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Authors
Schlueter, R.D.
Halbach, K.

Publication Date
1993-09-01
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R. Schlueter and K. Halbach

September 1993

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098
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Skew Harmonics Suppression in Electromagnets with Application to the Advanced Light Source (ALS) Storage Ring Corrector Magnet Design*

R. Schlueter and K. Halbach

Advanced Light Source
Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

September 1993

Paper presented at the 13th International Conference on Magnet Technology
Victoria, B.C., Canada
September 20-24, 1993

*This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy, under Contract No. DE-AC03-76SF00098.
Skew Harmonics Suppression in Electromagnets with Application to the Advanced Light Source (ALS) Storage Ring Corrector Magnet Design

R.D. Schluter and K. Halbach
Lawrence Berkeley Laboratory, Berkeley, California 94720

Abstract - An analytical expression for prediction of skew harmonics in an iron core combined function regular/skew dipole magnet due to arbitrarily positioned electromagnet coils is developed. A structured approach is presented for the suppression of an arbitrary number of harmonic components to arbitrarily low values. Application of the analytical harmonic strength calculations coupled to the structured harmonic suppression approach is presented in the context of the design of the ALS storage ring corrector magnets, where quadrupole, sextupole, and octupole skew harmonics were reduced to less than 1.0% of the skew dipole at the beam aperture radius $r=3.0$ cm.

I. INTRODUCTION

Harmonics suppression, required in many accelerator physics magnet applications, is usually performed by shimming an iron dominated magnet. Conductor dominated magnets necessitate a different approach. Such is the case for the ALS combined function regular/skew dipole corrector magnets. The effect of field errors in the ALS storage ring on the electron beam's dynamic aperture has been analyzed [1] and harmonics suppression requirements for these magnets have been tabulated [2]: skew quadrupole $(sQ)$, sextupole $(sS)$, and octupole $(sO)$ components at $|r|=3.0$ cm are not to exceed 1.4%, 1.0%, and 1.4%, respectively, of the skew dipole $(sD)$.

II. ANALYTICAL EXPRESSIONS FOR SKEW HARMONIC COMPONENTS

Fig. 1a shows the gap portion of a wide $[z]$-direction magnet with current filaments of magnitude $\pm I$ at $z_0$ and $z_0^*$, respectively, giving rise to a skew $[z]$-direction dipole. The return paths for the currents on the outsides of the iron yoke are not shown. The width $[x]$-direction of the iron pole face yoke is assumed effectively infinite in the analytical model. Exponential decay of field errors as one moves toward the origin from the corner of the iron pole insures that this approximation will not result in large errors of calculated harmonic components near the location of the beam axis (the origin in Fig. 1a).

<table>
<thead>
<tr>
<th>iron</th>
<th>regular/skew dipole magnetics</th>
<th>iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$f(z_0)$</td>
<td>$f(z)$</td>
</tr>
<tr>
<td>$t = \infty$</td>
<td>$\gamma^*(z_0)$</td>
<td>$\gamma^*(z)$</td>
</tr>
</tbody>
</table>

Fig. 1a. $z$ plane (electromagnet skew dipole geometry).
Fig. 1b. $t$ plane (Schwarz-Christoffel transformed geometry).

We shall use a Schwarz-Christoffel transformation to conformally map the gap region between $z=(x, \pm h)$ of Fig. 1a to the upper half $t$ plane shown in Fig. 1b. Choosing $t = \infty$ at $z = +\infty$ results in the transformation $dz/dt = 1/k$. Choosing $t = a = 0$ at $z = -\infty$ makes $1/k$ take on the value $2\pi$ since at $t = 0, \text{Im}(\Delta x) = \text{Im}(\int_{z_0}^{z} dt') = \frac{\pi}{k}$. Integrating with respect to $t$ gives $z = \frac{2\pi}{k} \ln t + c, \Rightarrow t = e^{\frac{\pi}{k}(z - 2)}$. Choosing $t = 1$ at $z = -ih$ makes $c$ take on the value $-ih$. The complete $t = z$ transformation is thus

$$z = \frac{2\pi}{k} \ln t - ih, \Rightarrow t = ie^\frac{\pi i}{k} = ie^\frac{\pi i}{k} = ie^\alpha,$$

where $k \equiv \frac{\pi}{k}$ and $\alpha \equiv k \pi$.

