Title
Numerical Investigations of Star Formation and Interstellar Clouds

Permalink
https://escholarship.org/uc/item/91s6x91k

Author
Myers, Andrew

Publication Date
2013

Peer reviewed|Thesis/dissertation
Numerical Investigations of Star Formation and Interstellar Clouds

By

Andrew Thomas Myers

A dissertation submitted in partial satisfaction of the requirements for the degree of

Doctor of Philosophy

in

Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:
Christopher McKee, Co-chair
Richard Klein, Co-chair
Leo Blitz
Daniel Kasen

Fall 2013
Abstract

Numerical Investigations of Star Formation and Interstellar Clouds

by

Andrew Thomas Myers

Doctor of Philosophy in Physics

University of California, Berkeley

Christopher McKee, Co-chair
Richard Klein, Co-chair

This thesis explores several related questions on the physics of star formation and interstellar clouds. Chapter 2 addresses the remarkable independence of the stellar initial mass function to gas metallicity across a wide a range of galactic and extra-galactic environments. I perform analytic calculations that suggest the temperature structure of a centrally heated, dusty gas cloud should be relatively insensitive to the dust-to-gas ratio over the range of variation probed by observations. I support these calculations with full radiation-hydrodynamic simulations. Chapter 3 investigates the fragmentation of magnetized, massive cores via direct numerical simulation, finding that a combination of magnetic fields and protostellar heating strongly suppresses core fragmentation. These results could explain why the stellar initial mass function so closely resembles the dense core mass function, even at very high core masses. Chapter 4 analyzes the magnetic field structure around a rotationally-dominated protostellar disk formed in the above simulations. I present a map of the column density, magnetic field vectors, and outflow lobes at various length scales and viewing angles that can be compared against observations. I find that the disk begins to influence the geometry of the magnetic field on ∼ 100 AU scales, which should begin to be probed in nearby stellar cores by the next generation of radio interferometers. Chapter 5 addresses the long-standing question of the high CH$^+$ abundance along diffuse molecular sight lines. I compute the CH$^+$ abundance in a turbulent, diffuse molecular cloud by post-processing numerical simulations of magnetohydrodynamic turbulence. The results agree well with observations of the column densities of CH$^+$ and of rotationally-excited H$_2$. Finally, Chapter 6 presents the results of radiation-magnetohydrodynamic simulations of star formation at the cluster scale. The results show that the magnetic field plays a role in determining the star formation rate, the degree of fragmentation, and the characteristic stellar mass.
I dedicate this dissertation to Heather.
Contents

List of Figures v
List of Tables xi
Acknowledgments xii

1 Introduction 1
   1.1 The Initial Mass Function 2
   1.2 The Core Mass Function 2
   1.3 The Magnetic Braking “Catastrophe” 3
   1.4 The CH+ problem 4
   1.5 Star Formation on the Cluster Scale 4

2 Metallicity and the Universality of the IMF 6
   2.1 Introduction 6
   2.2 Simulation Setup 8
      2.2.1 Numerical Techniques 8
      2.2.2 Refinement criteria 11
      2.2.3 Initial conditions 11
   2.3 Results 13
      2.3.1 Temperature and density structure 13
      2.3.2 Fragmentation and Star Formation 15
   2.4 Discussion 18
      2.4.1 Analytic Model 18
      2.4.2 Comparison to simulations 22
      2.4.3 Predictions for star-forming regions 24
   2.5 Conclusions 26

3 The Fragmentation of Magnetized, Massive Star-Forming Cores with Radiative Feedback 29
   3.1 Introduction 30
   3.2 Numerical Setup 31
      3.2.1 Equations and Algorithms 31
<table>
<thead>
<tr>
<th>6.3.1</th>
<th>Global Evolution</th>
<th>102</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3.2</td>
<td>Star Formation</td>
<td>108</td>
</tr>
<tr>
<td>6.3.3</td>
<td>The Initial Mass Function</td>
<td>112</td>
</tr>
<tr>
<td>6.3.4</td>
<td>The Protostellar Mass Function</td>
<td>114</td>
</tr>
<tr>
<td>6.3.5</td>
<td>Core Magnetic Field Structure</td>
<td>116</td>
</tr>
<tr>
<td>6.3.6</td>
<td>Turbulent Core Accretion</td>
<td>118</td>
</tr>
<tr>
<td>6.3.7</td>
<td>Mass Segregation</td>
<td>120</td>
</tr>
<tr>
<td>6.3.8</td>
<td>Multiplicity</td>
<td>124</td>
</tr>
<tr>
<td>6.4</td>
<td>Discussion</td>
<td>126</td>
</tr>
<tr>
<td>6.5</td>
<td>Conclusions</td>
<td>128</td>
</tr>
</tbody>
</table>

**Bibliography**

130
List of Figures

2.1 Projections through the simulation volumes at $t_{ff} = 0.5$. The left panels show the column density of the entire core, defined as $\int \rho \, dx$. The middle column is also the column density, zoomed in to show the middle 5000 AU. The right column shows the column density-weighted temperature, $\int \rho T_{\text{gas}} \, dx / \int \rho \, dx$, at the same scale. The rows, from top to bottom, show runs “Solar,” “0.2 Solar,” “0.05 Solar,” and “High $\Sigma$”. Stars are represented by circles drawn on the plots, with the size of the circle corresponding to the size of the star. Stars with masses between 0.05$M_{\odot}$ and 1$M_{\odot}$ are the smallest, intermediate mass stars with $1M_{\odot} < m < 5M_{\odot}$ are larger, and stars with masses greater than 5$M_{\odot}$ are the largest. 14

2.2 Star particle statistics as a function of time for all four runs. The top panel shows the total mass in stars (the top set of lines) and the mass of the most massive star (bottom, dotted set). The middle plot shows $f_{\text{max}}$, the fraction of total stellar mass that is in the most massive star. The bottom plot is the total stellar luminosity. 16

2.3 Cumulative mass distributions from all four runs at $t_{ff} = 0.5$. $f(> m)$ the fraction of the total stellar mass that is in stars with masses greater than m. 19

2.4 Temperature scaling exponent $k_T$ as a function of $\delta$ for the two different values of $\Sigma$ considered in the simulations. 23

2.5 Temperature profiles from the simulations (dots) and analytic theory (solid lines). To show both values of $\Sigma$ on the same plot, we have normalized $r$ by the size of the core $R_c$. The simulations profiles are averaged over $t_{ff} = 0.2$ to $t_{ff} = 0.5$. 25

2.6 Contours of the mean core temperature $T(\bar{\rho})$ at the early stages of collapse as a function of $\Sigma$ and $\delta$ for a core with mass $M = 300M_{\odot}$. The turnover of the $T = 30$ K contour at high $\Sigma$ and $\delta$ is due to $k_T$ becoming large; see Equation (2.24) and Figure 2.4. 27
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Column density through the simulation volume at 6 different times for runs BR (left), BI (middle), and HR (right). Projections are taken along the $x$ direction, and the initial magnetic field is oriented in the positive $z$ direction. We have set the viewing area of the images to be 0.3 by 0.3 pc to show the global evolution of the entire core. Star particles are portrayed as black circles, with the size of the circle corresponding to the mass of the star. The smallest circles represent stars with masses between $0.05 M_\odot$ and $1.0 M_\odot$. The next size up represents masses between $1.0 M_\odot$ and $8.0 M_\odot$, and the largest represents stars with masses greater than $8.0 M_\odot$.</td>
</tr>
<tr>
<td>3.2</td>
<td>Same as Figure 3.1, but zoomed in to show the central 5000 AU around the most massive star in each simulation. Projections are still taken along the $x$ direction through the entire simulation volume.</td>
</tr>
<tr>
<td>3.3</td>
<td>Top - Face-on view of the disk in the high-resolution version of run BI at 0.2 $t_{ff}$. The colors correspond to the column density through a sphere of radius 100 AU centered on the star particle. The arrows show the direction of the mean in-plane velocity of the disk gas. Bottom - the black circles show the mean angular velocity $\omega$ in the disk as a function of cylindrical radius $R_z$. The red line corresponds to a Keplerian profile normalized using the mass of the star. We have also shown the sink particle accretion zone in green to demarcate the radius at which our sink particle algorithm begins to alter the fluid properties.</td>
</tr>
<tr>
<td>3.4</td>
<td>A density slice taken through the center of the computational domain perpendicular to the $x$-axis at 0.3 free-fall times. The $y$- and $z$- components of the magnetic field lines are over-plotted in white with evenly-spaced anchor points along the $y$-axis.</td>
</tr>
<tr>
<td>3.5</td>
<td>Number of stars $N_<em>$ (top), total stellar mass $M_</em>$ (middle), and mass of the most massive star $M_p$ (bottom) for all three runs as a function of free-fall time. In this figure and throughout the rest of this paper, we only count a sink particle as a star if it has passed the minimum merger mass of $0.05 M_\odot$, ensuring its permanence as the simulation proceeds.</td>
</tr>
<tr>
<td>3.6</td>
<td>Fraction of total stellar mass that is in stars with mass less than $m$ for all three runs at $t_{ff} = 0.6$.</td>
</tr>
<tr>
<td>3.7</td>
<td>Maps of the average gas temperature, taken at the same times as Figures 3.1 and 3.2. The averages were taken along the $x$ direction through a 5000 AU cube around the most massive star. Run BR is on the left and run HR is on the right.</td>
</tr>
<tr>
<td>3.8</td>
<td>Left two panels - phase diagrams showing the amount of mass in each $\rho - T_{eff}$ bin at different times, for runs BR and BI. Right two panels - the same, but with $\rho - T_g$ bins for runs BR and HR. The snapshots are taken at the same times as the above figures. The islands of low density material at $T_g \sim 10^2$ K and $T_{eff} \sim 10^4$ K correspond to gas in the ambient medium and should be ignored.</td>
</tr>
</tbody>
</table>
3.9 Histograms of star formation distance $r_*$ for runs BI, HR, and BR. Here, $r_*$ is the distance each star was from the most massive star when it formed, computed for every star that forms over the entire history of each simulation. In run BR, there is only one secondary fragment, which forms a distance of $r_* \approx 3600$ AU from the primary. .......................... 55

3.10 Logarithm of the column density (normalized to $\rho(0) \ell_0$) through the filament in each run at the point at which the maximum density has reached $2 \times 10^7$ times the initial value. Top Left - $J_{\text{max}} = 0.125$. Top Right - $J_{\text{max}} = 0.25$. Bottom Left - $J_{\text{max}} = 0.375$. Bottom Right - $J_{\text{max}} = 0.5$. The top panels have only one filament, while the bottom two show clear signs of artificial fragmentation. Each image shows a region of size $\approx 0.07 \ell_0$. ............................................ 62

4.1 Top - Projections taken through the simulation volume such that the viewing angle of the disk is approximately edge on. From left to right, the physical size of the regions shown are 10,000 AU, 1,000 AU, and 200 AU. The color scale of each window represents gas column density, normalized using the same color bar. The black arrows are the density-weighted mean plane-of-sky components of the magnetic field. The thick white dotted line shows the projected orientation of the disk angular momentum vector. The lighter dotted lines show the 1-σ variation in the disk orientation averaged over the last 5 outer edge orbital periods, to give a sense of the amount of precession. The white contours show the regions where the maximum velocity is greater than 15 km/s. Bottom - Same, but with an approximately face-on viewing angle. To prevent confusion, we have suppressed the outflows and disk orientation vectors in the bottom panels since they are mainly pointed out of the page. ............................................ 67

4.2 The relative orientation of $L_{\text{disk}}$ and $B$ as a function of radius for five different time snapshots. The flat red line corresponds to our initial conditions, in which B is uniform. The grayscale curves indicate different snapshots from our simulation. The lightest curve corresponds to $t = 3,200$ years. From there, the time increases from light to dark in steps of 800 years, until the final time of $t = 6,400$. .... 69

4.3 Density-weighted mean rotational velocity as function of radius for the same time snapshots as Figure 4.2. The red line corresponds to a Keplerian velocity profile associated with a 3.5 solar mass star. Because the mass of the star is different in each snapshot, we have divided each curve by $\sqrt{M/3.5M_\odot}$. .......................... 70

5.1 Logarithm of the total column density $N_H$ through the computational domain along the $x$ direction. The simulation data has been scaled such that the mean $N_H$ is $1.83 \times 10^{21}$ cm$^{-2}$. The size of the box is indicated on the $x$ and $y$ axes. . 76
5.2 Heating (red) and cooling (blue) rates per unit volume versus $T$ for $n_H = 30 \text{ cm}^{-3}$ and the standard chemical abundances shown in Table 5.1. The solid blue curve shows $n_H \Lambda_{\text{tot}}$, while the dashed, dashed-dotted, and dotted curves are $n_H \Lambda_{\text{H}_2}$, $n_H \Lambda_{\text{C}^+}$, and $n_H \Lambda_{\text{O}}$, respectively. The solid, dashed, and dashed-dotted red lines show the mean values of $\Gamma_{\text{Turb}}$, $\Gamma_{\text{PE}}$, and $\Gamma_{\text{CR}}$, respectively.

5.3 Blue - the circles show the mass-weighted distribution of $v_d$ divided by its mean value $\langle v_d \rangle$ from the $M = 3$, $M_A = 0.67$, $R_{\text{AD}}(\ell_0) \approx 1000$ AD simulation. The error bars show the $2\sigma$ temporal variation in distribution over 2 box crossing times, and solid line shows the best-fit log-normal. Green - same, but for the $M = 3$, $M_A = 0.67$ ideal simulation, with $v_d$ computed from equation (5.22). The agreement between the two curves is quite good over more than 3 standard deviations.

5.4 Distribution of $\log v_d$ for our model in physical units. The blue circles are the simulation data, and the solid line is a best-fit normal distribution with a mean of 4.04 and a standard deviation of 0.89.

5.5 Blue solid line - mass-weighted differential distribution function of $\log T$. Green solid line - the volume-weighted distribution of same. Dotted lines - the colors have the same meaning as before, but the distribution of $\log T_{\text{eff}}$ has been plotted instead.

5.6 Same as Figure 5.5, but showing the cumulative rather than the differential distribution functions. Specifically, $P (\ > \log T )$ shows the fraction of the mass or volume with values of $\log T$ greater than the corresponding value on the x-axis. As before, the solid lines show the distribution of $\log T$ and the dotted lines the distribution of $\log T_{\text{eff}}$.

5.7 Left - the color scale shows the fraction of the total mass in each logarithmically-spaced $T - v_d$ bin. The black contours show the region of phase space in which 99, 90, 50, 10, and 1 percent of the CH$^+$ is found. Middle - same, but for $T - n_H$. Right - same, but for $v_d - n_H$.

5.8 Blue - average histogram of CH$^+$ column densities over 2500 lines of sight passing through the simulation volume. The error bars indicate the $1-\sigma$ range of variation over all 50 samples of 50 sight lines each. Red - histogram of the CH$^+$ column densities from the 50 sight line sample in Weselak et al. (2008).

5.9 CH$^+$ column density versus total column density $N_H$ integrated along the same eight sight lines shown in Table 5.3.2. The quantities $N_{\text{CH}^+}(d)$ and $N_H(d)$ are the CH$^+$ and total column densities integrated up though path length $d$ along each ray. The x-axis has been normalized by the total column density $N_H(\ell_0)$ to fit all the rays on the same plot.
5.10 Excitation diagram for the first 5 rotational levels of H$_2$ from our model compared to observed values. The y-axis shows the mean column densities in each rotational level divided by the degeneracy factors $g_J$. The x-axis is the energy of each level expressed as a temperature. Red circles - Lacour et al. (2005). Green circles - Gry et al. (2002). Blue circles - Ingalls et al. (2011). Black circles - our model. We have scaled our results up by a factor of 3 to match the mean H$_2$ column density of the observations. The data have been shifted slightly along the x-axis for clarity. 93

6.1 Turbulent initial conditions for our three main runs. The colors indicate column density, while the mass-weighted, plane-of-sky magnetic field orientations are over-plotted as black arrows. ................. 101

6.2 Density-weighted mean temperature (left) and column density (right) for the Hydro run. Projected sink particle positions have been over-plotted as white dots. 103

6.3 Same as Figure 6.2, but for the Weak MHD run instead. ......................... 104

6.4 Same as Figure 6.2, but for the Strong MHD run instead. ......................... 105

6.5 T-$\rho$ phase plots for all three runs. The columns, from left to right, correspond to the Hydro, Weak, and Strong runs, while the rows, from top to bottom, show the state of the simulations at the points at which 5, 10, 15, and 20 $M_\odot$ of stars have formed. The colors show the amount of mass in each T-$\rho$ bin. ................. 106

6.6 T-$\rho$ phase plots for all three runs. The columns, from left to right, correspond to the Hydro, Weak, and Strong runs, while the rows, from top to bottom, show the state of the simulations at 0.1, 0.2, 0.3, and 0.4 $t_{ff,\rho}$. The colors show the amount of mass in each T-$\rho$ bin. ......................... 107

6.7 Star formation efficiency (SFE) versus free-fall time for the Hydro (blue), Weak (green) and Strong (red) runs. The solid lines are from the high-resolution simulations, the dashed from the low. The black dotted lines demonstrate the slope of the low-resolution curves computed at SFE = 0.02, which we use to determine the star formation rate below. The free-fall time has been computed using the mean density. ......................... 109

6.8 Same as Figure 6.7, but showing the number of stars, $N_*$, instead of the SFE. 110

6.9 The solid lines refer to the median sink particle mass in the Hydro (blue), Weak (green), and Strong (red) runs. The shaded regions correspond to the middle 50% of the sink particle mass distribution - i.e. the bottom edge of the shaded region traces out the 25th percentile mass and the top the 75th percentile mass. ........ 113

6.10 Differential mass distributions for the Hydro, Weak, and Strong runs. The blue histograms are the simulation data, while the black solid and dotted lines are log-normal distributions (Equation (6.6)) with either the simulated value of $m_c$ (solid), or the Chabrier (2005) value (dotted). ......................... 114
6.11 Protostellar mass distributions in our simulations at $M_\ast \approx 20 \, M_\odot$ compared to the theoretical PMFs in McKee & Offner (2010). The blue histograms are the simulation data. The green solid curve is the PMF associated with the TC model, the red solid curve the CA model, and the blue solid curve is the IS model. The green and red dotted curves are the 2CTC and 2CCA PMFs, respectively.

6.12 K-S test results comparing our simulated protostar populations to the 2CTC (solid) and 2CCA (dotted models. The y-axis shows the $p$-value returned by the test, and the x-axis shows time.

6.13 Top - zoomed in views of the four most massive protostars in the Strong field calculation at $t = 0.4 \, t_{\text{ff}}$. The window size has been set to 3000 AU. The color scale shows the logarithm of the column density, and the black arrows show the mass-weighted, plane-of-sky magnetic field vectors. The masses of the protostars have been indicated in each panel. Bottom - same, but for the Weak MHD run.

6.14 Same as Figure 6.13, except convolved with a 1000 AU Gaussian beam. The size of the beam is indicated by the white circle.

6.15 Plots of $\dot{m}_\ast$ versus $m_\ast$ for the four most massive protostars in each run. The solid colored lines correspond to the individual protostars, while the black solid line is the average $\dot{m}_\ast$ of all four. The black dashed line is the theoretical prediction of the TC model (see text), while the black dotted line is the best $\dot{m}_\ast \propto m_\ast^{1/2}$ power-law fit to the data. These lines overlap almost exactly for the Hydro case. The accretion rates have been smoothed over a 500-year timescale for clarity.

6.16 Stellar density $n_\ast$ versus stellar mass $m_\ast$ for all the stars in our simulations. The blue circles correspond to the Strong run, the green plus symbols to Weak, and the red crosses to Hydro. The colored lines show the median stellar density in each run.

6.17 Same as Figure 6.16, but for the stellar surface density $N_\ast$ instead.
List of Tables

2.1 Simulation Parameters ................................................................. 12
2.2 K-S test results. ................................................................. 18
2.3 Light-to-Mass Ratios ................................................................. 22
3.1 Simulation Parameters ................................................................. 36
5.1 Standard Physical and Chemical Model Parameters .......................... 78
5.2 Data from 8 randomly selected rays cast through the problem domain. .... 91
6.1 Simulation Parameters ................................................................. 99
6.2 Summary of the star formation in each run. .................................. 112
6.3 Stellar surface density around > 1 $M_{\odot}$ versus < 1 $M_{\odot}$ stars ........ 123
6.4 Multiple star systems ................................................................. 125
Acknowledgments

First and foremost, I would like to thank Tom Myers, Nancy Myers, Anna Myers, and Heather Hardison for many years of love and support, and to apologize to Heather in particular for too many late nights.

This dissertation would not exist without hours of stimulating discussions with my mentors, colleagues, and collaborators, including, but not limited to: Chris McKee, Richard Klein, Mark Krumholz, Andrew Cunningham, Pak Shing Li, Charles Hansen, Stella Offner, Jeff Oishi, Nathan Roth, and Aaron Lee. I would also like to thank my qualifying exam and dissertation committee members: Stuart Bale, Leo Blitz, and Dan Kasen, for their guidance and patience. I have benefited greatly from a number of gifted and conscientious teachers in Berkeley Physics and Astronomy Departments, particularly Steven Stahler and Eugene Chiang. The support staff in the Physics and Astronomy Departments have greatly eased my path as a graduate student, despite my best efforts to the contrary. I would particularly like to thank Anne Takizawa and Claudia Trujillo for digging me out of multiple ditches on multiple occasions. I also thank Peter Williams for his work on the \texttt{ucastrothesis} \LaTeX{} template, which greatly simplified the preparation of this manuscript. Finally, I would like to thank Matt Turk and the other \texttt{yt} developers for years of hard work that greatly expedited the work presented in this thesis.

This work would not have been possible without the generous support of the National Science Foundation and the United States Department of Energy. Computational resources were provided by NASA Advanced Supercomputing, the National Institute for Computational Science, and the National Energy Research Scientific Computer Center.
Chapter 1

Introduction

The theoretical principles that underly the structure and evolution of mature stars have been relatively well-understood now for many decades. The theory of stellar birth, on the other hand, remains an incompletely understood problem, despite its importance to many different fields of modern astrophysics. For instance, it is well-known that a star’s mass largely determines its subsequent evolution, including whether or not it will end its life as a supernova. Questions like “What sets a star’s initial mass?”, on the other hand, or even “What physical processes are most relevant to that question?”, are still debated in the literature.

The principal difficulty in formulating a theory of star formation is the variety and complexity of the different physical processes involved. The giant molecular clouds (GMCs) of gas and dust out of which stars form are turbulent, magnetized, self-gravitating fluids in which the gravitational, magnetic, and kinetic energies are all of comparable importance. In addition, stars exert a variety of different feedback processes onto their parent clouds as they assemble, in forms such as ionizing and non-ionizing radiation and protostellar outflows. These feedback processes in turn alter the chemical, thermal, and mechanical state of the gas, and likely play a role in determining both the star formation rate and stellar initial mass function, two of the most important observables of the star formation process. Furthermore, the interplay of these processes can give rise to effects that one would not notice when considering them in isolation. Because of this complexity, numerical simulation offers an attractive way forward.

An additional issue that contributes to the intractability of problems in star formation is the vast dynamic range involved. A typical molecular cloud might have a particle density of \( n \sim 10^2 \text{ cm}^{-3} \), but the Sun has a particle density of \( n \sim 10^{24} \text{ cm}^{-3} \), a span of 22 orders of magnitude. Even a first core may have a particle density of some \( 10^{11} \text{ cm}^{-3} \). A star-forming cloud may have many such cores, but a mean density that remains \( \sim 10^2 \text{ cm}^{-3} \). This renders adaptive methods that can follow the collapse of isolated regions without wasting computational effort on the bulk of the cloud especially attractive.

In my thesis, I investigate a variety of issues in the theory of star formation and the interstellar medium. A common theme will be the use of numerical techniques to treat multiple physical effects simultaneously, with a focus on magnetic fields, turbulence, and gas...
thermodynamics. In this introductory chapter, I briefly describe how each chapter of my thesis relates to broader issues in the field.

### 1.1 The Initial Mass Function

One of the most basic observable properties of a population of stars is the distribution of stellar masses. For a population of zero-age stars (where age is defined from the time the stars enter the main sequence), this is called the initial mass function (IMF). The IMF is frequently expressed as a number of stars per logarithmic bin around mass $m$

$$\Psi(\log m) = dN/d\log m.$$  \hfill (1.1)

In 1955, Salpeter proposed a power-law for the IMF for stars more massive than the Sun:

$$\Psi(\log m) \propto m^{-1.35}.$$  \hfill (1.2)

As observations improved, it became clear that the IMF was not a single power-law over all masses, but that there exists a characteristic stellar mass, $m_c$, such that the IMF turns over at a few tenths of a solar mass. This functional form is commonly fit by either a multiple-part power law (e.g. Kroupa 2002) or a log-normal distribution with a Salpeter tail at masses larger than $1 M_\odot$. (e.g. Chabrier 2005). Today, this basic form for the IMF has been observed in wide variety of Galactic environments, from disk stars, to open clusters like the Pleiades, to embedded regions such as the Orion Nebula Cluster still undergoing active star formation, and even to globular clusters (Bastian et al. 2010), and has been observed in nearby Milky Way satellites as well (Sabbi et al. 2008; Andersen et al. 2009).

