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LONGITUDINAL DISPERSION IN PROCESS EQUIPMENT

Theodore Vermeulen

December 1963
LONGITUDINAL DISPERSION IN PROCESS EQUIPMENT

by Theodore Vermeulen, Ph.D. #

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Abstract

The effects of longitudinal dispersion upon equipment performance can be calculated by several available methods, among them the diffusion model indicated here. Countercurrent systems and packed-bed operations are discussed, with reference to data from the author's laboratory. Different types of dispersion are seen to occur in the laminar and turbulent flow regimes. The need is expressed for additional experimental work, and for new correlations of existing performance data after correction for longitudinal dispersion.

# The work described was done in the Lawrence Radiation Laboratory, supported by the U. S. Atomic Energy Commission, and contributed to by T. Miyauchi, G. L. Jacques, C. R. Wilke, J. E. Cotter, N. N. Li, T. T. Word, W. E. Dunn, A. N. Hennico, and J. S. Moon.
"Longitudinal dispersion" or "axial mixing" describes the effective sum of all departures from ideal plug flow. The true flow of a liquid or gas through a pipe or other flow path can often be described as the net result of an assumed plug-flow behavior with a superimposed longitudinal dispersion. Mathematical analysis on this basis provides a one-dimensional approximation for behavior that may actually be two- or three-dimensional, perhaps with statistical fluctuations. In terms of our existing ability to compute and correlate, the one-dimensional analysis permits a necessary, and hopefully an adequate, advance in accuracy over the more accessible plug-flow approach.

Longitudinal dispersion needs to be considered in "miscible displacement" of one fluid by another, as in oil-bearing sands or in product pipelines; in continuous-flow chemical reactors; and in countercurrent heat-transfer and mass-transfer equipment, which will be the principal topic of the present discussion. Description of longitudinal dispersion has two parts: mathematical relation between the extent of "mixing" and the physical result that is measured or predicted; and correlation of the parameter measuring the extent of dispersion, which normally is considered to have a single average value for an entire apparatus.

Definitions

Several abstract models of longitudinal dispersion have been proposed. These include:

1. Diffusion.
2. Random walk (in a forward direction only).\(^1\)
3. Filament flow; for instance, Taylor diffusion for laminar flow in pipes.
4. Mixing cells in series, with or without backflow between cells.

In all these models, a "number of dispersion units" (NDU) or "column Péclet number" (PB) can be identified that increases with the length of the apparatus. For long flow conduits the NTU values given by the different models all tend to become proportional to the length, and hence to approach the same degree of validity.
Because of this asymptotic similarity between models, it is convenient to use a **diffusional** terminology even if another model applies more directly to the particular system. Thus we define a "mixing length", \( l \), such that

\[
(NDU) = \frac{h}{l}
\]

where \( h \) is the total height or length of the apparatus. The mixing length relates the longitudinal-dispersion coefficient \( E \) to the mean velocity.

\[
E = \frac{l}{U}
\]

In a packed column, a superficial dispersion coefficient \( E_0 \) is \( l \) times the superficial velocity \( U_0 \), and a linear coefficient \( E_\ell \) is \( l \) times the mean linear velocity \( U = \frac{U_0}{\varepsilon} \), with \( \varepsilon \) the void-fraction. A local Péclet number \( P \) can also be defined, relative to a characteristic dimension \( d \) (usually \( d_w \), the wall-to-wall diameter, in an unpacked column; or \( d_p \), the volume-average spherical diameter in a packed column):

\[
P = \frac{d}{l} = \frac{d}{U/E}
\]

(A "longitudinal-dispersion group" has also been defined that is the reciprocal of \( P \).\(^{12} \))

If the column length is expressed in dimensionless units based on \( d \) — that is, in terms of \( h/d \), or \( B \) — the NDU (or column Péclet number) for single-phase flow can be expressed as

\[
(NDU) = \frac{h}{U/E} = \frac{P}{h/d} = \frac{PB}{E}
\]

The larger the NDU is, the more closely the system approaches ideal plug-flow behavior. For most purposes, longitudinal dispersion is a disadvantage. Hence a small value of \( P \) or of NDU can be viewed as unfavorable, and a large one as desirable.

