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FINANCIAL INTERMEDIATION

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A NEW LOOK AT THE THEORY OF FINANCIAL INTERMEDIATION

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1. Introduction

A theory of the behavior of financial intermediaries cannot be separated from the issue of solvency. Current regulation aims at controlling banks' equity capital, reserves, and exposure to risks in order to enhance the soundness of the banking system and, in particular, to limit the chance of bank failure.\textsuperscript{1} Most existing models of financial intermediaries, however ignore the issue of bankruptcy, by assuming that the bank never goes insolvent but may incur illiquidity costs.\textsuperscript{2} It is usually assumed that when the bank's money balances become negative, due to deposit withdrawals, it must borrow funds at a penalty rate. Another approach suggested in the literature is to impose a safety-first constraint to bound the likelihood of failure to a prespecified probability.\textsuperscript{3}

In contrast to the above mentioned papers which ignore the effect of insolvency on the market value of the bank's liabilities, Kareken and Wallace (1978) and Dothan and Williams (1980) incorporate insolvency in a state preference approach to determine the value of the bank's equity. Our paper focuses on the valuation of the financial intermediary's liabilities in a partial equilibrium framework using continuous-time contingent claim valuation procedures.\textsuperscript{4} The effect of future interest rate uncertainty on default free bonds is incorporated in the valuation of equity and the determination of the yield required by the depositors. The approach followed in this paper is more

\textsuperscript{1} See, for example, Maisel (1981) and Cargill and Garcia (1982).

\textsuperscript{2} See, for example, Pyle (1972), Baltensperger (1980), Spellman (1982) and Tobin (1982).

\textsuperscript{3} See, for example, Kahane (1977) and Koehn and Santomero (1980).

\textsuperscript{4} The valuation models will be consistent with the general equilibrium capital asset pricing model in its continuous time version.
powerful than the state preference approach and yields further analytical results and testable hypotheses.

Two critical questions must be addressed before developing any model of the banking firm. First, what objective function should the bank optimize, and second, what are the sources of uncertainty faced by the bank? The most popular objective function in the literature is the expected utility of terminal wealth.\(^1\) Tobin (1982), on the other hand, makes a case against expected utility maximization and advocates that risk neutrality is a more appropriate assumption since the bank is managed with a long-run perspective.\(^2\) Short-run survival is not viewed by Tobin as a crucial issue, since the bank is assumed to incur high enough illiquidity costs in order to avoid the risk of insolvency. More recently O'Hara (1983) argues that unless the bank is not closely held, agency problems must be considered. She thus assumes that the bank's manager maximizes the expected utility of his/her total compensation subject to a risk sharing agreement constraint which specifies stockholders' dividend as a function of net worth, profits, and the risk of the loan portfolio.

We follow Kareken and Wallace (1978) and Dothan and Williams (1980) by assuming that the bank's objective function is the maximization of the market value of the bank's equity. At the same time we incorporate a protective behavior of each claimholder who disallows actions that may reduce the market value of their claims. Depositors are assumed to follow their best interest by adjusting their required yield to reflect changes in expected risks, and by

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2Risk neutrality leads to expected profit maximization.
writing a contract that protects their interests against adverse actions which could be taken by value maximizing equityholders. In our framework the sometimes conflicting interests of all claimholders will be considered in a consistent framework where only market values are involved.

The major source of uncertainty in many models of the banking firm is associated with deposits withdrawals.\(^{1}\) This may stem from disintermediation, changes in market's interest rates or their volatility. In a competitive market, however, each bank is a rate-taker rather than a rate-setter.\(^{2}\) Hence, any quantity of deposits can be obtained for the "right" price, which reflects the ability of the bank to pay back its liabilities. Pyle (1971), Hart and Jaffee (1974) and Koehn and Santomero (1980) assume the bank to be rate-taker and suggest to solve the bank's exposure-to-risk problem by using a portfolio theory approach. Such an approach, it should be remembered, assumes an expected utility maximization behavior which is hard to justify for a publicly owned bank.

The bank in our model is assumed to be engaged in maturity intermediation. It holds assets with long maturities relative to the maturity of its deposits, and thus repackages illiquid assets in a form of liquid deposits. In this framework uncertainty stems from the exogenously-determined future interest rates on a default-free bond.\(^{3}\) The future rate on bonds affects the endogeneously-determined required rate on future deposits. The amount of deposits relative to equity financing also affects the financial risk to which

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\(^{1}\)See, for example, Klein (1971), Spellman (1982) and Tobin (1982).

\(^{2}\)For a discussion of this issue see Pyle (1972) and Hart and Jaffee (1974).

\(^{3}\)Later on in the paper we shall introduce risky loan as another source of uncertainty.
depositors and equity-holders are exposed and, hence, the yields required by claimholders. All rates and values of risky claims in our model are endogenously determined in a way consistent with equilibrium conditions, and are utility-independent.

This paper will not address the issue of why banks exist in a competitive environment with complete markets. It is assumed that individuals demand maturity intermediation, and competing banks can efficiently fulfill this service. It is sufficient for our purpose to assume market segmentation that makes it too costly for individuals to engage in home-made maturity intermediation. It can also be assumed there is a cost to divisibility, and individuals can only buy liquid assets with small denomination from specialized financial intermediaries.

In Section 2 the framework and assumptions for our model are presented. The model is developed in Section 3. In Section 4 the required interest rate paid on deposits is determined. The implications of the model for the capital adequacy problem are discussed in Section 5. The effects of deposit insurance on the value of equity and on capital adequacy are analyzed in Section 6. In Section 7 alternative policies for the asset structure are evaluated. Finally, Section 8 summarizes our results and suggests some testable hypotheses.

