.1 A Word on Success Probability

The success of a packet transmission depends on the number of concurrent transmission, as well as the transmission rate targeted. It is well-known that CSMA suffers from hidden and exposed terminal problems. The hidden terminal problem can be solved by using a small idle threshold, and the closest concurrent transmissions are far enough not to interfere with its packet transmission. This achieves a success probability of one. However, this actually cause the network to operate sub-optimally because of two reasons. First, a conservative idle threshold increases the number of exposed terminals. Second, these interfering devices might not be present to begin with. This is dependent on $\lambda_a$. Indeed, the success probability balances the trade-off between hidden and exposed terminals.

For example, for $\lambda_a = 1$, the success probability when network throughput
is maximized is 0.8567. In contrast, the success probability is 0.9654 when $\lambda_a = 10$. When the offered load in the system is small, the hidden terminal problem is less pronounced since the nodes are adequately separated spatially. In this case, nodes should transmit more aggressively at a higher rate. On the other hand, when the offered load in the system is high, it is likely that hidden terminals will be present. Therefore, the system should operate in a manner such that the success of packet delivery is close to unity.

As a result, optimal network throughput is not necessarily achieved when success probability close to one. Furthermore, we observed from simulations that the targeted success probability is dependent on the intensity of offered load. This is in contrast to many proposals in the literature, where system parameters are adapted to achieve close to success certainty. We shall postpone the analysis of optimal probability of success and intensity of attempts to our future work.

### 3.4.2 Verifying Impact of Transmit Power

In Section 3.3.2, we show by analysis that when we seek to jointly adapt $T_i$ and $T_d$, transmit power is in fact a scaling factor in network throughput, assuming thermal noise is negligible. We verify this with simulation to show that it is indeed the case. Recall for $\lambda_a = 10$ and $P_t = 1$, the maximum $C$ achieved is 0.2472 bps/sec/m²/Hz, with set at $T_i = 8$ and $T_d = 8.6$. Simulations are also ran for $\lambda_a = 10$ and $P_t = 10$. The maximum $C$ achieved is 0.2392 bps/sec/m²/Hz, with values of $T_i = 70$ and $T_d = 80$. The discrepancy, in part is due to simulations
inaccuracies, as well as the resolution over which values of \( T_i \) and \( T_d \) are searched.

### 3.4.3 Heuristic for Adapting Idle and Packet Detection Threshold

To gain insights on the joint adaptation of idle and packet detection threshold, it is useful to understand how these two parameters related to each other. For instance in [32], the authors make the assumption that the aggregate interference at the source device is identical to the interference at its intended destination. Following this assumption, the idle threshold is chosen such that it equals the ratio of received signal power at its intended destination to the target SINR, i.e. \( T_i = \frac{P_t f(d)}{\overline{\text{SINR}}_{\text{target}}} \). In other words, the idle threshold equals the packet detection threshold. On this contrary, the correlation between the transmit and receive devices depends on how far they are spatially and temporally separately, as well as transmit power used and the attempts intensity.

According to our simple model, the idle threshold is related to packet detection threshold through the success probability. For a particular value of success probability \( \beta \), then

\[
T_d = F^{-1}(\beta)
\]

where \( F \) is the cumulative distribution function of the conditional aggregate interference process \( I \), whose statistics depends on the intensity of actual transmissions \( \lambda \). Note that \( \lambda \) has a one-to-one map with \( T_i \) defined according to the fix point equation (3.1).
In many proposals, the targeted success probability for packet delivery is close to one. For instance, the authors in [25] set the carrier sense range far enough to enclose the interference range, thus this eliminates all hidden terminals and ensures that packet delivery is always successful. Similarly, in [32] where the spatial backoff algorithm adapts the carrier sense threshold and rate according to positive or negative feedback, implicitly targets a success probability of one as well. Instead, as discussed in Section 3.4.1, the success probability when network throughput is maximized depends on attempts intensity $\lambda_a$ and is smaller than one. Through simulations, we found the success probability that achieves optimal network throughput is 0.9654 when $\lambda_a = 10$ and $P_t = 1$, and 0.7978 when $\lambda_a = 1$ and $P_t = 10$. We plot in Figure 3.3 the success probability versus $T_i$ maximized over $T_d$. Maximum network throughput is achieved when $T_i = 8$ and $T_d = 8.6$ for $\lambda_a = 10$ and $P_t = 1$, and $T_i = 2$ and $T_d = 2.6$ for $\lambda_a = 1$ and $P_t = 10$. Observe that the success probability is approximately flat over a wide range of $T_i$. In general, the success probability is typically high for a wide range of $\lambda_a$.

For a fixed attempt intensity $\lambda_a$, there is a corresponding optimal success probability $\beta$ that maximizes network throughput. Equation (3.9) shows the relationship between idle threshold to the packet detection threshold for a certain success probability. This equation also specifies how idle and packet detection threshold should be adapted in order to meet the targeted success probability. As previously discussed, the target success probability is obtained by simulation.

