A PERSPECTIVE ON RELATIVISTIC
NUCLEAR COLLISIONS*

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A PERSPECTIVE ON RELATIVISTICNUCLEAR COLLISIONS*

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By studying nuclear collisions at high energy, we expect to cross the boundaries that have so far circumscribed the study of nuclear physics. We expect to be able to create nuclear matter under extreme conditions such as those of high temperature, pressure, particle and energy density.

These studies began some few short years ago with perhaps too great an optimism for discovering new phenomena associated with these extremes. No exotic phenomena have been discovered so far. This will be interpreted readily by the pessimist as indicating that there are none to be discovered. The realist must face the fact, perhaps nowhere else encountered in physics, that the phenomena that he wishes to discover are created, if at all, in a system that is destroyed in the same collision that creates the conditions under which the phenomena can occur. For example, even if a shock wave is produced as two nuclei interpenetrate, we cannot insert sensors into the nuclei to detect the head pressure. Instead we must be content to make measurements on the fragments that are collected at infinity, long after the nuclear material has blown apart and therefore after the information has, at the very least, been partially obscured by successive interactions among the fragments as they leave the collision site.

The early experiments concentrated on making the simplest measurements, the so called single-particle inclusive measurements, in which only one product type is selected for study, irrespective of what else happens:

\[ A + B \rightarrow C + X \quad (X = \text{anything}) \]

*Text of invited paper at Saturne Workshop, Roscoff, France, May 1979.*
Perhaps, the lesson learned earlier by particle physicists—that multi-particle experiments lead to the big breakthroughs—should have been taken more to heart! Now we know that difficult experiments involving simultaneous measurements on several or many particles are needed. And we begin to realize that theoretical studies must embrace not merely what happens in the center of the hot dense matter, but how the fragments propagate to infinity. There are enormous difficulties implied by these last two sentences that will challenge both our persistence and our intellects. Moreover, there are no guarantees. It could be in the nature of things that final state interactions will obscure whatever happens in the dense medium. So it is with a certain fatalism that one enters this field.

Speculations as to the new phenomena that might be formed in high energy nuclear collisions have been discussed already at other conferences and in the literature and are perhaps already familiar to you. I shall make no mention of them (except briefly in the last section when I shall remark on quark matter). Concerning research in the energy range up to several GeV/nucleon kinetic energy in the laboratory (<500 MeV in C.M.) such as is available at the Bevalac and Saturne, I will discuss first a successful type of experiment in an unexpected area, the production of nuclei far from stability in peripheral collisions. Secondly, I will describe the expectations for the explosive disassembly of dense nuclear matter and some evidence for it. Then I will come to the theory of pion interferometry, a subject which recognizes the need to go beyond the single-particle inclusive measurements and the need to take account of final state interactions which tend to obscure the information from the dense region. Finally I will discuss my own research, which has to do with much higher energy than that available at the Bevalac or Saturne. It is an assessment of the possibility of determining the form of the hadronic spectrum in the high mass region through nuclear collisions at ultra-relativistic energies. This has exciting implications both in particle physics and in cosmology.
NUCLEI AT THE LIMIT OF STABILITY

It has turned out that a relativistic nuclear accelerator is an ideal instrument for efficiently producing nuclei far from stability. David Scott expressed the motivation for doing so in the following way. There are some 300 stable nuclei that define the valley of stability. During the last half century some additional 1300 radioisotopes have been identified and studied along its slopes. Various theoretical methods have been employed to estimate the existence of an additional 6000, stretching out on the plains. If this is so, then in our present state of ignorance, an attempt at extrapolating our current knowledge to the description of the mass surface on which they lie is somewhat analogous to attempting to infer the topography of the United States from a detailed survey of the Grand Canyon!

