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Multi-Bunch Beam-Break-Up Studies for a SWFEL/TBA*

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Abstract
A set of parameters minimizing BBU is obtained for a high current, low energy "drive beam" in a standing wave free electron laser two-beam acceleration (FELfBA). A large reduction in the transverse wake function is obtained by making the cavity-iris junction gradual by means of a cone. BBU is examined under various BBU reduction schemes.

1. INTRODUCTION
Multi-bunch beams have been shown to exhibit unstable transverse beam dynamics due to cumulative transverse wake fields. The purpose of this work is to design a cavity of low transverse wakes and to produce a set of optimized parameters minimizing BBU for a high current, low energy "drive beam" of the SWFEL/TBA[1,2]. The physical set-up is shown in Figure 1.

Table 1. Parameters of a SWFEL for 17.1 GHz with cylindrical cavities

<table>
<thead>
<tr>
<th>cavity</th>
<th>dimension (radius x length)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= 1.5 cm x 88 cm</td>
</tr>
<tr>
<td>iris</td>
<td>(radius x length) = 0.4 cm x 3 cm</td>
</tr>
<tr>
<td>junction</td>
<td>= 2 cm length; cone ~30°</td>
</tr>
<tr>
<td>beam</td>
<td>energy ( \gamma = 16.6 )</td>
</tr>
<tr>
<td></td>
<td>average current ( I = 1 , \text{kA} )</td>
</tr>
<tr>
<td></td>
<td>pulse length = 100 ns</td>
</tr>
<tr>
<td>focusing</td>
<td>focusing length ( \lambda_p = 52 , \text{cm} ); (( k_p = 0.12 , \text{cm}^{-1} ))</td>
</tr>
</tbody>
</table>

II. BASIC BEAM-BREAK-UP EQUATIONS
A. The Model
Let the planar wiggler magnetic fields lie in the \( y-z \) plane as indicated in Figure 1, with the particle wiggle in the \( x-z \) plane. These wiggle oscillations in a cylindrical cavity excite the TE11p FEL mode. The beam dynamics in the wiggle plane and that perpendicular to it can be treated separately as a first order approximation. In this paper we consider BBU in the \( y \)-direction, as that is the most severe case.

Assuming one macro particle per bunch and equal spacing between bunches the transverse displacements, \( \xi_j \), of the \( j \)-th bunch with energy \( \gamma_j \) can be represented in a standard BBU form.

\[
\frac{\partial^2}{\partial z^2} \left[ \gamma_j (z) \frac{\partial}{\partial z} \xi_j \right] + \gamma_j (z) k_{p_j}^2 (z) \xi_j = \frac{I_b}{I_0} \sum_{k=0}^{j} W((j-k)s_b) \xi_k,
\]

where \( z \) is the axial coordinate in cm, \( s_b \) is the distance between bunches in cm, \( W \) is the transverse wake function in \( \text{cm}^{-2}, I \) is the average current in \( \text{kA} \), and \( I_0 = mc^2/e = 17.05 \, \text{kA} \).

B. Analytic Estimates
For a purely sinusoidal wake function, \( W = W_0 \sin(\alpha \phi s/c) \) the oscillation amplitude of displacements of the \( j \)-th bunch can be estimated using the discrete Laplace Transformation as in Reference [3]. If only the first bunch is displaced,
Reduction in BBU can be obtained for smaller wakes, higher \( \gamma k_p \), smaller charge, and shorter device lengths.

The wakes, in reality, are far from a single sinusoidal wave. Any finite quality factor \( Q \) for the cavity, or any spread in frequency of the wakes reduces the amplitude for larger \( s \). In this case, we can reduce BBU for the same output energy FEL by making the pulse length longer, since the output energy of FELs depend on the charge of the pulse. If we take into account the quality factor the wake potential is reduced by \( e^{-\omega_r/Q(s/c)} \), where \( \omega_r \) is the wake modes, and the beam displacements are reduced by the same factor. In the presence of frequency spread \( \Delta \omega \) of the wake potential, the displacements are also reduced, although depending on the model, roughly by the factor \( e^{-\Delta \omega(s/c)} \).