To further simplify (3.9), we appeal to the Gaussian property of the
Figure 3.3 Success Probability vs Idle Threshold with Optimal Packet Detection Threshold
interference process at the intended destination. We write the inverse cumulative distribution function as

\[ T_d = m_{I|N_0}(d, L/2) + \eta + k_d \times \sigma_{I|N_0}(d, L/2), \]  

(3.10)

where \( k_d = \sqrt{2\text{erf}^{-1}(2p_{\text{suc}} - 1)} \) and \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \) is the error function. Common values of \( k_d \) are 1, 2, and 3, when \( p_{\text{suc}} \) is 0.8414, 0.9773 and 0.9987 respectively. Since the success probability is generally high when network throughput is maximized, and is assumed to always be \( k_d \) is a positive number. We approximate \( m_{I|N_0}(d, L/2) \) by \( \lambda H_1 \), and \( \sigma_{I|N_0}(d, L/2)^2 \) by \( \lambda H_2 \). Finally, we approximate (3.9) as

\[ T_d \approx \lambda H_1 + \eta + k_d \sqrt{\lambda H_2}. \]  

(3.11)

Hence (3.11) provides a simple heuristic to adapt jointly packet detection and idle threshold.

Similarly, since the interference process at the source device is also Gaussian, we can express the probability of a device observing an idle medium as

\[ T_i = m_I + \eta + k_i \times \sigma_I, \]  

(3.12)

where \( k_i = \sqrt{2\text{erf}^{-1}(2p_{\text{idle}} - 1)} \). Using Campbell’s theorem, \( m_I \) and \( \sigma_I^2 \) is \( \lambda H_1 \) and \( \lambda H_2 \) respectively. We write (3.12) as

\[ T_i = \lambda H_1 + \eta + k_i \sqrt{\lambda H_2}. \]  

(3.13)

Depending on the probability of idle, \( k_i \) can take either positive or negative values.

Finally, using (3.11) and (3.13), we have

\[ T_d \approx T_i + (k_d - k_i) \sqrt{\lambda H_2}. \]  

(3.14)
It is interesting to compare this result to [32] where the authors suggesting $T_i$ equals $T_d$. The additional factor in (3.14) accounts for the variation in interference at the intended destination.

In Figure 3.4, we plot the optimal packet detection threshold that maximizes network throughput for each idle threshold value obtained through simulation. Consider the plot for $\lambda_a = 1$ and $P_t = 10$. We see for small values of $T_i$, $T_d$ is larger than $T_i$. Now refer to (3.14). When $T_i$ takes on small values, the probability of an idle medium is small as well. As a result, $k_i$ takes on negative values and the additional factor is positive. Similarly, when $T_i$ takes on large values, $T_d$ is smaller than $T_i$. Since the probability of an idle medium is higher, and this results in the factor being negative, i.e. $k_i$ is larger than $k_d$. Our results matches the simulations.
well. Consider next the plot for $\lambda_a = 10$ and $P_t = 1$. Note that $H_2$ is smaller in this case, therefore a smaller $T_i$ is needed for the network to approach ALOHA. Note also because of the smaller variance in $H_2$, the second factor in (3.14) is small. Likewise, this nearly affine relationship between $T_i$ and $T_d$ is also evident from the plot.

### 3.4.4 Achieving Optimal Operation

In addition to the heuristic between idle and packet detection threshold, we observed from simulation results that the packet detection threshold is always larger than the idle threshold when network throughput is maximized. This is in contrast to other studies, for example [32], which suggests setting the idle threshold to be equal to the packet detection threshold. This results in a reduction of transmission rate is to account for the variation in the interference process. In instances when $\lambda_a$ is small, equality between idle and packet detection threshold is achieved. For example, when $\lambda_a = 0.1$, network throughput is maximized when $T_i = T_d = 0.4$. The reason is under these levels of low offered load, the variance due to concurrent transmissions is so small such that the aggregate interference from concurrent transmissions at the source device is highly correlated to the interference observed at its intended destination.

With this, we reduced further the set of feasible idle and packet detection thresholds for optimal network operation. This is indicated in Figure 3.4 by the set of points that lies about the diagonal line. Then, we apply this condition to (3.14),
and we get $k_s$ larger than $k_i$. Thus, the probability of observing an idle threshold is upper bounded by the corresponding success probability. Since we indicated earlier that the success probability for optimal network operation is smaller than one, thus this suggest that CSMA outperforms ALOHA. More regarding the CSMA and ALOHA comparison will be discussed later.

For a certain attempt intensity and targeted success probability $\beta$, a device shall increase its $T_d$ if success probability is smaller $\beta$. On the other hand, if the success probability is larger than $\beta$, the $T_d$ is reduced thus increasing the packet transmission rate. In other words, we update $T_d$ using

$$T_d(n) = T_d(n - 1) + \theta_1 \cdot (\beta - p_{suc}(n - 1)), \quad (3.15)$$

where $n$ denote the time index and $\theta_1 > 0$ is step size. Subsequently, once the target success probability is met, we update $T_i$ next using

$$T_i(n) = T_i(n - 1) + \theta_2 \cdot C'(T_i(n - 1)), \quad (3.16)$$

where $\theta_2 > 0$ is step size. And we can express $C$ in terms of $\lambda$, which in directly related to $T_i$,

$$C(T_i(n)) = \lambda(n) \times \beta \times \log_2 \left( 1 + \frac{P_t f(d)}{\lambda(n) H_1 + \eta + k_s \sqrt{\lambda(n) H_2}} \right) \quad (3.17)$$

bps/sec/m$^2$/Hz. Note that $T_i(n)$ should be smaller than $T_d(n)$ as discussed earlier.

