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J.R. Trow
(Ph.D. Thesis)

March 1985

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H- Production in a Multicusp Microwave Plasma*

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Ph.D. Thesis
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University of California
Berkeley, CA 94720

March, 1985

* This work was supported by the Director, Office of Energy Research, Office of Fusion Energy, Development & Technology Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.
H⁻ Production in a Multicusp Microwave Plasma

By

John Robert Trow
B.S. (Kansas State University) 1977

DISSERTATION
Submitted in partial satisfaction of the requirements for the degree of

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Physics

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GRADUATE DIVISION

OF THE

UNIVERSITY OF CALIFORNIA, BERKELEY

Approved:

[Signatures and dates]

..................................................
H⁻ Production in a Multicusp Microwave Plasma*  
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Abstract

High energy neutral beams are necessary for the continued development of magnetically confined fusion plasma devices. Neutral beams based on positive ions are not efficient at beam energies of 100 keV or above, however negative ion based neutral beam systems are efficient, even at high beam energies.

Volume production of H⁻ has many advantages over the other methods, chiefly: simplicity of design and operation, and no need for alkalai metals. Since volume production requires a low electron temperature (~ 1 eV) but also requires molecular intermediates only formed by more energetic electrons (≥ 20 eV), double plasma devices with a separate hot electron region are desirable. Therefore an experiment was undertaken to examine H⁻ production by volume processes in a multicusp microwave discharge, part of the cusp field being enhanced to produce an ECR (electron cyclotron resonance), that would also isolate the hotter plasma formed there. This arrangement is analogous to the "magnetic filters" used in some other negative ion sources.

* This work was supported by the Director, Office of Energy Research, Office of Fusion Energy, Development & Technology Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.
This work describes the experiment set up and the results obtained, which are a survey of the behavior of this type of device. Also included is a discussion of the volume processes associated with H⁻ production including numerical estimates, based on the experimental measurements, which indicate H⁻ production is by dissociative attachment of cold electrons to vibrationally excited hydrogen molecules, and loss is by mutual neutralization with positive ions. The experimental observations are consistent with this model. These are also the same mechanisms used in the models of Bacal and Hiskes.

Since magnetic fields generated by samarium cobalt permanent magnets were an important part of this experiment a set of field calculations was undertaken and is included here as a separate chapter.

This device is shown to be a viable scheme of H⁻ (or O⁻) production and is worthy of further development. There are several more quantities which still need to be measured listed in the conclusion, along with suggested improvements.
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Dedication

To my parents
Leo and Hilga Trow
Pity them, for after all they invested in me
they didn't even get a quarter back!
Acknowledgements

In the course of this experiment I have received the help of many skilled professionals, all of whom I wish now to thank, a few of whom I also wish to honor by name; doing so with the full knowledge that I shall miss naming some who truly also deserve the honor; to them I apologize.

First I wish to thank my advisor Prof. Robert Pyle for his long-suffering, for his direction and advice, and especially for insisting that the time to finish had come. I thank Prof. Wulf Kunkel for introducing me to plasma physics, and for his help along the way. Also of the staff scientists who helped, a word of thanks to Dr. Klaus Halbach for teaching me about permanent magnet calculations and to Drs. Ka-Ngo Leung, John Hiskes, and Ken Ehlers for talking with me about the physics of $H^-$ production and ion sources.

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I offer my appreciation and thanks to the members of TID, the technical illustrators and photographics services, for producing the many fine figures in this work. Finally a special recognition, and thanks to Martha Duenas who patiently and skillfully typed this lengthy dissertation through several editions.
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Chapter I

Introduction

Plasma ion sources from which high current D\(^-\) (negative deuterium ion) beams can be extracted are needed to produce the higher energy neutral beams necessary for the next generation of magnetically confined plasmas for continuing fusion energy development. High energy beams of neutral particles can carry energy, momentum, and the particles themselves into the plasma core through the strong magnetic fields used for containment, which would divert charged particle beams away. (Fig. 1-1) Once inside the plasma these particles are collisionally ionized and thereafter trapped by the confining fields.

The most straightforward use for neutral beams is bulk heating of the plasma, but there are also other applications.\(^1\),\(^2\) Since the beams do carry momentum they can be used to drive currents in fusion plasmas, leading to longer pulse duration in tokamaks. Injected neutrals with a chosen velocity can charge exchange with plasma ions, what was the plasma ion now being a neutral particle and hence unconfined, leaves. This pumping of ions is used to tailor ion velocity distributions, which leads to greatly enhanced end cell plugging in mirror machines. Thermonuclear fusion means that the plasma ions are required to have a temperature that is high enough so that a sufficient number of them will undergo collisions energetic enough to overcome the Coulomb barrier, and fuse. For such a plasma to become self sustaining (ignite) the energy in the charged particles produced by these reactions must balance all plasma losses. The very high temperature required for ignition may be reduced by relying on the collisions between the very high energy particles from neutral...
Fig. 1-1 Schematic of a Neutral Beam Line.
beams and the plasma ions rather than collisions between plasma ions themselves to produce the initial fusion reactions. Such a scheme may be used to do the final plasma heating up to true thermonuclear ignition.

All of these uses require the beam to penetrate a certain depth into the plasma, but of course not out the other side; this determines the beam energy required. As plasmas become larger and denser, ever more energetic beams are required. Present plans call for beams with energies in excess of 100 keV and even up to 1 MeV with many megawatts of power required (hence several 10's of ampere equivalents). Presently all systems utilize collisions with either gas or plasma to neutralize the beam ions; this imposes a maximum obtainable neutral fraction less than unity, since subsequent collisions can cause reionization. Neutralizing positive ions by electron capture becomes increasingly less efficient as beam energy is increased, whereas neutralizing negative ions by stripping off the weakly bound extra electron remains very efficient even at high energies. For negative ions an ideal neutralizer can be had by using photons whose energy is sufficient to strip the extra electron, but not enough to cause subsequent ionization. Such a photodetachment scheme promises efficiencies in excess of 95% for any energy. (Fig. 1-2). Higher energy neutral beam systems of any reasonable efficiency require negative ions.

Hydrogen has 3 stable positive ions: H\(^+\), H\(_2\)^+, H\(_3\)^+ (likewise for deuterium), hence positive hydrogen ion beams are composed of a mixture of these, resulting in a neutral beam with H\(^0\) (D\(^0\)) atoms at full, 1/2, and 1/3 the acceleration energies. This is undesirable
Fig. 1-2  Maximum Obtainable Neutral Fraction vs Beam Energy
since these low energy neutrals will not penetrate to the plasma core and deposit energy where required, and may even be detrimental by heating the outer plasma layers. Hydrogen has only one stable negative ion $H^-$, hence for negative based neutral beams this problem is eliminated.

**D^- Sources**

Presently $D^-$ sources\(^4,5\) are of three types: double charge exchange systems, surface production/conversion sources, and volume production sources.

Double charge exchange systems convert a low energy positive ion beam (typically 1 to 10 keV) into a negative ion beam by capturing two electrons in an alkalai or alkaline earth vapor cell (Fig. 1-3).\(^6\) This beam is then further accelerated, and then neutralized. Approximately 10% of a 10 keV proton beam can be converted to $H^-$ in sodium vapor, and up to 50% of a 250 eV proton beam in strontium vapor.\(^7\) Much less current can be extracted from positive ion sources at these low energies leading to accel decel systems. High current beams at such low energies will be highly divergent due to space charge effects in the beam and large scattering angles in the vapor cells, decreasing efficiency and causing high electrode loading. Metal vapors are hard to confine, resulting in contamination, which will greatly reduce the voltage holding capabilities of the accelerator structures, and in the introduction of these metal impurities into the fusion plasma.

Surface $D^-$ sources rely on production from a cesiated metal surface which is inserted into a hydrogen plasma (Fig. 1-4). A cesium
Fig. 1-3 Schematic of a Double Charge Exchange Neutral Beam System
Fig. 1–4 Schematic of a Surface Converter Negative Ion Source
layer adsorbed onto a metal substrate produces a low work function surface which enhances the formation and subsequent escape of $D^-$ ions. Optimum $D^-$ production requires a partial monolayer of cesium on the surface. The converter surface is biased negatively with respect to the plasma to expel the $D^-$ ions formed (self extraction) and to also attract positive ions which bombard the surface, some of these back scatter as $D^-$ or sputter off $D^-$ ions which have formed at the surface. This positive ion flux represents a large drain on the plasma and also requires much additional power to maintain converter bias (10's of amps at 100's at volts). Ion bombardment also erodes the cesium layer requiring either a pulsed operation, where a fresh cesiated surface is prepared subsequent to each pulse, or for some kind of equilibrium between cesium injected into the plasma, and cesium adhered to the surface, which is not readily controlled. The $D^-$ ions are extracted across a weak magnetic field which bends away the electrons but only slightly effects the ions. The relatively large transverse energy of the $D^-$ from the converter (mainly due to the backscattering contribution) and the inherent divergence of the self extraction scheme degrades beam optics. Even so these sources have yielded the highest $D^-$ currents yet obtained and are the major area of negative ion source work.

It has been observed that some hydrogen plasmas have a significant $H^-$ density in the center. Much simpler negative ion sources which require no cesium or other metal vapors would result from large high density plasmas of this type; also volume formed $H^-$ has a low temperature which would yield a low divergence beam. Here the problem
is extracting these ions from the volume of the plasma through the positive plasma potential generated to retard electron losses and maintain quasineutrality. The other acute problem, which is also of concern in the other types of H\(^-\) sources, is electron supression; simple schemes to extract negatively charged particles, from a plasma will also draw out large electron currents.

Study of volume production indicates that H\(^-\) production is by multiple electron collisions with hydrogen molecules in the discharge. One or maybe several energetic electron collisions produce a suitable intermediate (excited molecule or molecular ion) which is then dissociated by attachment of a low energy electron resulting in H\(^-\)\(^{10}\).

The identity of the intermediate has not been firmly established yet because no means of measuring the populations of the likely candidates, so as to correlate them with H\(^-\) densities, are now at hand; although methods are under development.

The plasmas used for H\(^-\) volume studies so far have chiefly been hot filament discharges. Energetic electrons (called primary electrons) are injected into hydrogen gas to collisionally ionize it. The vast majority of the electrons in the discharge are low energy thermal electrons (\(T_e < 2\) eV) but a high energy tail stretches up all the way to the filament voltage (typically around 50 volts). This tail is produced by the primary electrons whose energy is continually decreased by a series of inelastic collisions. These high energy electrons are the chief producers of the molecular intermediates, but also constitute the major destruction mechanism of H\(^-\) since collisions with energetic electrons detach its weakly bound extra
electron. This naturally leads to investigation of double plasma discharges\textsuperscript{11,12} where energetic electrons are confined to one region but neutrals, ions, and thermal electrons are allowed into a second region for $H^-$ production. This region of less energetic electrons will also have a lower plasma potential which makes $H^-$ extraction easier.

**Multicusp Microwave Plasma**

This experiment does not use a hot filament discharge but microwaves ($10.6$ GHz $- 3$ cm). The hot tungsten filaments used so far, have relatively short lifetimes, and filament discharges have long turn on times since the filaments must first reach emission temperatures (this is especially undesirable for pulsed operation); hence alternatives merit investigation. Also this discharge will have a different electron distribution, so by comparison and contrast more can be learned about volume processes.

Microwaves are that part of the electromagnetic spectrum with wavelengths comparable to the size of ordinary objects ($\text{ie } \lambda \sim 1$ cm and $\nu \sim 10^{10}$ hz). For use in this thesis, in conjunction with plasmas, the word microwave will mean an alternating electric field with a frequency that is much greater than any collision process involved, therefore the electrons will undergo many unperturbed oscillations in this field before colliding. Elastic collisions, by momentarily decoupling the electron's velocity from the oscillating field, add part of it to the particle's random (thermal) velocity since, statistically, the direction of travel immediately after
colliding is arbitrary. The electron is immediately swept back into oscillation by the field; thus by a random walk process, energy is transfered from the microwave field to the electrons. This is to be contrasted with high frequency discharges where the electron will undergo several collisions before it could have otherwise completed one oscillation in the electric field. These are in many ways really more of a d.c. discharge which frequently reverses direction.

In this experiment coupling is enhanced in one region by a cyclotron resonance producing magnetic field (Fig. 1-5). There is a distinctly brighter plasma confined to this gyroresonance zone hence this magnetic field also serves to separate a small hot plasma from the rest of the volume. This benefit is probably lessened somewhat by the fact that a significant microwave field exists even in the cooler plasma region resulting in subsequent electron heating.

Multicusp magnetic fields from permanent magnets (Fig. 1-6) have been successfully used for several years to improve performance of filament driven, low temperature plasmas. This arrangement reduces the effective loss area at the wall to a narrow strip along each cusp while maintaining a field free plasma in the center. Also a relatively flat central radial density profile is obtained since transport near the walls has been greatly reduced. For this work a line cusp arrangement was used.

This dissertation describes the microwave plasma experiment assembled, a survey of the trends pertinent to \( H^- \) production that it exhibited, and a discussion of some relevant physics theory. Chapter II considers the overall nature of the discharge namely energy
Iron yoke with permanent magnets

Hot plasma trapped in ECR zone

Gas jet

Microwaves in \( v \sim 10^{10} \)

Intermediates from hot plasma

Cool plasma confined by multicusp field

Cool electrons

Local intermediates

\( H^- \)

Sample beam extractor

Mass analysis

Disc probe

Fig. 1-5 Schematic of the Multicusp Microwave Experiment
Fig. 1-6 Basic Multicusp Arrangements

- Line cusp
- Checkerboard
- Ring Cusp
flow from the microwaves to the electrons and into the plasma. Chapter III considers the formation and loss mechanisms of the principal particles that are present with emphasis on H⁻ formation. Chapter IV looks at a set of calculations that were performed to gain understanding of the magnetic fields obtainable from the very strong permanent magnets now available. Chapter V describes the apparatus and Chapter VI its utilization in performing the experiment. Chapter VII discusses the results obtained and compares them to the theory discussed in Chapter III and to results from filament discharge experiments.
Chapter II

Overall Nature of the Discharge: Energy Flow and Plasma Confinement

In this chapter some general aspects of the discharge will be evaluated theoretically and a simple model will be used to estimate what temperatures and electron densities could be expected in the small ECRH plasma in this experiment. In the next chapter these will be combined with experimentally measured values for the relevant parameters in the nonresonant plasma to examine H\(^-\) formation and loss and determine the principal mechanisms.

Plasma Potential and Energy Transport

Consider a steady state plasma bounded by a continuous perfectly conducting wall, which will serve as the reference potential (\(\varphi_{\text{wall}} = 0\)). The problem of interest, namely transport to the wall, has already been solved numerous times\(^\text{15}\), but it is necessary here to extend it to the case of a plasma composed of several species of ions. The electrons will be assumed to have a maxwellian velocity distribution (temperature \(T_e\)) but the ions' temperatures will be considered negligible and ions will be treated as a set of cold interpenetrating fluids. The plasma will assume a positive potential with respect to the wall in order to retard electron loss and enhance ion transport to maintain steady state. The electrons will be assumed to be isothermal and hence described by a Boltzmann spatial distribution

\[
n(z) = n_e^0 e^{\left(-\varphi_p - \varphi_{\text{wall}}\right)/T_e} \]

\(n_e^0\) is a constant and \(\varphi_p\) is the plasma potential.
(temperatures will be given in units of energy, specifically, electron volts) where $z$ is the coordinate normal to the wall, and $q_e$ is the absolute value of the electron's charge. Negative ions are present but only in very small quantities ($n \sim 0.005 n_e$). Since they also are confined by the plasma potential it is assumed they will also assume a Boltzmann distribution, the total negative charge density would then be

$$\rho^-(z) = n^0_e q_e (\delta - \delta_p)/T_e + \sum_m n^0 q_m (\delta - \delta_p)/T_m.$$ 

To first order this is equivalent to electrons only with an effective temperature

$$T_{\text{eff}} = \left(\frac{n^0_e}{n^0_e} \frac{1}{T_e} + \sum \frac{n^0 m q_m}{n^0 q_e} \frac{1}{T_m}\right)^{-1}$$

where $n^0 = \frac{1}{q_e} (n^0_e q_e + \sum n^0 m q_m)$

which is lower since the negative ion temperature will be lower. For this work the effect of negative ions will be negligible.

A simple model of the transition at the wall will be used: the plasma itself will be assumed to be quasineutral and separated from the wall by a thin, non-neutral sheath with a well defined sheath edge. Motion across the sheath will be considered collisionless and formation and loss of charged particles in the sheath will be considered negligible. As has already been shown the concept of a well defined sheath edge is not compatible with this cold ion model and it will be assumed the ions acquire a net drift due to a potential gradient which extends on into the plasma. The ion spatial distribution in the sheath can be gotten from the equation of continuity and the equation of motion.
\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{v}_s) = 0
\]

\[
n_s m_s \left( \frac{\partial}{\partial t} + \vec{v}_s \cdot \nabla \right) \vec{v}_s = n_s q_s (-\nabla \phi)
\]

(index \(s\) for ion species). For a one dimensional, steady state case these can be simplified and rewritten as

\[
n_s(z) = n_s(a) \frac{v_s(a)}{v_s(z)} \frac{d}{dz} \left( \frac{1}{2} m_s v_s^2 + q_s \phi \right) = 0
\]

Let point \(a\) represent the sheath edge and assume the velocity that the ions acquire is due solely to the potential drop in the plasma preceding the sheath, then the ion distribution can be written as

\[
n_s(z) = n^0_s e^{(\phi(a) - \phi_p)/T_e} \sqrt{\frac{(\phi_p - \phi(a))}{(\phi_p - \phi(z))}}
\]

where it has been assumed that beyond the sheath edge the plasma is quasineutral and ion composition does not vary. The only quantity necessary here is the ion contribution to the charge density which is

\[
\rho_{ion}(z) = \sum q_s n_s(z)
\]

but note from the previous equation that the only part of the summation that depends on the particular species is it's reference density, all else factors out, leaving a trivial summation

\[
\sum q_s n^0_s = n^0_e
\]

due to quasineutrality. Thus the ion charge distribution is independent of the ion composition under these assumptions and the result will be taken from the derivation of Chen\(^{15}\) for a single ion species.
In the plasma next to the sheath where the small accelerating field is assumed to exist the ion motion is still governed by the same equation of motion, hence

\[ \frac{d}{dz} \left( \frac{1}{2} m_s v_s^2 + q_s \phi \right) = 0 \]

but the equation of continuity is different now since ion production and loss due to processes other than transport cannot be negligible. From the equation of motion it follows immediately that the ions will be streaming out at their respective sound speeds upon reaching the sheath

\[ v_s(a) = \sqrt{\frac{T_e}{m_s}} \]

and the density determined by Boltzmann's equation will be

\[ n_s(a) = n_s^0 e^{-1/2} \]

The plasma potential can now be calculated from the steady state requirement that net current to the wall must be zero. All ions entering the sheath will be lost, but only those electrons with enough energy to overcome the Coulomb barrier at the wall will escape. To preserve the maxwellian velocity distribution collisions or some equivalent mechanism must replenish the fast electrons as quickly as they are lost. It will also be assumed that any departure from a maxwellian in the sheath will not be enough to alter the electron spatial distribution significantly from a Boltzmann profile. Equating the ion charge flux at the sheath boundary to the charge flux carried
across it by electrons with sufficient energy to reach the wall yields
\[ n_e^0 e^{-1/2} q_e \sqrt{\frac{m_e}{2\pi T_e}} \int_0^\infty \frac{2q_e \phi_p}{m_e} v e^{-1/2m_e v^2/T_e} \, dv \]
\[ = \sum n_s^0 e^{-1/2} q_s \, T_e / m_s \]
This can be evaluated and simplified to yield
\[ \phi_p = - \frac{T_e}{q_e} \log \left( \sum \frac{n_s^0 q_s}{n_e^0 q_e} \sqrt{\frac{2\pi m_e}{m_s}} \right) \] (1)
This is shown in Chapter VII to agree well with the measured value.

Now the second quantity to be determined is the average energy removed by each electron lost. That can be determined by dividing the energy flux by the electron flux and has been determined by Loveberg. 16
\[ \Delta \epsilon = q_e \phi + 2T_e \]
The ions will also carry out an additional \( 1/2 T_e \) acquired during their acceleration, before entering the sheath. The total energy lost by the plasma by flux to the wall is thus
\[ \left( \frac{dn_e}{dt} \right)_{\text{transport}} = \left( \frac{dn_e}{dt} \right)_{\text{transport}} QT_e \]
where \( Q \) is defined as follows:
\[ Q = \frac{5}{2} + \log \left( \sum \frac{n_s^0 q_s}{n_e^0 q_e} \sqrt{\frac{m_s}{2\pi m_e}} \right) \] (2)
Microwave Discharges

Consider the single particle equation of motion for collisional oscillation in a microwave field with a d.c. applied magnetic field present.

\[ \frac{d\mathbf{v}}{dt} = \frac{q}{m} (\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) - v_m \mathbf{v} \]

where \( v_m \) is the collision frequency for momentum transfer. In this discharge the dominant collision for electrons is with neutral hydrogen molecules. For a 7 eV Maxwellian electron distribution the ratio of the average collision frequency to pressure is 5.7 MHz/mTorr.

