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Managing a closed-loop supply system with random returns and a cyclic delivery schedule

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ABSTRACT

Motivated by an industry example, we develop a mathematical framework to address the inventory replenishment and capacity planning problem for a closed-loop supply system with random returns. The provider needs to deliver new or refurbished products to a group of clients under a fixed cyclic schedule, and also collects back a random portion of the used products in the subsequent delivery cycle for refurbishment. We first address the product replenishment strategy, in which only a random portion of the delivered products will be returned for refurbishment and the supplier must regularly purchase new products to replace the lost units. We then analyze the capacity decision problem where the provider uses his facility to refurbish the returned products for reuse, and the provider could incur extra refurbishing cost to handle the returned product at the end of each cycle due to insufficient capacity. Our models provide a simple decision support tool for making effective replenishment and capacity decisions in managing such a closed-loop supply system.

1. Introduction

We recently visited a supply company that offers a variety of facility products such as cloth table napkins used at local restaurants. The supply company delivers some fixed amount of the clean napkins to the individual restaurant owners and then collects back the soiled napkins based on some pre-determined weekly delivery and collection schedule. The exact delivery quantity and delivery interval depend on the specific usage of each individual restaurant. The soiled napkins returned are then cleaned, packed and recycled for future use. This supply company offers a variety of napkin styles, e.g., white or black, and each style can be shared among a number of local restaurants. One interesting feature of this problem is that the amount of soiled napkins returned is only a (random) fraction of the delivered quantity, and it is difficult and expensive for the supply company to count the exact quantity of each type of the soiled napkins from each individual restaurant before they are sorted and combined with those returned from the other restaurants.

The above example motivates us to study the inventory and capacity management issues in a closed-loop supply recovery system with random returns. Specifically, we consider a closed-loop system involving a single product supplied by a service provider to its clients. The provider delivers a fixed quantity of this product to its clients using a cyclic delivery schedule, and at the same time, collects back the used products from clients delivered in the previous cycle. These used products are then refurbished by the provider, and recycled back to the clients for future use. An important feature of this system is that only a (random) fraction of the delivered products will be recycled back for refurbishment, as some of these products are lost or deemed non-usable due to normal wear and tear. Our model applies to an operating environment in which a firm uses recyclable components or products that will be returned and refurbished for future use under a regular delivery and collection schedule. In another application of our problem setting, Straus uses reusable glass bottles for their creamery products, and they deliver new inventory of creamery products and pick up the empty used bottles from the farmers markets (such as Whole Foods, Mother's Market and others) on a regular basis.[1]

The provider faces two important operations issues. First, the provider needs to determine an effective replenishment strategy for their products. As a random portion of the products (new or refurbished) is lost during each delivery cycle, the provider must

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[1] See http://strausfamilycreamery.com/values-in-action/reusable-glass-bottles for more information.

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regularly purchase new products from its supplier to replace the lost units in order to maintain an adequate supply of the products to its clients. We refer to this as the optimal replenishment problem.

Second, the provider needs to maintain an appropriate capacity in its facility to refurbish the returned products, so that the refurbished products can be recycled back to the closed-loop supply system in a timely manner. Since the amount of returned products is random, the provider must plan the capacity effectively to minimize the operating cost of refurbishment. We assume that any returned products that cannot be refurbished at the end of each time period due to capacity constraint can be either carried over to the next period, or processed using overtime (or possibly outsourced) at a higher unit operating cost. The provider must carefully take into account the random return rate of the used product in planning the refurbishing capacity in order to balance between excess idle capacity and the higher operating cost due to overtime processing. We refer to this as the optimal capacity problem.

We develop an analytical model to address these two operational problems. For the optimal replenishment problem, we assume that the number of returned products follows a Binomial distribution based on the delivery quantity and the average return rate of the products. We first develop a decision model to determine the optimal replenishment policy. Then, we analyze how the different model parameters affect the optimal replenishment policy and provide some numerical results to illustrate several basic insights derived from our model.

For the optimal capacity problem, we develop a simple approximation scheme to estimate the amount of returned products to be refurbished under some fixed refurbishment policy. We also perform a set of numerical experiments to demonstrate the accuracy of this approximation scheme and compare the effectiveness of this approximation scheme with two other simple approaches in selecting the optimal refurbishment capacity that minimizes the average capacity and processing costs. Our numerical results show that the approximation scheme provides near-optimal performance and can be used as an effective decision support tool in capacity planning for the facility.

The paper is organized as follows. In Section 2 we briefly review the existing research literature on managing inventory and capacity in closed-loop supply chains. In Section 3 we describe our model setup and introduce the basic notation used in our model. We then analyze the optimal replenishment problem and the optimal capacity problem in Section 4 and Section 5, respectively. In Section 6 we provide our concluding remarks and illustrate two important applications of our models in managing such a closed-loop supply system.

2. Literature review

Our research is concerned with the management of closed-loop supply chains with recoverable products. Product recovery management is an important element in sustainable supply chain management, in which the manufacturer can recover, reuse, and remanufacture some of the used products (or its components) in a closed-loop supply chain. It has been well documented that an effective recovery and remanufacturing process coupled with an efficient closed-loop supply chain can be profitable for a number of popular product categories including single-use cameras, toner cartridges, glass bottles, computer chips, and automobile batteries; see Ayres, Ferrer, and Leynseele (1997), Davis (1996), Ginsburg (2001), and Kodak (2001). Given the importance of sustainable supply chain management with recoverable manufacturing systems, research interest in managing closed-loop supply chains has been increasing in the past two decades. Researchers have studied various operational and strategic issues arising from a closed-loop supply chain ranging from production planning, inventory control, quality management, reverse logistics network design, product recovery management to pricing strategy, and competition between new and remanufactured products. We refer the readers to Guide and van Wassenhove (2006) and Verter and Boyaci (2007) for two special journal issues on some earlier research on closed-loop supply chains and reverse logistics, as well as Atasu, Guide, and van Wassenhove (2008) and Souza (2013) for two recent reviews.

Broadly speaking, Kenne, Dejax, & Gharbi (2012) describe four categories of return items that have been commonly analyzed in closed-loop supply chains: reuseable items (such as returned pallets that do not require any rework and can immediately put back into use), repair services (where products are sent back to customers after repair), remanufacturing (an industrial process where used products are put back into the system after refurbishing) and recycling of raw materials and waste. Our paper focuses on some specific inventory and capacity management issues for repair services or remanufacturing under an operating environment as described in Section 1.

There exists an extensive literature on using quantitative models for studying inventory and capacity management issues in closed-loop supply chains. We refer to a recent comprehensive review article by Akcali and Cetinkaya (2011) who classify this existing quantitative literature based on the product characteristics and quantitative nature of the models. Our paper falls into the category of stochastic demand and/or return models for single-item problems. However, our problem context consists of some unique features that have not been addressed in the extant state-of-the-art literature in this area.

First, the products are continuously used and refurbished in the closed-loop system, which does not require coordination between new and recycled products until the amount of recycled products in the system becomes low and a replenishment of new products is required. Second, the system has a cyclic delivery schedule and the facility needs to refurbish all returned products at the end of each delivery cycle. This cyclic nature of demand (along with uncertain product returns) has implications on the capacity investment needed to reintroduce used products back into the system, and provides a different set of issues from the general closed-loop supply chains. To the best of our knowledge, closed-loop supply systems with joint inventory replenishment, cyclic scheduling, and capacity planning decisions have not been addressed in the existing literature.