What makes a relativistic beam of nuclei so ideal for launching a large scale program of producing nuclei out to the limits of stability? Note first that because the frequency associated with the transit time of a relativistic nucleus across a nuclear dimension is large compared to single particle or collective frequencies, then peripheral collisions can produce large fragments with rather low excitation energy. Therefore there appears to be appreciable possibility, born out by experiment, of producing nuclei sufficiently close to their ground state by simply shearing off some nucleons in a peripheral collision leaving the spectator fragments rather cold. The high velocity of the fragment, close to the beam velocity and dispersed only by the much smaller transverse velocity associated with the Fermi momentum, assures that the products are strongly focussed in a small solid angle, which yields a gain of $\sim 10^3$ over conventional experiments. Furthermore the high velocity assures that the products can emerge from very thick targets, yielding a total boost in yield of $\sim 10^6$. In a recent one-day run, the most successful so far, about 20 new isotopes were identified.\textsuperscript{1} The analysis is incomplete and I have no material from this run. However from an earlier experiment using $^{40}$Ar at $\sim 200$ MeV/nucleon, the production of many isotopes including 3 previously unknown ones is shown in Figure 1.

![Fig. 1. Isotopes produced by the fragmentation of $^{40}$Ar at $E = 205$ MeV/nucleon. Evidence for three new ones is shown in boxes. (Ref. 1)](image-url)
EXPLOSIVE DISASSEMBLY OF NUCLEAR FIREBALL

Central collisions between nuclei are expected to lead to the formation of dense regions of matter. The succession of n-n collisions, and creation of pions will tend to bring the matter to equilibrium, which state we refer to as a nuclear fireball. The sudden creation of hot dense matter usually leads to an explosion. Particles at the surface and directed outward can move freely outward. Those directed inward will be scattered by the dense medium and some of them will be reflected. As a result, in a short time an outward net flow will be generated. Some of the kinetic energy of thermal motion will be converted to energy of a blast wave. Additional energy is released to the blast as the density and temperature fall and energy stored in compression, resonances, and pions becomes available. The conversion will continue as long as collisions are sufficiently frequent. However, at a sufficiently low density, collisions will cease, and the particles, now possessing a thermal distribution characterized by the final temperature and a net radial flow, will move freely until they reach the detectors. This is the picture applied by Siemens and Rasmussen\(^2\) to some Bevalac data. Anticipated features are:

1) a peaking in velocity distribution around a mean blast velocity in contrast to purely thermal distribution,

2) a reduction in the temperature from that expected by energy conservation,

3) a smaller apparent temperature for pions compared to nucleons because the common blast velocity implies a lower energy for the lighter particles.

Although several possible important effects, such as pre-freeze-out emission and the development of gradients in the fireball are omitted in this simple model, there does seem to be some evidence for its main features as seen in the data\(^3\) of Figure 2. The initial temperature ought to be \(\sim 90\) MeV, whereas the final temperature fitted to the experimental spectra is \(44\) MeV, indicating appreciable conversion of thermal energy to radial energy. And there is a distinct difference in the slope of the high energy tail between pions and nucleons. It is possible that a detailed analysis of data taken over a wide dynamic range can give a fairly accurate picture of the dynamical path followed in the evolution of nuclear matter from high density to freeze-out.
This subject concerns the measurement of correlations in the momentum and energy of two identical pions and is analogous to the technique of intensity interferometry of light developed by Hanbery-Brown and Twiss to measure the radii of stars. It has been employed in hadron collisions to measure the space-time structure of pion reproduction in p-p annihilation. Its possible usefulness in the context of nucleus-nucleus collisions is the subject of a very careful and detailed analysis by Gyulassy, Kauffmann, and Wilson, who have also considerably extended the theory and its rigor.

Incoherent production

First, let us see (non-rigorously) what is meant by an ideal Bose-Einstein correlation. Suppose that pions are produced, independently within some space-time region defined by a distribution function \( \rho(x,t) \). We have in mind either photons emitted by a star, or pions produced as two nuclei...
collide, and we imagine that the pions are produced singly in uncorrelated nucleon-nucleon collisions as the nuclei interpenetrate. In this latter case \( \rho(x,t) \) would have the spatial dimensions of the overlapping nuclei and a time duration corresponding to the collision time. The wave function for observing two identical pions (say \( \pi^-, \pi^- \)) in the counters with four-momenta \( k_1 \) and \( k_2 \), given that they were produced at \( x_1 \) and \( x_2 \) and, with the neglect of interactions between the pions or with the nuclear medium, is

\[
\langle 12 \rangle \equiv \langle k_1 k_2 | x_1 x_2 \rangle = \frac{1}{\sqrt{2}} \left( e^{-i k_1 x_1} e^{i k_2 x_2} + (x_1 \leftrightarrow x_2) \right)
\]

where the wave function is symmetrized for bosons and

\[
k_x = \omega t - k \cdot x, \quad \omega = \sqrt{p^2 + m^2} \pi
\]