For \( \mathbf{B} = \mathbf{B}_0 \) and \( \mathbf{E} = E_x e^{-i\omega t} \mathbf{x} + E_z e^{-i\omega t} \mathbf{z} \) this can easily be solved to yield

\[ v_x = \frac{\frac{v}{\omega} - \Omega}{(\frac{v}{\omega} - \Omega)^2 + \Omega^2} \left( \frac{qE_x}{m} \right) \]

\[ v_y = \frac{\Omega v_x}{(v-\omega)} \]

\[ v_z = \frac{1}{v-\omega} \left( \frac{qE_z}{m} \right) \]

where \( \Omega = \frac{qB}{mc} \) is the gyrofrequency. The average power transferred to the oscillating particle is

\[ p = <qE \cdot v> = \frac{q^2}{m} \text{Re} \left( \frac{\frac{v}{\omega} - \Omega}{(\frac{v}{\omega} - \Omega)^2 + \Omega^2} \right) + \frac{q^2}{m} \text{Re} \left( \frac{1}{v-\omega} \right) \]

\[ p = \frac{q^2}{m} \left< E_x \right> \frac{v}{2} \left( \frac{1}{v^2 + (\omega-\Omega)^2} + \frac{1}{v^2 + (\omega+\Omega)^2} \right) + \frac{q^2}{m} \left< E_z \right> \frac{v}{(\omega^2 + v^2)} \]

(3a)
This can be rewritten in terms of an effective field strength

\[ p = \frac{q^2}{mv} \langle E_e^2 \rangle \] (3b)

\[ \langle E_e^2 \rangle = \frac{v^2}{2} \left( \frac{1}{v^2 + (w-\Omega)^2} + \frac{1}{v^2 + (w+\Omega)^2} \right) \langle E^2 \rangle \]

This can be applied to the effective electric field parallel to the applied magnetic field and also when there is no applied magnetic field, by letting \( \Omega = 0 \)

\[ \langle E_e^2 \rangle = \frac{v^2}{v^2 + \omega^2} \langle E^2 \rangle \]

Figure 2-1 shows the tremendous enhancement (5 to 6 orders of magnitude) of coupling obtained for resonance (\( \omega = \Omega \)) for \( v = 1.1 \times 10^8 \) and \( v = 2.8 \times 10^8 \) (gas pressures 20 to 50 mTorr) which corresponds to the operating range in this experiment. Note that at resonance

\[ \langle E_e^2 \rangle = \frac{1}{2} \langle E^2 \rangle \] for all \( v << \omega \).

**Simple 1-D Steady State Discharge**

A very simple theory\(^{17}\) will be developed as a basis for understanding the small hot plasma formed in the gyroresonance zone. Figure 2-2 shows a sketch of this region (see also Figs. 4-14, 4-18, and 5-3). The pertinent facts are that particles are constrained to move along the strong magnetic field lines, and that the characteristic length is about 1 centimeter.

Consider a plasma composed of a single type of positive ion which is formed by electron collisions with neutral gas particles in the discharge and lost to the walls. The electron density equilibrium
Fig. 2-1 Enhancement of the Effective Microwave Electric Field by Gyroresonance (3.8 kgauss for 10 Ghz).
Fig. 2-2 Sketch of enhanced cusp field region. The ECR zone is shown in red. The magnet pole faces are the white bars above and below it (compare with Fig. 4-4). Microwaves enter through the two rectangular waveguides.
equation is

\[
\frac{dn_e}{dt} = n_e n_0 \langle \sigma_{\text{ion}} v_e \rangle - n_e / \tau = 0 \tag{4a}
\]

\[
n_0 \langle \sigma_{\text{ion}} v_e \rangle \tau = 1 \tag{4b}
\]

where \( \langle \sigma_{\text{ion}} v_e \rangle \) is the ionization rate coefficient and \( \tau \) is the characteristic loss time. The average ion loss time will be approximated as the time to travel the characteristic length at the sound speed.

\[
\tau = L / v_s \tag{5}
\]

for a characteristic length \( L \) and the sound speed given by

\[
v_s = \sqrt{\frac{T_e}{m_i}}
\]

Equation (4b) provides a functional relationship between \( n_0 \) and \( T_e \), since both \( \tau \) and \( \langle \sigma_{\text{ion}} v_e \rangle \) depend on \( T_e \). Notice also input power has not yet been included in this model \( T_e \) is independent of it, but assumes the value necessary to balance ionization with loss at each pressure. Figure 2-3a shows the solution obtained using the ionization rate for \( e^- + H_2 \rightarrow 2e^- + H_2^+ \) (see Appendix C for method used to produce this and subsequent graphs).

If we assume an input power \( P \) per unit volume the energy balance equation is

\[
P = n_e n_0 \langle \sigma_{\text{ion}} v_e \rangle \epsilon_1 + Q T_e n_e / \tau = (\epsilon_1 + Q T_e) n_e / \tau \tag{6}
\]

where \( Q \) is as defined in equation (2) and \( \epsilon_1 \) is the effective
ionization energy. Not all inelastic electron-neutral collisions are ionizing; many result in electron excitation and for neutral molecules in vibration and rotation or even dissociation. Most of the energy into electronic excitation is lost by radiative decay, while vibrational and rotational energy is distributed among all the neutrals by neutral-neutral collisions. Therefore much more energy than that necessary to ionize a neutral must be transferred from the microwaves to the discharge to result in one electron-ion pair. Miles et al. have looked at electron energy deposition in hydrogen theoretically by summing over all inelastic collision paths for both the original electron and the secondaries until the energy is expended; they assumed no electron heating mechanism, but that all the energy was present with the initial electron. For electrons starting with 100 eV or more the effective energy expended for each ionization was independent of the initial energy and was about 35 eV. As initial electron energy is lowered this effective ionization energy sharply increases, and passes 100 eV at about 30 eV initial electron energy. The actual problem including microwave heating would be far more difficult, but Miles' work indicates that the effective ionization energy will be quite high. As an approximation $\varepsilon_i = 100$ eV will be used. Equation (6) can be solved to yield $n_e$ as a function of $n_o$ using $T_e$ obtained from (2)

$$n_e = \frac{P}{n_o <\alpha_1 v> \varepsilon_1 + Q T_e / \tau}$$

Note that $n_e$ is directly proportional to $P$. This is shown in Figure 2-3b for $P = 300$ watts/cm$^3$. Since total input power was
Fig. 2-3  Predicted Electron Temperature and Density
about 750 watts and the volume of the gyroresonant zone was about 2.5 cm$^3$ this is the right magnitude for the available power.

**Applicability of this Model**

Now consider whether this model can be applied to the small gyroresonance zone. Several aspects will be considered and shown to fit with this model. At the same time a good understanding of this small plasma will be obtained.

First, even though the model does not directly consider the mechanism for energy transfer namely randomization of the electron's oscillatory motion in the microwave electric field by collisions, is it compatible with this? The model considers the electrons to be characterized by a temperature $T_e$ and for the ionization and loss rates to be determined by it. For the electron energy to be maxwellian some randomizing agent must be present, this is the collisions; even though they are predominantly with gas particles and not other electrons they do cause random amounts of kinetic energy to be decoupled from oscillation. Since the model uses fluid equations, by necessity the electrons should undergo several collisions before being lost for these to be applicable. Figure 2-4 shows the average number of collisions predicted for an electron in the time it takes it to travel to the wall at the sound velocity.

$$N_{\text{coll}} = v_T$$

for both electron neutral collisions $^{19}(v_{eo})$ and electron ion collisions $^{20}(v_{ei})$ calculated from

$$v_{eo} = n_0 \langle \sigma_{eo} v \rangle.$$
Fig. 2-4  Average Number of Collisions of an Electron with Neutrals (+) and with Ions (*) before Loss.
\[ \nu_{ei} = n_1 \langle \sigma_{ei} \rangle \quad \nu = \frac{n_1 e^4 \log(A)}{6 \pi \sqrt{3} m_e c^2 (T_e)^{3/2}} \]

\[ \nu_{ee} = \frac{1}{2} \nu_{ei} \]

The electron will make on average at least 10 collisions with a neutral before it is lost which should be enough to justify \( T_e \) and at higher pressure electrons will be exchanging energy in collisions with each other.

Secondly how do the particles' random energies compare with that tied up in oscillation? The oscillation kinetic energy is

\[ \epsilon_{osc} = \frac{m}{2} (\nu_x^2 + \nu_y^2) = \frac{m}{4} (\nu_x \nu_x^* + \nu_y \nu_y^*) \]

\[ = \frac{e^2 E_x^2}{4m} \left( \frac{\nu^2 + \omega^2 + \Omega^2}{(\nu^2 + \omega^2 - \Omega^2)^2 + 4\omega^2 \nu^2} \right) \]

(7)

The collision frequency \( \nu \) is considered here as a function of \( \epsilon_{osc} \). Microwave power up to 750 watts is delivered from 2 rectangular waveguide openings (TE\(_{01}\) mode) each 2.3 x 1 cm. For a waveguide of these dimensions with no additional reflected field we have average field strengths

\[ 100 \ \text{v/cm} < E < 500 \ \text{v/cm}. \]

Fig. 2-5 shows the oscillation energy at resonance for these microwave fields as a function of magnetic field. The actual resonance region is very thin, about .5 cm for the region from 3 to 4 kgauss. As seen in the figure at even the high electric field an electron must be within a region less than .1 kgauss wide to have energies in excess of
Fig. 2-5 Oscillation Energy of an Electron in a Microwave Field showing Resonance Enhancement.
100 eV. The gyroradius for an electron is
\[
\rho_g = \frac{\nu_{osc}}{\Omega} = \frac{c \sqrt{2} \epsilon_{osc} m}{e B} = 3.4 \times 10^{-3} \sqrt{\frac{\epsilon}{B}}
\]
for \( \rho_g \) in cm, \( \epsilon \) in eV and B in kgauss. Physically the region for energies in excess of 100 eV will be only about .01 cm wide which equals the gyroradius it would have at 3.8 kgauss hence an electron could not stay in resonance if it were that energetic. Some electrons with several 10's of eV of oscillatory energy might exist in a narrow band right at resonance but most in the region will have only a few eV of oscillatory energy; so indeed for most of the region high random energy electrons will account for ionization. For calculations a Maxwellian velocity distribution is used and results are seen not to be grossly in error.

To look at this another way consider an electron at gyroresonance, how much random energy can be acquired by the time it drifts to the wall? A good estimate would be

\[ \epsilon = p \tau \]

Using (3b) for the average power transferred to each electron and \( v=10^8 \) we get a staggering energy transferred, in excess of of \( 10^5 \) eV in time \( \tau \). Of course energy transfer of this magnitude does not take place, but this does mean particles can be off resonance, with the effective field reduced by several orders of magnitude, and still be adequately heated. From (3a), ignoring the second term which is negligible near resonance since \( \omega^2 \approx 10^4 \times v^2 \), we can get the field necessary to impart a power \( p \) from

\[
\Omega \approx \omega \pm \sqrt{\frac{e^2}{2mp} <E^2> - v^2}
\]
Choosing as a reasonable necessary power $p = (\epsilon_i + QT_e)/\pi$ we get

$$\Delta B_{\text{broadening}} \approx \frac{mc}{e} \sqrt{\frac{e^2 \nu L <E^2>}{2m(\epsilon_i + QT_e)V_S}} - \nu^2$$  \hspace{1cm} (8)

Figure 2-6 shows this for $E = 100$ V/cm and $E = 500$ V/cm, note the tremendous effect of increasing microwave field strength on size of the effective resonance zone. Also note how rapidly this broadening increases with pressure (due to increase in $v_{eo}$) ranging from about 100 gauss at 5 mTorr to over 900 gauss at 50 mTorr for 500 V/cm. This means that not only does $n_e$ in the gyroresonance zone increase with pressure but so does the effective volume.

Thirdly the model predicts that moderate powers will produce densities well in excess of $10^{12}$ cm$^{-3}$. Because of the geometry the electromagnetic modes of interest are those traveling across the magnetic field. The microwave frequency for this experiment was fixed at 10.6 Ghz, thus the cutoff density for the ordinary mode is

$$n_{e}^\text{ord} = \frac{m_e}{4\pi e^2} \omega^2 = 1.4 \times 10^{12} \text{cm}^{-3}$$

and the upper cutoff density for the extraordinary mode is

$$n_{e}^\text{ext} = \frac{m_e}{4\pi e^2} \omega(\omega + \Omega) = n_{e}^\text{ord} (1 + \frac{\Omega}{\omega}) = 2 n_{e}^\text{ord}$$

Beyond cutoff the microwave field will be an evanescent mode with a skin depth

$$\delta = \frac{c}{\sqrt{\omega^2 - \omega_0^2}} = \frac{c}{\sqrt{\frac{4\pi e^2}{m_e} n_e - (2\pi \times 10^{10})^2}}$$  \hspace{1cm} (9a)
Fig. 2-6  Pressure Broadening of the Effective Resonance Zone for Microwave Fields of 500 V/cm (upper) and 100 V/cm (lower).
for the ordinary mode and

$$\delta = \frac{c}{\sqrt{\frac{\omega_p^2}{\omega_p^2 + \Omega^2 - \omega^2} - \omega^2}}$$  \hspace{1cm} (9b)

for the extraordinary mode. These are plotted in figure 2-7. Neither mode will propagate through the dense resonance plasma, nor will the evanescent field penetrate all the way, however on the steep density and field gradients along the edge the waves will convert to other plasma wave modes which will propagate.\textsuperscript{22,23}

Fourthly is the assumption of loss of ions only to the walls valid? The mean free paths for destruction by the fastest processes namely

$$\begin{align*}
H_2^+ + e^- & \rightarrow 2H \\
H_2^+ + e^- & \rightarrow H^+ + H + e^- \\
H_2^+ + H_2 & \rightarrow H_3^+ + H
\end{align*}$$

are defined by

$$L = \frac{v_s}{n_i<\sigma v>}$$

where the rate $<\sigma v>$ is calculated using the electron temperature for the first two, but ion temperature, which is not really known, is needed for the last. It should lie in the range

$$0.1 \text{ eV} < c_{\text{ion}} < 1 \text{ eV}$$

in which range $<\sigma v>$ for conversion to $H_3^+$ varies from $2 \times 10^{-9}$ to $5 \times 10^{-9}$ cm$^3$/sec. Figure 2-8 shows considerably more than one centimeter is required for ion loss, but that most of the $H_2^+$ are
Fig. 2-7 Penetration Depth (l/e) for the two Cross Field Electromagnetic Modes. The Extra-ordinary mode is for 3.8 kgauss (gyroresonance).
Fig. 2-8 Mean Free Path for Conversion of $H_2$ into $2H$ (*), $H^+ + H$ (†), and $H_3^+$ (#).
converted to other ion forms. The equilibrium fraction of $H_2^+$ can be gotten by equating the rate for ionization with the rate for loss
\[
\frac{n_e}{n_o} <a_{1on}v> = n_2^+ \left( n_0 <a_{2-3}v_{ion}> + n_e <a_{2-1}v_e > + \frac{1}{\tau} \right)
\]
\[
\frac{n_2^+}{n_e} = \frac{n_o <a_{1on}v>}{n_0 <a_{2-3}v_{ion}> + n_e <a_{2-1}v_e > + \frac{1}{\tau}}
\]
(10)

(this type of equation and notation is explained in Chapter III in the section entitled Reaction Rates). This fraction is shown in Fig. 2-9.

The fact that most $H_2^+$ is converted to another ion type is not serious for the model it only means the effective ion mass is somewhat different but this would have only a very small effect if included since it only appears in $V_a$.

Finally using the formulas for curved field drift\textsuperscript{24} an estimate will be made of how far a particle would drift before it is lost to the wall.

\[
D \frac{mc}{q} \left( v_{||}^2 + \frac{1}{2} v_\perp^2 \right) \frac{R_c \times B}{|R_c|^2 |B|^2}
\]

where $R_c$ is the radius of curvature of the field. Converting to MKSA units and substituting in the appropriate values.

\[
\frac{v_{drift}}{v} = \frac{mv}{qBRC} = \frac{1.6 \times 10^{-27} \times 3 \times 10^4}{1.6 \times 10^{-19} \times .4 \times .01} = .08
\]

The particle will thus drift about .08 *L which is negligibly small.

Overall this model fits remarkably well in spite of its simplicity and although no direct measurements have been made on the gyroresonant zone plasma it fits the observed patterns.
Fig. 2-9  $H_2^+$ Fraction of Positive Ions
(1) Gyroplasma visibly larger and much brighter as gas pressure increased (both \( n_e \) and width of gyrozone predicted to increase).

(2) Power reflection becomes a serious problem as pressure increased (dense plasma extends in front of waveguide openings).

(3) Estimates made in the next chapter based on \( n_e \) and \( T_e \) from this model agree well with measurements.

The gyroresonance plasma can be expected to have electron densities of several times \( 10^{12} \) \( \text{cm}^{-3} \) with density and physical size increasing and electron temperature decreasing with gas pressure.

**Nonresonant Volume Plasma**

The plasma in the nonresonant region of the chamber is composed of particles formed in the outer layers of the gyroresonant plasma and also of those formed out in the volume. There is still a very large microwave field out here but the effective field density \( <E_e^2> \) is reduced by \( 10^{-5} \) to \( 10^{-7} \). The characteristic length is now 10 cm (ignoring multicusp field) and \( T_e \) is relatively low (1-2 eV) hence

\[
\tau = L/v_a \approx 1 \times 10^{-5} \text{ sec}
\]

The gas pressure is in the same range hence

\[
2 \times 10^7 < v_{eo} < 2 \times 10^8 \text{ sec}^{-1}
\]

The microwave heating rate (per particle) from (3) is

\[
p = \frac{e^2}{2m} \left( \frac{v}{\omega^2 + v^2} \right) <E_e^2>
\]

thus the expected average energy transfer is

\[
\epsilon = p \tau = .5 \text{ to } 5 \text{ eV}
\]

for the range of \( v_{eo} \) indicated and \( E = 100 \text{ v/cm} \). This is about
right since the volume plasma will run even with no gyroresonant zone present for pressures exceeding 10 mtorr ($\nu > 4 \times 10^7$ sec$^{-1}$).

The chamber walls are covered by a multicusp arrangement of permanent magnets. The effect of this field can be thought of as increasing the number of encounters with the wall a particle will survive before it is lost. This will increase average confinement time for any electron and hence increase the energy transfer from the nonresonant field to it. This increase in energy transferred from the microwave field does not all go into increased electron temperature but is again transferred by collisions to the neutrals increasing gas temperature, excitation levels, and resulting in more local ionization. This is to be contrasted with the primary electrons in filament discharges where increased confinement means more energy loss time to the plasma by ionization and sharing of energy with thermal electrons, but no increase in primary electron energy since they start with all the energy they are going to get.

At very high pressures bright crescent shaped plasmas form in the multicusp fields. This is probably due to collisional broadening of the gyroresonance in the cusps since the magnetic field strength there is up to nearly 3 kgauss, and there is a large increase in field strength as the cusp is approached hence electrons could well be subject to mirror confinement.

**Improved Gyroresonant Zone**

In chapter IV a design for a much improved gyroresonant zone is presented. Here a considerable increase of field strength will occur along field lines before they reach the wall, hence there will be
mirror confinement. The zone is to be physically much larger and much farther from the wall. With a good design for a microwave antenna this design should produce an excellent double plasma for H⁻ production since $T_e$ in the resonant plasma should be much higher and particles should drift out into the volume rather than being lost. Nearly complete absence of microwaves in the volume should be obtainable due to greatly improved absorption.
Chapter III

Formation and Loss of Discharge Components Relevant to H⁻

Introduction

In this chapter the collisional processes in the plasma relevant to H⁻ formation and loss will be considered to determine which processes are the most important, those will be used to obtain an equilibrium equation for H⁻ density, and hence relate it to controllable parameters. Relatively complete computer models for the hydrogen plasmas used as ion sources are research projects in themselves. Chan et al.²⁵ are continuing too develop a computer model that predicts the hydrogen species densities (H⁺, H₂⁺, and H₃⁺) in positive ion sources and in order to include an accurate electron distribution, have included all relevant inelastic processes so that the high energy electron tail can be constructed by accounting for the collisional energy loss of the injected filament electrons. Hiskes and Karo.²⁶ and Bretagne et al.²⁷ have separately considered the entire electron distribution as a whole by numerically solving the Boltzmann equation. Their works clearly show the high energy tail and nearly maxwellian low energy distribution of filament sources but of course knowledge of the ion population has been lost.

None of the cross sections known around 1970 could account for the relatively high density of H⁻ found in low to moderate power hydrogen discharges. Since then studies (to be discussed presently) have shown that dissociative attachment cross sections for various excited molecular states or molecular ions are several orders of magnitude larger than for ground state hydrogen, and are high enough to account
for the H\(^-\) densities reported. These have however also complicated
the picture since they also need to be formed by an additional
collisional processes. Bacal, Hiskes, and co-workers\(^{28,29}\) have been
developing a model for volume H\(^-\) production which agrees rather well
with measurements. This model is based on H\(^-\) production by
dissociative attachment of low energy electrons to highly vibrationally
and rotationally excited molecules which in turn are formed by
collisions of energetic electrons with hydrogen molecules. Their
collective works will be referenced several times in this chapter and
the final result is very similar to theirs.

Reaction Rates
The determination of which processes are dominant and consequently
what the dependence of the H\(^-\) density on measurable parameters is,
depends on determining the rate at which each process proceeds; for
this reason this section might well be called plasma chemistry. The
collision processes are all binary with measured or calculated cross
sections (\(\sigma\)) from which a rate coefficient can be calculated by
averaging \(\sigma v\) over the appropriate velocity distribution, here assumed
to be Maxwellian. If there is a high energy tail to the distribution
it is assumed to be negligible in effect. As an example consider the
basic ionization equation

\[
e^- + H_2 \rightarrow e^- + H_2^+ + e^-\]

the rate is

\[
\left( \frac{dn_2^+}{dt} \right)_{\text{form}} = n_e n_o \sigma_{\text{ion}} v
\]
where <> indicates an average over the electron velocity distribution. As a useful shorthand \( \sigma v \) will be replaced by an alphanumerical symbol for the process, since many rates will be used; for this case

\[
<\sigma_{\text{ion}} v> \rightarrow <\text{Ion } 2^+>
\]

The other type of process to be considered, which is the dominant loss process for nearly all species, is drifting to the wall. Since the mean free paths for significant momentum transfer of any species exceeds the chamber size this loss rate will be estimated to be the average particle velocity divided by the characteristic length, for example

\[
H_3^+ + \text{wall} \rightarrow H_2 + H
\]

\[
\left( \frac{dn_3^+}{dt} \right)_{\text{loss}} = \frac{1}{\tau_3} n_3^+
\]

\[
\tau_3 = \frac{L}{v} = \frac{L}{\sqrt{2e/m}}
\]

For the species of interest these are summarized in Table 3-1 for \( L = 10 \text{ cm} \), \( \epsilon_{\text{ion}} = .1 \text{ ev} \) and \( T_{\text{gas}} = 500 \text{ K} \). Gas temperature and ion energy are inferred from measurements on other (filament driven) plasmas.