2. The Framework and Assumptions

In order to focus on the issue of capital adequacy and the resulting required rate of interest on deposits, a few simplifying assumptions are made. First, the framework for analysis will be a two-period model. Initially it is assumed that all the bank's assets are in the form of two-period default-free loans. The maturity value of a dollar of investment is \( \exp(2r_a) \) where \( r_a \) is the deterministic, continuously compounded, periodic interest on the bank's
assets. Money deposited is committed, however, for only one period. The promised interest rate on deposits at the initial period is denoted by \( r_0 \). At the end of the first period deposits must be renewed and the associated promised rate is \( r_1 \). Since we assume a competitive market where interest rates are affected by market forces and where each individual bank is a price taker, there is no uncertainty concerning the size of deposits, beyond that associated with their cost. Any quantity of deposits can be acquired for the "right price," and the uncertainty at present concerns the right price for refinancing deposits.\(^1\)

A decision variable of the bank is the proportion \( \alpha \) of all initial assets that will be financed by equity. The bank is not expected to raise additional equity at the end of the first period. Since markets are assumed to be competitive, and the bank's assets are assumed to be default free, there are no economies of scale and the bank can be analyzed per one dollar of assets.

The source of uncertainty in our model is the rate to be paid on deposits for the second period. Since the maturity value of the bank's assets is assumed to be known with certainty, therefore, if the bank is solvent, the rate on funds deposited at the beginning of the second period should also be the default-free interest rate. Current uncertainty concerning the second period interest rate coupled with the financial risk introduced by the ratio of equity financing affects the required interest rate on the initial deposits.

In order to derive analytical solutions, it is assumed that financial markets are complete in a sense that any financial claim can be replicated in

\(^1\)The procedure to derive the "right price," or the market-based interest rate that should be paid on deposits in the bank, is described below.
the market-place by a combination of other financial assets. Therefore, in order to eliminate arbitrage opportunities, the price of any asset is identical to the investment in the replicating portfolio. In addition, it is assumed that trading in financial markets is continuous. This assumption while not necessary, facilitates some of the derivations and numerical examples.

3. The Model

In the model considered here there is uncertainty at time zero with respect to the exogeneously-determined one-period default-free market rate that will be paid between time 1 and time 2, \( i_1 \). This uncertainty is resolved at time 1, when the rate \( i_1 \) is established in the market-place. Hence, at time 1, given the value of \( i_1 \), the outcome to depositors and shareholders is known with certainty and depositors at time 1 receive the market rate \( r_1 = i_1 \).

At time 2 the value of the bank's assets is known, with probability one, to be equal to \( \exp(2r_a) \). At that time the value of deposits, \( D_2 \), is equal to

\[
D_2 = \begin{cases} 
(1-\alpha)\exp(r_0+i_1) & \text{if } (1-\alpha)\exp(r_0+i_1) < \exp(2r_a) \\
\exp(2r_a) & \text{otherwise}
\end{cases}
\]

where \( \alpha \) is the initial proportion of equity financing to total assets of the bank, \( r_a \) is the promised interest rate on the bank's assets, and \( r_0 \) is the

\[\text{It is sufficient that the market is complete for the competing banks, while individuals face an incomplete financial market.}\]
one-period interest rate promised to depositors as of time zero. The first line in expression (1) is the value of deposits in the case the bank is solvent, and has enough assets to cover its obligations to depositors. If the bank is insolvent, all its assets belong to depositors. It should be noted that depositors at time 2 have a claim for \((1-\omega)\exp(r_0 + i_1)\), where \((1-\omega)\exp(r_0)\) describes the claim of the original depositors at time 1. When the bank is solvent, deposits are refinanced at time 1 with the promised rate being the default-free rate, \(i_1\), since there is no uncertainty with respect to the bank's assets at time 2.

At time 1, when the information \(i_1\) is revealed in the market place, the depositors know with certainty whether or not the bank will be able to cover its initial obligations to the first period depositors. Since they know the certain value of the assets at time 2, \(\exp(2r_a)\), and the risk-free discount rate between time 1 and 2, \(i_1\), they can determine the value of the assets at time 1 to be \(\exp(2r_a - i_1)\). Hence, default by the bank can be determined at time 1 when the value of the assets at that time, \(\exp(2r_a - i_1)\) is less than the promised payment to the original depositors \((1-\omega) \exp(r_0)\). Therefore, the value of the deposits at time 1 will be given by

\[
D_1 = \begin{cases} 
(1-\omega) \exp(r_0) & \text{if solvent} \\
\exp(2r_a - i_1) & \text{if insolvent} 
\end{cases}
\]

\(^1\)It is shown below that in a competitive market \(r_0\) should be endogenously determined to reflect the specific risk faced by depositors, with \(r_0 \geq i_0\) where \(i_0\) is the given first period default-free rate.

\(^2\)Remember that \(i_1\) is the interest rate on the default-free bond purchased at time 1 with maturity at time 2. This rate, however, is unknown at time zero.
Since at time 0 the rate \( i_1 \) is random, the outcome to depositors is stochastic. Hence, \( r_0 \) must be endogeneously determined as a function of the risk faced by depositors, and it is a function of the uncertainty, as of time 0, concerning time 1 value of the pure-discount bond, \( \exp(-i_1) \), and of the fraction of total assets initially financed by equity.\(^1\)

The value of the bank's assets at time 1 is \( \exp(2r_a - i_1) \) with certainty, and is also identically equal to the value of the bank's liabilities plus equity at that time. Hence, the value of equity at time 1 follows:

\[
S_1 = \exp(2r_a - i_1) - D_1 = \begin{cases} 
\exp(2r_a - i_1) - (1-\alpha)\exp(r_0) & \text{if solvent} \\
0 & \text{if insolvent}
\end{cases}
\]