In summary, the device first adapts its packet detection threshold to achieve the targeted success probability $\beta$. Subsequently, it can adapts its idle threshold using the gradient-descent method to find the optimal network throughput.
3.5 Discussion

3.5.1 Effects of Thermal Noise on Network Throughput

We study the performance of CSMA when the network is noise-limited and interference-limited. In Figure 3.5, we plot network throughput $C$ at optimal packet detection threshold $T_d$ versus idle threshold $T_i$ for different values of thermal noise $\eta$. When $\eta = 0.0001$, the network is interference-limited. We see the maximum network throughput achieved is 0.1902 bps/sec/m$^2$/Hz, at $T_i = 2$ and $T_d = 2.6$. When $\eta = 0.5$, the network is noise-limited. In this case, the maximum network throughput achieved is 0.1694 bps/sec/m$^2$/Hz at $T_i = 4$ and $T_d = 4.8$. As expected, thermal noise degrades the performance of CSMA networks.
Another interesting conclusion is, the effectiveness of CSMA over ALOHA decreases when increasing thermal noise is observed. For instance, when the network is noise-limited, the gain in CSMA network throughput compared to ALOHA is 18.87%. On the other hand, when the network is interference-limited, CSMA gained an increase of 9.75% over ALOHA. Therefore, when the network is noise-limited, ALOHA is an adequate access mechanism, since CSMA only achieves marginal gains. Indeed, CSMA is more well-suited in interference-limited environments.

3.5.2 Comparison between CSMA and ALOHA

We compare the CSMA and ALOHA protocols taking into account of spatial reuse. In fact, ALOHA is a special case of CSMA for $T_i$ set at infinity. As discussed in the beginning of this dissertation, CSMA is an enhancement to ALOHA since it avoids unnecessary packet failures by first sensing the medium. How does CSMA perform against ALOHA under different network conditions? We look specifically at two scenarios. First, under high offered loading, i.e. $\lambda_a = 10$ and $P_t = 1$. Second, under low offered loading when $\lambda_a = 1$ and $P_t = 10$.

These two scenarios are plotted in Figure 3.6. We observe a gain of 5.91% for CSMA over ALOHA when the network is operating with high offered load. The reason is when the attempts intensity is high, devices are more efficient in finding quiet regions in space and time. Also, these devices are using a smaller transmit power, therefore a smaller inhibitory impact on its neighbors. Thus, this causes
more concurrent transmissions and a higher level of aggregate interference. On the other hand, when the network is operating with low offered load, there is a 18.76% gain for CSMA over ALOHA. This is mainly due to the fact that the devices are spaced further apart in space and in time. At the same time, the higher transmit power used prevents any nearby devices from interfering. This results in fewer concurrent transmissions and devices are allow to send a higher rate assuming the same success probability.
3.6 Conclusion

We considered the joint adaptation of transmit power, idle threshold and per packet transmit rate for a wireless network using physical carrier sensing. Using the simple model developed in the previous chapter, we found the ratio of idle threshold to transmit power, as well as the ratio of packet detection threshold to transmit are the key parameters that influences network throughput. Transmit power is in fact a scaling factor, assuming negligible thermal noise. Based on this insight, we reduce the search space and narrow our focus on the joint adaption of idle and packet detection threshold to optimize network throughput.

Through extensive simulations, we found that the target SINR to achieve maximum network throughput is much smaller than what is currently used in wireless networks. In other words, today’s systems are extremely poorly optimized in terms of network throughput. We found that it is much more efficient to have more packet transmissions, but at a much smaller rate.

Contrary to many studies in the literature, we found the success probability when network throughput is maximized is smaller than one. In fact, the success probability depends on the transmission attempts intensity. The higher the attempts intensity, the closer the success probability is closer to one. Subsequently, we developed a heuristic to jointly adapt the idle and packet threshold, while subjecting to a target success probability. We also discovered that the packet detection threshold is higher than the idle thresholds for most network scenarios. This suggests CSMA is superior to ALOHA. Lastly, we developed an distributed
algorithm to achieve optimal network operation.

To fully understand the impact of CSMA, we also studied the performance of CSMA in noise-limited and interference-limited cases. We conclude that CSMA is more well-suited in an interference-limited environment. Although in a noise-limited environment, CSMA still outperforms ALOHA, but the gain is marginal. Finally, we compared the performance of CSMA and ALOHA with different levels of offered load. We conclude that CSMA outperforms ALOHA in most cases, and the amount gained is dependent on the attempts intensity.
Chapter 4

Application to Cognitive Radio Networks

4.1 Introduction

In recent years, the increasing demand for wireless access and service has led to a severe imbalance in spectrum utilization across all frequency bands. In some cases, spectrum is allocated to legacy devices which are no longer in operation; on the other hand, high utilization can be seen in the unlicensed ISM bands, where popular systems like Wi-Fi and Bluetooth operate. In an effort to reverse this trend, there has been much interest in cognitive radio systems recently [34]. These devices are capable of finding regions in space, time and frequency where the spectrum might be underutilized. This is commonly known as dynamic spectrum access in the literature. These devices have the distinct
trait of limiting degradation on the licensed primary users. This new paradigm in communications does not preallocate spectrum, time or space to any devices in the cognitive network. Rather, these devices should be capable of adapting their behavior around the legacy systems.