A particularly interesting feature of the IMF is that it appears to be remarkably invariant across such a wide range of environments. Chapter 1 addresses one particular aspect of this problem: the apparent invariance of the IMF with respect to gas metallicity $Z_\odot$. This behavior is puzzling, since the presence of metals directly impacts the amount of dust, and the dust is the dominant opacity source in the neutral interstellar medium (ISM). The dust opacity is frequently considered important for setting $m_c$, whether by controlling the density at which the gas becomes optically thick to its own cooling radiation (Low & Lynden-Bell 1976), the coupling between the gas and dust (Larson 2005; Elmegreen et al. 2008a), or the efficiency of feedback due to protostellar heating (Bate 2009b; Krumholz et al. 2010). In chapter 1 I argue, using a mix of numerical and semi-analytic techniques, that if the temperature structure in star-forming clouds is primarily determined by protostellar feedback, the characteristic fragment mass should depend only weakly on the gas metallicity.

### 1.2 The Core Mass Function

Another interesting feature of the IMF is the extent to which it resembles the mass function for dense *cores* (the CMF) - roughly spherical, over-dense regions in star-forming...
1.3. THE MAGNETIC BRAKING “CATASTROPHE”

clouds identified using either mid-infrared extinction (e.g. Alves et al. 2007) or thermal dust emission (e.g. Reid & Wilson 2006; Nutter & Ward-Thompson 2007; Enoch et al. 2008). The functional forms of these distributions are very similar, but with the IMF shifted down in mass by a factor of a few, which is usually explained as a core efficiency factor accounting for the fraction of mass ejected from the core by outflows. One possible interpretation of these observations is that whatever processes are responsible for setting the IMF operate in the gas phase, with each core collapsing to form a single star or possibly a low-order multiple system.

However, this correspondence between the CMF and IMF continues up to very high mass cores (Reid & Wilson 2006) which contain many thermal Jeans masses, $M_J$, of material, where

$$M_J = \frac{\pi}{6} \frac{c_s^3}{G^{3/2} \rho^{1/2}} \approx 0.1 \left( \frac{T}{10 \text{ K}} \right)^{3/2} \left( \frac{n}{10^6 \text{ cm}^{-3}} \right)^{-1/2}.$$ (1.3)

A typical massive core may have a mass of $\sim 100 M_\odot$ and density of $n \sim 10^6 \text{ cm}^{-3}$ (Swift 2009), and thus contains $\sim 1000$ Jeans masses. A possible challenge to the above scenario, then, is that such cores should fragment strongly as the collapse, forming a small cluster instead of a single system. In Chapter 3, I address this question using radiation-magnetohydrodynamic simulations of massive core collapse. This work suggests that, when the effects of magnetic fields and protostellar feedback are both taken into account, massive cores with typical field strengths do not fragment strongly.

1.3 The Magnetic Braking “Catastrophe”

The individual cores from which stars form are magnetized and in general contain some initial angular momentum. As a core collapses, material that falls into the center tends to spin up, which twists the magnetic field lines. The field lines resist this torsion, which has the effect of removing angular momentum from the central region of the core, a phenomenon known as magnetic braking. Ideal MHD simulations of core collapse have frequently found that this effect is so efficient that it prevents the formation of a Keplerian disk in cores with the observed degrees of spin and magnetization (e.g. Price & Bate 2007; Mellon & Li 2008; Krasnopolsky et al. 2012). Because disks are commonly observed around young stars (Haisch et al. 2001) and brown dwarfs (Luhman et al. 2005), this failure of ideal magnetohydrodynamics to reproduce the observed disk fraction has sometimes been termed the “Magnetic Braking Catastrophe.”

Recently, the extent of this failure has been called into question, and simulations that include either misalignment between the core magnetic field and angular momentum vectors (Hennebelle & Ciardi 2009) or significant turbulence (Seifried et al. 2012; Santos-Lima et al. 2012b), seem to form disks for realistic magnetizations, although the precise mechanism by which this occurs is currently under debate (Seifried et al. 2013; Santos-Lima et al. 2013). In Chapter 3, I report the formation of a Keplerian disk around a massive star in an ideal MHD simulation, and in Chapter 4 I investigate the detailed structure of the magnetic field around such a disk. This is particularly interesting in light of the Atacama Large Millimeter...
1.4. THE CH$^+$ PROBLEM

The interstellar medium is home to a bewildering variety of molecular species. The CH$^+$ ion was one of the first of these molecules detected (Douglas & Herzberg 1941), and today is frequently observed along diffuse molecular sight lines in absorption against bright background O and B stars (e.g. Gredel et al. 1993; Gredel 1997; Crane et al. 1995; Weselak et al. 2008; Sheffer et al. 2008). The high abundance of CH$^+$ has remained a problem for chemical models of diffuse clouds, however, because the reaction that forms it:

$$C^+ + H_2 \rightarrow CH^+ + H \quad \Delta E/k = -4640\text{K},$$  \hfill (1.4)

has a high energy barrier and cannot proceed at the $\sim 100$ K temperatures expected in such clouds.

The pioneering work of Falgarone & Puget (1995) suggested a potential answer to this puzzle: that the intermittent dissipation of interstellar turbulence provides the energy source to power reaction 5.1. Later semi-analytic models based on this principle (e.g. Joulain et al. 1998; Godard et al. 2009) have had a large degree of success in reproducing large column densities of CH$^+$. However, these models frequently assume constant density and involve tunable free parameters. In Chapter 5, I compute the CH$^+$ abundance in a turbulent, diffuse molecular cloud by post-processing numerical simulations of magnetohydrodynamic turbulence. The results agree well with observations of the column densities of CH$^+$ and of rotationally-excited H$_2$ and largely confirm the basic picture originally proposed in Falgarone & Puget (1995).

1.5 Star Formation on the Cluster Scale

Many theoretical (e.g. Shu 1977; McKee & Tan 2002, 2003) and numerical (Hennebelle & Fromang 2008; Krumholz et al. 2007b, 2010; Myers et al. 2011; Cunningham et al. 2011; Myers et al. 2013) models of star formation consider stars forming in isolation. However, this is not the mode of star formation most commonly encountered in nature: stars generally form in the presence of other stars (Lada & Lada 2003). Interaction affects can be important; for example, Krumholz et al. (2012b) found that the presence of only a few massive stars affected the temperature structure of the entire cluster. While theoretical models of star formation in isolation may be a helpful building block, a true understanding of star formation will require consideration of the more characteristic clustered mode of formation.

In the last chapter of my thesis, I present simulations of star cluster formation in magnetized, dense molecular cloud clumps. These simulations include the effects of both radiative and outflow feedback and are aimed at quantifying the effects of the magnetic
field strength on the resulting cluster properties. I find that magnetic fields of the typically observed strengths reduce the star formation rate by a factor of $\approx 2.4$, reduce the number of fragments by a factor of $\approx 2$, and increase the characteristic fragment mass by a factor of $\approx 2.4$ relative to the zero-field case. These simulations also reproduce many of the observed features of stellar clusters, including the pattern of mass segregation observed in the Orion Nebula Cluster and the companion frequency observed in nearby Class I sources.
Chapter 2

Metallicity and the Universality of the IMF

Abstract

The stellar initial mass function (IMF), along with the star formation rate, is one of the fundamental properties that any theory of star formation must explain. An interesting feature of the IMF is that it appears to be remarkably universal across a wide range of environments. Particularly, there appears to be little variation in either the characteristic mass of the IMF or its high-mass tail between clusters with different metallicities. Previous attempts to understand this apparent independence of metallicity have not accounted for radiation feedback from high-mass protostars, which can dominate the energy balance of the gas in star-forming regions. We extend this work, showing that the fragmentation of molecular gas should depend only weakly on the amount of dust present, even when the primary heating source is radiation from massive protostars. First, we report a series of core collapse simulations using the ORION AMR code that systematically vary the dust opacity and show explicitly that this has little effect on the temperature or fragmentation of the gas. Then, we provide an analytic argument for why the IMF varies so little in observed star clusters, even as the metallicity varies by a factor of 100.  

2.1 Introduction

The stellar initial mass function - the distribution of stellar masses at birth - appears to be remarkably constant across a wide range of star-forming environments. While there is some evidence that the IMF is different in giant elliptical (van Dokkum & Conroy 2010; Treu et al. 2010) and dwarf galaxies (Lee et al. 2009), in our own galaxy the IMF is practically universal.
(Kroupa 2002; Chabrier 2003; Bastian et al. 2010). In particular, while zero-metallicity stars likely had a much different mass distribution than present-day stars (Abel et al. 2002; Bromm et al. 2002), the IMFs observed for population I and II stars appear to vary little with the metallicity of the star-forming region. Bastian et al. (2010) review the literature on this topic and find no evidence for a systemic dependence of the IMF on metallicity. For example, in the Milky Way disk, extreme outer galaxy clusters with metallicities of $\sim 0.1$ solar have the same IMF as nearby clusters with approximately solar metallicity (Yasui et al. 2008). Likewise, globular clusters, which have metallicities that are often 10-100 times lower than the solar value, have the same IMF as regions of present-day star formation once dynamical evolution is taken into account (De Marchi et al. 2000, 2010). Outside our own galaxy, Schmalzl et al. (2008) and Sirianni et al. (2002) find no difference between the Milky Way disk IMF and that of the Small Magellenic Cloud, which has a metallicity of $\sim 0.2$ solar. In short, the IMF appears to be insensitive to metallicity across a wide range galactic environments, and understanding why is an important challenge for theories of star formation.

One potential explanation was offered by Elmegreen et al. (2008b). They focus on the characteristic mass of the IMF plateau, which should be related to the Jeans mass $M_J \propto T^{3/2} \rho^{-1/2}$ of the region. They calculate $M_J$ for a prestellar core under the assumption that $T$ is set by the balance between molecular cooling and heating from gas-dust coupling, finding that for a given dust temperature, the gas temperature depends only weakly on metallicity. However, this skirts an important question: what if the dust temperature itself is a strong function of metallicity? In regions where stars are already forming, radiative feedback should dominate the dust’s energy balance even before the onset of nuclear burning, especially when the stars are massive. This is not a minor effect. Most stars are born in clusters (Lada & Lada 2003), and the cluster mass function is $dN/dM \propto M^{-2}$ (Zhang & Fall 1999; Fall et al. 2009; Chandar et al. 2010), which implies equal mass per logarithmic bin. Thus, 2/3 to 3/4 of stars are born in clusters of 1000 $M_\odot$ or more, and essentially all of these clusters are expected to contain at least one early O star. Hence, the majority of stars do not form in isolated environments like Elmegreen et al. consider, but form rather in the presence of a massive star that will affect the gas temperature distribution. If this heating were strongly metallicity dependent, then $M_J$ would be as well.

Other work has considered the effect of protostellar feedback on the form and apparent universality of the IMF. Bate (2009b) argued that the lack of variation in the IMF is the result of self-regulating feedback from radiating protostars, but did not explain why this effect should be independent of metallicity when the material around the stars is optically thick. Additionally, while Bate includes radiative feedback, he underestimates the luminosity by a factor of 20 and the temperature in the core by more than a factor of 2 (Offner et al. 2009). He also does not form any massive stars, which if present would dominate the heating rate.

High-mass stars are known to strongly affect the temperature and fragmentation of molecular gas. Heating from their extremely high luminosities, which can reach $10^5$ times the solar value, is capable of raising $M_J$ significantly and preventing further fragmentation. Observations using ground-based interferometers reveal that a number of $\sim 100 M_\odot$ massive cores appear to remain single, compact objects when observed at 1700 AU resolution, even
though the cores initially contained roughly $10^3$ Jeans masses worth of material (Bontemps et al. 2010). Similarly, Longmore et al. (2011) show observations of a young, massive star-forming region within the IRAS 18032-2137 complex that suggest a typical fragment size of $> 1M_\odot$. Numerical simulations (Krumholz et al. 2010, hereafter KCKM10) find that this radiative suppression of fragmentation is most pronounced in regions of high surface density, where the accretion rates are high and the core is effective at trapping radiation in the optically thick regions near protostars. Krumholz & McKee (2008) argued for an effective threshold surface density above which a region will fragment into a few massive objects rather than a few small ones. This potentially explains why the IMF corresponds so well to the core mass function even at the high-mass end, where based on isothermal assumptions the core should fragment into many small objects instead of a few massive ones.

However, the strength of this effect could in principle depend on the opacity of the dust, since that determines the matter-radiation coupling, and hence on the metallicity of the region. It is not obvious that the above effect would work at all in globular clusters, which have far fewer metals, and presumably far less dust, than present-day Milky Way star-forming regions. The purpose of this paper is to gauge the importance of metallicity to the fragmentation of star-forming molecular gas at small scales, where the collapse is highly non-isothermal due to radiative feedback. To do this, we have conducted a series of core collapse simulations where we vary the dust opacity and show that it makes little difference to the core’s temperature or its fragmentation. We also provide a simple analytic argument based on the work of Chakrabarti & McKee (2005) for why the temperature profiles of prestellar cores should depend only weakly on the dust opacity. This work sheds light on why the IMF should be so similar in regions with different metallicity, even when radiation feedback from massive stars on the gas cannot be ignored.

The outline of the paper is as follows. In Section 2.2 we describe our simulation setup, including the initial conditions and numerical methods used. In Section 2.3 we show the results of our simulations, which demonstrate how fragmentation is insensitive to metallicity variations of a factor of 20. In Section 2.4 we apply the work of Chakrabarti & McKee (2005) to show that the temperature profile of a dusty, centrally-heated core should depend only weakly on its metal content. Finally, we present our conclusions in Section 2.5.

### 2.2 Simulation Setup

#### 2.2.1 Numerical Techniques

To perform our simulations, we have used ORION (Truelove et al. 1998; Klein 1999), a parallel, adaptive mesh, radiation-hydrodynamics code for astrophysical applications. The equations and methods used are almost identical to those in KCKM10, so we describe them only briefly here, emphasizing the differences. In short, ORION tracks the four conserved quantities of the combined gas-radiation fluid: the density $\rho$, the momentum per unit volume $\rho v$, the non-gravitational gas energy density $\rho e$, and the radiation energy density $E$. It
updates these quantities conservatively, including the effects of gravity and diffuse radiation, but not magnetic fields or ionizing radiation. The radiation transport is solved in the flux-limited diffusion approximation using the mixed-frame formulation (Mihalas & Klein 1982), retaining terms of order $v/c$. For a full description of the equations solved by the radiation module as well as accuracy tests of the mixed-frame treatment, see Krumholz et al. (2007b) and Shestakov & Offner (2008).

In addition to the gas and radiation, the simulation domain also includes star particles, which are placed on fine level grids whenever the Jeans density in a cell exceeds a critical value (see below). Star particles interact with the gas by accreting mass from the simulation volume according to the algorithms in Krumholz et al. (2004) and emitting radiation according to the protostellar model described in the appendices of Offner et al. (2009). The full set of equations solved for the fluid is

\[
\frac{\partial}{\partial t} \rho = -\nabla \cdot (\rho v) - \sum_i \dot{M}_i W(x - x_i), \tag{2.1}
\]

\[
\frac{\partial}{\partial t} (\rho v) = -\nabla \cdot (\rho vv) - \nabla P - \rho \nabla \phi - \lambda \nabla E - \sum_i \dot{p}_i W(x - x_i), \tag{2.2}
\]

\[
\frac{\partial}{\partial t} (\rho e) = -\nabla \cdot [(\rho e + P)v] - \rho v \cdot \nabla \phi - \kappa_{0P} \rho (4\pi B - cE) + \lambda \left( \frac{2\kappa_{0P}}{\kappa_{0R}} - 1 \right) v \cdot \nabla E - \sum_i \dot{\mathcal{E}}_i W(x - x_i), \tag{2.3}
\]

\[
\frac{\partial}{\partial t} E = \nabla \cdot \left( \frac{c \lambda}{\kappa_{0R} \rho} \nabla E \right) + \kappa_{0P} \rho (4\pi B - cE) - \lambda \left( \frac{2\kappa_{0P}}{\kappa_{0R}} - 1 \right) v \cdot \nabla E - \nabla \cdot \left( \frac{3 - R_2}{2} v E \right) + \sum_i L_i W(x - x_i), \tag{2.4}
\]

where $\kappa_{0R}$ and $\kappa_{0P}$ are the Planck and Rosseland mean opacities computed in the co-moving frame of the fluid and $B = c a_R T_g^4/(4\pi)$ is the Planck function. The flux limiter $\lambda$ and $R_2$, which is related to the Eddington factor, are dimensionless numbers that enter into our approximation of the radiative transfer. For details, refer to Krumholz et al. (2007b) and the references therein.

In the above equations, the sums are taken over all the particles in the simulation. $L_i$ is the luminosity of star $i$, while $\dot{M}_i$, $\dot{p}_i$, and $\dot{\mathcal{E}}_i$ are the rates at which mass, momentum, and energy are transferred from gas to stars. $W(x - x_i)$ is a kernel that distributes the transfer
over a radius of 4 fine level cells around the star. The star particles are updated according to

\[
\frac{d}{dt} M_i = \dot{M}_i, \quad (2.5)
\]

\[
\frac{d}{dt} x_i = \frac{p_i}{M_i}, \quad (2.6)
\]

\[
\frac{d}{dt} p_i = -M_i \nabla \phi + \dot{p}_i, \quad (2.7)
\]

where \( \phi \) is the gravitational potential given by

\[
\nabla^2 \phi = -4\pi G \left[ \rho + \sum_i M_i \delta(x - x_i) \right]. \quad (2.8)
\]

Note that we do not include the effects of protostellar outflows; for discussion of high-mass star formation with outflows, see Cunningham et al. (2011).

We adopt a polytropic equation of state:

\[
P = \frac{\rho k_B T_g}{\mu m_{\text{H}}} = (\gamma - 1) \rho \left( e - \frac{v^2}{2} \right), \quad (2.9)
\]

where \( T_g \) is the gas temperature, \( \mu = 2.33 \) is the mean molecular weight for molecular gas with cosmic abundances, and \( \gamma \) is the ratio of specific heats. We have taken \( \gamma = 5/3 \), appropriate for a gas of molecular hydrogen with the rotational levels frozen out, but this choice is essentially irrelevant because \( T_g \) is determined primarily by radiative effects.

The above is all identical to KCKM10. The only differences between the numerical schemes employed in those simulations and ours are:

1. We have used the dust opacity model described in Cunningham et al. (2011), based on the work of Semenov et al. (2003). This opacity model was created with stellar winds in mind and includes line cooling effects at high temperatures. These effects are irrelevant at \( T_g \lesssim 1000 \) K, which includes practically all of the gas under discussion here.

2. We have turned off mergers between sink particles with masses greater than \( 0.05 \, M_\odot \).

3. We have modified the Planck and Rosseland mean dust opacities by a multiplicative constant \( \delta \) to allow for dust-to-gas mass ratios other than solar.

Item (2) requires some discussion. As mentioned above, each sink particle is surrounded by an accretion zone of 4 fine level cells from which it draws gas. In KCKM10, if a sink particle moved within another sink’s accretion zone, the particles would merge regardless of their masses. Because the finest resolution in our simulations is \( \sim 7 \) AU, this would mean that any stars that ever moved within a distance of 28 AU would be combined. Because this may not be realistic, we have imposed a mass limit of \( 0.05 \, M_\odot \) above which sink particles will no longer merge. This limit is chosen to roughly correspond to the mass at which a
protostar’s core temperature becomes high enough to dissociate molecular hydrogen and initiate second collapse (Masunaga et al. 1998; Masunaga & Inutsuka 2000). Before this, the sinks are more like extended balls of gas with radii of a few AU than stars, so it is more likely that they will merge. After second collapse, the sinks represent objects that are much smaller, and mergers should be less likely. Although our initial conditions are also slightly different (see below), this choice accounts for most of the differences between the simulations reported here and those in KCKM10.

2.2.2 Refinement criteria

The computational domain is a cube of side $L_{\text{box}}$ that is discretized into a coarse grid of $N_0$ cells, so that the resolution on the coarse grid is $x_0 = L_{\text{box}}/N_0$. The AMR functionality of ORION automatically identifies regions that need more resolution and covers them with a finer grid. With $L$ levels of refinement and a refinement ratio of 2, the resolution of the finest level is $\Delta x_L = x_0/2^L$. In this work, we have chosen these parameters such that $\Delta x_L$ is always $\sim 7\ AU$.

In generating the grids, a cell is tagged for refinement if

1. It is within a distance $16\Delta x_L$ of the nearest star particle.
2. It has a density greater than the Jeans density, given by

$$\rho_J = J^2 \frac{\pi c_s^2}{G\Delta x_L^2},$$

where $c_s$ is the sound speed and we use $J = 1/8$ (Truelove et al. 1997).
3. The local gradient in the radiation energy is greater than a critical value, given by

$$0.15 \frac{E}{\Delta x_L}.$$ 

This procedure is repeated recursively until the final level is reached. Taken together, these conditions ensure that the regions near the star particles are always tracked with the highest available numerical resolution.

2.2.3 Initial conditions

Our initial conditions also follow the approach laid out in KCKM10, and have been chosen to resemble the structures from which massive stars are believed to form. Observations of the internal structure of infrared dark clouds (IRDCs) using sub-mm interferometry reveal the presence of peaks in the local density distribution termed massive cores (Swift 2009). They are measured to have masses in the range of $\sim 100 M_\odot$, radii of about $\sim 0.1\ pc$, and temperatures of about $\sim 20\ K$. Massive cores are observed to be centrally concentrated, and unlike low-mass prestellar cores are highly turbulent, with virial ratios of order unity.
Table 2.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>( \delta )</th>
<th>( M )</th>
<th>( \Sigma )</th>
<th>( R )</th>
<th>( \sigma_v )</th>
<th>( t_{ff} )</th>
<th>( L_{\text{box}} )</th>
<th>( L )</th>
<th>( N_0 )</th>
<th>( \Delta x_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>1.0</td>
<td>300</td>
<td>2.0</td>
<td>0.1</td>
<td>3.59</td>
<td>30.2</td>
<td>0.40</td>
<td>6</td>
<td>192</td>
<td>6.71</td>
</tr>
<tr>
<td>0.2 Solar</td>
<td>0.2</td>
<td>300</td>
<td>2.0</td>
<td>0.1</td>
<td>3.59</td>
<td>30.2</td>
<td>0.40</td>
<td>6</td>
<td>192</td>
<td>6.71</td>
</tr>
<tr>
<td>0.05 Solar</td>
<td>0.05</td>
<td>300</td>
<td>2.0</td>
<td>0.1</td>
<td>3.59</td>
<td>30.2</td>
<td>0.40</td>
<td>6</td>
<td>192</td>
<td>6.71</td>
</tr>
<tr>
<td>High ( \Sigma )</td>
<td>0.2</td>
<td>300</td>
<td>10.0</td>
<td>0.045</td>
<td>5.37</td>
<td>9.03</td>
<td>0.18</td>
<td>5</td>
<td>168</td>
<td>6.85</td>
</tr>
</tbody>
</table>

Note. — Col. 8: linear size of computational domain. Col. 9: maximum refinement level. Col. 10: number of cells per linear dimension on the coarsest level. Col. 11: linear cell size on the finest level.

To model these objects, we initialize the simulation volume to contain a sphere of gas with radius \( R \), total mass \( M \), surface density \( \Sigma = M/\pi R^2 \), and constant initial temperature \( T_{\text{core}} = 20 \text{ K} \). Following the theoretical models of McKee & Tan (2003), we give the core a power law density profile of

\[
\rho(r) \propto r^{-k_{\rho}} \tag{2.12}
\]

where we have taken \( k_{\rho} \) to be 1.5. This is consistent with observations of clumps of molecular gas - structures a few pc in size with thousands of solar masses of material - which reveal power-law density profiles with scaling exponents between 1 and 2 (Beuther et al. (2007), Caselli & Myers (1995), Mueller et al. (2002)), and with higher resolution observations of individual cores, which find power law density profiles with slopes between 1.5 and 2 (Longmore et al. 2011; Zhang et al. 2009). We stress, however, that the artificiality of the initial conditions, in which the density initially lacks structure and the core is considered in isolation, is a potential source of uncertainty.

The cores are placed in cubic volume with \( L_{\text{box}} = 4 \times R \), surrounded by a background medium with density \( \rho_{\text{bg}} = 0.01 \times \rho_{\text{edge}} \), where \( \rho_{\text{edge}} \) is the density at the edge of the initial core:

\[
\rho_{\text{edge}} = \left( \frac{3 - k_{\rho}}{4\pi} \right) \frac{M}{R^3} \tag{2.13}
\]

To maintain pressure balance, the temperature of the background gas is set to \( T_{\text{bg}} = 100 \times T_{\text{core}} = 2000 \text{ K} \). The opacity of the ambient gas is set to a tiny value to ensure that it does not interfere with the temperature structure of the collapsing core.