The value of equity at time 1 has a structure similar to a call option to buy the bank's assets at time 1 for the promised payment to the original depositors. It is also similar to an option to buy a fraction \( \exp(2r_a) \) of a time 1 default-free bond maturing at time 2, \( \exp(-i_1) \). Indeed, define the promised payment to initial depositors by

\[
U \equiv (1-\alpha) \exp(r_0)
\]

and rewrite (3) as follows:

\[
S_1 = \begin{cases} 
\exp(2r_a)[\exp(-i_1) - U \exp(-2r_a)] & \text{if } U \exp(-2r_a) \leq \exp(-i_1) \\
0 & \text{if } U \exp(-2r_a) > \exp(-i_1)
\end{cases}
\]

\(^1\)The ratio of equity financing is usually referred to in the literature as the capital to assets ratio.
Figure 1: The value of deposits (D₁) and equity (S₁) at time 1 as a function of the price of a default-free bond at time 1, \(\exp(-i₁)\).

\[ U \equiv (1-\alpha) \exp(r₀) \]
Figure 1 describes the values of equity and deposits at time 1 as a function of the value at time 1 of a one-period, default-free pure discount bond maturing at time 2. The value of equity is zero if the bank does not have enough resources to meet its obligation to the initial depositors, i.e. when the value of the second period pure discount bond is below the critical number \( U \exp(-2r_a) = (1-\alpha) \exp(r_0 - 2r_a) \). The equity increases in value as the default-free interest rate decreases. Equity is also bounded from above and its time 1 upper limit is given by\(^1\)

\[
\exp(2r_a - i_1) - (1-\alpha)\exp(r_0)
\]

As was shown above, the value of the bank's equity is equivalent to buying \( \exp(2r_a) \) units of a call option on the one-period pure-discount bond issued at time 1, with a striking price equal to \( U \exp(-2r_a) \). In order to model the valuation formula, additional assumptions are made concerning the stochastic process generating \( i_1 \).

One approach to value future pure-discount bonds and the contingent claims on these bonds is suggested by Brennan and Schwartz (1982). The basic assumption underlying Brennan-Schwartz model is that the prices of all default-free securities can be written as functions of the yield on the currently maturing pure discount bond and the yield on a consol bond, and time to maturity. In other words, the whole term structure of interest rates can be determined in terms of the (very) short term rate, and the long-term rate. Both rates follow a joint stochastic process. Their formulation is more

\(^1\)At time 2 the value of equity is limited by

\[
S_2 < \exp(2r_a) - (1-\alpha)\exp(r_0 + i_1)
\].
general than the single state-variable models of Black and Scholes (1973) and Courtadon (1982). However, there is no closed-form solution to the Brennan-Schwartz model, only a numerical solution to the differential equation coupled with the boundary conditions. Based on their simulations it can be seen that the call option on a pure-discount bond is an increasing function of the bond value and time to maturity, and a decreasing function of the long-term interest rate.

By imposing additional restrictions Ball and Torous (1983) derive an analytical solution for the value of a European call option on a pure discount bond. They assume that the excess holding return on the pure discount bond follows a Brownian bridge process. This specification incorporates the terminal constraint that the market price of the default-free, pure discount bond must equal its face value at maturity.

By following the Ball and Torous procedure, and assuming markets to be frictionless with continuous trading, the present value of equity can be expressed at time $t$ as follows:

$$ S = [P(2) N(h_1) - P(1) U \exp(-2r_a) N(h_2)] \exp(2r_a) $$

where

\footnote{This is similar to assuming that the price dynamics of the default-free pure discount bonds are described by a nonstandardized transformed Brownian bridge process, given by

$$ \frac{dP(T)}{P(T)} = \alpha_T[P(T),t] \ dt + \sigma_T dZ(t,T) \quad 0 \leq t < T $$

where $P(T)$ is the time $t$ value of the bond maturing at time $T$, $\alpha_T[P(T),t]$ is the instantaneous expected rate of return on the bond, $Z(t,T)$ is a standard-dized Brownian motion process, and $\sigma_T$ is the instantaneous standard deviation of the bond excess holding return. It should be noted that the instantaneous expected rate of return is stochastic and depends on the price level, and also on the time left to maturity. The instantaneous standard deviation is, however, constant over the life of the bond.}
\[ h_1 = \left[ \ln\left( \frac{P(2)}{P(1)} \right) - \ln P(1) + 2r_a + \left( \sigma^2/2 \right) \tau \right] \sqrt{\tau} \]

\[ h_2 = h_1 - \sigma \sqrt{\tau} \]

\[ \sigma^2 = \sigma^2_1 + \sigma^2_2 - 2 \rho \sigma_2 \sigma_1 \]

and where \( N(\cdot) \) is the cumulative standard normal distribution, \( P(T) \) is the time \( t \) value of a default-free pure discount bond maturing at time \( T (T=1,2) \), \( \sigma^2_T \) is the instantaneous variance of the excess holding return of the bond maturing at \( T \), \( \rho \) is the instantaneous correlation coefficient between the rates of return for the two bonds, \( \tau = 1-t \) is the time left to maturity of the first period bond, and \( t \) is the current time.

The valuation formula in (4) is similar to the Black-Scholes valuation model for a European call option as amended by Merton (1973) for the case of stochastic bond prices. The model described in (4) is also consistent with the one derived under the assumption of risk neutrality. Rather than assuming continuous trading in financial claims, assume that all investors are risk neutral in a market with discrete trading. The value of equity is then the discounted value of future expected cash flows to equityholders. (See Cox and Ross (1976)).

Equation (4) describes the current value of the stock as a function of the current prices of the pure discount bonds maturing at time 1 and 2, their

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1The subscript \( t \) for the current time is omitted from \( S, P(1) \) and \( P(2) \).