The use of cognitive radios enables operation in previously unavailable spectrum, and could potentially increase the system performance by multiple fold. While the basic idea behind cognitive radio is simple, it has many technical challenges. Among the many are the problem of sensing and identifying usable spectrum, access coordination among cognitive radios, and policing of cognitive radios.

The problem of identifying usable spectrum is a challenging one. An ideal cognitive radio is capable of monitoring multiple frequency bands constantly, in order to take full advantage of its capability. In practice, most wireless devices only have a single transceiver, thus eliminating the possibility of sensing and communicating simultaneously. Furthermore, a very wide-band front-end would be needed in order to monitor all the available frequency bands. This itself would incur a huge processing cost and requires a large amount of power drain. At last, even if a potential frequency band is identified, there will be a fair amount of overhead incurred to coordinate the transmitter and receiver to operate in the same frequency and time [35]. Under such practical constraints, we consider a simpler system, where a cognitive secondary network operates within a single frequency band.

Legacy radio systems could be over designed to ensure robustness and
maintain good link quality. In order to achieve this, these systems might operate above their targeted signal to interference plus noise level. This leads to unnecessary spectrum resource wastage. On the other hand, this provides an opportunity for a secondary network to co-exist and exploit the excess spectrum resources. This is conditioned on the fact that the secondary network limits the amount of degradation on the primary users.

An important architecture design consideration is whether the access strategy for the secondary network should be centralized or distributed. With a centralized scheme, there is a central controller responsible for coordinating all secondary communications. This keeps the impact on the licensed primary users under tight control. However, a significant amount of network planning and control signaling is necessary. This is the system architecture proposed by IEEE 802.22 [36]. Rather, we take the distributed approach [37], where no centralized controller is present. Secondary devices access the medium based on the conditions of their local environment. They do not exchange information, nor coordinate with other devices. While this approach is simple, it could lead to a large impact on the licensed primary users. Careful consideration must be taken for the aggregate impact of the secondary users on each licensed primary user.

We discuss some recent works on cognitive radio and dynamic spectrum access in the next section. In section III, we present our network model and performance metrics. In section IV, we present our proposed cognitive MAC protocol for secondary users. In the subsequent sections V and VI, we present our simulation
setup and some results. In the last section, we conclude with a summary of our findings and suggest some future work.

4.2 Related Work

In the literature, many interesting works related to utilizing the unused spectrum or white spaces in time, space and frequency can be found. We highlight a few here. In [35], the authors consider utilizing unused spectrum available in time and frequency. In their proposed schemes, they accounted for the inability of secondary users to sense all available frequency bands. The state of each frequency band in every time slot is based on a Markov model. The authors do not consider spatial reuse in their model.

The approach proposed in [38] is similar to the RTS/CTS mechanism adopted in CSMA/CA. The basic idea is an exchange of control messages is required between the transmitter-receiver pair prior to permitting secondary users to transmit. This increases the signaling overhead, and lower network throughput.

The authors in [39] [40] [41] study the limits of detecting the signals from a single primary transmitter with secondary users located spatially. This can be difficult, as the primary signals are likely to be weak and possibly below the noise floor. They also gain insights on the block out region around the single primary transmitter.

In [42], a single primary transmitter and a group of secondary users is considered. The authors investigated the detection of primary signal under shad-
owing and fading. They proposed a scheme of detecting the primary user’s signal via collaboration among the secondary users.

For an overview or survey on cognitive radio and dynamic spectrum access, readers are referred to [43] [44] [45].

4.3 Network Model

We consider a system where a primary network interacts with a secondary network in the same channel of bandwidth $W$ Hertz. This could also model the operation in a single frequency channel within a multi-channel system. This channel is licensed to the primary network, whose spectrum usage depends on the positions and traffic statistics of the primary users. In turn, users in the secondary network yearn to exploit the channel during periods or regions of low usage.

We assume users always have a packet to send, and they communicate over discrete time slots. Each packet can span over a single or multiple slots. We assume packet transmissions are omni-directional. Packets are transmitted with fixed power $P_i$, where $i = 1, 2$ correspond to transmission power from primary and secondary users respectively. We let $f(r)$ denote the attenuated signal power at Euclidean distance $r$ away from the transmitter. Specifically, we adopt for simplicity $f(r) = (1 + r)^{-\alpha}$ where $\alpha$ is the path loss exponent. In addition, we denote thermal noise power as $\eta$.

Consider only a primary network in the system, whereby users are spatially distributed according to a two-dimensional homogeneous Poisson point pro-
cess $N_1$ on the infinite plane with mean intensity $\lambda_1$ users per meter$^2$. A primary user sends a packet with a time duration of $k$ slots to its intended primary receiver at fixed distance $r_1$ meters. This packet transmission is a success if the total interference plus thermal noise power at the intended primary receiver is less than $T_{s1}$. We call $T_{s1}$ the packet success threshold. In other words, the signal to interference plus thermal noise power ratio $\gamma_1$ should at least be $\frac{P_1 f(r_1)}{T_{s1}}$, in order for a packet transmission to be successful. In the absence of the secondary network, we define the probability of successful packet transmission as

$$p_{s1} = \Pr(I_1 + \eta \leq T_{s1}), \quad (4.1)$$

where $I_1$ is the total received interference power from other concurrent transmitting primary users. More precisely, $I_1$ is a filtered Poisson shot noise process, but solving for its exact statistics is not trivial. Several approximations for the statistics of the Poisson shot noise have been proposed in the literature [46]. We generate the exact statistics of $I_1$ through extensive Monte Carlo simulations. To this end, we adopt network throughput $C_1$ as the performance metric for the primary network, which we write

$$C_1 = \lambda_1 \times r_1 \times p_{s1} \times R_1 \quad (4.2)$$

bits-m/sec/m$^2$/Hz, where $R_1$ is the average number of bits per primary packet transmission per Hertz. We model $R_1$ with Shannon’s capacity formula, which we write

$$R_1 = \log_2(1 + \gamma_1), \quad (4.3)$$
where the targeted received signal to interference plus noise power ratio (SINR) at the intended primary receiver is

$$\gamma_1 = \frac{P_1 f(r_1)}{T_{s1}}.$$ (4.4)