Then the probability for seeing two identical pions emitted by the source is

\[
P(k_1 k_2) = \int d^4x_1 d^4x_2 \rho(x_1 t_1) \rho(x_2 t_2) |\psi_{12}|^2
\]

\[
= 1 + |\rho(k_1 - k_2, \omega_1 - \omega_2)|^2
\]

The correlation function refers to the frequency of such measurements compared to uncorrelated measurements of a pion with \( k_1 \) and another with \( k_2 \): this ratio is

\[
R(k_1 k_2) = 1 + |\rho(q, \omega)|^2
\]

where

\[
q = k_1 - k_2, \quad \omega = \omega_1 - \omega_2
\]

and \( \rho(q, \omega) \) is the Fourier transform of the source \( \rho(x,t) \). If, for example, the pions are produced at points distributed uniformly in a sphere of radius \( R \) and during a period \( T \), then

\[
\rho(q, \omega) = e^{i \omega T/2} \frac{\sin T/2}{\omega T/2} \frac{3}{(qR)^3} [\sin qR - qR \cos qR]
\]

Thus in this idealized situation, Bose statistics impose a correlation on the momenta and energy of two observed identical pions (or photons) if they are produced independently. The width of the distribution function in momentum and energy (illustrated in Figure 3) provides a measurement of the space-time region from which they were emitted. That such measurements begin to be possible is illustrated in Figure 4 were \( \pi^- \bar{\pi}^- \) correlations are
Fig. 3. The $\pi-\pi$ correlation function is illustrated schematically for a) pions produced in independent processes that are uniformly distributed in a sphere of radius $R$ and in a time $T$, b) pions produced in a coherent process, and c) both coherently produced pions and independently produced pions.

Fig. 4. Measured $\pi-\pi$ correlation for equal energy pions. Counters were triggered on a range of central collisions (Ref. 5).

measured in events that are triggered for central collisions. The conditions under which the ideal Bose correlation function should hold, have been derived rigorously by Gyulassy et al. They find that the number of independently produced pions must be large and that they must be produced over a period of time large compared to $1/m_{\pi}$.

Coherent production

If instead of the independent production of pions such as would correspond to a cascade picture of the collision of two nuclei we suppose that there is some coherent process that produces the $\pi$'s, such as a collective pionic instability (which would be related in a static situation to an abnormal phase of nuclear matter called a pion condensed state), then a very different result emerges. In the case of correlated production there is no intensity interference

$$R(k_1, k_2) \equiv 1$$
Partially coherent production

More generally, of all the pions produced in a collision, only some fraction, if any, will be produced by a coherent process, while the others are produced independently in unrelated n-n collisions that occur during the duration of the nuclear collision. For pions of momentum $k$ this fraction is denoted by $D(k)$. It can be measured as the intercept of $R$ when $q = \omega = 0$

$$R(k,k) = 2 - [D(k)]^2$$

After such a determination has been made, then the full function $R(k_1k_2)$ can be used to measure the space-time structure of the source, as before.

Final state interactions

The above results can all be modified by final state interactions, either between the pions or with the nuclear material (Figure 5). Most important are the long-range Coulomb fields in both cases. Gyulassy et al. have derived results analogous to DWBA in which the $\pi-\pi$ interaction, through their Coulomb repulsion (proportional to $\omega$) is taken into account in first order, while their interaction with the nuclear medium (proportional to $Z\alpha$) is calculated to all orders.

Impact parameter average

Generally the experiments are set up to attempt to distinguish central from peripheral reactions, but there is of course no way to isolate a single partial wave or impact parameter. An average over impact parameters very much convolutes the problem of deducing the space-time structure of the production process. Therefore as many exclusive triggers in the experiment as possible are essential.