Relative to the diffusion model, the cell model gives a number of mixing cells \( (n_c) \) approximately half the NDU. Thus the length of a cell \( L_c \) is about twice the mixing length: \( L_c \approx 2l \).
**Single-Phase Flow**

The diffusion equation, for any one component in a homogeneous fluid phase, has the general one-dimensional form:

\[
dc/dt + Ed^2c/dz^2 - Udc/dz + \Phi = 0
\]

(5)

where \( \Phi \) is the rate of depletion by reaction or material-transfer. This equation can be simplified in two ways, each for a different application. The first involves transient response to a changed input level; here \( dc/dt \) is of major interest, and \( \Phi \) vanishes. The second involves steady-state behavior of a reacting or transferring component, for which \( dc/dt = 0 \) and \( \Phi \) is expressed as a function of \( c \).

In dimensionless units, for a first-order rate process, Equation 5 becomes

\[
(PB)^2dX/d\theta + d^2X/dZ^2 - PB dX/dZ - N_0 PB(X-X^*) = 0
\]

(5a)

or

\[
dX/d\theta + (1/\bar{E}^2) d^2X/db^2 - (1/\bar{E}) dX/db - (N_0/PB)(X-X^*) = 0
\]

(5b)

where \( \theta = tU/l \); \( Z = z/h \); \( B = z/d \); and \( N_0 = \kappa h/U \), the "true" overall number of transfer (or reactor) units. For heat-transfer problems \( X \) can be considered a dimensionless temperature, running from zero at the initial bed temperature to unity at the final value. For material-transfer or chemical-reaction problems, \( X \) is a dimensionless concentration, generally \( c/c_0 \) (with \( c_0 \) the maximum, or feed, value). Also, \( X^* \) is the local equilibrium-limit value for \( X \).

**Unsteady State**

The transient problem applies to miscible displacement, and to unsteady-state methods of measuring the longitudinal-dispersion coefficient \( E \) by injection and observation of heat or of a tracer species. For the \( F \) curve (in Danckwerts's nomenclature), or response to a step-function input from \( c_1 \) to \( c_0 \), the exact solution has been given by Miyauchi and Brenner. An approximate solution due to Danckwerts and others is

\[
X = \frac{c - c_1}{c_0 - c_1} = \frac{1}{2} \left\{ 1 + \text{erf} \left[ \frac{(PB + 1)T - PB}{2[(PB + 1)T]^{1/2}} \right] \right\}
\]

(6)

with \( T = \theta/PB = t U/h \). The family of breakthrough curves calculated from Equation 6 is shown in Figure 1. Similar, but not
identical, sets of curves are obtained for the exact diffusion solution and for each of the other models referred to above.

Steady State

The extent of steady-state removal of a solute from the phase of interest, for the case of an irreversible change ($X^* = 0$), is related to the effective number of piston-flow (plug-flow) transfer units by the appropriate integration of Equation 5a:

$$X = \exp \left[ -N_0 \tilde{\rho} \right]$$

(7)

Allowance for longitudinal dispersion, with use of Equation 5a, gives the relation

$$X = \frac{2q e^{\frac{1}{2}(\tilde{\rho}B + q)}}{(2N_0 + \tilde{\rho}B + q)e^q - (2N_0 - \tilde{\rho}B + q)}$$

(8)

with $q = [(\tilde{\rho}B)^2 + 4N_0\tilde{\rho}B]^{1/2}$. For perfect mixing ($\tilde{\rho} = 0$), this reduces to

$$X = \frac{1}{1 + N_0}$$

(8a)

Figure 2 shows the relation between $N_0\tilde{\rho}$, or $X$, and the various possible combinations of $N_0$ and $\tilde{\rho}B$.

Epstein has related the performance (in terms of the residual concentration, taken here as $X$) to the number of mixing cells ($n_c$) and the number of transfer units per cell ($N_0/n_c$). Hofmann and Schoenemann have plotted curves of $X$ against $\epsilon h/U$ or $\epsilon h/U n_c$, for series of equal-sized cells in which a first-order reaction occurs; these curves are applicable to the present problem. Elsewhere, Miyauchi has shown that $\tilde{\rho}B$ and $n_c$ can be related empirically:

$$\tilde{\rho}B = 2(n_c + \frac{1}{n_c} - 2)$$

(9)

For the limiting condition of perfect mixing, with $n_c = 1$, Equation 9 gives $\tilde{\rho}B = 0$ as expected. Also, at asymptotically large values, $\tilde{\rho}B \approx 2n_c$.

