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**Author**
Willia, Beverly Hill.

**Publication Date**
1953-10-01
HIGH-ENERGY PARTICLE DATA
Volume III
KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

W. Peter Trower
January 1965
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HIGH-ENERGY PARTICLE DATA
Volume III
KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

W. Peter Trower
January 1965
HIGH-ENERGY PARTICLE DATA
Volume III

KINEMATICS OF PARTICLES AS A FUNCTION OF MOMENTUM

Contents

Introduction .................................................. 1

Ellipse-Plotting Method for Solution of the Relativistic Two-Body
Problem .......................................................... 3

Acknowledgments ................................................ 7

Other Volumes in UCRL-2426 Series .......................... 8

Kinematics of Elementary Particles (Graphs)
(\(\beta\), 1-\(\beta\), \(P_\beta\), \(H_\beta\), \(T\), and \(\gamma\)-1):

- Electrons (1 keV/c to 10 MeV/c) ......................... 9
- \(\mu^\pm\) Leptons (100 keV/c to 100 BeV/c) ............... 13
- \(\pi^0\) Mesons (100 keV/c to 100 BeV/c) .................. 19
- \(\pi^\pm\) Mesons (100 keV/c to 100 BeV/c) ................ 25
- \(K^\pm\) Mesons (1 MeV/c to 100 BeV/c) ................. 31
- \(K^0\) Mesons (1 MeV/c to 100 BeV/c) .................. 36
- Protons (1 MeV/c to 100 BeV/c) ......................... 41
- \(\Lambda\) Hyperons (1 MeV/c to 100 BeV/c) ............... 46
- \(\Sigma^+\) Hyperons (1 MeV/c to 100 BeV/c) .............. 51
- \(\Xi^-\) Hyperons (1 MeV/c to 100 BeV/c) ............... 56
- Deuterons (1 MeV/c to 100 BeV/c) ....................... 61
- Alpha Particles (10 MeV/c to 100 BeV/c) .............. 66

Dynamics of Collisions with a Proton Target (Graphs)
(\(\eta\), \(\eta M_\rho\), \(\pi K^2\), \(\beta\), 1-\(\beta\), \(\gamma\)-1, and \(w\)):

- Photons: \(\gamma + p\) (1 MeV/c to 100 BeV/c) ............ 70
- Electrons: \(e^\pm + p\) (1 MeV/c to 100 BeV/c) ............ 75
- \(\pi\) Mesons: \(\pi^\pm + p\) (1 MeV/c to 100 BeV/c) ........ 80
- \(K\) Mesons: \(K^\pm + p\) (1 MeV/c to 100 BeV/c) ........ 85
- Protons: \(p + p\) (10 MeV/c to 100 BeV/c) ............... 90
- \(\Lambda\) Hyperons: \(\Lambda + p\) (10 MeV/c to 100 BeV/c) .. 94
- \(\Sigma\) Hyperons: \(\Sigma^+ + p\) (10 MeV/c to 100 BeV/c) .. 98
- \(\Xi\) Hyperons: \(\Xi^- + p\) (10 MeV/c to 100 BeV/c) .... 102
Mnemonic Device for Relativistic Kinematic Formulas  
(Frank S. Crawford, Jr.)  

106

Some Simple Rules for Relativistic Kinematics (Frank T. Solmitz).  
108

Solid Angle Subtended by a Finite Rectangular Counter  
(Frank S. Crawford, Jr.)  

110
INTRODUCTION

The first section of this volume displays the kinematic quantities \( T, (\gamma-1), H_p, \beta \) or \((1-\beta)\), and \( P\beta \) as functions of particle momentum for the most common elementary particles (the notation is given below). The second part presents the dynamic quantities \( \pi \lambda^2, \eta M_p, \beta \) or \((1-\beta)\), \( \eta \), \((\gamma-1)\), and \( w \) as functions of incident-particle momentum for the most common elementary particles interacting with a proton. The particles \( \Sigma^- \), \( \Sigma^0 \), \( \Xi^0 \), and \( n \) were not plotted, being so close in mass to one of the particles that has been included that they cannot be distinguished graphically.

Now a word about notation. In this publication, mass, energy, and momentum are expressed in terms of energy: \( M \) is an abbreviation for \( M c^2 \), and momentum \( P \) is an abbreviation for \( P c \). The total energy \( (W) \) equals the kinetic energy \( (T) \) plus a rest energy \( (M) \). Capital letters refer to quantities relative to the laboratory (LAB) system, and lower-case letters refer to quantities relative to the center-of-mass (c.m.) system. The Greek letters \( \beta, \gamma, \) and \( \eta \) are used for the transformation quantities that relate motion of the c.m. system to the LAB system. The numerical subscripts designate the particle to which a quantity refers:

<table>
<thead>
<tr>
<th>LAB</th>
<th>Name</th>
<th>c.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Kinetic energy</td>
<td>( t )</td>
</tr>
<tr>
<td>( P )</td>
<td>Momentum</td>
<td>( p )</td>
</tr>
<tr>
<td>W</td>
<td>Total energy</td>
<td>( w )</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>Angle</td>
<td>( \theta )</td>
</tr>
</tbody>
</table>
We are concerned only with reactions of the form \( M_1 + M_2 \rightarrow M_3 + M_4 \), and of these possible cases we discuss only that one in which \( M_2 \) is at rest and bombarded by \( M_1 \) with a momentum \( P_1 \). The final state of the reaction is represented by \( M_3 \) and \( M_4 \).

This 1965 revision of UCRL-2426 Volume III has attempted to correct the errors and omissions found in the 1963 edition. These errors were found in the text and headings but none were discovered in the plots.

W. P. Trower
ELLIPSE- PLOTTING METHOD FOR SOLUTION OF THE RELATIVISTIC TWO-BODY PROBLEM

A useful graphic solution for the relativistic two-body problem is known as the ellipse-plotting method. One may wish to know, for a given $P_1$ in the LAB system, what the momenta $P_3$ and $P_4$ are that correspond to a $\theta_3$ or $\theta_4$ (where $\theta_3 = \pi - \theta_4$). We first perform the following calculations, recalling that the quantities described above in the Introduction can all be obtained from the graphs constituting the bulk of this volume:

(a) Find the total energy in both systems from

$$w = \left[ M_1^2 + M_2^2 + 2M_2 \left( P_1^2 + M_1^2 \right)^{1/2} \right]^{1/2},$$

$$W = \left[ P_1^2 + M_1^2 \right]^{1/2} + M_2,$$

where $w$ is the invariant mass of the system and is plotted in the dynamics-of-collision plots.