Outlook

In principle the measurement of the $\pi-\pi$ correlation function can measure the fraction of pions produced in coherent processes and the space-time structure of the region where they are produced. Provided that the distortion of final-state interactions can be realistically evaluated it appears that we have a means of detecting coherent production mechanisms, among which the pionic condensate is a likely candidate.
THE HADRONIC MASS SPECTRUM

The hadronic mass spectrum as it is known today is displayed in Figure 6 where the number of hadronic states in unit (pion) mass interval is plotted as a function of the mass. Notice that the spectrum increases exponentially until masses of $\sim 14 \, m_\pi \approx 2 \text{ GeV}$. Thereafter, rather fewer resonances have been identified. However, we believe that this is due to the difficulty in producing and identifying them, rather than in their non-existence, because in the region above $m \approx 12 \, m_\pi$, the ratio of resonance width to the spacing between resonances is $\sim 100$. The range of theoretical expectations is also shown on the figure. We note that the confinement of quarks almost implies by itself an infinite spectrum of resonances. In particular a lower bound can be obtained by what I refer to as the rigid quark bag, which pictures a hadron as consisting of free valance quarks in a rigid container which is the

![Fig. 6](image_url)

Fig. 6. The hadronic spectrum is plotted as the density of states as a function of mass of the state. The resonances with reasonably certain determination of spin and isospin as of 1976 are represented by the histogram. Only the baryons and mesons are plotted, not the anti-baryons. The curves are explained in the text.
same size for all masses. In this case the asymptotic form of the spectrum varies as \( m^{-1} \). The bootstrap hypothesis probably generates hadronic states maximally. In a thermodynamic formulation, Hagedorn obtains an exponential spectrum for the bootstrap. I hasten to mention that it is conceivable that a more realistic quark model could also yield an exponential spectrum.

Now the full enormity of the problem of determining the asymptotic spectrum can be stated: if a new resonance could be found every day, it would take 100 years to determine the one point on the exponential curve at \( m = \frac{1}{16} m \). Clearly it is impossible if approached in the usual way of searching for the individual resonances. Is there another way, in principle, and is it likely to be practical? That is the subject of this research.

One could question whether it is interesting to know the spectrum in a region where it is impossible to detect the individual resonances. It is nevertheless a fundamental property of matter that relates both to the subatomic world and to the cosmos, to times going back to the beginning of the universe, and forward to the ultimate fate of black holes. In particle physics, it may someday provide an asymptotic constraint on the theory of hadronic structure (see Figure 7).

Nuclear collisions at ultra-relativistic energies offer the only experimental way of (perhaps) producing almost macroscopic quantities of nuclear matter at enormous energy density such as probably existed in the early universe and such as may exist in the late stages of evolution of black holes. Of course the properties of matter depend on its composition. Now the crucial point—at high enough energy density, the composition of matter must depend on the number and masses of particles and resonances that can be created at that energy—that is, it depends on the hadronic spectrum whether or not the individual resonances are, or ever can be, resolved!

Fig. 7. The thermal history can be traced backward in time with reasonable assurance until temperature \( T \gtrsim 100 \text{ MeV} \) when it is dominated by the unknown hadron spectrum.
Our goal in investigating this subject is to estimate what signals, if any, can be propagated from the dense medium to the asymptotic world of the counting apparatus, and at what energy they become strong.

**Perspective**

A complete dynamical description of a nuclear collision at energies sufficient to create particles is obviously out of the question. It is presumably a field theoretical problem involving a Lagrangian describing all of the elementary fields and their interactions. Although there is a good candidate for the Lagrangian, that of quantum chromodynamics, its solution in the context of nuclear collisions involves the most brutal problem in this theory, the infrared behavior and the nature of confinement. On the other hand, while a many-body system poses enormous problems if a microscopic account is sought, certain bulk properties can be inferred without solving the detailed equations of motion. This is the approach that we explore. We conceptualize the collision as progressing through three stages. We shall say nothing about the first stage, in which the high-energy primary collisions between nucleons are taking place, other than to assume that it leads through a succession of collisions to the stopping of one nucleus by the other. Statistical models have been applied to hadron-hadron collisions. We shall apply a statistical thermodynamic description to the second stage of the nuclear collision; after a sequence of hadron-hadron collisions have brought the system to an assumed state of equilibrium. Our view is that the hard parton collisions that seem to play an important part in the high-energy hadron-hadron collisions may not be amenable to a statistical theory, whereas the singular behavior of the parton collisions become irrelevant if equilibrium is achieved in a high energy nuclear collision. Factors that would tend to bring about equilibrium are:

1) an enormous phase space is opened up by particle production;

2) the strong interactions of pions and nucleons especially in the resonance region and the corresponding high velocity of the pions (near c) provides a fast mechanism for thermalization;

3) thermalization may require only 3 or 4 collisions anyway;

4) the finite size of a nuclear complex inhibits the disassembly of its interior;

5) particle creation increases the density.