Flow in Pipes

Sir Geoffrey Taylor has provided much of the background for our knowledge of axial dispersion in pipes. Distinctly different mechanisms apply in the laminar and turbulent regimes of flow.
In laminar flow, velocity distribution is the cause, and its effect is reduced in the event molecular diffusivity produces appreciable transverse mixing between slow and fast filaments. For a circular cross-section, the limiting breakthrough for lengths of pipe short enough to have no appreciable effect of molecular diffusivity (but long enough for a parabolic velocity profile to be fully established) is given by

$$X = 1 - \frac{1}{4T^2}$$

(10)

This result is seen to be independent of the pipe length; and it corresponds to a value of $\bar{P}_B$ of about 12, evaluated for the diffusion model by matching the midpoint slope $dX/d(t/t_X=0.5)$. Where transverse diffusion controls, larger values of $\bar{P}_B$ are encountered; in such cases the step-input response corresponds more directly to the diffusion model, the dispersion coefficient being given by $E = U^2d_w^2/192D_m$ with $D_m$ the molecular diffusivity.

For turbulent flow, eddy "diffusion" provides the mechanism for the dispersion process. Taylor's work and other experimental studies have been analyzed by Levenspiel with the conclusion that $\bar{P}$ (or $d_w/l$) is generally between 3 and 4.

**Flow in Packed Beds**

The transition from laminar to turbulent behavior is less well defined in packed-bed flow phenomena; but it nevertheless produces striking changes in both axial- and radial-dispersion behavior, especially for liquids. The transition occurs in the range of $U_0d_p/\nu$ values between 20 and 200, as indicated also by breaks in the curves for friction factor and for the $j$-factor in heat and mass transfer; and by studies of wakes behind individual spheres.

The essential difference in flow through interstices of the packing is shown schematically in Figure 3. In the laminar regime, individual filaments retain their identity through channels of continually varying cross-section and thus (unlike those in a circular pipe) tend to vary in their velocity from one point to another in their flow path. Each void space is considered to
have many different "filaments" passing through it, with an essentially similar pattern of behavior occurring in every such space. Radial dispersion occurs in the migration of the filaments as they separate around each successive obstacle in their longitudinal path. Likewise, axial dispersion occurs due to the velocity distribution (as in Taylor diffusion); but its effect is gradually reduced through an interchange of fast-moving and slow-moving filaments, that is equivalent to remixing.

A simplified physical model based on the velocity changes of the individual filaments, termed "segmented laminar flow", is sketched in Figure 4. The width depicted for each entire diagram is of the order of one void space, but the total height may be equivalent to 40 or 50 particle diameters. A filament is considered to enter a new "segment" each time its velocity crosses the mean velocity. If the distribution of velocities is the same at each cross section of the bed, one or more other filaments will be reduced in velocity at the cross section where a particular filament changes from a slow-moving to a fast-moving segment. In the idealized model, the random spacing of velocity-interchanges is replaced by a uniformly spaced sequence; with complete mixing occurring at each segment boundary, no mixing or radial diffusion between, and each filament having a constant velocity determined by a suitable velocity-distribution function within the entire segment. A velocity function that gives a good fit to experimental data has not yet been found, but the model seems to provide a realistic description of liquid flow in the laminar range.

For turbulent flow each void space appears to function as a nearly perfect mixing cell, so that the cell model probably provides the best physical description of this case. Because of transverse molecular diffusion in gases, the laminar-range dispersion coefficient is nearly unchanged from the turbulent-range value.

The results of measurements in liquids, made at Berkeley over an extended period, are shown in Figure 5. The experimental data were all from the step-input response, using dilute aqueous NaN0₃ solution as tracer, and monitoring the concentration downstream through its electrical conductivity. The experimental columns were about 6 inches in diameter and 26 inches high. The data were evaluated from the diffusional (or, equivalently, random-
walk) model for dispersion behavior. The very low Reynolds-number range has been explored in a search for effects of transverse diffusion that might relate liquid behavior more closely to that of gases. No such effect is observed; the possibility is being investigated that a rise in $P$ tends to occur due to transverse diffusion, but is obscured by compensatory effects of channeling arising from non-uniform packing density or orientation. It appears that gas-phase behavior has not been explored with quite the same degree of completeness as for liquids in the laminar range; on the other hand, the turbulent-flow range for liquids requires further study in order to refine the present accuracy.