To mimic the effects of turbulence, we also give the cores an initial Gaussian random velocity field with a power spectrum \( P(k) \propto k^{-2} \). We scale each component of the velocity to have a one-dimensional root-mean-squared dispersion, \( \sigma_v \), equal to the velocity at the surface
of a singular polytropic sphere (McKee & Tan 2003):

\[
\sigma_v = \sqrt{\frac{GM}{R(2k_\rho - 1)}}. \tag{2.14}
\]

The corresponding virial parameter \( \alpha_{\text{vir}} = 5\sigma_v^2 G M / R \) (Bertoldi & McKee 1992) is 5 for \( k_\rho = 3/2 \), so that the turbulent kinetic energy is initially larger than the gravitational potential energy. However, the turbulence decays as the core collapses, so the virial parameter drops.

Starting from these conditions, we have performed a series of four core collapse simulations, summarized in Table 2.1. For three of the runs, we chose a surface density of \( \Sigma = 2 \, \text{g cm}^{-2} \), which is comparable to the surface densities observed in massive star-forming regions in the Galaxy. We vary the parameter \( \delta \), which represents the dust-to-gas mass ratio relative to that expected at solar composition. The values of \( \delta = 1.0, 0.2, \) and 0.05 are representative of nearby star clusters, extreme outer galaxy star clusters, and low-metal globular clusters, respectively. We have also conducted a run at \( \Sigma = 10 \, \text{g cm}^{-2}, \delta = 0.2 \), which is characteristic of extragalactic super star clusters. Motivated by the above observations, we have chosen to set the mass \( M \) of all our cores to \( 300 M_\odot \). This is higher than in KCKM10, but it will allow us to form massive stars more quickly and thereby gauge the effect of radiative feedback at less computational expense. Because the mass of the cores is the same in the \( \Sigma = 2 \, \text{g cm}^{-2} \) and \( \Sigma = 10 \, \text{g cm}^{-2} \) runs, the High \( \Sigma \) run has a slightly smaller radius. In the rest of the paper, we will refer to these runs by the names given in Table 2.1.

We run all the simulations out to \( 0.5 t_{\text{ff}} \), where \( t_{\text{ff}} \) is the gravitational free-fall time evaluated at the mean density \( \bar{\rho} = 3M/4\pi R^3 \):

\[
t_{\text{ff}} = \sqrt{\frac{3\pi}{32G\bar{\rho}}}. \tag{2.15}
\]

By this time, the basic similarity of all the runs has been established. We emphasize that in general, the numerical methods and resolution have been held constant across all the runs, so that any differences between them should be attributable to variation in \( \delta \) or \( \Sigma \).

### 2.3 Results

#### 2.3.1 Temperature and density structure

Figure 2.1 shows the result of evolving the above cores out to half a global free-fall time. The leftmost panels show the column density in units of \( \text{g cm}^{-2} \), zoomed out to show the entire simulation volume. The large scale morphology of the collapse is practically identical between the \( \Sigma = 2 \, \text{g cm}^{-2} \) runs, as that is set primarily by the magnitude of the initial velocity perturbations. The High \( \Sigma \) run shows a slight tendency towards more filamentary structure than the others, since the initial Mach number must be larger to pump the higher
2.3. RESULTS

Figure 2.1: Projections through the simulation volumes at $t_{ff} = 0.5$. The left panels show the column density of the entire core, defined as $\int \rho dx$. The middle column is also the column density, zoomed in to show the middle 5000 AU. The right column shows the column density-weighted temperature, $\frac{\int \rho T_{\text{gas}} dx}{\int \rho dx}$, at the same scale. The rows, from top to bottom, show runs “Solar,” “0.2 Solar,” “0.05 Solar,” and “High $\Sigma$”. Stars are represented by circles drawn on the plots, with the size of the circle corresponding to the size of the star. Stars with masses between $0.05M_\odot$ and $1M_\odot$ are the smallest, intermediate mass stars with $1M_\odot < m < 5M_\odot$ are larger, and stars with masses greater than $5M_\odot$ are the largest.
surface density cloud into virial equilibrium. Nonetheless, the differences between the High $\Sigma$ run and the others are only minor at this scale.

The middle panels again show the column density, with the same units and color scale as before, but zoomed in to show the central 5000 AU around the most massive object. At this scale, some differences between the runs are apparent. In particular, the shape of the accretion flow around the stars appears to be different in the three $\Sigma = 2 \, \text{g cm}^{-2}$ runs. This is because radiation pressure can be important near the massive star(s), and its magnitude varies with the opacity.

The rightmost panels show the column density-weighted gas temperature at the same 5000 AU scale. The most striking feature is that the temperature of the gas surrounding the central condensation of stars is quite similar in the three $\Sigma = 2 \, \text{g cm}^{-2}$ runs. Changing the opacity by a factor of twenty appears to have little impact on the temperature of the bulk of the gas. On the other hand, the High $\Sigma$ run has noticeably more gas heated to a hundred degrees or higher. The “wall” of hot gas visible on the left-hand side of some of the runs is the hot, diffuse medium that was initially outside the core; it is more visible in the High $\Sigma$ run because the core is smaller, so that the 5000 AU frame captures a larger fraction of the volume. The stars are near the edge of the core at $t_{\text{ff}} = 0.5$ in all the runs because the random velocity perturbations we used happened to advect them that way. With a different realization of the turbulence, this would not necessarily happen.

### 2.3.2 Fragmentation and Star Formation

At the end of 0.5 free-fall times, about 40 $M_\odot$ of gas has been turned into stars, or about 13% of the total core. At that point, all the runs have a massive star of $\sim 10 M_\odot$. In the three $\Sigma = 2.0 \, \text{g cm}^{-2}$ runs, this star forms a binary system with a massive companion of $\sim 6 M_\odot$. In the High $\Sigma$ run, the secondary is only $3 M_\odot$, with the missing mass spread out among the other objects.

A major difference between these simulations and those of KCKM10 and Cunningham et al. (2011), is that we form many more objects during the collapse. This difference is largely due to the choice of merger criterion - by turning off mergers beyond a threshold mass, objects that would have formed a single star here form several. At the opening stage of the collapse, the initial velocity perturbations create a network of dense filaments, visible in the leftmost panels of Figure 2.1. Because the core is centrally concentrated, gas falls into the center, forming a star with about $3 M_\odot$ after approximately 0.15 free-fall times. At this time, additional sinks begin to form in the gas that is falling on to the star. Some of these sinks are small enough that they will merge onto the central star, but many of them accrete enough mass that they surpass the 0.05$M_\odot$ threshold and become “stars” in their own right. These objects fall into the center of the core’s gravity well and begin to undergo complex N-body interactions with each other. They stay in the center for several orbits before being thrown out. At that time, they have will have grown to approximately 0.1 to 1 $M_\odot$, and will take away with them mass that would have merged with the central star(s) in the simulations without merger suppression. This results in the most massive star growing much less rapidly
Figure 2.2: Star particle statistics as a function of time for all four runs. The top panel shows the total mass in stars (the top set of lines) and the mass of the most massive star (bottom, dotted set). The middle plot shows $f_{\text{max}}$, the fraction of total stellar mass that is in the most massive star. The bottom plot is the total stellar luminosity.
after $\sim 0.2\tau_{\text{ff}}$ than in KCKM10 or Cunningham et al. (2011). Thus, with mergers suppressed, we form a massive star or binary plus a system of a few dozen low-mass stars, as opposed to having most of the stellar mass in one system.

This is similar to the concept of fragmentation-induced starvation laid out in Peters et al. (2010), where low-mass stars compete with the most massive star for the available gas and thereby stunt its growth. Unlike traditional competitive accretion, the massive star is starved of gas and grows slowly instead of running away from its low-mass competitors. Because our simulations start from turbulent initial conditions, however, the fragmentation happens in the dense filaments that feed the central star system rather than in a disk as in the simulations of Peters et al. (2010), which did not include the effects of turbulence.

We emphasize that the handling of stellar mergers is artificial in both cases. The large number of massive stars observed to have close companions (Sana et al. 2008) shows that merging all stars with separations less than 28 AU is clearly unrealistic. A detailed study of how mergers actually occur in regions of high-mass star formation is beyond the scope of this paper. We stress that the absolute number and masses of stars formed is affected by the choice of sub-grid model, so these results should be interpreted cautiously. However, because the merger criterion is the same for all the runs presented in this paper, any differences in the mass distribution of the objects formed between the four runs is due to changes in $\Sigma$ and $\delta$, not the merger criterion.

With that caveat, Figure 2.2 shows the properties of the star particles formed in the simulations as a function of time. To show all the runs on the same plot, we have normalized the time by $\tau_{\text{ff}}$, but in physical units the High $\Sigma$ simulation evolves about 3 times faster than the others. The top panel shows the total mass in stars, which is quite similar across all the runs. This is to be expected, because the star formation rate is set by the global properties of the flow, which are essentially independent of radiative effects. The middle panel shows $f_{\text{max}}$, the fraction of the total stellar mass that is in the most massive star. There is a period around $0.3\tau_{\text{ff}}$ where the Solar run levels off, but this appears to be a temporary phenomenon. By $0.5\tau_{\text{ff}}$, $f_{\text{max}}$ is very similar across all the runs. Overall, there appears to be little difference between the runs, either in the total mass converted to stars or in the fraction of that mass that ends up in the most massive star. The bottom panel of Figure 2.2 shows the total luminosity of all the stars in the simulation. The value of $10^4 L_\odot$ we find is typical of observed massive protostars (e.g. Cesaroni et al. (2007)). Here, we can see a difference between the High $\Sigma$ run and the others - because of the higher accretion rates, the total luminosity is higher in the High $\Sigma$ run, although the difference decreases with time as the stars become more massive and more of the radiant output comes in the form of nuclear luminosity. The nuclear luminosity is never dominant, however, and thus the luminosity as a function of time is roughly flat after about $0.2\tau_{\text{ff}}$, even though we are forming more stars and the stars are growing more massive. We will make use of the luminosity averaged after over $0.2\tau_{\text{ff}}$ to $0.5\tau_{\text{ff}}$ in Section 2.4 below.

Figure 2.3 shows the cumulative mass distribution of the stars in the four runs - that is, for each value of $m$, the y-axis shows the fraction of the total stellar mass than is is stars with masses greater than $m$. Visually, there appears to be little difference between the curves,
2.4 DISCUSSION

Table 2.2: K-S test results.

<table>
<thead>
<tr>
<th>Run</th>
<th>Solar</th>
<th>0.2 Solar</th>
<th>0.05 Solar</th>
<th>High Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>–</td>
<td>0.1</td>
<td>0.62</td>
<td>0.02</td>
</tr>
<tr>
<td>0.2 Solar</td>
<td>–</td>
<td>–</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td>0.05 Solar</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.02</td>
</tr>
</tbody>
</table>

particularly for the Σ = 2 g cm$^{-2}$ runs. To test whether the initial mass function is indeed the same in the different runs, we have performed two-sided Kolmogorov-Smirnov (K-S) tests between each pair of distributions. The results are shown in Table 2.2. At the 10% level, we cannot reject the null hypothesis that the three Σ = 2 g cm$^{-2}$ have the same underlying initial mass function. The High Σ distribution, on the other had, does seem to be statistically different from the others.

In contrast to the minor effect reported here, Krumholz et al. (2007a) found that isothermal runs fragmented completely differently from radiative ones, and KCKM10 found a major difference in fragmentation between runs with low and high surface density. To summarize, the differences in the fragmentation of all of our runs are minor. To the extent that there are significant differences, they are due to changes in the surface density, rather than to changes in the metallicity. At least within the range of parameters considered here, metallicity appears to have little effect on either the temperature or the fragmentation of molecular gas.

2.4 Discussion

2.4.1 Analytic Model

The above simulations suggest that metallicity plays little role in the fragmentation of star-forming gas. To understand why, consider a simple model system like the initial conditions above: a core of gas and dust with radius $R_c$, mass $M$, surface density $\Sigma = M/\pi R_c^2$, and a power law density profile $\rho(r) \propto r^{-k}\rho$. We would like to understand what happens to the temperature of this core once stars have started to form, so we will place a point source of luminosity $L$ in the center to represent the combined radiant output of the central collection of stars. We assume that the dust opacity follows a power law in the far IR regime:

$$\kappa_\nu = \delta \kappa_0 \left( \frac{\lambda_0}{\lambda} \right)^\beta$$

$$= \delta \kappa_0 \left( \frac{T}{T_0} \right)^\beta , \text{ for } 3 \text{ mm} < \lambda < 30 \mu m$$

where the subscript “0” refers to an arbitrary reference value and $\delta$ is the dust-to-gas mass ratio relative to solar. The $T$ in this equation is the dust temperature, which we assume is
Figure 2.3: Cumulative mass distributions from all four runs at $t_{ff} = 0.5$. $f(>m)$ the fraction of the total stellar mass that is in stars with masses greater than $m$. 
identical to the radiation temperature. We will adopt the dust opacity model of Weingartner & Draine (2001), for which \( \kappa_0 = 0.27 \text{ cm}^2 \text{ g}^{-1} \) at \( \lambda_0 = 100 \mu\text{m} \) and \( \beta = 2 \); however, we have verified that using the opacities of Semenov et al. (2003) (as used by ORION) or Pollack et al. (1994) makes little difference. For reference, the corresponding \( T_0 \) is 144 K.

The emission from such a system is considered in Chakrabarti & McKee (2005) (hereafter CM2005). They find that even though cores do not have sharply delineated photospheres as stars do, the radiation they emit is still well-described by

\[
L = \tilde{L} 4\pi R_{ch}^2 \sigma T_{ch}^4 \tag{2.17}
\]

where \( \tilde{L} \) is a constant of order unity and \( T_{ch} \) and \( R_{ch} \) are a characteristic temperature and the radius from which radiation with frequency \( \nu_{ch} = kT_{ch}/h \) has an optical depth of 1. These are given by

\[
\tilde{R}_c = \frac{R_c}{R_{ch}} = \left\{ \frac{(L/M)\Sigma^{(4+\beta)/\beta}}{4\sigma\tilde{L}} \left[ \frac{(3-k_\rho)\delta\kappa_0}{4(k_\rho - 1)T_0^\beta} \right]^{4/\beta} \right\}^{\frac{\beta}{\alpha}} \tag{2.18}
\]

and

\[
T_{ch} = \left\{ \frac{L/M}{4\sigma\tilde{L}\Sigma^{3-k_\rho}} \left[ \frac{4(k_\rho - 1)T_0^\beta}{(3-k_\rho)\delta\kappa_0} \right]^{\frac{2}{k_\rho - 1}} \right\}^{\frac{1}{k_\rho - 1}} \tag{2.19}
\]

where \( \alpha = 2\beta + 4(k_\rho - 1) \). Rather than using the expression for \( \tilde{L} \) given in CM2005, we will adopt the more accurate expression from Chakrabarti & McKee (2008), which they report gives excellent agreement with results from the DUSTY code, based on the work of Ivezic & Elitzur (1997):

\[
\tilde{L} = 1.6 \tilde{R}_c^{0.1}. \tag{2.20}
\]

A final result we will take from CM2005 is that the temperature profile in the vicinity of the photosphere is also well-described by a power law:

\[
T(r) = T_{ch} \left( \frac{r}{R_{ch}} \right)^{-k_T} \tag{2.21}
\]

We can solve the above equations simultaneously to get that the temperature as a function
of density (or, equivalently, radius) in the core is

\[ T(\rho) = T_c (\rho/\rho_{\text{edge}})^{k_T/k_\rho}, \]

\[ T(r) = T_c (r/R_c)^{-k_T}, \]

\[ T_c = \left[ \frac{L/M}{4\sigma L} \right]^{\frac{k_\rho - 1 + 3k_T}{3}} \times \frac{(3 - k_\rho)\delta \kappa_0}{4(k_\rho - 1)T_0^3} \times \sum \frac{(4 + \beta)k_T + k_\rho - 3}{2}, \] (2.22)

or, specializing to \( k_\rho = 3/2 \) and \( \beta = 2 \),

\[ T_c = \left[ \frac{L/M}{4\sigma L} \right]^{\frac{k_T + 1/4}{3}} \times \frac{3\delta \kappa_0}{4T_0^3} \times \sum \frac{2k_T - 1}{3} \times \frac{k_T - 1/4}{3}. \] (2.23)

We can see from this expression that the scaling of \( T_c \) with \( \delta \) actually goes to zero for \( k_T = 0.5 \). This is surprising at first, since both \( \widetilde{R}_c \) and \( T_{\text{ch}} \) scale with \( \delta \), to the -2/3 and -1/3 powers, respectively, for our assumed parameter values. However, we can understand this in the following way: take a point \( r \) that is outside the effective photosphere of the core. Since radiation with frequency \( \nu_{\text{ch}} \) is thin here, this point’s temperature will be determined roughly by radiative equilibrium with a luminosity source with effective radius \( R_{\text{ch}} \) and temperature \( T_{\text{ch}} \). If we then lower \( \delta \), \( R_{\text{ch}} \) will decrease, since the core will be less effective at trapping radiation and photons with frequency \( \nu_{\text{ch}} \) will be able to travel further through the core’s envelope before being absorbed. This will make the effective emitting area smaller, which will tend to lower the flux at point \( r \). However, because it is closer to the central heating source, the temperature at the new value of \( R_{\text{ch}} \) will be higher as well, and that will tend to increase the flux at \( r \). If the temperature always scales as the -0.5 power of radius, then these two effects will cancel out exactly; the luminosity seen at \( r \), \( L \approx 4\pi\sigma R_{\text{ch}}^2 T_{\text{ch}}^4 \), will be the same, and the temperature will be the same as well.

Is \( k_T \) close to 0.5? CM2005 give a fitting function for the temperature scaling exponent \( k_T \) as a function of \( \tilde{R}_c \):

\[ k_T = \frac{0.48k_\rho^{0.005}}{\tilde{R}_c^{0.02k_\rho^{1.09}}} + \frac{0.1k_\rho^{5.5}}{\tilde{R}_c^{0.7k_\rho^{1.09}}}. \] (2.24)

This expression shows that as long as \( \tilde{R}_c > 1 \), meaning that the photospheric radius is less than half the core radius, \( k_T \) is indeed quite close to 0.5 and depends only weakly on \( \tilde{R}_c \). This expression assumes that \( T_{\text{ch}} \lesssim 300 \) K, so that most of the flux is emitted at wavelengths longer than 30 \( \mu \)m and the opacity is well approximated by a power law in frequency. We have verified that this condition holds under the circumstances considered in this paper. Since dust sublimes at \( \sim 1000 \) K, this condition also implies that \( R_{\text{ch}} \) is larger than the dust destruction radius.

Note that for a given \( \Sigma \) and \( L/M \), \( \tilde{R}_c \) is a function of \( \delta \) only. From our simulation results, we can calculate the light-to-mass ratio associated with \( \Sigma = 2 \) and 10 g cm\(^{-2} \) from the
2.4. DISCUSSION

Table 2.3: Light-to-Mass Ratios

<table>
<thead>
<tr>
<th>Run</th>
<th>L/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>80.7</td>
</tr>
<tr>
<td>0.2 Solar</td>
<td>69.8</td>
</tr>
<tr>
<td>0.05 Solar</td>
<td>65.4</td>
</tr>
<tr>
<td>High Σ</td>
<td>152.6</td>
</tr>
</tbody>
</table>

Note. — Averaged over $t = 0.2 t_{ff}$ to $t = 0.5 t_{ff}$.

luminosities shown in the bottom panel of Figure 2.2, averaged over the period from $t = 0.2 t_{ff}$ to $t = 0.5 t_{ff}$. The results are shown in Table 2.3. The “M” here refers to the gas mass only, not the mass in stars. In the $\Sigma = 2$ g cm$^{-2}$ runs, the light-to-mass ratio appears to depend weakly on $\delta$, but the effect is small and we will simply use the average over the three metallicities.

In Figure 2.4, we plot $k_T$ as a function of $\delta$ for the two values of $\Sigma$ represented in our simulations from Equation (2.24). Note that $k_T$ remains relatively close to 0.5 for a wide range of values for $\delta$ for both of the surface densities we have considered. It is therefore not surprising that the simulations show such a weak dependence on $\delta$.

Note that for most of the parameter space, the value of $k_T$ is actually a bit less than 0.5, meaning that the temperature actually scales inversely with $\delta$. When the temperature falls off more slowly than $k_T = 0.5$, the temperature at the smaller value of $R_{ch}$ associated with a lower $\delta$ will actually be larger than what is required to keep the temperature outside of the photosphere at a constant value, so $T$ will increase slightly. Once $\delta$ increases much past solar, however, $k_T$ increases dramatically and $T$ begins to rise with $\delta$. This suggests that regions with super-solar metallicity may in fact have larger characteristic stellar masses than regions of solar or sub-solar metallicity.

2.4.2 Comparison to simulations

We can also calculate the temperature profiles expected for the cores in the above simulations. The procedure is as follows. Using the light-to-mass ratios from Table 2.3 along with the known simulation values of $\Sigma$ and $\delta$, we can compute $k_T$ using Equations (2.18) and (2.24). Then, we use this value along with Equation (2.22) to get the temperature as function of radius. In Figure 2.5, we plot these relations and compare them to the profiles calculated from the simulations. To get the simulation profiles, we calculated the density-weighted mean temperature in spherical shells of radius $r$ around the most massive star, and plotted the result versus $r$. The density weighting ensures that the hot, diffuse gas surrounding the
Figure 2.4: Temperature scaling exponent $k_T$ as a function of $\delta$ for the two different values of $\Sigma$ considered in the simulations.
cores does not interfere with the result inside the core, although its presence can be seen in the temperature rise as \( r \) approaches the core radius \( R_c \). We have averaged the simulation profiles over all the snapshots from \( t_{\text{ff}} = 0.2 \), when the luminosity has roughly leveled off, to \( t_{\text{ff}} = 0.5 \), and the error bars show the standard error over all the snapshots. The error bars are larger close to the central massive star because of the presence of dynamically interacting stars within in central few hundred AU of the simulation volume. The important things to note are

1. The simulation results confirm the power law nature of the temperature profile and agree closely with the predicted slopes

2. The effect of varying the metallicity is quite small compared to the effect of changing the surface density, both in the analytic calculation and in the simulations.

The analytic expression is systematically higher than the simulations by roughly 10%. It does, however, agree with the jump in \( T \) from \( \Sigma = 2 \) g cm\(^{-2} \) to \( \Sigma = 10 \) g cm\(^{-2} \) in quite well. The discrepancy is likely due to differences in the treatment of the radiative transfer between our simulations and CM2005. ORION treats the radiation in a frequency-averaged gray approximation in terms of the Plank and Rosseland mean opacities, while CM2005 used the DUSTY code (Ivezic & Elitzur 1997), which includes frequency information about the photons. However, CM2005 also assumed a spherically symmetric, static core with no density perturbations, so it is not clear which result is more accurate. Whatever the case, both methods agree that the temperature profiles are not particularly sensitive to \( \delta \).

### 2.4.3 Predictions for star-forming regions

Star-forming regions in the Milky Way and other galaxies sample a large range of surface densities and metallicities. A question we can ask is: in what range of parameter space is the gas temperature insensitive to changes in the metallicity? To answer this question, we will use the relationship between the light-to-mass ratio and the surface density given in Krumholz & McKee (2008):

\[
L/M \approx 3.6 M_2^{-0.33} \Sigma_0^{0.67} T_{b,1}^{0.16} \left( \frac{L_{\odot}}{M_{\odot}} \right)
\]  

(2.25)

where \( M_2 = M/100 M_{\odot} \), \( \Sigma_0 = \Sigma/1 \) g cm\(^{-2} \), and \( T_{b,1} \) is the background core temperature divided by 10K. In what follows, we will take \( M_2 = 3 \) and \( T_{b,1} = 2 \). This expression is calculated by assuming that the core converts its gas into stars at a rate of a few percent per free-fall time and is not accurate once massive stars have formed. However, it is still relevant to the early stages of collapse, and will allow us to gauge whether the core temperature becomes high enough to slow fragmentation. Note that the star formation efficiency assumed here is lower than what we see in the simulations; because we have not included the effects of outflows or other turbulence-driving mechanisms, our star formation rates are higher than observed.
Figure 2.5: Temperature profiles from the simulations (dots) and analytic theory (solid lines). To show both values of $\Sigma$ on the same plot, we have normalized $r$ by the size of the core $R_c$. The simulations profiles are averaged over $t_\Pi = 0.2$ to $t_\Pi = 0.5$. 
Using this expression, we can eliminate the dependence on \( L \) in the equation for \( T_c \) to get the edge temperature as a function of \( M, \Sigma \) and \( \delta \) only. We plot the expected temperature for a \( M = 300 M_\odot \) core as a function of these parameters in Figure 2.6. To characterize the core by a single temperature, we use Equation (2.22) to evaluate \( T(\rho) \) at the mean density \( \bar{\rho} = 3 \rho_{\text{edge}}/(3 - k_\rho) \). Note that we have assumed the temperature is determined by protostellar feedback; gas with sufficiently low metallicity would be warm anyway owing to the lack of coolants. Omukai et al. (2010) considered this effect, and concluded that it should be dominant for metallicities lower than 0.01 \( Z_\odot \), so we will limit our analysis to values of \( \delta \) greater than 0.01. Figure 2.6 includes a range of surface densities that span the conditions typical of star formation, from low-mass star-forming regions like Taurus and Perseus (0.1 g cm\(^{-2}\)), to regions of active massive star formation in the galaxy (1 g cm\(^{-2}\)), to extra-galactic super star clusters (10 g cm\(^{-2}\)). For all those environments, one would need to change \( \delta \) by at least two orders of magnitude to get a factor of 2 change in the temperature. Hence, we expect there to be very little variation in the fragmentation of cores across environments with different metallicities over the range currently probed by observations. Note that Figure 2.6 shows the mean core temperature only when the dominant source of heating is protostellar feedback. In reality, cores in the upper left hand region of the parameter space would likely be hotter than the indicated 5 - 10 K due to heating from cosmic rays or the decay of turbulence, so the true range of temperature variation is even smaller than indicated in Figure 2.6.

