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GLUE BALLS AND MEIKTONS WHICH DECAY TO MULTI-KAON FINAL STATES

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ABSTRACT

We observe in the usual spherical cavity approximation to the bag model that TM gluon modes couple predominantly in the s-channel to $\bar{s}s$ quarks. We compute the spectrum of glueballs and meiktons containing TM gluons, which have unique decays to states of two, three, or four kaons.

The lack of a clear, simple signature is the chief obstacle to experimental verification of the glueball spectrum expected in Q.C.D. This difficulty also applies to most of the $\bar{q}qg$ states, which we call meiktons (pronounced "make-ton", from the classical Greek for a mixed thing — the terms hermaphrodite or hybrid have also been used), expected in bag and potential models and lattice calculations. In this paper we propose a striking experimental signature for certain excited glueball and meikton states and we present the results of a calculation of the masses of those states. We find, in the spherical cavity approximation of the bag model, that they often decay to final states of two, three, or four K mesons.

These decays are striking not only because of their high K multiplicities. For example, we find an $I = 1, J^{PC} = 1^{+}$ meikton which decays prominently to $\phi\pi$ and $I = 1, J^{PC} = (0,1,2)^{++}$ states which decay prominently to $\phi\rho$. These would be extremely rare decay modes of $I = 1 \bar{q}q$ mesons since for such mesons they would be OZI suppressed. Similarly, we find strange meiktons which decay to final states containing $KK\bar{K}$... and $\phi$-like meiktons decaying to $KK\bar{K}...$

We first discuss the basis for the expectation that certain excited meiktons and glueballs decay in this fashion. We then present the spectrum of the lightest of these states, calculated to $O(a_s)$ in the bag model. We conclude by discussing the phenomenological implications. The $\psi(1440)$ resonance, the recently seen $\Omega(2220)$, the candidate $D'(1526)$, and the $\phi\phi$ candidate resonances at 2160 and 2320 MeV could be examples of these excited states.

In the bag model the lowest energy gluon mode is the TE (transverse electric) mode with axial vector quantum numbers $J^{PC} = 1^{+}$ and energy $E_{TE} = 2.74/R$ in a sphere of radius R. The TM (transverse magnetic) mode has vector quantum numbers $J^{PC} = 1^{-}$ and a higher energy $E_{TM} = 4.49/R$. The TE gluon couples in the s-channel to $u\bar{u}, d\bar{d}$, and $s\bar{s}$ in an approximately flavor symmetric way, as may be seen in...
Table 1 of Ref. (3). Because of this flavor symmetric coupling we suggested in Ref. 3 that meiktons containing TE gluons should have larger branching ratios into final states with strange quarks than we would expect for ordinary mesons. This paper is motivated by the fact, also recorded in Table I of Ref. (3), that the TM gluon s-channel coupling is much stronger to $\bar{s}s$ than to $\bar{u}u$ and $\bar{d}d$. Therefore we expect glueballs and meiktons with TM gluon constituents to decay often to final states rich in kaons.

This expectation is based on a decay mechanism in which the coupling of the gluon to quark-antiquark pairs gives rise to a $\bar{q}q\bar{q}q$ component of the wave function which can fall apart into two mesons if above threshold. We can calculate the fraction of the state in the $\bar{q}q\bar{q}q$ component using cavity perturbation theory. In the case of a meikton containing a TM gluon we would have a fraction:

$$a_s \times \text{(COLOR-SPIN)} \times \left[ \frac{L^{\text{TM}}(m_qR)}{2E_s(m_qR) - E_{\text{TM}}} \right]^2$$