It should be noted that the data of Figure 5 are somewhat controversial. The numerous investigators of axial dispersion have not obtained exact agreement either with one another or with the present data$^{17-21}$. Virtually all workers agree as to the presence of the turbulent-to-laminar transition; but they differ regarding the extent of the Péclet-number drop and also the precise Reynolds-number range in which it occurs.

Measurements on different packings have been brought together, vertically and horizontally, by introducing empirical functions of the void-fraction $\epsilon$. The correction used on the ordinate (multiplying the Péclet number by $\epsilon$) was first proposed by Ebach and White$^{18}$. The Péclet number for turbulent flow through a bed of randomly packed spheres (at $\epsilon = 0.4$) is around 2.3; this is equivalent to a mixing length of around $0.44 \, d_p$, or to a mixing-cell length of $0.88 \, d_p$, which corresponds almost exactly to the average thickness of one layer of packing. For laminar flow, at $\epsilon = 0.4$, the Péclet number of 0.52 corresponds to 4.3 layers of packing and confirms that the mixing-cell model can not apply in this range.

Radial-dispersion behavior should be noted here, since it provides further confirmation of the transition in flow regimes. For sphere-packed beds in turbulent flow$^{20}$ or gas flow$^{22,23}$, the radial Péclet number $(d_p \, U_0 / E_r)$ is about 4; for laminar liquid flow$^{20}$, about 12. If considered in terms of "mixing lengths", the former is seen to be consistent with complete remixing in every void, and the latter with a geometrical spreading of intact filaments.
Special Cases

Pulsed packed beds have recently been investigated at Berkeley with the objective of comparing both longitudinal dispersion and material-transfer rates with those in unpulsed beds. The interesting result is that the axial-mixing length is often reduced by pulsing, due to the increased turbulence that is produced. Figure 6 shows experimental values for packing Péclet number, plotted against a "combined Reynolds number" in which the pulse velocity (2 \( \omega A \), with \( \omega \) the frequency and \( A \) the amplitude) is added empirically to the steady-state superficial velocity. At high steady-state velocity the pulsing has almost no effect; for zero net flow, the mixing length will become very large (\( P \rightarrow 0 \)).

Fluidized beds of particulate type have been studied by Cairns and Prausnitz, with water as the fluidizing medium. For particles of relatively low density (glass, \( \Delta \rho = 1.45 \)), as the water flow rate increases, the mixing length increases almost monotonically from the packed-bed value to the open-pipe valve. For particles of high density (lead, \( \Delta \rho = 10.3 \)) the mixing length increases more rapidly in the high-velocity fluidized range, and later decreases.

Two-Phase Countercurrent Flow

Data are reported here for both gas-liquid and liquid-liquid systems. In each phase of such systems, due mainly to velocity-distribution effects, the Péclet number is less than the corresponding single-phase value. Tracer-injection methods can be used, with tracers that do not transfer to the coexisting phase; however, the sampling or measurement of tracer breakthrough curves is generally less accurate because of interference from the second phase.

Liquid-liquid results for pulsed packed beds are qualitatively similar to those without pulsation.

Liquid-Liquid Systems: Unpulsed

Continuous Phase. Figure 7 shows the results of numerous measurements of step-input response, in water as the continuous phase. Kerosene, or in a few cases (open squares) diisobutyl ketone, was used as dispersed phase. Density-difference effects have not been studied, and may enter in the eventual correlation;
also, the effect of continuous-phase viscosity is inferred from dimensional considerations but has not been proved.

**Dispersed Phase.** Figure 8 shows Péclet-number values for the aqueous phase dispersed, mostly with kerosene as the continuous phase\(^7\). The values undergo very little drift with changes in velocity, and are usually substantially higher than the continuous-phase Péclet numbers. Much additional work is needed before a general correlation can be attempted.

**Gas-Liquid Systems**

**Liquid Phase.** Results for the air-water system, obtained in a 2-ft. by 6-ft. packed section with 1-in. Berl saddles and 1-in. and 2-in. Raschig rings\(^{26}\), are shown in Figure 9. Because the variation is relatively small, the choice of plotting coordinates is rather arbitrary.