2See Black and Scholes (1973) and Merton (1973, pp. 163-167) and especially his expression (40).
volatilities and the correlation coefficient of their excess holding returns. It is also a function of the return on the bank's assets, \( r_a \), and of the promised payment to initial deposit holders \( U \), and the latter is a function of the capital adequacy ratio for the bank and the discount factor for the pure\-discount bond maturing at time 1. By substituting for \( U \) in Equation (4) we obtain

\[
(4') \quad S = P(2) \exp(2r_a) N(h_1) - P(1)(1-\alpha) \exp(r_0) N(h_2)
\]

and

\[
h_1 = \left\{ \ln \left[ \frac{P(2)}{P(1)} \right] - \ln[(1-\alpha) \exp(r_0)] + 2r_a + \frac{\sigma^2}{2} \tau \right\} / \sigma \sqrt{\tau}
\]

In this form \( S \) is expressed in terms of the risk adjusted present value of the bank's assets minus the risk adjusted present value of the obligation of the bank to its initial depositors, where the risk adjustment factors are \( N(h_1) \) and \( N(h_2) \), respectively.

The model can be simulated to find the sensitivity of the equity value to changes in the various parameters. However, simple comparative statics results, expressed by the direction of the first-order partial derivatives, can be obtained:

\[
(5a) \quad \frac{\partial S}{\partial P(2)} = N(h_1) \exp(2r_a) \geq 0
\]

\[
(5b) \quad \frac{\partial S}{\partial P(1)} = -(1-\alpha) \exp(r_0) N(h_2) \leq 0
\]

\[
(5c) \quad \frac{\partial S}{\partial U} = -P(1)N(h_2) \exp(2r_a) \leq 0
\]

\[
(5d) \quad \frac{\partial S}{\partial \sigma^2} = (1-\alpha) \exp(r_0) P(1)Z(h_2) \frac{\sqrt{\tau}}{2\sigma} \geq 0
\]
where \( Z(\cdot) \) is a standard normal density function and

\[
Z(h_2) = \frac{1}{\sqrt{2\pi}} \exp(-h_2^2/2)
\]

(5e) \[
\frac{\partial S}{\partial \alpha} = \frac{\partial S}{\partial \theta} [-\exp(r_0 - 2r_a)] = P(1) \exp(r_0)N(h_2) \geq 0
\]

(5f) \[
\frac{\partial S}{\partial \tau} = P(1)(1-\alpha) \exp(r_0)[Z(h_2) \frac{\sigma}{2\sqrt{\tau}} - \frac{1}{\tau} \ln(P(1))N(h_2)] > 0
\]

The value of equity is a positive function of the value of the 2-period bond but a negative one of the 1-period bond. The value of equity is expected to go down with an increase in the 2-period discount rate, ceteris paribus, but to go up with an increase in the 1-period discount rate. These results are obvious; if after determining the rate paid on short term deposits, the market rate on alternative financial assets goes up, the market value of the deposits declines and, hence, the equity value increases. Still, a more interesting result is that the value of equity is also expected to increase with an unanticipated increase in the combined volatility of the two bonds. However, an unanticipated increase in variance of one bond does not necessarily lead to an increase in the value of equity. If the movements in bond's values are highly correlated, part of the increase in the specific volatility might be offset.¹

¹For example, if \( \sigma_1^2 = \sigma_2^2 \) and \( p = 1 \), the combined variance \( \hat{\sigma}^2 \) is zero. In such a case \( N(h_1) = N(h_2) = 1 \) and the value of equity will be

\[
S = P(2) \exp(2r_a) - P(1)(1-\alpha) \exp(r_0)
\]

In this case the value of equity is simply the discounted value of the asset value of the bank minus the discounted value of the obligation to initial depositors.
4. The Determination of the Required Interest Rate on Deposits

As a special case consider the value of equity at time of issuance, \( t = 0 \). Then, by definition

\[ P(2) = \exp(-2r_a) \]

and

\[ P(1) = \exp(-i_0) \].

By substituting these terms into (4') the value of equity at \( t = 0 \) is given by

\[ (4'') \quad S_0 = N(h_1) - (1-\alpha) \exp(r_0-i_0)N(h_2) \]

where

\[ h_1 = [i_0 - r_0 + \frac{\sigma^2}{2} - \ln(1-\alpha)]/\sigma \]

If in addition we assume away interest rate uncertainty, \( \sigma_1 = \sigma_2 = 0 \), and hence \( \hat{\sigma} = 0 \), then the equity value is equal to \( \alpha \) and \( r_0 = i_0 \). In general, however, \( S_0 \) increases with \( \hat{\sigma} \) ceteris paribus. But since \( S_0 \) should be equal to \( \alpha \), higher \( \hat{\sigma} \) necessarily implies a higher \( r_0 \) to be paid to depositors.

In section 2 it was claimed that there is a "right price" to be paid to the bank's depositors. With the second period rate being the default-free rate \( r_1 = i_1 \), the relevant question then is how the rate for the first period depositors, \( r_0 \), should be determined? Given the assumption that claimholders protect their interest in the bank, we expect depositors to require a rate \( r_0 \) such that the risk-adjusted present value of their claim is equal to \( 1-\alpha \). They try to limit the value of equity not to be more than \( \alpha \). Equation (4'') can be used to solve for \( r_0 \) by imposing the constraint \( S_0 = \alpha \). Since the term \( \alpha \) also is included in \( N(h_1) \) and \( N(h_2) \) there is no way to express \( r_0 \) as an explicit function of the other terms. Equation (4'') can only be solved by trial and error for \( r_0 \). The rate on initial deposit, \( r_0 \), should be equal
to or greater than the interest rate on the first-period default-free bond, \( i_0 \).