In this dissertation, we assume the protocol parameters for the primary network are fixed; specifically, the average intensity of primary users $\lambda_1$, the transmission power $P_1$, and the packet success threshold $T_{s1}$. We will consider different values for $T_{s1}$ in our simulations. Large values of $T_{s1}$ correspond to small target SINR at the receiver, and vice versa. However, the measured SINR at the receiver, which depends on the $\lambda_1$ and $P_1$ could be much higher. This gap between the measured and target SINR is potentially unused spectrum resources, and can be exploited by the secondary users.

We consider a secondary network seeking to exploit these unused spectrum resources. The prerequisite for operation in the shared channel is ensuring a tolerable degradation on the primary network. Secondary users are spatially distributed according to another two-dimensional homogeneous Poisson point process $N_2$ on the infinite plane with mean intensity $\lambda_2$ users per meter$^2$. Each secondary user senses the channel in the beginning of each time slot. Based on this sensing outcome, the secondary user makes a local decision, whether it is permitted to send a packet using the rest of the slot or reattempts at a later time.

We assume packet transmissions from the primary users are longer than those from the secondary users. If we do not consider fading, it is reasonable to assume the interference due to the primary network experienced by a secondary
user is constant over a time slot. In other words, secondary users seek to find regions in space where the spectrum is underutilized. This could also model a primary network whose users are making persistent and constant transmissions. An example is television signal broadcasting.

**Clustering of Secondary Users**

One phenomenon that results from secondary users measuring their incident received power due to the primary network in the beginning of the time slot, is the spatial clustering of eligible secondary users. For each realization of the primary users, the interference observed by secondary users are correlated spatially. Therefore, secondary users that sense an idle channel, and become eligible for packet transmissions tend to be in close proximity to each other. If too many eligible secondary users send a packet at the same time, their combined effect will have a detrimental impact on the primary users. Likewise, this affects the performance of the secondary network; since, the transmitting secondary users are also in close proximity to each other. Our targeted cognitive MAC protocol addresses this problem. Recall the targeted MAC protocol for the secondary user is distributed, and requiring only local channel state information.
4.4 Cognitive MAC

We now describe the proposed medium access strategy for secondary users. This can be separated into two stages, namely sensing for white spaces and sensing for access. Sensing for white spaces involves finding regions where the spectrum is underutilized by the primary network, while sensing for access involves actual packet transmission by secondary users. This strategy assumes knowledge of $T_{s1}$ by the secondary users.

4.4.1 Sensing for White Spaces

In the first segment of the slot, secondary users sense the channel by measuring their incident received power. This measurement include the aggregate interference from primary users plus thermal noise power. If this measurement falls below an idle threshold $T_i$, then the secondary user is eligible for packet transmission. Otherwise, the secondary user is inhibited and retries in the next time slot. Thus, the probability of a secondary user finding spatial opportunities is given by

$$p_i = \Pr (I_1 + \eta \leq T_i).$$  \hfill (4.5)

Note that the statistics of $I_1$ depends on $\lambda_1$ and $P_1$. While it may be easier for secondary users to find regions of underutilized spectrum with a higher $T_i$, the increased number of transmitting secondary users will result in a larger impact on the primary network.
In our model, we assume $I_1$ to be statistically larger than thermal noise power $\eta$. This is reasonable since, $I_1$ is the aggregate signal power from infinitely many primary users on the infinite plane; moreover, we are interested in a reasonably dense primary network. For these reasons, the *silence periods* to measure the primary network in isolation will suffice. However, for a sparse primary network, the statistics of $I_1$ at a secondary user is typically dominated by the closest primary user. Signals from the primary users could fall below thermal noise power; in this case, detection of the primary signals is necessary.

### 4.4.2 Sensing for Access

To address the clustering issue and the aggregate impact of the transmitting secondary users on primary receivers, an eligible secondary user waits for a random amount of time, and senses the channel again. If the subsequent measured received (primary and secondary interference plus thermal noise) power is below $T_{s1}$, the secondary user sends the packet in the remaining slot. Otherwise, the secondary is prohibited from transmitting and restart the access procedure in the next time slot.

The design philosophy for this access mechanism is two-fold. First, secondary transmissions are staggered in time. This prevents the eligible secondary users from sending simultaneously. The second sensing requirement limits the total amount of interference due to the secondary network. This prevents the accumulated effect from the secondary users from degrading the performance of the
primary network beyond tolerable limits. The other practical benefit is no sophisti-
catical receivers is needed by the secondary users to distinguish the received power
from the multiple primary and secondary users.