In the third stage the collision complex, called a nuclear fireball, disassembles. During this stage, the fastest particles escape first while the whole fireball expands and the constituents eventually cease to interact. Thereafter resonances decay freely and only stable particles survive to be seen in the counters. We develop a quasi-dynamical model for this disassembly stage during which we know that much information concerning the conditions in the initial high density stage will be lost due to subsequent interactions. We want to be able to identify any surviving signals from the dense stage.
The Initial Fireball

We calculate the temperature and composition of the initial fireball assuming a completely inelastic central collision between symmetric nuclei. We describe the fireball as a relativistic gas of hadrons in equilibrium, and we enforce the important conservation laws of baryons number, strangeness, electric charge, energy, and momentum. The hadron states are assumed to be distributed in mass according to an unknown spectrum \( \rho(m) \) and the composition of the fireball and its temperature are determined by thermodynamics, the conservation laws, and the spectrum of hadrons. To determine how the properties of the fireball and the spectra of stable particles emitted in its disassembly depend on the hadronic spectrum, \( \rho \), we investigate three possible forms for \( \rho \) (see Figure 6): 1) \( \rho \) consists of only the particles and resonances presently known; 2) \( \rho \) is the spectrum of the rigid quark bag model; 3) \( \rho \) is an exponential spectrum corresponding to the bootstrap hypothesis. We use Hagedorn's form, namely

\[
\rho(m) = \frac{1}{m^3} e^{m/T_0}
\]

We note also that the quark hypothesis could conceivably imply an exponential spectrum. In any case, the last two forms of \( \rho \) represent the theoretical extremes.

Neither the temperature nor composition of the initial fireball are observables because any conceivable experiment looks at the products of the collision after the fireball has disassembled. Nonetheless, it is interesting to look at the calculated populations because they are the starting point of the subsequent expansion or decay of the fireball. They also give us a glimpse of what the temperature and composition of the universe might have been at the beginning of time. The temperature to which the matter would be heated for a given energy input is shown in Figure 8 for these three assumptions. Since we assume for simplicity that the nuclear collisions are perfectly inelastic, then the c.m. collision energy including rest mass is the total fireball energy. For the case of the "known" spectrum, the temperature of matter is by far highest at energies greater than several GeV. Because energy goes into making additional resonances in the quark bag spectrum that were not present in the "known" spectrum, the temperature is lower at corresponding energy. For the exponential spectrum, as first discovered and emphasized by Hagedorn, the temperature is limited to a maximum value corresponding to the constant \( T_0 \) in the spectrum. This limiting temperature of matter, if composed of hadrons having an exponential spectrum, is a truly remarkable property. No matter how much energy is injected the temperature cannot be raised.

To display the composition of the fireball it is necessary to make some arbitrary groupings because there are so many discrete resonances, not to mention the continua. Therefore, each family (defined by its baryon and strangeness quantum members) is divided into light particles comprising the lightest five, and heavy particles comprising all the rest, including continuum resonances in the case of the exponential and quark bag spectra.
Fig. 8. For the three hadronic spectra considered, the temperature of hot hadronic matter as a function of energy per nucleon is plotted. Assuming a colliding beam central collision between identical N=Z nuclei in which the nuclei are stopped by each other, this is the c.m. collision energy per nucleon including the rest mass.

Fig. 9. The fireball populations corresponding to the three assumptions of the underlying hadronic spectrum. The light and heavy members of the family of ordinary mesons (π), a strange mesons (K), and ordinary baryons (N) are plotted as a function of collision energy. Light refers to the first five multiplets (if that many) of each family and are denoted by <. Heavy refers to all others, including the continuum where applicable, and are denoted by >. Anti-particles, in the case of the "known" and "bag" spectrum, approach the particle populations at high energy and can be identified in the figure by this property. In contrast, for the exponential spectrum, the heavy baryons dominate.
Figure 9 shows remarkable differences in the initial composition of the fireball, depending on the underlying hadronic spectrum. For both the known and the rigid quark bag spectrum, the heavy baryons and antibaryons dominate the composition at very high energy, with heavy mesons being the next most populous group.