One research stream of particular relevance to our work is on inventory and production planning for remanufacturing using pull or push management strategies. For example, van der Laan, Salomon, Dekker, and van Wassenhove (1999) analyzed a hybrid system for a single durable product in which the outputs of the manufacturing process of new products and the remanufacturing process can be used to fulfill customer demands. The main decision is whether the returned products should be remanufactured as soon as available (push strategy) or as late as convenient (pull strategy). van der Laan and Teunter (2006) further developed heuristics for finding near-optimal pull and push strategies. Heyman (1977); Kelle and Silver (1989); Muckstadt and Isaac (1977) and van der Laan, Dekker, Ridder, and Salomon (1996), among others, have also studied inventory models that use push strategy to manage single-item products with random returns. In contrast to these existing models, our model assumes that all returned products will be remanufactured when available using a fixed regular capacity, but need to be fully processed at the end of each delivery and collection cycle.

Finally, some literature on cyclic scheduling is also of relevance to our research. First, there exists an extensive literature on the inventory routing problem where a vendor needs to replenish the inventories of a set of customers repeatedly, and an effective cyclic delivery schedule is deployed for the product delivery to minimize the transportation and inventory costs. We refer to Ekici, Ozener, and Kuyzu (2014) for some discussions and references in this research stream. Our paper assumes that the cyclic schedule is pre-determined and does not explicitly address the construction of effective cyclic construction, but instead focuses on the optimal inventory replenishment strategy due to the (random) loss in the returned products. Second, one research stream in the literature has examined the performance of cyclic schedules in a re-entrant manufacturing environment. For example, Zhang and Graves (1997) have studied the behavior of cyclic schedules in a stochastic re-entrant flow shop where the machines can be subject to random failures, and found effective cyclic schedules that would minimize task delays. In contrast, our research focuses on finding the optimal operating capacity for a given cyclic schedule subject to random returns.

3. Model setup

Consider a firm that must make one delivery of some fixed quantity of a (new) product to a number of clients in a cyclic schedule, e.g., weekly deliveries. Each cycle consists of a fixed number of delivery periods (e.g., 5 days in a weekly delivery cycle), and each client is being assigned to one of these delivery periods based on her geographical location or delivery quantity. When the firm delivers the products to the client locations in each cycle, he also collects back (used) products from his clients delivered in the previous cycle.

The used products by the clients are subject to random returns, and some returned products are not reusable due to severe wear and tear. All returned products collected from the clients in each period are combined together before refurbishment, and it is not possible (or economical) for the firm to separate the returned products from individual clients to determine her specific return/reusable proportion. For simplicity, our model assumes the same probability for a new product delivered in a cycle to be returned and reusable in the subsequently period, and we refer it as the reusable probability.

All returned products collected from the clients in a period need to be first pre-processed (e.g., cleaned and inspected), and all reusable products can then be refurbished at the firm's refurbishment facility in the next period. All refurbished products are recycled back for future use as new products. The refurbishment facility has a fixed capacity. In the event that the amount of reusable products collected in a period exceeds the refurbishment capacity, the extra reusable products will be refurbished in the subsequent periods. At the end of a delivery cycle, any leftover amount of reusable products will be refurbished at an extra cost (e.g., via overtime over the weekends) and will be available for delivery as new products at the start of the next cycle. However, the returned products collected during the last period of a cycle are only available for refurbishment in the first period of the next cycle, as the resources required to pre-process the returned products are not available until the start of the next cycle.

We introduce the following notation:

M	=	number of periods in a cycle,
Q	=	refurbishment capacity per period,
d_m	=	delivery quantity of new products for period m in each cycle,
d	=	$\sum_{m=1}^{M} d_m$ = total delivery quantity of new products in one cycle,
p	=	the reusable probability of a new product,
R_m^n	=	amount of reusable products collected during period m in cycle n,
X_m^n	=	amount of available new products at the beginning of period m in cycle n,
Y_m^n	=	amount of reusable products to be refurbished at the beginning of period m in cycle n,

The sequence of events in cycle n is as follows. (1) At the beginning of period m, $m = 1, 2, \ldots, M$, d_m units of new products are delivered during the period, and reusable products collected from the previous period are pre-processed for refurbishment. (2) During period m, the firm delivers d_m units of new products and also collects back R_m^n units of reusable products, and the facility refurbishes the available reusable products collected from the previous periods up to the maximum capacity of Q. (3) All refurbished products are recycled back to the inventory of new products. (4) Any reusable products leftover at the end of period M will be refurbished at an extra cost. Fig. 1 provides a timeline of the event sequences for product delivery, collection and refurbishment.

Using the above notation, the amount of reusable products refurbished at period m in cycle n is equal to $\min(Y_m^n, Q)$. Therefore, the amount of new products and reusable products to be refurbished at the beginning of period m ($m = 1, 2, \ldots, M$) in cycle n are given by the following relationships:

$$\begin{aligned} X_1^n &= X_M^{n-1} + \min(Y_M^{n-1}, Q) - d_m \\ Y_1^n &= (Y_M^{n-1} - Q)^+ + R_M^{n-1}, \end{aligned} \tag{1}$$

and for $m \geq 2$,

$$\begin{aligned} X_m^n &= X_{m-1}^n + \min(Y_{m-1}^n, Q) - d_{m-1} \\ Y_m^n &= (Y_{m-1}^n - Q)^+ + R_{m-1}^n. \end{aligned} \tag{2}$$

Since any reusable products left at the end of each cycle will be refurbished (e.g. via overtime), (1) is reduced to

$$\begin{aligned} X_1^n &= X_M^{n-1} + Y_M^{n-1} - d_m \\ Y_1^n &= R_M^{n-1}. \end{aligned} \tag{3}$$

We substitute (2) repeatedly into (3) to obtain

$$\begin{aligned} X_1^{n+1} &= X_M^n + Y_M^n - d_m \\ &= [X_{M-1}^n + \min(Y_{M-1}^n, Q) - d_{M-1}] + [(Y_{M-1}^n - Q)^+ + R_{M-1}^n] - d_m \\ &= X_{M-1}^n + Y_{M-1}^n + R_{M-1}^n - (d_{M-1} + d_m) \\ &= \ldots \\ &= X_1^n + Y_1^n + (R_1^n + \cdots + R_{M-1}^n) - (d_1 + \cdots + d_m) \\ &= X_1^n + (R_1^n + \cdots + R_{M-1}^n + R_M^{n-1}) - (d_1 + \cdots + d_m). \end{aligned} \tag{4}$$

As each new product delivered in a cycle has the same reusable probability p in the subsequent cycle, the amount of reusable products collected during period m, R_m^n, follows a Binomial distribution with parameters (d_m, p). Also, since any reusable products collected at the end of the time period will be refurbished during the next time period, the total amount of reusable products refurbished in cycle n, $(R_1^n + \ldots + R_{M-1}^n + R_M^{n-1})$, follows a Binomial distribution with parameters (d, p). Therefore, we can rewrite (4) as

$$X_1^{n+1} = X_1^n - B_n = \cdots = X_1^1 - \sum_{j=1}^{n} B_j, \tag{5}$$

where B_j represents the amount of products lost in cycle j, and B_j's are i.i.d. random variables with a Binomial distribution with parameters $(d, 1-p)$. Therefore, the total amount of products lost in n cycles, $\sum_{j=1}^{n} B_j$, follows a Binomial distribution with parameters $(nd, 1-p)$, with $E(\sum_{j=1}^{n} B_j) = n(1-p)d$ and $Var(\sum_{j=1}^{n} B_j) = np(1-p)d$.

