Lawrence Berkeley National Laboratory
Recent Work

**Title**
OPTIMUM MULTIPLE SCATTERING

**Permalink**
[https://escholarship.org/uc/item/9045c4z](https://escholarship.org/uc/item/9045c4z)

**Author**
Greiner, Douglas E.

**Publication Date**
1964-06-01
University of California

Ernest O. Lawrence Radiation Laboratory

OPTIMUM MULTIPLE SCATTERING

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

Berkeley, California
OPTIMUM MULTIPLE SCATTERING

Douglas E. Greiner

June 1964
OPTIMUM MULTIPLE SCATTERING

Douglas E. Greiner

Lawrence Radiation Laboratory
University of California
Berkeley, California
U.S.A.

INTRODUCTION

The purpose of this paper is to study the best utilization of multiple scattering data to determine the scattering rigidity of a particle.

VARIABLES

Consider the projected image of a track as shown in Fig. 1. We define:

\( y_k \) = measured ordinates a distance \( t \) apart.
\( \lambda_i \) = distance between the \( i^{th} \) and \( i+1^{st} \) deflections in the \( k^{th} \) cell.
\( \omega_i \) = \( i^{th} \) angle of deflection in the \( k^{th} \) cell
\( \phi_k \) = angle between projection of track and \( y \) axis at \( k^{th} \) ordinate
\( n_k \) = number of deflections in the \( k^{th} \) cell
\( \sigma_k \) = noise associated with the \( k^{th} \) ordinate

Assuming \( \delta_k \) to be a random variable, Barkas has defined two more independent variables \( \chi_k \) and \( \psi_k \):

\[
\chi_k = \frac{\sum_{i=1}^{n_k} \sum_{j=1}^{i} \lambda_i}{2}, \quad \psi_k = \frac{\sum_{i=1}^{n_k} \sum_{j=1}^{i} \omega_i}{2} \tag{1}
\]

Any order difference \( \geq 2 \) can be expressed in terms of a linear combination of these variables:

\[
D_r^k = \sum_{i=1}^{r} (a_i \psi_{k+i-1} + b_i \chi_{k+i-1}) + \sum_{i=0}^{r} c_i \delta_{k+i} \tag{2}
\]
where

\[ a_i = \frac{(-1)^{i-1}(r-2)(2i-4)}{(r-i)!}(i-1)! \]

\[ b_i = \frac{(-1)^{i-1}(r-1)}{(r-i)!}(i-1)! \]

\[ c_i = \frac{(-1)^{i-1}}{(r-i)!} \]

(3)

Hence any difference product average, \( \langle D_k^r D_k^s \rangle \) is a linear combination of
\( \langle \psi_k^2 \rangle \), \( \langle \chi_k^2 \rangle \) and \( \langle \delta_k^2 \rangle \).

**CALCULATION OF SCATTERING RIGIDITY**

If we assume a gaussian distribution for second differences we can relate the mean squared noise-free second difference, \( \langle s \rangle \), to the scattering rigidity \( r \).

\[ r^2 = \frac{k^2 + 3}{(573)^2} \frac{\pi}{2\langle s \rangle} \]

(4)

Here one must choose the appropriate scattering "constant" \( K \). It is easily shown that:

\[ \langle s \rangle = \frac{8}{3} \langle \psi_k^2 \rangle = 8 \langle \chi_k^2 \rangle \]

(5)

Hence any difference product average is a linear combination of \( \langle s \rangle \) and \( \langle \delta_k^2 \rangle \).

Solving between two difference product averages we have

\[ \langle s \rangle = A(\langle D_k^r D_k^s \rangle + B(D_k^t D_k^u)) \]

\[ \langle \delta_k^2 \rangle = C(\langle D_k^r D_k^s \rangle + D(D_k^t D_k^u)) \]

(6)

Where A, B, C, and D are functions of \( r, s, t, u, m \) and \( n \).