Steady state density relations are established by equating the formation rate with the loss rate

\[
\left( \frac{dn}{dt} \right)_{\text{total}} = \sum \left( \frac{dn}{dt} \right)_{\text{form}} - \sum \left( \frac{dn}{dt} \right)_{\text{loss}} = 0
\]

where of course the summations are over all processes. This will be what is meant whenever equilibrium equations are referred to.
<table>
<thead>
<tr>
<th>Species</th>
<th>Velocity (cm/sec)</th>
<th>Loss time (sec)</th>
<th>Rate (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(_2) at 500K*</td>
<td>(V_m=2\times10^5)</td>
<td>(T_m=5\times10^{-5})</td>
<td>(2\times10^4)</td>
</tr>
<tr>
<td>H(^+) at .1 eV</td>
<td>(V=4.4\times10^5)</td>
<td>(T=2.3\times10^{-5})</td>
<td>(4.4\times10^4)</td>
</tr>
<tr>
<td>H(_2) at .1 eV</td>
<td>(V_2=3.1\times10^5)</td>
<td>(T_2=3.2\times10^{-5})</td>
<td>(3.1\times10^4)</td>
</tr>
<tr>
<td>H(^+) at .1 eV</td>
<td>(V_3=2.5\times10^5)</td>
<td>(T_3=4\times10^{-5})</td>
<td>(2.5\times10^4)</td>
</tr>
</tbody>
</table>

*Various excited molecular states are of interest and the probability that the molecule will be de-excited in a wall collision is high.
Positive Ions in the Volume

Since the plasma must be quasineutral ion densities cannot be deduced apart from \( n_e \), however the relative density of each can be gotten from rate equations. Nearly all \( H_3^+ \) and \( H^+ \) come from conversion of \( H_2^+ \). It is interesting to compare the predicted ratios for both the volume and the gyrozone. Consider the following in equilibrium

\[
\begin{align*}
e^- + H_2 & \rightarrow e^- + H_2^+ + e^- \quad (a) \\
e^- + H^+ & \rightarrow e^- + H^+ + H \quad (b) \\
H^+ + H_2 & \rightarrow H_3^+ + H \quad (c) \\
H^+ \text{ wall} & \rightarrow H, H_2 \quad (d) \\
H_3^+ \text{ wall} & \rightarrow H, H_2 \quad (e)
\end{align*}
\]

These combine to give the following equilibrium equations:

\[
\begin{align*}
n_e \ n_0 <\text{Ion } 2^+> &= n_2^+ n_0 <2 + 3> + n_2^+ n_e <2 + 1> \quad (a,c,b) \\
n_e n_2^+ <2 + 1> &= n_1^+ 1/\tau_1 \quad (b,d) \\
n_0 n_2^+ <2 + 3> &= n_3^+ 1/\tau_3 \quad (c,e)
\end{align*}
\]

Since ion energy and gas temperature have been assumed to be about the same in both the gyroresonant zone and the volume the conversion rate to \( H_3^+ \) is fixed.

\[
n_2^+ n_0 <2 + 3> = n_2^+ 5 \times 10^{14} \times 2.4 \times 10^{-9} = 1.2 \times 10^6 n_2^+
\]

The conversion rates to \( H^+ \) are

Volume rate

\[
n_e n_2^+ <2 + 1> = 10^{11} \times 5.8 \times 10^{-8} n_2^+ = 5.4 \times 10^3 n_2^+
\]

Gyrozone Rate

\[
n_e n_2^+ <2 + 1> = 5 \times 10^{12} \times 1 \times 10^{-7} n_2^+ = 5.5 \times 10^5 n_2^+
\]
The conversion ratios are

\[
\text{volume alone } \frac{n_1^+}{n_3^+} = \frac{5.4 \times 10^3 n_2^+}{1.2 \times 10^6 n_2^+} = 4 \times 10^{-3}
\]

\[
\text{Gyrozone } \frac{n_1^+}{n_3^+} = \frac{5 \times 10^5 n_2^+}{1.2 \times 10^6 n_2^+} = 0.4
\]

The actual ratio observed in the volume is about 0.15. (See chapter VII) This indicates that about 3/8 of the ions are formed in the gyroresonance zone.

The processes for \(H^-\) formation and loss will now be considered and the reaction rates estimated for each so the most significant can be determined and incorporated into an equilibrium equation for \(H^-\) density. Then each of the species involved in these processes which are not measured will be similarly treated. Estimates will be based on the following set of experimental facts for the volume plasma:

- \(L\) (characteristic length) \(\sim 10\) cm
- \(n_0 = 10^{15}\) (30 mtorr)
- \(n_e = 10^{10}\)
- Ions: \(H^+ \sim 15\%\) \(H_2O^+ \sim 15\%\) \(H^+ < 1\%\) \(H_3^+ \sim 70\%\)
- \(T_e = 2\) eV
- \(T_i \sim .1\) eV (indicated by measurements in other discharges)

and on the following results from the calculations in the previous chapter for the gyroresonant plasma:

- \(n_0 = 10^{15}\)
- \(n_e = 5 \times 10^{12}\)
ions: \( H^+ \sim 24\% \quad H_2^+ \sim 40\% \quad H_3^+ \sim 36\% \)

\( T_e = 7 \text{ eV} \)

\textbf{H^- Formation}

The principal mechanisms are dissociative attachment (abbreviated DA) to:

Vibrationally and rotationally excited states

\[
e^- + H_2(v,j) \rightarrow H + H^- \quad (f)
\]

Metastable electronic state

\[
e^- + H_2(c^3 \pi_u) \rightarrow H + H^- \quad (g)
\]

Dissociative Recombination into ion pairs (abbreviated DR)

\[
e^- + H_3^{+} \rightarrow H_2^{+} + H^- \quad (h)
\]

\[
e^- + H_2^{+} \rightarrow H^+ + H^- \quad (i)
\]

Reaction (f) is very complicated since each vibrational and rotational state is different and there are 294 such combinations for \( H_2 \) in the ground electronic state which are stable. Wadehra and Bardsley\textsuperscript{31} have calculated the peak DA cross section for each of the vibrational rotational states available and Wadehra\textsuperscript{32} has calculated the rate coefficients for several representative states. Calculations by Bottcher and Bucky\textsuperscript{33} for DA to \( H_2(c^3 \pi_u) \) do not show much dependence on vibrational excitation in the cross section magnitude but the threshold is significantly decreased for each level. So here again careful consideration would mean treating all seven vibrational levels; rotational effects are as yet unknown. Calculations by Bottcher\textsuperscript{34} for DR in \( H_2^+ \) again show little dependence on vibrational energy but as yet vibrational dependence of DR in \( H_3^+ \) is
unknown. Since the rate for each of these depends on the density of each intermediate all must be retained until later for comparison.

**H^- Destruction**

The principal loss mechanisms for H^- are:

Collisional Detachment (ED for electrons)

\[ e^- + H^- \rightarrow e^- + H + e^- \] (j)

\[ \text{ion}^+ + H^- \rightarrow \text{ion}^+ + H + e^- \] (k)

Mutual Neutralization (MN)

\[ \text{ion}^+ + H^- \rightarrow \text{neutral} + H \] (l)

Associative Detachment (AD)

\[ H^- + H \rightarrow H_2(v) + e^- \] (m)

Charge exchange (CX)

\[ H^- + \text{neutral} \rightarrow H + \text{ion}^- \] (n)

Collisional detachment processes require a projectile with kinetic energy at least equal to the electron affinity (.75 eV) hence only electrons will be important. Hickman\(^{35}\) has developed a scaling rule for mutual neutralization

\[ \langle \sigma v_{\text{ion}} \rangle \approx \left( 4.38 \times 10^4 (\text{E.A.})^{0.4} \sqrt{m_{\text{reduced}} T/300} \right)^{-1} \] (2)

Where E.A. is the electron affinity in eV (.75 for H^-), the reduced mass is in atomic units (ie \( m_e = 1 \)) and T is in degrees K. This shows there is a weak dependence on ion species through \( m_{\text{reduced}} \) which for simplicity will be ignored. Associative detachment is the inverse of the formation process and is fairly large at low relative velocities due to an \( H_2^- \) intermediate state resonance.\(^{36}\)
Charge exchange rates are two orders of magnitude lower than MN for energies of interest and hence it will be dropped.

\[ H^- \text{ Equilibrium Equation} \]

Equating formation rates for processes (f)-(i) with rates for loss processes (j), (l) and (m) yields:

\[
\sum n_{e} n_{v}^{s} <DAv>^{s} + \sum n_{e} n_{v}^{s} <DAw>^{s} + n_{e} n_{3}^{+} <DR3> + n_{e} n_{2}^{+} <DR2> = n^{-} (n_{e} <ED> + n_{e} <MN> + n_{a} <AD>)
\]

where the summations are over all vibrational and rotational states, \( n_{a} \) means density of atomic hydrogen neutrals, and all MN have been lumped together using \( n_{e} = \sum n_{i}^{+} \)

Solving for \( n^{-} \) yields

\[
\frac{\sum n_{v}^{s} <DAv>^{s} + \sum n_{w}^{s} <DAw>^{s} + n_{3}^{+} <DR3> + n_{2}^{+} <DR2>}{<ED> + <MN> + n_{a}/n_{e} <AD>}
\]

\[ (3a) \]

**Contribution from Gyroresonant Zone**

All the species upon which \( H^- \) depends are formed both in the volume and in the gyroresonant zone, where formation rates are much higher due to the greatly increased electron density and temperature. What is the flux of particles from a small volume in the gyrozone to some representative small volume, a sphere of radius \( a \), on the center line of the chamber a distance \( h \) from the gyrozone? Assume the gyrozone is of uniform thickness \( t \) and has a constant creation rate for the species of interest \( (\frac{dn}{dt}) \). If this beam of particles is attenuated by some process with attenuation constant \( \alpha \) then the flux...
should be (assuming \( t \) is small enough so all escape)
\[
d\Gamma = \frac{1}{\omega a^2} \left( \frac{dn}{dt} \right) e^{-\alpha r} \frac{\Omega}{4\pi} t \, dx \, dy
\]

From the geometry (see Fig. 3-1) \( \Omega \) and \( r \) can be evaluated and the total flux from the gyrozone is
\[
\Gamma = 4 \frac{t}{4\pi} \left( \frac{dn}{dt} \right) x_m \int_0^{y_m} \int_0^{x_m} \frac{e^{-\alpha \sqrt{h^2 + x^2 + y^2}}}{h^2 + x^2 + y^2} \, dx \, dy
\]
\[
= G(\alpha, h) \left( \frac{dn}{dt} \right)
\]

This has been integrated numerically and \( G(\alpha, h) \) is shown in figure 3-2.

Now \( x_m = 3 \text{cm} \) \( y_m = 1 \text{cm} \) and \( t = .3 \text{ cm} \) so beyond \( h = 5 \), \( h^2 >> x^2 + y^2 \) so both the numerator and denominator can be expanded. By first expanding the square root in the exponential, and then expanding each exponential in the product formed, the integral can be reduced to algebraic terms and then integrated.

\[
G(\alpha, h) = \frac{1}{\omega h^2} e^{-\alpha h} x_m y_m t \left( 1 - \frac{1}{3h} \left( \frac{\alpha}{2} + \frac{1}{h} \right) (x_m^2 + y_m^2) + \frac{1}{h^3} \left( \frac{1}{n} - \frac{\alpha}{8} \right) \left( \frac{1}{5} x_m^4 + \frac{2}{9} x_m^2 y_m^2 + \frac{1}{5} y_m^4 \right) + \ldots \right)
\]
\[
G(\alpha, h) = \frac{9^2}{\omega h^2} e^{-\alpha h}
\]

This approximation is also shown in figure 3-2. For even \( h = 5 \text{ cm} \) the second order correction has become only about 5% and the first about 13%, so the approximation will be used in the estimations to follow.

The contribution this flux makes to the density of species \( s \) at \( h \) is
Fig. 3-1 Geometry involved in Determining $G(\alpha, H)$. 

\[ r = \sqrt{h^2 + x^2 + y^2} \]

\[ \Omega = \pi a^2 / l^2 \]
Fig. 3-2 $G(\alpha, h)$ (solid curves) and the approximation, from top to bottom: 
$\alpha = 0, .05, .1, \text{ and } .2$. 
\[ n'_s = \tau_s \frac{1}{v_{gs}}, \text{ } g \text{ designates gyrozone} \]

This will be a significant contribution if it is comparable to or larger than local production

\[ n' > \left( \frac{dn}{dt}\right)_{\text{volume}} \ast \tau_{\text{loss}} \]

For most species loss in the volume is by flow to the walls hence

\[ \tau_{\text{loss}} = \frac{L}{v_{\text{th}}} \text{ yielding} \]

\[ \left( \frac{dn}{dt} \right)_{\text{gyro}} > \frac{\pi \hbar L}{N \gamma m} e^{-ah} \frac{v_q}{v_{\text{th}}} \left( \frac{dn}{dt} \right)_{\text{vol}} \]

which at 20 cm should be in the neighborhood of

\[ \left( \frac{dn}{dt} \right)_{\text{gyro}} > 4000 \left( \frac{dn}{dt} \right)_{\text{volume}} \]

(5)

The formation rates are of the form

\[ \left( \frac{dn}{dt} \right)_{\text{form}} = n_e n_0 <\sigma v> \]

hence for the chosen values of \( n_e \) for the two regions the criterion becomes

\[ <\sigma v>_{\text{gyro}} > 8<\sigma v> \text{ volume} \]

which is easily met by processes requiring energetic electrons.

For each relevant species the formation and loss mechanisms will be considered and the rates estimated to see which is significant in both volume and gyrozone. Using the fastest processes equilibrium densities in the volume alone will be calculated then additional flux from the resonance region calculated and added in if important.
Atomic Hydrogen Formation

\[ \begin{align*}
e^- + H_2 & \rightarrow e^- + H + H \quad (o) \\
e^- + H_2^+ & \rightarrow H^+ + H + e^- \quad (p) \\
e^- + H_3^+ & \rightarrow H_2 + H \quad (r) \\
e^- + H_3^+ & \rightarrow H + H + H \quad (s)
\end{align*} \]

The rate at which the fastest channels of each reaction proceeds will be evaluated numerically for the densities given and for the rate coefficients evaluated for the typical electron temperatures given for both the volume and the gyroresonant zone.

Volume rates:

\[ \begin{align*}
2n_e n_o <\text{Dis}> & = 2 \times 10^{15} \times 10^{15} \times 2.4 \times 10^{-10} = 4.8 \times 10^{15} \\
1n_e n_2^{+2} \rightarrow 1 & = 1 \times 10^{10} \times 10^{8} \times 4 \times 10^{-8} = 4 \times 10^{10} \\
2n_e n_2^{+\text{DR2}} & = 2 \times 10^{10} \times 10^{8} \times 2.4 \times 10^{-8} = 4.8 \times 10^{10} \\
3n_e n_3^{+\text{DR3}} & = 3 \times 10^{10} \times 10^{10} \times 3.4 \times 10^{-8} = 1 \times 10^{13}
\end{align*} \]

Hence only neutral dissociation (o) is important.

Gyroresonant rates:

\[ \begin{align*}
2n_e n_o <\text{Dis}> & = 2 \times 5 \times 10^{12} \times 10^{15} \times 6.7 \times 10^{-9} = 6.7 \times 10^{19} \\
1n_e n_2^{+2} \rightarrow 1 & = 1 \times 5 \times 10^{12} \times 2 \times 10^{12} \times 1 \times 10^{-7} = 1 \times 10^{18} \\
3n_e n_3^{+\text{DR3}} & = 3 \times 5 \times 10^{12} \times 5 \times 10^{12} \times 7 \times 10^{-9} = 5 \times 10^{17}
\end{align*} \]

Again it appears that only neutral dissociation is important.
Atomic Neutral Loss

\[ H + H \rightarrow (\text{wall}) \rightarrow H_2, \]  
\[ H + H \rightarrow H_2 \]  
\[ H + H^- \rightarrow H_2(v) + e^- \]  

The neutrals in filament driven discharge are observed to have approximately .1 eV energy. The wall loss time is thus

\[ \tau_a = \frac{L}{v_{th}} \approx 10/5 \times 10^{-5} = 2 \times 10^{-5} \text{ sec.} \]

The probability that a hydrogen atom striking a copper wall at 300\(^0\) K will not return but will be converted to \(H_2\) is \(P_a = .14\). This large return flux will be considered uniform and included in the model by reducing the loss rate to \(P_a \tau_a\). The associative detachment cross section is about \(10^{-16} \text{ cm}^2\). A value for the association cross section is not at hand but should not exceed that value and may well be less since it involves two neutral particles.

Volume rates:

\[ 1 \times n_a \frac{P_a}{\tau_a} = .14 \times 5 \times 10^4 \times n_a = 7 \times 10^3 n_a \]  
\[ 2 \times n_a \times n_a \sigma_v = 2 \times 10^{-16} \times 2.8 \times 10^5 \times n_a^2 = 5.6 \times 10^{-11} n_a^2 \]  
\[ 1 \times n_a \times n^- \sigma_v < \times n_e \times 10^{-16} \times 5 \times 10^5 \times n_a < .5 n_a \]

Both loss to the wall and volume recombination will be retained, although volume recombination is insignificant here the fact that it depends on \(n_a^2\) shows how saturation is accomplished.

Atomic Neutral Equilibrium

Equating the dissociation (o) and recombination rates (t,u) yields:

\[ 2 \times n_e n_o <\text{Dis}> = n_a P_a \frac{1}{\tau_a} + n_a^2 <\text{Assoc}> \]
\[ n_a = 2^n_e n_a \langle \text{Dis} \rangle \tau_a / P_a - 1/4(2^n_e n_o \langle \text{Dis} \rangle)^2 (\tau_a / P_a)^3 \langle \text{Assoc} \rangle + ... \]

Volume Alone:

\[ n_a \approx n_e n_o \langle \text{Dis} \rangle \tau_a / P_a = 6.8 \times 10^{-4} n_o \quad (6a) \]

For flux from gyrozone to volume, the attenuation constant \( \alpha \) will come from association which gives,

\[ \alpha = n_a \sigma = 4 \times 10^{12} \times 10^{-16} = 4 \times 10^{-4} \]

hence no effective attenuation. The atomic density which it alone would produce at \( h \) is

\[ n_a' = G(o, h) \left( \frac{\text{dn}}{\text{dt}} \right)_{\text{gyro}} \frac{1}{v_m} \]

\[ n_a' \approx \frac{9}{\pi h^2} \frac{1}{5 \times 10^5} 9 \times 10^4 n_o = \frac{0.054}{h^2} n_o \]

Which is seen not to be important except very near the resonance zone.

**H\(_2\)(v) Formation**

The determination of the vibrational population (not even mentioning the rotational structure superimposed on that) of H\(_2\) in a collisional environment is an extremely involved process. Collisions with thermal electrons and other H\(_2\) molecules produce \( \delta v = \pm 1 \) changes in the vibrational state. This type of collision with molecules is the dominant factor in shaping the low level vibrational population. The very high vibrational states are populated by direct excitation from the following processes.

\[ e^- + H_2 \rightarrow e^- + H_2^* \rightarrow e^- + H_2(v) + h\nu \quad (w) \]

\[ e^- + H_2^+(v) \rightarrow H_2(v) + H \quad (x) \]

\[ e^- + H_2(v) + H^+ \quad (y) \]
Reaction (w) is known as E-V singlet excitation (EV); reaction (z) as proton transfer (PT). These high vibrational states are again subject to the $\Delta v=\pm 1$ processes listed above which tend to degrade the high vibrational states. Those vibrational states in between of course are affected by both types of processes.

The role of $H_3^+$ is not well understood. It is created by a two step process:

\[ H_3^+(v) + H_2 \rightarrow H_2(v) + H_3^+(v'<v) \quad (z) \]
\[ H_3^+(v) + \text{Wall} \rightarrow H_2(v) + H \quad (aa) \]

The rate for (c) is so fast that all but a tiny fraction of the $H_2^+$ are converted. The two channels listed above leading from the dissociative recombination of $H_3^+$ (x and y) to $H_2(v)$ are only part of a set of at least five:

\[ e^- + H_2 \rightarrow e^- + H_2^+ + e^- \quad (a) \]
\[ H_2^+ + H_2 \rightarrow H_3^+(v) + H + 1.6 \text{ eV} \quad (c) \]

Mitchell\textsuperscript{39} et al have measured the branching ratios for the first two channels for $E_e < 1$ eV and found about 1/3 result in $H_2 + H$ with $H_2(v)$ unknown. Calculations by Kulander and Guest\textsuperscript{40} indicate that about 1 eV of additional energy, kinetic energy from the electron or vibrational energy already in the $H_3^+$, is needed for formation of $H_2 + H$ so that above 1 eV electron energy this may slowly become
dominant over $H + H + H$. Thus for $Te \sim 1-2$ eV a fraction on the order of $1/2$ will result in $H_2$ in any form, and only a small part of that in really high vibrational states. Smith and Futrell\textsuperscript{41} have measured the vibrational populations of $H_3^+$ coming from a small accelerator ion source operating by processes (a) and (c) and found that about 5% of the $H_3^+$ were in high vibrational states ($E_v > 2.5$ eV), which corresponds to $H_2(v=6)$, ($v=6$ is the beginning of the high vibrational levels needed for DA). However since the fractional yield of $H_2$ ($v\geq 6$) is not known for any of these processes it will simply be accounted for by including a probability $P$ (which will be considered to be $.01$ or less) as an unknown variable in each rate equation.