4. The optimal replenishment problem

Let an N-replenishment policy denote the policy that the facility will replenish the amount of lost products once every N cycles. We formulate the decision problem as to determine the optimal N-replenishment policy with the corresponding initial amount of available products which minimizes the total expected ordering and holding costs subject to the service constraint that the probability of a stockout occurred during the N-replenishment cycle is

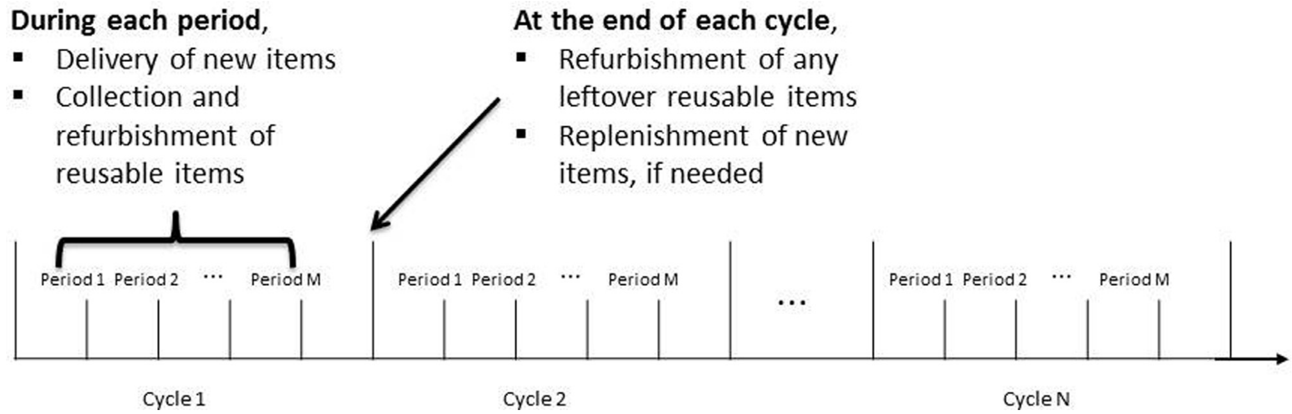

Fig. 1. A timeline for the delivery, collection and refurbishment sequence.

no more than α, where α is the desired service level specified by the manager.

Suppose that the initial amount of new products at the beginning of the replenishment cycle is given by x_0, i.e., $X_1^1 = x_0$. For any given N, a stockout occurs during the next N cycles if $X_m^n < d_m$ for some $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$. It is clear from Eq. (5) that X_1^n is decreasing in n, i.e., the available amount of new products for delivery at the beginning of each cycle decreases within the N-replenishment cycle. This implies that the probability of a stockout is the highest during the last cycle N, i.e., $P(X_m^N < d_m) \geq P(X_m^n < d_m)$ for all $m = 1, 2, \ldots, M$, and $n = 1, 2, \ldots, N-1$. Thus, we can simply approximate the probability of a stockout occurred during the N-replenishment cycle as $P(X_1^{N+1} < 0)$, or equivalently, $P(\sum_{j=1}^{N} B_j > x_0)$ in view of (5). Furthermore, we approximate the total amount of products lost in N cycles, $\sum_{j=1}^{N} B_j$, by a normal distribution with mean of $N(1-p)d$ and variance of $Np(1-p)d$. For a fixed N, we can set the initial amount of new products under the N-replenishment policy as

$$x_0 = N(1-p)d + z_\alpha \sqrt{Np(1-p)d}, \tag{6}$$

such that the stockout probability is no more than α, where z_α is the normal constant corresponding to the specified service level α.

We consider the following costs incurred for every N cycles: (1) a fixed ordering cost s, (2) procurement cost of new products to replenish lost products in the expected amount of $N(1-p)d$ with unit product cost of c, and (3) holding (opportunity) cost of initial capital investment in new products (equal to cx_0) at the beginning of each replenishment cycle with a unit holding cost per cycle of h. Then, the total expected cost during the N cycles is equal to $s + cN(1-p)d + hN(cx_0)$. Using (6), we can express the average total cost per cycle under an N-replenishment policy as:

$$\begin{aligned} g(N) &= \frac{1}{N}\{s + cN(1-p)d + hN(cx_0)\} \\ &= \frac{s}{N} + c(1-p)d + hc\left[N(1-p)d + z_\alpha\sqrt{Np(1-p)d}\right]. \end{aligned} \tag{7}$$

Therefore, our decision problem is to find the optimal N that minimizes the average total cost per cycle $g(N)$ with the corresponding optimal initial amount of new products x_0 given in (6).

Consider N as a continuous variable for now. Taking the first derivative of $g(N)$ with respect to N, we obtain

$$g'(N) = \frac{-s}{N^2} + hc(1-p)d + \frac{hcz_\alpha}{2}\sqrt{\frac{p(1-p)d}{N}}. \tag{8}$$

We can derive the following analytical properties for the average cost function $g(N)$.

Proposition 1. *There exists a unique solution y^* to the first-order condition $g'(y^*) = 0$. Furthermore, the function $g(N)$ is unimodal, and y^* minimizes $g(N)$.*

Proof of Proposition 1. We take the second derivative of $g(N)$ with respect to N and obtain

$$g''(N) = \frac{2s}{N^3} - \frac{hcz_\alpha\sqrt{p(1-p)d}}{4N^{3/2}} = \frac{1}{N^{3/2}}\left(\frac{2s}{N^{3/2}} - \frac{hcz_\alpha\sqrt{p(1-p)d}}{4}\right). \tag{9}$$

Therefore, $g''(x^*) = 0$ when $x^* = (\frac{8s}{hcz_\alpha\sqrt{p(1-p)d}})^{2/3}$, with $g''(x) > 0$ when $x < x^*$ and $g''(x) < 0$ when $x > x^*$. This implies that $g'(x)$ is increasing in x when $x < x^*$ and is decreasing in x when $x > x^*$. Also, it is straightforward to show from (8) that $g'(x) > 0$ when $x > x^*$ and that $g'(x) \to -\infty$ as $x \to 0$. Thus, there exists a unique $y^* \in [0, x^*]$ such that $g'(y^*) = 0$.

Furthermore, $g'(N) < 0$ when $N < y^*$ and $g'(N) > 0$ when $N > y^*$. This implies that $g(N)$ is decreasing in N when $N < y^*$ and is increasing in N when $N > y^*$. In other words, the function $g(N)$ is unimodal, and y^* minimizes $g(N)$. □

It follows from Proposition 1 that the optimal N^* can be readily found. In particular, we can easily compute the unique solution to the first-order condition $g'(y^*) = 0$ numerically. Then, the optimal N^* is given by one of the two neighboring integers by comparing the values of $g(N)$ at these two integer points.

We next provide some results to show how the different model parameters would affect the optimal replenishment cycles N^*. Specifically, we show how the optimal first-order solution y^* given in Proposition 1 changes with respect to the different model parameters.

Proposition 2. *y^* is strictly increasing in s and α, while y^* is strictly decreasing in h and d. Also, y^* is strictly increasing in p for $p > 1/2$.*

Proof of Proposition 2. As shown in the proof of Proposition 1, $g'(N) < 0$ when $N < y^*$ and $g'(N) > 0$ when $N > y^*$. Since the value y^* minimizes the cost function $g(y)$, we have $g'(y^*; s) = 0$. We use the envelope theorem to take the derivative of $g'(y^*; s)$ with respect to s (at a neighborhood of y^*) and obtain

$$\frac{dg'(y^*; s)}{ds} = \frac{\partial g'}{\partial y}\frac{\partial y}{\partial s}|_{y=y^*} + \frac{\partial g'}{\partial s}|_{y=y^*} = 0. \tag{10}$$

Since y^* minimizes $g(y)$, we have $\frac{\partial g'}{\partial y}|y = y^* > 0$. Also, we can differentiate $g'(y^*)$ given in (8) with respect to s and obtain $\frac{\partial g'(y^*)}{\partial s} = \frac{-1}{y^{*2}} < 0$. Therefore, it follows from (10) that $\frac{\partial y}{\partial s}|_{y=y^*} > 0$, which shows that y^* is strictly increasing in s.