**Gas Phase.** Similarly, gas-phase Péclet numbers for air-water are shown as a function of superficial liquid and gas velocities in Figure 10. These values are exploratory in nature, and a broader study is needed in order to arrive at a general parametric knowledge of the system behavior.

**Use of Péclet-Number Data for Design Purposes**

For accurate correlation and prediction of the performance of either countercurrent or cocurrent\(^{28}\) equipment, for absorption, extraction, or direct-contact or shell-and-tube heat exchange, it appears essential to account separately for axial dispersion and for the material- or heat-transfer behavior since these are dependent upon different influences of the physical variables. The first step in this accounting should be the reinterpretation of previous data on equipment performance, to correct for axial dispersion in those cases where it is significant. This study will need to be reinforced by further data on both the dispersion and the transfer operation.

Two-phase systems have been described theoretically in terms of the diffusion model\(^3\),\(^{27}\) and the mixing-cell model with back flow.\(^{10}\) The latter method can accommodate variations in transfer coefficients, dispersion coefficients, and equilibrium partition
ratios; but it requires the iterative use of a high-speed digital computer, and provides no direct insight into the relations between the parameters. The exact solution of the diffusion model, available in closed analytical form, also requires use of a computer; but here the available results have been put into an empirical framework that clarifies the influence of longitudinal dispersion.

The differential equation (Equation 5b, without the accumulation term) is written for each of the two phases. For the feed phase, the dimensionless concentration is \( X = \frac{c_x}{c_{x0}} \), with \( c_{x0} \) the feed value; and the rate term is \( N_{ox} \alpha_x B (X-Y) \). For the solvent phase, for a solute-free incoming solvent with linear equilibrium behavior represented by the partition ratio \( m \), the dimensionless concentration is \( Y = \frac{mc_y}{c_{x0}} \), and the rate term is \( N_{ox} \Lambda \beta_y B (X-Y) \); here the dimensionless "extraction factor" or capacity ratio is given by \( \Lambda = m \frac{U_{ox}}{U_{oy}} \) with \( U_{ox} \) and \( U_{oy} \) the superficial velocities of the two phases. The solution for either concentration, at any location in the column, is obtained as a function of the four parameters \( N_{ox}, \Lambda, \beta_x B, \) and \( \beta_y B \).

Qualitatively the effect of longitudinal dispersion on counter-current absorption or extraction is shown in Figure 11, which compares plug-flow and axial-dispersion behavior for the same extent of extraction. In plug flow the concentrations in each phase vary smoothly from inlet to outlet. With axial dispersion, however, each concentration undergoes a "jump" at the inlet end if the diffusion model correctly applies. Also, the overall driving force for mass transfer is reduced, so that a larger \( N_{ox} \) will be required to achieve the same extraction \( (N_{ox} > N_{oxP}) \).

The outlet concentration in the raffinate phase, \( X_1 \), is re-expressed in terms of \( N_{oxP} \) by the Colburn relation \(^{29} \); for \( \Lambda \neq 1 \),

\[
N_{oxP} = \frac{1}{1-\Lambda} \ln \frac{1-\Lambda}{X_1} + \Lambda
\]

or, for \( \Lambda = 1 \),

\[
N_{oxP} = \frac{(1-X_1)}{X_1}
\]

A "number of overall dispersion units", \( N_{oxD} \), is now defined empirically to represent the difference between the reciprocals of the
apparent and true NTU's:

\[ \frac{1}{N_{oxD}} = \frac{1}{N_{oxP}} - \frac{1}{N_{ox}} \]  

(12)

Examination of the limiting case where \( N_{oxP} = N_{oxD} \) leads to adoption of the empirical form:

\[ N_{oxD} = \frac{\nu \Lambda}{\Lambda - 1} + \frac{\Lambda}{f_x P_x} + \frac{1}{f_y P_y} \]  

(13)

If \( \Lambda = 1 \), the fraction \( (\nu \Lambda)/(\Lambda - 1) \) reduces to unity. Here \( f_x \) and \( f_y \) are weighting factors, given only as functions of \( N_{ox} \) and \( \Lambda \) as shown in Figure 12.