(b) Find the transformation quantities from

$$\gamma = \frac{w}{W}, \quad \beta = \frac{P_1}{W}, \quad \text{and} \quad \eta = \beta \gamma = \frac{P_1}{w}.$$

(c) Find the total energies of particles 3 and 4, in the c.m. system, from

$$w_3 = \frac{w^2 + M_3^2 - M_4^2}{2w}, \quad w_4 = w - w_3.$$

(d) Find the momentum, in the c.m. system, from

$$p = p_3 = p_4, \quad p = \left( w_3^2 - M_3^2 \right)^{1/2} = \left( w_4^2 - M_4^2 \right)^{1/2}.$$

(e) Finally, compute $\eta w_3, \eta w_4,$ and $\gamma p.$

---

The construction of the ellipse follows directly from the Lorentz transformation of the c.m. to the LAB quantities:

\[
\begin{pmatrix}
\gamma & 0 & 0 & \eta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\eta & 0 & 0 & \gamma
\end{pmatrix}
\begin{pmatrix}
p \cos \theta \\
p \sin \theta \\
0 \\
w
\end{pmatrix}
= \begin{pmatrix}
\gamma p \cos \theta + \eta w \\
p \sin \theta \\
0 \\
\eta p \cos \theta + \gamma w
\end{pmatrix}
= \begin{pmatrix}
P \cos \Theta \\
P \sin \Theta \\
0 \\
w
\end{pmatrix}

Taking the quantities associated with particle 3 for our construction, we can write

\[
P_x = P_3 \cos \theta_3 = \gamma p \cos \theta_3 + \eta w_3,
\]
\[
P_y = P_3 \sin \theta_3 = p \sin \theta_3,
\]

which are simply the parametric equations of an ellipse. The term \((\gamma p)\) is a measure of the eccentricity, the degree to which the circle in the c.m. system has been dilated because of momentum \(P_1\). The quantity \(\eta w_3\) represents the translation of the center of the coordinate origin in going from the LAB to the c.m. system and is due to momentum \(P_1\). When \(\theta_3 = 0\), \(P_x\) is the semimajor axis, and when \(\theta_3 = \pi/2\), \(P_y\) is the semiminor axis.

Having calculated all the necessary quantities, we construct a plot by performing the following geometrical operations (see Fig. 1):

(a) Construct a Cartesian base (y ordinate and x abscissa).

(b) Draw a circle of radius \(p\) with its center at the origin.

(c) Mark off an arc on the circle corresponding to the desired \(\theta_3\).

(d) Through this point on the circle, draw a line parallel to the x axis. We then have steps (c) and (d) represented in Fig. 1 by two dashed lines.

(e) Multiply the value for \(x\) at \(y\) by \(\gamma\), and plot the new point \((\gamma x, y)\).

(f) Repeat (c) through (e), picking enough points to generate an ellipse.

(g) On the x axis, locate a point (3) at a distance \(\eta w_3\) to the left of the y axis, and a point (4) at a distance \(\eta w_4\) to the right of the y axis. The distance between points (3) and (4) equals the LAB momentum of the bombarding particle \(P_1\), and is the vector sum of \(\vec{P}_3\) and \(\vec{P}_4\).

(h) Draw verticals through points (3) and (4), and locate points \((3')\) and \((4')\) at distance \(M_3\) and \(M_4\) above (3) and (4) respectively.
(i) From points (3) and (4) draw momentum vectors $\vec{P}_3$ and $\vec{P}_4$ to points on the ellipse corresponding to the c.m. angles desired.

(j) Measure LAB angles $\Theta_3$ and $\Theta_4$ with a protractor.

(k) If desired, $\vec{P}_3$ and $\vec{P}_4$ can be measured, and their values converted to $T_3$ and $T_4$ respectively, by reference to the graphs. Or alternatively, $T_3$ and $T_4$ can be ascertained by construction by applying Pythagoras' theorem as follows:

Place one tip of a pair of dividers on point (3), and extend the other tip a distance $P_3$ on the ellipse. Pivot the dividers about point (3) until the other tip touches the major axis of the ellipse, at point (a). Then pivot the dividers about point (a), and open them so that the other tip is on point (3'). The dividers now show a separation

$$W_3 = \left[ P_3^2 + M_3^2 \right]^{1/2}.$$  

Next, pivot the dividers about point (3') to where the other tip is vertically below points (3') and (3), at point (b). Point (b) then lies at a distance $T_3$ below point (3), since $T_3 = W_3 - M_3$. Now an energy scale plotted downward from point (3) permits a direct reading of $T_3$. This procedure, once mastered, is simple to perform. It is demonstrated in Fig. 2.

It is interesting to note that in the extreme relativistic limit a particle going backward in the c.m. system (i.e., $\theta_3 = \pi$) approaches a constant momentum in the LAB system; namely,

$$\left( M_3^2 - M_2^2 \right) / 2M_2.$$
Figs. 1-2. Typical diagram for a reaction of the type $\pi^- + p \rightarrow K^+ + \Sigma^-$. $M_1$ is the bombarding particle with kinetic energy $T_1$. $M_3$ and $M_4$ are the rest masses of the resulting particles.
ACKNOWLEDGMENTS

The author is indebted to Dr. Arthur H. Rosenfeld, Dr. M. Lynn Stevenson, and Dr. Robert D. Tripp for their comments and suggestions during the preparation of this collection of graphs. The actual presentation was made possible through the patient art work of Mr. Robert Stevens and the tireless editing of Mr. Howard Rogers.

This work was done under the auspices of the U. S. Atomic Energy Commission.
OTHER VOLUMES IN THE UCRL-2426 SERIES

Vol. I (1965 Revision), Kinematics of Particles as a Function of Kinetic Energy, January 1965

KINEMATICS OF ELEMENTARY PARTICLES
(GRAPHS)
(β, 1-β, Pβ, Hρ, T, and γ-1)
ELECTRONS

$T$ in MeV
$H_{\rho}$ in kgauss-cm
$P\beta$ in MeV/c

$1$ keV/c to $10$ keV/c

$\beta$, $(y-1)$, $T$, $P\beta$, $H_{\rho}$

$M_e = 0.510976$ MeV

$= 1$ m

$P$ (keV/c)
ELECTRONS

T in MeV
Hp in kgauss-cm
Pβ in MeV/c

10 keV/c to 100 keV/c

β, (γ-1), T, Pβ, Hp

M_e = 0.510976 MeV
= 1 m

P (keV/c)
ELECTRONS

100 keV/c to 1 MeV/c

$T$ in MeV
$H_{p}$ in kgauss-cm
$P_{\beta}$ in MeV/c

$\beta$, $(1 - \beta)$, $(\gamma - 1)$, $T$, $P_{\beta}$, $H_{p}$

$M_{e} = 0.510976$ MeV

$= 1 \text{ m}$

$P$ (keV/c)
ELECTRONS

$T \text{ in MeV}$

$H_p \text{ in kgauss-cm}$

$P \beta \text{ in MeV/c}$

$1 \text{ MeV/c to } 10 \text{ MeV/c}$

$(1 - \beta), (\gamma - 1), T, P \beta, H_p$

$M_e = 0.510976 \text{ MeV}$

$= 1 \text{ m}$

UCRL-2426
Vol. III (1965 Rev.)