Using a similar argument, it is straightforward to show that $\frac{\partial y}{\partial \alpha}|_{y=y^*} < 0$, $\frac{\partial y}{\partial h}|_{y=y^*} < 0$ and $\frac{\partial y}{\partial d}|_{y=y^*} < 0$. Since z_α decreases as α increases, we prove that y^* is strictly increasing in α, and is strictly decreasing in h and d.

Finally, we can differentiate $g'(y^*)$ given in (8) with respect to p and obtain

$$\frac{\partial g'(y^*)}{\partial p} = -hcd + \frac{hcz_\alpha}{4}\left(\sqrt{\frac{p(1-p)d}{y^*}}\right)^{-1/2}\frac{(1-2p)d}{y^*}.$$

This implies that $\frac{\partial g'(y^*)}{\partial p} < 0$ when $p > 1/2$. Following the same argument as above, we can deduce that $\frac{\partial y}{\partial p}|_{y=y^*} > 0$ when $p > 1/2$. Therefore, y^* is strictly increasing in p for $p > 1/2$. This completes the proof. □

Since the optimal replenishment cycle N^* is given by one of the two neighboring integers around y^*, we expect the relationships provided in Proposition 2 for y^* will also apply to N^*. Specifically, the optimal N^* increases as the fixed ordering cost s increases, as a longer replenishment cycle would reduce the frequency of incurring the fixed ordering cost. Also, the optimal N^* decreases as the unit holding cost h is higher, as a shorter replenishment cycle would reduce the initial amount of new products needed, which reduce the overall holding cost. Similarly, the optimal N^* decreases as the total amount of products delivered in each cycle d increases. A larger value of d would require a higher initial amount of new products, which has the same effect as a smaller unit holding cost. Also, the optimal N^* increases as a lower service level is required, i.e., higher α. To explain this result, a lower service level (higher stockout probability α) would require less safety stock and smaller initial amount of new products, which also has the same effect as a smaller unit holding cost. Thus, the optimal N^* increases.

Finally, the optimal N^* increases as the reusable probability of a product p increases as long as $p > 1/2$ (which is applicable to our specific problem here). The intuition for this is as follows. Note that the average amount of products lost in a cycle is proportional to $(1-p)$, whereas the variance is proportional to $p(1-p)$. For $p > 1/2$, both the mean and variance of products lost is strictly decreasing in p. Consequently, it follow from (6) that a higher value of N would not lead to large increase in initial inventory x_0 for a given service level. Moreover, from Eq. (7), the balance between lower fixed cost per cycle and the holding cost can be achieved with a higher value of N as the third term is decreasing in p. Conversely, if $p < 1/2$, an increase in p would lead to higher variance in the amount of products lost and, as such, the tradeoff of lowering the fixed cost using a higher N is negated by an increase in the holding (capital) cost.

Numerical Example:

We provide the following example to illustrate some of our analytical results. Let $s = 1000$, $c = 1$, $h = 0.01$, $p = .9$, $M = 2$, $d_1 = 10,000$, $d_2 = 20,000$, and $\alpha = 0.05$ (with $z_\alpha = 1.645$). In this example, the optimal solution to the first-order condition y^* in Proposition 1 is given by $y^* = 5.756$. Direct computation of the average cost gives $g(5) = 3352$ and $g(6) = 3349$. Therefore, the optimal replenishment cycle $N^* = 6$, with the corresponding initial stocking quantity $x_0 = 18,209$ and the optimal minimum cost $g^* = 3349$.

We next illustrate how the reusable probability p can affect the optimal replenishment cycle N^*, the initial amount x_0 and the corresponding optimal cost g^*. Here, we used the same values for all model parameters as given in the above numerical example, but varied the value of p from 0 to 1. The results are summarized in Table 1. We observe the intuitive results that optimal N^* increases and the optimal cost g^* decreases as p increases, showing the fact that a higher return rate increases the length of the replenishment cycle and reduces the optimal cost, as the facility does not need to replenish the products as frequent. Interestingly, the initial stocking quantity x_0 decreases as p increases when N^* is constant, i.e., less initial stock is required as p increases when the replenishment cycle is constant. However, x_0 is not monotone in general when p increases, as the optimal N^* increases as p increases (for $p > 1/2$).

Table 1
Impact of p on N^*, g^* and x_0.

p	N^*	g^*	x_0
.05	2	29,571	57,088
.10	2	28,041	54,121
.15	2	26,511	51,144
.20	2	24,982	48,161
.25	2	23,452	45,174
.30	2	21,922	42,185
.35	2	20,392	39,192
.40	2	18,862	36,197
.45	3	17,331	49,746
.50	3	15,786	45,247
.55	3	14,241	40,746
.60	3	12,696	36,242
.65	3	11,151	31,735
.70	3	9606	27,226
.75	4	8052	30,247
.80	4	6492	24,228
.85	5	4927	22,727
.90	6	3349	18,209
.95	8	1747	12,176

5. The optimal capacity problem

We next analyze the optimal capacity problem to minimize the average total operating cost of refurbishing the reusable products. Let Q be the maximum refurbishment capacity per period. Also, the facility can use overtime to refurbish all used products left over at the end of each cycle n at a higher cost, and this amount is given by $(Y_M^n - Q)^+$.

We consider three basic cost components in our model: (1) capacity cost of $c_1 Q$, which is incurred regardless of whether all available capacity Q is utilized in each period; (2) normal operation cost of $v(R_1^n + \cdots + R_M^n)$ for refurbishing all reusable products returned in each cycle; and (3) extra operating cost of $c_2(Y_M^n - Q)^+$ for refurbishing any reusable products left over at the end of each cycle. Since the normal operation cost is independent of the capacity Q, we assume $v = 0$ to simplify our notation. We also assume that $0 < c_1 < c_2$, because otherwise, it would be cheaper to simply refurbish all reusable products at the end of each cycle rather than to acquire any regular capacity, i.e., $Q^* = 0$. Then, the expected operating cost per cycle is given by

$$G(Q) = Mc_1 Q + c_2 E(Y_M^n - Q)^+, \tag{11}$$

and the optimal capacity problem is to select the optimal Q^* that minimizes $G(Q)$.