Hiskes and Karo\textsuperscript{42} have calculated that approximately 20% of $H_2^+$ neutralized on an iron wall will return as $H_2(v\geq 6)$. If this can be any indication for $H_3^+$, wall neutralization may be very important for $H_2(v)$ formation, especially in sources with large surface to volume ratios. This process will not be included in these calculations however, because of lack of information.

Hiskes\textsuperscript{43} has calculated the cross sections for E-V singlet excitation (reaction r). Since to a good degree of approximation the vibrational states are separable from the electronic states the cross sections may be written as

$$\sigma(E_e,v) = g(v) \sigma(E_e)$$  \hspace{1cm} (7)

The electron energy determines the total cross section for excitation to higher electronic states with subsequent radiative decay back to
the ground electronic state. Multiplying the Franck-Condon factors for populating each vibrational level of the excited state by the factor for ending up in level \( v \) of the ground state after radiative decay from that vibrational level then summing over all levels of the excited state will yield the total population expected in state \( v \).

The total cross section is found by summing these over all accessible excited electronic states. The result is still pretty much energy independent as far as relative population and can still be reasonably approximated by (7). \( v=6 \) will be considered as a representative, and for comparison the total population in high vibrational states will be approximated by

\[
\langle \sigma(v) \rangle = s \langle \sigma(v=6) \rangle v
\]

\[
s = 1 + \frac{1}{\sigma_6} \sum_{v>6} \sigma_v = 5.7
\]

Now to compare the rates of all processes which form high vibrational states

Volume Rates:

\[
s n_e n_o < E V_6 > = 5.7 n_e * 10^{15} * 5 \times 10^{-12} = 2.9 \times 10^{13} \quad (w)
\]

\[
n_e n_3^+ < DR > p_{DR} = n_e^2 * 3.4 \times 10^{-8} * p_{DR} = 3.4 \times 10^{12} p_{DR} \quad (y)
\]

\[
n_3^+ n_o < PT > p_{PT} = n_e * 10^{15} * 3 \times 10^{-10} * p_{PT} = 3 \times 10^{15} p_{PT} \quad (z)
\]

Only DR can be safely neglected. PT will be retained, but it does not appear to be the major source.

Gyroresonant Rates:

\[
s n_e n_0 < E V_6 > = 5.7 * 5 \times 10^{12} * 10^{15} * 1.5 \times 10^{-10} = 4.3 \times 10^{18} \quad (w)
\]
Since the probabilities $p^g$ are probably .1 or less both DR and PT can be neglected. Note the rates predicted are over 4 orders of magnitude higher than in the volume.

**$H_2(v)$ Loss**

\[ H_2(v) + \text{wall} \rightarrow H_2 \]  
\[ e^- + H_2(v) \rightarrow e^- + H + H \]  
\[ e^- + H_2(v) \rightarrow H^- + H \]

Reaction (cc) is vibrational excitation from electron collisions which carries the molecule beyond the bound vibrational levels. The cross sections for $\delta v = v^* - v'$ should roughly be independent of $v^*$, so using the measured $<\sigma v>$ ($v = 0 \rightarrow v = 1$) rate coefficients, this rate can be inferred for high vibrational states.

**Volume Rates:**

\[ n_v / \tau_m = 2 \times 10^4 n_v \]  
\[ n_e n_v <\text{Dis}> = n_v * 10^{-6} * 10^{-10} = 6 n_v \]  
\[ n_e n_v <\text{DA}> = n_v * 10^{-10} * 10^{-8} = 100 n_v \]

So only losses to the wall are important.

**$H_2(v)$ Equilibrium**

Equating wall losses (bb) to the volume formation mechanisms (w,z) yields the expected density of $v=6$ for the volume alone.

\[ n_v \tau_m = s n_e n_o <E V_6> + n_3 n_o <\text{PT}> P_{PT} \]
Thus

\[ n_v = n_e n_0 \langle \text{ExV} \rangle_{\text{eff}} \tau_m \quad (8) \]

with \( \langle \text{ExV} \rangle_{\text{eff}} = \langle E V_6 \rangle + \frac{n_3^+}{n_e} \langle PT \rangle P_{PT} \)

\[ n_v = (1.5 \times 10^{10} + 1.5 \times 10^{11} P_{PT}) \quad (8a) \]

For the flux from the gyro resonant zone, none of the destruction processes are large enough to produce a significant attenuation, so again use \( \alpha = 0 \). The \( v=6 \) excited population that the flux from the gyrozone alone would yield is

\[ n_v' = \Gamma \frac{1}{v_m} = G(o,h) \left( \frac{dn}{dt} \right)_{\text{gyro}} \frac{1}{v_m} \quad (9) \]

\[ n_v' \approx \frac{g}{\pi h^2} s n^g_e n_0 \langle E V_6 \rangle g \frac{1}{v_m} \]

This is shown in figure 3-3(a). Evaluated at the representative values this becomes

\[ n_v' = \frac{6 \times 10^{12}}{h^2} \quad (9a) \]

which at \( h=20 \) cm will still be slightly larger than the locally produced population. The total density is

\[ n_v = n_e n_0 \langle \text{ExV} \rangle_{\text{eff}} \tau_m + \frac{G(o,h)}{v_m} n^g_e n_0 \langle E V \rangle \quad (10) \]
Fig. 3-3  Estimated Contribution to the Excited State Density by the Resonance Zone for $H_2(V)$ (*) and $H_2(c^3u_0)$ (+).
**H₂\(^{(c^3\pi_u)}\) Formation**

This is a metastable electronic state, so upon it is superimposed a whole vibrational rotational structure. As was mentioned earlier, the DA cross sections for each of these level are different, hence as was for \(H₂(v)\), a proper solution should include a series of coupled equations to yield the final vibrational population, however there is not really enough knowledge of the electronic state "alone" for it to be understood, let alone adding details. Here the state will be discussed with complete ignorance towards vibrational levels.

The formation mechanisms are

\[
\begin{align*}
\text{(ee)} & : e^- + H₂ \rightarrow e^- + H₂\left(c^3\pi_u\right) \\
\text{(ff)} & : e^- + H₂\left(c^3\pi_u\right) + H \rightarrow e^- + H^+ + H₂\left(c^3\pi_u\right) \\
\text{(gg)} & : e^- + H₂\left(c^3\pi_u\right) + H \rightarrow e^- + H^+ + H₂\left(c^3\pi_u\right)
\end{align*}
\]

There is no experimental data on producing this state by electron collisions with energy less than kilvolts. Miles et al have produced semiempirical fits to a large number of cross sections for many of the results of electron collisions with \(H₂\) molecules, included is excitation of the \(H₂\left(c^3\pi_u\right) v=0\) level. This cross section exceeds the EV singlet excitation cross sections at energies below about 30 volts and has a maximum about 20 times higher than that for excitation of the \(v=6\) level. Their semiempirical curves, fit well experimental data on other processes and agree with the high energy experiments but there are no low energy experiments to compare with for this state. A more recent calculation by Lee et al, who numerically solved a distorted wave approximation agree with Miles fit at high energy but is much smaller at low energy. The discrepancies
have not been resolved and point to the need for experiments of this cross section. Since Miles technique fit well many other $e^- + H_2$ cross sections and since other calculations of this cross section seem to agree that it should be higher at low energies than Lee et al indicate Miles fit will be used.

Kulander and Guest\textsuperscript{40} have calculated that an excess energy of 2.4 eV above the $H_3^+$ ground state is needed to form $H_2(c^3\pi_u)$ + H in the dissociative recombination of that ion. Since this is the same size as the energy needed to produce high vibrational states the rates should be comparable however nothing is known beyond this conjecture, so again an unknown probability $P_n$ will be included

Volume rate:

\[
\begin{align*}
\text{Volume rate:} \\
\text{Gyroresonant rate:}
\end{align*}
\]

Hence only the collisional excitation is important

Gyroresonant rate:

\[
\begin{align*}
\text{Gyroresonant rate:} \\
\text{Volume rate:}
\end{align*}
\]

Again only direct excitation will be retained

\[
\begin{align*}
\text{Volume rate:} \\
\text{Gyroresonant rate:}
\end{align*}
\]
\[ H_2(c^3\pi_u) + e^- \rightarrow H + H^- \quad (kk) \]
\[ H_2(c^3\pi_u) + H_2 \rightarrow H_2 + H_2 \quad (11) \]

For low energy electrons the collisional de-excitation rate should be dominated by collisional transfer to the close lying \( H_2(a^3\Sigma_g) \) state which radiatively decays with a life time of about \( 10^{-8} \) sec

\[ H_2(c^3\pi_u) + e^- \rightarrow H_2(a^3\Sigma_g) + e^- \rightarrow H_2 + h\nu + e^- \quad (jj') \]

Hiskes \(^{10}\) has estimated the rate coefficient for this process to be between \( 1.5 \times 10^{-6} \) and \( 7 \times 10^{-6} \) for \( T_e = 1 \) eV. For this discussion the average of those values, \( 4 \times 10^{-6} \), will be used. The de-excitation rate for collisions with other hydrogen molecules could not be found, but since the interaction energies are a small fraction of an eV, and this collision involves two neutral molecules, the cross section should be \( 10^{-18} \) cm\(^2\) or less, hence \( \langle C_{De} \rangle \approx \sigma_{uv} \approx 2 \times 10^{-11} \)

Volume rates

\[ n_\pi / \tau_m = 2 \times 10^4 n_\pi \quad (ii) \]
\[ n_e n_\pi \langle DeEx \rangle = 10^{10} \times 4 \times 10^{-6} n_\pi = 4 \times 10^4 n_\pi \quad (jj') \]
\[ n_e n_\pi \langle DA \rangle = 10^{10} n_\pi 10^{-10} = n_\pi \quad (kk) \]
\[ n_o n_\pi \langle CD_{De} \rangle < 5 \times 10^{14} n_\pi 2 \times 10^{-11} < 1 \times 10^3 n_\pi \quad (11) \]

Collisional de-excitation via the close lying \( H_2(a^3\Sigma_g) \) state and loss to the walls dominate. The loss to the walls may be somewhat lessened since some molecules may survive in the excited state after hitting the wall and will return to the discharge but the probability far this is unknown.

\( H_2(c^3\pi_u) \) Equilibrium

The expected \( H_2(c^3\pi_u) \) density from volume processes alone (ee,
The attenuation constant for flux from the gyrozone is determined by collisional de-excitation both with electrons and with molecules,

\[ \alpha = n_e \sigma_{\text{De-Ex}} + n_o \sigma_{\text{De-De}} = 1.2 \times 10^{-3} \]

which is negligible. The excited density that this flux alone would produce at \( h \) is

\[ n'_e = \frac{1}{v} \Gamma = G(o,h) \left( \frac{dn}{dt} \right) \frac{1}{v_m} \]

\[ n'_e = \frac{3}{wh^2} n_e n_o <\text{Ex}> g \left( \frac{1}{v_m} \right) \]

This is shown in figure 3-3(b). Evaluated at the representative values this becomes

\[ n'_e \approx \frac{9}{wh^2} \frac{1}{2 \times 10^5} \times 3.8 \times 10^{18} = 5.7 \times 10^{12} \frac{1}{h^2} \]

This flux will be significant throughout the volume and at the end of the chamber, \( h = 20 \text{ cm} \), will still exceed what would have been there otherwise. The total population is

\[ n_e = \frac{n_e n_o <\text{Ex}>}{n_e <\text{De-Ex}> + 1/\tau_m} + \frac{G(o,h)}{v_m} g \left( n_e n_o <\text{Ex}> \right) \]

Return to Equilibrium Equation

The densities needed for utilization of (2) are now all at hand.

First consider the three loss terms:
Volume Rates:
\[ n_e \langle ED \rangle = 8.2 \times 10^{-8} n_e \]  
\[ n_e \langle MN \rangle = 9.8 \times 10^{-7} n_e \]  
\[ n_a \langle AD \rangle = n_e n_o \langle Dis \rangle \langle AD \rangle \frac{\tau_a}{P_a} = 1.7 \times 10^{-7} n_e \]

Here \( \langle MN \rangle \) obviously dominates, however these three have different dependences on the discharge parameters. \( \langle ED \rangle \) is a rapidly increasing function of \( T_e \), \( \langle \text{Dis} \rangle \) depends on the atomic neutral density and energy which also increase with \( T_e \); in contrast \( \langle MN \rangle \) is proportional to \( T_i^{-1/2} \) and since \( T_i \) should be a monotonically increasing function of \( T_e \) it should decrease if \( T_e \) is raised. Since all three are proportional to \( n_e \) they can be combined into an effective detachment rate which will be only weakly dependent on \( n_o \) since AD should not exceed the other two anywhere in the operation range

\[ \langle \text{Det} \rangle_{\text{eff}} = \langle ED \rangle + \langle MN \rangle + n_o \frac{\tau_a}{P_a} \langle \text{Dis} \rangle \langle AD \rangle \]
\[ n_e \langle \text{Det} \rangle_{\text{eff}} \approx 1.1 \times 10^{-6} n_e \]

\( H^- \) ions from dissociative attachment are fairly energetic. Wadehra has calculated the \( H^- \) energies from dissociative attachment to vibrationally excited \( H_2 \) getting an average \( H^- \) energy of from .2 to .4 eV depending on the rotational and vibrational level of the parent. At .3 eV an \( H^- \) would travel

\[ L_d = \frac{V(\epsilon=.3)}{n_e \langle \text{Det} \rangle_{\text{eff}}} = \frac{7.6 \times 10^5}{7.9 \times 10^3} = 96 \text{ cm} \]

In spite of this losses to the wall should not be important since in the operating range the plasma potential exceeds 5 volts effectively trapping the \( H^- \). This does mean the \( 1/h^2 \) dependence in excited
states from the gyrozone will not have as noticeable an effect, since
the volume will be somewhat averaged over.

Now consider the relative size of the source terms (in equation 3)
for h=20. \( \text{H}^- \) from vibrationally excited molecules:

\[
\left( \frac{dn}{dt} \right)_{\text{DA, vib}}^{\text{form}} = n_e \sum_v n_v^s <DA_v>^s = n_e \sum <DA_v>^s \left( n_e n_o <Ex_v>^s \right)_{\text{eff}} \tau_m
\]

Again the summation will be only approximated. As was mentioned
previously, the formation cross sections have nearly identical energy
dependence, differing only in magnitude (hence the same ratio for both
volume and gyroresonance zone), and since the \( \text{H}^- \) formation takes
place only in the volume, and hence at a single temperature, the above
summation can be reasonably approximated by a single vibrational state
and a multiplier:

\[
n_e \sum_v n_v^s <DA_v>^s = S n_e \frac{n_v^{6,v=6} <DA_v>^{v=6}}{<Ex_v>^{v=6} <DA_v>^{v=6}} = 4.2
\]

The population at \( v=6 \) can be obtained by adding equations (8a) and
(9a) and then dividing out the comparison factor \( s \) used there.

At \( h = 20 \),

\[
n_v^{v=6} = \frac{(1.5 \times 10^{10} + 1.5 \times 10^{10})}{5.7} = 5.3 \times 10^9
\]

\[
\left( \frac{dn}{dt} \right)_{\text{DA, vib}}^{\text{form}} = 4.2 \times 10^{10} \times 5.3 \times 10^{10} \times 2.5 \times 10^{-9} = 5.6 \times 10^{11}
\]

\( \text{H}^- \) from metastable electronic states:
Here the summation cannot be done since no information could be found on excitation to the various vibrational levels. It will be assumed that the excitation rate used was to all the vibrational states, and since the DA rates for all the vibrational states are comparable, they will be approximated by the $v=0$ term only; hence the summation will be neglected altogether. From equations (11) and (12a) this becomes at $h=20$

\[
 n_v = 1.4 \times 10^{10} + 2.5 \times 10^{9} = 1.7 \times 10^{10}
\]

\[
 \left( \frac{dn}{dt} \right)_{DA_{3H}}^{u,v} = 10^{10} \times 1.7 \times 10^{10} \times 8 \times 10^{-11} = 1.4 \times 10^{10}
\]

$H^-$ from molecular ions:

\[
 \left( \frac{dn}{dt} \right)_{DA_{H^+}} = n_e n_3^+ \langle DR3 \rangle = 10^{10} \times 10^{10} \times 8 \times 10^{-11} = 8 \times 10^9
\]

\[
 \left( \frac{dn}{dt} \right)_{DA_{H_2}} = n_e n_2^+ \langle DR2 \rangle = 10^{10} \times 10^8 \times 7 \times 10^{-11} = 7 \times 10^7
\]

Due to the very high DA rate only vibrational excitation appears to be important hence,

\[
 n_{-} = \frac{\sum n_v^{<DA_vv>} \left\langle n_e n_0 <Ex_vv>^s \tau_m + G(o,h) \frac{n_e^n g n_0 <Ev^{gs}>^g_s}{v_m} \right\rangle}{\langle Det \rangle_{eff}^{<MN> + <ED> + n_a/n_e <AD>}}
\]

(14a)
This can be rewritten as

\[ n^- = \frac{n_0 <\text{Attach}>_{\text{eff}}}{<\text{Det}>_{\text{eff}}} \]  

(14b)

where the effective attachment rate coefficient is

\[ <\text{Attach}>_{\text{eff}} = \sum <\text{DA } v>^S \left( <\text{Ex } v>^S + \frac{G(o,h)}{\tau_m} \frac{n_e^g}{n_e} <EV>_g^S \right) n_e \tau_m \]

Numerically this gives

\[ \frac{n^-}{n_e} = \frac{1}{n_e^2 <\text{Det}>_{\text{eff}}} \left( \frac{dn^-}{dt} \right)_{\text{form}} \]

\[ = \frac{1}{10^{20} \times 1.1 \times 10^{-6}} \times 5.6 \times 10^{11} = .005 \]

Which is in good agreement with the measurements of the ratio of extracted negative ion and electron currents.
Multicusp magnetic fields (also called multipoles or magnetic buckets) are widely used to increase the efficiency of the filament driven plasma ion sources now in use. This arrangement greatly reduces the loss rate to the walls while still maintaining a large field free plasma in the center, resulting in greater arc efficiency and a flatter density profile in the center. Magnetic fields can also greatly enhance electromagnetic wave coupling to plasmas by the many resonances produced (i.e., electron cyclotron, ion cyclotron, upper hybrid and lower hybrid).

A multicusp field from permanent magnets that would produce a very high field region where electron cyclotron resonance would occur was desired, thus a theoretical study of some of the cusp shaped magnetic fields obtainable from permanent magnets was undertaken. Rare earth cobalt (REC) permanent magnets are ideal for such uses. They possess a large remnant field which is very nearly independent of applied fields, even reverse fields of comparable magnitude\textsuperscript{45} (Fig. 4-1). The remnant field is a property which is characteristic of the material and the manufacturing process; it is not the surface field, which depends both on the remnant field and the magnet's shape. Although the permeability is slightly anisotropic it is very nearly unity in all directions, thus REC magnets can be treated as regions of "free space" with frozen in magnetization.
Fig. 4-1 Comparison of two REC Alloys with Alnico.
The calculations undertaken fall into 3 categories: (1) infinitely long cylindrical arrays of rectangular bar magnets with no steel present, (2) any collection of finite length, rectangular bar magnets with no steel present and (3) steel pole pieces energized by REC magnets. A 2-dimensional theory for REC magnets with no steel will be developed which will cover cases (1) and (2), then a separate theory will be considered for case (3).

**REC with no Ferromagnetic Materials Present**

The total magnetic field\(^4\) is

\[
\vec{B} = \mu_0 (\vec{H} + \vec{M} (\vec{H}))
\]

where \(\vec{H}\) is field component due to external sources and \(\vec{M}\) is the magnetization, with \(\vec{B} = \mu_0 \vec{H}\) for free space. For REC \(\vec{M}\) is very nearly constant for \(\mu_0 \vec{H} < 6-10\) kgauss. For no time variation or external currents, Maxwell's equations become:

\[
\nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{H} = 0.
\]

Since \(\nabla \times \nabla \) (any scalar) = 0 we may use

\[
\vec{H} = -\frac{1}{\mu_0} \nabla \varphi
\]

which yields

\[
\nabla^2 \varphi = \mu_0 \nabla \cdot \vec{H} = -\mu_0 \nabla \cdot \vec{M} = \mu_0 \rho
\]

(hence defining an effective magnetic charge density). This may be solved by utilizing the fact that

\[
\nabla^2 \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) = -4\pi \delta(\vec{r} - \vec{r}'),
\]

whereupon integration over all space leads to\(^4\)
The first integral is over the volume of magnetized material; the second takes into account the discontinuity at the surface. This can be simplified by using the divergence theorem to obtain a substitution for the surface integral:

\[
\int \int \frac{\mathbf{M} \cdot \mathbf{n}}{| \mathbf{r} - \mathbf{r}' |} \, d^2 r' = \int \int \nabla' \left( \frac{\mathbf{M}}{| \mathbf{r} - \mathbf{r}' |} \right) \, d^3 r'
\]

The right hand term can be expanded into two terms one of which cancels the first term of (3), yielding

\[
\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \int \frac{\mathbf{M}(\mathbf{r}') \cdot \mathbf{n}(\mathbf{r}')}{| \mathbf{r} - \mathbf{r}' |^3} \, d^3 r'
\]

The magnetic field is gotten from equations (1) and (2). In this work only the field outside the magnetic materials is important where \( \mathbf{M} = 0 \) hence \( \mathbf{B} = -\nabla \psi \).