5. **The Capital Adequacy Issue**

One of the major areas of regulation concerns the capital adequacy of the banks. The Federal Reserve Bank, the Comptroller of the currency, the FDIC, and state banking agencies require their member banks to maintain a minimum ratio of equity to total assets. The rationale behind such a regulation is to reduce the insolvency risk of the bank.

It is easy to show in our framework that increasing the proportion of equity financing decreases the risk of insolvency of the bank, *ceteris paribus*. The critical point between solvency and insolvency at time 1 is whether the value of the assets is greater or smaller than the obligation to the initial depositors, i.e. \( \exp(2r_a) \exp(-i_1) > (1-\alpha) \exp(r_0) \). An increase in \( \alpha \) reduces the right hand side of the inequality and, thus, given the distribution of \( \exp(-i_1) \), the probability of insolvency goes down.

Our conclusion is contrary to that of Kahane (1977) and Koehn and Santomero (1980). Both papers use a portfolio framework to show that if \( \alpha \) increases, the risk of bankruptcy may also increase. Their result stems from changing the riskiness of the assets in response to a change in \( \alpha \). They fail, however, to consider a possible response by deposit-holders to a change in the riskiness of the assets; after all such a change will be against their interest and they will require a higher yield, \( r_0 \). However, if a ceiling on interest rate is imposed, our model will also predict an increase in the risk of assets in response to increase capital adequacy requirement. This issue is further discussed below.
In our framework, shareholders have an incentive to increase unexpectedly the riskiness of assets since wealth is transferred from depositors to equityholders. However it is expected that depositors protect their interests by disallowing such a move. If the risk of the assets is expected to increase, \( r_0 \) is also expected to increase in a competitive market.

In Section 4 it was shown that for each level of equity, \( \alpha \), there is a required rate \( r_0 \) that compensates for the financial risk of the bank. (See Figure 2.) There is an infinite number of combinations \((\alpha, r_0)\) such that \( S_0 = \alpha \), for which the market is in equilibrium. Therefore, imposing a constraint on capital adequacy should be reflected in the interest rate on deposits. If the capital adequacy requirement is effective and more equity must be raised, interest rates offered to depositors should be less than without the regulation.

If, in addition, regulators impose ceiling on the interest rate paid to depositors, it can be shown that one of the constraints is ineffective. (See Figure 2.) Given a capital adequacy ratio constraint \( \tilde{\alpha} \), the depositors require \( \tilde{r}_0 \). If regulators impose a ceiling on interest rates \( r^*_0 \), such that \( r^*_0 < \tilde{r}_0 \), the bank must increase its equity to \( \alpha^* > \tilde{\alpha} \) in order to reduce the financial risk, until the required rate by depositors is just equal to the ceiling imposed by the regulators. If, on the other hand, the required capital adequacy ratio \( \tilde{\alpha} \) is greater than \( \alpha^* \) needed to satisfy the interest rate ceiling, market forces will drive interest rate on deposit below the ceiling.

By imposing a specific interest rate on deposits, and simultaneously imposing a minimum capital to assets ratio, a disequilibrium may occur. The interesting case is when the imposed interest rate \( r^*_0 \) is greater than the equilibrium rate \( \tilde{r}_0 \) associated with the minimal capital adequacy ratio \( \tilde{\alpha} \). Then, \( S_0 < \tilde{\alpha} \) and the bank will face difficulties in raising equity capital. A
Figure 2 The Equilibrium Relationship between the Equity to Assets Ratio and the Required Rate of Interest on Deposits.
possible response from the equity holders is to raise the risk of the bank's assets. This move illustrates the kind of perverse or vicious circle which could develop from imposing various inconsistent regulatory measures. As a matter of fact, the regulatory agency could further react to the bank's adjustment by increasing the minimal capital adequacy ratio, which would further induce higher risk exposure from the bank.

6. The Effects of Deposit Insurance

With full coverage deposit insurance the depositors should only receive the risk-free rates, \( i_0 \) and \( i_1 \) on the first and second period deposits, respectively. For simplicity of exposition it is assumed that the cost of deposit insurance per period is a constant fraction \( g \) of deposits.\(^1\) Hence at time zero, the bank has to pay \( g \) \((1-\alpha)\) as insurance premium. For that premium the insurance company, say the FDIC, guarantees the initial depositors will receive \((1-\alpha) \exp(i_0)\) at time 1.

With an initial investment of \( \alpha + g \) \((1-\alpha)\) the shareholders have a claim on the residual value of the bank at time one:

\[
S_1 = \begin{cases} 
\exp(2r_a - i_1) - (1-\alpha) \exp(i_0) & \text{if solvent} \\
0 & \text{if insolvent.}
\end{cases}
\]

\(^1\)For in-depth analysis of deposit insurance schemes in a continuous time framework see Merton (1977, 1978) and Pyle (1983). They also introduce auditing costs into the analysis.
Expression (6) is similar to (3) with \( r_0 = i_0 \). The value of equity at time zero, \( S_0 \), is now given by\(^1\)

\[
S_0 = N(h_1) - (1-\alpha) N(h_2)
\]

where

\[
h_1 = \frac{\sigma - \ln(1-\alpha)}{2} \quad \text{and} \quad h_2 = \frac{\sigma}{2} - \frac{\ln(1-\alpha)}{\sigma}.
\]

The equilibrium pricing of equity satisfies

\[
S_0 = \alpha + g \, (1-\alpha)
\]

which means that equity is fairly priced and also that the insurance premium is set at a fair market value. Condition (8) imposes a constraint on the insurance premium \( g \). In order to avoid a corner solution \( g \) must be less than the value of equity for the limit case where the investment by shareholders tends to zero. Figure 3 depicts the two sides of equation (8). The intersection of the two functions yields the equilibrium value of \( \alpha \) for a given \( g \) set by the FDIC. A value of \( \alpha \) below \( \alpha^* \) means that the market value of equity \( S_0 \) is above the initial investment in equity \( \alpha + g \, (1-\alpha) \) and the FDIC actually subsidizes the equityholders. It is the case where equityholders shift the burden of financial risk to the FDIC which then is not properly compensated by the rate \( g \).