5.1. $M = 2$ case

For this special case with $M = 2$, we substitute (2) and (3) into (11) to obtain

$$\begin{aligned} G(Q) &= 2c_1 Q + c_2 E\left[(Y_1^n - Q)^+ + R_1^n - Q\right]^+ \\ &= 2c_1 Q + c_2 E\left[(R_2^{n-1} - Q)^+ + R_1^n - Q\right]^+ \\ &= 2c_1 Q + c_2 \left\{ P(R_2^{n-1} \le Q) \sum_{i=Q+1}^{d_1} (r_1 - Q) P(R_1^n = r_1) \right. \end{aligned}$$

$$+P(R_2^{n-1} > Q)\sum_{r_1=Q+1}^{d_2}\sum_{r_2=2Q-i+1}^{d_1}(r_1+r_2-2Q)$$
$$\left.\times P(R_1^n = r_1)P(R_2^{n-1} = r_2)\right\}. \quad (12)$$

We use normal approximations for the amount of returned products, R_1^n and R_2^{n-1}, which follow a Binomial distribution with parameters (d_1, p) and (d_2, p), respectively. Then, we can approximate (12) as

$$G(Q) = 2c_1Q + c_2\left[\int_0^Q\int_Q^\infty (r_1-Q)f_1(r_1)dr_1f_2(r_2)dr_2\right.$$
$$\left.+\int_Q^\infty\int_{(2Q-r_2)^+}^\infty (r_1+r_2-2Q)f_1(r_1)dr_1f_2(r_2)dr_2\right], \quad (13)$$

where f_i is the density function of a normal distribution with mean of pd_i and variance of $p(1-p)d_i$, $i = 1, 2$. We can derive the following result:

Proposition 3. *The function $G(Q)$ given in (13) is strictly convex in Q. Furthermore, there exists a unique solution Q^* to the first-order condition function $G'(Q) = 0$ that minimizes the $G(Q)$.*

Proof of Proposition 3. Taking the derivatives of $G(Q)$ given in (13) with respect to Q, we obtain

$$G'(Q) = 2c_1 - c_2F_2(Q)[1-F_1(Q)]$$
$$-2c_2\int_Q^\infty [1-F_1(2Q-r_2)]f_2(r_2)dr_2, \quad (14)$$

and

$$G''(Q) = \left\{-c_2f_2(Q)[1-F_1(Q)] + c_2F_2(Q)f_1(Q)\right\}$$
$$+\left\{4c_2\int_Q^\infty f_1(2Q-r_2)f_2(r_2)dr_2 + 2c_2[1-F_1(Q)]f_2(Q)\right\}$$
$$= c_2f_2(Q)[1-F_1(Q)] + c_2F_2(Q)f_1(Q)$$
$$+4c_2\int_Q^\infty f_1(2Q-r_2)f_2(r_2)dr_2.$$

Clearly, $G''(Q) > 0$, which shows that $G(Q)$ is strictly convex in Q. Also, it is easy to show that $G'(0) = 2c_1 - 2c_2 < 0$ since $c_1 < c_2$, and $G'(Q) = 2c_1$ as $Q \to \infty$. Thus, there exists a unique solution Q^* to the first-order condition function $G'(Q) = 0$ that minimizes the $G(Q)$. □

Proposition 3 shows that the optimal Q^* can be readily found. In particular, we can easily compute the unique solution to the first-order condition $G'(Q^*) = 0$ numerically. Then, the optimal Q^* is given by one of the two neighboring integers by comparing the values of $G(Q)$ at these two integer points.

Proposition 4. *Q^* is strictly decreasing in c_1, while Q^* is strictly increasing in c_2.*

Proof of Proposition 4. We differentiate $G'(Q)$ given in (14) with respect to c_1 and obtain $\frac{\partial G'(Q^*)}{\partial c_1} = 2 > 0$. Therefore, Q^* is strictly decreasing in c_1. Similarly, we can easily show from (14) that $\frac{\partial G'(Q^*)}{\partial c_2} < 0$. □

Proposition 4 shows the intuitive result that the firm would decrease the capacity Q as the unit regular operating cost increases. On the other hand, the firm would increase the capacity Q as the unit overtime operating cost for processing the leftover products increases. Since the return rate of the reusable products are random, the firm employs overtime to refurbish products at a higher unit cost of c_2 to hedge against the risk of having insufficient regular capacity. A higher value of c_1 makes the hedge more attractive, thus lowering the optimal capacity Q. Similar, a higher value of c_2 has the opposite effect, and the optimal capacity Q decreases.

5.2. M > 2 case

We next extend our analysis to the general case with M periods, with $M > 2$. We first introduce some notation. Let Z_i be the amount of reusable products left at the end of period i in cycle n, i.e., $Z_i = (Y_i^n - Q)^+$. (For simpler notation, we ignore the superscript n on Z_i without causing any confusion.) To simplify our exposition, we also consider Z_i as a continuous random variable here. Let q_i denote the probability that there is no leftover products at the end of period i, i.e., $q_i = P(Z_i = 0)$. Also, let $h_i(.)$ denote the density function for the event $(Z_i|Z_i > 0)$, i.e., the amount of leftover products in period i given that there are leftover products in period i.

We note the following relationships between periods i and $(i+1)$. If there is no leftover products at the end of period i, then there is no leftover products at the end of period $(i+1)$ if and only if the amount of reusable products returned in period $(i+1)$ does not exceed Q. If there are leftover products at the end of period i, given by $(Z_i|Z_i > 0)$, then there is no leftover products at the end of period $(i+1)$ if and only if the amount of reusable products returned in period $(i+1)$ does not exceed $Q-(Z_i|Z_i > 0)$. Effectively, the available capacity in period $(i+1)$ in this case is reduced by the amount of $(Z_i|Z_i > 0)$. Therefore, we can easily establish the following relationships:

$$q_1 = \int_0^Q f_1(r_1)dr_1 = F_1(Q)$$

$$E(Z_1) = \int_Q^\infty (r_1-Q)f_1(r_1)dr_1,$$

and for $i = 2, 3, \ldots, M$,

$$q_i = q_{i-1}F_i(Q) + (1-q_{i-1})\int_0^\infty F_i(Q-x)h_{i-1}(x)dx \quad (15)$$

$$E(Z_i) = q_{i-1}\int_Q^\infty (r_i-Q)f_i(r_i)dr_i$$
$$+(1-q_{i-1})\int_0^\infty\int_{Q-x}^\infty [r_i-(Q-x)]f_i(r_i)dr_ih_{i-1}(x)dx. \quad (16)$$

Then, the expected amount of leftover products at the end of each cycle is then given by $E(Z_M)$, and the corresponding expected operating cost is equal to

$$G(Q) = Mc_1Q + c_2E(Z_M). \quad (17)$$

The density function $h_i(.)$ depends on Q, and is generally very complex, even for small values of M. We next develop a recursive scheme that can be used to approximate the values of q_i and $E(Z_i)$. In particular, we simply replace the random variables, $Z_i|Z_i \geq 0$, by $E(Z_i|Z_i > 0)$, and then approximate the value of $E(Z_i|Z_i > 0)$ by $\tilde{E}(Z_i|Z_i > 0)$ using the following recursive scheme:

$$\tilde{q}_1 = F_1(Q)$$

$$\tilde{E}(Z_1|Z_1 > 0) = \frac{\int_Q^\infty (r_1-Q)f_1(r_1)dr_1}{1-\tilde{q}_1},$$

and for $i = 2, 3, \dots, M$,

$$\tilde{q}_i = \tilde{q}_{i-1} F_i(Q) + (1 - \tilde{q}_{i-1}) F_i(Q - \tilde{E}(Z_{i-1}|Z_{i-1} > 0)) \quad (18)$$

$$\tilde{E}(Z_i) = \tilde{q}_{i-1} \int_Q^{\infty} (r_i - Q) f_i(r_i) dr_i + (1 - \tilde{q}_{i-1}) \int_{Q-\tilde{E}(Z_{i-1}|Z_{i-1}>0)}^{\infty} \left[r_i - (Q - \tilde{E}(Z_{i-1}|Z_{i_1} > 0)) \right] \times f_i(r_i) dr_i \quad (19)$$

$$\tilde{E}(Z_i|Z_i > 0) = \frac{\tilde{E}(Z_i)}{1 - \tilde{q}_i}. \quad (20)$$

5.3. Numerical experiments

We conducted a set of numerical experiments to illustrate the accuracy of the above approximation scheme. We first analyze how the pattern of returns affects the performance of the approximation scheme. We assume a 5-day process cycle, i.e., $M = 5$, and that the (random) reusable quantity follows a normal distribution with a coefficient of variation of 0.1. In particular, we consider the following four scenarios for the reusable quantity R_i in period i:

- $S1 : R_1 = R_2 = R_3 = R_4 = R_5 = N(100, 10)$
- $S2 : R_2 = R_3 = R_4 = R_5 = N(100, 10), R_1 = 0$
- $S3 : R_1 = R_2 = R_4 = R_5 = N(100, 10), R_3 = 0$
- $S4 : R_1 = R_2 = R_3 = R_4 = N(100, 10), R_5 = 0$.