**Complex Representation**

Construct an analytic function:

\[ \psi = s + i\varphi \]

where \( \varphi \) is the scalar magnetic potential just discussed and \( s \) is the complementary real function called the stream function. We will work in the complex plane \( w = x + iy \) (chosen instead of the customary \( z = x + iy \) to avoid confusion with the 3rd rectangular coordinate \( x, y, z \)).

For \( \psi \) to be analytic the Cauchy-Riemann condition must be satisfied:

\[
\frac{\partial s}{\partial x} = \frac{\partial \varphi}{\partial y} \quad \frac{\partial s}{\partial y} = -\frac{\partial \varphi}{\partial x}
\]
\[ B = -\nabla \phi \text{ (in free space) leads to } B = \nabla x(s^2), \text{ hence } s \text{ and the vector magnetic potential are related.} \]

Consider \[ d\phi = \left( \frac{\partial s}{\partial x} i + \frac{\partial \phi}{\partial x} \right) \, dx + \left( \frac{\partial s}{\partial y} i + \frac{\partial \phi}{\partial y} \right) \, dy \]

This can be reorganized using the Cauchy-Riemann equations into

\[ d\phi = i(\frac{\partial \phi}{\partial x} - \frac{\partial s}{\partial y})(dx + i \, dy) = -i (B_x - iB_y) \, dw \]

hence

\[ B^* = B_x - iB_y = \frac{i \, d\phi}{dw} \quad (6) \]

**Field Lines**

The field lines are defined to be a set of curves to which the vector \( B \) is everywhere tangent

\[ \frac{dy}{dx} = B_y / B_x = -\frac{\partial s}{\partial x} / \frac{\partial s}{\partial y} \]

This leads to

\[ \frac{\partial s}{\partial y} \, dy = -\frac{\partial s}{\partial x} \, dx \]

and hence

\[ ds = \frac{\partial s}{\partial x} \, dx + \frac{\partial s}{\partial y} \, dy = 0 \]

so curves of constant \( s \) are the field lines. The same is not true of the \( z \) component of the vector potential unless it is independent of \( z \); hence though similar, \( s \) and \( A_z \) are not always equal, and lines of constant \( A_z \) are not always the field lines.
Infinitely Long Magnets (i.e. true 2-d)

We can simply (4) by integrating out z.

\[ \phi(x,y) = \frac{\mu_0}{2\pi} \int \int \frac{M_x(x-x') + M_y(y-y')}{(x-x')^2 + (y-y')^2} \, dx' \, dy'. \] (7)

and rewrite it as

\[ \phi(x,y) = \text{Re} \left( \frac{\mu_0}{2\pi} \int \int \frac{M}{(w-w')} \, dx' \, dy' \right) \] (8)

where \( M = M_x + i M_y \) and \( \text{Re} \) denotes the real part. This results in an acceptable complex representation:

\[ \phi(w) = -\frac{\mu_0}{2\pi i} \int \int \frac{M(x',y')}{(w-w')} \, dx' \, dy' \] (9a)

\[ B^*(w) = \frac{\mu_0}{2\pi} \int \int \frac{M(x',y')}{(w-w')^2} \, dx' \, dy' \] (9b)

Consider rotating the magnetization by a fixed angle \( \alpha \)

\[ M_2(x',y') = e^{i\alpha} M_1(x',y') \]

This yields

\[ B^*_2(x,y) = e^{i\alpha} B^*_1(x,y) = (e^{-i\alpha} B_1)^* \] (10)

This is the easy axis rotation theorem, which shows that the field produced by magnet assemblies of the same geometry, differing only in having the direction of magnetization rotated, will produce fields which have the same intensity as a function of space \(|B|\), but the field direction will be rotated, by the same angle, but in the opposite sense. (Compare Figs. 4-7, 4-8 and 4-9).

The field variable \( w \) can be separated from the source variable \( w' \) by expanding \( 1/(w-w') \) into a power series, this results in
\[ \phi = \frac{\mu_0}{2\pi} \sum w^n \iint \frac{M(x',y')}{w^n+1} \, dx'dy' = \sum \phi_n w^n, \text{ for } w < w' \quad (11a) \]

\[ B^* = i \frac{d\phi}{dw} = \sum b_n w^{-1}, \text{ where } b_n = i\phi_n \quad (11b) \]

For constant magnetization \( M = B_r/\mu_0 \) (\( B_r \) is the remnant magnetic field)

\[ \phi_n = \frac{B_r}{2\pi} \sum \iint \frac{dx'dy'}{(x'+iy')^n+1} \quad (12) \]

Hence the solution of the field has been reduced to a set of related integrations over the cross sectional area of each magnet, from these both the field intensity \( |B| = |B^*| \) and the field lines \( s = \text{Re}(\phi) \) can be obtained.

**Case 1 Cylindrically Symmetric Array of Magnets**

**Multipole**

Consider the field inside an infinitely long, circular cylinder bounded by \( T \) rows of permanent magnets of identical shape and of equal magnetization, but with the direction of magnetization successively rotated by an equal angle between rows. There will be \( N \) magnet periods; that is in relative coordinates (radial and tangential direction at each magnet location) the magnetization undergoes \( N \) complete rotations in passing around the circumference (whereas in fixed coordinates there will be \( N+1 \) complete rotations see Fig. 4-2).

Let the field due to all the rows be represented as

\[ \phi = \sum \frac{-1}{n} b_{mn} w^n \quad (13a) \]
Fig. 4-2 Illustration of the Terms Used to Describe Magnet Arrangements.
where the subscript m denotes the individual magnet rows and

\[ B_{mn} = \frac{B}{2\pi} \int \int \frac{n \, dx \, dy'}{m \, (x'^{1/2} + iy')^{n+1}} \]  

Since by definition all the magnet rows have an identical cross section, if each set of B's were evaluated in its own relative coordinates (see Fig. 4-3), the sets of integrals would be equal for all the rows. Transformation back to a set of reference coordinates, (the relative coordinates for row 0) consists of a complex rotation for the field variable \( w_m \), which was not integrated over,

\[ w_m = e^{-i\alpha} w_0 \]

where \( \alpha = \frac{2\pi}{T} \), and an additional "rotation" of the B's themselves, since each has been expressed in a peculiar complex representation based on relative coordinates

\[ B_0^* = B \cdot (x_0 - i y_0) = B \cdot [e^{i\alpha}(x_m - iy_m)] = e^{i\alpha} B_m^* \]

In its own relative coordinate representation

\[ B_{rm} = e^{iN\alpha} B_r \]

All these combine to yield

\[ B_{mn} = B_{on} e^{i(N-n)\alpha} \]

(14)

Summing over all the magnets, and all orders of the expansion:

\[ B^* = \sum (B_{on} w^{n-1} e^{i(N-n)2\pi m/T}) \]
Fig. 4-3 Illustration of the Relative Coordinates of two Rows of Magnets. The Actual Origin Used was the Axis of the Cylinder but for Clarity the Unit Vectors have been shown Centered on the Magnets.
but \[ \sum_{m=0}^{T-1} e^{i2\pi m(N-n)/T} = \begin{cases} T & \text{if } N-n = \nu T \text{ for } \nu \text{ an integer} \\ 0 & \text{if not} \end{cases} \]

which is easily proven since it is a simple geometric series. The total field is therefore

\[ B* = \sum_{\nu} T B_{\nu \nu} w^{n-1}, \quad \text{where } n = N + \nu T \quad (15) \]

Evaluation of (13c) for the reference row \((m=0)\) and for rectangular magnets (see appendix D), and substitution into (15) yields

\[ B*(w) = -\frac{B_{\nu \nu}}{w} \sum_{\nu} \frac{T}{n-1} w^{n-1} \left\{ \frac{\sin \left[ (n-1) \tan^{-1} \left( \frac{a}{2r_0} \right) \right]}{\left( r_0^2 + \left( \frac{a}{2} \right)^2 \right) (n-1)/2} \right\} \]

\[ -\frac{\sin \left[ (n-1) \tan^{-1} \left( \frac{a}{2r_1} \right) \right]}{\left( r_1^2 + \left( \frac{a}{2} \right)^2 \right) (n-1)/2} \quad (16) \]

\( \phi \) is nearly the same, but with \(-1 \frac{w^n}{n}\) replacing \(w^{n-1}\).

The following computer generated plots show \(|B*| = \text{const. and} \ Re \phi = s = \text{const. curves.} \) The computer program was written in BASIC for the HP 9845B small computer which was available. Since there are always \(2N\) identical cusps only a sector of \(\pi/N\) radians need be solved, and for many cases these cusps are symmetric about their center line hence only a \(\pi/2N\) radian sector need be solved. The program summed the series on a lattice of points (Fig. 4-4) in this wedge of symmetry. Many more terms were included for those points near the periphery than at the center in order to keep truncation errors low, but to conserve computing time. The program then plotted the curves
Fig. 4-4 An Example of the Type of Grid Used for Cylindrically Symmetric Problems.
by taking each mesh "square" of 4 points and determining if the curve of interest passed through any of the sides, if so it estimated the crossing point by linear interpolation from the corners, and then connected the points on the 2 sides by a straight line.

The following series of seven figures present a good pictorial tutorial on the different fields produced by varying magnet size and orientations. All have 5 magnet periods (10 cusps) on a 21 cm diameter cylinder, and all the magnets have $B_{\text{rem}} = 9.4$ kgauss (strength of Sm Co$_5$ magnets now available). The direction of magnetization is indicated by the "V" inside each magnet block. The field lines drawn (top half of each figure) are uniformity spaced at .2 kgauss-cm (the 10 lines are 0.2-2.0) the field intensity lines (bottom half of each figure) are in 1 kgauss steps starting at 1 kgauss. Figures (4-5) and (4-6) have two 1.27 x 2.54 cm magnets per period. The other five examples have 2.54 cm square magnets. The first three figures represent the "standard" bucket geometry, namely all magnets are magnetized either radially inward or radially outward (two magnets per period). Fig. (4-7) through Fig. (4-9) demonstrate the easy axis rotation theorem; note the identical intensity plots. Rotation allows the cusp to be shifted away from the magnet face which may be desirable for cooling or access to the cusp region, but note the most intense field is in front of the magnet so this shift is to a weaker field region. Note the skewed cusp that results from the $\pi/4$ rotation. Figs. (4-10) and (4-11) show 4 magnets per period, note the great increase in field uniformity. Here a rotation by $\pi/2$ reproduces Fig. (4-10) so only the $\pi/4$ rotation is shown. Due to
Fig. 4-5  Standard Bucket Arrangement with Tall, Narrow Magnets.
Fig. 4-6 Standard Bucket with Short, Wide Magnets.
Fig. 4-7 Standard Bucket with Square Magnets. Compare this with the next two Figures.
Fig. 4-8  Bucket with two Square Magnets per Period with the Magnetization Rotated by 45°.
Fig. 4-9  Bucket with two Square Magnets per Period with the Magnetization Rotated by 90°.
Fig. 4-10  Bucket with four Square Magnets per Period  Compare with the Next Figure.
Fig. 4-11 Bucket with four Square Magnets per Period with the Magnetization Rotated by 45°.
increased uniformity of field strength the loss in shifting the cusp is not nearly so severe. This arrangement (Fig. 4-11) is the most satisfactory since in addition access can be obtained between the magnets to both the cusp which is open to the plasma and the center of the magnetic field which is shielded from the plasma.

Case 2 Finite Length Magnets in Free Space

We will only consider uniform magnetization perpendicular to z. Going back to (3) note that now the volume integral will vanish so

\[ \phi = \frac{B_r}{4\pi} \int \frac{b \cdot \hat{n}}{|r - r'|} \, d^2r' \]

(17)

where \( \hat{b} \) is the unit vector in the direction of magnetization and \( \hat{n} \) is the outwardly pointed unit vector normal to the surface. In complex notation this becomes

\[ \hat{b} \cdot \hat{n} = Re (b n^*) = Re (b^* n) \quad \text{for} \quad b = b_x + i b_y, \quad n = n_x + i n_y. \]

We can still develop a complex representation since we will only be interested in projections onto some plane perpendicular to \( \hat{z} \) (hence 2-d). A suitable one is

\[ \phi = -\frac{B_r}{4\pi} \iint \frac{b n^*}{|r - r'|} \, d^2r' \]

(18)

with an implied summation over all surfaces where \( \hat{b} \cdot \hat{n} \) does not vanish. For our case \( d^2r' \) will be \((\cos(\alpha)dx' + \sin(\alpha)dy')dz'\).

Consider the field lines (really their projection onto our chosen plane)
Comparison of (19) with (17) shows that \( s \) would be the scalar potential for the complementary problem (magnets with the same geometry but magnetization rotated by \( \pi/2 \) about \( z \)).

For this case the complex notation was not used beyond yielding the last result (19), \( \hat{B} \) and \( s \) were calculated directly from the integrals.

\[
\hat{B} = -\nabla \phi(x,y,z) = -\frac{B}{4\pi} \sum \iint_{\text{surfaces}} \frac{\hat{B} \cdot \hat{n}}{s} \frac{r' - \hat{r}}{|r' - \hat{r}|^3} d^2r'
\]

This was evaluated for rectangular magnets (see Appendix D) with \( x-x' \) replaced by \( x \) alone for convenience (likewise \( y \) and \( z \)).

\[
B_x = \frac{B}{4\pi} \left[ \frac{b x}{2} \text{Sgn}(xyz) \cos^{-1} \left( \frac{x^2 r^2 - y^2 z^2}{(x^2 + z^2)(x^2 + y^2)} \right) - b_y \log(z + r) \right]
\]

\( B_y \) is the same as \( B_x \) but with \( x \) and \( y \) interchanged and

\[
B_z = -\frac{B}{4\pi} \left[ \frac{b x}{2} \log \left( \frac{r - x}{r + y} \right) + \frac{b y}{2} \log \left( \frac{r - y}{r + x} \right) \right]
\]

All are evaluated between the same limits.

\[
x = x \pm \frac{1}{2} \quad y = y \pm \frac{1}{2} \quad z = z \pm \frac{1}{2}
\]

The stream function is

\[
s = -\frac{B}{4\pi} \sum \iint (b \times z) \cdot \hat{n} \frac{d^2r'}{|r' - \hat{r}|}
\]

this yields
\[ s = -\frac{B}{4\pi} \left\{ \begin{array}{l} b_y \left[ z \log(y+r) + y \log(z+r) \\ + |x| \text{Sgn}(z) \cos^{-1} \left( \frac{x^2 + y^2 + yr}{\sqrt{x^2 + y^2 (y+r)}} \right) - z \right] \\ - b_x \left[ z \log(x+r) + x \log(z+r) \\ + |y| \text{Sgn}(z) \cos^{-1} \left( \frac{x^2 + y^2 + xr}{\sqrt{x^2 + y^2 (x+r)}} \right) - z \right] \end{array} \right\} \] (21)

evaluated between the same limits as \( B \). Since both \( B \) and \( s \) are linear functions the result for several magnets is just the linear superposition. Each magnet's field can be solved in a suitable relative coordinate system then rotated into absolute coordinates.

These were incorporated into a program which shows the contours

\[ B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \text{const} \quad \text{and} \quad s = \text{const}. \]

in any chosen plane \( z = \text{const} \), for an array of finite length magnets with centers in the plane \( z=0 \) but not necessarily all of the same length.

The two examples chosen are both planar arrays of very small magnets arranged as a magnetic filter. These filters are used in filament discharge to divide the chamber into a driver plasma and a cool electron plasma; only the cold electrons diffusing out along with the ions pass through, hence they filter out the primary electrons. The upper figure in each of the following two pairs shows field lines in 50 gauss-cm steps the lower shows field intensity from 100 to 500
gauss in 100 gauss steps. Fig. 4-12 shows the conventional arrangement of magnetization, (ie alternating directions but lying in the same plane as the array). This arrangement produces weak cusps between the magnet rows (which are desirable here in order to increase ion throughput). Fig. 4-13 shows how an array can be constructed to have the field more intense on one side than the other by pairing diagonally magnetized blocks.

Case 3  Ferromagnetic Pole Pieces "Energized" by REC Magnets

Satisfactory regions of high magnetic field were not obtained in the studies of REC magnets alone so steel pole pieces were added to channel the flux of many magnets into a small gap. The word steel will be used to mean any ferromagnetic material whether or not it contains iron. For unsaturated steel, infinite permeability is a very good approximation (even beyond saturation \( \mu \) is still very large, \( \mu > 100 \)). In this limit the magnetic field lines must be perpendicular to the pole boundary hence it becomes a magnetic equipotential (the problem is an exact analog of a perfectly conducting electrode). REC magnets can be treated as regions of free space (since \( \mu_{rel} \approx 1 \)) that deposit flux into the pole pieces. The problem can be solved by first solving the electrostatic problem with the pole pieces at \( \phi = \pm 1 \) then multiplying the potential field by an amount which makes the flux leaving each pole piece equal to that which was inserted by the magnets.

The problem of interest was the strong cusp field produced by two "L" shaped pole pieces. (see Fig. 4-14) This arrangement is symmetric
Fig. 4-12 Conventional Filter Arrangement.
Fig. 4-13 Asymmetric Filter Arrangement.
Fig. 4-14 Illustration of the Geometry of the Pole Pieces and Magnets.
about its midplane so the electrostatic problem consists of a single pole piece at $\phi=1$ and the axis of symmetry at $\phi=0$. In order that both the field lines and the field intensity could be readily obtained a complex potential is again to be used.

Consider the simple, complex potential

$$\phi = s + i\phi = \frac{v_0}{v} \log(t), \text{ in the plane } t = u + iv$$

This can be inverted to give

$$t = u + iv = e^{\frac{vs}{v_0}} (\cos(\frac{v\phi}{v_0}) + i \sin(\frac{v\phi}{v_0})) \quad (22)$$

Note that contours of constant $s$ are circles and that equipotentials are radial "spokes" (see Fig. 4-15). The real axis is split at 0 with the positive half being $\phi=0$ and the negative half $\phi=1$.

This contains all the desired physics; namely, a pair of equipotentials ($\phi=1$ and $\phi=0$) which can represent the pole piece and the axis of symmetry, and the correct complex potential field solution between them. Since the method of conformal mapping retains the orthogonality of the curves of constant $s$ and $\phi$, it can be used to transform this solution into the desired one. The desired map will bend the negative semiaxis into the shape of the desired pole piece while the positive semi axis is left straight. The point $t=0$ will be mapped to $-\infty$ so the positive semiaxis will be mapped to the entire real axis in the new complex plane. A Schwartz-Christofel transformation can be constructed which can duplicate either end of the pole piece, but with parallel sides extending from there to infinity. Since the field between the two pole pieces in the parallel
Fig. 4-15 The Complex Field $\Phi = 1/w \log(t)$ where $\Phi = s + i\phi$
region away from the ends is uniform, two separate maps can be made and then cut and spliced together. The small error due to the improper field produced above the pole pieces will be ignored.

The map that produces the square ends of the pole pieces can be solved exactly. (see Fig. 4-16)

\[ \frac{dw}{dt} = k \frac{1}{t} \sqrt{(1 + t)(a + t)} \]

\((w=x+iy \text{ is the final complex plane, and } t = u+iv \text{ is gotten from equation (22)})\).

When integrated this yields

\[ w = k \left[ \frac{a+1}{2} \log (2X + 2t + a+1) - a \log ( aX + (a+1) t + 2a) \right] \]

(23)

where \( X = \sqrt{(t +1)(t + a)} \).

By evaluating \( w \) for \( t = -1 \) and \( t = -a \) the integration constants \( k \) and \( a \) can be evaluated in terms of the slab thickness \( T \) and the spacing \( G \).

\[ k = \frac{G}{\pi^2 a} \]

\[ a = 2(T_G + 1)^2 -1 + \sqrt{(2(T_G + 1)^2 -1)^2 -1} \]

The net flux can easily be evaluated by determining the \( s \) values at the top and bottom where this is to be cut and spliced (flux = \( s_2 - s_1 \)).

The other end cannot be solved analytically but the map was generated numerically from (See Fig. 4-17)

\[ \frac{dw}{dt} = k \frac{1}{t} (1 + t)^{c-1} (1 + \frac{t}{a})^{a-1} (1 + \frac{t}{b})^{b-1} (1 + \frac{t}{c})^{c-1} (1 + \frac{t}{d})^{d-1} \]

(24)

The constant \( k \) can easily be evaluated by considering \( t \to 0 \).
Fig. 4-16 Desired Shape for the Real Axis of the $t$ plane after Transformation into the $w$ plane to Produce the Pole Piece End.
Fig. 4-17 Desired Shape for the Real Axis of the \( t \) plane after Transformation into the \( w \) plane to Produce the Strong Cusp Producing Pole Pieces.
\[ w = k \int \frac{dt}{t} \]

\[ k = \frac{G}{\pi} \]

All other constants (i.e., a through d) were evaluated by trial and error.

Since we have generated a map \( w(t(s,\phi)) \), \( s \) and \( \phi \) are a natural set of parameters yielding an orthogonal coordinate frame in \( w \) hence for each unique value of the complex potential \( \phi = s + i\phi \), a point \( w = x + iy \), is generated. The field is plotted "backwards" since the coordinates are gotten from the potentials. The field lines can easily be plotted once the map is solved since \( s \) is one of the parameters. The field intensity is a little more difficult.

\[ B^* = i \frac{d\phi}{dw} = i \frac{d\phi}{dt} \left( \frac{dw}{dt} \right) = \frac{V_0}{\pi t} \left( k_1^2 (1 + t) (1 + \frac{t}{a})^2 \ldots \right)^{-1} \]  

This can also be evaluated at every point \( w(s,\phi) \) and the field plotted as before by interpolation between the corners of the grid.