**CALCULATION OF ERROR**

As \( \frac{1}{r^2} \propto \langle s \rangle \) we have \( \frac{\sigma(r)}{r} = \frac{1}{2} \frac{\sigma(\langle s \rangle)}{\langle s \rangle} \). Using the variables defined above it is possible to define an independent contribution to \( s \) from each cell.

The calculation is tedious but straightforward. The final result can be put in the form,
\[
\frac{\sigma(r)}{r} = \frac{1}{(n)^{1/2}} (a+b + c \lambda^2)^{1/2}
\]

where \( \lambda = \frac{\langle \delta^2 \rangle}{\langle \delta \rangle} \) and \( a, b \) and \( c \) depend on the choice of difference product averages used to obtain \( \langle \delta \rangle \) and \( \langle \delta^2 \rangle \). Figure 2 shows this error as a function of \( \lambda \) for two different combinations of difference products.

We have calculated this error for all possible combinations of order \( \leq 3 \) and found the combinations which yield the smallest error.

OVERLAPPING CELLS

In order to use a cell length longer than the measurement cell length \( \alpha \), we can ignore the intermediate points or form differences of the form below for each cell.

\[
D^2_{k,n} = Y_{k+2n} - 2Y_{k+n} + Y_k
\]

It can be shown that these differences are related in the same manner to the signal and noise as the differences for unit cell length. The two equations:

\[
D^2_{k,n} = \sum_{l=0}^{n-2} (l-1) D^2_{k+l} + \sum_{l=n-1}^{2n-2} (2n-l-1) D^2_{k+l}
\]

\[
D^r_{k,n} = D^r-1_{k+n-1,n} - D^r-1_{k,n}
\]

(9)

(10)

can be used to relate the signal to the variables \( \chi, \psi \) and \( \delta \).

The error calculation proceeds exactly as earlier.

Figure 3 shows fractional error as a function of number of overlaps for several initial noise to signal ratios.

ELIMINATION OF SPURIOUS SCATTERING

The assumption that \( \delta \) is an independent variable is violated by the presence of spurious scattering which is correlated in some manner to cell
length.

In a region of cell lengths where \( \langle s \rangle \propto t^3 \) the spurious scattering contribution is small and the signal is increasing as required by our assumptions.

This suggests the following method for determining scattering rigidity.

1. Measure ordinates of the track at a short cell length.
2. Calculate \( \langle s \rangle \) at several multiples of the measurement cell length.
3. When in the region where \( \langle s \rangle \propto t^3 \), calculate rigidity and error.
4. Test other cell lengths in this region for smaller error.

We have applied this method to tracks of known momenta and found good agreement up to several BeV/c.

**REFERENCE**


\[
\langle s \rangle = 1.25 \left( \langle D_k^2 D_k^2 \rangle - 0.4 \langle D_k^3 D_{k+1}^3 \rangle \right)
\]
\[
\langle \delta^2 \rangle = -0.042 \left( \langle D_k^2 D_k^2 \rangle + 2.0 \langle D_k^3 D_{k+1}^3 \rangle \right)
\]

\[
\sqrt{n} \frac{\Delta r}{r}
\]

\[
\langle s \rangle = 0.73 \left( \langle D_k^2 D_k^2 \rangle + 1.5 \langle D_k^2 D_{k+1}^2 \rangle \right)
\]
\[
\langle \delta^2 \rangle = 0.045 \left( \langle D_k^2 D_k^2 \rangle - 4.0 \langle D_k^2 D_{k+1}^2 \rangle \right)
\]

\[
\frac{\langle \delta^2 \rangle}{\langle s \rangle} = \lambda
\]
FIG. 3

USING LINEAR COMBINATIONS OF

$D_k^2 D_k^2$, $D_k^3 D_k^3$

INITIAL $\lambda = 5$

INITIAL $\lambda = 1$

$\sqrt{n \Delta r} \over r$

NUMBER OF OVERLAPS