For the filament of magnitude $+I$ at $z_0$ and its image in the $t$ plane about the real axis, which makes the magnetic field perpendicular to the air-iron interface, the complex magnetic potential $F(t) \equiv A + iV$ (where $\nabla \times A = H, \nabla V = -H$) is given by $F(t) = \frac{1}{2\pi} \ln((t^* - t)(t - t^*))$.

The magnetic field is related to the complex potential by $H^*(z) = i \frac{dF}{dt}$. For this case we have

$$H^*(z) = \frac{1}{2\pi i} \frac{d}{dz} \left[ \frac{1}{e^\alpha - e^{\alpha}} + \frac{1}{e^\alpha + e^{\alpha}} \right].$$

(2)

For current filaments of magnitude $\pm I$ at $z_0$ and $z_0^*$, respectively, and their images in the $t$ plane about the real axis, we have for $H^*(z) \equiv H^*(z)/e^\alpha$:

$$\hat{H}^*(z) = \frac{e^\alpha}{e^\alpha - e^{\alpha}} + \frac{e^\alpha}{e^\alpha + e^{\alpha}} + \frac{-e^\alpha}{e^\alpha - e^{\alpha}} + \frac{-e^\alpha}{e^\alpha + e^{\alpha}} + \frac{1}{1 - \frac{1}{e^{\alpha}}} \frac{1}{\text{Sinh}(\alpha - a_0) - \text{Sinh}(\alpha - a_0')},$$

where the relations $t(z_0) = i e^{\alpha}, t(z_0^*) = i e^{\alpha}, t^*(z_0) = -i e^{\alpha}$, and $t^*(z_0^*) = -i e^{\alpha}$ have been used.
For a pair of current sheets of magnitude ±I' from \( z_1 \) to \( z_2 \) and from \( z'_1 \) to \( z'_2 \), respectively, and their images in the \( t \) plane about the real axis, we have for \( H^*(z) \equiv H^*(z) / 2\pi i \) by integrating the terms in (3) with respect to \( z_0 \) or \( z'_0 \), as appropriate:

\[
H^*(z) = \ln \left( \frac{e^{\alpha_0 - \alpha} - 1}{e^{\alpha_0 - \alpha} + 1} \right)_{\alpha = \alpha_0}^{\alpha = \alpha_0'} - \ln \left( \frac{e^{\alpha_0 - \alpha} - 1}{e^{\alpha_0 - \alpha} + 1} \right)_{\alpha = \alpha'_0}^{\alpha = \alpha_0}.
\]

We now decompose these expressions for \( H^*(z) \) into harmonic components. Define \( G(z) \equiv \frac{1}{\sinh(z)} \). Then,

\[
G(\alpha - \alpha_0) = \sum_0 \alpha^n a_n
\]

where

\[
\frac{\partial^n}{\partial \alpha^n} \left( \frac{1}{\sinh(\alpha - \alpha_0)} \right) = (-1)^{n+1} G^n(\alpha_0)
\]

and where the first equality in (6) is true for any function \( f(\alpha - \alpha_0) \) that can be expanded in the power series \( \sum_0 a_n \). The second equality in (6) is true whenever \( f(\alpha - \alpha_0) \) is an odd function, and where the exponent \( n \) in \( G^n(\alpha_0) \) refers to differentiation with respect to \( \alpha_0 \).

Therefore, the multipole components for a pair of current filaments of magnitude ±I at \( z_0 \) and \( z'_0 \), respectively, are given by

\[
H^*(z) = \frac{I'}{\pi} \sum_0 \alpha^n \left( -1 \right)^{n+1} I m \left\{ \frac{\partial^n G(\alpha_0)}{\partial \alpha^n} \right\} = \frac{I'}{\pi} \sum_0 \alpha_n \left( -1 \right)^{n+1} \left\{ 2i \int \left[ \frac{\partial^n G(\alpha_0)}{\partial \alpha^n} \right] \right\}. \quad (7)
\]