Therefore, the first sensing outcome relative to $T_i$, is an instantaneous
measure of total interference due to the primary network. This influences the
number of secondary users eligible for transmission. The second sensing outcome
relative to $T_{s1}$, attempts to enable secondary packet transmissions gradually until
total interference in the system exceeds $T_{s1}$. The success of each primary or sec-
ondary packet transmissions depends on the received signal to total interference
plus thermal noise power at its intended receiver. Measuring of received incident
power at the transmitter is only an estimate on the interference environment at its
intended receiver. This assumes the interference at the transmitter and receiver
are correlated, despite that it may not be the case when the distance between the
transmitter and receiver is sufficiently large. Finally, this protocol does not require
any signaling or control message exchanges.

We adopt a similar metric for the performance of the secondary network.
Specifically, the network throughput for the secondary network is given as

$$C_2 = \lambda_{2,a} \times r_2 \times p_{s2} \times R_2$$  \hspace{1cm} (4.6)

bits-m/sec/m$^2$/Hz, $\lambda_{2,a}$ denote the intensity of transmitting secondary users per
meter$^2$, and $R_2$ is the average number of bits per secondary packet transmission
per Hertz, which we write

$$R_2 = \log_2 (1 + \gamma_2)$$,  \hspace{1cm} (4.7)
and the target SINR at the intended secondary receiver is

\[ \gamma_2 = \frac{P_2 f(r_2)}{T_{s2}}, \]

where \( T_{s2} \) is the packet success threshold for the secondary user and the probability of a successful secondary packet transmissions is given by

\[ p_s = \Pr(I_1 + I_2 + \eta \leq T_{s2} \mid I_1 + \eta \leq T_i), \]

where \( I_1 \) and \( I_2 \) are the total received interference from other concurrent primary and secondary transmitters respectively.

4.4.3 Intensity of Transmitting Secondary Users

A secondary user transmits a packet if two conditions are met. First, the presence of a white space. This is based on comparing the total received interference power from other primary users to an idle threshold, \( T_i \). Second, the total interference (primary plus secondary) plus thermal noise power falls below a second threshold, \( T_{s1} \). We can write the intensity of transmitting secondary users as

\[ \lambda_{2,a} = \lambda_2 \times p_i \times \Pr(I_1 + I_2 + \eta \leq T_{s1} \mid I_1 + \eta \leq T_i) \]

users per meter\(^2\). It is also clear that the statistics of \( I_2 \) is a function of \( P_2, \lambda_{2,a} \) and \( I_1 \). Thus, (4.10) is a fixed point equation in \( \lambda_{2,a} \). However, any attempts to find an analytical solution for (4.10) may be futile; since the density of \( I_2 \) depends not only on the primary users, but also among other secondary users. Recall we model the positions of the secondary users by a two-dimensional homogeneous Poisson
point process on an infinite plane. The set of eligible secondary users after the initial sensing are no longer homogeneous. Furthermore, the transmissions from the eligible secondary users are correlated, and the set of transmitting secondary users are no longer Poisson. Nonetheless, (4.10) captures the dynamics between the primary and secondary users, and also among the secondary users. We take the Monte Carlo simulation approach to gain further insights on this problem.

4.4.4 Estimating Access Threshold

We assumed a priori that users from the secondary network know the value of the $T_{s1}$, which is the success threshold for primary users. This knowledge may be known in variety of ways. First, this may be provided as part of the design specifications for the secondary network. Alternatively, this information may also be propagated from the primary to secondary network through beacon messages. Finally, since the aggregate interference from primary network is Poisson shot noise and approximately Gaussian, we have

$$T_{s1} \approx m_{I_1} + k_i \cdot \sigma_{I_1},$$

(4.11)

where $m_{I_1}$ and $\sigma_{I_1}$ denote the mean and standard deviation of $I_1$, and $k_i = \sqrt{2} \text{erf}^{-1}(2\gamma - 1)$ and $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the error function, and $\gamma$ is probability of sensing a white space. These parameters can be calculated from the past samples of $I_1$ obtained during the first sensing phase. Also, $k$ is a predetermined design parameter. For instance, if requirements for successful packet transmission for the primary network is 99.9%, then we set to $k = 1$. In fact, these statistics
can also be found if the intensity of primary users is known a priori, that is,

\[ m_{I_1} \approx \frac{2\pi \lambda_a P_1}{(\alpha - 1)(\alpha - 2)}, \]  

(4.12)

\[ \sigma^2_{I_1} \approx \frac{2\pi \lambda_a P^2_1}{(2\alpha - 1)(2\alpha - 2)}. \]  

(4.13)

4.5 Simulation Setup

We simulate a discrete time system. The network consists of primary and secondary users placed randomly on the plane of size \( L \times L \) according to a Poisson point process with mean \( \lambda_1 \) and \( \lambda_2 \) respectively. To address the edge users, we consider a two-dimensional torus structure. In other words, the points in the center square of size \( L \times L \) is repeated 8 times around it. The total interference power incident on each user is then due to any points within an area of \( L \times L \) relative to that user lying in the middle.

We assume users always have a packet to send. Primary users transmit packets of \( k \) slots, and remain immobile until that packet transmission is completed. The set of users locations in each time slot for the secondary users, and every \( k \) time slots for the primary users changes according to its associated point process. This is despite the success or failure of the previous transmission. Each receiver associated with each primary transmitter is located at fixed distance \( r_1 \), and with uniform angular distribution. Recall a packet transmission is a success if the total interference (primary plus secondary) plus thermal noise power at the intended
primary receiver falls below $T_{s1}$. Let $P_1$ and $\alpha_1$ denote the transmission power and path loss exponent for signals in the primary network.