$P (\text{MeV/c})$
\[ \mu^\pm \text{ LEPTONS} \]

- T in MeV
- \( H_p \) in kgauss-cm
- \( P_\beta \) in MeV/c

100 keV/c to 1 MeV/c

\[ \beta, (\gamma - 1), T, P_\beta, H_p \]

\[ M_{\mu^\pm} = 105.655 \text{ MeV} = 206.77 \text{ m} \]

UCRL-2426
Vol. III (1965 Rev.)
μ± LEPTONS

T in MeV
Hp in kgauss-cm
Pβ in MeV/c

1 MeV/c to 10 MeV/c

β, (γ - 1), T, Pβ, Hp

M_μ± = 105.655 MeV
= 206.77 m

UCRL-2426
Vol. III (1965 Rev.)
μ± LEPTONS

T in MeV
Hρ in kgauss-cm
Pβ in MeV/c

10 MeV/c to 100 MeV/c

Mμ± = 105.655 MeV
p = 206.77 m
\[ \pm \text{LEPTONS} \]

1 BeV/c to 10 BeV/c

\( (1 - \beta), (\gamma - 1), T, P\beta, H_\beta \)

\( M_{\mu^\pm} = 105.655 \text{ MeV} \)
\( \mu = 206.77 \text{ m} \)

\( T \) in MeV

\( H_\beta \) in kgauss-cm

\( P\beta \) in MeV/c

\( P \text{(BeV/c)} \)
\[ \mu^\pm \) LEPTONS

\begin{array}{c}
\text{T in MeV} \\
\text{H}_p \text{ in kgauss-cm} \\
\text{P}_\beta \text{ in MeV/c}
\end{array}

10 \text{ BeV/c to 100 BeV/c}

\begin{align*}
(1 - \beta), (\gamma - 1), T, P_\beta, H_p
\end{align*}

\[ M_{\mu^\pm} = 105.655 \text{ MeV} \]

\[ = 206.77 \text{ m} \]
$\pi^0$ MESONS

100 keV/c to 1 MeV/c

$\beta$, $(\gamma - 1)$, $T$, $P\beta$

$M_{\pi^0} = 135.00 \text{ MeV}$

$= 264.42 \text{ m}$

$P (\text{keV/c})$

$T \text{ in MeV}$

$P\beta \text{ in MeV/c}$
$\pi^0$ MESONS

$1 \text{ MeV/c to } 10 \text{ MeV/c}$

$\beta, (\gamma - 1), T, P_\beta$

$M_{\pi^0} = 135.00 \text{ MeV}$

$= 264.42 \text{ m}$

$P (\text{MeV/c})$

$P_{\beta} \text{ in MeV/c}$

$T \text{ in MeV}$

$\text{UCRL-2426}$

$\text{Vol. III (1965 Rev.)}$
\( \pi^0 \) MESONS

10 MeV/c to 100 MeV/c

\( \beta, (1 - \beta), (\gamma - 1), T, \ P\beta \)

\( M_{\pi^0} = 135.00 \text{ MeV} \)

\( = 264.42 \text{ m} \)

\( P \ (\text{MeV/c}) \)
π^0 MESONS

UCRL-2426
Vol. III (1965 Rev.)

100 MeV/c to 1 BeV/c

Pβ in MeV/c

(1 - β), (γ - 1), T, Pβ

M_{π0} = 135.00 MeV

= 264.42 m

P (MeV/c)
\( \pi^0 \) MESONS

1 BeV/c to 10 BeV/c

\( (1 - \beta), (\gamma - 1), T, P\beta \)

\( M_{\pi^0} = 135.00 \text{ MeV} \)

\( = 264.42 \text{ m} \)

\( P(\text{BeV/c}) \)
\( \pi^0 \) MESONS

10 BeV/c to 100 BeV/c

\((1 - \beta), (\gamma - 1), T, P\beta\)

\(M_{\pi^0} = 135.00 \text{ MeV}\)

\(= 264.42 \text{ m}\)

\(P (\text{BeV/c})\)
\[ \pi^\pm \text{MESONS} \]

\[ \beta, (\gamma - 1), T, P\beta, H_\rho \]

100 keV/c to 1 MeV/c

\[ M_{\pi^\pm} = 139.59 \text{ MeV} \]
\[ = 273.18 \text{ m} \]

\[ T \text{ in MeV} \]

\[ H_\rho \text{ in kgauss-cm} \]

\[ P\beta \text{ in MeV/c} \]

\[ P \text{ (keV/c)} \]

UCRL-2426
Vol. III (1965 Rev.)

-25-

[Graph showing the relationship between T, H_\rho, P\beta, and P]
\[ \pi^\pm \text{MESONS} \]

10 MeV/c to 100 MeV/c

\[ \beta, (1 - \beta), (\gamma - 1), T, P\beta, H_p \]

\[ M_{\pi^\pm} = 139.59 \text{ MeV} = 273.18 \text{ m} \]
\[100 \text{ MeV/c to } 1 \text{ BeV/c}\]
**π⁺ MESONS**

T in MeV

H₀ in kgauss-cm

Pβ in MeV/c

1 BeV/c to 10 BeV/c

(1 - β), (γ - 1), T, Pβ, H₀

Mₚ⁺ = 139.59 MeV

= 273.18 m

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UCRL-2426

Vol. III (1965 Rev.)
$\pi^\pm$ MESONS

$10 \text{ BeV/c to } 100 \text{ BeV/c}$

$(1 - \beta), (\gamma - 1), T, P\beta, H_p$

$H_p$ in kgauss-cm

$P\beta$ in MeV/c

$T$ in MeV

$M_{\pi^\pm} = 139.59 \text{ MeV}$

$= 273.18 \text{ m}$

$P \text{ (BeV/C)}$
$K^\pm$ MESONS

$T$ in MeV

$H_0$ in kgauss-cm

$P\beta$ in MeV/c

1 MeV/c to 10 MeV/c

$\beta$, $(\gamma - 1)$, $T$, $P\beta$, $H_0$

$M_{K^\pm} = 493.9$ MeV

$= 966.58$ m
$K^\pm$ MESONS

$T$ in MeV

$H_\rho$ in kgauss-cm

$P_\beta$ in MeV/c

$10 \text{ MeV/c to 100 MeV/c}$

$\beta, (\gamma - 1), T, P_\beta, H_\rho$

$M_{K^\pm} = 493.9 \text{ MeV}$

$= 966.58 \text{ m}$

$P (\text{MeV/c})$
K\textsuperscript{±} MESONS

1 BeV/c to 10 BeV/c

\( (1 - \beta), (\gamma - 1), P\beta, H_p, T \)