Thus, the multipole components for a pair of current sheets of magnitude ±I' from \( z_1 \) to \( z_2 \) and from \( z'_1 \) to \( z'_2 \), respectively, the multipole components are given by

\[
H^*(z) = \frac{I'}{\pi} \sum_0 \alpha^n \left( -1 \right)^{n+1} I m \left[ \frac{\partial^n G(\alpha_0)}{\partial \alpha^n} \right] = \frac{I'}{\pi} \sum_0 \alpha_n \left( -1 \right)^{n+1} \left\{ 2i \int \left[ \frac{\partial^n G(\alpha_0)}{\partial \alpha^n} \right] \right\} = \sum_0 \alpha^n \left( -1 \right)^{n+1} \left\{ 2i \int \left[ \frac{\partial^n G(\alpha_0)}{\partial \alpha^n} \right] \right\}. \quad (8)
\]

where \( S \equiv \sinh(\alpha_0) \) and \( C \equiv \cosh(\alpha_0) \).

For a pair of current sheets of magnitude ±I' from \( z_1 \) to \( z_2 \) and from \( z'_1 \) to \( z'_2 \), respectively, the multipole components are given by

\[
H^*(z) = \frac{I'}{\pi} \sum_0 \alpha^n I m(kb_n), \quad (9)
\]

where \( \alpha \equiv k z = \frac{z_2 - z_1}{\lambda} \) and

\[
kb_n = \int_{\alpha_0}^{\alpha_0'} \frac{1}{\sinh(\alpha_0')} d\alpha_0 = - \ln \left( \frac{e^{\alpha_0} - 1}{e^{\alpha_0} + 1} \right)_{\alpha_0}^{\alpha_0'}, \quad \text{and} \quad kb_n = \left( -1 \right)^{n+1} \frac{n!}{\partial \alpha^n} \left( \frac{1}{\sinh(\alpha_0')} \right)_{\alpha_0'}^{\alpha_0}, \quad n \geq 1. \quad (10)
\]
A base arrangement of coil packets is selected, from which harmonics may be calculated according to (9) by superposing contributions from each packet at its respective location. Let \( x_{i \text{base}} \) = the base \( x \)-coordinate of the center of packet \( i \), for \( i = 1, 2, 3 \). An \( i \times j = 3 \times 3 \) matrix \( M \) calculated from \( \frac{d}{dx} \) of (9) relates changes in position of the \( i \) packets to changes in the \( j \)-length solution vector \( \vec{H} \) consisting of \( sQ, sS, \) and \( sO \):

\[
\vec{H} = M \Delta \vec{x} + \vec{H}_{\text{base}}
\]  

where \( \vec{H}_{\text{base}} \) is comprised of \( sQ, sS, \) and \( sO \) harmonic strengths when coil packets are at their base positions and \( \Delta \vec{x} \) are perturbations of packets from those positions. A [hopefully improved] guess at a desired coil arrangement is found from inverting the matrix:

\[
\vec{z}_{\text{new}} = \vec{z}_{\text{base}} + \Delta \vec{x}, \quad \Delta \vec{x} = M^{-1}(\vec{H}_{\text{desired}} - \vec{H}_{\text{base}})
\]  

Nonlinearities over coil packet position perturbations \( \Delta \vec{x} \) necessitate iteration. Several points worth noting:

1. The \( \vec{z}_{\text{new}} \) may move toward a local maximum, in which case a converged solution may not be possible for the chosen \( \vec{z}_{\text{base}} \). The curves in Fig. 3 will indicate this.
2. There may be no solution possible for a chosen set of coil packet widths regardless of \( \vec{z}_{\text{base}} \). A set of widths for which a solution does exist can be selected using Fig. 3.
3. Operating near a local maximum is to be avoided only if one is depending on the perturbation of that particular packet to significantly change that particular harmonic.
4. One must insure that matrix element variations over packet excursions from base positions will not cause \( M \) to become singular, and thus the problem, intractable. Experimental uncertainties and the effect of matrix element variations on the condition of \( M \) are discussed in [3].
5. Higher order harmonics may also be nulled following the techniques described herein [3].