where COLOR-SPIN depends on the flavor and spin of the state, and the remaining notation is as in Ref. 3: $L^{\text{TM}}$ is the TM gluon-quark-antiquark vertex, $E_{\text{TM}}$ is the gluon mode energy and $E_s(m_qR)$ is the mode energy of a quark with mass $m_q$ in an s-wave mode in a cavity of radius $R$. Using the values of $m_qR$ found below ($m_u = m_d = 0$, $m_s = 2/R$) it turns out that the $\bar{s}s$ enhancement over $\bar{u}u$ or $\bar{d}d$ of $-25$ due to the vertex factor $L$ ($L^{\text{TM}}(m_qR = 0) = 1.2$, $L^{\text{TM}}(m_qR = 2) = 0.58$) is cancelled by the energy denominator, giving approximate flavor symmetry in the mixing fractions. In the TE gluon case the analogous calculation gives an enhancement of $\bar{u}u$ or $\bar{d}d$ over $\bar{s}s$ instead of flavor symmetry. However there is good reason to doubt the validity of this calculation, particularly in the TM gluon case. The cancellation of the $\bar{s}s$ enhancement in that case occurs because of a fortuitous near cancellation of the lowest order mode energies, $E_{\text{TM}} = 2E_s$ for $m_qR = 0$ (equality occurs for $m_qR = 0.4$), so that the energy denominator is small and varies rapidly with $m_qR$. But if other $O(a_s)$ energy shifts, e.g. that due to the self energies, are larger than this small energy denominator, then one must include these shifts in the energy denominator when calculating the mixing fraction. This can lead to substantial modifications of the naive perturbative result. For example, if one includes the $O(a_s)$ self energies in the energy denominator (i.e. $[E_{\text{TM}} - 2E_s] \rightarrow [E_{\text{TM}} + a_sC_{\text{TM}} - 2(E_s + a_sC_q)]$ where the notation of Ref. 3 has been used) then, for the values of the self energies used below, the enhancement of $\bar{s}s$ over $\bar{u}u$ or $\bar{d}d$ due to the vertex is not cancelled. Similarly the flavor symmetry in the TE case is restored. Now this perturbative mixing corresponds to mixing between physical meiktons and $\bar{q}q\bar{q}q$ states. Because of the uncertainties in the bag model calculation, e.g. the gluon self energies and possible projections against spurious states (see below), we do not know the masses of these states, and in particular the mass differences, well enough to calculate this mixing accurately. But in most cases we expect that the larger coupling of the TM gluon to $\bar{s}s$ implies larger mixing with $\bar{q}q\bar{q}q$ states containing strange quarks. Even with the pessimistic estimate in which the TM gluon mixes flavor symmetrically we would still have the striking signatures discussed below such as apparent violations of the OIZ rule.

It should be noted that the enhancement of $\bar{s}s$ over $\bar{u}u + \bar{d}d$ occurs for a large range of $m_qR$ and is thus not sensitive to the precise parameters we use in our fits. For example the enhancement due to the vertex is $\sim 8$ for $m_qR = 1.5$, $\sim 12$ for $m_qR = 2$ (as shown above), $\sim 15$ for $m_qR = 2.5$, and $\sim 50$ for $m_qR = \infty$. The sensitivity to the cavity shape, however, remains to be studied. This latter issue is more serious for glueballs than for meiktons containing s-wave quarks and antiquarks (like those we consider below), since it is the gluon modes which require a nonspherical shape (see the discussion in Ref. 3).

In Ref. 3 we computed the spectrum of $\bar{q}q\bar{q}q$ TE meiktons, four nonets with
$J^{PC} = 1^{--}, (0, 1, 2)^{++}$, and four glueball states, $TE^2$ with $J^{PC} = (0, 2)^{++}$ and $TM$ with $J^{PC} = (0, 2)^{--}$. The calculations were to $O(a_s)$ in the spherical cavity approximation, with self energies determined empirically. The calculations agree with those of other authors, though varying treatments of the self energies lead to different predictions for the masses. We identified the $TE-TM$ $0^{++}$ glueball with $\pi(1440)$ from which we determined the sum of self energies $C_{TE} + C_{TM}$. The $2^{++}$ mass was then predicted unambiguously to be 2.3 GeV (although see the discussion below for a possible problem), while the $(0, 2)^{++}$ glueball and the $q_8 q_8 TE$ meikton masses were determined by the unknown ratio $C_{TE}/C_{TM}$, which we considered between 1/2 and 2. $C_{TE}/C_{TM}$ will be fixed when the spin of the glueball candidate $\Omega(1700)$ is measured: for $J(\Omega) = 2$ we have $C_{TE}/C_{TM} \sim 1/2$ while for $J(\Omega) = 0$, $C_{TE}/C_{TM} \sim 2$.