The composition corresponding to the exponential Hagedorn spectrum is remarkably different from the other two. The light meson population rises to a maximum of about 10% and then declines. The heavy baryon populations rise sharply above 3 GeV. Above this energy the fireball is composed almost entirely of heavy baryons! In both the "known" and "rigid bag" worlds all particle-anti-particle populations approach each other at high energy (with anti-particles being slightly less numerous). In the "exponential" world all anti-particles and mesons have vanishing populations at asymptotic energies. It is a world that is dominated by heavy baryons at high energy density!

This remarkable property of the exponential spectrum can easily be understood, both intuitively and mathematically. At high energy density, the phase space associated with heavy particles becomes enormous because of the exponential spectrum, and as already noted, far exceeds the phase space associated with kinetic motion (related to temperature). Since, however, baryon conservation is enforced, at any given energy the largest phase space is achieved by committing the energy to the conserved particle type, to the exclusion of all others. Hence at very high energy density such as might be produced in high-energy nuclear collisions or such as what existed in the earliest instants of the universe, or in the late stages of black holes, matter is dominated by heavy baryons if the spectrum of hadrons is exponential and if no phase change to quark matter can occur.

**Isoergic Equilibrium Expansion of the Fireball**

We have seen that the initial conditions in the nuclear fireball are very sensitive to the underlying hadronic spectrum at sufficiently high energy density. However, the only possible observations that can be made occur after it has expanded and come apart. The simplest assumption that we can make is to assume, as in cosmology, that the expansion occurs through a sequence of equilibrated states. At some point during the expansion, the density will fall below a critical value where the interactions cease to maintain equilibrium. This is referred to as the freeze-out density. Thereafter relative populations do not change except by decay of isolated resonances.

The above assumption will provide some insight, but it is oversimplified because there is nothing to prevent some of the fast outward moving particles from leaving the equilibrated region of the fireball before it has expanded to the freeze-out density. Therefore, there are two indistinguishable components to the particles that reach the counters, those that escape from the fireball during the expansion and prior to freeze-out and those that remain in thermal contact until the freeze-out. This is the scenario that we shall model in the next section in order to calculate the spectra of stable particles. However, as a first orientation, we consider the simpler isoergic expansion in which no pre-freeze-out radiation of particles occurs.
The fall in temperature of a 20-GeV/nucleon fireball as it expands, and the corresponding evolution of the particle populations for the two extreme worlds, are shown in Figures 10 and 11. Although the initial temperatures are quite different at this energy, as shown in Figure 8, the known and rigid bag worlds cool during the expansion and are virtually equal over the scale shown in Figure 10. This contrasts with the exponential (Hagedorn) world where the temperature remains essentially constant until the fireball has expanded to roughly normal nuclear density. Thereafter, all three worlds cool in much the same way. At the same time, the particle populations, initially very different, become more similar as the density falls. This is so because as the energy density decreases only the lower mass particles, common to all three worlds, can remain populated. Therefore, at some sufficiently low density the three worlds must become indistinguishable if all of the material remains in the expanding fireball and stays in thermal contact. However, the point is that some particles can escape in the early stage and the populations in the high density phase of the expansion are very

**Fig. 10.** The temperature of the fireball as it expands with constant energy equal to 20 GeV/baryon. The ordinate is the reciprocal of the total hadron density in units of the density of normal nuclei (0.17 fm$^{-3}$). On this scale, 2 corresponds to a density such that each hadron has a share of the volume corresponding to a sphere with radius equal to the pion Compton wavelength 1.4 fm.
different especially in the two extreme worlds. This strongly suggests that particle radiation from the early, high-temperature stage will be very different depending on the underlying hadron spectrum, and in particular, that these differences will be reflected most in the high energy tails of the spectra of stable particles!

Quasi-Dynamical Expansion

The hint gained in the preceding study, that the pre-freeze-out radiation from the three worlds will be different because of the differing populations in the early, high-temperature phase of the expansion, provides the motive for attempting to follow the time development of the expansion. The assumption of hydrodynamics has been made for single- or several-component models of hadron-hadron collisions and, therefore, provides a dynamical description. Here, however, we deal with a system containing many species which would make a detailed hydrodynamical model very difficult to implement.
Instead, we develop an expansion model which incorporates the main physical processes that would govern the expansion, namely the populations in the fireball, their velocity distributions, the mean free path, and the mean resonance lifetime.