\( M_{K^\pm} = 493.9 \text{ MeV} \)

\( = 966.58 \text{ m} \)
K± MESONS

T in MeV

Hp in kgauss-cm

Pβ in MeV/c

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hp

M_{K±} = 493.9 MeV

966.58 m
K° MESONS

1 MeV/c to 10 MeV/c

\[ \beta, (\gamma - 1), \, T, \, P\beta \]

\[ M_{K^0} = 497.8 \text{ MeV} \]
\[ = 974.42 \text{ m} \]

\[ \text{T in MeV} \]
\[ P\beta \text{ in MeV/c} \]

\[ P \text{ (MeV/c)} \]
$K^0$ MESONS

$T$ in MeV

$P\beta$ in MeV/c

$\beta, (\gamma - 1), T, P\beta$

$M_{K^0} = 497.8$ MeV

$M_{K^0} = 974.42$ m

10 MeV/c to 100 MeV/c
K⁰ MESONS

100 MeV/c to 1 BeV/c

β, (1 - β), (γ - 1), T, Pβ

Mₖ⁰ = 497.8 MeV

= 974.42 m

T in MeV

Pβ in MeV/c

P (MeV/c)
K⁰ MESONS

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ

M_K⁰ = 497.8 MeV

= 974.42 m

P (BeV/C)
PROTONS

T in MeV
Hp in kgauss-cm
P\beta in MeV/c

1 MeV/c to 10 MeV/c
\beta, (\gamma - 1), T, P\beta, Hp

M_p = 938.213 MeV
= 1836.12 m
PROTONS

10 MeV/c to 100 MeV/c

\( \beta, (\gamma - 1), P\beta, H_p, T \)

\( M_p = 938.213 \text{ MeV} \)

\( = 1836.12 \text{ m} \)

- T in MeV
- \( H_p \) in kgauss-cm
- \( P\beta \) in MeV/c
PROTONS

$T$ in MeV

$H_p$ in kgauss-cm

$P \beta$ in MeV/c

100 MeV/c to 1 BeV/c

$\beta$, $(1 - \beta)$, $(\gamma - 1)$, $T$, $P \beta$, $H_p$

$M_p = 938.213$ MeV

$= 1836.12$ m

$P$ (MeV/c)
PROTONS

1 BeV/c to 10 BeV/c
$(1 - \beta)$, $(\gamma - 1)$, $T$, $P\beta$, $H\rho$

$T$ in MeV
$H\rho$ in kgauss-cm
$P\beta$ in MeV/c

$M_p = 938.213$ MeV
$= 1836.12$ m
PROTONS

UCRL-2426
Vol. III (1965 Rev.)

$T$ in MeV

$H_p$ in kgauss-cm

$P\beta$ in MeV/c

$10 \, \text{BeV/c to 100 BeV/c}$

$(1 - \beta), (\gamma - 1), T, P\beta, H_p$

$M_p = 938.213 \, \text{MeV}$

$= 1836.12 \, \text{m}$
\( \Lambda \) HYPERONS

\[ T \text{ in MeV} \]

1 MeV/c to 10 MeV/c
\( \beta, (\gamma - 1), T, P \beta \)

\[ M_\Lambda = 1115.36 \text{ MeV} \]
\[ = 2182.80 \text{ m} \]

\( P \) (MeV/c)
$\Lambda$ HYPERONS

10 MeV/c to 100 MeV/c

$\beta, (\gamma - 1), T, P\beta$

$M_\Lambda = 1115.36$ MeV

$P_{1/2} = 2182.80$ m

$T$ in MeV

$P\beta$ in MeV/c

$P$ (MeV/c)
HYPERONS

100 MeV/c to 1 BeV/c

\[ \beta, (1 - \beta), (\gamma - 1), T, P \beta \]

\[ M_\Lambda = 1115.36 \text{ MeV} \]
\[ = 2182.80 \text{ m} \]
### A Hyperons

- **T in MeV**
- **Pβ in MeV/c**

1 BeV/c to 10 BeV/c

\((1 - \beta), (\gamma - 1), T, P\beta\)

\[M_\Lambda = 1115.36 \text{ MeV} = 2182.80 \text{ m}\]

UCRL-2426
Vol. III (1965 Rev.)
\( \Lambda \) HYPERONS

10 \( \text{BeV/c} \) to 100 \( \text{BeV/c} \)

\( (1 - \beta), (\gamma - 1), T, P\beta \)

\( M_\Lambda = 1115.36 \text{ MeV} \)
\( = 2182.80 \text{ m} \)

\( T \text{ in MeV} \)
\( P\beta \text{ in MeV/c} \)

\( P \) (\( \text{BeV/C} \))

UCRL-2426
Vol. III (1965 Rev.)
$\Sigma^+$ HYPERONS

$T$ in MeV

$H_\rho$ in kgauss-cm

$P\beta$ in MeV/c

1 MeV/c to 10 MeV/c

$\beta$, $(\gamma - 1)$, $T$, $P\beta$, $H_\rho$

$M_{\Sigma^+} = 1189.40$ MeV

$= 2321.83$ m
\[ \Sigma^+ \text{ HYPERONS} \]

10 MeV/c to 100 MeV/c

\[ \beta, (\gamma - 1), T, \beta, H_p \]

\[ M_{\Sigma^+} = 1189.40 \text{ MeV} \]
\[ = 2321.83 \text{ m} \]

\( P \) (MeV/c)
\( \Sigma^+ \) HYPERONS

100 MeV/c to 1 BeV/c

\( \beta, (\gamma - 1), T, P\beta, H\rho \)

\( M_{\Sigma^+} = 1189.40 \text{ MeV} = 2321.83 \text{ m} \)