In particular, scenario $S1$ depicts the situation where the facility has the same return characteristic in all 5 days, scenario $S2$ depicts the situation where the facility has no return in the early periods of the cycle, scenario $S3$ depicts the situation where the facility has no return in the middle periods of the cycle, and scenario $S4$ depicts the situation has no return in the late periods of the cycle.

We set $c_1 = 1$ and $c_2 = 10$. We simulated the system for 10,000 cycles and recorded the average amount of leftover products at the end of period 5, $E(Z_5)$. We then computed the expected operating cost $G(Q)$ given in (17) over the 10,000 cycles for different values of Q. We further calculated the value of $\tilde{E}(Z_5)$ using the approximation scheme given in (18)–(20) for each value of Q.

Fig. 2 shows the approximation error, $E(Z_5) - \tilde{E}(Z_5)$ for different values of Q. Observe that the approximation scheme consistently underestimates the expected leftover amount at the end of each cycle, i.e., $E(Z_5) - \tilde{E}(Z_5) > 0$ for all values of Q. This can be explained by the fact that the approximation scheme simply uses the expected leftover amount in each period, rather than the distribution of the leftover amount, to compute the leftover amounts in subsequent periods. In other words, the approximation scheme ignores the tails of the distribution of leftover amounts, resulting in underestimating the expected leftover amount at the end of a cycle.

Also, the approximation schemes performs the worst for scenario $S4$. Here, the leftover amounts accumulate over the first 4 periods, and the tails of the distribution of leftover amount have a more significant impact on $E(Z_5)$. Furthermore, the system has only period 5 to clear out the leftovers and to smooth out the tail effect of the distribution of leftover amount. Consequently, the approximation scheme performs relatively poor in such situations as it simply uses the expected leftover amount in each period, rather than the distribution of the leftover amount, to compute the leftover amount in the subsequent periods.

We also conducted a similar set of numerical experiments with higher degrees of variability of the reusable quantities in each period. In particular, Fig. 3 provides the results for the same four scenarios as described earlier with the amounts of reusable products collected follow a normal distribution with a coefficient of variation of 0.3. The basic observations are similar to those as provided in Fig. 2.

Table 2 shows the optimal value of Q based on the simulation result, denoted by Q^*, as well as the optimal value of Q based on our approximation, denoted by $\tilde{Q}^*$, for all four scenarios. We also show the corresponding average operating costs using Q^* and $\tilde{Q}^*$, denoted by $G(Q^*)$ and $G(\tilde{Q}^*)$, respectively. We further computed the percent increase in the optimal capacity value, $\Delta Q = \frac{\tilde{Q}^* - Q^*}{Q^*}$, as well as the percent increase in the average operating cost in using $\tilde{Q}^*$ instead of Q^*, i.e., $\Delta G = \frac{G(\tilde{Q}^*) - G(Q^*)}{G(Q^*)}$. The results in Table 2 suggests that the approximation scheme gives near-optimal solutions in most cases. For instance, even for scenario $S4$ where the approximation scheme performs the worst, $\tilde{Q}^*$ is still relative close to Q^*, and the average operating cost using $\tilde{Q}^*$ is about 10 percent higher than the minimum expected operating cost $G(Q^*)$. Thus, it seems that the approximation scheme can be used to determine near-optimal capacity values in most realistic cases.

Comparisons with Other Methods We next conducted a set of numerical experiments to assess the performance of the approximation scheme as a decision support tool for selecting the optimal capacity. In this set of experiments, we assume $M = 5$ and the expected amount of reusable quantity collected in period i is given by r_i, i.e., $E(R_i) = r_i$, $i = 1, 2, \dots, M$. In our numerical experiments, r_i is randomly selected from a uniform distribution with support on $[100 - \beta, 100 + \beta]$. (For actual applications, the service provider can easily estimate r_i by multiplying the delivery quantity d_i by the reusable probability p.) Then, we assume that the (random) amount of reusable products R_i follows a normal distribution with a mean of r_i with a fixed coefficient of variation cv. We consider different scenarios with different values of cv and β, where cv measures the variability of the amount of reusable products within a given period, and β measures the variability of the average amount of reusable products across periods within a cycle. We set $c_1 = 1$ and consider different values of c_2.

For each scenario, we randomly generated 100 cases for the values of $r_i, i = 1, \dots, M$. For each case with given values of r_i, we ran a simulation of 10,000 cycles to calculate the average operating cost for any given value of capacity level Q, from which we determine the optimal capacity Q^*. We then used the approximation scheme to estimate the average cost for each given value of Q, from which we selected the optimal Q based on the estimated average cost.

We also evaluated two other simple approaches in selecting the capacity Q: (i) first, we consider the simple approach of using the average of $\sum_{i=1}^5 r_i$; (ii) second, we use an aggregate newsvendor approach to select the capacity level Q. Specifically, we consider that the aggregate 5-period demand follow a normal distribution with mean of $\sum_{i=1}^5 r_i$ and standard deviation of $cv\sqrt{\sum_{i=1}^5 r_i^2}$. In this approach, we set the unit understocking cost to $c_2 - c_1$, as it corresponds to the unit additional overtime cost due to insufficient capacity, and set the unit overstocking cost to $5c_1$, as it corresponds to the unit cost due to excess capacity over the 5-period cycle. We then apply the classical newsvendor fractile ratio to select the capacity level.

For each approach, we computed the deviation of the selected capacity level from the optimal capacity level by $\Delta Q = \frac{|Q - Q^*|}{Q^*}$. Also, we used a simulation of 10,000 cycles to estimate the average operating costs $G(Q)$ and $G(Q^*)$ as given in (17). We then computed the deviation of this average cost from the optimal average cost by $\Delta G = \frac{G(Q) - G(Q^*)}{G(Q^*)}$. For each approach, the average values of ΔQ and ΔG over the 100 random cases in each scenario are shown in Table 3.

Table 3 shows that the approximation scheme performs very well in all scenarios. The deviations of the selected capacity level