B. ALS Storage Ring Corrector Magnets

1. Coil Design

Experimental results of harmonics strength dependence on ALS corrector magnet coil packet position for packets of various widths compared with its 2-D analytical counterpart confirmed that (9) models the observed harmonics behavior very well. This is true because field errors decay exponentially as one moves inside the magnet from the corners of the iron poles. Minor differences are attributable to 3-D fringe fields and to C-shape asymmetry, which has the effect of making the magnitudes of experimentally measured positive and negative peaks of the \( sQ \) vs. \( x \) plot (cf. Fig. 3) differ by \( \sim 16\% \).

The coil design outline follows:

- The 24 turns of width 7.85 mm/turn in each of four layers make total wound coil width 188.4 mm, leaving 31 mm of play for coil positioning within the 220 mm mold gap.
- Three independently positionable coil packets provide the degrees of freedom to null \( sQ, sS,\) and \( sO \).
- To null \( sQ \), the 24 coils were initially split 13/11, rather than positioning equally-split coils far off-center and suffering a decrease of the fundamental’s magnitude. For the base arrangement, the 24 coils were further split into three packets 13/3/8, centered at \( x = 55.0 \) mm, 23.6 mm, and 66.8 mm. Experimentally obtained \( sQ, sS, \) and \( sO \) were \( +1.6\%,-1.5\%\), and \( -1.0\%\), respectively. Additionally, this base position features a well-conditioned matrix \( M \) whose elements have units of \( \text{H/mm packet displacement from base position} \):

\[
M = \begin{pmatrix}
-0.005 & 0.006 & -0.0075 \\
0.013 & -0.014 & 0.0002 \\
-0.007 & -0.009 & -0.0004
\end{pmatrix}
\]  

Units of \( M \) can be converted to \( \% \ sQ/mm packet displacement from base position \), using the normalized skew dipole strength of ‘4.0’. Packets #2 and #3 abut in this base position. However, \( sQ \) is positive (by design), so iterations drive the 3\textsuperscript{rd} packet in the positive direction from its base position to null \( sQ \). Simultaneous change in \( sS \) is minimal since \( M_{2,2} \) is small. Since \( M_{2,2} \) is large, a smaller positional perturbation of the 2\textsuperscript{nd} packet nulls \( sS \), without overlapping the physical space occupied by the 3\textsuperscript{rd} packet.

- Packet base positions and experimentally obtained values for \( sQ, sS, \) and \( sO \) are input into a matrix solver code which inverts \( M \) and performs the operations given by (11) to give a predicted positioning of the packets which will null \( sQ, sS, \) and \( sO \).

- The output configuration is experimentally tested and the three components are brought to \( \leq 0.25\% \) of the fundamental in just two iterations. The final configuration comprises packets centered at -56.6 mm, 18.6 mm, and 66.8 mm, yielding experimentally obtained values for \( sQ, sS, \) and \( sO \) of \( -0.02\%, -0.13\%, \) and \( -0.10\% \), respectively.

- Corrector magnets do not operate in isolated environments; nearby iron structures slightly affect skew harmonics. A final, single corrector magnet design was chosen to accommodate the various anticipated environments.

2. Tolerances

- \( x, y, \), and \( Z \)-positioning Displacements. The effect of a millimeter \( x \)-displacement of a given coil packet on \( sQ, sS, \) and \( sO \) is given by appropriate elements of \( M \) in (13) or alternatively, is calculable using (9). Likewise, the effect of a coil stack-up displacement in the \( y \) direction on \( sQ, sS, \) and \( sO \) is calculable using (9). Experiments of coil packet \( Z \)-position displacements with respect to the iron core showed virtually no effect on harmonics over the range of coil movement possible \( \pm 0.127 \) cm (0.050").