\( T \) in MeV

\( H\rho \) in kgauss-cm

\( P\beta \) in MeV/c
$\Sigma^+$ HYPERONS

T in MeV

1 BeV/c to 10 BeV/c

$P\beta$ in MeV/c

$M_{\Sigma^+} = 1189.40$ MeV

$= 2321.83$ m

$$
\begin{align*}
T & \text{ in MeV} \\
H_p & \text{ in kgauss-cm} \\
P\beta & \text{ in MeV/c}
\end{align*}
$$
\[ T \text{ in MeV} \]

\[ H_\rho \text{ in kilogauss-cm} \]

\[ P_\beta \text{ in MeV/c} \]

\[ 10 \text{ BeV/c to 100 BeV/c} \]

\[ (1-\beta), (\gamma-1), P, P_\beta, H_\rho \]

\[ M_{\Sigma^+} = 1189.40 \text{ MeV} \]

\[ = 2321.83 \text{ m} \]

\[ \Sigma^+ \text{ HYPERONS} \]
-56-

\[ E^+ \text{ HYPERONS} \]

1 MeV/c to 10 MeV/c

\[ \beta, (\gamma - 1), T, P\beta, H_p \]

\[ M_{\Xi^-} = 1318.4 \text{ MeV} \]
\[ = 2580.16 \text{ m} \]

\[ T \text{ in MeV} \]
\[ H_p \text{ in kgauss-cm} \]
\[ P\beta \text{ in MeV/c} \]
HYPERONS

$T$ in MeV

$H_p$ in kgauss-cm

$P\beta$ in MeV/c

$\beta, (\gamma - 1), T, P\beta, H_p$

$M_{\Xi^-} = 1318.4$ MeV

$M_{\Xi^-} = 2580.16$ MeV

UCRL-2426

Vol. III (1965 Rev.)
$\Xi^-$ HYPERONS

$T$ in MeV

$H_\rho$ in kgauss-cm

$P\beta$ in MeV/c

$\beta$, $(1 - \beta)$, $(\gamma - 1)$, $T$, $P\beta$, $H_\rho$

$P$ (MeV/c)

$M_{\Xi^-} = 1318.4$ MeV

$= 2580.16$ m

100 MeV/c to 1 BeV/c

UCRL-2426

Vol. III (1965 Rev.)
$\Xi^{-}$ HYPERONS

1 BeV/c to 10 BeV/c

$M_{\Xi^{-}} = 1318.4$ MeV

$= 2580.16$ m

$T$ in MeV

$H_\rho$ in kgauss-cm

$P\beta$ in MeV/c

$(1 - \beta), (y - 1), T, P\beta, H_\rho$
$\Xi^-$ HYPERONS

$T$ in MeV

$H_p$ in kgauss-cm

$P\beta$ in MeV/c

10 BeV/c to 100 BeV/c

$(1 - \beta), (\gamma - 1), T, P\beta, H_p$

$M_{\Xi^-} = 1318.4$ MeV

$= 2580.16$ m

$P$ (BeV/C)
DEUTERONS

1 MeV/c to 10 MeV/c

$T$, $H_p$, $P_{\beta}$

$M_d = 1875.49$ MeV
to $3670.40$ MeV

$P$ in MeV/c

$T$ in MeV

$H_p$ in kgauss-cm

$P_{\beta}$ in MeV/c
DEUTERONS

$T$ in MeV
$H_p$ in kgauss-cm
$P\beta$ in MeV/c

$10 \text{ MeV/c to } 100 \text{ MeV/c}$

$\beta$, $(\gamma - 1)$, $T$, $P\beta$, $H_p$

$M_d = 1875.49 \text{ MeV} = 3670.40 \text{ m}$
DEUTERONS

T in MeV
H_p in kgauss-cm
P_β in MeV/c

100 MeV/c to 1 BeV/c

β, (γ - 1), T, P_β, H_p

M_d = 1875.49 MeV
    = 3670.40 m

P (MeV/c)
T in MeV
Hp in kgauss-cm
Pβ in MeV/c

1 BeV/c to 10 BeV/c
(1 - β), (γ - 1), T, Pβ, Hρ

Md = 1875.49 MeV
= 3670.40 m
DEUTERONS

T in MeV

10 BeV/c to 100 BeV/c

(1 - β), (γ - 1), T, Pβ, Hp

Md = 1875.49 MeV

Hp in kgauss-cm

= 3670.40 m

Pβ in MeV/c

P (BeV/C)
ALPHA PARTICLES

100 MeV/c to 1 BeV/c

\[ M_a = 3727.23 \text{ MeV} \]
\[ = 7294.47 \text{ m} \]

\[ P(\text{MeV/c}) \]

- T in MeV
- \( H_p \) in kgauss-cm
- \( P\beta \) in MeV/c

\[ \beta, (\gamma - 1), T, P\beta, H_p \]
T in MeV

$P_{\beta}$ in MeV/c

$H_p$ in kgauss-cm

$1 \text{ BeV}/c$ to $10 \text{ BeV}/c$

$(1 - \beta), (\gamma - 1), T, P_{\beta}, H_p$

$M_a = 3727.23 \text{ MeV}$

$= 7294.47 \text{ m}$
DYNAMICS OF COLLISIONS WITH A PROTON TARGET

(GRAPHS)

\( (\eta, \eta M_p, \pi \kappa^2, \beta, 1-\beta, \gamma-1, \text{ and } w) \)
PHOTONS: $\gamma + p$

$\eta M_p$ and $w$ in MeV

$1 \, b = 10^{-24} \text{ cm}^2$

1 MeV/c to 10 MeV/c

$\beta, (\gamma - 1), \eta, \eta M_p, w, \pi x^2$

$M_\gamma = 0 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
PHOTONS: $\gamma + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ b} = 10^{-24} \text{ cm}^2$

10 MeV/c to 100 MeV/c

$\beta$, $(\gamma - 1)$, $\eta$, $\eta M_p$, $w$, $\pi \chi^2$

$M_\gamma = 0 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

$P (\text{MeV/c})$
\[ \eta M_p \text{ and } w \text{ in MeV} \]

\[ 1 \text{ mb} = 10^{-27} \text{ cm}^2 \]

PHOTONS: \( \gamma + p \)

100 MeV/c to 1 BeV/c

\[ \beta, (\gamma-1), \eta, \eta M_p, w, \pi \]

\[ M_Y = 0 \text{ MeV} \]

\[ M_p = 938.213 \text{ MeV} \]

\[ P \text{ (MeV/c)} \]
PHOTONS: $\gamma + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

$1 \text{ BeV/c}$ to $10 \text{ BeV/c}$

$(1-\beta), \beta, (\gamma-1), \eta, \eta M_p, w, \pi_k^2$

UCRL-2426
Vol. III (1965 Rev.)