By the same reasoning the present value of the FDIC obligation to depositors can be modelled. The FDIC is obligated to pay at time 1 the amount

\[
F_1 = (1-\alpha) \exp(i_0) - \exp(2r_a - i_1)
\]

if this value is positive, i.e. when the

---

\(^1\)Equation (7) is derived from (4''') with \( r_0 = i_0 \).
Figure 3: The relationship between the Equity to Asset Ratio, $\alpha$, and the Value of Equity with Deposit Insurance.
value of the bank's assets at time 1 is below its obligation to pay the initial depositors. Based on \( F_0 = S_0 - \alpha \) and equation (7) the present value of the FDIC contingent liability is given by\(^1\)

\[
F_0 = N(-h_1) + (1-\alpha) N(-h_2)
\]

and \( h_1 \) and \( h_2 \) are defined in expression (7). The equilibrium premium for the FDIC should satisfy the condition:

\[
F_0 = g (1-\alpha)
\]

or

\[
g = \frac{N(-h_1)}{(1-\alpha)} + N(-h_2)
\]

A policy implication of the above analysis is that the FDIC, when setting \( g \), should simultaneously fix the equity-to-assets ratio according to (10) if it does not want to transfer wealth to equityholders. The capital adequacy requirement is therefore a function of the rate \( g \), and the higher \( g \) the lower the required \( \alpha \). It will also be a function of the future interest rate variability as measured by \( \hat{\sigma} \). From the discussion in Sections 5 and 6, it is clear that capital adequacy is not actually needed to protect depositors, but rather to protect the FDIC.

7. The Effects of Assets Composition

Until now the bank's assets have been assumed to be held in illiquid, two-period, default-free bond. The bank in our framework has dealt only with maturity intermediation by providing liquidity to depositors against the

\(^1\)This is the time-zero value of the FDIC obligation. The expression to evaluate \( F \) at any point \( t \) in the interval \( 0 < t < 1 \) can be easily derived.
illiquid default-free assets. This assumption is now relaxed to accommodate other alternatives for financial intermediation, and to study the relationship between the asset structure and the value of liabilities and equity.

7.1 The Case of Risky Assets with no Finite Maturity

Let us assume the bank invests all the proceeds in a single risky asset which value follows a log-normal distribution over time. The equity position at time 1 is then

$$
S_1 = \begin{cases} 
V_1 - (1-\alpha)\exp(r_0) & \text{if solvent} \\
0 & \text{if insolvent}
\end{cases}
$$

(11)

This corresponds to the classical case of equity valuation as discussed in Galai and Masulis (1976). The value of equity is isomorphic to a call option on asset $V$. The valuation formula for time $t$ ($0 \leq t < 1$) equity is given by:

$$
S_t = V_t N(d_1) - (1-\alpha) \exp(r_0) P(1) N(d_2)
$$

(12)

where

$$
d_1 = \ln\left[\frac{V_1}{(1-\alpha)\exp(r_0) P(1)}\right] + \frac{\hat{\sigma}^2 \tau}{2\hat{\sigma} \sqrt{t}}
$$

$$
d_2 = d_1 - \hat{\sigma} \sqrt{t}
$$

and where $\tau = 1-t$, $\hat{\sigma}^2 = \sigma_V^2 + \sigma_1^2 - 2\rho_{V,1} \sigma_V \sigma_1$, $\sigma_V^2$ is the instantaneous variance of the rate of return on the risky asset, $\sigma_1^2$ is the instantaneous variance of the rate of return on the one-period bond, and $\rho_{V,1}$ is the instantaneous correlation coefficient between the rates of return on $V$ and $P(1)$.

In equilibrium, at time zero, we should find $r_0$ that reflects the asset's risk and such that $S_0 = \alpha$. Most of the results derived above can be easily
modified to apply to this case. The major empirical implication concerns the magnitude of $\sigma_v^2$ compared to the variance of the excess holding-period return on the one-period bond $\sigma_2^2$, and the correlation coefficient between the returns on the risky assets and the bond. We should, therefore, expect for any given $\sigma$, the rate $r_0$ to be greater when the bank invests in risky assets. It is clear now that investing in a portfolio of assets will lower the risk as measured by $\hat{\sigma}^2$ and hence may lead to reduction in the required interest rate on deposits.

7.2 Combining Risky Assets with a Liquid Asset

Another interesting case is where the bank invests a proportion $\beta$ in a one-period, default-free bond, and proportion $1-\beta$ in a risky stock whose price follows a log-normal distribution. The balance sheet of the bank at time zero is

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities and Equity</th>
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</thead>
<tbody>
<tr>
<td>Risky Asset</td>
<td>(1-\beta)</td>
</tr>
<tr>
<td>Risk-free Asset</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Deposits</td>
<td>(1-\alpha)</td>
</tr>
<tr>
<td>Equity</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

The value of the assets of the bank at time $t$ $(0 \leq t \leq 1)$ is given by

$$A_t = (1-\beta) V_t + \beta P(1) \exp(i_0)$$

where $V_0 = 1$ and $P(1)$ is the discount factor for a default-free bond maturing at time $1$.

The value of equity at time $t$, $S_t$, can be found by slightly modifying Rubinstein's (1983) displaced diffusion option pricing model:

$$(13) \quad S_t = (1-\beta) V_t N(d_1) - [(1-\alpha) \exp(r_0) - \beta \exp(i_0)] P(1) N(d_2)$$
where

\[
    d_1 = \frac{\ln\{(1-\beta)V_t / [(1-\alpha)\exp(r_0) - \beta \exp(i_0)]\} - \ln P(1) + \frac{1}{2} \sigma^2 \tau}{\sigma \sqrt{\tau}}
\]

\[
    d_2 = d_1 - \sigma \sqrt{\tau}
\]

and \( \sigma \) is the instantaneous standard deviation of the stock's rate of return.