It seems from experimental studies of the $\eta nn^9$ and $\eta nnn^{10}$ channels that $\pi(1440)$ decays predominantly to $K\bar{K}n$, contrary to what one might naively expect of glueballs which must be flavor singlets. However this is precisely what the lowest order diagrams, shown in Fig. 1, lead us to expect for a pseudoscalar glueball since it contains a TM mode. Figure (1a) favors $\bar{s}s(Uu + \bar{d}d)^{F3}$ because of the $L^{nm}$ vertex while Fig. (1b) favors $\bar{s}s$ overwhelmingly over $\bar{u}u + \bar{d}d$ both because of $L^{sm}$ and for kinematical reasons (helicity conservation) special to a $J = 0$ state which has been discussed elsewhere. This example shows that it is perilous to use flavor symmetry to decide whether a given state is a glueball.

Because of the interesting experimental signatures they may possess we have extended the calculations of Ref. (3) to include the $(TM)^2$ glueballs with $J^{PC} = (0, 2)^{++}$ and the $s$-wave meikton nonets $q_8 q_8 TM$ with $J^{PC} 1^{--}, (0, 1, 2)^{++}$. We use fit I and the approximations of Ref. (3). For the $(TM)^2$ glueballs our computations agree with those of Carlson et al. The $q_8 q_8 TM$ spectrum has not previously been computed. The results are presented in Table 1 for $C_{TE}/C_{TM} = 1/2, 1, 2$.

There is a complication which must be mentioned at this point. In addition to the $q_8 q_8 TM$ states there are $L = 1$ meiktons with compositions $q_8 q_8 TE^*, q_8 q_8 TE$, and $q_8 q_8 TE$. Here $TE^*$ is the $J^{PC} = 2^{--}$ transverse electric gluon state with mode energy $3.87/R$ (i.e. slightly less that that of the TM mode), and the $p$ (anti) quarks have either $J = 1/2$ or $J = 3/2$ with mode energies $3.81/R$ and $3.20/R$ respectively — to be compared with $2.04/R$ for the $s$ (anti) quarks. Barring dramatic differences in self energies these states will have masses in the same range as the $q_8 q_8 TM$ states. However roughly a third of these states are spurious, or at least are pushed to higher masses, and should be projected out. We have not done this projection — in fact it is not clear that it is uniquely defined because there are a number of states for each of the spurious quantum numbers — but we simply assume that after the projection there remain states which are substantially composed of $q_8 q_8 TM$. This is reasonable because only a part (1/3?) of the spurious excitation of $q_8 q_8 TE$ involves excitation of the gluon mode, and this part itself is divided between TM and TE.* We use fit I and the approximations of Ref. (3). For the $(TM)^2$ glueballs our computations agree with those of Carlson et al. The $q_8 q_8 TM$ spectrum has not previously been computed. The results are presented in Table 1 for $C_{TE}/C_{TM} = 1/2, 1, 2$.

The same caveat applies to the $(TM)^2$ glueballs. These are doubly excited, and thus may be mixed by the projection against spurious states with $TM-TE^*$, $TE^*-TE^*$, $TM-TE^*$, $TM-TE^*$, and $TE^*-TE^*$ (TM* has $J^{PC} = 2^{++}$, TE** has $J^{PC} = 3^{--}$). We again assume that a state consisting mainly of $TM^2$ survives the projection. Given the premise of this paper it would be interesting to calculate the masses of the $TM^2$ states.

A final comment should be made while discussing the excited glueballs. The singly excited glueballs are either $TE-TM (J^{PC} = (0, 1, 2)^{--})$ or $TE^*-TE^* (J^{PC} = (1, 2, 3)^{--})$ while the spurious states have quantum numbers $1^- \times (0, 2)^{++} = 1^{--}$, $(1, 2, 3)^{--}$. Projecting against these spurious states leaves a $0^{++}$ which is unambiguously TE-TM, and a $2^{--}$ state which will be a mixture of TE-TM and TE-
TE*. Previous studies,\textsuperscript{2,14} (including our own) have overlooked this mixed nature of the 2\(^{-}\) which means that the 2\(^{-}\) mass is somewhat uncertain. Furthermore the predominance of \(\bar{s}s\) decays will be lessened, although, because of the TE* couplings mentioned above (see footnote) not removed completely.