$M_\gamma = 0 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

$P (\text{BeV/c})$
PHOTONS: $\gamma + p$

$\eta_{M_{p}}$ and $w$ in MeV

$1 \mu b = 10^{-30} \text{ cm}^2$

10 BeV/c to 100 BeV/c

$(1-\beta), (\gamma-1), \eta_{b}, \eta_{M_{p}}, w, \pi \chi^2$

$M_{\gamma} = 0 \text{ MeV}$

$M_{p} = 938.213 \text{ MeV}$

$P (\text{BeV/c})$
\( \eta M_p \) and \( w \) in MeV

1 b = \( 10^{-24} \) cm\(^2\)

1 MeV/c to 10 MeV/c

\( \beta, (\gamma-1), \eta, \eta M_p, w, \pi \)

\( \eta M_p \) and \( w \) in MeV

\( \beta, (\gamma-1), \eta, \eta M_p, w, \pi \)

\( M_{\pm} = 0.510976 \) MeV

\( M_p = 938.213 \) MeV
ELECTRONS: $e^\pm + p$

$\eta$ and $w$ in MeV

$1 \, b = 10^{-24} \, \text{cm}^2$

10 MeV/c to 100 MeV/c

$\beta, (\gamma-1), \eta, \eta_M, w, \pi^2$

$M_{e^\pm} = 0.510976 \, \text{MeV}$

$M_p = 938.213 \, \text{MeV}$

$P (\text{MeV/c})$
ELECTRONS: $e^\pm + p$

100 MeV/c to 1 BeV/c

$\eta m_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

$M_{e^\pm} = 0.510976 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
ELECTRONS: $e^\pm + p$

$nM_p$ and $w$ in MeV

$1$ BeV$/$c to $10$ BeV$/$c

$(1-\beta)$, $\beta$, $\gamma-1$, $n$, $nM_p$, $w$, $\pi^2$

$M_{e^\pm} = 0.510976$ MeV

$p^+ = 938.213$ MeV

$nM_p = 10^{-27}$ cm$^2$
Electrons: $e^{\pm} + p$

$10 \text{ BeV/c to 100 BeV/c}$

$(1-\beta), (\gamma-1), \eta, \eta M_p, w, \pi^2$

$E_{e^{\pm}} = 0.510976 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

$\eta M_p$ and $w$ in MeV

$1 \mu b = 10^{-30} \text{ cm}^2$
\[ \pi \text{ MESONS: } \pi^\pm + p \]

\[ M = 938.213 \text{ MeV} \]

\[ M_{\pi^\pm} = 139.59 \text{ MeV} \]

1 MeV/c to 10 MeV/c

\[ \beta, (\gamma-1), \eta, \eta M_p, w, \pi k^2 \]

\[
\begin{align*}
\eta M_p &= 139.59 \text{ MeV} \\
M_p &= 938.213 \text{ MeV}
\end{align*}
\]

\[ 1 \text{ mb} = 10^{-27} \text{ cm}^2 \]

\begin{align*}
\eta M_p &\approx 139.59 \text{ MeV} \\
M_p &\approx 938.213 \text{ MeV}
\end{align*}
$\pi$ MESONS: $\pi^{\pm} + p$

$\eta M_p$ and $w$ in MeV

$1 \ b = 10^{-24} \ cm^2$

$10 \ MeV/c$ to $100 \ MeV/c$

$\beta, (\gamma-1), \eta, \eta M_p, w, \pi \lambda^2$

$M_{\pi^{\pm}} = 139.59 \ MeV$

$M_p = 938.213 \ MeV$
\[ \pi \text{ MESONS: } \pi^\pm + p \]

\[ \eta_M \text{ and } w \text{ in MeV} \]

\[ 1 \text{ mb} = 10^{-27} \text{ cm}^2 \]

\[ 100 \text{ MeV/c to } 1 \text{ BeV/c} \]

\[ (1-\beta), \beta, (\gamma-1), \eta, \eta_M, p, \pi^2 \]

\[ M_{\pi^\pm} = 139.59 \text{ MeV} \]

\[ M_p = 938.213 \text{ MeV} \]
$\eta M_p$ and $w$ in MeV

$1$ mb $= 10^{-27}$ cm$^2$

$1$ BeV/c to $10$ BeV/c

$(1-\beta), (\gamma-1), \eta, \eta M_p, w, \pi \lambda^2$

$M_{\pi^\pm} = 139.59$ MeV

$M_p = 938.213$ MeV

UCRL-2426
Vol. III (1965 Rev.)
π MESONS: $\pi^\pm + p$

$\eta_M$ and $w$ in MeV

$1 \mu b = 10^{-30}$ cm$^2$

10 BeV/c to 100 BeV/c

$(1-\beta), (\gamma-1), \eta, \eta_M, w, \pi k^2$

$M_{\pi^\pm} = 139.59$ MeV

$M_p = 938.213$ MeV

UCRL-2426
Vol. III (1965 Rev.)
$\eta M_p$ and $w$ in MeV

$1 \text{ b} = 10^{-24} \text{ cm}^2$

$1 \text{ MeV/c to 10 MeV/c}$

$\beta, (\gamma - 1), \eta, \eta M_p, w, \pi^2$

$M_{K^\pm} = 493.9 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
K MESONS: $K^\pm + p$

10 MeV/c to 100 MeV/c

$\beta, (\gamma-1), \eta, nM_p, w, \pi k^2$

$M_{K^\pm} = 493.9$ MeV

$M_p = 938.213$ MeV

$\eta M_p$ and $w$ in MeV

$1 \text{ b} = 10^{-24}$ cm$^2$

$P$ (MeV/c)
$\eta M_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

100 MeV/c to 1 BeV/c

$(1-\beta), \beta, (\gamma-1), \eta, \eta M_p, w, \pi x^2$

$M_{K^\pm} = 493.9 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

$P (\text{MeV/c})$
K MESONS: $K^\pm + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

$1 \text{ BeV/c to 10 BeV/c}$

$(1 - \beta), (\gamma - 1), \eta, \eta M_p, w, \sqrt{s}$

$M_{K^\pm} = 493.9 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
K MESONS: $K^\pm + p$

$\eta M_p$ and $w$ in MeV

$1 \mu b = 10^{-30} \text{ cm}^2$

10 BeV/c to 100 BeV/c

$(1-\beta), (\gamma-1), \eta, \eta M_p, w, \pi k^2$

$M_{K^\pm} = 493.9 \text{ MeV}$

$M = 938.213 \text{ MeV}$

$p (\text{BeV/C})$
PROTONS: $p + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ b} = 10^{-24} \text{ cm}^2$