These two maps were incorporated into a third computer program.

Since the field due to the square end is not really of interest it was not displayed but the flux that would be lost from it was calculated and used to scale the potential field for the other end. The two examples shown correspond to the small strong cusp which was built and installed on the plasma source to provide an ECRH zone (this was done before the program was finished) and to a greatly enlarged very strong cusp which has not yet been built. The small cusp used was built from 6mm thick steel angle and was 5 cm along the cusp, 3 cm high and 6 cm deep. A block of Sm Co₅ magnets (5 cm x 5 cm x 2 cm) of appro-
Approximately 7 kgauss remnant field provided the flux. The computer output (Fig. 4-18) shows field lines spaced by .5 kgauss cm and field intensity in steps of 1 kgauss from 1 to 10. The large cusp is to be made of Vanadium Permendur with the yoke 2.5 cm thick. It will be 10 cm along the cusp 15 cm high and 15 cm deep. The flux will come from block of SmCo₅ magnets (10 cm x 15 cm x 10 cm) of 9.4 kgauss remnant field. The computer output (Fig. 4-19) shows, field lines spaces by 1 kgauss–cm and field intensity in steps of 1 kgauss from 1 to 12. The important point is the gyroresonant zone (3.8 kgauss for 10 GHz) which in the second design is much larger and further from the wall and the field lines which pass through it must go through fields of < 6 kgauss before reaching the wall thus there will be a strong mirroring effect.

Summary

Three relatively straightforward theories were developed to calculate the fields obtainable from arrangements of REC magnets and a limited class of ferromagnetic flux concentrators. Two of the fields exhibited are actually in use on the experiment, and their fields, measured at one or two points, agree very well with the calculations.
Fig. 4-18 Field of the Small, Trial Strong Cusp.
Fig. 4-19 Field of the Proposed Improved Strong Cusp.
Chapter V

Apparatus

The experimental apparatus consisted of the vacuum system, the microwave system, and the plasma diagnostics, with their associated power supplies and measuring circuitry.

The Vacuum System

The plasma was produced in a water cooled, cylindrical, copper discharge chamber of 21 cm diameter and depth (Fig. 5-1), which was connected via a 2.54 cm thick copper plate, the base plate, to a 30 cm deep stainless steel analyser chamber of the same diameter (Fig. 5-2). The other end of the discharge chamber was closed by a copper plate, the end plate, which contained the microwave antenna, a gas jet, and a permanent magnet and steel yoke assembly, 5.08 cm wide, which produced the electron gyroresonance zone. In addition to this small strong cusp, ten rows of 2.54 cm square cross section, permanent magnets produced a multicusp confining field around the side of the cylinder, and four rows of 1.27 x 1.91 cm cross section, magnets produced cusp fields across the end plate which joined the side fields. The base plate contained no confining magnets.

The end plate was configured as follows (Fig. 5-3 see also Fig. 2-2). The microwave antenna was a pair of rectangular wave guides terminated flush with the inside of the vacuum wall and lying on either side of the gyroresonance zone in the end plate. A small gas jet, a .3 cm diameter copper cylinder extending .3 cm beyond the wall with a .08 cm hole in the side which sprayed gas across the gyroresonance zone, stood on one side between the waveguide and the
Fig. 5-1 Input Waveguide, End Flange, and the Discharge Chamber.
Fig. 5-2 Schematic of the Vacuum Chamber.
Fig. 5-3 Inside view of the End Flange showing the Gyroresonance Zone.
pole pieces ends. Microwaves were not confined to this region alone, nor were they absorbed completely by the gyroresonant plasma, so they filled the entire volume of the discharge chamber.

There were eight access ports piercing the side of the chamber between the magnet rows; two diametrically opposed pairs (with 3.81 cm id) on the chamber midplane, and two diametrically opposed pairs (with 1.9 cm id) 5 cm from the base plate. On them were mounted (Fig. 5-4) a barocell and a high pressure ion gauge \( (10^{-6} \text{ to } 1 \text{ torr}) \) to measure base vacuum and neutral gas pressure, two view ports, a liquid nitrogen cooled cold trap, (all of these had some sort of microwave barrier), a moveable disc probe with shielded coaxial stem, and a spark starter.

The cold trap (Fig. 5-5) was a capped stainless steel cylinder mounted inside another stainless steel tube to which it was joined at the top but there was a vacuum gap from there on down to provide thermal insulation. The end of the inner cylinder came to within about 1.5 cm of the inside edge of the discharge chamber; there was a .6 cm thick pierced copper disc plug between it and the discharge to serve as a microwave barrier, without which microwave heating would have rendered the trap useless. The plug contained a hexagonal array of .3 cm holes and was about 66% transparent. Before the trap was mounted a glass plate was mounted over the barrier and a full power discharge run in the chamber; a maximum of 3 \( \text{ mw/cm}^2 \) was indicated by the microwave detector.

The sparker (Fig. 5-6) was simply a .03 cm dia tungsten wire attached to a high voltage vacuum feed through with the tip in the
Fig. 5-4 Discharge Chamber Flange Usage.
Fig. 5-5 Liquid Nitrogen Cold Trap Detail.
Fig. 5-6 Sparker Detail.
center of a .1 cm dia hole in a thin metal cover at the edge of the discharge chamber. Power was supplied by a high voltage pulse supply (Fig. 5-7) which consisted of a variable a.c. transformer, a 13 kV transformer and rectifier unit, a capacitor and a 5C22 hydrogen thyratron tube trigger. This tube was in turn triggered either by a remote switch hung by the discharge chamber, or by a logic pulse, for automatic restarting after chopping the plasma; however this pulser was very slow with a maximum rep rate of about 1/2 per second.

Construction of a faster spark supply utilizing a variable 400 volt power supply and a photo-flash transformer was started but abandoned since pulsed operation was not pursued.

The copper base plate (Fig. 5-8) contained the extractor snout for the mass analyser, a covered slot with a moveable probe, and a microwave shielded 5 x 15 cm pumping channel with a sliding cover. When the barrier was open, vacuum conductance was approximately 600 liter/sec, which was used to establish a low base pressure, or for cleaning the chamber by running a high gas flow rate discharge. The barrier was closed during experimental operation resulting in a low conductance of approximately 1 to 5 liter/sec and low gas flow rate. Also when open an excellent end view of the plasma was obtained revealing a bright plasma in the gyroresonant zone and the well outlined arches of the multicusp field.

The analyser chamber contained the variable field, permanent magnet mass analyser and had six access ports on its side (all 3.81 cm id). These held (Fig. 5-9) a low pressure nude gauge, a rotation feedthrough for the analyser, electrical feedthroughs for the analyser,
Fig. 5-7  High Voltage Pulse Generator Circuit.
Fig. 5-8 Inside View of the Discharge Chamber.
Fig. 5-9 Analyser Chamber Flange Usage.
a thermocouple pair feedthrough for temperature measurements, a thermocouple vacuum gauge, and an argon gas inlet for bringing the entire vacuum system up to atmospheric pressure. The end of the analyser chamber was completely closed by a glass plate with a high transparancy mesh microwave barrier, since there was some microwave leakage around the analyser snout assembly. A 15 cm diameter vacuum line joined to the chamber bottom and connected to the vacuum pump assembly, a 500 liter/sec turbomolecular pump (Balzers TPC 510) backed by a rotary vane mechanical pump, which was located outside the laboratory building. Base vacuum pressures were $1 \times 10^{-6}$ torr in the discharge chamber and $4 \times 10^{-7}$ torr in the analyser chamber. With the base plate barrier closed discharge pressures in to 5-50 mtorr range could be maintained with pressures in the analyser chamber approximately 100 times less ($0.05 - 0.2$ mtorr).

The Microwave System

The microwave system (Fig. 5-10) consisted of an oscillator, a modulator, an amplifier, power measuring circuitry, and various power supplies and protection circuitry. Both the oscillator and the amplifier were vacuum tubes, hence each needed several high voltages, plus filament power, and some form of cooling to be supplied. The oscillator was air cooled by a fan, but the amplifier was water cooled. The oscillator was a Varian X-13 reflex klystron with mechanical, course frequency tuning, and some electrical fine tuning. Operating power was supplied by a separate PRD model 809A klystron power supply. Maximum output was around 200 milliwatts. The frequency was checked by a high Q, adjustable cavity, frequency meter.
Fig. 5-10 The Microwave Circuit.
mounted on a side arm of the output waveguide. The amplifier was a Varian 922-70 klystron which was incorporated in a self contained cabinet with its protection circuitry and high voltage power supplies. Maximum obtainable output was from 800 to 900 watts. This unit was originally a part of an experimental radar installation for the military and had been obtained from surplus. The klystron could be self excited by a feedback network, but this was removed and replaced by the frequency stable reflex klystron oscillator since the amplifier has a very narrow frequency band and slight frequency changes greatly decreased power. The microwave signal could be modulated by a PIN diode modulator which was between the oscillator and the amplifier. A PIN diode is basically a current controlled microwave shunt, somewhat analogous to a bipolar transistor. This modulator served as a remotely controlled on-off switch, and microwave power level control, and could be used to chop the microwaves; however this mode of operation was not found necessary; all data was taken with a continuous discharge. A simple control circuit which supplied the correct voltage signals to drive the PIN modulator was built and located in the experimental electronics rack (see Fig. 5-11). Besides the output and reverse power monitors incorporated in the klystron cabinet a pair of IN518 diode detectors and a differential amplifier monitored forward, reflected, and net microwave power at the discharge chamber (Fig. 5-12). Only the differential amplifier which yielded net microwave power, was calibrated (output was 1 volt/kwatt), but the raw signal from the reflected power diode was monitored, and an attempt to minimize it for all cases was made.
Fig. 5-11 PIN Modulator Control Circuit.
Fig. 5-12 Net Microwave Power Differential Amplifier.
Microwaves were conducted by standard X-band (WR-90) hollow rectangular wave guide, which was filled with room air at atmospheric pressure, except for two short curved sections of waveguide extending back from the discharge chamber end plate, which were under vacuum, the vacuum seal being made by a 1/4 wavelength thick quartz waveguide window on each one. The single waveguide from the klystron cabinet fed a four port, sidewall waveguide power divider which distributed power to the two input waveguide sections. Reflected power was directed to the fourth part which had a sliding microwave short that served as a tuning element. An E-H tuner just before this splitter served to minimize reflected power. All of this microwave plumbing is barely visible at the extreme left edge of figure 5-1. In actual operation all three of these tuning arms were used together to minimize the microwaves reflected from the discharge. The location of the microwave feeds immediately adjacent to the gyroresonant zone, which probably contained the densest plasma, led to high minimum reflected microwave powers.

Protection equipment included a ferrite isolator to absorb reverse microwave power heading towards the reflex klystron, and a water cooled circulator to similarly protect the klystron. A phototransistor arc detector to protect the klystron from breakdown arcs in the waveguide (which never happened), a reverse power monitor, a cooling water flow monitor, a high voltage overload and a body current monitor were all connected to a klystron protection circuit, which in turn was connected to a second PIN modulator switch between the reflex klystron and the klystron.
The Mass Analyzer

Variable magnetic field momentum analysers are a common plasma diagnostic. If a particle of mass \( m \) and charge \( q \) is accelerated electrostatically by a potential \( V \), then injected into a uniform transverse magnetic field it will follow a circular path of radius

\[
r = c \frac{mv}{qB} = \frac{c}{B} \sqrt{\frac{2m}{q} V}
\]

For plasmas where the charge to mass ratio is different for each different charged particle this device will yield the fractional populations. These devices are often called mass analysers since a great many plasmas have only singly charged ions (\( q=e \)).

A variable magnetic field is typically produced by varying the current through a pair of coils wound around the pole pieces. Such devices are straightforward to design and operate, but coils generate heat that must be dissipated, which is a major problem if they need to be mounted inside a vacuum chamber. Since variable magnetic multipoles for focusing high energy particle beams\(^50\) have already been made from steel pole pieces, and the strong permanent magnets now readily available, it seemed appropriate to build a variable dipole field for a mass analyser this way also. The magnet assembly consisted of a magnetized rotor and a steel yoke assembly (Fig. 5-13). The rotor was a rectangular block of permanent magnets surrounded by rounded side pieces to form a cylinder. The two side pieces on the magnet poles were steel; the other two were nonmagnetic. The rotor fit into circular cutouts in the steel yoke which connected it to the rectangular pole pieces which had 4.6 x 10.2 cm faces and a .6 cm gap. As the rotor is turned between
Fig. 5-13 Analyser Magnet and Yoke Assembly.
the yoke side pieces, the fraction of the total flux which passes through the yoke to the pole piece gap varies, so that when the magnetization axis of the block in the rotor is perpendicular to the yoke pieces, the magnetic field across the gap is maximum, and when the magnetization axis is parallel to the yoke pieces, the transverse gap field is zero (Fig. 5-14). The transverse gap field plot in the figure was for the first set of magnets used; the maximum has since been increased to 2800 gauss by using a slightly better set of magnets. Since the yoke is not saturated and magnets that are 50% stronger are available, it is reasonable to predict that this could be raised to 4000 gauss by using those magnets. The longitudinal field in the gap was too small to accurately measure and did not degrade performance.

The rotor was turned by a worm gear drive, located outside the vacuum, via a shaft which passed through a Wilson seal. It was necessary to include a universal joint inside the vacuum chamber, since the axis of rotation was not aligned with the axis of the vacuum port (Fig. 5-15). The rotor has four equilibrium points, the two maxima (of absolute field strength) and the two zero field points. When starting away from these points the magnets first exert a restoring torque on the shaft but this diminishes to zero then reverses sign as the next equilibrium is approached. Since the mechanical linkage had some play (mostly in the u-joint) this leads to a sudden jerk in the rotor as the torque reverses and consequently a jump in the magnetic field. This could have been eliminated by a rigid rotational drive, but at least was rendered innocuous by addition of a spring brake for the rotor.
Fig. 5-14 Measured $B_{\text{gap}}$ vs Magnetic Rotation Angle.
Fig. 5-15 Inside View of the Analyzer Chamber showing the Analyzer as Mounted.
The rest of the analyser (Fig. 5-16) consisted of a Hall probe (Siemens SV230) to measure the gap field, a beam extractor assembly and a shielded, insulated graphite block into which was drilled a depression which served as the collector cup. The beam path was of 3.9 cm radius and focused at 155°, which was where a 0.08 cm wide defining aperture was located. Immediately behind this was the collector cup. The entire region in which the beam traveled was enclosed by a conducting box held at acceleration potential to intercept particles not traveling along the collection path. Since the analyser is a potential well, these particles could not leave it anyway, and if not intercepted they would be deflected back into the analyser by the fringe electric field, and might thus reach the collector.

A small particle beam was extracted from the plasma by a two-electrode extractor snout assembly, which was mounted in the base plate approximately 2.8 cm off center. The extractor (Fig. 5-17) was made of two concentric cylinders, the outer one copper, the inner one steel (to provide magnetic shielding) each wrapped with kapton foil insulation. The end of each cylinder on the plasma side was closed except for a 0.08 cm diameter hole on the center. The first electrode contacted the plasma, and was given a suitable low extraction bias. Its chief purpose was to shield the plasma from the high acceleration bias on the second electrode. Originally only a single extraction electrode was used, but when extracting electrons and negative ions (positive bias) a visibly different plasma formed in front of the extractor, sometimes becoming a centimeter or more in radius. This
Fig. 5-16 Analyser Insert Detail.
Fig. 5-17 Detail Cutaway of the Extractor Assembly as Mounted in the Base Plate.
phenomena has also been observed in other discharges and reported. With the extraction electrode screening the acceleration electrode this has not been observed.

The associated circuitry is shown schematically in Fig. 5-18. Electrical bias was supplied by two regulated D.C. power supplies and a battery box. The accelerator electrode and the faraday cage of the insert were electrically connected to one supply and the extractor electrode to the other. Currents of several tenths of micro-amperes reached the cup. The isolated cup was biased to the same potential as the insert by a shielded battery box grounded through an picoammeter, which served as a very sensitive current to voltage converter. The output signal was fed to the y input of an x-y recorder and the Hall probe voltage was fed to x, which thus yielded plots of cup current vs magnetic field. Since cooling of the magnet was of no concern, the field could be swept very slowly to allow for the time constant of the electrometer; even time constants of seconds could be used for noisy plasmas, or the field could be left fixed on a certain peak which could then be monitored as a function of time. (Sometimes over 1 hour continuously!)

Probes

There were two electric probes used as plasma diagnostics: a small moveable shielded collector in a channel machined across the center, of the front, of the base plate, and a disc probe with a coaxial support, which could be inserted into the plasma from one of the small diagnostic parts approximately 5 cm from the base plate.
Fig. 5-18 Mass Analyser Power Supplies and Circuitry.
In order to measure the parameters in the discharge volume many probe designs were tried; none were totally satisfactory. The probe used (Fig. 5-19) was a .4 cm molybdenum disc mounted across the end of its supporting tungsten wire. The wire fit inside an alumina tube which was inside a copper shielding tube, which was inside yet another alumina tube. Between the end of the copper tube and the probe disc was an alumina disc insulator. The shielding tube was attached to a sliding microwave short which was housed in a vacuum pipe section outside the discharge chamber. This shorted outer conductor served to prevent microwave coupling to the inner conductor. Disc probes were necessary, as even very short wires nearly always were heated to glowing by the microwaves, and a distinct knee in the characteristic trace was never observed for them.

The associated circuitry consisted of a regulated D.C. power supply, a polarity reversing switch, a 10 kΩ series resistor and an xy recorder (Fig. 5-20). This probe yielded excellent characteristic traces, did not melt in the microwave environment, and did not leak microwaves out into the lab; however it still represented a microwave power loss, and could not be moved without perturbing the plasma.

Since the microwave wavelength was only about 3 cm, and a microwave field filled the entire discharge chamber it was impossible to move a probe immersed in the plasma more than a fraction of a centimeter without causing noticeable, and often catastrophic changes, such as greatly increased microwave reflection or the probe structure being heated enough to glow. To overcome this, and allow radial density profiles, a probe was incorporated in a covered slot in the
Fig. 5-19 Disc Probe Assembly.
Fig. 5-20 Probe Circuitry. Lower Circuit is Peculiar to Slot Probe.
base plate so it could be shielded from the microwaves, and yet sample some plasma in order to give relative density information (see Fig. 5-8). Originally the channel was covered by 2 molybdenum strips which left a .07 cm slot open across the plate. Through this slot a .02 cm diameter tungsten wire probe shielded by a .05 cm diameter alumina tube protruded about .2 cm into the discharge chamber. This could still not be moved without microwave arcs forming across the gap and the reflected microwave power being modulated. This arcing was due to the large potentials at microwave frequency that this arrangement could support. Although the two cover strips were electrically shorted in the D.C. sense, since the shortest conducting path from the edge of cover to the other was under the slot, a distance which is comparable to a wavelength, they were uncoupled at microwave frequencies. The next slot covering tried was copper mesh, but power modulation was still high. The last cover tried was a nearly solid copper sheet penetrated only by a single row of 29 holes of .16 cm diameter, spaced .6 cm apart. This nearly worked, but power modulations of about 5% still occur. These are not random fluctuations but for a given pressure and net power occur on each radial scan at the same probe locations. Apparently there still is enough microwave coupling to the slot for it to act as a sort of tuning element. This should be reducable by yet smaller holes.

When the continuous slot had to be abandoned it was no longer possible for the probe, to penetrate into the plasma and still be moveable, so a small planar probe which is actually more nearly a biased ion collector, was used instead. The collector was a small
tungsten rectangle approximately .5 cm square which was shielded by a grounded conducting barricade so that it could only sample one hole at a time (Fig. 5-21). The position was monitored by a helical potentiometer, which was mechanically connected to the probe stem. Both this probe and the disc probe used the same detection circuitry. (Fig. 5-20)

Although it was intended for the probe slot to pass through the center of the discharge, the analysis chamber was mounted slightly off center on the base plate, hence the line of sampling holes is also off center. The large mounting block which attaches the probe's shield to the tube which moves it, does not allow the collector to sample the last hole on that side of the array. These do produce a slightly asymmetric looking trace, but it is still good enough for comparing different discharges.
Fig. 5-21 Slot Probe Assembly as Mounted in the Base Plate.
Chapter VI
Operation and Experimental Technique

Operating the Microwave Equipment

Starting up the microwave system consisted of turning on and warming up both of the microwave tubes and their power supplies, adjusting the reflex klystron to nearly correct frequency, and then after applying high voltage to the amplifier klystron, slightly adjusting the frequency to optimize output power.

Initially the manual variable attenuator in the reflex klystron waveguide arm is adjusted to minimum attenuation to allow monitoring the tube's output, while the PIN diode modulator was biased to maximum attenuation, to prevent the signal from reaching the amplifier. A cross guide coupler in this arm diverted a small fraction of this signal through the frequency meter and into its microwave diode detector (see Fig. 5-10). The frequency meter was a tunable, high-Q parasitic cavity which strongly attenuated the signal in a very narrow frequency band, which could be adjusted by changing cavity size. The reflex klystron frequency was adjusted by tuning the reflector voltage until a relative minimum of the signal was achieved, this would be the center of the attenuation band of the cavity. If the reflex klystron was grossly detuned (e.g. installing a new tube) then a mechanical screw on the klystron tube itself also had to be adjusted. This screw changed the size of the oscillator cavity inside the tube and hence set the range of frequencies over which it would oscillate. Here the best procedure appeared to be to first get the tube to oscillate as it was, and then adjust the frequency measuring cavity until its signal suddenly dipped, indicating the frequency, then to change the tube
adjustment screw a little and note which way the frequency changed. This procedure determined which way to rotate the coarse adjustment screw on the reflex in order to move the frequency towards the desired one. Small adjustments were then made, followed by tuning the reflector voltage to obtain optimum signal after each adjustment (the tube did not oscillate at the same voltage setting for each cavity size, hence this voltage also had to be adjusted). Small steps were necessary to prevent over tightening the tube adjustment which could destroy the tube. Thus the tube could be slowly "walked" to the proper adjustment.