### III. TRANSVERSE WAKES

For a Gaussian bunch of 1 Coulomb electrons with width \( \sigma \), the transverse wake of the dipole \((m=1)\) mode in an ideal pill-box cylindrical cavity can be evaluated as following.

\[
W(r=a,s) = -\left(\frac{1}{\pi \varepsilon_0 L}\right) \alpha^2 e^{-s^2/2 \sigma^2} \sum_{n,p} J_n^2(j_n \alpha) - J_1^2(j_n \alpha) \times \left\{ I^p \cos(v_{np}L) \right\} \frac{1}{2v_{np}a} \text{Im}\left\{ \frac{1}{\sqrt{2}} \frac{v_{np} \sigma}{\sqrt{2}} - \frac{is}{\sigma \sqrt{2}} \right\},
\]

where \( v_{np} = \omega_{np}/c \), \( \omega_{np} \) being the cavity mode frequencies, \( \alpha = \sigma R \) and \( j_n \) are zeros of the Bessel function \( J_1 \) and \( w \) is the complex error function. For the cavity of Figure 2(a) with \( \sigma = 3 \hbox{ mm} \), the wake shown in Figure 2(a') can be reproduced with 50 modes.

Wake functions are obtained for a cylinder of radius 1.5 cm and length 87.5 cm using ABCI [4] for the dipole mode \((m=1)\). Transverse wakes for (a) a step junction cavity and (b) a cone junction cavity are shown in Figure 2. By making the junctions gradual, a dramatic reduction in the wakes is achieved for the gradual junction effectively introduces spread in frequencies of wake modes, resulting in similar wake functions as in a de-tuned structure [5]. Smoothing the corners of the step junctions does not adequately reduce the wake. Due to computational limits, we have obtained wakes only up to 3 m. Since with reasonable \( Q \) values, the wakes at large \( s \) should damp away, we have assumed the wakes beyond 3 m to be negligible.

For long cavities the transverse momentum kicks depend only on the junction conditions and are independent of the cavity lengths. The reason for this is that for long cavities the wakes are superposition of the individual wakes generated due to the two junctions. This has been verified numerically.

### IV. NUMERICAL BBU RESULTS

Using the wake functions of Figure 2(b'), we have integrated the basic model equation numerically using the 4-th order Runge-Kutta method. Uniform displacements of the pulse were assumed as an initial condition.

The maximum amplitudes of a 100 nsec pulse of 17.1 GHz are shown in Figure 3 with respect to device lengths for the parameters presented in Table 1. Assuming negligible wake beyond 3 m, the beam displacements are small for a 60 m device but become too large for a 100 m device as shown in curve A of Figure 3.
Now we consider some possible damping mechanisms. First consider BNS damping [8]. The idea is to compensate that wake by increasing or decreasing the energy of the bunch, thus introducing a small change to $k_B$ with $s$.

The sign of the slope, $k_B$ with $s$, is crucial. However, with either an increment (B in Figure 3) or decrement of $\gamma(s)$, we obtain only a slight reduction in BBU.

Next we assumed that the wake beyond $1.74 \, m$ is negligible. When we set the wakes beyond $1.74 \, m$ to zero (C in Figure 3), then an external $Q=100$ is introduced instead of zeroing the wakes beyond $1.74 \, m$ (D in Figure 3). These gave the same good results indicating virtually no growth for $100 \, m$ device. Lastly we introduced some spread in $k_B$. With 2% spread in $k_B$, all transverse motion are damped out (E in Figure 3).

V. DISCUSSION

The transverse dynamics of the beam depends on $\gamma k_B$ strongly, yielding smaller BBU for larger $\gamma k_B$. The presented BBU results are for $\gamma k_B=2.0 \, cm^{-1}$. On the other hand the fluctuation level of output power amplitude and phase, in beam energy, depends strongly on $k_w$. For $17.1 \, GHz$ the best sensitivity is obtained around $k_w=0.16$. For a pulse of $1kA, 100 \, nsec, a_w=1.4$ is all that is needed to obtain an output energy of about $10 \, J/m$. With such parameters, and with natural FEL focusing alone, the value $\gamma k_B = a_w k_w$ (where the bar is used for an rms value,) is too small to reduce BBU to an acceptable level. This dilemma can be eliminated by introducing strong focusing in the drift region to minimize BBU (which does not affect the power extraction performance).

The BBU results presented here can be considered as an upper limit since we did not take into account the wiggler motion. Also, we believe that the BBU can be controlled better with rectangular cavities since we have more freedom to change the junction conditions. Nevertheless, even with the excessive estimate of BBU obtained in this work, we can find suitable SWFEL parameters as can be seen in Reference [7].

Acknowledgments

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VI. REFERENCES