Unlike \( \delta \), \( \Sigma \) does make a significant difference in the core temperature. In regions like Taurus (\( \Sigma \sim 0.1 \) g cm\(^{-2}\)), we do not predict protostellar feedback to be able to heat the gas much above 10 K. Hence, collapse there will likely be isothermal, and may be more prone to fragmentation to produce low-mass stars. Regions like of higher surface density (\( \Sigma \sim 1 \) g cm\(^{-2}\)), are not isothermal, which may bias them towards forming high-mass stars.

2.5 Conclusions

We have performed a series of numerical experiments using ORION in which we follow the collapse of massive cores past the onset of star formation to the subsequent heating of the gas by radiative feedback, varying the dust opacity by a factor of twenty. We find that the opacity makes little difference to either the temperature or the fragmentation of the cores as they collapse. Our simulations consider surface densities of \( \Sigma = 2 \) g cm\(^{-2}\), characteristic of massive star-forming regions in the Milky Way, and \( \Sigma = 10 \) g cm\(^{-2}\), characteristic of extra-galactic super star clusters.

We have also presented an analytic argument for why the IMF should be relatively independent of the metallicity of the star-forming region, even when the heating is dominated by a central source as in high-mass star-forming cores. This helps to explain why there do not appear to be significant differences between the IMFs of disk stars and globular clusters, or between those of the Milky Way and the SMC, despite large differences in metallicity. We find that the metallicity should only weakly influence the IMF over a large range of star-forming environments.
Figure 2.6: Contours of the mean core temperature $T(\bar{\rho})$ at the early stages of collapse as a function of $\Sigma$ and $\delta$ for a core with mass $M = 300M_\odot$. The turnover of the $T = 30$ K contour at high $\Sigma$ and $\delta$ is due to $kT$ becoming large; see Equation (2.24) and Figure 2.4.
Acknowledgments

We thank Charles Hansen, Stella Offner, and Andrew Cunningham for helpful discussions. This project was initiated during the ISIMA 2010 summer program, funded by the NSF CAREER grant 0847477, the France-Berkeley fund, the Institute for Geophysics and Planetary Physics and the Center for Origin, Dynamics and Evolution of Planets. We thank them for their support. Support for this work was also provided by: the DOE SciDAC program under grant DE-FC02-06ER41453-03 (ATM), an Alfred P. Sloan Fellowship (MRK), NSF grants AST-0807739 (MRK), CAREER-0955300 (MRK), and AST-0908553 (CFM, RIK and ATM), the US Dept. of Energy at LLNL under contract DE-AC52-07NA (RIK), NASA through Astrophysics Theory and Fundamental Physics grant NNX09AK31G (RIK, CFM, and MRK), and through a Spitzer Space Telescope Theoretical Research Program grant (MRK and CFM). Support for computer simulations was provided by an LRAC grant from the NSF through Teragrid resources and NASA through a grant from the ATFP. We have used the YT software toolkit (Turk et al. 2011) for data analysis and plotting.
Chapter 3

The Fragmentation of Magnetized, Massive Star-Forming Cores with Radiative Feedback

Abstract

We present a set of 3-dimensional, radiation-magnetohydrodynamic calculations of the gravitational collapse of massive (300 $M_\odot$), star-forming molecular cloud cores. We show that the combined effects of magnetic fields and radiative feedback strongly suppress core fragmentation, leading to the production of single star systems rather than small clusters. We find that the two processes are efficient at suppressing fragmentation in different regimes, with the feedback most effective in the dense, central region and the magnetic field most effective in more diffuse, outer regions. Thus, the combination of the two is much more effective at suppressing fragmentation than either one considered in isolation. Our work suggests that typical massive cores, which have mass-to-flux ratios of about 2 relative to critical, likely form a single star system, but that cores with weaker fields may form a small star cluster. This result helps us understand why the observed relationship between the core mass function and the stellar initial mass function holds even for $\sim 100 M_\odot$ cores with many thermal Jeans masses of material. We also demonstrate that a $\sim 40$ AU Keplerian disk is able to form in our simulations, despite the braking effect caused by the strong magnetic field.  

1This chapter has been previously published as Myers, McKee, Cunningham, Klein, & Krumholz 2013, ApJ, 766, 07, and is reproduced with the permission of all coauthors and the copyright holder. Copyright 2013 American Astronomical Society.
3.1 Introduction

Massive stars, which have mass $> 8M_\odot$, make up $< 1\%$ of the total stellar population, but their numbers belie their impact. Both the total luminosity and the ionizing luminosity of a star are highly super-linear functions of mass. Thus, massive stars have a much stronger impact on their birth environments than low-mass stars do. Since most stars form in clusters that contain at least one early O star, massive stars have an important impact on the formation of their low-mass neighbors, whether by altering the thermal properties of their parent clumps by heating the dust, or by destroying them outright via photoionization. The latter process is so bright that it allows observation of the star formation rate in other galaxies. Finally, massive stars end their lives in supernova explosions, which produce heavy elements and add large amounts of energy to the interstellar medium (ISM), contributing to the driving of its turbulence on large scales. Understanding the life cycle of massive stars from their births to their deaths is thus an important problem for many branches of astrophysics.

Unfortunately, the first stage of this process - the birth of massive stars - remains an incompletely understood problem. Observationally, regions of massive star formation in our own galaxy tend to lie farther away from Earth than regions of low-mass star formation, meaning that observers have not yet been able to probe the formation process for high-mass stars at the same level of detail as they have for low-mass stars. Theoretically, the central difficulty is the large number of mutually interacting physical processes involved. Massive stars form out of a supersonically turbulent, self-gravitating fluid with dynamically significant magnetic fields. Massive protostars also deeply impact their surroundings as they form through a variety of feedback processes, including magnetically-launched outflows, radiation pressure, radiative heating, and ionization. Because of the complexity of these processes, simulations of massive star formation are able to include at most a few of these effects at one time. In the past several years, there has been much work done on massive star formation that ignored the effects of magnetic fields, both with (e.g. Krumholz et al. 2007a, 2010, 2009; Cunningham et al. 2011) and without (e.g. Girichidis et al. 2011) radiative feedback. There has also been much work on simulating massive star formation that included the magnetic field, but did not include radiative feedback (e.g. Seifried et al. 2011, 2012; Li & Nakamura 2006; Wang et al. 2010; Hennebelle et al. 2011). Thus far only two published simulations of massive star formation have included both radiation and magnetic fields, and these provide only a limited picture of how fragmentation in massive cores works. Peters et al. (2011) treat direct stellar radiation and ionization chemistry, but neglect the dust-reprocessed radiation field, which is mainly responsible for regulating fragmentation. Commerçon et al. (2011) include dust-reprocessed light, but because they do not employ a subgrid stellar model they are forced to halt their calculations when $\lesssim 1\%$ of the core material has collapsed, and as a result they cannot study the fragmentation of the bulk of the gas.

In this paper, we attempt to fill that gap. We present the results of 3-dimensional, adaptive mesh refinement (AMR), radiation-magnetohydrodynamic (R-MHD) simulations that treat the dust-processed radiation from protostars in the flux-limited diffusion (FLD) approximation. In particular, we focus on the fragmentation of isolated, massive cores in the
relatively early stages of star formation - up to the point at which about 10% of the core gas has turned into stars. The question of how massive cores fragment is an important one for any theory of star formation in which the initial mass function (IMF) is set in the gas phase, e.g. the turbulent fragmentation scenario originally laid out in Padoan & Nordlund (2002). Observations of the core mass function (CMF) in galactic star-forming regions reveal that it looks like a scaled-up version of the stellar initial mass function (IMF) (Alves et al. 2007; Nutter & Ward-Thompson 2007; Enoch et al. 2008). This relationship appears to continue even up to $\sim 100 M_{\odot}$ (Reid & Wilson 2006). This correspondence - that the CMF has the same form as the IMF but is shifted up in mass by a factor of $\sim 3$ - has a natural explanation if massive cores do not fragment strongly as they collapse, but instead simply convert $\sim 1/3$ of their mass into single massive stars or systems.

The purpose of this paper is to address the question of how massive cores fragment via direct numerical simulation. Our outline is as follows: in section 3.2, we describe our numerical setup, including the equations and algorithms used as well as our initial and boundary conditions. In section 3.3, we present our results, focusing on the evolution of our cores over a period of 0.6 mean-density free-fall times. In section 3.4, we discuss our results, in which the magnetic field and the radiative transfer together have a significant impact on the fragmentation of the cores in a way one would not predict from either process considered in isolation. We summarize our conclusions in section 3.5.

3.2 Numerical Setup

3.2.1 Equations and Algorithms

We solve the equations of mass, momentum, and energy conservation on a hierarchy of AMR grids. We assume that the motion of the gas is governed by the ideal MHD equations and treat the radiation using the mixed-frame approach of Krumholz et al. (2007b). At any time, the computational domain consists of a fluid made up of gas, dust, and radiation, plus some number of sink particles that represent stars. The fluid quantities are described by a vector of state variables $(\rho, \rho v, E, B, E_R)$ defined at every grid cell, where $\rho$ is the gas density, $\rho v$ the momentum, $E$ the non-gravitational energy density (i.e. the total of the kinetic, thermal, and magnetic energy densities), $B$ the magnetic field, and $E_R$ the radiation energy density. The particles are characterized by their position $x_i$, momentum $p_i$, mass $M_i$, and luminosity $L_i$, which is determined via the protostellar evolution model described in McKee & Tan (2003) and Offner et al. (2009). The equations governing the evolution of the R-MHD fluid-particle system are:
3.2. NUMERICAL SETUP

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) - \sum_i M_i W(x-x_i) \quad (3.1) \]

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v} \mathbf{v} - \frac{1}{4\pi} \mathbf{B} \mathbf{B}) - \nabla P_T - \rho \nabla \phi - \lambda \nabla E_R - \sum_i \dot{M}_i W(x-x_i) \quad (3.2) \]

\[ \frac{\partial E}{\partial t} = -\nabla \cdot [(E + P_T) \mathbf{v} - \frac{1}{4\pi} \mathbf{B} (\mathbf{v} \cdot \mathbf{B})] - \rho \mathbf{v} \cdot \nabla \phi - \kappa_0 P \rho (4\pi B_T - cE_R) + \lambda \left( \frac{2\kappa_0 P}{\kappa_0 R} - 1 \right) \mathbf{v} \cdot \nabla E_R - \sum_i \dot{E}_i W(x-x_i) \quad (3.3) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) \quad (3.4) \]

\[ \frac{\partial E_R}{\partial t} = \nabla \cdot \left( \frac{c \lambda}{\kappa_0 R \rho} \nabla E_R \right) + \kappa_0 P \rho (4\pi B_T - cE_R) - \lambda \left( \frac{2\kappa_0 P}{\kappa_0 R} - 1 \right) \mathbf{v} \cdot \nabla E_R - \nabla \cdot \left( \frac{3 - R_2}{2} \mathbf{v} E_R \right) + \sum_i L_i W(x-x_i). \quad (3.5) \]

In the above equations, the total pressure \( P_T \) is \( P_{\text{gas}} + B^2/8\pi \), and we use an ideal equation of state, so that

\[ P_{\text{gas}} = \frac{\rho k_B T_g}{\mu m_H} = (\gamma - 1) \rho \epsilon, \quad (3.6) \]

where \( k_B \) is the Boltzmann constant, \( T_g \) the gas temperature, \( \mu \) the mean molecular weight, \( \gamma \) the ratio of specific heats, and \( \epsilon \) the thermal energy per unit mass. We take \( \mu = 2.33 \) and \( \gamma = 5/3 \), appropriate for molecular gas of solar composition that is too cold to store energy in rotational degrees of freedom. The corresponding value for the gas’s specific heat capacity is \( c_v = k_B/(\gamma - 1) \mu m_H \approx 5.3 \times 10^7 \) erg g\(^{-1}\) K\(^{-1}\).

The summations in the gas-sink interaction terms are taken over all the particles in the domain, and \( W(x-x_i) \) is a weighting kernel that distributes the transfer of mass, momentum, and energy over a radius of 4 fine-level cells around sink particle \( i \). The values for \( M_i, \dot{M}_i, \) and \( \dot{E}_i \), or the rates of mass, momentum, and energy transfer between the sink particles and the fluid, are computed by fitting the flow around each sink particle to a magnetized Bondi-Hoyle flow; see Lee et al. (2013) for details. The star particle states themselves are
updated according to the following equations:

\[
\frac{d}{dt} M_i = \dot{M}_i, \quad (3.7)
\]
\[
\frac{d}{dt} x_i = \frac{p_i}{\dot{M}_i}, \quad (3.8)
\]
\[
\frac{d}{dt} p_i = -M_i \nabla \phi + \dot{p}_i. \quad (3.9)
\]

Because our sink particle algorithm destroys information about the fluid flow inside the 4 fine cell accretion zone around each particle, we are not able to properly follow the dynamics of particles that pass within that distance of each other. We therefore adopt the following criterion to handle mergers between sink particles that pass within one accretion radius of each other (40 AU in most of the simulations presented here): we merge the two sinks together only if the smaller sink is less than 0.05\(M_{\odot}\) in mass. This threshold roughly corresponds to the mass at which second collapse occurs (Masunaga et al. 1998; Masunaga & Inutsuka 2000). Before that point, sink particles represent hydrostatic cores of several AU in size, which could be expected to merge together. After that point, they have collapsed down to roughly solar size scales, and will not necessarily merge simply because they pass within 40 AU of each other.

The gravitational potential \(\phi\) in the above expressions obeys the Poisson equation with a right-hand side that includes contributions from both the fluid and the star particles:

\[
\nabla^2 \phi = -4\pi G \left[ \rho + \sum_i M_i \delta(x - x_i) \right], \quad (3.10)
\]

where \(G\) is the gravitational constant.

The radiation-specific quantities are the speed of light \(c\), the comoving frame specific Planck- and Rosseland-mean opacities \(\kappa_{0R}\) and \(\kappa_{0P}\), and the Planck function \(B_T = c a_R T_9^4/(4\pi)\), where \(a_R\) is the radiation constant. Finally, the flux limiter \(\lambda\) and Eddington factor \(R_2\) are two quantities that enter the flux-limited diffusion approximation we use to compute the radiative transfer. In this work, we adopt the Levermore & Pomraning (1981) approximation:

\[
\lambda = \frac{1}{R} \left( \coth R - \frac{1}{R} \right), \quad (3.11)
\]
\[
R = \frac{|\nabla E|}{\kappa_{0R} \rho E}, \quad (3.12)
\]
\[
R_2 = \lambda + \lambda^2 R^2. \quad (3.13)
\]

We obtain the dust opacities \(\kappa_{0P}\) and \(\kappa_{0R}\) from a piecewise-linear fit to the models of Semenov et al. (2003); see Cunningham et al. (2011) for the exact functional form.

We solve the above equations using a new version of our astrophysical AMR code ORION, which allows us to simultaneously include the magnetic field and the radiative feedback.
ORION solves the above equations in a number of steps, which we summarize below. First, we solve the ideal MHD equations by themselves (Equation (3.4), the first two terms of Equation (3.1) and the first three terms of Equations (3.2) and (3.3)) using a Godunov-type scheme with the HLLD approximate Riemann solver (Miyoshi & Kusano 2005). Specifically, we use the dimensionally unsplit, AMR Constrained Transport (CT) scheme described in Li et al. (2012), which makes use of the unigrid CT scheme from the open-source astrophysical MHD code Pluto (Mignone et al. 2012). This portion of the update algorithm uses a face-centered representation for the magnetic field $B$, and we use the Chombo\(^2\) AMR library to provide support for the face-centered fields. Next, we incorporate self-gravity in the manner of Truelove et al. (1998) and Klein (1999). To solve the Poisson equation (Equation (3.10)), we use an iterative multigrid scheme also provided by Chombo. In the third step, we update Equations (3.2), (3.3), and (3.5) for the radiative terms using the operator-split approach described in Krumholz et al. (2007b). Briefly, this technique first solves the radiation pressure, work, and advection terms explicitly, and then implicitly updates the gas and radiation energy densities for the terms involving diffusion and the emission/absorption of radiation. This update is handled by the iterative process described in Shestakov et al. (2005), which uses pseudotransient continuation to reduce the number of iterations required for convergence. We then complete the update cycle by calculating the new sink particle states using the above equations and computing their interactions with the fluid using the algorithms described in Lee et al. (2013).

Finally, we point out some important numerical caveats: our treatment of the radiation in this work focuses on the diffuse, dust-processed component of the radiation field, and it treats that radiation as gray. Massive stars, however, put out large numbers of ionizing photons, and these photons have a dramatic impact on the surrounding environment. Furthermore, treating the diffuse component of the field as gray and ignoring the direct component of the non-ionizing radiation both lead us to underestimate the radiation pressure force by a factor of a few (Kuiper et al. 2011). However, since both of these effects are most significant for stars more massive than $\sim 20M_\odot$, and since our conclusions are mainly based on the evolution of the cores prior to the most massive star reaching that point, we do not believe that our qualitative conclusions will be significantly altered by a more accurate treatment of the radiative transfer. We have also not included the effects of protostellar outflows in any of the runs in this paper. We shall do so in future work.

### 3.2.2 Refinement and Sink Creation

The computational domain is a cube with side $L_{\text{box}}$ that is discretized into a coarse grid of $N_0$ cells, so that the resolution on the coarse grid $\Delta x_0 = L_{\text{box}}/N_0$. Our code operates within an AMR framework that automatically adds and removes finer grids as the simulations evolve. With $L$ levels of refinement and a refinement ratio of 2, the resolution of the finest level is $\Delta x_L = \Delta x_0/2^L$. In this work, we have chosen these parameters such that $\Delta x_L$ is 10 AU.

\(^2\)https://commons.lbl.gov/display/chombo/
Any cell that meets one or more of the following criteria is flagged for refinement:

1. The density in the cell exceeds the magnetic Jeans density, given by
   \[ \rho_{\text{max}} = \frac{\pi J_{\text{max}}^2 c_s^2}{G \Delta x_l^2} \left( 1 + \frac{0.74}{\beta} \right), \tag{3.14} \]
   where \( c_s \) is the isothermal sound speed, \( \Delta x_l \) the cell size on level \( l \), \( \beta = \frac{8 \pi \rho c_s^2}{B^2} \) and \( J_{\text{max}} \) is the maximum allowed number of magnetic Jeans lengths per cell, which must be small to avoid artificial fragmentation. Throughout this work, we take \( J_{\text{max}} = 1/8 \). Note that this is identical to our previous work except for the inclusion of the magnetic field. Because the field provides additional support against collapse, we do not need to resolve the flow as highly in the presence of magnetic fields to prevent artificial fragmentation. For a derivation and numerical justification of this relation, see the Appendix (3.6), but we note that it is roughly equivalent to including the magnetic energy density along with the thermal energy in the expression for the Jeans length.

2. The cell is within \( 16 \Delta x_l \) of a sink particle.

3. The gradient in the radiation energy density exceeds
   \[ \nabla E_R > 0.25 \frac{E_R}{\Delta x_l}. \tag{3.15} \]

This procedure is repeated recursively until the final level is reached. At that point, if there are still any cells on the finest level that exceed the magnetic Jeans density, then the excess matter is removed from the cell and placed into a new sink particle, which then evolves according to the algorithm in section 3.2.1 above. Taken together, these three conditions ensure that the regions where star formation is happening are always tracked with the highest available numerical resolution.

The application of these criteria to simulations of self-gravitating, isothermal gas requires special care, because such simulations have a fundamental problem: They do not converge. Isothermal gas tends to produce long, thin filaments, which do not fragment strongly (Inutsuka & Miyama 1992; Truelove et al. 1998) and are thus non-trivial to decompose into point particles. Convergence studies by Boss et al. (2000) and Martel et al. (2006) suggest that there is no well-defined, converged solution for fragmentation and sink particle creation in this case, because the correct solution is collapse to singular filaments rather than singular points. As a result, for any choice of the finest resolution, application of the Truelove criterion to a collapsing isothermal gas will result in producing artificial fragments at the finest grid scale. This does not mean that all fragmentation in isothermal simulations is artificial: As we shall see below, our isothermal simulation produces about the same total mass in stars and the same amount of mass in the most massive star as our radiative simulations; on the other hand, it produces many more low-mass stars. In view of this over-fragmentation problem
3.2. NUMERICAL SETUP

Table 3.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Run</th>
<th>RT</th>
<th>$M$ ($M_\odot$)</th>
<th>$R$ (pc)</th>
<th>$\sigma_v$ (km/s)</th>
<th>$t_{\text{ff}}$ (kyr)</th>
<th>$\mu_\Phi$</th>
<th>$\bar{B}$ (mG)</th>
<th>$\bar{\beta}$</th>
<th>$L_{\text{box}}$ (pc)</th>
<th>$N_0$</th>
<th>$L$ (AU)</th>
<th>$\Delta x_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR</td>
<td>Yes</td>
<td>300</td>
<td>0.1</td>
<td>2.3</td>
<td>30.2</td>
<td>$\infty$</td>
<td>0.0</td>
<td>$\infty$</td>
<td>0.4</td>
<td>256</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>BR</td>
<td>Yes</td>
<td>300</td>
<td>0.1</td>
<td>2.3</td>
<td>30.2</td>
<td>2.0</td>
<td>1.6</td>
<td>0.05</td>
<td>0.4</td>
<td>256</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>BI</td>
<td>No</td>
<td>300</td>
<td>0.1</td>
<td>2.3</td>
<td>30.2</td>
<td>2.0</td>
<td>1.6</td>
<td>0.05</td>
<td>0.4</td>
<td>256</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Note. — Col. 2: whether this run includes radiative transfer. Col. 8: mean magnetic field in the core. Col. 9: mean plasma $\beta = 8\pi\rho c_s^2/B^2$ in the core. Col. 10: resolution of the base grid. Col. 11: number of levels of refinement. Col. 12: maximum resolution at the finest level.

In isothermal simulations of star formation, it is essential to carry out a resolution study to verify that the conclusions being drawn from such simulations are physical and not numerical. Interestingly enough, while much of the fragmentation in isothermal simulations is ultimately caused by the numerical mesh, proper adjustment of the finest level of resolution may nonetheless enable isothermal simulations to give a qualitatively correct picture of fragmentation in the absence of radiative feedback. Without protostellar heating, molecular gas still becomes non-isothermal at some density $\rho_{\text{crit}}$ at which energy can no longer be efficiently radiated away. Masunaga & Inutsuka (1999) find that, for our choice of initial temperature and dust opacity, $\rho_{\text{crit}} \sim 10^{-13}$ g cm$^{-3}$. Past that point, the thermal pressure inside the filament starts to become more important relative to gravity. Eventually, gravitational contraction begins to slow, the timescale for cylindrical collapse becomes large compared to that for spherical collapse, and fragmentation will occur. Unfortunately, the results from such a simulation cannot be validated with a convergence study: increasing the resolution makes the fragments that form smaller than appropriate for the actual, non-isothermal case.

We stress that the non-convergence of the number of fragments in isothermal simulations is not a consequence of our particular sink particle algorithm. Using more stringent sink creation criteria, like those proposed in Federrath et al. (2010a), has the benefit of producing fewer spurious fragments, but some will still be present, and their properties will still ultimately be determined by the numerical mesh. Furthermore, one cannot get around this problem by suppressing sink formation entirely within filamentary structures, since once the Truelove criterion is violated the filament will fragment artificially anyway. To get a converged answer on the number of fragments formed in self-gravitating, turbulent media, one must include some sort of deviation from isothermality and a fine enough numerical mesh to resolve the resulting fragments.
3.2. NUMERICAL SETUP

3.2.3 Initial and Boundary Conditions

We begin with three cores that are identical except that we include a different combination of physical processes in each run. The parameters for these simulations are summarized in Table 6.1. Run HR includes the radiative transfer physics but has no magnetic field, run BI has a magnetic field but no radiation, and run BR has both a magnetic field and the radiative transfer. For run BI, we have dropped Equations (3.3) and (3.5) and adopted the isothermal equation of state \(P_{\text{gas}} = \rho c_s^2\) instead of Equation (3.6).