Once the source frequency was right, the manual waveguide attenuator was turned to maximum attenuation, and high voltage was applied to the amplifier klystron. There was a built in timing circuit to allow adequate tube warmup time after filament power was supplied before voltage could be applied. With voltage on, the PIN diode attenuator was now biased to minimum attenuation; this resulted in a small output of about 20-30 watts since some input power could still get through the manual attenuator. This manual attenuator was now slowly opened, and the output power monitor carefully watched. Power would rise until the tube saturated, further increase in input signal resulted in diminished output. The manual attenuator was carefully adjusted until that optimum setting was reached. The amplifier klystron was very narrow band, so the reflex klystron reflector voltage, hence frequency, would be readjusted gently until output was maximum. The manual attenuator was also again gently readjusted to reach optimum. After the first 3 to 5 minutes both had to be readjusted a final time. Now this attenuator would be left
alone and all power adjustment would be done by the PIN modulator. Since these two attenuators were in series this prevented the input power going past saturation.

The PIN modulator was controlled by a voltage signal from a module mounted in the experiment's electronics rack. This module could be switched to supply the voltages necessary for maximum attenuation, low attenuation or a variable voltage, so as to allow varying microwave power from about 100 watts up to maximum. This unit could also be gated by a logic pulse to switch between full attenuation and the level set by the variable control.

Whenever the microwave equipment was on, a portable microwave radiation monitor (Narda model 8606) was also turned on. If any section of waveguide had been disassembled since the last running, or if any new vacuum feedthrough was installed on the chamber, then it was checked for dangerous microwave leakage. Since established safe microwave levels are only 5 mw/cm² at 10 Ghz, and 1 kwatt was applied, a very small gap between metal pieces, or any coaxial electrical feedthrough could easily cause serious leakage. Microwave safety was verified by checking all around the suspect area when the microwave amplifier was first turned on (ie. low power), and then increasing microwave power in steps, checking all around each time until full power was achieved. If the new device contacted the plasma, its microwave leakage was often drastically changed from the vacuum level by the presence of a discharge. If any area had a detectable, but safe level of microwave leakage the monitor could be left there with a preset alarm in case changes in the plasma suddenly produced dangerous leakage.
Starting and Running the Discharge

While the microwave equipment was warming up, the vacuum chamber was also prepared. If gas flow was to be high for cleaning out the chamber, which was the usual case when turning on the system, the sliding barrier across the pumping slot in the base plate was left open and the nude ion guage in the analyzer chamber had to be turned off, otherwise electrons coming through the pump slot initiated a local plasma around the high voltage grid and collector of this device. The input line gas value would be then opened, and the needle valve adjusted to yield the desired chamber pressure. Then the microwave amplifier would be powered up, and adjusted as described in the previous subsection. Often the discharge self started, but if not it could be initiated by the sparker.

Once the discharge was on, the waveguide system had to be "tuned" to maximize the net power to the discharge. The net microwave power was indicated by a differential amplifier connected to the pair of microwave diode detectors that monitored forward and reflected power at the waveguide input to the discharge chamber. This was displayed on a digital volt meter in the experiment's electronics rack. Also the reflected power signal was monitored by an oscilloscope and another digital voltmeter. This allowed the net power, the reflected power and the modulation of the reflected power caused by relaxation oscillations in the plasma to be monitored. These oscillations are caused by the reflectivity of the plasma being a function of electron density, which is itself a function of the absorbed microwave
power. Similar oscillations in other microwave plasmas are widely reported. The two adjustable shorted waveguide tuning arms, which made up the E-H tuner, and the tuning arm on the waveguide power splitter (see Figs. 5-1 and 5-10), were adjusted while observing these power indicators until the quietest reflected signal, at the lowest obtainable level was achieved. This tuning was a sensitive function of gas pressure and microwave power, and often required a lot of patience. There were some pressures where it was just not possible to run a quiet discharge at all, and at some other pressures it would be relatively quiet only over some limited power range. At most pressures power could be varied with little or no tuning required, but gas pressure could not be varied at all without at least a little adjusting being required, if not a complete returning. This finicky behavior was the most annoying aspect of the discharge.

Some additional tuning was required in the first five minutes or so as things heated up and the dimension changed due to thermal expansion. The E-H tuner was water cooled, otherwise it never settled into steady state, but slowly the net power drifted around due to this thermal feedback between reflected power and tuning. Also the input waveguide section of the vacuum chamber was water cooled to protect the vacuum windows from over heating. The discharge sometimes suddenly detuned itself, that is net power dropped substantially and did not recover (mode change). This was most likely when operating at very low gas pressure (p < 15 mTorr), or if trying to operate in a regime of high relaxation oscillation noise. It did sometimes rapidly detune after having run quietly for several minutes; this was not
really understood. The best prevention was patient careful tuning after each new power or pressure change. Once it settled in, the discharge could be run for hours with only a little periodic tuning if no changes in power or gas pressure were made.

In order to be able to operate the mass analyser, which required lower gas pressures than those at which the discharge was run, the cover for the pumping slot had to be lowered. Since this greatly changed the pumping rate for the discharge chamber the needle valve had to be adjusted. The discharge would be extinguished by cutting off the microwaves, and the gas would first be shut off and the system pumped out then the cover would be closed, the needle valve adjusted to the desired pressure and the discharge restored by switching back on the microwave power and discharging the sparker. Oddly, even though the discharge usually started itself when first turned on, it almost always required a spark to restart, if subsequently shut off. This is probably due to fact that initially the microwave system is not yet properly tuned, and a small local hot spot on the metal wall is formed by nonuniform microwave field strength. Bright pinpoint microwave arcs can be observed in the chamber if the microwaves are injected with no gas present, and can sometimes be seen even with a plasma going.

The inside of the chamber soon acquired a silvery coating after running which must have been some of the soft solder used to repair the end plate in the vicinity of the gyroresonant zone (this is visible in figures 5-3 and 5-8). Since large amounts of power are deposited on the wall there, some of the low melting temperature
solder could easily be vaporized and deposited elsewhere on the walls. This also was probably the cause of the large amount of water (oxygen in a hydrogen plasma is rapidly converted to water) in the discharge before the trap was installed.

Using the Mass Analyser

Once the discharge was running with the pumping slot covered and the analyser chamber at good vacuum (typically .2 mTorr or less), the power supplies for the extractor electrodes would be switched on (see Fig. 5-17). A fixed acceleration voltage of ± 200 V was used. Higher voltages yielded higher collected currents but reduced the maximum particle mass that could be discerned (since maximum obtainable field was fixed at 2800 gauss). The value chosen was the highest voltage which still allowed m=20 to be clearly detected. Since positive ions are expelled from the discharge by the positive plasma potential the outer extractor electrode was simply left floating for collecting them. The case for negative ion collection was more difficult: at each pressure the outer electrode bias was optimized to yield a maximum H⁻ signal. This was done by first applying about +10 V of bias and then sweeping the magnetic field until the H⁻ peak was detected; the magnetic field was then adjusted until it coincided with the center of the H⁻ peak. Then the bias voltage was then varied, and the H⁻ signal recorded as a function of it. The bias was reset to the voltage which yielded maximum H⁻ signal. It was found that this optimum bias did not depend on microwave power, but only gas pressure, hence it was left fixed for each pressure.
The mass analyser signals were larger, and were reproducible if the analyser field was swept from maximum intensity to 0 rather than vice versa. This was probably a hysteresis effect. The Hall probe signal and the collector cup output were connected to an x-y recorder, the peak heights were measured on the chart produced and that was used as the ion signal data. Polarities were reversed for opposite charges, so the two traces appear mixed together. Positive traces were taken in red, and negative in black to make them easily distinguishable. A typical trace is shown in figure 6-1. Because of the disparity in sizes the electrometer was switched to a scale that was 10 times less sensitive to record the H\textsuperscript{+} and electron peaks and a scale either 10 or 100 times more sensitive to record the H\textsuperscript{−} peak. The negative peaks were narrower, and the field often had to be reversed back and forth over them several times in an attempt to record the peak, in spite of this it is evident that quite often the electron peaks were still clipped off. In addition to the x-y recorder trace, the drain current on the outer electrode, when it was biased positively, was recorded since it was a reliable monitor of extracted negative current.

The absolute value of magnetic field at which H\textsuperscript{+} and H\textsuperscript{−} signals were collected was not the same. This was because of the difference in effective extraction voltage for the two particles due to the plasma potential (it accelerates H\textsuperscript{+} but retards H\textsuperscript{−}), hence

\[ B_\pm = \frac{c}{r} \sqrt{\frac{2m}{e}} (V \pm V_p). \]

The magnetic field was monitored by a Hall probe which was linear, but had an offset (ie. Hall probe signal for B=0 was not 0), so
Fig. 6-1 Typical Mass Analyzer Output. The Species from left to right are: e⁻, H⁻, H⁺, H₂⁺, H₃⁺, N⁺, O⁺, OH⁺, and H₂O⁺. The e⁻, H⁻, and H₃⁺ peaks have been scaled as indicated.
operationally the mass analyser equation could be written as

\[(V_{hp} - V_o) = k \sqrt{A(V_{acc} + V_p)}\]

where A is the mass of the species in any useful unit, say in terms of the proton mass. There are three unknowns, but with two negative \((e^-, H^-)\) and five positive \((H^+, H_3^+, O^+, OH^+, H_2O^+)\) peaks available this equation could easily be solved. At least one positive and one negative species must be included otherwise the product \(k \sqrt{V_{acc} + V_p}\) could not be divided. This was tried as a means of measuring \(V_p\) but there was a random scatter of 2-3 volts to the results, the reason for this was not discovered. Since a probe which would work was finally made, this study was not continued, however it still remains as an option for measuring \(V_p\) where insertion of probes is not possible.

Using the probes

The disc probe was normally retracted and only inserted into the plasma briefly for measurements since it was heated by the microwaves. Bias was supplied by a regulated power supply; current was measured across a 1 kΩ series resistor. The bias signal was input to the x amplifier of an x-y recorder and the voltage on each side of the resistor was input to the differential inputs of the y channel of the recorder (see Fig. 5-19). A typical trace is shown in figure 6-2, as can be seen the signal always become slightly noisy for probe bias beyond about 4 volts. A simple analysis\(^{15}\) was used. The electron current was determined from the x-y output by fitting a straight line to the ion saturation current (negative asymptotic line) and then using it as the base line and measuring the distance to the current
curve with a ruler. There was a slight leakage current which is noticeable as the slope to the ion "saturation" current, but this method also automatically subtracted it. These numbers were then input to a computer along with the proper scaling factor to convert them to net electron current. The computer then displayed $\log_{10}(I_{e-})$ as a function of $V_{probe}$, from which the dividing line between those points belonging to the transition region and those belonging to the electron saturation region was determined visually and input back to the computer. Each group of points was then least squares fit to a line of the form

$$\log_{10}(I_{e-}) = m V_p + b$$

The electron temperature is then just the inverse of the slope of the line for the transition region, the plasma potential is taken to be the probe voltage where the two lines intersect, and the electron density is calculated from

$$I_{e-}(V = V_p) = \frac{1}{4} n_e \sqrt{\frac{8T_e}{\pi m_e}}$$

The factor of $1/4$ is the product of two factors of $1/2$. One factor of $1/2$ is for a disc with diameter much larger than a debye length, since $1/2$ of the particles will be headed towards it and $1/2$ away, the other is from the average of the cosine from the velocity over a hemisphere. Due to the shadow effect of the probe and possibly some collisionality this is only approximate. The computer fit to the data shown in Fig. 6-2 is shown in Fig. 6-3.

The slot probe was biased at a fixed negative potential (-25 V) to act as an ion collector and was moved from one edge to the other to produce a radial profile.
Fig. 6-2 A Typical Probe Trace.
Fig. 6-3 Analysis of the Previous Probe Trace.
Chapter VII

Results and Conclusions

In this chapter data from the mass analyser and the probe are presented. As has already been mentioned there are only limited ranges of pressure in which a reasonably quiet stable discharge could be maintained. From these ranges a set of pressures which were roughly evenly spaced were chosen (20, 30, 44, 52 mTorr). At each of these the discharge was operated at the same set of net microwave powers (450, 550, 650, 750 watts); power could not be regulated to better than about ± 20 watts at the higher pressures.

Experimental Results

Figure (7-1) through (7-6) show the experimental measurements as a function of gas pressure for each different net microwave power. The first four show that the volume plasma only weakly depended on pressure and only density showed much dependence on net power. This is to be contrasted with figure (7-5) which shows the nature of the extracted H\(^{-}\) current. The salient feature is the strong increase of H\(^{-}\) signal with pressure \(n_0\) above 30 mTorr whereas total negative signal again only varies slightly (see Fig. 7-6). The measured value for the plasma potential is in good agreement with the measured value for \(T_e\) and the ion composition. At the beginning of Chapter II a very simplified theory of the sheath was presented but figure (7-7) shows there is excellent agreement (\(V_p\) is the measured value, \(\phi_p(T_e, M_i)\) is calculated from the measured values for \(T_e\) and the ion composition using equation (2-2)).
Fig. 7-1 Electron Density from the Probe.
Fig. 7-2 Electron Temperature from the Probe.
Fig. 7-3 Plasma Potential from the Probe.
Fig. 7-4 Positive Ion Composition.
Fig. 7-5 Analyzer H⁻ Signal.
Fig. 7-6 Mass Analyzer Extractor Snout Drain Current.
Fig. 7-7 Comparison of the measured Plasma Potential to that calculated from the measured Ion Composition and Temperature.
\( n^- = n_o \frac{<\text{Attach}>_{\text{eff}}}{<\text{Det}>_{\text{eff}}} = \frac{\sum <\text{DA}>^S \left( n_e n_o <\text{EV}>^S + \frac{dn}{dt} \text{gyro} \right)^{\text{form}}}{<\text{Det}>_{\text{eff}}} \tau_{m} \)

Destruction will be dominated by mutual neutralization for which the rate coefficient depends on ion temperature and effective ion mass. Since \( T_e \) and \( V_p \) change only a little it is quite reasonable to assume \( T_i \) also to be fairly constant, and as was shown the ion composition does not vary too much; therefore, the effective detachment rate coefficient should be fairly constant. In the numerator all the quantities except the contribution to \( H_2(v) \) from the resonance zone depend on volume parameters and are hence only going to vary a little. The estimates in chapter III were based on \( n_o = 10^{15} \) which corresponds to about 29 mTorr. It was found that at those parameters the production from the gyroresonance zone and the local production were equal. As was shown in Figure (3-3) production in the gyrozone increases sharply with pressure while that from the volume should remain roughly constant, hence above about 30 mTorr \( H_2(v) \) from the gyrozone should begin to become dominant. This is what is seen in Figure (7-5).

Figure (7-8) shows the ratio of the analysed \( H^- \) current to that of the analysed electron current and figure (7-9) the ratio of it to extractor drain. As was mentioned the very narrow electron peaks were subject to being clipped off; this is the reason for the scatter of
Fig. 7-8  H⁻ Fraction of the Analyzed Negative Current.
each of the sets of points in the figure but still it indicates that
the ratio is several parts of 1000 as was estimated. Figure (7-9) was
included to show that the fraction really is a steadily increasing
function of pressure, since drain current should be a reliable monitor
of extracted negative current. Figure (7-10) shows the ratio of
analysed electron signal to the extractor drain current, again the
scatter is evident but the ratio is seen to be roughly $10^{-6}$ and
possibly improves slightly with higher pressure. The extractor was
1.27 cm in diameter. These two facts can be combined to estimate the
analyser efficiency or equivalently to convert the analysed currents
into extracted current densities

$$J \approx \left( \frac{10^6}{\text{Area}} \right) I = 8 \times 10^5 \left( \text{cm}^{-2} \right) I$$

The estimated maximum extracted H$^-$ current density is .78 mA/cm$^2$.
This is very low compared to actual long pulse negative ion sources
which achieve 10's of mA/cm$^2$ but their extraction is at several
hundred volts whereas here it was only 6 to 9 volts.

**Effect of Multicusp Confinement**

Figure (7-11) shows a radial density profile taken at 30 mTorr
pressure before the side magnets were mounted; figure (7-12) shows one
at 30 mTorr with side magnets. The reflected microwave power
modulation induced by probe motion discussed in Chapter V is very
apparent in the second profile; net microwave power dropped at the
hole just right of center then recovered and dropped again four holes
further left and beyond. The asymmetry due to the misalignment of
base plate and chamber is much more pronounced in the profile with
side magnets since it passes through the multicusp field nearer to the
Fig. 7-9 Analyzer H⁻ Signal Divided by the Mass Analyzer Extractor Snout Drain Current.
Fig. 7-10  Analyzer Electron Signal Divided by the Mass Analyzer Extractor Snout Drain Current.
Fig. 7-11  Radial Profile with no Side Magnets Mounted (left side of graph corresponds to left side of chamber as viewed in Fig. 5-8).
Fig. 7-12  Radial Profile with Strong Side Magnets Mounted (see Fig. 4-7).
cusp on the left side than on the right hence the density remains higher there. In spite of their poor quality these figures still clearly show that the central density is proportionally much higher when side magnets are present.

Some mass analyser data was taken before the side magnets were mounted, the disc probe was not perfected until afterwards. Without side magnets positive ion signals were only about 15-20% of their heights at the same power and pressures with side magnets, the H⁻ peaks were less than 7%! This extra factor of two in the enhancement of H⁻ by side magnets is probably due to somewhat higher positive ion temperatures (hence lower <MN>) that the improved confinement will produce.

It was easier to tune the microwaves to the discharge and the maximum net power achievable at the higher gas pressures was also higher. This is due to the much lower plasma densities indicated by the extracted ion currents.

The effect of H₂O

The water concentration in the discharge was far higher than desired, so the cold trap was designed and built to remove it. The results from using it were at first surprising, but are in agreement with the theory so far developed.

A 750 watt net microwave power discharge at 43 mTorr was used for these observations. The mass analyser was set on the H₂O peak and liquid nitrogen was poured in the trap. After a few minutes the H₂O peak was down to about 10% of its previous height (now only 1-2%
of the positive ion population). The trap was then rapidly blown dry
with nitrogen and the fractional population of H₂O jumped to over
50%. This was repeated for the mass analyser set for H⁻. Upon
introduction of liquid nitrogen the H⁻ signal dropped to about 75%
of its original value. After several minutes the trap was again
rapidly blown and the H⁻ signal briefly jumped up to over 2.5 times
its original value (see Fig. 7-13). The probe was inserted to monitor
the plasma before and during the period the trap was cold and in the
blow down afterwards. The electron temperature before and during was
1.5 eV and the density was 2.5 x 10^{10} with no significant difference
between them. The probe was swept manually and hence was too slow to
follow the blow down. A trace which was taken on the downward slope
of the peak showed temperature had dropped to 1.4 eV and density risen
to 2.8 x 10^{10}.

Three things happened when the trap was rapidly blown: T_e
dropped, n_e rose, and the effective ion mass jumped up due to the
dominance of the massive water ions. This experiment was conducted in
the region where H₂(v) from the resonance zone is dominant hence
n⁻ depends only weakly on n_e. The dissociative attachment rate
coefficient is a rapidly decreasing function of temperature in this
range; the change from 1.5 to 1.4 eV increased it from 3x10^{-9} to
5x10^{-9}. As was determined in chapter III the chief destruction
mechanism is mutual neutralization. If the change due to the effect
of increased ion mass due is evaluated by equation 3-2 the loss rate
coefficient is seen to decrease from 1x10^{-6} to 7x10^{-7}. All
combined these predict
Fig. 7-13 Analyzer $H^-$ Signal vs Time. LN$_2$ was added at $t = 150$ and removed at $t = 650$. 
\[ \frac{n^{-'}}{n^-} = \frac{<\text{DA}>' <\text{MN}>}{<\text{DA}> <\text{MN}>'} = \frac{5.1 \times .99}{3.1 \times .73} = 2.2 \]

which is in perfect agreement with the observation, since no doubt the peak change in \(T_e\) and ion composition were somewhat larger. For the cooling down stage only the change in effective ion mass due to the elimination of water was noticed. The predicted effect is

\[ \frac{n^{-'}}{n^-} = \frac{<\text{MN}>}{<\text{MN}>'} = \frac{.92}{.99} = .92 \]

The extra decrease measured can easily be explained by a slight drop in \(T_e\) (only about .05 eV is needed) which was too small to accurately measure. There does indeed appear to be a change of this size in \(T_e\) but this change is far below any reasonable estimate of the error in the determination of probe temperature so it cannot be reported as fact.

This experiment indicates that rather than being detrimental \(H_2O\) aids \(H^-\) production. The mass analyser has revealed no other negative ion signals within a factor of 20 of the size of the \(H^-\) peak for masses out to around 24 or so (see Fig. 6-1); water impurity leads to no impurity in the beam!
Summary of Results

This experiment was undertaken to gain experience with a microwave driven hydrogen discharge of a type that would be suitable for use as an ion source for high current beams. The emphasis was on H\textsuperscript{-} production. The results may be listed as follows.

(1) H\textsuperscript{-} can be produced by a microwave driven discharge, and the behavior is consistent with predictions based on production by dissociative attachment to vibrationally excited molecules.

(2) In comparison to present day ion sources, the operating pressures were high. The discharge had a cut off pressure below which it could not maintain itself (characteristic of all microwave discharges), and the H\textsuperscript{-} current increased rapidly with increasing gas pressure.

(3) Water in the discharge was found not to be ruinous, but to the contrary, appears to be beneficial to H\textsuperscript{-} production.