For simplicity of exposition the instantaneous risk of default-free bond is assumed to be zero in this formulation. Obviously, risky deposits require \((1-\alpha)\exp(r_0) > \beta \exp(i_0)\).\(^1\)

Once again, the qualitative results derived in previous sections still hold. Now, the effect of the asset structure on the required rate on deposits, \( r_0 \), can be analyzed. The higher is the proportion of the risk-free asset, \( \beta \), the lower is the required rate \( r_0 \). Equation (13) can be used to simulate the trade-off between additional equity and more asset liquidity for any given risk faced by depositors.

The issue of reserve requirement is a special case of the one discussed above. Regulators very frequently require financial institutions to hold a fraction \( \beta \) of assets in liquid, zero-interest bearing assets, i.e. cash reserves.\(^2\) Usually the amount of liquid reserves is a function of total deposits, or \( \beta = \gamma(1-\alpha) \) Where \( \gamma \) is the reserve requirement coefficient. Equation (13) can then be easily modified and the time-zero value of equity becomes

\[
    S_0 = [1 - \gamma(1-\alpha)] N(d_1) - [(1-\alpha) \exp(r_0) - \gamma(1-\alpha) \exp(-i_0)] N(d_2)
\]

\(^1\)Otherwise, deposits are riskless and \( r_0 = i_0 \).

\(^2\)For example, since the Monetary Act of 1980 the Federal Reserve Bank requires depository institutions to hold in reserve balance an amount equal to 3% of demand deposits up to 25 million dollars of deposits and 12 percent deposit balances above this limit.
\[ d_1 = \frac{\ln[(1-\gamma(1-\alpha))/(1-\alpha)[\exp(r_0 - i_0) - \gamma \exp(-i_0)] + \frac{1}{2} \sigma^2}{\sigma_Y} \]

The equilibrium condition for equity holders is, as before, \(S_0 = \alpha\). However, now, the assets earn less than the market rate on similar portfolios in terms of the future pay-out and risk structure. The current real value of the bank's assets is reduced by \(\gamma(1-\alpha)[1 - \exp(i_0)]\). Actually, in this set-up equilibrium cannot be reached for both equity and deposit holders. Since if \(\alpha = S_0\), the initial depositors of \((1-\alpha)\) have a claim whose present value \(D_0\) is smaller than \((1-\alpha)\) and the difference is equal to the reduction in the value of the bank's assets due to reserve requirements. This is the case where the whole burden of regulation is incurred by deposit holders. Regulators, very frequently, impose reserve requirements coupled with a ceiling on interest rates on deposits. The ceiling is intended, in our framework, to assure the equity holders that the burden of reserve requirements and the "leakage" in the value of the bank's assets fall on the deposit holders. A constrained equilibrium may be achieved but it can be expected that total deposits in the banking system with reserve requirements will be less than it would be with no regulation. Reserve requirements in the form of zero-interest paying assets may be destabilizing.

7.3 The Case of Finite Maturity Risky Assets

In reality a major proportion of the bank's assets are invested in business loans. The loans have finite life and if they are not paid back at maturity time the borrowing corporation is declared insolvent. It is assumed that all the bank's assets are invested in one corporate loan with a maturity
at $T > 1$. It is further assumed that the corporation has a single loan outstanding, and that the value of its assets $V$ follows a log normal distribution over time. $M$ denotes the face value of the loan per one dollar of initial market value at time 0.

Under the assumptions needed to derive the Black-Scholes formula the loan can be priced as a contingent claim on the firm's assets with striking price $M$. The value of the bank's equity is therefore similar to an option on a contingent claim. By modifying Geske's (1979) compound option pricing model a formula for pricing the bank's equity is derived.\(^2\)

\[
S_t = \exp(-rT') \left[ M N_2(d_{12}, d_{22}, \sqrt{\tau/T'}) + V N_2(d_{11}, -d_{21}, -\sqrt{\tau/T'}) - (1-\alpha)\exp(r_0 - rt) N(d_{12}) \right]
\]

where

\[
d_{12} = \frac{\ln V/V + (r - \frac{\sigma^2_V}{2})\tau}{\sigma_V \sqrt{\tau}}
\]

\[
d_{11} = d_{12} + \sigma_V \sqrt{\tau}
\]

\[
d_{22} = \frac{\ln (V/M) + (r - \frac{\sigma^2_V}{2})T'}{\sigma_V \sqrt{T'}}
\]

\[
d_{21} = d_{22} + \sigma_V \sqrt{T'}
\]

and $T'$ is the time to maturity of the loan ($= T-t$), $\tau$ is the time to maturity of initial deposits ($= 1-t$), $\sigma^2_V$ is the instantaneous variance of the rate of

---

\(^1\)See Merton (1974) and Galai and Masulis (1976).

\(^2\)While Geske prices a call option on another call option, the problem here is of a call option on the value of the underlying asset minus a prior call on the asset. See also Johnson (1983).
return on the assets of the borrowing firm, and $N_2(\cdot)$ is the standardized cumulative bivariate normal distribution. $V$ is the value of the borrowing firm's assets, and $\bar{V}$ is the critical value of $V$ for which the time-one value of the loan is equal to the promised payment to initial depositors, i.e.

$$L_1 = M \exp(-r(T-1)) \; N(d_2) + \bar{V} \; N(-d_1) = (1-\alpha)\exp(r_0)$$

$$d_1 = \frac{\ln \left( \frac{\bar{V}}{M} \right) + \frac{\sigma^2_V}{2}(T-1)}{\sigma_V \sqrt{\frac{1}{T}}} \quad \text{and} \quad d_2 = d_1 - \sigma_V \sqrt{\frac{1}{T}}.$$  

It is also assumed here, for simplicity of exposition, that the instantaneous risk-free interest rate is constant over time.