With the exception of the magnetic field, our initial conditions are almost identical to the those in Myers et al. (2011) and (with the exception of the protostellar outflows) Cunningham et al. (2011). In all of our runs, we begin with an isolated sphere of gas and dust with mass \(M_c = 300 \, M_\odot\), radius \(R_c = 0.1 \, \text{pc}\), and temperature \(T_c = 20 \, \text{K}\). The density follows a power-law profile proportional to \(r^{-1.5}\), so that the density at the edge of the core is

\[
\rho_{\text{edge}} = \frac{3M_c}{8\pi R_c^3}. \tag{3.16}
\]

The surface density of these cores, \(\Sigma_c = M_c/\pi R_c^2 \approx 2.0 \, \text{g cm}^{-2}\), is chosen to resemble that observed in galactic regions of high-mass star formation. For example, McKee & Tan (2003) inferred a mean \(\Sigma \approx 1 \, \text{g cm}^{-2}\) from the sample of high-mass clumps in Plume et al. (1997). The corresponding mean density is \(\bar{\rho} \approx 4.8 \times 10^{-18} \, \text{g cm}^{-3}\), or \(\bar{n}_H = 2.4 \times 10^6 \, \text{H nuclei cm}^{-3}\). This value determines the characteristic timescale for gravitational collapse, given by

\[
t_H = \sqrt{\frac{3\pi}{32G\bar{\rho}}} \approx 30.2 \, \text{kyr}. \tag{3.17}
\]

While these initial parameters are to an extent chosen for computational convenience (higher densities mean shorter free-fall times, which mean fewer total time steps need to be taken) they are consistent with sub-mm interferometric observations of massive cores (Swift 2009). Furthermore, the \(r^{-1.5}\) density profile agrees with observations of star-forming regions at the \(\sim 1 \, \text{pc}\) clump scale (Beuther et al. (2007), Caselli & Myers (1995), Mueller et al. (2002)) and the \(\sim 0.1 \, \text{pc}\) core scale (Longmore et al. 2011; Zhang et al. 2009). Similarly, a recent mid-infrared extinction study (Butler & Tan 2012) observed 42 massive cores in 10 different IRDCs and (after envelope subtraction) reported a mean \(k_\rho\) of \(\approx 1.6\). They also report that the power-law profile was a better fit to their observations than the less centrally concentrated Bonnor-Ebert profile.

Our cores are placed at the center of a cubic box with side length equal to 0.4 pc, so that the sides are far enough removed from the core that there is minimal interaction from the boundaries. The parts of the box that are not covered by the core are filled with a hot, diffuse medium with \(\rho_m = \rho_{\text{edge}}/10\) and \(T_m = 200 \, \text{K}\), so that the ambient medium will be in thermal pressure equilibrium with the core. We set the opacity of this confining gas to zero so that it will not cool as the simulation proceeds. The initial condition on \(E_R\) is given everywhere by \(a_R T_R^4\), where the radiation temperature \(T_R\) is also set to 20 K.
For boundary conditions, we choose outflow for the MHD update, meaning that in advancing the hyperbolic subsystem we set the gradients of \( \rho, \rho v, E, \) and \( B \) to zero at the domain boundary. For the radiation update, we use Marshak boundary conditions, meaning that the entire simulation volume is bathed in a blackbody radiative flux corresponding to 20 K, while radiation generated within the simulation volume may escape freely. Finally, in solving Equation (3.10) for \( \phi \), we require that \( \phi = 0 \) at the boundaries.

We also give the core an initial 1D velocity dispersion of \( \sigma_c = 2.3 \text{ km s}^{-1} \), chosen to put the core into approximate virial balance. If we take the virial ratio \( \alpha \) to be \( 5\sigma_c^2 R_c / GM_c \) (Bertoldi & McKee 1992), then \( \alpha \approx 2.1 \). Thus, there is initially slightly more kinetic energy than gravitational potential energy in each of our cores. We choose a slightly super-virial value for \( \alpha \) because we do not drive the turbulence by adding kinetic energy after the simulations begin. Although the virial parameter greater than unity at \( t = 0 \), it has decayed to \( \approx 1.0 \) by the time the simulations end. The velocities themselves are drawn from a Gaussian random field with power spectrum \( P(k) \propto k^{-2} \), appropriate for the highly supersonic turbulence found in molecular cloud cores. We include the perturbations in the following manner: first, we generate a \( 1024^3 \) perturbation cube using the method of Dubinski et al. (1995) with power on scales ranging from \( k_{\text{min}} = 1 \) to \( k_{\text{max}} = 512 \). We then place the cube over the simulation volume and either coarsen or interpolate the perturbation data so that we can represent perturbations at all levels of refinement. We have made no attempt to filter out compressive modes from the initial velocity field. The precise mixture of solenoidal and compressive components have been found to be important for gravitational fragmentation in unforced core collapse simulations (Girichidis et al. 2011) and on the overall rate of star formation in simulations with driven turbulence (Federrath & Klessen 2012), but we do not explore this effect here.

In our MHD runs, we also give the cores an initial magnetic field pointing in the \( z \) direction. The importance of this field is best expressed in terms of the mass-to-flux ratio:

\[
\mu_\Phi = M/M_\Phi, \tag{3.18}
\]

where

\[
M_\Phi \approx \frac{\Phi}{2\pi G^{1/2}} \tag{3.19}
\]

is the magnetic critical mass and \( \Phi \) is the magnetic flux threading the core. Cores with \( \mu_\Phi > 1 \) are unstable against gravitational collapse, while cores with \( \mu_\Phi < 1 \) are expected to be stable. Measurements of Zeeman splitting in both the OH molecule (Troland & Crutcher 2008), which probes densities of \( 10^3-4 \text{ cm}^{-3} \), and the CN molecule (Falgarone et al. 2008), which probes higher densities of \( 10^5-6 \text{ cm}^{-3} \), show that the mean value of \( \mu_\Phi \) is approximately 2, a value supported by theoretical arguments as well (McKee 1989). Note, however, that there may be substantial scatter in the magnetic field strength such that many dark molecular cloud cores have much more supercritical values of the mass-to-flux ratio (Crutcher et al. 2010). In this paper, we adopt \( \mu_\Phi = 2 \) for all of our MHD runs, and defer a more extensive parameter study on the effects of the magnetic field strength to a later work.
In the absence of more detailed information about the magnetic field geometry, we will assume that the spatial dependence of the initial $B$ field follows the cylindrically symmetric profile

$$B(R_z) = B_{\text{edge}} \left( \frac{R_z}{R_c} \right)^{-1/2} \hat{z},$$

(3.20)

where $R_z$ is the distance to the z axis and the value of $B_{\text{edge}}$ is chosen to give the desired mean mass-to-flux ratio for overall core:

$$B_{\text{edge}} = \frac{3 \sqrt{GM_c}}{2 \mu \Phi R_c^2}.$$  

(3.21)

For $\mu = 2$, $B_{\text{edge}} \approx 1.2$ mG. Using this form for the initial magnetic field is clearly an idealization, but it does have the advantage that it 1) satisfies the condition $\nabla \cdot B = 0$, and 2) ensures that the mass-to-flux ratio in the central flux tube ($\sim 5.6$ above critical) does not greatly exceed the mean value for the overall core, consistent with the Zeeman measurements discussed above.

Our initial conditions do not include any explicit rotation on top of the random turbulent perturbations described above. However, these perturbations do include some incidental angular momentum. In fact, as found by Burkert & Bodenheimer (2000), Gaussian random turbulence alone may be sufficient to account for the observed rotational properties of prestellar cores. When we apply the technique in that paper to measure $\beta_{\text{rot}}$ for our cores, we get $\beta_{\text{rot}} = 0.012$, in line with the values observed in Goodman et al. (1993). Note, however, that as discussed in Dib et al. (2010), the rotational properties of cores measured in projection by observers may differ substantially from the actual 3D values. In fact, if we calculate $E_{\text{rot}}/E_{\text{grav}}$ from our initial conditions using the full 3D velocity and density information, we get $\approx 0.002$, lower than $\beta_{\text{rot}}$ by a factor of 6. Thus, while the rotation in our initial conditions is consistent with observations, it is significantly lower than in other simulations that impose solid-body rotation in addition to random turbulence, such as those of Seifried et al. (2012). Finally, as we do not chose the direction of the angular momentum vector in our cores explicitly, there was no imposed choice about the initial orientation of the core angular momentum vector $L$ with respect to $B$. It turns out to be misaligned with the magnetic field by $\theta \approx 60$ degrees.

While the above initial conditions are clearly somewhat artificial, they do capture the essential observed properties of high-mass dark-cloud cores. The most unrealistic aspect of our initial conditions is probably our imperfect treatment of the initial turbulence. While we include perturbations to the velocity field, there are no corresponding perturbations to the density at time $t = 0$. Thus, while the velocity field soon creates filamentary structures reminiscent of those expected from turbulence, these filaments do not have the same properties they would in a self-consistent realization of a turbulent density-velocity field, as discussed in Krumholz et al. (2012b) and Federrath & Klessen (2012). Krumholz et al. (2012b) found that this difference can have an important impact on e.g. the overall star formation rate, so we mention it here as a caveat. Another caveat is that our initial velocity field does not
include any infall motions at \( t = 0 \). This probably has the effect of encouraging fragmentation somewhat, since the accretion rates and therefore the protostellar heating rates would be higher if infall were included from the beginning. Ideally, one would generate initial conditions for massive cores from larger simulations at the clump scale, which would then contain self-consistent density perturbations and infall. We are considering these issues in simulations of massive star formation at the cluster scale that are now in progress. The goal of this paper is to examine an idealized case first to elucidate the underlying physics.

We wish to emphasize that we have chosen the above runs to as far as possible create a controlled experiment where we have isolated the effect of only one physical process. Runs BR and BI are identical expect for the presence of the radiative feedback, and runs HR and BR are identical except for the presence of the magnetic field. Thus, we can isolate the effect of the radiative feedback by comparing the first set of runs, and the effect of the magnetic field by comparing the second.

\section{Results}

Here, we summarize the main results of our calculations. The simulations presented here were run on the NASA supercomputing platform Pleiades on 128 to 512 processor cores and took a total of about 700,000 CPU hours.

\subsection{Density Structure}

The time evolution of the large-scale structure of cores BR, HR, and BI is shown in Figure 3.1. In all three runs, the imposed velocity perturbations create a system of filaments embedded within the collapsing core that feed gas into the central region where the massive star is forming. In the MHD runs the velocity perturbations rearrange the field lines so that the filaments are primarily perpendicular to the field. At this scale, the primary difference between the runs is that the filamentary structure created by the velocity perturbations in run HR is much more pronounced than in either of the runs with a magnetic field, despite the fact that all three runs have the same sonic Mach number of \( \sim 15 \). There are two reasons for this behavior. First, even though the cores in runs BR and BI are highly supersonic, they are only marginally super-Alfvenic, with \( M_A \approx 1.9 \). The presence of the faster magnetic signal speeds means that although shocks parallel to the magnetic field lines can be as strong as in run HR, flows perpendicular to the field that would be strong shocks in run HR are only weak shocks - or not shocks at all - in the other two runs. The overall effect is that, even ignoring gravity, the density contrasts imposed by the turbulence in the MHD runs are smaller than the hydro only run. Second, in all three runs, over-densities created by the turbulence can grow due to the self-gravity of the gas. However, in the presence of the magnetic field, these dense regions are only able to grow by drawing in material along the field lines, whereas there is no such restriction in the hydrodynamic case. The combined effect is that density distribution in the cores at a given time is broader in run HR than in the other two - that is,
Figure 3.1: Column density through the simulation volume at 6 different times for runs BR (left), BI (middle), and HR (right). Projections are taken along the $x$ direction, and the initial magnetic field is oriented in the positive $z$ direction. We have set the viewing area of the images to be 0.3 by 0.3 pc to show the global evolution of the entire core. Star particles are portrayed as black circles, with the size of the circle corresponding to the mass of the star. The smallest circles represent stars with masses between $0.05M_\odot$ and $1.0M_\odot$. The next size up represents masses between $1.0M_\odot$ and $8.0M_\odot$, and the largest represents stars with masses greater than $8.0M_\odot$. 


Figure 3.2: Same as Figure 3.1, but zoomed in to show the central 5000 AU around the most massive star in each simulation. Projections are still taken along the $x$ direction through the entire simulation volume.
3.3. RESULTS

Figure 3.3: Top - Face-on view of the disk in the high-resolution version of run BI at 0.2 \( t_{\text{ff}} \). The colors correspond to the column density through a sphere of radius 100 AU centered on the star particle. The arrows show the direction of the mean in-plane velocity of the disk gas. Bottom - the black circles show the mean angular velocity \( \omega \) in the disk as a function of cylindrical radius \( R_z \). The red line corresponds to a Keplerian profile normalized using the mass of the star. We have also shown the sink particle accretion zone in green to demarcate the radius at which our sink particle algorithm begins to alter the fluid properties.

the dense regions are more dense and the diffuse regions more diffuse. Finally, we note in passing that at this scale the effect of the radiative heating has essentially no effect on the morphology of the core; the gas structure in runs BR and BI appears practically identical.

The situation is different when we zoom in to show the central 5000 AU of the simulation volume as in Figure 3.2, where the center is defined as the location of the most massive star in the simulation. At this scale, we begin to see clear differences in the gas morphology between runs BR and BI. In both cases, the gas collapses into a network of filaments, and there is a rough correspondence between the filaments in BR and those in BI. However, the filaments in run BR are much fatter and more diffuse than in run BI. This is easily understood as a consequence of radiative heating. For an isothermal, magnetized filament like the ones in BI, both the magnetic and pressure forces scale the same way with filament size as gravity in the virial theorem (see the Appendix (3.6) for a more detailed discussion). Thus, either the total pressure (magnetic plus thermal) is initially enough to halt collapse, or else it will never be and the filament will collapse until something causes the equation of state to deviate from isothermality (Inutsuka & Tsuribe 2001). This behavior is clearly seen in run BI, where the filaments contract until they reach the density at which our code creates sink particles. In run BR, on the other hand, radiative feedback from the central protostar has already caused the gas to become non-isothermal, and thus filaments close to the protostar stop collapsing before much sink creation takes place.

We can also isolate the effect of the magnetic field on the gas morphology by comparing runs BR and HR. There are two main differences. First, without the magnetic field to
3.3. RESULTS

help support it, the main filament of gas feeding the central protostar has already begun to fragment into self-gravitating, spherical "beads" by 0.3 free-fall times. These beads have a characteristic size of a few hundred AU, and are therefore well-resolved in our runs. The type of grid-induced filament fragmentation discussed in section 3.2.2 in the context of isothermal simulations is thus not a concern in runs BR and HR. Second, beginning around the same time, we can see the presence of a dense, $\sim 200$ AU disk around the most massive star in run HR. This disk is centrifugally dominated with a roughly Keplerian velocity profile. We do not see a similar disk in either of our runs with a magnetic field, at least at the 10 AU resolution of the simulations presented here. This is the well-known magnetic braking effect, where at $\mu_\Phi = 2$ the field is so efficient at removing angular momentum from the center of the core that it suppresses the formation of a Keplerian disk (Allen et al. 2003; Hennebelle & Fromang 2008; Mellon & Li 2008). However, if we repeat run BI with three more levels of refinement so that the maximum resolution is 1.25 AU and the sink accretion radius is 5 AU, we do in fact begin to see a rotationally-dominated disk beginning around $\sim 0.15 t_{ff}$. By about $\sim 0.2 t_{ff}$, when the star has reached a mass of about $3.5M_\odot$, the disk has grown to $\sim 40$ AU and developed a Keplerian velocity profile, as shown in Figure 3.3. This would lie entirely within the sink particle accretion zone in our simulation with 10 AU resolution, so it is not surprising that we do not see it there. While magnetic braking has certainly removed angular momentum from the material accreting onto the disk, allowing it to fall much closer to the central protostar than would be the case without a magnetic field, we do not find that it suppresses the formation of a disk entirely at high resolution.

Several other researchers have already reported forming disks in MHD simulations of star formation. In a study of magnetic braking in low-mass cores, Hennebelle & Ciardi (2009) found that the efficiency of magnetic braking depends on the angle between the initial magnetic field and the core's angular momentum vector, with a 90 degree misalignment lowering the value of $\mu_\Phi$ at which disk formation is suppressed by a factor of $2-3$ relative to the aligned case. Santos-Lima et al. (2012b,a) studied this problem numerically as well, arguing that the presence of turbulence increases the rate of magnetic diffusion in the inertial range, allowing parcels of gas that have lost magnetic flux to fall onto a disk. Seifried et al. (2012) also found that the presence of turbulent perturbations reduces the efficiency of magnetic braking enough to form a Keplerian disk at $\mu = 2.6$, although they disagree that flux loss is involved. In our disk, we find $\mu$ averaged over a 100 AU sphere around the most massive star has risen to $\sim 20$ by the snapshot displayed in Figure 3.3, although we have not verified that this is due to the mechanism proposed by Santos-Lima et al.

Finally, we briefly mention one more difference between our magnetic and non-magnetic runs: the presence of episodic outflows in runs BI and BR. Around $0.3t_{ff}$, we begin to find material in those runs with radial velocities of $\sim 10$ km s$^{-1}$ away from the primary star. These outflow velocities increase with time, such that by $0.6t_{ff}$ (when the primary has grown to $> 20M_\odot$) they can be as large as 40 km s$^{-1}$, which is roughly the Keplerian speed at the grid scale. $\sim 10$ km s$^{-1}$ outflows have been observed previously in non-radiative MHD simulations of massive cores (e.g. Seifried et al. 2011; Hennebelle et al. 2011). However, because the outflow launching mechanism is badly under-resolved in our simulations, we shall
3.3. RESULTS

not discuss outflow properties in detail here.

3.3.2 Magnetic Field Structure

Although the magnetic field lines are initially oriented in the \( z \) direction, this is not an equilibrium configuration, and as the simulations proceed they settle into a new, quasi-equilibrium “hourglass” shape shown in Figure 3.4, which resembles the morphology in the dust polarization maps of Girart et al. (2009) and Tang et al. (2009). Here, we take a density slice through the center of the domain aligned to be perpendicular to the \( x \) direction. On top of that slice, we show the planar components (that is, the \( y \)- and \( z \)-components) of the magnetic field lines. This slice is taken from run BR at 0.3 free-fall times, but the overall shape of the field lines is similar at other times as well, provided enough time has passed for the initial conditions adjust to the new equilibrium. Because the Alfvén Mach number of the initial turbulence is \( \sim 2 \), the lines are able to be bent somewhat by the turbulent perturbations, but this is not a large effect. In the slice shown in Figure 3.4, we can see a dense filament in red, with the field lines adjusting so that the magnetic field tends to be perpendicular to the axis of the filament.

3.3.3 Fragmentation and Star Formation

The most dramatic difference between the three runs is in the fragmentation. In all three cases, there is a primary with a mass of about \( 23 M_\odot \). In run BR, there is also a secondary star with less than \( 1 M_\odot \) of material. In runs BI and HR, however, the filaments that feed the primary object have fragmented into dozens of stars by the end of 0.6 free-fall times, with typical masses of \( 0.2 M_\odot \) but ranging up to \( \sim 11 M_\odot \). This filament fragmentation takes place beginning around 0.2 to 0.3\( t_{\text{ff}} \). By 0.5\( t_{\text{ff}} \), these stars have fallen into the central region and undergone significant N-body interactions with each other. After that time, the positions of the sinks in Figures 3.1 and 3.2 no longer correspond to the places they were born - many of the sinks have been ejected towards the outer regions of the core.

We summarize the properties of star particles in all three runs in Figures 3.5 and 3.6. Note that we only count a sink particle as a star once it has passed the minimum merger threshold of \( 0.05 M_\odot \). Thus, the extra stars in runs BI and HR are not temporary objects that will eventually accrete onto the primary. While the exact value of this threshold is somewhat arbitrary, we point out that in runs BI and HR, there are a few dozen small sink particles that do not meet this threshold by 0.6 free-fall times, while in run BR there are none. Thus, we do not believe that the basic conclusion that fragmentation is dramatically suppressed in run BR compared to the others is sensitive to the exact numerical value of value of the minimum merger mass.

One possible explanation for the difference between runs BR and HR is that extra fragmentation in run HR is due to disk fragmentation that is not present in the other runs because the magnetic field has removed much of the angular momentum from the central region. However, this is not the case. From Figure 3.2, we can see that run HR has already
Figure 3.4: A density slice taken through the center of the computational domain perpendicular to the x-axis at 0.3 free-fall times. The y- and z- components of the magnetic field lines are over-plotted in white with evenly-spaced anchor points along the y-axis.
3.3. RESULTS

Figure 3.5: Number of stars $N_\ast$ (top), total stellar mass $M_\ast$ (middle), and mass of the most massive star $M_p$ (bottom) for all three runs as a function of free-fall time. In this figure and throughout the rest of this paper, we only count a sink particle as a star if it has passed the minimum merger mass of $0.05 \, M_\odot$, ensuring its permanence as the simulation proceeds.
Figure 3.6: Fraction of total stellar mass that is in stars with mass less than $m$ for all three runs at $t_{\text{ff}} = 0.6$. 
undergone significant fragmentation in filaments well before the disk has grown large enough to fragment. In fact, most of the stars in run HR form at distances of a few thousand AU or greater from the central star - well outside the disk. Whatever the cause for the difference in fragmentation between the runs with a magnetic field and run HR, it is not due to the presence of a disk in one and not in the others.

As discussed in section 3.2.2, although much of the fragmentation in run BI is numerical in that it comes from filaments that collapse down to $\rho_{\text{max}}$, it is still possible to choose $\Delta x_L$ such that the fragment masses are roughly correct. From Equation (3.14), $\rho_{\text{max}}$ in run BI ranges from $\sim 10^{-14}$ (for $\beta \to \infty$) to $\sim 10^{-13}$ g cm$^{-3}$ (for $\beta = 0.01$), and so the density at which sink creation occurs in our simulations roughly mimics the density at which molecular gas can no longer cool efficiently. Thus, we expect that the fragmentation in run BI is qualitatively similar to what would happen in massive cores if there was no protostellar feedback: the filaments would fragment a bit after they reached densities of $\sim 10^{-13}$ g cm$^{-3}$, and one would end up with many more fragments than would be formed in the presence of radiative heating. Furthermore, some of the protostellar properties in run BI do indeed appear to be converged. If we compare both the total mass in stars and the mass of the primary in run BI to the high-resolution version of BI at 0.2 $t_{\text{ff}}$, we find that they differ by only 6% and 5% respectively, over a factor of 8 difference in resolution. Thus, while the number and mass distribution of the fragments in run BI are not converged, quantities that depend mainly on the overall accretion rate do seem to be.

In addition to the fragmentation, we also find that the magnetic field slows down the overall rate of star formation by about a factor of about 3, consistent with Padoan & Nordlund (2011) and Federrath & Klessen (2012). At 0.6 free-fall times, the total mass in stars in run BR is about $\sim 20M_\odot$, almost all of which is in the primary, compared to over $60M_\odot$ in the run HR. Almost all of the “extra” star formation in the run HR has gone into stars other than the primary, which contains only $\sim 40$% of the total stellar mass at 0.6 free-fall times. The mass of the most massive star, on the other hand, is approximately the same in all three runs, probably because our initial conditions place the same amount of mass in position to quickly collapse towards the center. Beginning at around $0.5t_{\text{ff}}$, there is an increase in the rate of star formation in run HR as compared to the others. This increase is associated with the fragmentation of a filament formed in the outer region of the core that by $\sim 0.5t_{\text{ff}}$ has begun to form stars, as shown in the bottom panels of Figure 3.1. The relative timescales here are roughly what one would expect from inside-out collapse given our initial conditions: for a power-law density profile with slope $-1.5$, the ratio of the free-fall time at $0.75R_c$ to that at $0.25R_c$ is about a factor of 1.5, which is approximately the delay we see here. Note that filament fragmentation in the outer regions of the core does not happen in either of the runs with a magnetic field - there, star formation only occurs close to the core center. We will discuss this difference further in section 3.4.

We mention here as a caveat that our 10 AU resolution means that we cannot resolve any binaries closer than $\sim 40$ AU, the accretion radius on one sink particle. Thus, we cannot rule out the possibility that the massive star present in run BR would in fact be a massive binary with a separation of $\lesssim 40$ AU if we had higher resolution. However, even if that were the
case, the fragmentation would still be qualitatively different than in runs HR and BI, where we form dozens of stars with a masses that sample the full IMF.

### 3.3.4 Thermal Structure

The difference in fragmentation between runs BI and BR is expected, since it is well-established that radiative feedback in massive cores reduces fragmentation by raising the thermal Jeans mass of the collapsing gas (e.g. Krumholz et al. 2007a, 2010). The difference in fragmentation between runs BR and HR, however, is more interesting. One possibility is that protostellar heating is somehow more efficient in the presence of magnetic fields. Figure 3.7 shows maps of the average temperature through a 5000 AU cube centered at the most massive star in runs BR and HR. We find that, contrary to this hypothesis, the heating in run HR is either similar to or slightly more widespread than in run BR, because accretion rates are higher in the absence of the field. This is not a dramatic effect, however. The total protostellar luminosity in run BR is typically smaller than that of run HR by only a factor of \( \sim 0.7 \). The temperatures, which in the optically thin limit scale like \( L^{0.25} \), would be lower by only a factor of \( \sim 0.9 \). At 0.25\( t_{\text{ff}} \), when the first fragmentation in run HR occurs, the mean \( T_g \) in the 5000 AU cube around the primary is 63.3 K in run HR and only 53.7 K in run BR, but despite the higher temperatures the gas in HR fragments while the gas in BR does not. So, the difference in the effectiveness of radiative heating between runs BR and HR cannot be responsible for the difference in fragmentation - it is too small and in the wrong direction.