\( N_{oxD} \), like \( N_{oxP} \), is a function of \( N_{ox} \), \( \Lambda \), \( P_x \), and \( P_y \); but it has a somewhat reduced dependence upon \( N_{ox} \). Splitting it into the terms of Equation 13 separates the effects of \( P_x \) and \( P_y \).

This framework allows explicit approximate calculations to be made in each of several directions. If \( X_1 \) is given, and \( \Lambda \), \( P_x \), and \( P_y \) are known, both \( N_{oxD} \) and \( N_{oxP} \) can be found, and \( N_{ox} \) then obtained through Equation 12. If \( N_{ox} \), \( \Lambda \), and one \( P \) are known, the other \( P \) can be solved for. Finally, if all four parameters are known through previous correlations, \( X_1 \) can be predicted as a step in process design.

If one of the two \( P \) terms is either zero or infinite, alternate exact methods of calculation are available for relating the remaining variables, and are to be preferred for reasons of accuracy.

**Applications**

**Liquid-Liquid.** Data of Colburn and Welsh\(^{30}\) have been selected for analysis, because the transfer resistances lie almost entirely in a single phase with the opposing phase providing an "infinite" reservoir. The system is isobutanol-water; each phase is initially free of the other, and the material-transfer resistance lies entirely in the respective "solvent" phase. For these cases, \( (1-Y) \) replaces \( X \) in Equations 7 and 8 and Figure 2. The latter is used with an experimental \( (1-Y) \) and a previously measured \( P \), to solve for the true \( N_{oy} \). The true HTU, obtained as \( \text{HTU} = h/N_{oy} \), is
plotted in Figure 13 along with the apparent HTU ($H_{oyp}$) obtained similarly from $N_{oyp}$. With the "solvent" being run as the continuous phase in each case considered here, it is seen that $H_{cp}$ varies from 1.1 to 1.8 times $H_C$. The average discrepancy in the dispersed phase will be substantially less.

**Gas-Liquid.** The desorption of oxygen by water, into an inert carrier gas, has been studied by Sherwood and Holloway $^{31}$. The resistance lies almost entirely in the liquid phase, and the desorption can be treated as irreversible. Hence Figure 2 again applies, with values of $E_xB$ obtained from the previously cited measurements $^{26}$. The difference between the true and apparent HTU, of the same magnitude as in the preceding example, is shown in Figure 14.