$10 \text{ MeV/c to } 100 \text{ MeV/c}$

$\beta, (\gamma -1), \eta, \eta M_p, w, \pi k^2$

$M_p = 938.213 \text{ MeV}$
PROTONS: p + p

100 MeV/c to 1 GeV/c

$\beta$, $(\gamma-1)$, $\eta$, $\eta M_p$, $w$, $\pi^2$

$M_p = 938.213$ MeV

$\eta M_p$ and $w$ in MeV

$1$ mb = $10^{-27}$ cm$^2$

$P$ (MeV/c)
\[ \eta M_p \text{ and } w \text{ in MeV} \]

\[ 1 \text{ mb} = 10^{-27} \text{ cm}^2 \]

\[ 1 \text{ BeV/c to } 10 \text{ BeV/c} \]

\[ \beta, (1 - \beta), (\gamma - 1), \eta, \eta M_p, w, M^2 \]

\[ M_p = 938.213 \text{ MeV} \]
\[
\begin{align*}
\eta_M \text{ and } w \text{ in MeV} \\
1 \mu b = 10^{-30} \text{ cm}^2
\end{align*}
\]

10 BeV/c to 100 BeV/c

\[
(1-\beta), (\gamma-1), \eta, \eta_M, w, \pi^2
\]

\[
M_p = 938.213 \text{ MeV}
\]

PROTONS: \( p + p \)

UCRL-2426
Vol. III (1965 Rev.)
\( \Lambda \text{ HYPERONS: } \Lambda + p \)

\( \eta M_p \) and \( w \) in MeV

\( \beta, (\gamma-1), \eta, \eta M_p, w, \pi \chi^2 \)

10 MeV/c to 100 MeV/c

\( b = 10^{-24} \text{ cm}^2 \)

\( M_\Lambda = 1115.36 \text{ MeV} \)

\( M_P = 938.213 \text{ MeV} \)

\( P (\text{MeV/c}) \)
$\eta M_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

$\beta, (\gamma - 1), \eta, \eta M_p, w, \pi^2$

$100 \text{ MeV}/c$ to $1 \text{ BeV}/c$

$M_\Lambda = 1115.36 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

$P (\text{MeV}/c)$
$\Lambda_{\text{HYPERONS: } \Lambda + p}$

$\eta^M_p$ and $w$ in MeV

$1$ mb $= 10^{-27}$ cm$^2$

$\beta, (1 - \beta), (\gamma - 1), \eta, \eta^M_p, w, x^2$

$P$ (BeV/c)

$M_\Lambda = 1115.36$ MeV

$M_p = 938.213$ MeV

UCRL-2426
Vol. III (1965 Rev.)
HYPERONS: $\Lambda + p$

10 BeV/c to 100 BeV/c

$(1-\beta), \beta, (\gamma-1), \eta, \eta M_p, w, \pi k^2$

$M_\Lambda = 1115.36$ MeV
$M_n = 938.213$ MeV

$\eta M_p$ and $w$ in MeV
$1 \mu b = 10^{-30}$ cm$^2$

$P$ (BeV/C)
\[ \eta M_p \text{ and } w \text{ in MeV} \]

\[ b = 10^{-24} \text{ cm}^2 \]

10 MeV/c to 100 MeV/c

\( \beta, (\gamma-1), \eta, \eta M_p, w, \pi x^2 \)

\( M_{\Sigma^+} = 1189.40 \text{ MeV} \)

\( M_p = 938.213 \text{ MeV} \)
$\Sigma$ HYPERONS: $\Sigma^+ + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ mb} = 10^{-27} \text{ cm}^2$

$P$ (MeV/c)

$100 \text{ MeV/c}$ to $1 \text{ BeV/c}$

$\beta, (\gamma-1), \eta, \eta M_p, w, \pi k^2$

$M_{\Sigma^+} = 1189.40 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$

UCRL-2426
Vol. III (1965 Rev.)
\[ \eta M_p \text{ and } w \text{ in MeV} \]

\[ 1 \text{ mb} = 10^{-27} \text{ cm}^2 \]

\[ 1 \text{ BeV/c to } 10 \text{ BeV/c} \]

\[ \beta, (1 - \beta), (\gamma - 1), \quad \eta, \eta M_p, \quad w, \quad w^2 \]

\[ M_{\Sigma^+} = 1189.40 \text{ MeV} \]

\[ M_p = 938.213 \text{ MeV} \]
\[ \eta M_p \text{ and } \omega \text{ in MeV} \]
\[ 1 \mu b = 10^{-30} \text{ cm}^2 \]

\[ M_{\Sigma^+} = 1189.40 \text{ MeV} \]
\[ M_p = 938.213 \text{ MeV} \]

\[ \Sigma \text{ HYPERONS: } \Sigma^+ + p \]

Graph showing the relationship between proton momentum (P in BeV/C) and \( (1-\beta), (\gamma-1), \eta, \eta M_p, \omega, \pi^2 \) for the range 10 BeV/c to 100 BeV/c.
$\Xi$ HYPERONS: $\Xi^- + p$

$\eta M_p$ and $w$ in MeV

$1 \text{ b} = 10^{-24} \text{ cm}^2$

10 MeV/c to 100 MeV/c

$\beta, (\gamma - 1), \eta, \eta M_p, w, \pi^2$

$M_{\Xi^-} = 1318.4 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
\( \eta M_p \) and \( w \) in MeV

1 mb = 10\(^{-27}\) cm\(^2\)

\( 100 \text{ MeV/c} \) to \( 1 \text{ BeV/c} \)

\( \beta, (\gamma-1), \eta, \eta M_p, w, \pi \lambda^2 \)

\( M_\Xi^- = 1318.4 \text{ MeV} \)

\( M_p = 938.213 \text{ MeV} \)

\( p \) (MeV/c)
$\Xi$ HYPERONS: $\Xi^- + p$

1 BeV/c to 10 BeV/c

$\beta, (1 - \beta), (\gamma - 1), \eta, \eta M_p, w, \frac{w}{\eta}$

$M_\Xi^- = 1318.4$ MeV

$M_p = 938.213$ MeV

$\eta M_p$ and $w$ in MeV

$1$ mb $= 10^{-27}$ cm$^2$
HYPERONS: $\Xi^- + p$

$\eta m_p$ and $w$ in MeV

$1 \mu b = 10^{-30} \text{ cm}^2$

10 BeV/c to 100 BeV/c

$(1-\beta), (\gamma-1), \eta, \eta m_p, w, \pi^2$

$M_{\Xi^-} = 1318.4 \text{ MeV}$

$M_p = 938.213 \text{ MeV}$
MNEMONIC DEVICE FOR RELATIVISTIC KINEMATIC FORMULAS

Frank S. Crawford, Jr.

In order to obtain slide-rule accuracy for kinetic energies, one should use formulae expressed in terms of kinetic energies, because if one uses the total energy, the rest mass often uses up the first few significant figures.