The difference, then, must be due to the direct support provided by the magnetic field in run BR. To quantify this effect, we define an effective temperature \( T_{\text{eff}} \) by

\[
\frac{3}{2} n k_B T_{\text{eff}} = \frac{3}{2} n k_B T_g + \frac{B^2}{8\pi},
\]

where \( n \) is the number of particles per unit volume. Expressed in terms of \( \beta \), we find

\[
T_{\text{eff}} = T_g \left(1 + \frac{2/3}{\beta}\right).
\]

In other words, \( T_{\text{eff}} \) is the temperature defined in terms of the thermal plus magnetic energy densities instead of just the thermal energy density. Note that this is actually more closely related to our criterion for creating a sink particle than the gas temperature because we have included the magnetic energy in defining the magnetic Jeans number (see the Appendix). While the concept of an effective temperature is clearly an oversimplification - for one, the magnetic field does not resist collapse isotropically the way thermal pressure does - we find that it is helpful in understanding our simulation results.

In Figure 3.8, we summarize the combined temperature and magnetic field structure of the cores in runs BR, BI, and HR. In the two right panels, we plot the total mass in each \( \rho - T_g \) bin for runs BR and HR over a series of time snapshots. In the two left panels, we instead use \( \rho - T_{\text{eff}} \) bins for the two runs with magnetic fields. The top row of the figure merely summarizes our initial condition. Although the core temperature starts at precisely
Figure 3.7: Maps of the average gas temperature, taken at the same times as Figures 3.1 and 3.2. The averages were taken along the $x$ direction through a 5000 AU cube around the most massive star. Run BR is on the left and run HR is on the right.
20 K in all three runs, the cylindrically symmetrical magnetic field profile means that there are a range of magnetic field strengths, and thus $T_{\text{eff}}$ covers a range of values. The blue diagonal lines represent the threshold at which the code lays down a sink particle. Thus, there can be no gas in any of the runs to the right of this line - any cell that exceeds this threshold has some of its gas converted into sink particles until it no longer violates the MHD Truelove criterion. This line is suppressed in the third column, because in the presence of magnetic fields, there is no single density at which sinks are created for a given temperature (see Equation 3.14).

We can get a sense of whether star formation is taking place from these plots by looking at whether there is any gas close to crossing this threshold. In run BR, there is hardly any gas close to the densities required for sink formation. Runs BI and HR, on the other hand, have significant amounts of gas close to that threshold by around $0.2$ to $0.3t_{\text{ff}}$. The phase diagram for run HR, in particular, bears a number of “finger" features that correspond to gas that is all at one $T_g$, but that stretches over a range of densities approaching that required for sink formation. These features are most prominent at $0.3t_{\text{ff}}$, but are visible before and after as well. The is precisely the time at which the main filament in run HR has broken up into a number of gravitationally unstable “beads", which collapse down until they form sink particles. The “fingers," then, correspond to gas in these beads that is collapsing isothermally, albeit at higher temperatures than the initial 20 K, with the precise value determined by the distance from the bead to the central protostar. This collapse is isothermal because the temperature changes on the evolution timescale of the most massive protostar, which for our problem is $t_{\text{ff}}$, while the timescale for local gravitational collapse in the bead is must faster. In contrast, we do not see this behavior in run BR, because a combination of magnetic and thermal support has rendered the main filament in that run stable against gravitational collapse at a density much higher than the sink creation value.

In one sense, Figure 3.8 restates what we already know - there is much fragmentation in runs BI and HR and hardly any in run BR. However, this plot can also help us untangle the effect of the magnetic field and the radiative feedback by telling us in which regimes each effect is more important. By comparing runs BR and BI, for instance, we can see that the primary effect of the radiation is to heat up the relatively dense regions in the core - i.e. to move material greater than about $1 \times 10^{-15} \text{ g cm}^{-3}$ up in the plot and away from the sink formation threshold. Alternatively, the slope of the $(\rho - T_{\text{eff}})$ phase diagrams for runs BR and BI show that the magnetic field is most effective at raising $T_{\text{eff}}$ at low density. Hence, we can begin understand that the reason the combination of the $B$ field and the radiative feedback is more effective at suppressing fragmentation than either considered in isolation is that they are effective in different regions, with the magnetic field mostly helping to support (or, at least, to slow the collapse of) material in the diffuse, outer parts of the core, and with radiation most effective in the dense material that is close to the central protostar.
Figure 3.8: Left two panels - phase diagrams showing the amount of mass in each $\rho - T_{\text{eff}}$ bin at different times, for runs BR and BI. Right two panels - the same, but with $\rho - T_g$ bins for runs BR and HR. The snapshots are taken at the same times as the above figures. The islands of low density material at $T_g \sim 10^2$ K and $T_{\text{eff}} \sim 10^4$ K correspond to gas in the ambient medium and should be ignored.
3.4 Discussion

3.4.1 Why do Magnetic Fields and Radiation Suppress Fragmentation?

We would like to understand the suppression of fragmentation in run BR in terms of the mass-to-flux ratio $\mu_\Phi$. The average $\mu_\Phi$ for the entire core is 2, but, because the core is centrally concentrated, it is greater through flux tubes passing near the center and lower through flux tubes passing through the diffuse, outer regions. It is illustrative to do the following analysis on our initial conditions: take the initial spherical region and exclude a cylindrical region of radius $R_z$ concentric with the sphere and extending through the entire domain. Then, compute $\mu_{\Phi,z}$, the mass-to-flux ratio in the remaining region. $\mu_{\Phi,z}$ is 2 when $R_z = 0$ and monotonically drops to 0 when $R_z = R_c$. How quickly $\mu_{\Phi,z}$ drops off with $R_z$ will give us a rough estimate of where we can expect the core to be subject to fragmentation. We find that, by a radius of $R_z \approx 0.73R_c$, $\mu_{\Phi,z}$ has dropped below 1, meaning that the region external to that cylindrical radius (corresponding to approximately 32% of the core volume and 19% of the mass) should be fairly well-supported against collapse. Furthermore, the point at which $\mu_{\Phi,z}$ has dropped to 1.5 is at only $0.44R_c$, meaning that $\sim 72\%$ of the core volume and $\sim 53\%$ of the mass has a mass-to-flux ratio below that value. While structures with a mass-to-flux ratio of 1.5 are supercritical and should collapse, they will still collapse more slowly than in the absence of the magnetic field, giving the radiative feedback more time act. This effect is not dramatic; the effect of the magnetic pressure force in the virial theorem is to dilute gravity along the field lines by a factor of $(1 - \mu_\Phi^{-2})$ (Shu & Li 1997), so that structures that are supercritical by a factor of 1.5 collapse approximately half as quickly as structures with $\mu_\Phi$ of infinity, and even the core as a whole collapses about 75% as fast at $\mu_\Phi = 2$.

Thus, even inside a supercritical core, the magnetic field can slow collapse in specific sub-regions with mass-to-flux ratios below unity, or even halt it altogether, and this mechanism is more effective at suppressing fragmentation in the outer regions of the core. We can see this effect operating in the bottom row of Figure 3.1 - while most of the star formation in run HR takes place towards the central region of the core, by 0.6 free-fall times we have begun to see signs of fragmentation of a filament in the outer regions as well. This does not take place in either of the runs with magnetic fields, including the one with no radiation, so this cannot be a radiative effect. Rather, it is due to the ability of the magnetic field to effectively suppress fragmentation in the outer regions of the core. To quantify this, we plot in Figure 3.9 the distribution of $r_*$, which measures how far away each star was from the central massive star at the time it formed. We compute this quantity for every star (other than the first) that forms over the history of each simulation. In run BI, most of the fragmentation takes place at distance of $\sim 2,000$ AU, and there are no stars that form at a distance greater than $\sim 5,000$ AU from the central object. In run HR, however, the most likely value of $r$ is roughly 5000 AU, while a significant fraction (about 1/3) of the stars form at distances of 10,000 AU or greater. Star formation at such large radii is completely suppressed by the magnetic field.
Figure 3.9: Histograms of star formation distance $r_*$ for runs BI, HR, and BR. Here, $r_*$ is the distance each star was from the most massive star when it formed, computed for every star that forms over the entire history of each simulation. In run BR, there is only one secondary fragment, which forms a distance of $r_* \approx 3600$ AU from the primary.
even without heating effects.

The following picture thus emerges: magnetic fields work to suppress fragmentation in the outer regions of the centrally-concentrated cores, either by slowing it down or halting it altogether. If they halt it altogether, then fragmentation is confined to the central region, where radiative heating is most effective. If magnetic fields merely slow fragmentation at large radii, then they still allow radiative heating more time to “win” by heating up filaments to the point at which they are too warm to collapse further. The combined result is that the magnetic field and the radiation are together far more effective at suppressing fragmentation than either process in isolation. Our work suggests that typical massive cores, which are centrally concentrated and have $\mu_{\Phi} \sim 2$, do not fragment strongly, as one would expect from the correspondence between the core and initial stellar mass functions.

Finally, while our simulations are based on ideal MHD, we do not expect that non-ideal effects will dramatically alter our conclusions. Ohmic dissipation, ambipolar diffusion, and reconnection diffusion (Santos-Lima et al. 2010; Lazarian 2011) are all capable of increasing $\mu_{\Phi}$, but they also all most important at high densities and/or regions where the magnetic field lines are most bent. Those are precisely the regions where the magnetic field is least important for the suppression of fragmentation, because they are all primarily associated with the inner region of the core where radiative heating is most effective.

3.4.2 Comparison to Commerçon et. al.

Commerçon et al. (2011) have also performed a set of radiation-magnetohydrodynamic simulations of massive core collapse using similar initial conditions to the ones considered here. They also found a synergistic effect between the radiative heating and the magnetic field, where the two effects in tandem lead to much less fragmentation than either considered in isolation. However, their work differs from our own in a few key respects. Most significantly, they have focused on the early stages of collapse, up to just past the point at which the first hydrostatic core forms, while we have focused on what happens to the remainder of the core after the first protostar has undergone second collapse. Thus, while we have both identified mechanisms by which a combination of radiative and magnetic effects suppress fragmentation, these mechanisms have different underlying causes and manifest themselves at very different times.

This difference in emphasis stems from our different recipes for representing protostellar feedback. In this work, we use sink particles to represent material that has collapsed to densities higher than we can follow on the grid, and compute a luminosity $L_i$ for each particle according to a sub-grid model. Commerçon et al. (2011), on the other hand, do not use sink particles, but instead employ higher resolution (2.16 AU), which allows them to follow the formation of a hydrostatic core. The feedback from the accretion shock on to this core can then be computed on the grid. They find the radius at which this shock releases its energy depends on the core’s magnetic field, since strong magnetic braking allows smaller first cores to form.

The approach of Commerçon et al. (2011) has the advantage of self-consistency. Fur-
3.4. DISCUSSION

thermore, it is probably more accurate than our technique at representing radiation from particles before the second core has formed. In fact, we do not include any radiative feedback until the sink particle mass exceeds $0.01 M_\odot$, and second collapse generally takes place at a few times that value, so except for a very short period of time, we ignore this radiation altogether. Also, because their resolution is higher and they do not have to handle sink particle mergers, they can resolve binaries that we cannot. However, their method has the downside that it cannot model the effect of the much larger (by a factor of $\sim 100$) accretion luminosities that occur after second collapse. Additionally, the lack of sink particles severely limits the integration time for Commerçon et al. (2011), since they cannot follow collapse past the point where they fail to resolve the Jeans density. This limited them to running for only a few percent of a free-fall time after the first hydrostatic object formed. In contrast, with our initial conditions the first core forms almost immediately, and we find that most fragmentation does not occur until around 20% to 30% percent of a free-fall time past that point. In a sense, our simulations pick up where those of Commerçon et al. (2011) left off, in that our simulations begin with a centrally concentrated core with one protostar that very quickly undergoes second collapse.

Sub-grid luminosity models have problems of their own related to unresolved binarity, as pointed out by Bate (2012). As mentioned in section 3.3.3, our use of sink particles means that we cannot resolve any binaries closer than 40 AU, and we cannot rule out the possibility that the central massive star in our simulations in fact represents an unresolved binary. However, for accretion luminosity-dominated stars, it matters little whether a sink particle represents a single star or a binary too tight to be resolved, because the energy released per unit mass accreted onto low-mass protostars is nearly independent of the stars' masses (Krumholz 2011). On the other hand, stars' internal luminosity scales with mass as roughly $M^{3.5}$, meaning that in the worst case where a sink particle should in fact represent an equal mass binary, the internal luminosity is overestimated by a factor of $2^{2.5} = 5.7$. While this is a potential concern, the internal luminosity does not become comparable to the accretion luminosity in our calculations until about $0.3 t_{ff}$, and by that time there are already clear differences in the fragmentation between runs HR and BR. Moreover, the alternative of not including a sub-grid luminosity model, as in Bate (2012), is far worse. Without such a model one omits both the accretion luminosity onto the stellar surface and the larger internal luminosity, and the resulting error is many orders of magnitude.

Finally, we mention one last difference between our work and Commerçon et al. (2011): their initial conditions contained much less kinetic energy than our own, with $\alpha_{\text{vir}} = 0.2$ versus $\alpha_{\text{vir}} = 2.3$. This could explain why we see a small disk in our high resolution run, and Commerçon et al. (2011) do not. If their $\beta_{\text{rot}} \approx 0.02 \alpha_{\text{vir}}$, as implied by Burkert & Bodenheimer (2000), then they would have $\beta_{\text{rot}} \approx 0.004$, smaller than our own by a factor of 3. On the other hand, Seifried et al. (2012) had $\beta = 0.04$, higher than ours by a factor of 4 again. Thus, the sequence of disk sizes seen in our papers, ranging from $\sim 100$ AU (Seifried et al. 2012) to $\sim 40$ AU (us) to unresolved (Commerçon et al. 2011), could simply be a consequence of different amounts of angular momentum in the cores. It is possible, however, that if Commerçon et al. (2011) extended their simulation to later times, that they too would
begin to resolve a disk in their $\mu_{\Phi} = 2$ run. We conjecture that this disk would be smaller than $\sim 40$ AU in radius.

### 3.4.3 Where is Fragmentation Suppressed?

An interesting question is: in what range of the $\Sigma - \mu_{\Phi}$ parameter space is fragmentation weak? As discussed in Crutcher et al. (2010), although the average molecular cloud core is marginally magnetically supercritical, it by no means follows that there are no cores with weak magnetic fields. The Bayesian analysis presented in that paper suggests the distribution of field strengths is quite flat, such that there may be many cores where the field is significantly weaker than the ones discussed here. In fact, for cores like ours with a mean density of $n_H = 2.4 \cdot 10^6 \text{ cm}^{-3}$, their result suggests that the total magnetic field strength should be evenly distributed between $\approx 0.0 \text{ mG}$ and $\approx 3.4 \text{ mG}$, roughly twice the value considered here. This implies that about 25% of cores like the ones in this paper would have values of $\mu_{\Phi}$ of 4 or greater. Our work suggests that there should be a tendency towards greater fragmentation in massive cores with such weak magnetic fields, with such cores being more likely to form clusters rather than isolated massive stars or binaries. A recent set of millimeter observations (Palau et al. 2013) studied the fragmentation of 18 massive cores with $\lesssim 1000$ AU resolution, and found that $\sim 30\%$ showed no signs of fragmentation, while $50\%$ did. They propose that variation in the magnetic field strength may be responsible for the determining the fragmentation, but confirmation of this view will have to wait for follow-up observations of the field.

Furthermore, the recent observations of Butler & Tan (2012) found a typical massive core surface density of $\sim 0.1 \text{ g cm}^{-2}$, over a factor of 10 lower than the $2 \text{ g cm}^{-2}$ cores considered here. These cores are below the surface density threshold for massive star formation $\sim 1 \text{ g cm}^{-2}$ identified in Krumholz & McKee (2008), which ignored magnetic fields. Could magnetic fields play a role in lowering the threshold for massive star formation? We plan to address these questions in future work.

### 3.5 Conclusions

We have presented a set of 3D, R-MHD simulations of the collapse of isolated, magnetized, massive molecular cloud cores that attempt to isolate the effects of the magnetic field and of the radiative feedback on core fragmentation. We find that the magnetic field and protostellar radiation can combine to largely suppress fragmentation throughout the core, so that the simulation that includes both magnetic fields and radiation results in only a single binary star system, while the runs that exclude either effect are subject to far more fragmentation. The explanation for this behavior is that magnetic fields and radiative heating are effective in different regimes, so that that each effect influences gas that the other misses. We find that massive cores with typical magnetic field strengths likely collapse to form single star systems, as suggested by the observed relationship between the CMF and IMF. We have also
reproduced the result found by other researchers that Keplerian disks can form in the presence of magnetic fields with $\mu_\Phi \sim 2$, provided that turbulence, which results in a misalignment between the magnetic field and angular momentum vectors, is present.

Acknowledgments

A.T.M. wishes to thank Pak-Shing Li, Louis Howell, Christoph Federrath, and Bo Zhao for helpful discussions, and the anonymous referee for constructive comments that improved the paper. Support for this work was provided by NASA through ATP grant NNG06-GH96G (R.I.K., M.R.K. and C.F.M.) and a Chandra Space Telescope grant (M.R.K.); the NSF through grants AST-0908553 and NSF12-11729 (A.T.M., R.I.K. and C.F.M.) and grant CAREER-0955300 (M.R.K.); an Alfred P. Sloan Fellowship (M.R.K); and the US Department of Energy at the Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344 (A.J.C. and R.I.K.) and grant LLNL-B569400 (A.T.M.). Supercomputing support was provided by NASA through a grant from the ATFP. We have used the YT toolkit (Turk et al. 2011) for data analysis and plotting.

3.6 Appendix

3.6.1 The MHD Truelove Condition

Simulations of isothermal, self-gravitating systems are subject to artificial fragmentation unless the Jeans length,

$$\lambda_J \equiv \left( \frac{\pi c_s^2}{G\rho} \right)^{1/2}, \quad (3.24)$$

is resolved by a sufficiently large number of grid cells,

$$J_{th} \equiv \frac{\Delta x}{\lambda_J} < J_{th,\text{max}}, \quad (3.25)$$

where $\Delta x$ is the width of a grid cell (Truelove et al. 1997). We have added the subscript “th" to the Jeans number $J$ to indicate that it is for the purely thermal case, in which there is no magnetic field. For the case they studied, Truelove et al. (1997) found that $J_{th,\text{max}} = 0.25$ was adequate to suppress artificial fragmentation, but in general it is a problem-dependent quantity. For AMR simulations, the Truelove condition is one of the criteria used to increase the refinement; for simulations in which sink particles are used as a sub-grid model for protostars, the Truelove condition is often used to determine when sink particles should be introduced—i.e., whenever the density exceeds

$$\rho_{\text{max}} = \frac{\pi J_{th,\text{max}}^2 c_s^2}{G\Delta x^2}. \quad (3.26)$$
Magnetic fields suppress fragmentation and therefore should allow one to defer refinement or the introduction of sink particles to higher densities. Federrath et al. (2010a) included the effects of the magnetic field in their refinement criteria, but it was used as a supplement to the thermal Truelove criterion, not as a replacement. For introducing sink particles, they did require that the total energy of a control volume be negative.

To generalize the Truelove condition to include magnetic fields, we begin with the expression for the maximum mass of an isothermal, magnetized cloud derived by Mouschovias & Spitzer (1976), as generalized by Tomisaka et al. (1988),

\[
M_{cr} = 1.18 M_{BE} \left[ 1 - \left( \frac{M_{\Phi}}{M_{cr}} \right)^2 \right]^{-3/2},
\]

where

\[
M_{BE} = 1.18 \left[ \frac{c_s^3}{(G^3 \rho)^{1/2}} \right]
\]

is the Bonnor-Ebert mass and

\[
M_{\Phi} \simeq \frac{\Phi}{2\pi G^{1/2}}
\]

is the magnetic critical mass. (Tomisaka et al. (1988) found that a factor 0.17 fit their numerical results better than 1/(2\pi), but we adopt the latter for simplicity.) Using the alternative form for the magnetic critical mass, \(M_B\), which is defined by

\[
\frac{M_B}{M_{cr}} = \left( \frac{M_{\Phi}}{M_{cr}} \right)^3,
\]

Equation (3.27) can be expressed as

\[
M_{cr} = \left[ 1.12 M_{BE}^{2/3} + M_{B}^{2/3} \right]^{3/2}
\]

(Bertoldi & McKee (1992), who wrote \(M_J\) for the Bonnor-Ebert mass). Evaluation of \(M_B\) gives \(M_B/M_{BE} = 0.76\beta^{-3/2}\). We define the critical radius by (Mouschovias & Spitzer 1976)

\[
M_{cr} = \frac{4}{3} \pi \rho R_{cr}^3,
\]

and then obtain

\[
R_{cr} = 0.39 \lambda_{J} \left( 1 + \frac{0.74}{\beta} \right)^{1/2}.
\]

We denote the critical radius in the absence of a magnetic field (\(\beta \to \infty\)) as \(R_{cr,th}\). As expected, we see that \(2R_{cr,th} \approx \lambda_{J}\).

In the non-magnetic case, the Jeans number is \(J_{th} = \Delta x/\lambda_{J} \propto \Delta x/R_{cr,th}\). We then generalize the Jeans number to the MHD case by writing

\[
\frac{J}{J_{th}} = \frac{R_{cr,th}}{R_{cr}}.
\]
so that
\[ J_{\text{th}} = J \left(1 + \frac{0.74}{\beta}\right)^{1/2}. \] (3.35)

Since the same relation applies to the maximum Jeans numbers, the MHD Truelove condition follows from Equation (3.26):
\[ \rho_{\text{max}} = \frac{\pi J_{\text{max}}^2 c_s^2}{G\Delta x^2} \left(1 + \frac{0.74}{\beta}\right). \] (3.36)

We note that if one expresses the Jeans length in terms of the energy density, \( u = \frac{3}{2} \rho c_s^2 \), as \( \lambda_J = \left(\frac{2\pi u}{3G\rho^2}\right)^{1/2} \) and then adds the magnetic energy density into \( u \), one obtains the same result as in Equation (3.36) except that the factor 0.74 is replaced by \( \frac{2}{3} \), a negligible difference. Our result is thus very similar to the approach advocated by Federrath et al. (2010a). The advantage of the present derivation is that it is directly tied to the maximum stable mass.

Magnetic fields can halt collapse perpendicular to the field, but they have no effect on gravitational instability parallel to the field (Chandrasekhar 1961). Gas that collapses along the field lines has a thickness
\[ H = \frac{\Sigma}{\rho_0} = \frac{\sqrt{2}}{\pi} \lambda_J, \] (3.37)
where \( \Sigma \) is the surface density, so that
\[ H = \left(\frac{0.45}{J_{\text{th}}}\right) \Delta x. \] (3.38)

In fact, a self-gravitating sheet cannot become thinner than \( 2\Delta x \), since a single layer of cells cannot exert a vertical gravitational force inside the layer. Hence, if it is important to follow the internal dynamics of gravitationally stable sheets, one should maintain \( 2\Delta x < H \), corresponding to \( J_{\text{th}} \leq 0.25 \) from Equation (3.38). This criterion does not apply to gravitationally unstable sheets \( (J > J_{\text{max}}) \), since they will either be refined or replaced by sink particles.

To test the MHD Truelove criterion, we carry out the same test used by Truelove et al. (1997), but with the addition of an initially uniform magnetic field. We begin with a cubic, periodic box of size \( \ell_0 \) filled with isothermal gas that has a spherically symmetric, Gaussian density profile
\[ \rho(r) = \rho(0) \exp \left[ -\left(\frac{r}{r_1}\right)^2 \right]. \] (3.39)
Here, \( \rho(0) \) is the central density and \( r_1 \) is a characteristic fall-off radius, which have taken to be 0.48 \( \ell_0 \). To this background density we also added a \( m = 2 \) azimuthal density perturbation with an amplitude of 10\%. We report our simulation results in units that have been normalized by \( \rho(0) \), \( \ell_0 \), and the central-density free-fall time, given by
\[ t_{\text{ff}} = \sqrt{\frac{3\pi}{32G\rho(0)}}. \] (3.40)
Figure 3.10: Logarithm of the column density (normalized to \( \rho(0) \ell_0 \)) through the filament in each run at the point at which the maximum density has reached \( 2 \times 10^7 \) times the initial value. Top Left - \( J_{\text{max}} = 0.125 \). Top Right - \( J_{\text{max}} = 0.25 \). Bottom Left - \( J_{\text{max}} = 0.375 \). Bottom Right - \( J_{\text{max}} = 0.5 \). The top panels have only one filament, while the bottom two show clear signs of artificial fragmentation. Each image shows a region of size \( \approx 0.07 \ell_0 \).
We have also set the core in initial rotation about the $z$ axis with an angular velocity $\omega$ such that $\omega t_{ff} = 0.53$. This value was chosen so that the core would make approximately 1 rotation in 12 free-fall times, as in the original Boss & Bodenheimer (1979) version of this problem. Finally, we have imposed an initially uniform magnetic field pointing in the $z$ direction, with a magnitude such that $M/M_\Phi \approx 2.25$. This field is strong enough to alter the morphology of the collapse, but not strong enough to halt it completely. This value of the mass-to-flux ratio is also comparable to that observed in real star-forming regions. The initial plasma beta $\beta_0$ is $\approx 1.57$ at the center of the domain and as low as 0.19 at the edges.