Secondary users communicate with each other over the same channel, occupied by the primary users. The exchange of messages are carried over one slot time packets. In the beginning of a slot, each secondary user measures the total incident power from other primary transmitter. This measurement is compared against an idle threshold $T_i$. If the interference falls below $T_i$, then the secondary user becomes eligible for transmission. Otherwise, it tries again in the next time slot. An eligible secondary user then measures the total incident power again after a random time. If the second measured interference falls below the success threshold $T_{s1}$, then the secondary user sends a packet. Otherwise, the secondary user starts all over again in the next time slot. Let $P_2$ and $\alpha_2$ denote the transmission power and path loss exponent for signals in the secondary network. The intended secondary receiver is located at fixed distance $r_2$, and with uniform angular distribution from the secondary transmitter. The packet transmission is a success if the total interference (primary plus secondary) plus noise at the intended secondary receiver falls below $T_{s2}$.

Typical simulations are run for 20,000 slots, with the primary and secondary networks having 100 nodes each for $\lambda_1 = \lambda_2 = 1.0$ and 150 secondary nodes for $\lambda_2 = 1.5$. We let $\eta = 0.01$. 
4.6 Results

We present simulation results for two different scenarios: first with the primary network operating close to optimal performance; and the second when the primary network is in suboptimal operation, and there is unused spectrum resources in the system.

In the first set of experiments, we study the effects of $\lambda_2$ and $T_i$ on the performance of the primary network. We take $T_{s1} = 1.0$, which implies the primary network is in high SINR region operation. Any secondary network activity will have significant consequences. In Figure 4.1, we plot the results for $C_1$ (PRI), $C_2$ (SEC) and $C_1 + C_2$ (SUM) for two values of $\lambda_2 = 1.0, 1.5$. We see an increasing number of eligible secondary users for $T_i = 0.3$ or larger. Note that once secondary users start packet transmissions, there is significant degradation on the primary network throughput. For example, consider $\lambda_2 = 1.0$ and $T_i = 0.7$, for small secondary network throughput of 0.0071 bits-m/sec/m$^2$/Hz, there is a 56% degradation on the primary network throughput; while a 80% degradation is observed on the primary network throughput for $\lambda_2 = 1.5$. This is expected, since there are more secondary users attempting to access the medium.

Refer to Figure 4.2. We now look at the intensity of successful transmissions for the secondary users, denoted by $\lambda_{2s}$, for $\lambda_2 = 1.0, 1.5$. Consider first $\lambda_2 = 1.0$, we see that at $T_i = 0.3$, some secondary users start to transmit, and they continue to be increasingly successful until $T_i = 0.65$; and beyond which, the intensity of successful transmissions for the secondary users start to degrade. There
Figure 4.1 Primary and secondary network throughput for different $T_i$ thresholds for $T_{s1}=1.0$

are two reasons for this degradation. First, there is an increasing number of secondary users start packet transmissions, and these secondary users interfere with each other. Second, the set of transmitting secondary users are in close proximity to each other, thus experiences stronger self-interference. Our proposed MAC protocol not only has take into account the interference from the primary network, but also the self-interference among the transmitting secondary users. Note that if the primary network is already operating closer to optimal performance, any activity from the secondary users leads to a large degradation on the primary users.

For $\lambda_2 = 1.5$, a similar behavior is observed for increasing $T_i$. Note the effect of self-interference among the secondary users is more pronounced. This is because more secondary users are present, and transmitting. One interesting
Figure 4.2 Intensity of successful transmission for secondary users for different $T_i$ thresholds, and $T_{s1}=1.0$

observation is for each value of $\lambda_2$, there exist an optimal $T_i$ which maximizes the network throughput of the secondary users. Thus suggesting adaptive schemes to pick $T_i$ may be desirable to maximize secondary network throughput.

In the next two figures, we consider $T_{s1} = 2.0$. This lies in the regime where the primary network is operating at a lower SINR compared to the case for $T_{s1} = 1.0$. For $T_i = 0$, the secondary network is considered absent in the system. Note that the primary network is operating in suboptimal operation compared to $T_{s1} = 1.0$, and the higher primary network throughput at $T_i = 0$ in Figure 4.1 than in Figure 4.3. Thus suggesting unused spectrum is available in the latter case for the secondary network.

In Figure 4.3, the network throughput of the primary network is unaf-
Figure 4.3 Primary and secondary network throughput for different $T_i$ thresholds for $T_{s1}=2.0$

affected when the secondary users start packet transmissions. This implies our cognitive MAC scheme is effective in locating the regions of low spectrum utilization, while ensuring minimal or no impact on the primary users. Consider $\lambda_2 = 1.0$ and $T_i = 1.0$, the primary network degrade by 4%, but the secondary network achieved a network throughput of 0.0356 bits-m/sec/m$^2$/Hz. This is more than 50% of the primary network throughput. We also see a higher secondary network throughput gain in $\lambda_2 = 1.5$ compared to $\lambda_2 = 1.0$. The reason being there are more secondary users on the plane looking for white spaces, and are successful with their packet transmissions. Note that at $T_i = 1.0$ and $\lambda_2 = 1.5$ the self-interference from other secondary users starts to have an impact on one another.