The following mnemonic device enables one to write down the exact relativistic (R) formula, if one remembers the nonrelativistic (NR) one:

(a) Write the correct NR formula.

(b) To the rest energy of each moving particle add one-half the total kinetic energy in the center-of-mass (c.m.) system.

Example 1:

A particle of rest mass $M_1$ and LAB system kinetic energy $T_1$ is incident on a stationary particle of rest mass $M_2$. Letting $c = 1$, what is the total kinetic energy $T$ in the c.m. system?

(a) NR: $T = T_1$ (for $M_2 + M_1$)

(b) R: $T = T_1 \left( \frac{M_2}{M_2 + M_1} \right)$

To solve this quadratic expression numerically for $T$, it is easier and faster to consider this as a recursion formula, and use a slide rule, rather than to rewrite and solve by radicals. For example, a 600-MeV proton on a proton gives

$$t_{n+1} = 600 \left( \frac{938}{938 + 938 + \frac{t_n}{2}} \right) = 300, 278, 279, 279, \ldots \text{MeV.}$$

Example 2:

Two particles of rest mass $M_1$ and $M_2$ share the kinetic energy $T = T_1 + T_2$ in their c.m. system. How do they divide up $T$?

(a) NR: \[ t_1 = t \left( \frac{M_2}{M_2 + M_1} \right); \]

(b) R: \[ t_1 = t \left( \frac{M_2 + \frac{t}{2}}{M_2 + \frac{t}{2} + M_1 + \frac{t}{2}} \right) = T \left( \frac{M_2 + \frac{t}{2}}{M_1 + M_2 + t} \right). \]

For instance, what is the kinetic energy of the \( \mu \) meson (rest mass 106 MeV) in the decay of a \( \pi \) meson (rest mass 140 MeV) \( \pi \rightarrow \mu + \nu? \)

\[ t_{\mu} = 34 \frac{(0 + \frac{34}{2})}{(0 + 106 + 34)} = 4.13 \text{ MeV}. \]

**Example 3:**

A single particle \( M_1 \) moves relative to the LAB system origin, where an infinite mass is located (so that the laboratory and c.m. systems are equivalent). Express the particle's kinetic energy in terms of its momentum and rest energy.

(a) NR: \[ T = \frac{p^2}{2M}; \]

(b) R: \[ T = \frac{p^2}{2(M + \frac{T}{2})}. \]

This example exposes the underlying root of the "mnemonic" in the exact relativistic formulae. We added an infinite mass at the origin so that we could use the "mnemonic" without modifying the phrase "in the c.m. system." One could perhaps say that the infinite mass provides the inertial frame in which special relativity is true.
I. Energy-Angular Distributions

In many problems in particle physics one has to transform an energy angular distribution from one frame of reference to another (say, from the LAB system to the c.m. system of a reaction). This transformation is given simply by

\[ \frac{1}{P} \frac{d^2 n}{dW d\Omega} = \frac{1}{P} \frac{d^2 n}{dW d\omega}, \]

where \( P, W, \) and \( \Omega \) are the momentum, total energy, and solid angle in one frame, and the corresponding lower case quantities refer to the other frame. In other words, \((P dW d\Omega)\) is Lorentz invariant.

II. Two-Body Decays

Consider particle 1 decaying into particles 3 and 4, with respective masses and momenta \( M_1, M_2, M_3, P_1, P_2, P_3;\) let the angle between the directions of particles 3 and 1 be \( \theta_2,\) and that between 4 and 1 be \( \theta_3,\) and let \( \theta = \theta_2 + \theta_3.\) Given two of the momenta and one angle, or both angles and one momentum, one can identify the type of decay process with the help of kinematics tables and graphs. However, one can often rule out certain possible identifications, even if only two quantities are measured, by the use of simple inequalities:

\[ P_2 \sin \theta_2 = P_3 \sin \theta_3 \leq p, \quad (a) \]
\[ P_1 \sin \theta_2 = P_3 \sin \theta \leq (M_1/M_2)p, \quad (b) \]
\[ P_1 \sin \theta_3 = P_2 \sin \theta \leq (M_1/M_3)p, \quad (c) \]

Here \( p \) denotes the momentum of particle 3 (or 4) in the rest frame of particle 1:

\[ p = \frac{1}{2M_1} \left[ (M_1+M_2+M_3)(M_1-M_2-M_3)(M_1+M_2-M_3)(M_1-M_2+M_3) \right]^{1/2}. \]
Equation (a) is a consequence of the invariance of the momentum components transverse to the direction of flight of particle 1. Similarly, Eq. (b) follows from consideration of the momentum components transverse to the direction of flight of particle 3, and from the consideration that the momentum of particle 1 (or 4), in the rest frame of particle 3, is \( \frac{M_1}{M_2} p \).
SOLID ANGLE SUBTENDED BY A FINITE RECTANGULAR COUNTER

Frank S. Crawford, Jr.

A geometry problem that arises in particle detection is the calculation of the solid angle $\Omega$ subtended by a "finite" detector at a source of particles. For a rectangular detector and a point source, a simple formula can be obtained for the integrated solid angle. First consider the special case in which the point source $P$ is located a distance $c$ perpendicularly above a corner of a rectangle of length $a$ and width $b$ (see Fig. 3a). Then we have

$$\tan \Omega = \frac{ab}{r^2_{\text{eff}}}$$

where $ab =$ area of rectangle,

$$r_{\text{eff}} = (cd)^{1/2} = \text{geometric mean of smallest and largest distances from } P \text{ to the rectangle},$$

$$d = (c^2 + a^2 + b^2)^{1/2}.$$

Thus the finite solid-angle formula is obtained from that of an infinitesimal detector by replacing $r^2$ by $r^2_{\text{eff}}$ and $\Omega$ by $\tan \Omega$.

The above holds only for the special case in which the perpendicular from $P$ to the plane of the detector intersects one corner of the detector.

We can now use this result to obtain the solid angle subtended by a rectangle oriented arbitrarily with respect to $P$. Let $\sigma$ be the intersection with the plane of the rectangle of the perpendicular from $P$ to the plane of the rectangle. If $\sigma$ lies inside the rectangle (Fig. 3b), it implies the four sub-rectangles $A$, $B$, $C$, and $D$; we simply apply the formula to them and add the results. If $\sigma$ lies outside the rectangle (Fig. 3c), then we apply the formula to the four rectangles $A + B + C + D$, $B + C$, $C + D$, and $C$, and find the desired quantity from $A = (A + B + C + D) - (C + D) - (B + C) + C$. 
Fig. 3. Geometrical solution for the solid angle subtended by a finite rectangular counter.
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