As in the non-magnetic case, the gas first collapses into a sheet and then into a filament, where the field is normal to the filament. The plasma beta in the sheet before it begins to collapse is of order unity. Since the magnetic energy and the gravitational energy are both independent of the radius of the filament, an isothermal filament with a supercritical mass-to-flux ratio will collapse indefinitely (Inutsuka & Tsuribe 2001). To see this, note that the gravitational energy per unit length is $-Gm_\ell^2$ (Fiege & Pudritz 2000), where $m_\ell$ is the mass per unit length, whereas the magnetic energy per unit length is of order $\pi r^2 B^2 \propto \Phi_\ell^2$, where $\Phi_\ell$ is the magnetic flux per unit length. (The exact expression for the magnetic energy depends on the structure of the field inside the filament.) Thus, the ratio of the two forces depends only on the mass-to-flux ratio per unit length, which is constant in ideal MHD. Note, however, that the scaling of the magnetic field is only valid if the field is perpendicular to the filament axis.

We have run four versions of this problem, each time using the AMR capabilities of our code to impose a different $J_{\text{max}}$. The results are shown in Figure 3.10. Because the free-fall time is a function of density, more or less well-resolved simulations of this problem will not be at the same stage of development at the same simulation time. We instead compare the runs at the point where they have all reached approximately the same maximum density of $2 \times 10^7 \rho(0)$. The simulation times at which the peak density reaches this value range from about 3.79 $t_{ff}$ in the best-resolved case to 4.02 $t_{ff}$ in the worst. We find that, as in the pure hydro version of this problem, $J_{\text{max}} = 1/4$ is sufficient to halt the onset of artificial fragmentation. The maximum thermal Jeans number, on the other hand, is larger then $J_{\text{max}}$ by a factor of $\approx 2$ in these runs. Refining on the less stringent magnetic Jeans number seems sufficient to accurately follow the collapse of magnetized gas for this problem, although other physical processes, such as B-field amplification via dynamo action, may require a higher resolution (Federrath et al. 2011).
Chapter 4

The (Mis-)Alignment of a Protostellar Disk with the Core Magnetic Field

4.1 Introduction

The interstellar medium of our galaxy appears to be threaded by dynamically significant magnetic fields. While the strength of these fields can be measured using observations of Zeeman splitting (e.g. Heiles & Troland 2005; Troland & Crutcher 2008; Falgarone et al. 2008), the best information about the field morphology comes from observing polarization effects due to the presence of interstellar dust grains. Non-spherical grains can be preferentially aligned with their long axes perpendicular to the local magnetic field direction by a variety of mechanisms (e.g. Cho & Lazarian 2007). Thus, optical starlight extinguished by these grains tends to be polarized parallel to the integrated field direction, while the sub-mm thermal emission from the grains themselves tends to polarized perpendicular to the field. Mapping the polarization vectors of electromagnetic radiation at these wavelengths thus provides information about the geometry of the plane-of-sky magnetic field.

These techniques provide some evidence that the magnetic field in the cold, neutral portions of the interstellar medium is remarkably well-aligned over a wide range of scales, from $\lesssim 0.1 - 1$ pc molecular cloud cores with $n \gtrsim 10^5$ cm$^{-3}$ to $\sim 100$ pc atomic regions with $n \sim 1$ cm$^{-3}$ (Li et al. 2009). These observations are most consistent with a model of the neutral ISM in which the turbulence is sub- to trans- Alfvénic from 0.1 to 100 pc, such that there is not much tangling of the magnetic field over those scales. A question is: at what scale, if any, does this alignment break down?

The recent set of observations by Hull et al. (2013) provides a strong clue. Hull et al. looked at a sample of nearby low-mass Class 0 sources using the Combined Array for Research in Millimeter-wave Astronomy (CARMA), measuring the magnetic field orientation from the polarization of the thermal dust emission and the outflow orientation via molecular line emission. They found strong evidence that the field and the outflow are not aligned at the $\sim 1000$ AU resolution of their observations; in fact, their data are consistent with a random draw from a flat distribution of misalignment angles.
Since the outflow orientation is presumably aligned with the disk angular momentum vector, this implies that protostellar disks are not preferentially aligned with the magnetic fields in their parent cloud cores. However, most models of disk winds suggest that the field should be toroidally wrapped around the disk in order to drive a bipolar outflow. This implies that, on the \( \sim 100 \) AU scales of protostellar disks, the alignment between the local magnetic field direction and that at larger scales should break down.

In this chapter, we investigate this effect using high-resolution magnetohydrodynamic (MHD) simulations of the formation of a protostellar disk/outflow system in a magnetized, turbulent molecular cloud core. We briefly describe our simulation setup in 4.2; for a complete description of our methods, see Myers et al. (2013) (Chapter 3). Although the core magnetic field and angular momentum vectors are initially misaligned by construction in this simulation, by the time a Keplerian disk forms the mean magnetic field orientation has become aligned with the disk angular momentum vector in the central few 100 AU around the protostar. In section 6.3, we present maps of the column density, magnetic field orientation vectors, and outflow lobes in our simulations at various scales and viewing angles. These maps show a departure from the characteristic "hour-glass" geometry often observed in high and low mass stellar cores (e.g. Girart et al. 2006; Rao et al. 2009; Girart et al. 2009; Tang et al. 2009) at \( \sim 100 \) AU that should be observable in nearby star-forming regions by the Atacama Large Millimeter Array (ALMA). Section 6.4 contains further analysis that demonstrates we only begin to see an alteration in the mean core magnetic field direction on scales where Keplerian rotation becomes significant.

4.2 Simulation Setup

The simulation presented in this chapter was originally published in Myers et al. (2013). Here, we briefly describe our initial conditions and numerical methods. For a full description of the equations and algorithms employed, see Myers et al. (2013) (Chapter 3) and the references therein.

We simulate the collapse of a centrally-concentrated, high-mass \( (M_c = 300 \, M_\odot) \) spherical core of gas using the equations of isothermal, ideal MHD with self-gravity. The radius of the core \( R_c \) is 0.1 pc, its surface density is \( \Sigma_c = 2 \) g cm\(^{-2}\), and its temperature is fixed at 20 K. For the density profile we take a \( \rho(r) \propto r^{-k_\rho} \) power law with index \( k_\rho = 1.5 \), similar to the median value in the mid-infrared extinction mapping study of massive cores in Butler & Tan (2012). The mean density is thus \( \bar{\rho} = 4.8 \times 10^{-18} \) g cm\(^{-3}\), or \( n_H = 2.4 \times 10^6 \) H nuclei cm\(^{-3}\). The cloud is also threaded by a cylindrically symmetric magnetic field that falls off with the 0.5 power of the cylindrical radius. The field is normalized so that the mass-to-flux ratio \( \mu_\Phi \) is 2 in critical units. The corresponding mean magnetic field in the cloud is 1.6 mG.

At time zero, we also initialize the velocity field with perturbations to model the large non-thermal velocity dispersions observed in molecular clouds. These perturbations are normalized so that the 1D root-mean-squared velocity dispersion \( \sigma_v \) is 2.3 km s\(^{-1}\), corresponding to a virial ratio \( \alpha_{\text{vir}} = 5 \sigma_v^2 R_c/GM_c \) of 2.1. We generate these perturbations in Fourier space and
impose a Burgers \((k = -2)\) power spectrum.

The cloud is placed inside a 0.4 pc cube and surrounded by a low density buffer medium to minimize the influence of the boundaries on the cloud’s collapse. We use outflow boundary conditions on the density, momentum, and magnetic field, and Dirichlet zero conditions on the gravitational potential \(\phi\). While we do not impose any solid body rotation on the core, the turbulent perturbations impart a net angular momentum to the cloud that is initially misaligned with the magnetic field vector by \(\approx 67\) degrees. The initial ratio of rotational kinetic energy to gravitational potential energy \(\beta_{\text{rot}}\) is 0.01, using the definition in Goodman et al. (1993) and Burkert & Bodenheimer (2000); see Myers et al. (2013) for details.

Our simulation uses an adaptive grid with 8 levels of refinement and a base grid of \(256^3\), for a maximum resolution of 1.25 AU on the finest level. This enables us to follow the collapse down to densities of \(\sim 10^{12} \text{ H nuclei cm}^{-3}\). To represent mass that has collapsed to yet higher densities, we use sink particles (Bate et al. 1995; Krumholz et al. 2004; Federrath et al. 2010a). Specifically, we adopt the method described in Lee et al. (2013). Our particles have an accretion kernel of 4 fine-level cells, corresponding to a physical radius of \(r_{\text{sink}} \approx 5\) AU.

### 4.3 Results

We evolve the system for a total of 6,400 years - about 20% of the core mean-density free-fall time of 30,000 years. At that point, the most massive sink particle in the calculation has reached \(M_\ast \approx 3.5\ M_\odot\). Beginning at about \(t = 5,000\) years, examination of the density structure reveals a flattened disk of radius \(r_{\text{disk}} = 40\) AU centered around the most massive sink particle. At about the same time, the rotational velocity profile around the particle also becomes Keplerian out to at least 100 AU (see Figure 4.3). In addition, the disk drives an outflow with a maximum velocity of \(\sim 30\ \text{km/s}\), comparable to the Keplerian velocity \(v_k = \sqrt{GM_\ast/r}\) at the sink accretion radius. At \(t = 6,400\) years, the mass of disk is \(\approx 0.5\ M_\odot\) and it has a scale height of approximately 10 AU.

Figure 4.1 shows the state of the core - including the column density, magnetic field vectors, and outflow orientation - at \(t = 6,400\) years. We show three different length scales and two viewing angles: face-on and edge-on. In the face-on view, the disk angular momentum vector \(L_{\text{disk}}\) points out of the page, where \(L_{\text{disk}}\) is calculated as the net angular momentum vector of all the gas within a 50 AU sphere around the most massive sink particle. Visually, this encompasses all the matter that appears to be associated with the disk, and using a larger or smaller radius by a factor of 2 or using a cylindrical geometry instead makes little difference to the orientation of the vector. For the edge-on view, the disk angular momentum vector lies in the plane of the image and its orientation is indicated by the dotted white line in Figure 4.1.

The leftmost panels show the inner 10,000 AU around the protostar. The spacing between the magnetic field vectors is about 500 AU - approximately the spatial resolution achieved in Hull et al. (2013). The field lines follow the characteristic pinched, "hour-glass" shape commonly observed in star-forming cores (e.g. Girart et al. 2006; Rao et al. 2009; Girart et al.
Although an outflow lobe is visible in the top left panel of Figure 4.1, it has only propagated $\sim 500$ AU owing the relatively early time at which these snapshots were taken. However, orientation of the disk angular momentum vector should track the outflow launching direction quite well. As expected from our initial conditions, we find that there is about a 70 degree misalignment between mean B-field direction and the outflow orientation at 1000 AU.
The middle panels show the same situation zoomed in by a factor of 10. The spacing of the magnetic field vectors is now $\sim 50$ AU. At this resolution, several interesting features become visible. First, we can now clearly distinguish two outflow lobes that are each being launched parallel to the disk angular momentum vector. However, the alignment of the outflow with the disk angular momentum vectors gets worse as one moves away from the launching region near the disk itself. Although precession may explain some portion of this effect, we do not find that it is the dominant factor. The light dotted lines show the $1\sigma$ variation in the disk orientation vector over the last 5 orbital periods of the outer edge of the disk, where we take period, $T$, to be the Kepler period for a 3.5 $M_\odot$ star at a distance of 40 AU ($T \approx 30$ years). While there is clearly some precession, it is not large enough to explain the orientation of the outflow lobe itself. Instead, the orientation must be affected by interactions with the core envelope, either due to the fast wind gas getting channeled by magnetic field lines, diverted by dense clumps of infalling gas, or both.

Additionally, the magnetic field geometry no longer follows an hour-glass form when viewed at this resolution. In the edge-on view, we instead see an "S-Curve" pattern, as discovered by Kataoka et al. (2012). The ultimate cause of this morphology is that Keplarian rotation tends to twist the initially straight magnetic field lines into a pitched helix shape. We can begin to see this at this resolution in face-on view in the middle bottom panel. Although the field in the disk is mostly toroidal, when viewed edge-on but at scales larger than the disk itself, the plane-of-sky component of the field will appear to be aligned with the disk/outflow axis. The S-Curve is thus a tell-tale sign of a rotationally dominated disk.

Unlike Kataoka et al. (2012), we find an S-Curve even though our core contains much more energy in the magnetic field than in rotation, with $E_B/E_{rot} \approx 20$. The reason is likely that our core is turbulent, while the velocity field in Kataoka et al. was laminar. The presence of velocity perturbations appears to aid the formation of Keplarian disks, although the precise mechanism by which this occurs is currently under debate in the literature (e.g. Hennebelle & Ciardi 2009; Seifried et al. 2012; Santos-Lima et al. 2012b,a). Note that the turbulence does not have to be supersonic for this effect to operate (Seifried et al. 2013). Because significant non-thermal velocity perturbations are ubiquitous in star-forming cores, it seems unlikely that the presence of an S-Curve could be used to gauge the ratio of magnetic to rotational energy, as suggested by Kataoka et al. (2012). If a significant fraction of cores eventually form disks as they collapse (Krumholz et al. 2013), then S-Curves should be a common feature of cores when observed at $\sim 100$ AU resolution, regardless of the ratio of $E_B$ to $E_{rot}$.

Finally, in the right-hand panels of Figure 4.1, we zoom in by another factor of 10. The arrow spacing now becomes $\sim 10$ AU. At this resolution, we can begin to probe the magnetic field structure within the disk itself. The toroidal wrapping of the field is particularly apparent in the face-on view in the bottom panel. Viewed edge-on, the orientation between the magnetic field vectors approaches 90 degrees near the disk center.
4.4 Discussion and Conclusion

We now analyze the misalignment angle between the disk and the core magnetic field more quantitatively as follows. We would like to determine the scale at which the magnetic field departs from its large-scale orientation, and verify that this is related to the presence of a rotationally-dominated disk. We take a series of 20 spheres concentric with the disk and having radii \( r \) logarithmically spaced over the range 5 AU (the sink accretion radius) to 20,000 AU (the approximate size of the core). In each of those spheres, we compute the mean magnetic field \( B \), the disk angular momentum vector \( L_{\text{disk}} \) defined as above, and \( \theta \), the angle between the two. Note that the orientation \( L_{\text{disk}} \) is quite similar to the initial angular momentum \( L \) of the core. The result is shown in Figure 4.2 for five different time snapshots.

The first snapshot is taken at \( t = 3,200 \) years, approximately 10% of a free-fall time. At that point, there is still misalignment between \( L_{\text{disk}} \) and \( B \) of at least 40 degrees over the

\[
\begin{align*}
\text{Figure 4.2:} & \quad \text{The relative orientation of } L_{\text{disk}} \text{ and } B \text{ as a function of radius for five different time snapshots. The flat red line corresponds to our initial conditions, in which } B \text{ is uniform. The grayscale curves indicate different snapshots from our simulation. The lightest curve corresponds to } t = 3,200 \text{ years. From there, the time increases from light to dark in steps of 800 years, until the final time of } t = 6,400.}
\end{align*}
\]
4.4. DISCUSSION AND CONCLUSION

Figure 4.3: Density-weighted mean rotational velocity as function of radius for the same time snapshots as Figure 4.2. The red line corresponds to a Keplerian velocity profile associated with a 3.5 solar mass star. Because the mass of the star is different in each snapshot, we have divided each curve by $\sqrt{M/3.5M_\odot}$.

entire core. By 4,800 years, the misalignment angle has dropped below 20% at about 100 AU. This snapshot corresponds to the time at which examination of the density field begins to reveal a flattened disk. For the next 1,600 years, $\theta$ versus $r$ maintains the same basic pattern: Outside of 1000 AU, the magnetic field maintains its initial alignment, which was misaligned with the core angular momentum vector by $\approx 67$ degrees. The initial misalignment is maintained down to $r \sim 1,000$ AU, but begins to drop as you approach 100 AU. Within 100 AU, as you begin to probe the disk itself, the misalignment increases again due to the toroidal wrapping of the field.

In Figure 4.3, we show the mean rotational velocity $v$ computed at the same times and in the same spheres defined above, compared against the Keplerian velocity profile $v(r) = \sqrt{GM_\odot/r}$. The distance at which $v$ stops following the rotationally-dominated profile in Figure 4.3 corresponds to the distance at which the mean magnetic field begins to depart from its initial alignment. This confirms that the misalignment between $B$ and $L_{\text{disk}}$ begins to break down at the scale at which the protostar is able to enforce a Keplerian velocity profile.
Our conclusion is that, even if the core magnetic field and core angular momentum vector start out misaligned, the formation of a rotationally dominated disk will force alignment between the mean field and the disk angular momentum vector on scales comparable to the distance at which the central protostar enforces Keplerian rotation. In our simulation, this is a few times the disk radius, or $\sim 100$ AU.
Chapter 5

The CH$^+$ Abundance in Turbulent, Diffuse Molecular Clouds

Abstract

The intermittent dissipation of interstellar turbulence is an important energy source in the diffuse ISM. Though on average smaller than the heating rates due to cosmic rays and the photoelectric effect on dust grains, the turbulent cascade can channel large amounts of energy into a relatively small fraction of the gas that consequently undergoes significant heating and chemical enrichment. In particular, this mechanism has been proposed as a solution to the long-standing problem of the high abundance of CH$^+$ along diffuse molecular sight lines, which steady-state, low temperature models under-produce by over an order of magnitude. While much work has been done on the structure and chemistry of these small-scale dissipation zones, comparatively little attention has been paid to relating these zones to the properties of the large-scale turbulence as found in numerical simulations. In this paper, we attempt to bridge this gap by estimating the CH$^+$ column densities through full, 3D, MHD turbulence simulations via post-processing. We find that the observed turbulence in diffuse molecular clouds is sufficient to account for their CH$^+$ abundances, and for their emission lines from rotationally excited H$_2$ molecules.  

5.1 Introduction

The CH$^+$ ion is commonly detected along sight lines towards bright O and B stars, with column densities $\gtrsim 10^{13}$ cm$^{-2}$ frequently reported in the literature (e.g. Gredel et al. 1993; Gredel 1997; Crane et al. 1995; Weselak et al. 2008; Sheffer et al. 2008). This prevalence is puzzling, however, because CH$^+$ is destroyed very efficiently by both atomic and molecular

---

$^1$This chapter will be submitted to the Monthly Notices of the Royal Astronomical Society as part of The CH$^+$ Abundance in Turbulent, Diffuse Molecular Clouds by Myers, McKee, & Li and is reproduced with the permission of all coauthors.
5.1. INTRODUCTION

hydrogen, and the only reaction that can form \( \text{CH}^+ \) rapidly:

\[
\text{C}^+ + \text{H}_2 \rightarrow \text{CH}^+ + \text{H} \quad \Delta E/k = -4640K,
\]

is strongly endothermic and can only proceed at temperatures of \( \sim 1000 \text{ K} \) or higher. For this reason, models of diffuse interstellar clouds with \( T \lesssim 100 \text{ K} \), like those of van Dishoeck & Black (1986), fail dramatically to reproduce these high CH\(^+\) columns, despite their success with other species.

Most proposed solutions to this problem have invoked an additional energy source to overcome this 4640 K activation barrier. Possibilities include hydrodynamic (Elitzur & Watson 1978, 1980) and magnetohydrodynamic (Draine & Katz 1986) shock waves, heating in turbulent boundary layers at cloud surfaces (Duley et al. 1992), and particularly dense photon-dominated regions (PDRs) surrounding bright stars (Duley et al. 1992; Sternberg & Dalgarno 1995); for an overview of these mechanisms and some of the problems they face confronting observations, see Gredel (1997). A particularly promising idea, pioneered by Falgarone & Puget (1995), is that the intermittent dissipation of turbulence heats small regions within diffuse clouds to the \( \gtrsim 1000 \text{ K} \) temperatures required for (1) to proceed. Drawing on laboratory experiments of unmagnetized, incompressible turbulent flows, they calculated that if the velocity dispersion in cold, mostly atomic clouds at a scale of 1 pc is 3 km s\(^{-1}\), then a few percent of the cloud could be heated to \( > 1000 \text{ K} \), a mass fraction sufficient to bring the CH\(^+\) abundance in line with observed values (Lambert & Danks 1986). This result was later found to be consistent with magnetized, compressible turbulence simulations as well (Pan & Padoan 2009). These pockets of warm gas may also explain the observed emission from the first few excited rotational states of \( \text{H}_2 \) detected in diffuse gas, which is often too large to be explained by UV pumping alone (e.g. Falgarone et al. 2005; Goldsmith et al. 2010; Ingalls et al. 2011).

Models that rely on turbulent heating alone can over-predict the abundance of other species, such as OH, which is already well modeled by cold cloud models (Federman et al. 1996). However, in addition to the direct heating effect, turbulence can give rise to net drift velocities between the ionic and neutral species in plasmas, enhancing the rates of ion-neutral reactions like (1) beyond those expected from the kinetic temperature alone (e.g. Draine 1980; Flower et al. 1985). Federman et al. (1996) approximated this effect by computing the rate of reaction (1) at the effective temperature \( T_{\text{eff}} \) given by:

\[
T_{\text{eff}} = T + \frac{\mu}{3k}v_d^2,
\]

where \( \mu \) is the reduced mass of (1), \( k \) is the Boltzmann constant, and \( v_d \) is the magnitude of the ion-neutral drift velocity. They proposed that MHD waves with amplitudes \( \sim 3 \text{ km s}^{-1} \) can enhance the predicted column densities of CH\(^+\) to the observed values even in gas that remains \( T \lesssim 100 \text{ K} \). A similar calculation was made in Spaans (1995), who computed the distribution of \( v_d \) from an analytic intermittency model. More recently, Sheffer et al. (2008) included this effect in their PDR models, finding a similar result. The appeal of these models
is that they have fewer problems over-producing molecules such as OH, which is not formed by an ion-neutral reaction.

The most successful models include both of these effects simultaneously. Joulain et al. (1998) and Godard et al. (2009) treat regions of intense dissipation, termed “Turbulent Dissipation Regions” (TDRs) by Godard et al., as magnetized vortices, taking their (axisymmetric) velocity profiles from that of a Burgers vortex, for which the vorticity as a function of radius is

\[ \omega(r) = \omega_0 \exp \left[ -\left( \frac{r}{r_0} \right)^2 \right]. \]  

(5.3)

Here, \( \omega_0 \) and \( r_0 \) are parameters describing the peak vorticity and characteristic fall-off radius of the vortex. These calculations follow the subsequent thermal and chemical evolution of parcels of gas trapped inside such a vortex, including both turbulent heating and ion-neutral drift. Godard et al. (2009) then construct models of entire sight lines by assuming they intersect some number of these vortex structures. These models have had a great deal of success reproducing the observed \( \text{CH}^+ \) and excited \( \text{H}_2 \) columns without overproducing species such as OH.

The goal of this paper is to provide a complementary approach to the above models, which concentrate on individual dissipation events. We post-process the gas temperature \( T \), drift velocity \( v_d \), and \( \text{CH}^+ \) abundance cell-by-cell through an output of a turbulence simulation from Li et al. (2012) that has been scaled to typical diffuse cloud conditions. This approach looses some of the detail of the above models, but it has the advantage of making fewer simplifying assumptions about, for example, the nature of the intermittent structures or the number of dissipation events along a line of sight. We find that \( \text{CH}^+ \) columns in excess of \( \sim 10^{13} \, \text{cm}^{-2} \) are readily obtained. We compare our results against a statistically homogenous sample of \( \text{CH}^+ \)-containing sight lines from Weselak et al. (2008) and against observations of rotationally excited \( \text{H}_2 \), finding good agreement with both.

5.2 Methodology

In this section, we provide an overview of our calculation, including our treatment of the heating and cooling rates, the drift velocity, and our calculation of the \( \text{CH}^+ \) abundance.