We shall assume that at any instant the particles that lie within a mean free path of the surface of the fireball, and are directed outward, will move into vacuum. Those that are unstable will decay within a resonance mean life into lighter stable and unstable particles so that in the immediate vicinity of the surface the density remains high. Therefore, we take this to define an instantaneous new surface and we assume that a new quasi-equilibrium state is established within three concentric zones in the outward moving material. Meanwhile, those of the original outward moving particles that are stable and moving faster than the unstable ones that established the position of the new surface escape to the vacuum. Their quantum numbers and energy are subtracted from those defining the state of the new quasi-equilibrated fireball. These steps are iterated until the density has dropped to the critical density or the fireball contains negligible energy and conserved quantum numbers in resonance states, whichever comes sooner. At that point the remaining particles move freely to the vacuum. In this way we model the effects of final state interactions on the spectra of observed particles.

We exhibit in Figure 12 the energy spectra of kaons produced in a central collision of equal mass nuclei with 10-GeV kinetic energy per nucleon. The particles are emitted almost isotropically, so we show the spectra at only one angle. Results for a collision of two mass-number 4 and two mass-number 200 nuclei are compared in the center of mass (colliding beams).

All spectra possess the common feature that they are concave upward. This corresponds to the emission of particles from a cooling object, the high-energy tails arising predominantly from emission from the early high-temperature stage, which is the stage during which the three worlds are most markedly different. The spectra are, so to speak, a convolution of the Fermi or Bose distributions over the history of the fireball. Asymptotically their slopes would characterize the initial temperature of the fireball. Therefore, the tails of the particle spectra are populated mainly by those particles that are emitted in the early stage of the disassembly, and for thisason, the tails are most sensitive in the underlying hadronic spectrum, as this figure shows.

**Quark matter**

There have been numerous speculations on the existence of quark matter. It is suggested that at high enough particle density the boundaries between the hadrons will be broken, and the quarks will be free to move throughout the high density region. Actually no one knows at this time whether quark matter is allowed to exist by the laws of nature, even if there are quarks, whether they are real quarks, not merely topological concepts. And no one will know until the confinement mechanism is understood. More precisely, it appears to be a law that only color singlet states of quarks are allowed as observable objects, e.g., the hadrons. But it is not known whether they must always be close-coupled to a color singlet (i.e., a hadron) or whether also a global color-singlet state is allowed (e.g., quark matter). In the language
of nuclear physics, we know that the ground state of an even-even nucleus has spin $J = 0$. Does this imply that nucleons are coupled pairwise to spin 0 or can they have non-zero spins which are coupled to total $J = 0$. In the nuclear case we know that there is nothing about the symmetries of the Hamiltonian that enforces pairwise coupling to zero spin. The analogous answer for quarks is not known.

If we do not know that quark matter is allowed to exist, we certainly cannot make quantitative predictions about how it could manifest itself. If a quark fireball existed, it would expand, being presumably not a stable state, and as it did so, it would begin to recondense into hadrons at some critical density. This process of recondensation involves directly the most
difficult problem of the quark theory of hadronic structure, how the quarks interact at low relative momentum, the so-called infrared problem. However, supposing that quark matter can exist, it is possible to make some qualitative comparisons between such a world and one in which quark matter is not allowed to exist. We have already seen how the temperature of a hadronic fireball changes as it expands, and we have seen how the spectra of observable stable particles reflect its thermal history. In particular, the high energy tails have a slope characteristic of the early high temperature, and the low energy part of the spectrum has a steep slope characteristic of the final lower freeze-out temperature. These features are recapitulated in Figure 13. Suppose in contrast that during the early high density phase of the fireball, a quark matter stage had been attained. We distinguish two possibilities in this eventuality. First, suppose that all of the matter is in the quark phase, and it all recondenses at the same critical density. The temperature would vary somewhat as in part b of Figure 13. Initially because of the smaller phase space associated with the quark flavors in comparison with hadron states, the temperature would be much higher at corresponding energy density. Since quarks cannot be radiated, we suppose in this scenario that no particles emerge during the quark phase. Therefore, when the quarks recondense, the temperature must be higher in comparison with the purely hadronic fireball. Particle spectra will be different in the absence of the high temperature tail but with an overall average higher temperature. Finally, in case c, the quark matter might have a halo of hadrons which can be radiated, or the quark matter may sputter out hadrons. The quark matter in this case has a lower temperature than case I. When it recondenses it