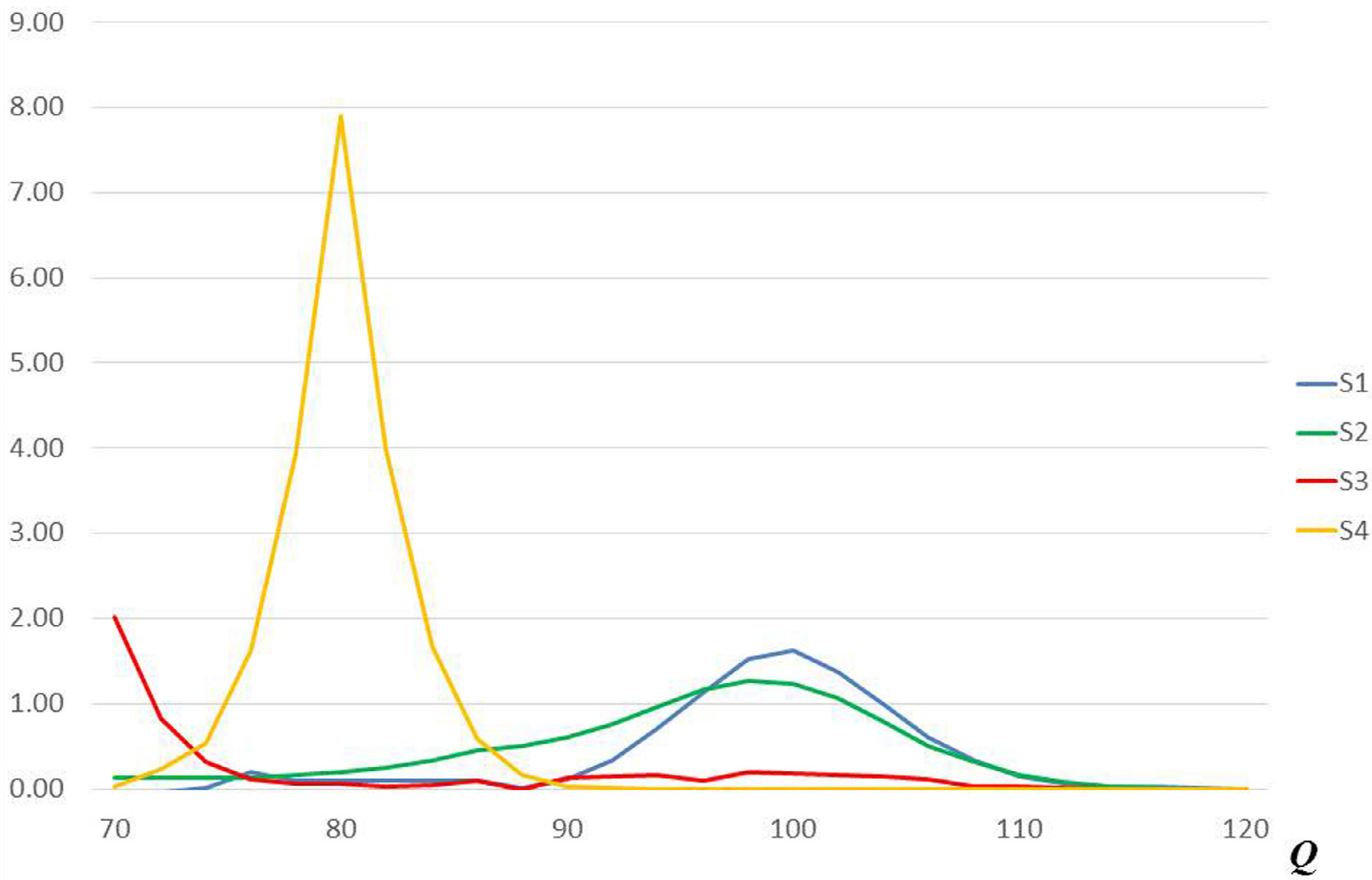

Fig. 2. Approximation error: $E(Z_5) - \tilde{E}(Z_5)$ when $R_i \sim N(100, 10)$.

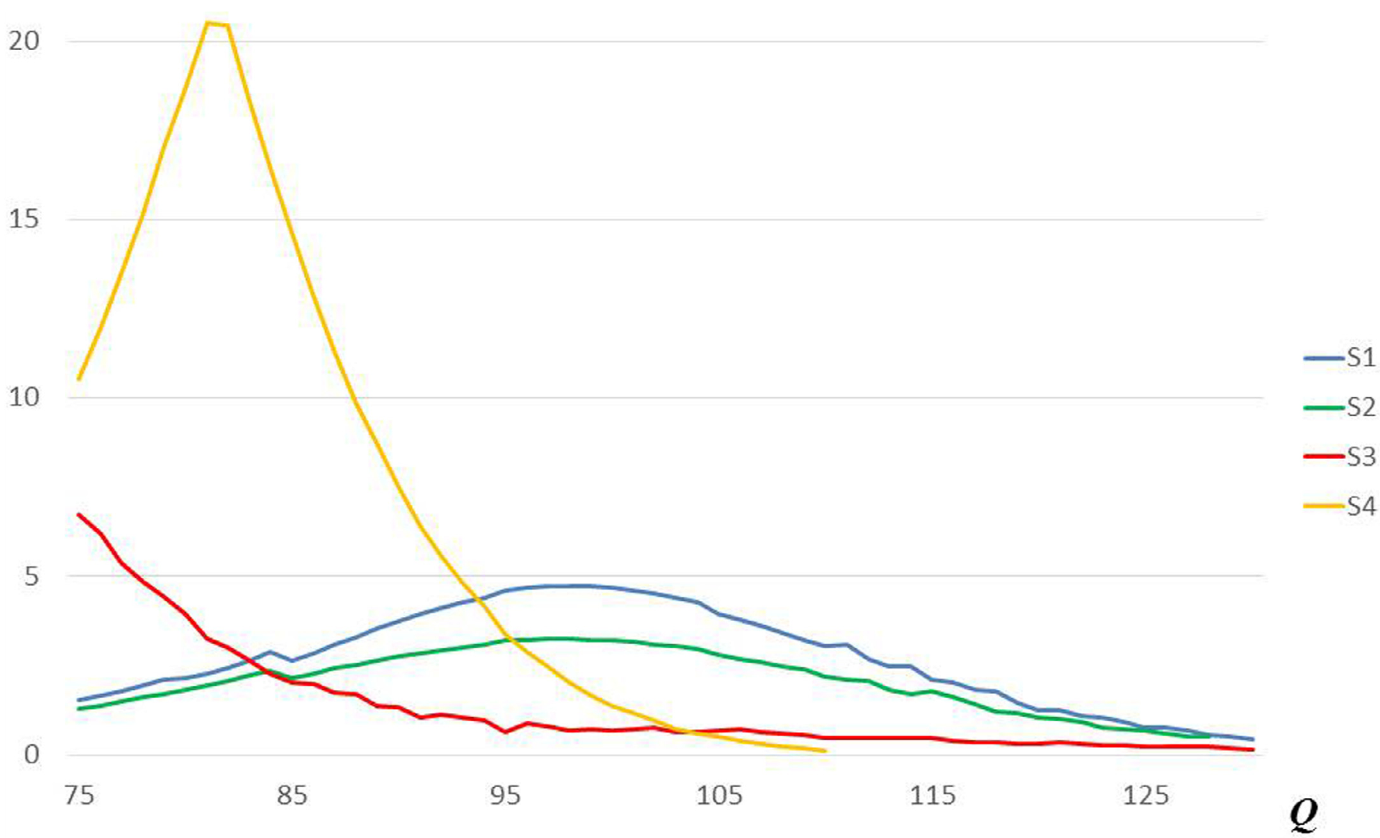

Fig. 3. Approximation error: $E(Z_5) - \tilde{E}(Z_5)$ when $R_i \sim N(100, 30)$.