5.2.1 Model Description

The \( \text{CH}^+ \) ion is believed to form in partially molecular environments. Indeed, in order for reaction (1) to proceed, at least some of the hydrogen must be in the form of \( \text{H}_2 \), and at least some of the carbon must be in \( \text{C}^+ \). Any plausible formation mechanism is thus not likely to be effective in either the outskirts of molecular clouds with visual extinction \( A_V < 0.1 \, \text{mag} \), where the hydrogen is almost all atomic, or deep in their interiors at \( A_V \) greater than a few, where almost all the carbon will be C and/or CO. This gas is sometimes referred to as the “dark gas” since it is difficult to observe (Grenier et al. 2005; Wolfire et al. 2010).
5.2. METHODOLOGY

We therefore model interstellar clouds in which the hydrogen has begun to turn to $\text{H}_2$, but the carbon is still primarily in the form of $\text{C}^+$. Snow & McCall (2006) classify such clouds as “diffuse molecular clouds.” We treat these regions as cubic boxes with length $\ell_0$, mean hydrogen nucleus number density $\bar{n}_\text{H}$, and one-dimensional velocity dispersion $\sigma_{\text{1D}}$. For simplicity, we set the relative abundances (relative to hydrogen nuclei) of molecular hydrogen $x(\text{H}_2)$ and ionized carbon $x(\text{C}^+)$ to be constant across the region. For the former, we adopt $x(\text{H}_2) = 0.16$, the mean observed molecular fraction from the sample of Weselak et al. (2008), which studied the correlation of $\text{CH}^+$ column density with that of atomic and molecular hydrogen. For the latter, we take $x(\text{C}^+) = 1.6 \times 10^{-4}$ from Sofia et al. (2004) $^2$. The mean mass per hydrogen nucleus is $\mu_\text{H} = 2.34 \times 10^{-24}$ g cm$^{-3}$.

Although chemical and physical models of diffuse gas often assume a constant $n_\text{H}$, the density distribution in the ISM in fact contains a wide range of fluctuations over many orders of magnitude due to the compressive effects of supersonic turbulence. To treat this, we use the results of a $512^3$ driven, turbulence simulation first published in Li et al. (2012). The density, magnetic field, and velocity at every point in our model are drawn from the corresponding cell in a data dump from this simulation, scaled to physical units by the process described below. A color plot of the column density through the simulation volume is shown in Figure 5.1. An important caveat to our calculation is that this simulation data is isothermal. We then calculate what the temperature would be if the intermittency in the isothermal case were the same as if the time-dependent heating and cooling effects were followed self-consistently.

5.2.2 Scaling to Physical Units

Simulations of magnetized, isothermal turbulent boxes are characterized by two dimensionless numbers: the 3D sonic Mach number $\mathcal{M} = \sigma_{\text{3D}}/c_s$ and the 3D Alfvénic Mach number $\mathcal{M}_A = \sigma_{\text{3D}}/v_A$. Here, $\sigma_{\text{3D}}$ is the three-dimensional non-thermal velocity dispersion in the box, $c_s = \sqrt{kT/\bar{m}}$ is the isothermal sound speed, where $T$ is the temperature and $\bar{m}$ the mean mass per particle, and $v_A = B_{\text{rms}}/\sqrt{4\pi\bar{\rho}}$ is the Alfvén velocity, where $B_{\text{rms}}$ is the root-mean-square magnetic field and $\bar{\rho}$ the mean density. In the simulation considered here, $\mathcal{M} \approx 9.4$ and $\mathcal{M}_A \approx 2.2$.

In the absence of further constraints, we would be free to scale $\bar{\rho}$, $\sigma_{\text{3D}}$, $c_s$, $B_{\text{rms}}$ and the size of the box $\ell_0$ at will as long as the dimensionless ratios $\mathcal{M}$ and $\mathcal{M}_A$ remained invariant (see McKee et al. (2010) for a more rigorous discussion of scaling laws for turbulent box simulations). However, to be consistent with observations of diffuse molecular gas, we impose several additional constraints. First, we require that the gas in the box obey a linewidth-size

---

$^2$Sofia et al. (2011) find using a different measurement technique that the gas-phase carbon abundance is lower than that adopted here by a factor of $\approx 0.43$. Since it is not clear which measurement is more accurate, we choose to adopt the higher value. The effects of a lower C abundance in our model are complex. On one hand, it directly reduces the $\text{CH}^+$ formation rate (see Section 2.5), since $\text{C}^+$ is one of the reactants in (1). On the other hand, it decreases the cooling rate due to $\text{C}^+$ (Section 2.3) and increases our estimate for the ion-neutral drift velocity (Section 2.4). The overall effect of decreasing our assumed carbon abundance by a factor of 0.43 is to increase the estimate for the $\text{CH}^+$ abundance by approximately 30%.
Figure 5.1: Logarithm of the total column density $N_H$ through the computational domain along the $x$ direction. The simulation data has been scaled such that the mean $N_H$ is $1.83 \times 10^{21}$ cm$^{-2}$. The size of the box is indicated on the $x$ and $y$ axes.

relation (e.g. McKee & Ostriker 2007):

$$\sigma_{1D} = \sigma_{pc} R_{pc}^{0.5},$$  \hspace{1cm} (5.4)

where $R_{pc}$ is the cloud radius in parsecs, and $\sigma_{pc} = 0.72$ km s$^{-1}$. The 1D non-thermal velocity dispersion $\sigma_{1D}$ is related to the 3D value by $\sigma_{3D} = \sqrt{3}\sigma_{1D}$, and in applying equation (5.4), which is meant for approximately spherical clouds, to our cubic simulation domain, we identify the cloud radius $R$ with $\ell_0/2$. Second, we require that the mean column density of hydrogen nuclei be fixed:

$$\bar{N}_H = \bar{n}_H \ell_0 = \bar{N}_{obs},$$  \hspace{1cm} (5.5)

where $\bar{N}_{obs} \approx 1.83 \times 10^{21}$ cm$^{-2}$ is the mean total column density from Weselak et al. (2008). This column corresponds to $A_V \approx 1$, consistent with our requirement that the gas be partially molecular. Note that equations (5.4) and (5.5) imply that we cannot independently choose
5.2. METHODOLOGY

$n_H$, $\ell_0$ and $\sigma_{1D}$; choosing a density fixes the box size $\ell_0$, which fixes the velocity dispersion through the linewidth-size relation. Numerically:

$$\ell_0 \approx 19.8 \left( \frac{30 \text{ cm}^{-3}}{\bar{n}_H} \right) \text{ pc} \quad (5.6)$$

$$\sigma_{1D} \approx 2.3 \left( \frac{30 \text{ cm}^{-3}}{\bar{n}_H} \right)^{0.5} \text{ km s}^{-1} \quad (5.7)$$

These relations also fix (along with the fact the $M_A = 2.2$) the rms magnetic field strength in the box:

$$B_{\text{rms}} = \sqrt{\frac{6\pi \mu_H \bar{N}_\text{obs} \sigma_{\text{pc}}^2}{(1 \text{ pc}) M_A^2}} \approx 5.3 \mu\text{G}. \quad (5.8)$$

Thus, only one of $\bar{n}_H$, $\ell_0$, $\sigma_{3D}$, and $B_{\text{rms}}$ may be set independently, with the others following from that choice.

The final remaining dimensional parameter describing our turbulence simulation $c_s$. The sound speed of a gas with $x(H_2) = 0.16$ and $x(\text{He}) = 0.1$ is

$$c_s(T) \approx 0.74 \sqrt{\frac{T}{100 \text{ K}}} \text{ km s}^{-1}. \quad (5.9)$$

However, because the temperature is an output of our model, we cannot set it arbitrarily. We thus compute the temperature using the process described below for a range of boxes, each scaled to a different $\bar{n}_H$, and select the one for which $M$ computed using the mass-weighted median temperature $T_{50,M}$ (the $T$ for which half of the mass in cloud is hotter) is $\approx 9.4$. We find that this occurs at $n_H \approx 30 \text{ cm}^{-3}$ and adopt that as our fiducial density. This is the same value adopted in the standard model of Joulain et al. (1998). However, individual sight lines passing through the simulation volume can have mean densities ranging from $\approx 5 \text{ cm}^{-3}$ to $\approx 180 \text{ cm}^{-3}$. The corresponding $T_{50,M}$ is $\approx 35 \text{ K}$, and the mass-weighted mean temperature is $\bar{T}_M \approx 65 \text{ K}$. We summarize the physical and chemical parameters describing this model in Table 5.1.

Our simulation data does not include the effects of self-gravity. As a consistency check, we compute the virial parameter $\alpha_{\text{vir}}$ to verify that turbulence indeed dominates self-gravity. Using the above parameters, the total mass $M$ in the box is $\approx 8000 M_\odot$. The virial parameter for a spherical cloud of mass $M$ and radius $R$ is $\alpha_{\text{vir}} = 5\sigma_{1D}^2 R/GM$ (Bertoldi & McKee 1992). Using $R = \ell_0/2$, we find $\alpha_{\text{vir}} \approx 7.4$, which is large enough that the effects of self-gravity are indeed negligible. We can also characterize the relative importance of gravity and magnetic fields in the box by computing the mass-to-flux ratio relative to critical, $\mu_\Phi \equiv M/\Phi$, where $M_\Phi \approx \Phi/2\pi\sqrt{G}$ and $\Phi$ is the magnetic flux threading the cloud. For our adopted parameters, $\mu_\Phi \approx 1.3$, so the cloud is marginally magnetically supercritical.
5.2. METHODOLOGY

Table 5.1: Standard Physical and Chemical Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{n}_H$</td>
<td>30 cm$^{-3}$</td>
</tr>
<tr>
<td>L</td>
<td>20 pc</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>2.3 km s$^{-1}$</td>
</tr>
<tr>
<td>$T_M$</td>
<td>65 K</td>
</tr>
<tr>
<td>$T_{50,M}$</td>
<td>35 K</td>
</tr>
<tr>
<td>$B_{\text{rms}}$</td>
<td>5.2 $\mu$G</td>
</tr>
<tr>
<td>x(H)</td>
<td>0.68</td>
</tr>
<tr>
<td>x(H$_2$)</td>
<td>0.16</td>
</tr>
<tr>
<td>x(He)</td>
<td>0.1</td>
</tr>
<tr>
<td>x($e^-$)</td>
<td>$1.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>x(C)</td>
<td>$1.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>x(O)</td>
<td>$3.2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

5.2.3 Heating and Cooling

The temperature in each cell is set by a balance between heating and cooling:

$$\Gamma_{\text{Turb}} + \Gamma_{\text{CR}} + \Gamma_{\text{PE}} = n_H \Lambda_{\text{tot}}(T).$$

(5.10)

where $\Gamma_{\text{Turb}}$, $\Gamma_{\text{CR}}$, and $\Gamma_{\text{PE}}$ are the heating rates per unit volume due to the dissipation of turbulence, cosmic ray ionizations, and the photoelectric effect on dust grains, respectively. $n_H \Lambda_{\text{tot}}(T)$ is the total cooling rate per unit volume, which we assume is dominated by electronic transitions of C$^+$ and O and by ro-vibrational transitions of the H$_2$ molecule. Note that throughout this paper, we use $\Lambda$ to represent the cooling rate coefficient (erg cm$^3$ s$^{-1}$) and $\Lambda$ for the cooling rate per molecule (erg s$^{-1}$).

$\Gamma_{\text{CR}}$ can be expressed as the product of three factors - the total cosmic ray ionization rate per H nucleus $\zeta_H$ (including both primary and secondary ionizations), the average energy deposited into the medium per ionization $\Delta Q$, and $n_H$. Both $\zeta_H$ and $\Delta Q$ are rather uncertain and can vary considerably over different Galactic environments. Typical values for $\zeta_H$ in dense gas are $\sim 1 \text{--} 5 \times 10^{-17}$ s$^{-1}$ (Dalgarno 2006), but there is evidence from H$_3^+$ observations that $\zeta_H$ is considerably higher in the diffuse gas under consideration here (Dalgarno 2006; Indriolo & McCall 2012), with values as large as $\sim 1 \times 10^{-15}$ s$^{-1}$ reported in the literature (Snow & McCall 2006; Shaw et al. 2008). In this paper, we adopt the value $4 \times 10^{-16}$ s$^{-1}$.

For $\Delta Q$, we use 10 eV, as estimated for diffuse molecular gas from Table 6 of Glassgold et al. (2012), although it is important to note that this value can vary by several eV depending on the precise physical and chemical conditions in the cloud. Combining these factors, the cosmic ray heating rate is:

$$\Gamma_{\text{CR}} = \zeta \Delta Q n_H \approx 1.9 \times 10^{-25} \left( \frac{n_H}{30 \text{ cm}^{-3}} \right) \text{ ergs cm}^{-3} \text{ s}^{-1}$$

(5.11)
For $\Gamma_{PE}$, we adopt the expression:

$$\Gamma_{PE} = 1.3 \times 10^{-24} n_H \epsilon G_0 \text{ ergs cm}^{-3} \text{ s}^{-1} \quad (5.12)$$

from Wolfire et al. (2003), where $G_0$ is the intensity of FUV light in units of the Habing (1968) field and $\epsilon$ is a heating efficiency factor given by equation (20) of Wolfire et al. (2003). For $n_H = 30 \text{ cm}^{-3}$, $T = 100 \text{ K}$, an electron fraction of $1.6 \times 10^{-4}$, and a FUV field of $G_0 = 1.1$ (Mathis et al. 1983), $\epsilon$ evaluates to $1.8 \times 10^{-2}$, yielding

$$\Gamma_{PE} = 7.6 \times 10^{-25} \left( \frac{n_H}{30 \text{ cm}^{-3}} \right) \text{ ergs cm}^{-3} \text{ s}^{-1} \quad (5.13)$$

The final heating process we consider is $\Gamma_{Turb}$. Dimensional arguments (e.g. Landau & Lifshitz 1959) and numerical simulations (e.g. Stone et al. 1998; Mac Low 1999) both suggest that the kinetic energy in a turbulent cloud $1/2 \rho \sigma_{3D}^2$ decays in roughly one crossing time $\ell_0/\sigma_{3D}$, so that the volume-averaged turbulent heating rate is approximately

$$\bar{\Gamma}_{Turb} \approx \frac{1}{2} \rho \sigma_{3D}^3 \approx 3.5 \times 10^{-26} \sqrt{n_H / 30 \text{ cm}^{-3}} \text{ ergs cm}^{-3} \text{ s}^{-1}, \quad (5.14)$$

where in the last step we have assumed the scaling given by equations (5.4) and (5.5). Locally, however, $\Gamma_{Turb}$ exhibits large fluctuations from place to place, a phenomenon known as intermittency. To calculate the spatial dependence of $\Gamma_{Turb}$, we follow the calculation in Pan & Padoan (2009). To summarize their argument, the work done against the viscous forces in a fluid with velocity field $\mathbf{v}$ is irreversibly converted into heat at rate per unit volume given by:

$$\Gamma_{Turb}(\mathbf{x}) = \frac{1}{2} \rho \nu \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} (\nabla \cdot \mathbf{v}) \delta_{ij} \right)^2. \quad (5.15)$$

This rate depends on the kinematic viscosity of the fluid, $\nu$. However, in our simulations, which were based on the Euler equations for a compressible gas, the viscosity was numerical in origin, and thus does not have its true microphysical value. Instead, we treat $\nu$ as a proportionality constant that takes whatever value is required so that the volume average of equation (5.15) equals equation (5.14). Once this constant has been determined, we can compute $\Gamma_{Turb}(\mathbf{x})$ for each cell in the simulation.

Note that the resolution of our simulation data $\Delta x = \ell_0/512 \approx 8 \times 10^3 \text{ AU}$ is significantly larger than the dissipation scale provided by the kinematic viscosity of interstellar gas of $\ell_d \sim 10 \text{ AU}$ (Joulain et al. 1998). If the turbulence were allowed to cascade down to these small scales, the distribution of the heating rate would be more intermittent than it is in our simulation (Pan & Padoan 2009). However, viscous dissipation is not the only process that removes energy from the turbulent cascade. Ion-neutral friction becomes significant at the much larger scale $\ell_{AD}$ (see Section 2.4), and is capable of dissipating most of the energy in the cascade at scales $\approx 10 \ell_{AD}$ (Li et al. 2012), which is comparable to the cell size in our simulation data. Below $\ell_{AD}$, a turbulent cascade can re-assert itself in the neutrals, allowing some of the energy to be dissipated on the $\sim 10 \text{ AU}$ scale set by viscosity. Thus, while our
procedure likely under-estimates the intermittency in the turbulent heating rate somewhat, it is not clear that the opposite assumption that all the energy in cascade makes it to ∼ 10 AU scales is more accurate.

For the C\(^+\) and O cooling coefficients, we adopt the formulas given by Wolfire et al. (2003):

\[
\lambda_{C^+}(T) = 3.6 \times 10^{-27} \exp\left(-92 \frac{K}{T}\right) \text{erg cm}^3 \text{s}^{-1}, \quad (5.16)
\]

\[
\lambda_O(T) = 2.35 \times 10^{-27} \left(\frac{T}{100 \text{K}}\right)^{0.4} \exp\left(-228 \frac{K}{T}\right) \text{erg cm}^3 \text{s}^{-1}, \quad (5.17)
\]

where we have scaled the overall numerical factors to account for our fractional abundances of carbon and oxygen of 1.6 \times 10^{-4} and 3.2 \times 10^{-4}, rather than the 1.4 \times 10^{-4} and 3.4 \times 10^{-4} used in Wolfire et al. (2003). The cooling rates per H nucleus are then \(\Lambda_{C^+}(T) = n_H \lambda_{C^+}(T)\) and \(\Lambda_O(T) = n_H \lambda_O(T)\).

For the cooling rate per H\(_2\) molecule in the low density limit, \(\Lambda_{H_2}(n_H \to 0)(T)\), we use the tables in Glover & Abel (2008), assuming an ortho:para ratio of 0.7 (see section 3.3) and the fractional abundances of H, H\(_2\), He and electrons listed in Table 5.1. At densities high enough for local thermodynamic equilibrium to be established, the H\(_2\) cooling rate per molecule \(\Lambda_{H_2, \text{LTE}}(T)\) becomes independent of the collision partner abundances, and we adopt the cooling rate from Coppola et al. (2012). We bridge these two limits following Hollenbach & McKee (1979):

\[
\Lambda_{H_2}(T) = \frac{\Lambda_{H_2, \text{LTE}}(T)}{1 + \Lambda_{H_2, \text{LTE}}(T)/\Lambda_{H_2}(n_H \to 0)(T)} \quad (5.18)
\]

The total cooling rate is then

\[
\Lambda_{\text{tot}}(T) = \Lambda_{C^+}(T) + \Lambda_O(T) + x_{H_2} \Lambda_{H_2}(T). \quad (5.19)
\]

Equation (5.10) becomes a non-linear equation for \(T\) in each cell, which we solve numerically using the \texttt{scipy.optimize.brenth} routine from the SciPy software library (Jones et al. 2001–).

The magnitudes of these cooling processes are summarized as a function of temperature in Figure 5.2 for our standard model parameters. For comparison, the (constant) values of the various heating rates are also displayed for our standard density of \(n_H = 30 \text{ cm}^{-3}\).

### 5.2.4 Ion-Neutral Drift

In the presence of ambipolar diffusion (the net slippage between the charged and neutral species in a plasma), ion-neutral reactions like (1) can proceed at rates faster than those expected from the kinetic temperature alone (e.g. Draine 1980; Flower et al. 1985). The relative importance of the ambipolar and inertial forces in a turbulent system can be characterized by the ambipolar diffusion Reynolds number \(R_{AD}(\ell_0)\) (Zweibel & Brandenburg 1997; Li et al. 2012):

\[
R_{AD}(\ell_0) \equiv \frac{4\pi \gamma_{AD}^2 \rho_i \rho_e \ell_0 \sigma_{3D}}{B_{\text{rms}}^2}, \quad (5.20)
\]
where $\rho_i$ and $\rho_n$ are the densities of the ionic and neutral components of the fluid and $\gamma_{AD}$ is the ion-neutral coupling constant given by $\langle \sigma v \rangle / (m_i + m_n)$. For $\text{C}^+$ and $\text{H}_2$, this evaluates to $8.47 \times 10^{13} \text{ cm}^3 \text{s}^{-1} \text{ g}^{-1}$ (Draine 1980). Applying equations (5.4) and (5.5) for our adopted degree of ionization and magnetic field strength, $R_{AD}(\ell_0)$ is

$$R_{AD}(\ell_0) \approx 6.3 \times 10^3 \sqrt{\left( \frac{n_H}{30 \text{ cm}^{-3}} \right)} \gg 1. \quad (5.21)$$

The corresponding length scale $\ell_{AD} = \ell_0 / R_{AD}(\ell_0)$ at which ambipolar dissipation becomes significant is $\approx 640$ AU for $n_H = 30$ cm$^{-3}$. Thus, ambipolar drift should not be significant on large scales in our model. However, as with the turbulent heating rate, there may be isolated regions in the tails of the drift velocity distribution where this effect is significant.

To proceed, we need a prescription for computing $v_d$. Unfortunately, two-fluid simulations of MHD turbulence are prohibitively expensive in the high $M$, strongly coupled regime considered here. To estimate the effects of $v_d$ on the production of $\text{CH}^+$, we instead use...
our ideal MHD data along with an approximate analytic expression for the drift velocity in the strongly coupled regime, which we corroborate with direct numerical simulations of turbulent ambipolar diffusion at lower \( M \). Specifically, if the system is weakly ionized, then the Lorentz force and the ion-neutral drag force dominate all the other terms in the ion momentum equation and the drift is given by \( v_d = |(\vec{\nabla} \times \vec{B}) \times \vec{B}| / 4\pi\gamma_{AD}\rho_i\rho_n \) (e.g. Shu 1992). If the effects of ambipolar diffusion are weak enough that they have only a minor effect on the geometry of the magnetic field, we can estimate the drift by computing \( |(\vec{\nabla} \times \vec{B}) \times \vec{B}| \) in the ideal limit:

\[
v_d \approx \frac{|(\vec{\nabla} \times \vec{B}) \times \vec{B}|_{\text{Ideal}}}{4\pi\gamma_{AD}\rho_i\rho_n}.
\]  

(5.22)

\[\text{Figure 5.3:} \text{ Blue - the circles show the mass-weighted distribution of } v_d \text{ divided by its mean value } \langle v_d \rangle \text{ from the } M = 3, M_A = 0.67, R_{AD}(\ell_0) \approx 1000 \text{ AD simulation. The error bars show the } 2\sigma \text{ temporal variation in distribution over 2 box crossing times, and solid line shows the best-fit log-normal. Green - same, but for the } M = 3, M_A = 0.67 \text{ ideal simulation, with } v_d \text{ computed from equation (5.22). The agreement between the two curves is quite good over more than 3 standard deviations.}\]

This procedure is illustrated in Figure 5.3. We take two simulations of \( M = 3, M_A = 0.67 \)
turbulence from Li et al. (2008), one which follows the ion and neutral fluids separately, and one which assumes ideal MHD. The AD simulation has $R_{AD}(\ell_0) \approx 1000$. We directly compute the time-averaged, density-weighted distribution of $v_d$ in the non-ideal simulation, and compare it to that of equation (5.22) computed using the ideal data with $\gamma_{AD}$ and $\chi_i$ chosen to match the AD simulation. The resulting distributions both have an approximately log-normal form:

$$P(\log v_d) d\log v_d = \frac{1}{\sigma_{\log v_d}\sqrt{2\pi}} \exp \left( -\frac{(\log v_d - \mu_{\log v_d})^2}{2\sigma_{\log v_d}^2} \right)$$

and agree with each other to within the error bars, which show the magnitude of the temporal fluctuation in the drift distribution computed over 2 crossing times. Equation (5.22) does slightly over-predict the simulated value of $v_d$ - both the mean $\mu_{\log v_d}$ and standard deviation $\sigma_{\log v_d}$ of the log-normal fit to the true drift distribution are smaller than those of the Lorentz Drift by 5% and 2%, respectively. This is likely because, though $R_{AD}(\ell_0)$ for the entire box is $\sim 1000$, there are still sub-regions in the box where the coupling less strong. We expect that this approximation should improve with increasing $R_{AD}(\ell_0)$.

Figure 5.4 shows the result of applying this procedure to our $M \approx 9.1$, $M_A \approx 2.2$ Ideal MHD data, scaled to physical units as described above. Here, we again find that the distribution of $\log(v_d)$ is approximately normal, with best-fit parameters $\mu_{\log v_d} = 4.04$ and $\sigma_{\log v_d} = 0.89$. Although the error in the best-fit lognormal parameters found above is small, it could potentially have a large impact on the CH$^+$ abundance, since that is primarily determined by the tails of the distribution. We can estimate the accuracy of our approximation as follows. First, we compute the mean CH$^+$ abundance (see Section 2.5 below) in a cloud with constant density $n_H = 30$ cm$^{-3}$ and kinetic temperature $T = 35$ K using equations (5.28) and (5.2), under the assumption that the distribution of $v_d$ is given by equation (5.23) with our best fit parameters. We then recompute the CH$^+$ abundance using values of $\mu_{\log v_d}$ and $\sigma_{\log v_d}$ that are lower by 5% and 2%, respectively - the error we found for the $M = 3$, $M_A = 0.67$ case above. While these errors are small, they could potentially make a large difference on the CH$^+$ abundance, since that is primarily determined by the high-velocity tail of the distribution. The result, however, is that the two abundances agree to within a factor of 2. As the $R_{AD}(\ell_0)$ in our target system is larger than 1000, we expect the true error to be somewhat less than this.

5.2.5 Chemistry

To assess the viability of turbulent dissipation as an energy source for CH$^+$ production, we perform a simple analytic estimate of the CH$^+$ abundance in a cell as a function of $n_H$ and $T_{\text{eff}}$ following Lambert & