Title
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A Simplified 1-D Model for Calculating CO$_2$ Leakage through Conduits

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**Problem Description**

In geological CO$_2$ storage projects, a cap rock is generally needed to prevent CO$_2$ from leaking out of the storage formation. However, the injected CO$_2$ may still encounter some discrete flow paths such as a conductive well or fault (here referred to as conduits) through the cap rock allowing escape of CO$_2$ from the storage formation. As CO$_2$ migrates upward, it may migrate into the surrounding formations. The amount of mass that is lost to the formation is called attenuation. This report describes a simplified model to calculate the CO$_2$ mass flux at different locations of the conduit and the amount of attenuation to the surrounding formations.

**Problem Setup**

The problem setup is shown in Figure 1.
We consider single-phase flow of CO₂ neglecting initial brine displacement. The boundary conditions of the system are: constant pressure at the top ($P_{top}$) which is set equal to hydrostatic pressure; constant pressure at the bottom ($P_{plume}$) set equal to hydrostatic pressure plus some overpressure due to CO₂ injection; and constant pressure at the far field set equal to hydrostatic pressure $P_e$. At steady state in this system,

$$q_j = q_{j-1} + q_{leak_j}$$  \hspace{1cm} (1)

where $q_j$ is the mass flow rate (kg/s) between grid blocks $j+1$ and $j$, and $q_{leak_j}$ is the leak-off mass flow rate (kg/s) from the $j$th grid block to the formation.

From Darcy’s law, we can write the following equations:

$$q_j = \rho \frac{\kappa \pi r^2}{\mu} \frac{P_{j+1}-P_j}{\Delta z_j}$$  \hspace{1cm} (2)

$$q_{leak_j} = \rho \frac{2\pi k_h j}{\mu} \frac{P_j-P_e(j)}{\ln(d/r)}$$  \hspace{1cm} (3)
where $r$ is the radius of the conduit, $d$ is the distance from the conduit to the far field where pressure is considered constant, $\rho$ is the density of CO$_2$, $\mu$ is the viscosity of CO$_2$, $k_j$ is the permeability of the formation adjacent to grid block $j$, and $\kappa$ is the permeability of the conduits. $P_j$ and $Pe(j)$ are the pressure and far-field pressure (hydro-static pressure), respectively, at grid block $j$, $h_j$ is the thickness of grid block $j$ and $\Delta z_j$ is the distance between the center of grid blocks $j$ and $j+1$.

These equations can be written for each grid block and combined with boundary conditions to calculate pressures and mass flow rates. A FORTRAN code was written to solve these equations. Below we will refer to this simplified model as the steady-state conduit model (SSCM).

**Code Testing**

An example problem is solved using three different approaches. Results are compared between the SSCM (described above), a TOUGH2 model of the same system at steady state, and the preliminary matlab code referred to as PMC (steady-state results).

The following properties are used for the test problem:

Water density (for establishing hydrostatic pressure): 998.057 kg/m$^3$
CO$_2$ density: 693.844 kg/m$^3$
CO$_2$ viscosity: 5.71336 e-5 Pa s

Conduit length: 140 m
Conduit radius $r$: 0.05 m
Distance $d$ from the conduit to far field: 1102 m, so $\ln(d/r) = 10$
Top pressure: atmospheric pressure ($P_{top} = P_{atm}$).
Bottom pressure: hydrostatic pressure plus 5000 Pa overpressure ($P_{plume}=Pe+5000$ Pa)
Conduit permeability: 1.e-12 m$^2$

- **Case 1**

For this case, the surrounding formation has a permeability of 1.e-13 m$^2$ at depth 130 – 140 m. The rest of the formation is impermeable. Discretization of the conduit at the depth of the permeable layer should be fine enough to capture the nonlinear pressure drops. To examine the discretization effect, runs with different grid-block size are also conducted. Results (mass flux in kg·m$^{-2}$ s$^{-1}$) are shown in Table 1. Comparisons of the three models are made for a grid size of 1 m. Additional comparisons are made between TOUGH2 and the SSCM for a grid size of 2 m, and between the SSCM and PMC for a grid size of 0.1 m.