In Figure 4.4, we see an increasing intensity of successful transmission for
secondary users for increasing values of $T_i$; since, there are more secondary users seeking out white spaces, and are successful in packet transmissions. Also note that the value of $T_i$ is not large enough to observe a degradation due to self-interference among the secondary users in Figure 4.2.

4.7 Conclusion

In this chapter, we considered an ad-hoc architecture for a secondary network on an infinite plane, whereby secondary users seek to exploit spatially the unused spectrum resources in the system. The amount of unused spectrum resources available is conditioned on how close the primary network is operating
to optimal performance. To this end, we proposed a simple, distributed MAC protocol for the secondary user to exploit this excess spectrum resources. We account for the total interference from all transmitting secondary users, and limit the degradation on the primary users. We have shown by simulations that when excess spectrum resources is available in the system, our simple protocol achieves higher spectral efficiency in the system with minimal degradation on the primary users. Careful parameters optimization for the secondary network is necessary for future work, in order for the secondary network to fully exploit these unused spectrum resources.

The text of this Chapter in part, is a reprint of material as it appears “On Physical Carrier Sensing for Cognitive Radio Networks,” in the Proceedings of the Allerton Conference on Communication, Control and Computing, 2007, E. C. Wong; R. L. Cruz. The dissertation author was the primary investigator and author of this paper.
Chapter 5

Closing Remarks

In this dissertation, we developed spatio-temporal models for a wireless network using physical carrier sensing to study its performance. We accounted for transmit power, packet duration, distance to intended destination, intensity of transmission attempts, idle threshold rate and per packet transmission rate in our analysis. Unlike previous studies, we considered an interference model based on infinitely many concurrent transmissions on an infinite plane. We adopt a threshold-based inhibiting model, and take into account the inhibitory effects around the source device when we calculate the success probability at the intended destination. We showed that our analysis matches the simulation reasonably well. In addition, we showed that with careful choices of protocol parameters, significant network throughput can be gained through physical carrier sensing.

One key advantage of physical carrier sensing is its simplicity, and therefore can be scaled easily to large wireless networks. However, the question of how
and which of these protocol parameters should be adapted is addressed in Chapter 3. From our analysis, we found that network throughput is a function of the ratio of idle threshold to transmit power, as well as the ratio of packet detection threshold to transmit power. Transmit power is in fact a scaling factor, i.e., we need not consider transmit power as an independent parameter to be optimized. Based on this, the problem reduces to the tradeoff between the number of concurrent transmissions and per packet transmission rate.

To study this problem, we make use of insights obtained through simulations. Assuming a certain attempt intensity, it is found that there is an associated SINR and success probability when network throughput is maximized. We found that the optimal SINR is much smaller than current systems, thus suggesting wireless networks today are not optimized. The optimal SINR depends on the intensity of transmission attempts. We also found that the success probability is high and approaches one when the intensity of transmission attempts increases. However, unlike what is suggested in other studies, the targeted success probability for maximum network throughput is not one. With this and the property of Gaussian process, we developed a heuristic to relate the idle threshold and packet detection threshold, which can be used to jointly adapt these two parameters. Finally, we proposed an algorithm which allows the network to achieve optimal operation.

Up to now, the focus is on a homogeneous network where every device adopts physical carrier sensing. In other words, each device can adapt its idle threshold or rate accordingly to react to network conditions. What if the network
consist of devices that do not adopt physical carrier sensing? In other words, these are passive devices e.g. broadcast TV stations. We considered these devices to be part of the primary network. In Chapter 4, we considered a secondary network consists of devices that adopt physical carrier sensing. Carrier sensing allows the secondary devices to find and exploit white spaces spatially. If all eligible secondary devices start to send a packet, their aggregate impact will cause the primary user to fail. In order to address this problem, we proposed a two-threshold physical carrier sensing. We showed that based on this modification to the CSMA protocol, the secondary network can achieved significant throughput while minimizing the degradation on the primary users.

A possible extension of our work is using physical carrier sensing to alleviate periods of high loads. For instance, certain parts of the network may experience momentary high levels of offered load. In such cases, there is degradation in the performance due to excessive interference. How should devices in these regions adapt the idle threshold? One possible drawback is when a sudden spike in offered load is experienced this will cause the devices to switch to a low idle threshold. However, if all the affected devices switches simultaneously, this causes the offered load in the network to oscillate. Hence, there may be instability issues in the system if due care is not taken.

Delay is another issue not addressed in our work. If a delay intolerant applicant is supported at a particular device, what should the appropriate idle threshold be in order to meet the delay constraints? If the targeted idle threshold
is set too low, a device would have to wait for a long time for the medium to be quiet. On the other hand, if the targeted idle threshold is set too high, packet transmissions will become increasingly likely to fail due to increased levels of interference. This would then result in increased delay due to multiple retransmissions. It is an interesting problem especially since applications like Voice-over-IP (VOIP) and video conferencing are becoming increasingly popular in recent days.

Finally, it is not uncommon to find a device running multiple applications. Each of these applications could have its own delay and bandwidth demands. This suggests for having application-specific idle thresholds in the devices. As a result, choosing the correct idle threshold becomes a design issue for each flow, rather than each device. This meant having different classes of traffic based on idle thresholds.
Bibliography