3
Fig. 4. Positioning error budget and design tolerances.

Tolerancing Implications. In Fig. 4 harmonics from +1 mm x, y, and Z positioning errors and coil width errors are tabulated. The total positioning errors budget (0.25% of sD for each harmonic) is allocated to distribute the mechanical assembly difficulty uniformly. Tolerances required to meet the error budget for each harmonic are tabulated. For the coil packet width error, two scenarios are investigated: registering from packet centers x<sub>c</sub>, and from packet inside edges. The latter approach is desired, as sQ rather than sS sets a relaxed packet width tolerance. Important error figures and suggested tolerances are found in columns e, p, u, and v for x<sub>c</sub>, w<sub>c</sub>, y<sub>c</sub> and Z<sub>c</sub>, respectively. The sS-limited packet x<sub>c</sub> position tolerance of .009" (column e) assumes x<sub>c</sub> positionings of the three coil packets are independent. The sQ-limited allowable coil width w, variation of .012" (column p) for the second coil packet (having three turns) assumes packet widths will be proportionately at variance from the ideal and that the design approach fixing inner x-positions of packets is employed. Tolerances on w<sub>c</sub> for the two larger coil packets scales proportionally with their respective x-direction widths. The sS-limited allowable coil y<sub>c</sub> position variation of .012" (column u) assumes that the three coil packets' y<sub>c</sub> positions will be uniformly at variance from the ideal. Harmonics introduced by Z<sub>c</sub> position errors (column v) are insignificant.

Rotating Coil Measurement System Alignment.

On-axis harmonics a<sub>n</sub> given by H<sup>*</sup>(z) = Σ<sub>n=0</sub><sup>N</sup>a<sub>n</sub>z<sup>n</sup> are suppressed below 1% of a<sub>0</sub> for n ≤ 3 as measured by a rotating coil which defines the on-axis position. If subsequently, the coil is off-axis by Δ<sub>x</sub>, where w = z − Δ<sub>x</sub>, then measured coefficients b<sub>n</sub> defined by H<sup>*</sup>(w) = Σ<sub>n=0</sub><sup>N</sup>b<sub>n</sub>w<sup>n</sup> can be related to the a<sub>n</sub> by expanding H<sup>*</sup>(z) in w:

H<sup>*</sup>(z) = Σ<sub>n=0</sub><sup>N</sup>a<sub>n</sub>z<sup>n</sup> = Σ<sub>n=0</sub><sup>N</sup>b<sub>n</sub>(w + Δ<sub>x</sub>)<sup>n</sup> = Σ<sub>n=0</sub><sup>N</sup>{w<sup>n</sup>Σ<sub>m=n</sub><sup>∞</sup>(a<sub>m</sub>Δ<sub>m−n</sub>/m!)m！} ≡ Σ<sub>n=0</sub><sup>N</sup>w<sup>n</sup>b<sub>n</sub>  

The b<sub>n</sub>'s can be thought of as measured values of the a<sub>n</sub>'s, in error due to the Δ<sub>x</sub> coil misalignment. We budget for a coil misalignment-induced error for the n<sup>th</sup> harmonic of f<sub>n</sub>% of a<sub>0</sub> at |r| = 3 cm. Defining e ≡ Δ<sub>x</sub>/r and p<sub>n</sub> ≡ e<sup>n</sup>a<sub>n</sub>/a<sub>0</sub>, we have

f<sub>n</sub> ≥ r<sup>n</sup>|b<sub>n</sub> − a<sub>n</sub>| = r<sup>n</sup>Σ<sub>m=n+1</sub><sup>∞</sup>(a<sub>m</sub>Δ<sub>m−n</sub>/m!)m！ 

= Σ<sub>m=n+1</sub><sup>∞</sup>(m!p<sub>m</sub>e<sup>−m</sup>/(m − n)!)l!  

(15)

For the storage ring corrector magnets, at |r| = 3 cm, p<sub>n</sub> ≤ 0.01 for n = 1, 2, 3, p<sub>n</sub> ≤ 0.03 for n = 4, 5, 6 and p<sub>n</sub> ≈ 0 for n > 6. Thus if f<sub>n</sub> = 0.2% for n = 1, 2, 3, allowable ε<sub>max</sub> = 0.016 ⟹ Δ<sub>x</sub> ≤ 0.048 cm (0.019"), at which coil misalignment-induced sQ, sS, and sO are 0.03%, 0.05%, and 0.20% of sD, respectively. If we choose f<sub>n</sub> = 0.12% for n = 1, 2, 3 allowable ε<sub>max</sub> = 0.016 ⟹ Δ<sub>x</sub> ≤ 0.030 cm (0.012"), at which coil misalignment-induced sQ, sS, and sO are 0.02%, 0.03%, and 0.12% of sD, respectively.

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