(4) A net microwave power of 750 watts produced a density around $3 \times 10^{10}$ cm\textsuperscript{-3} in a volume of 100 cm\textsuperscript{3}. It is not feasible to scale from this to denser discharges since the gyroresonance zone plays a crucial part, and what part of the power it absorbs is not known, but it is probably not a linear function of net power.

(5) The magnetic bucket configuration enhances the extracted H\textsuperscript{-} current of this device.

(6) By using steel pole pieces an ECR plasma can be created and confined to one end while a cooler plasma will fill the rest of the volume.
A workable variable field mass analyser was constructed from permanent magnets which, though plagued by a weak mechanical design, proved this concept: it provided a uniform variable field and generated no heat.

Summary and Conclusion

This experiment was motivated by the desire to find an alternative to filament driven ion sources, since neither the short filament lifetime, the long turn on time, nor the complexity of design that having to provide electrically isolated vacuum feedthroughs with water cooling for the filaments necessitates, are desirable. The microwave powered device was certainly simpler and could be operated far longer without maintainence.

The key feature of this device was the very strong cusp which yielded the small crescent shaped ECR zone in which hotter electrons could be formed, but yet kept separate from the rest of the plasma. An enlarged and much stronger cusp, like the one shown in Chapter IV, would greatly improve performance. It would contain a much larger ECR zone, and the plasma formed there would be subject to mirror confinement rather than being quickly lost to the wall at the magnetic poles. There would also be enough space between the magnets themselves and the ECR zone for both the vacuum wall and a microwave antenna, so that microwaves could be directed right into the resonant plasma. This should greatly reduce the reflected microwave power problem and with a good antenna design reduce microwave indensities beyond the resonance zone to negligible proportions. The mirror
confinement will increase microwave efficiency, lower the gas pressures necessary to operate the discharge, and raise the hot electron temperature. These improvements may also be obtainable by confining the microwaves to the present ECR zone by a properly designed barrier (grating or mesh), which would still allow particles to pass. If the microwave frequency were lowered, the field necessary for gyroresonance would also be lowered; this could result in an ECR zone in each bucket field cusp. For the same density to be obtainable however, the plasma would have to be that much more beyond cut off density since it also is lower at lower frequencies. The important thing, however, is the total production of excited states in the ECR zone which depends on both the temperature and the total volume of the hot plasma; so since much more volume would now be resonant, lower densities might well be acceptable. This scheme might be a useful alternative.

Further experimental work on this device is needed. No measurements have yet been made on the hot ECR plasma, a knowledge of both the electron density and temperature of that region are necessary. Several properties of the non-resonant volume plasma are also needed, namely: ion energies, atomic neutral fraction, and gas temperatures. Another useful experiment is injection of electrons from a separate cathode into the non-resonant volume which would allow some control, independent of the ECR plasma, of the electron temperature and plasma potential. Both for this experiment and all other volume production work the actual mechanism needs to be verified. Dissociative attachment to highly vibrationally excited molecules appears most likely but actual measurements of vibrational populations and
correlation to $H^-$ production is essential. The extracted $H^-$ current densities were very low, but in this experiment a very low voltage was used on the extraction electrode; much higher current densities will be obtained when beam extraction at high voltage like on other negative ion sources is used. Work on developing this type of $H^-$ ion source should continue with emphasis on an enlarged ECR zone and on large scale extraction of negative ion beams.
Appendix A

Some Aspects of Microwave Safety

Microwave safety was always a concern in this experiment since nearly a kilowatt of continuous power was delivered to an apparatus which had to be frequently disassembled and into which various devices electrically connected to the outside had to be inserted. The exact nature of the hazard afforded by microwaves and hence the definition of "Safe" levels of exposure is still a topic of debate. The most widely accepted view, and the official view behind the established exposure limits, is that harm is done only by deposition of energy in body tissues faster than those regions can remove it, that is the danger is in heat damage to effected areas of the body. With this guideline the official standard is $5 \text{ mWatt/cm}^2$ for frequencies above 1500 Mhz (1.5 Ghz).

This is however not the only point of view. Some believe that at even lower power levels the alternating electric field of the microwaves may interfere with the activities of the nervous system or possibly even alter the activities of the body's cells by changing the diffusion rates of the various ions dissolved in the protoplasm. This is the official view held in the USSR and is the basis for the much stricter standards set there.

The problem of preventing 5 mWatts out of 500 watts (which is 1 part in $10^5$) from escaping is difficult. All joints must have good, continuous, conducting contact around their entire periphery. Since waveguide flanges are designed with this in mind they usually did not present a problem unless damaged, but that only has to be a scratch, a
loose screw, or a cracked solder joint so whenever disassembled all joints were carefully checked as power was being applied.

The chief problem was vacuum seals. Small flanges with rubber "O" ring seals would not do, hence only conflat flanges with copper gaskets were used. If a window or a device in which microwave filters could not be installed was to be mounted a microwave barrier had to be added. The most satisfactory was a moderately thin metal plate with a hexagonal array of circular holes. If a plane wave is incident on an infinite plate of thickness $t$ with a rectangular array of holes (possibly staggered) of diameter $d$ with spacing between neighbors being $a$ and $b$ in the two array dimensions the transmitted signal will be

$$P_{\text{out}} = 10^{A/10} P_{\text{in}}$$

where $A$ is the attenuation in decibels and is given by

$$A = 20 \log_{10} \left( \frac{3a b \lambda}{2 \pi d^2} \right) + 40 \pi 1.841 \log_{10}(e) \frac{t}{d} \sqrt{1 - \left( \frac{\pi d}{1.841 \lambda} \right)^2}$$

The first term represents the fraction that is not reflected back by an infinitely thin sheet with the same hole pattern. The second term is the attenuation in a perfectly conducting circular waveguide operating below its cut off frequency $\lambda = \frac{\pi d}{c}$. For a hexagonal array

$$b = \frac{\sqrt{3}}{2} a$$

and the optical transparency is

$$\tau = \frac{\pi d^2}{4ab} = .906 \left( \frac{d}{a} \right)^2$$

This gives a useful attenuation formula
180\[A = 20 \log_{10}\left(\frac{3\lambda}{8\pi d}\right) + 32 \frac{t}{d} \sqrt{1 - \left(\frac{1.71d}{\lambda}\right)^2}\]

and for reasonably small holes the square root is negligible. Since reflecting most of the power back into the discharge is desirable the first term should be approximately 15 or larger yielding

\[\frac{3\lambda}{8\pi d} \geq 6\]

\[d \leq \frac{3\lambda}{48\pi}\]

\(\tau = .7\) is the highest transparency which can be reasonably machined hence

\[d \leq .1\lambda\]

This was found to be a useful rule of thumb.

In use only small sheets a few wavelengths across were used and these were only part of a large non planar boundary however barriers designed by the above to give 50 db always proved adequate.

The best way found to incorporate such a microwave barrier in a circular access part was to press fit a plug in the opening flush to the inside of the vacuum wall. Another satisfactory way when using conflat flanges was to replace the copper gasket with a perforated metal disc (either soft copper or aluminum can be used).

Another possible microwave barrier is a conducting mesh (screen). Meshes are treated as 2 independent orthogonal arrays of conducting wires. Since most are square and since any arbitrary field can be divided into orthogonal components only the effect of a field parallel to or perpendicular to an infinite wire grating need be considered. Lamb\(^{58}\) published a solution for a perfectly conducting grating in
1898 (somewhat before microwave shielding was a problem). For wires of
diameter $d$ and spacing $s$ the reflected power is

$$E_{\text{parallel}}\quad r = \frac{1}{1 + \left(\frac{2s}{\lambda} \log \left(\frac{s}{d}\right)\right)^2}$$

$$E_{\text{perpendicular}}\quad r = \frac{\left(\frac{s^2d^2}{2}\right)^2}{1 + \left(\frac{s^2d^2}{2}\right)^2}$$

These can be rewritten to give the transmitted power attenuation in db

$$E_{\text{parallel}}\quad A = -10 \log_{10} \left[ \frac{\left(\frac{2s}{\lambda} \log \left(\frac{s}{d}\right)\right)^2}{1 + \frac{2s}{\lambda} \log \left(\frac{s}{d}\right)^2} \right]$$

$$E_{\text{perpendicular}}\quad A = -10 \log_{10} \left[ \frac{1}{1 + \left(\frac{s^2d^2}{2}\right)^2} \right]$$

and the optical transparency is

$$\tau = \left(\frac{s-d}{d}\right)^2$$

For $d^2 \ll \lambda s$ the perpendicular attenuation vanishes (as was shown by
Hertz also in the 19th century). This is always the case for useful
screens so only the parallel formula need be considered.

The large end flanges of the chamber (20 cm id) had standard
rubber "o" ring seals. There was sufficient warping when the chamber
was soldered together that the entire chamber had to be put back in a
lathe and the flanges refaced before an adequate microwave seal could
be obtained.
Any feedthrough structure which had to be electrically isolated from the chamber wall presented a real problem since there is no cutoff frequency for coaxial structures, hence feedthrough could not be simply designed to be low pass filters. It was found that if a very thin insulator was used (on the order of .005 cm thick) microwave leakage was usually tolerable but this had to be verified for each case. Also such thin insulators are liable to be punctured by the high microwave electric fields in such thin coaxial structures resulting in an electrical short. Coaxial band reject filters are a continuing area of engineering research but published solutions are quite complex, and were found too limited in applicability to be incorporated into the to design a useful experimental structure.
Appendix B

Cross Sections Used and Calculation of Rate Coefficients

Binary collisions and the rates at which they proceeded in the plasma formed the basis of chapter II and III. The rate at which such a process proceeds is

\[ R = n_a n_b \frac{\int_{-\infty}^{\infty} f(v_{rel}) \sigma_{ab}(v_{rel}) v \, 2\pi v^2 \, dv}{\int_{-\infty}^{\infty} f(v_{rel}) \, 2\pi v^2 \, dv} \]

For collisions between electrons and molecules or ions the relative velocity is the electron's velocity since the much heavier "target" is essentially stationary. If the electrons are assumed to have a maxwellian velocity distribution and the cross section is known as a function of electron energy, then since

\[ v^3 \, dv = \frac{2e}{m} \frac{3/2}{2} \sqrt{\frac{2}{mc}} \, dc \]

the rate can be written as

\[ R = n_a n_b \sqrt{\frac{8}{m \pi T_e^3}} \int_{0}^{\infty} \sigma(\epsilon) e^{-\epsilon/T_e} \, \epsilon \, d\epsilon \]

Defining the rate coefficient

\[ <\sigma \, v> = \frac{R}{n_a n_b} \]

this becomes

\[ <\sigma \, v> = 6.69 \times 10^{-7} T_e^{-3/2} \int_{0}^{\infty} \sigma(\epsilon) \epsilon e^{-\epsilon/T_e} \, d\epsilon \]

For experimental measurements \( \sigma(\epsilon) \) was known for a finite set of
discrete energies; the theoretical cross sections used were not of a form to be readily integrated either, hence the integral was evaluated numerically using Simpson's rule \((m=2\) ie fitting to a parabola). The cross sections were evaluated by linear interpolation between data points and beyond the experimental points available an applicable theoretical curve was used (normalized to the experimental values) or if none was available the form 

\[
\log(\sigma) = \log(\sigma(\varepsilon_{\text{max}})) + b(\varepsilon - \varepsilon_{\text{max}})
\]

(where \(\varepsilon_{\text{max}}\) is the energy of the highest experimental point and \(b\) is determined from fitting to the last several points) was assumed.

Chan \(^5\) has already compiled a set of tables and graphs for the processes used for modeling positive ion sources this supplied all the necessary rates not specifically dealing with negative ions. This left the excitation rates to vibrational states and \(\text{c}^3\text{\Pi}_u\) state, all the dissociative attachment rates, and the electron collisional detachment rate to be calculated. The cross sections used are shown in figures (8-1, 3 and 5) with "*" representing data points. (The structure on the electron detachment cross sections is real and caused by a resonance resulting from \(\text{H}^2-\) formation). The corresponding rates are shown in figures (8-2, 4 and 6). The ratio of the electron neutral collision frequency to gas pressure is shown in figure (8-7) and the rate averaged over the electron distribution in figure (8-8).

The following sources were used for the cross sections.

\[
\begin{align*}
e^- + \text{H}_2 & \rightarrow e^- + \text{H}_2(v) & \text{Ref 43} \\
e^- + \text{H}_2 & \rightarrow e^- + \text{H}_2(\text{c}^3\text{\Pi}_u) & \text{Ref 18} \\
e^- + \text{H}_2(v) & \rightarrow \text{H} + \text{H}^- & \text{Ref 31}
\end{align*}
\]
For reactions where both species were molecular the rate coefficient was approximated by

$$<\sigma \nu_{\text{ion}} > = \sigma \sqrt{\frac{2\epsilon}{m}}$$

where \(\epsilon\) was the probable interaction energy. Proton transfer was an exception where no cross sections and only a single measurement at room temperature for the rate coefficient is known.
Fig. B-1  Electron Collisional Excitation Cross Sections.
Fig. B-2  Electron Collision Excitation Rate Coefficients.
Fig. B-3 Dissociative Attachment Cross Sections.
Fig. B-4 Dissociative Attachment Rate Coefficients.
Fig. B-5 Electron Collisional Detachment Cross Section.
Fig. 8-6 Electron Collisional Detachment Rate Coefficients.
Fig. B-7 Electron Collision Frequency in Hydrogen Gas divided by the Gas Pressure.
Fig. 8-8  Average Electron Collision Frequency in Hydrogen Gas divided by the Gas Pressure.
Appendix C

The Computer Model of the Gyroresonance Zone

In Chapter II a series of graphs of various quantities as a function of neutral gas density were presented. They were calculated by first determining $T_e$ and $n_e$ as set forth in equations (2-4b) and (2-6). The rate coefficients were evaluated as explained in appendix B for $T_e$ ranging from 0 to 25 eV in .25 eV steps. For index $I$

$$T_e = .25^*I$$

and equation (2-4b) yielded

$$n_0(I) = \frac{\tau}{\langle \sigma_{ion} v_e \rangle(I)}$$

For each $I$ the coordinate $n_0(I), T_e$ was marked on the $n_0-T_e$ plane by a symbol yielding figure 2-3(a). This set of $n_0$ and $T_e$ was combined with the appropriate ionization energy and $Q(T_e)$ and using equation (2-6) a set $n_e(I)$ was calculated and plotted on the $n_0-n_e$ plane to produce fig. 2-3(b). For each subsequent graph appropriate reaction rates at this same set of $T_e$'s were combined with the corresponding $n_0(I), n_e(I)$ yielding a set of discrete points which were plotted.

Equation (2-7) was a special case. To produce figure (2-5) $\epsilon_{osc}$ was treated as the independent variable. Using $v(\epsilon_{osc})$ and $E=100$ v/cm, $\Omega$ (and hence $B$) was calculated. The right and left sides of the peak had to be calculated separately; they correspond to the two solutions to the quadratic equation obtained for $\Omega^2$ from equation (2-7).
Appendix D

Evaluation of the Magnetic Field Integrals for Rectangular Magnets

Case 1  Cylindrically Symmetric Array

For the geometry indicated in figure D-1

\[ \beta_{on} = \frac{B_r}{2\pi} \frac{q}{r_i} \int_{r_i}^{r_o} \int_{q/2}^{r_o} \frac{n \, dx' \, dy'}{(x' + iy')^{n+1}} \]

Since \( n = N + \pi T \) it will not be equal to 1. After two straightforward integrations this becomes

\[ \beta_{on} = \frac{-i B_r}{2\pi (n-1)} \left( \frac{1}{(x' + i\frac{a}{2})^{n-1}} - \frac{1}{(x' - i\frac{a}{2})^{n-1}} \right) \]

evaluated at the limits \( x' = r_i \) and \( x' = r_o \). The two terms have the same magnitude but different complex phases.

\[ \beta_{on} = \frac{-i B_r}{2\pi(n-1)} \left( \frac{1}{(x')^2 + \left(\frac{a}{2}\right)^2} \right)^{(n-1)/2} \left[ e^{-i(n-1)\tan^{-1}(a/(2x'))} - e^{i(n-1)\tan^{-1}(a/(2x'))} \right] \]

This can immediately be combined to give the final result

\[ \beta_{on} = \frac{-8r}{2\pi(n-1)} \left( \frac{\sin[(n-1)\tan^{-1}(\frac{a}{2x'})]}{((x')^2 + (\frac{a}{2})^2)^{(n-1)/2}} \right) \]

which when evaluated between \( x' = r_i \) and \( x' = r_o \) gives equation (4-16).
Fig. D-1 Integration Geometry for Case 1.
Case 2  Finite Length Rectangular Magnets

First evaluate the magnetic field integral

\[ B = -\frac{B}{4\pi} \sum_s \oint \frac{A \cdot n_s}{|r - r'_s|} \, d^2r'_s \]

Choose as a relative coordinate system one where \( \alpha \) is a multiple of \( \pi/2 \) (i.e., either \( x \) or \( y \) is normal to any side) (see Fig. 0-2 for geometry). The integrals for each of the four surfaces are so similar that only one need be done, the others can be generated by properly switching \( x \) and \( y \) or changing limits, and by including the correct \( b_n \) for each one.

Therefore consider the top face, \( \hat{n} = \hat{x} \).

\[ \oint \frac{((x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}) dy'dz'}{((x-x')^2 + (y-y')^2 + (z-z')^2)^{3/2}} \]

For simplicity replace \( x-x' \) by \( x \), do similarly for \( y \) and \( z \), and use the notation

\[ r = \sqrt{x^2 + y^2 + z^2} \]

Integrating out \( dy' \) leaves

\[ \int \left( \frac{xy \hat{x}}{r(x^2+z^2)} - \frac{y \hat{y}}{r} + \frac{zy \hat{z}}{r(x^2+z^2)} \right) \, dz \]

The last two integrals are elementary the first one is a little more difficult

\[ \int \frac{yz \, dz}{r(x^2+z^2)} = \int \frac{yrdr}{r(r^2-y^2)} = \frac{1}{2} \int \left( \frac{1}{r-y} - \frac{1}{r+y} \right) \, dr = \frac{1}{2} \log \frac{r-y}{r+y} \]

\[ -\int \frac{dz}{r} = -\log(z+r) \]
Fig. D-2 Integration Geometry for Case 2.
Using the algebraic sign function

\[ Sgn(w) = \begin{cases} 
1 & \text{if } w \geq 0 \\
-1 & \text{if } w < 0
\end{cases} \]

the first integral becomes

\[
\int \frac{xydz}{r(x^2 + z^2)} = \int \frac{xy}{r(r^2 - y^2)} \frac{Sgn(z)dr}{\sqrt{r^2 - x^2 - y^2}}
\]

\[
= Sgn(z) \frac{x}{2} \int \left( \frac{1}{r-y} + \frac{1}{r+y} \right) \frac{dr}{\sqrt{r^2 - x^2 - y^2}}
\]

\[
= \frac{1}{2} Sgn(zx) \left[ -\cos^{-1}\left( \frac{y(r-y) - x^2}{(r-y)x^2 + y^2} \right) + \cos^{-1}\left( -\frac{y(r+y) + x^2}{(r+y)x^2 + y^2} \right) \right]
\]

\[
= \frac{1}{2} Sgn(xyz) \cos^{-1}\left( \frac{x^2 - y^2 z^2}{(x^2 + z^2)(x^2 + y^2)} \right)
\]

For this single face the field is

\[
B_x = -\frac{b_x}{4\pi} \left[ \frac{1}{2} Sgn(xyz) \cos^{-1}\left( \frac{x^2 - y^2 z^2}{(x^2 + z^2)(x^2 + y^2)} \right) \right]
\]

\[
B_y = -\frac{b_x}{4\pi} (-\log(z+r))
\]

\[
B_z = -\frac{b_x}{4\pi} \left( \frac{1}{2} \log\left( \frac{r-y}{r+y} \right) \right)
\]

As was already mentioned the other three faces can easily be determined from this and the result is as in eq. 4-20.

Now the other integral

\[
s = -\frac{B}{4\pi} \sum \int \int (\hat{b} \times \hat{z}) \cdot \hat{n} \frac{d^2 r'}{|r-r'|}
\]
Again only one of the four need be considered

\[
\mathcal{S} = \int \int \frac{dy' \, dz'}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} = \int \int \frac{dy \, dz}{r} = \log (y+r) \, dz
\]

using the previous notation. This last integral could not be found in any table. Starting from similar integrals a solution was found by trial and error.

\[
\log(y+r)dz = z \log(y+r) + y \log(z+r)
\]

\[
+ |x| \, \text{Sgn}(z) \cos^{-1} \left( \frac{x^2 + y^2 + yr}{\sqrt{x^2 + y^2} (y+r)} \right) - z
\]

This was verified by differentiation with careful consideration taken of the algebraic signs of \(x, y\) and \(z\). So for this face

\[
s = -\frac{b}{4\pi} \left[ z \log (y+r) + y \log (z+r)
\right.

\[
+ |x| \, \text{Sgn}(z) \cos^{-1} \left( \frac{x^2 + y^2 + yr}{\sqrt{x^2 + y^2} (y+r)} \right) - z
\]

Again by proper permutations the other three can easily be generated and the results is as in equation 4-21.
References

6. Figure 1-3, courtesy of K. Ehlers.
26. J. Hiskes and A. Karo, UCRL-87779, (Lawrence Livermore National Laboratory, 1982).


46. The magnetic field calculations in chapter IV are in rationalized MKSA units.


48. Ibid., pg. 194.

51. The development for Case 1 follows very closely that presented in ref. 50 sect. 4.
52. This arrangement were suggested by K. Halbach.
55. ANSI C95-1, 1982.
59. C. F. Chan, LBID-632 (Lawrence Berkeley Laboratory, 1983).
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