<table>
<thead>
<tr>
<th>Number of gridblocks</th>
<th>Grid size (m)</th>
<th>CO$_2$ mass flux (kg·m$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SSCM top</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>0.03624</td>
</tr>
</tbody>
</table>
Results from these three models are very similar, especially between TOUGH2 and the SSCM. The mass flux along the conduit using the SSCM is shown in Figure 2.

![Mass Flux](image)

**Figure 2.** Mass flux along the conduit using the steady-state conduit model with a grid block size of 0.1 m for Case 1.

![Mass Flux](image)

**Figure 3.** Mass flux along the conduit using the PMC model with a grid block size of 1 m for Case 1.
It seems that when the discretization is not fine enough, the PMC has some numerical issues (see Figure 3, mass flux for 1 m grid block size). However, this disappears when a finer discretization is used (0.1 m).

Results show most of the attenuation happens at the lower portion of the conductive formation. The upper portion of the conductive formation does not contribute much to the attenuation as has been pointed out previously (Minkoff, SIAM, Santa Fe, 2006). Therefore, a much finer discretization is needed for the conduits that sit at the lower part of the conductive formation.

- Case 2

The difference between Case 2 and the previous case (Case 1) is that the surrounding formation has a permeability of 1.e-15 m² at depth 130 – 140 m. Results are shown in Table 2.

| Number of gridblocks | Grid size (m) | CO₂ mass flux (kg m⁻² s⁻¹) | | | | | |
|----------------------|--------------|----------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                      |              | SSCM top                   | SSCM bottom      | TOUGH2 top       | TOUGH2 bottom    | PMC top          | PMC bottom       |                  |
| 14                   | 10           | 0.03633                    | 0.04598          |                  |                  |                  |                  |                  |
| 70                   | 2            | 0.03630                    | 0.05265          | 0.03642          | 0.05254          |                  |                  |                  |
| 140                  | 1            | 0.03630                    | 0.05313          | 0.03629          | 0.05299          | 0.03592          | 0.06557          |                  |
| 1400                 | 0.1          | 0.03630                    | 0.05330          |                  |                  | 0.03626          | 0.05470          |                  |

In this case, a finer discretization is not as necessary as it is in Case 1 because of the much smaller attenuation due to the large contrast between the low formation permeability and high conduit permeability.

- Case 3

The difference between this case and Case 1 is that, in this case, the surrounding formation has a permeability of 1.e-13 m² located at depth 120 m – 130 m. Results are shown in Table 3.

| Number of gridblocks | Grid size (m) | CO₂ mass flux (kg m⁻² s⁻¹) | | | | | |
|----------------------|--------------|----------------------------|------------------|------------------|------------------|------------------|------------------|                  |
|                      |              | SSCM top                   | SSCM bottom      | TOUGH2 top       | TOUGH2 bottom    | PMC top          | PMC bottom       |                  |
| 14                   | 10           | 0.03624                    | 0.04029          |                  |                  |                  |                  |                  |
| 70                   | 2            | 0.03624                    | 0.04173          |                  |                  |                  |                  |                  |
| 140                  | 1            | 0.03624                    | 0.04196          | 0.03624          | 0.04329          | 0.03593          | 0.05005          |                  |
| 1400                 | 0.1          | 0.03624                    | 0.04211          |                  |                  | 0.03620          | 0.04271          |                  |

Again, we see an overshoot in the PMC solution (Figure 4), even with a grid size 0.1 m. This is not observed in other models.
The discretization for this case is not very critical because more flux (caused by leak-off) leads to more pressure drop at the bottom layer, therefore the overpressure at the bottom of the leak-off layer is relatively smaller.

![Figure 4. Mass flux along the conduits using the PMC with a grid block size of 0.1 m for Case 3.](image)

**Conclusions**

From the comparison among the three model results, we can conclude that the steady-state conduit model (SSCM) provides a more accurate solution than the PMC at a given discretization. When there is not a large difference between the permeability of the surrounding formation and the permeability of the conduits, and there is leak-off at the bottom formation (the formation immediately above the CO₂ plume), a fine discretization is needed for an accurate solution.

Based on this comparison, we propose to use the SSCM in the rapid prototype for now given it does not produce spurious oscillations, and is already in FORTRAN and therefore can be easily made into a dll for use in GoldSim.

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