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**Fig. 13.** Temperature history as fireball expands. (a) No quark phase, only hadrons. Particle spectrum for a pure hadronic fireball. (b) Initial quark phase. Particle spectrum with quark phase present and high densities. (c) Initial quark phase with hadron halo. Particle spectrum with quark phase present at high density.
does so at an intermediate energy between case a and b. Particle spectra have an extremely hot tail corresponding to the sputtering of hadrons during the quark phase.

This possibly exhausts the usefulness of speculating about quark matter and how signals of it would appear different than for purely hadronic fireballs. We return to purely hadronic matter.

The Mass Degree of Freedom

Returning now to the discussion of the calculated signals from a hadronic nuclear fireball, which are sensitive to the underlying mass spectrum of hadrons, we note from Figure 12 that the calculated kaon spectrum in the three worlds is sensitive to the mass of the colliding nuclei, especially in the case of the exponential world. This is therefore an important degree of freedom to exploit. The reason why the mass or more particularly the size of the colliding nuclei effects the outcome is because it brings into play in different ways, according to the underlying hadron spectrum, the factors that govern the disassembly of the fireball; the mean free path in the fireball, the mean resonance lifetime, and the ratio of the emitting surface to volume reservoir. Figure 14 illustrates schematically how this comes about. In Figure 14a, the situation is illustrated in the case that the hadron spectrum has few high mass resonances (e.g., the known spectrum). Then the hot fireball contains many light and, therefore, fast particles (of Figure 11). Because of the resulting high particle density, the mean free path is short, and therefore, the fireball can radiate only from a thin skin. Therefore, there will be approximate scaling in the spectra of emitted stable particles with respect to the target-projectile mass. Also, the core will rapidly expand and rapidly cool (Figure 10).

In contrast, if the hadron spectrum is rich in high mass resonances (e.g., the exponential world), the fireball will consist of a few heavy and

Fig. 14. Target-projectile mass as a variable in the disassembly of a fireball. (a) Few high mass resonances in hadronic spectrum cause many, fast, light particles in fireball. (b) Many high mass resonances in hadronic spectrum cause few, heavy, slow particles in fireball.
therefore, slow particles. In this case, as illustrated in Figure 14b, the mean free path will be long, with the result that the spectra of emitted particles will not scale with mass of the target-projectile.

Moreover, the core will be a slowly expanding reservoir of particles of almost constant temperature (cf Figure 10).

Figures 15 and 16 show the ratio of pions and kaons to nucleons in a high energy (1.6-GeV) slice of the spectra as a function of mass of projectile-target as calculated for a collision of 10-GeV/nucleon kinetic energy in the center of mass. We see that there are qualitative differences in these ratios depending on the hadronic spectrum and that there are just the right number of combinations to provide a unique and strong signature. This is an easy signature to read, since it involves simply a ratio of particle number in a high energy bin, and is much more useful than the single-particle inclusive spectra because nature provides only one answer, not the three compared in Figure 12.

![Particle ratio at 90°, 1.6 GeV kinetic energy](image)

Fig. 15. Ratio of emitted pions to nucleons at 1.6 GeV kinetic energy as a function of total nucleon number in a symmetric collision of equal mass nuclei at 10 GeV/nucleon in c.m. The ratio behaves differently depending on the assumed hadronic spectrum.
Summary

We have developed a quasi-dynamical model for the disassembly of a high energy hadronic fireball. We find that some signals from the early high density stage are expected to survive the expansion stage. Indeed, certain signals, the \( \pi / N \) and \( K / N \) ratio at high kinetic energy, are so qualitatively unique in their dependence on target-projectile mass that we believe they will survive a refinement of the collision dynamics. Therefore, we feel optimistic that a determination of the asymptotic form of the hadron spectrum can be made by studying nuclear collisions. The required energies are high (~10 GeV/nucleon in C.M.), but they are within the reach of present technology.
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