The magnitude of the corrections in these two examples shows that any accurate correlation of HTU or mass-transfer coefficients must be based on values that are corrected for axial-dispersion effects.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>linear amplitude of pulse in column (cm., or ft.)</td>
</tr>
<tr>
<td>B</td>
<td>h/d, (dimensionless)</td>
</tr>
<tr>
<td>c</td>
<td>concentration (gm.-moles/liter, or lb.-moles/cu. ft.)</td>
</tr>
<tr>
<td>d</td>
<td>characteristic length (cm., or ft.)</td>
</tr>
<tr>
<td>Dm</td>
<td>molecular diffusion coefficient (sq. cm./sec. or sq. ft./hr.)</td>
</tr>
<tr>
<td>E</td>
<td>axial-dispersion coefficient (sq. cm./sec. or sq. ft./hr.)</td>
</tr>
<tr>
<td>f</td>
<td>weighting factor in Equation 13 (dimensionless)</td>
</tr>
<tr>
<td>G</td>
<td>gas-phase mass velocity [(gm./sec.)/sq. cm., or (lb./hr.)/sq.ft.]</td>
</tr>
<tr>
<td>h</td>
<td>total height of column (cm. or ft.)</td>
</tr>
<tr>
<td>Ho</td>
<td>height of overall transfer unit (cm., or ft.)</td>
</tr>
<tr>
<td>k</td>
<td>rate coefficient for reaction or material transfer (1/sec., or 1/hr.)</td>
</tr>
<tr>
<td>l</td>
<td>mixing length (cm., or ft.)</td>
</tr>
<tr>
<td>L</td>
<td>liquid-phase mass velocity [(gm./sec.)/sq. cm., or (lb./hr.)/sq. ft.]</td>
</tr>
<tr>
<td>Lc</td>
<td>length of a mixing cell (cm., or ft.)</td>
</tr>
<tr>
<td>m</td>
<td>solute partition coefficient (dimensionless)</td>
</tr>
<tr>
<td>n_c</td>
<td>number of mixing cells (dimensionless)</td>
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<tr>
<td>No</td>
<td>number of overall transfer- (or reaction- or dispersion-) units (dimensionless)</td>
</tr>
<tr>
<td>P</td>
<td>local Peclet number; d/l (dimensionless)</td>
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<tr>
<td>PB</td>
<td>$PB = [1 + 4(N_0/PB)]^{1/2}$ (dimensionless)</td>
</tr>
<tr>
<td>q</td>
<td>rate of reaction or material transfer (gm.-mole/liter/sec., or lb.-mole/cu. ft./hr.)</td>
</tr>
<tr>
<td>t</td>
<td>time (sec., or hr.)</td>
</tr>
<tr>
<td>T</td>
<td>number of column void-volumes; t U/h (dimensionless)</td>
</tr>
<tr>
<td>U</td>
<td>flow velocity (cm./sec., or ft./hr.)</td>
</tr>
<tr>
<td>X</td>
<td>relative x-phase concentration, $c_x/c_x^0$ (dimensionless)</td>
</tr>
<tr>
<td>Y</td>
<td>relative y-phase concentration, $c_y/c_x^0$ (dimensionless)</td>
</tr>
<tr>
<td>z</td>
<td>length in column (cm., or ft.)</td>
</tr>
<tr>
<td>Z</td>
<td>relative length; z/h (dimensionless)</td>
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<tr>
<td>ε</td>
<td>fractional void-volume in packing (dimensionless)</td>
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<tr>
<td>Λ</td>
<td>capacity ratio; $μ U_x^0/U_y^0$ (dimensionless)</td>
</tr>
<tr>
<td>ν</td>
<td>kinematic viscosity (sq. cm./sec., or sq. ft./hr.)</td>
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<tr>
<td>θ</td>
<td>relative time; tU/l (dimensionless)</td>
</tr>
<tr>
<td>ω</td>
<td>pulse frequency (1/sec., or 1/hr.)</td>
</tr>
</tbody>
</table>
Subscripts
- c: mixing cell
- c: continuous-phase
- d: dispersed-phase
- D: dispersion unit
- l: longitudinal
- L: liquid
- m: molecular
- o: overall
- p: particle
- P: piston-flow (plug-flow)
- r: radial
- w: wall-to-wall
- x: x-phase
- y: y-phase
- 0: feed-inlet end of column, inside
- 1: solvent-inlet end of column, inside

Superscripts
- 0: feed-inlet end of column, outside
- 1: solvent-inlet end of column, outside
- *: equilibrium value
References

Figure 1. Breakthrough curves calculated from the diffusion equation with approximate boundary conditions.
Figure 2. Effect of column Péclet number ($P_e B$) and true NTU ($N_{ox}$) on apparent NTU ($N_{oxP}$) in single-phase flow with chemical reaction.
Figure 3. Schematic representation of laminar and turbulent flow through interstices of a packed bed.
Figure 4. Realistic (a) and idealized (b) models of segmented laminar flow through a single channel of a packed bed.
Figure 5. Péclet-number correlation for single-phase liquid flow through packed beds.
Figure 6. Péclet number for liquid flow through pulsed packed beds.
Figure 7. Péclet number for the continuous phase in countercurrent liquid-liquid systems.
Figure 8. Péclet number for dispersed phase (water) in countercurrent liquid-liquid systems.
Figure 9. Péclet number for the liquid phase in countercurrent air-water flow.
Figure 10. Péclet number for the gas phase in countercurrent air-water flow.
Figure 11. Illustrative concentration profiles in countercurrent extraction, calculated without (---) and with (----) axial dispersion. $N_{oxP} = 1.3$, $\Lambda = 0.8$. The solid curves correspond to $\text{P}_x\text{P} = 1.5$, $\text{P}_L\text{P} = 3$. 
Figure 12. Weighting factors for the $P_x B$ and $P_y B$ terms in the calculation of $N$.
Figure 13. Comparison of continuous-phase \( H \) and \( H_{CP} \) for isobutanol-water extraction system.

\[ H_{CP} \text{ values from Colburn and Welsh} \]

\[ G_c = 1000 \text{ lb/hr} \cdot \text{ft}^2 \]
Figure 14. Comparison of $H$ and $H_p$ for water phase in oxygen desorption at constant gas-phase mass velocity $[G = 230 \text{ (lb./hr.)/ft}^2]$
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