Table 2
Performance of the approximation scheme for scenarios $S1$–$S4$.

cv	Scenario	Q^*	$\tilde{Q}^*$	ΔQ (percent)	$G(Q^*)$	$G(\tilde{Q}^*)$	ΔG (percent)
0.1	$S1$	108	108	0.0	560.1	560.1	0.0
	$S2$	108	108	0.0	560.0	560.0	0.0
	$S3$	106	106	0.0	554.8	554.8	0.0
	$S4$	86	80	−7.0	436.3	481.1	10.3
0.2	$S1$	116	114	−1.7	621.9	623.7	0.3
	$S2$	116	114	−1.7	619.6	620.6	0.2
	$S3$	112	110	−1.8	608.0	608.2	0.0
	$S4$	90	82	−8.9	470.8	526.4	11.8
0.3	$S1$	125	122	−2.4	681.4	682.5	0.2
	$S2$	123	121	−1.6	679.2	680.7	0.2
	$S3$	117	116	−0.9	663.5	663.7	0.0
	$S4$	95	82	−13.7	508.9	614.3	20.7

Table 3
Performance of the three approaches of setting optimal capacity level.

r_i	c_2	cv	Approximation		Simple average		Newsvendor	
			ΔQ (percent)	ΔG (percent)	ΔQ (percent)	ΔG (percent)	ΔQ (percent)	ΔG (percent)
U[50,150]	2	0.1	2.0	0.2	4.1	0.6	6.0	1.3
		0.2	1.3	0.1	3.6	0.4	8.3	1.8
		0.3	2.7	0.2	3.1	0.3	13.1	2.6
	10	0.1	4.0	4.6	15.3	27.6	13.9	22.4
		0.2	3.1	1.6	19.9	28.0	17.3	19.8
		0.3	3.6	1.5	24.5	34.2	20.7	22.5
U[0,200]	2	0.1	2.0	0.2	9.3	1.9	11.3	2.8
		0.2	3.0	0.2	10.6	1.2	13.5	2.4
		0.3	6.0	0.8	7.1	1.2	16.0	3.3
	10	0.1	4.1	4.8	20.9	44.0	19.4	38.4
		0.2	5.1	4.4	26.6	42.7	23.9	33.8
		0.3	5.1	6.4	27.2	41.8	23.2	29.6

from the optimal level and the corresponding average cost are much smaller than the other two approaches in all scenarios. In particular, the average deviations from the optimal capacity level range from 1.3 percent to 5.0 percent, while the average deviations from the minimum average cost range from 0.1 percent to 6.4 percent in all scenarios. We also observe that the approximation scheme performs worse as the unit overtime cost c_2 increases. This is expected as the approximation scheme tends to underestimate the expected leftover amount at the end of each cycle, as discussed earlier. As c_2 increases, this under-estimation increases the error in estimating the expected overtime cost, resulting in a higher deviation from the optimal capacity level and a higher average cost.

It is interesting to point out that the performance of the approximation scheme remains about the same as the coefficient of variation cv is higher or as the variability of the average return amounts within one cycle is higher, i.e., $r_i \sim U[50, 150]$ versus $r_i \sim U[0, 200]$. Our results thus suggest that the performance of the approximation scheme is rather robust with respect to both the degree of variability of the amount of reusable products within a given period and the variability of the average amount of reusable products within a cycle.

Our numerical cases further suggest that the approximation scheme performs the worst when the average return amounts in early periods are substantially higher than those in late periods of the cycle. For instance, among the 100 randomly generated cases for the scenario with $r_i \sim U[0, 200]$, $cv = 0.2$ and $c_2 = 10$, the approximation scheme gives the worst performance for the case with $(r_1, r_2, r_3, r_4, r_5) = (183, 34, 157, 78, 3)$, with $\Delta Q = 11.5$ percent and $\Delta G = 18.1$ percent. This observation is consistent with our earlier result that the approximation scheme performs the worst for scenario $S4$.

We further note that the newsvendor method does better than the simple average method for $c_2 = 10$, but the comparison is reversed when $c_2 = 2$. We can explain this observation as follows. The newsvendor method uses the aggregate capacity and aggregate demand to estimate the optimal capacity level. Generally, this underestimates the optimal capacity level, as some available capacity in the early periods of the cycle could be lost if the return quantities for the early periods are low. The simple average method does not use the cost parameters c_1 and c_2 in determining Q, and would perform badly if the relative difference between the underlying overstocking and understocking capacity costs is large. In particular, when $c_1 = 1$ and $c_2 = 10$, the simple average method generally gives a value of Q that is much larger than the optimal capacity level, and performs worse than the newsvendor method. On the other hand, when $c_1 = 1$ and $c_2 = 2$, the simple average method generally gives a value of Q that is closer to the optimal capacity level than that by the newsvendor method, and thus performs better than the newsvendor method.

We conclude this section by illustrating the benefit of applying our model to determine the optimal capacity for one specific (simplified) scenario using the data provided by the supply company that motivates our study. The company uses a 5-day delivery/collection cycle with daily delivery quantities $(d_1, d_2, d_3, d_4, d_5) = (41400, 3950, 7000, 33630, 26920)$ for one particular product. The company estimates that about 90 percent of the delivered products are returned and reusable in each cycle, so we set the average return quantities $(r_1, r_2, r_3, r_4, r_5) = (37, 260, 3555, 6300, 30, 267, 24, 228)$ in our model. We also approximate the return quantity variability in our model with $cv = 0.1$. The company uses overtime and pays a 50 percent wage premium for workers to refurbish all leftover reusable products at the end of each week, so we normalize the unit cost as $c_1 = 1$ and $c_2 = 1.5$. The company sets its regular daily refurbishment capacity equal to the average return quantity, i.e., $Q = 20, 322$, which gives an average operating cost of 122,401. Using simulation, the optimal capacity $Q^* = 15, 879$ with the optimal operating cost of

115,407, which represents about a 6 percent cost reduction. Our approximation scheme gives $\tilde{Q}^* = 15,734$, which gives an average operating cost of 115,422.

6. Applications and concluding remarks

In this paper we develop a simple mathematical framework to analyze some inventory replenishing and capacity planning issues in a product recovery system with random returns and cyclic delivery schedules. Our models can be used as an effective decision support tool to select appropriate operating policies in managing such systems. In particular, we can apply our models to provide useful information for addressing the following two issues of particular importance.

First, we can apply our model to evaluate how a change in the return rate of reusable products can impact the underlying inventory replenishment and capacity cost in the existing system. As illustrated in Table 1, our model can estimate how an increase in the reusable probability p can affect the optimal replenishment cycle and the associated cost. As it could be costly to monitor the actual return rate from individual clients, it becomes important to design an appropriate incentive system to entice clients to improve the current return rate of used products. The information provided by our analysis would be useful for the manager to evaluate the associated costs and benefits.

As another application, the manager needs to incorporate new clients into the existing delivery and collection schedule, since new clients constantly arrive for service from the facility. Our models can be useful in determining the most effective method of assigning a new client into the existing schedule as well as understanding how the new schedule would impact the corresponding inventory replenishment and capacity planning decisions. We use the following simple example to illustrate some underlying tradeoffs in the assignment decision.

The facility has an existing 5-day weekly delivery schedule with delivery quantity $(d_1, d_2, d_3, d_4, d_5) = (90, 110, 70, 100, 85)$. The facility now needs to add a new client with delivery quantity of 50 units to the existing schedule, and the new client can be assigned to any day of the week. To accommodate this new client, the facility must adjust its processing and delivery capacity accordingly. For the unit processing costs defined in Section 5, we assume $c_1 = 1$ and $c_2 = 2$. We also assume that the facility has a daily delivery capacity of 120 units, and any extra delivery above 120 units will require a third-party transportation service at an extra unit cost of c_T. Thus, the facility faces a tradeoff between the additional capacity cost and transportation cost in adding the new client to the existing schedule. For simplicity, assume that the return probability $p = 0.8$ for all existing and new clients, and that the return quantity R_i follows a normal distribution with a mean of $r_i = 0.8d_i$ and a standard deviation of $\sqrt{0.8(1-0.8)d_i}$. In this simple example, it seems reasonable to assign the new client either to Day 1 to minimize additional capacity cost (as it is always beneficial to receive the return units as early as possible) if the additional transportation cost is relatively low, or to Day 3 to minimize any extra transportation cost if the additional transportation cost is relatively high. Indeed, our analysis shows that it is optimal to assign the new client to Day 1 when $c_T < 0.50$, but it is optimal to assign the new client to Day 3 when $c_T > 0.50$.

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