We now consider the decays and possible candidates for the \(\bar{q}q_{\pi}TM\) and TM\(^2\) states. For the range of \(CTE/CTM\) considered the 2\(^{++}\) TM\(^2\) state has a mass ranging from 1.94 to 2.64 GeV. For \(M > 2.04\) GeV we expect it to have a substantial decay to \(\phi\phi\) in a relative s-wave. This state might be identified with the \(2^{-}\) mass is somewhat uncertain. Furthermore the predominance of \(\bar{s}s\) decays will be lessened, although, because of the TE* couplings mentioned above (see footnote) not removed completely.

The newly discovered \(\xi(2220)\) seen in \(p \to \gamma K \bar{K}\) could be identified with the "\(\omega\)" 2\(^{++}\) or 0\(^{++}\) meiktons, which in Table 1 are at 2320 and 1900 MeV for \(CTE/CTM = 1/2\). In this case we expect \(\xi\) to decay prominently to \(K^*\bar{K}\) and \(\phi\omega\) (see Table 2). As discussed below the \(\phi\omega\) decay is of particular interest.

Some expected two body decay modes are shown in Table 2. They respect G-parity selection rules but not their SU(3) extension, since the assumed decay mechanism breaks SU(3) badly. There are many striking signatures among the final states listed in Table 2. For instance, \(\rho^0 (1^{-+}) \to \phi \pi\) is a clean channel which is OIZ forbidden for a \(\bar{q}q\) meson, and the "\(\rho\)" to \(\phi\pi\) decays found in the other partial waves.\textsuperscript{17} Similarly \(\phi\eta\) and \(\phi\omega\) are OIZ forbidden decays for \(\omega\)-like and \(\phi\)-like \(\bar{q}q\) mesons (in the latter case they are like the "semi-forbidden" \(\eta \to \gamma \pi n\) transition).\textsuperscript{19} The \(\phi K\) and \(\phi K^*\) decays of the "\(K^*\)" states also provide a good signature; they would not be prominent in the decays of strange \(\bar{q}q\) mesons, since they could occur only by an OIZ rule violation (like \(\eta \to \gamma \pi n\)) or by creation of an extra \(\bar{s}s\) pair from the vacuum which is suppressed. The \(K\bar{K}K\bar{K}\) and \(\phi\phi\) decays of the "\(\phi\)" meiktons are obviously spectacular.

Finally it must be said that the reliability of cavity perturbation theory as applied here is by no means established. The convergence of the expansion remains to be demonstrated. The existence of states containing "valence" gluons, which are required in the bag model, has not yet been established experimentally. This could be
accomplished by verifying the bag model glueball spectrum or by finding some of the predicted meiktons.

The discussion of decays given here raises another problem: efforts to compute baryon strong interaction widths in cavity perturbation theory tend to underestimate badly the measured values. Clearly fixed cavity perturbation theory is inadequate to treat the effects of bag fission which must occur as the decay proceeds. However, our prediction for the decay of the TM gluon to strange quarks refers to the initial instant of the decay which occurs in the original cavity, and we may therefore hope it is at least qualitatively correct.

Furthermore if we carry out a semiquantitative analysis similar to that of Ref. 20 (in which \( \Delta \rightarrow \rho \pi \) was investigated in cavity perturbation theory) we find, at least for the TM meiktons, a much larger contribution to the width (\( \Gamma \sim 50-100 \) MeV) than that found in Ref. 20 for the \( \Delta(1238) \) (\( \Gamma \sim 2-5 \) MeV). This is because the mixing with \( \bar{q}q\bar{q}q \) occurs at lower order for the meiktons, and because the larger number of open channels increases the recoupling factors. This result gives us confidence that our mechanism is important. Of course it is possible that the other mechanisms needed to understand \( \Delta \rightarrow \rho \pi \) may be operative in this case also and may dilute the \( \bar{s}s \) dominance somewhat, although we still would expect a significant \( \bar{s}s \) component.

In conclusion if there are glueballs and meiktons containing valence gluons and if the spherical cavity estimate of the TM gluon couplings is qualitatively correct, then there exists a rich and unique spectrum of new hadrons which can be identified by their prominent decays to final states containing two, three, or four \( K \) mesons.