The value of the bank's equity is now directly related to the riskiness of the borrowing firm. The value of equity is an increasing function of the firm's asset risk ceteris paribus. Expected risk will be reflected in the required rate on deposits, $r_0$, but unexpected changes in $\sigma_V$ will increase the value of equity at the depositors' expense. An important role in determining $r_0$ is played by the debt to equity ratio of the borrowing firm. If the borrowing firm unexpectedly changes its capital structure and borrows additional funds with the same priority as the original loan, from another source, the market value of the initial loan decreases. Hence, the value of initial deposits will surely decline. The value of equity will also decline but at a slower rate, since it is positively affected by the increase in the financial risk of the borrowing firm, ceteris paribus. The model (15) can be used to estimate the effects of the financial risk of a borrowing company\(^1\) on the

\(^1\)The model is also applicable to measure the effects of country risk and its financial risk on the economics of the lending banks.
theoretically required rate on deposits. With effective interest rate ceilings, the bank will tend to reject loans to firms with high business risk or financial risk, since the bank cannot provide a fair, required compensation to its depositors. The equityholders have an incentive to increase unexpectedly the risk of the bank's assets, so it is expected that depositors protect their interest, and if they cannot do so, there will be a run against the bank.\footnote{The run on Continental Illinois National Bank and Trust Co. is a good example of the process described in this subsection.}

If the bank is solvent at time 1, it has to refinance itself. The rate \( r_1 \) for the second period deposit will be determined by a similar model to (15).

8. Summary and Conclusion

Financial intermediaries are engaged in repackaging financial instruments. They convert primary securities, usually long-maturity, less liquid investments, into short-term liquid instruments. A micro-economic model of a financial intermediary is presented,\footnote{The crucial macro-economic question of the rationale for financial intermediaries is ignored in our paper.} which focuses on the asset-transformation activity. We consider the combined effects of the intermediary financial structure and asset structure on the rates paid to depositors and on the valuation of its equity.

In efficient, competitive and unregulated financial markets, banks are expected to pay a fair interest rate on deposits. A model for determining the
fair rate is proposed, for different sources of uncertainty. First, uncertainty is only associated with future interest rates, which is basic to financial intermediation, and show how it affects the bank's solvency and the resulting required rate on deposits, given its financial structure.

Then, the bank is assumed to invest its fund in risky assets, either securities with infinite horizons, or corporate loans with finite horizon. In the latter we allow for default on the asset as well as the liability side of the bank. The qualitative results for the various structures are similar, though different factors specific to the assets are introduced, and modified modeling is required in each case.

The model is expanded to deal with regulation. In particular, reserve requirements, capital adequacy requirements, interest rate ceiling and deposit insurance are considered. We show that reserve requirements, if they are in the form of non-interest bearing reserves, may be disruptive to the banking industry. The cost will be incurred by depositors. Capital adequacy requirements alone should not affect the bank activity in an efficient market. Interest rate ceiling per se is also non-disruptive as long as the maximum rate is above the default-free rate and there is no capital adequacy constraint. Both regulatory measures however might be inconsistent and lead to perverse destabilizing sequential moves from the bank and the regulatory agencies. In general the regulation is not necessarily protecting the depositors, but rather the shareholders, unless some market imperfections are introduced.

Deposit insurance, while assuring depositors of a default-free account, gives incentive to shareholders to increase the risk of the bank's assets, and also to increase the financial leverage of the bank. In order to avoid wealth shifting from the insurance company to shareholders, the fixed-premium deposit insurance should be complemented by capital adequacy requirements and/or
reserve requirements. In a sense, regulation triggers further regulation with the risk of destabilizing inconsistencies.

The major testable hypotheses stemming from the derived model are as follows:

1. With no effective regulation, the rate paid on deposits will be positively related to the variance of the bank's assets and inversely related to the ratio of equity to total assets. Cross-country comparisons may provide the data for testing this hypothesis.

2. With interest rate ceiling, the lower is the ceiling, the higher is the ratio of equity to total assets and also the riskiness of the bank's assets.

3. Compulsory insurance with a fixed premium per dollar of deposit will motivate banks to increase the risk of their assets and their financial risk by lowering the equity to asset ratio.

4. The higher the proportion of liquid assets, including the reserves held at the central bank, the lower is the interest paid on deposits and/or the lower the capital to assets ratio.

5. Hedging activity to reduce assets' risk may be triggered by imposing lower interest rate ceiling.

The framework suggested in this paper to value the claims issued by financial intermediaries is general enough to accommodate further extensions. ¹ The important feature of our approach is that many of the issues that were previously dealt with by establishing an ad-hoc model can be analyzed in a uniform and consistent way.

¹For example, transaction costs, or a production function for the bank, can be introduced and their effect on the required rate by depositors can be analyzed.
## Appendix 1

Types of Assets of the Financial Intermediary
and the Suggested Model for Equity Valuation

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<td>Brownian Bridge Process</td>
</tr>
<tr>
<td></td>
<td>Ball and Taurus (1983)</td>
</tr>
<tr>
<td>Risky security (i.e. market portfolio)</td>
<td>Brownian Process</td>
</tr>
<tr>
<td></td>
<td>Black and Scholes (1973)</td>
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<tr>
<td>Combining Risky securities with default-</td>
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<td>free assets</td>
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<td>Risky Loans</td>
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<td></td>
<td>Geske (1979)</td>
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