ACKNOWLEDGMENT

We wish to thank San Fu Tuan for a conversation which provided the impetus for this work. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

FOOTNOTES

F1. It is plausible but not proven that the bag model meiktons may be identified with the states discussed in Ref. (5).
F2. See Ref. (6) for a recent review of meson spectroscopy.
F3. There would also be substantial \( \bar{s}s \bar{s}s \) if \( s \) were well above the 4\( K \) threshold.
F4. We thank Carsten Petersen for pointing out an error in our original calculation.
F5. The diagrams are as for the \( \bar{q}q \bar{q}q \) \( TE \) meiktons (see Ref. 3) except for differing intermediate quark modes in some cases due to parity conservation.
F6. In fact the \( TE^* \) gluon, which couples to \( \bar{q}_{1L2}q_{3R2} \) and its charge conjugate, favors \( \bar{s}s \) over \( \bar{u}u \) or \( \bar{d}d \) by \( \sim 4 \), so that a reduced \( \bar{s}s \) enhancement will still apply to any \( \bar{q}q \bar{q}q \) \( TE^* \) component.
F7. The \( J^{PC} = 1^{--} \bar{s}s \) pair formed from the TM gluon is of course in the color octet. With exchange of a soft gluon it acquires the quantum numbers of the \( \phi \). \( \phi \)'s may also form by final state interaction of \( K\bar{K} \) pairs.
REFERENCES

   P. Hasenfratz et al., Phys. Lett. B55, 299 (1980);
   T. Barnes and F. E. Close, Phys. Lett. 116B, 365 (1982);
13. C. Rebbi, Phys. Rev. D12, 2407 (1975); D14, 2362 (1976);
    T. DeGrand and R. Jaffe, Ann. Phys. 100, 425 (1976);
    T. Banes, F. E. Close and S. Monaghan, Phys. Lett. 110B, 159 (1981);

FIGURE CAPTION

Fig. 1 Lowest order glueball decay mechanisms.
Table 1. Masses of $\text{TM}^2$ glueballs and $\bar{q}q_8$ TM meiktons at $O(\alpha_s)$ using fit I of Reference 3. All masses are in GeV. The radii of the states are $\sim 5-6 \text{ GeV}^{-1}$.

<table>
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<tr>
<th>State</th>
<th>$C_{\text{TE}}/C_{\text{TM}} = 1/2$</th>
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<th>$C_{\text{TE}}/C_{\text{TM}} = 2$</th>
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<td>(CTM = 1.62)</td>
<td>(CTM = 1.08)</td>
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<td>1.55</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>2++</td>
<td>2.64</td>
<td>1.94</td>
</tr>
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<td>1+-</td>
<td>$\rho/\omega$</td>
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<td></td>
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<td>$\phi$</td>
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<td>2.33</td>
</tr>
</tbody>
</table>

Table 2. "Signature" decays of the $\bar{q}q_8$ TM meiktons into two $L = 0$ mesons in a relative s-wave, as expected from the decay mechanism discussed in the text.

<table>
<thead>
<tr>
<th></th>
<th>1+-</th>
<th>0++</th>
<th>1++</th>
<th>2++</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$\phi\eta, K^<em>\bar{K}^</em>$</td>
<td>$\phi\rho, K\bar{K}, K^*\bar{K}$</td>
<td>$\phi\rho, K\bar{K}^<em>, K^</em>\bar{K}$</td>
<td>$\phi\rho, K^<em>\bar{K}^</em>$</td>
</tr>
<tr>
<td></td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$\phi\eta', K^<em>\bar{K}^</em>$</td>
<td>$\phi\omega, K\bar{K}, K^*\bar{K}$</td>
<td>$\phi\omega, K\bar{K}^<em>, K^</em>\bar{K}$</td>
<td>$\phi\omega, K^<em>\bar{K}^</em>$</td>
</tr>
<tr>
<td></td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
<td>$K^<em>\rho^</em>$, $K^<em>\bar{K}^</em>$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi\eta, \phi\eta'$</td>
<td>$\phi\phi, \phi\omega^\dagger$</td>
<td>$\phi\omega^\dagger$</td>
<td>$\phi\phi, \phi\omega^\dagger$</td>
</tr>
</tbody>
</table>

$^\dagger$These decays may be suppressed relative to the others in the table since they involve the TM gluon coupling to $\bar{u}u$ and $\bar{d}d$, but they are included because they are not OZI suppressed for meikton decays unlike the corresponding decays of their ordinary meson counterparts.
Figure 1
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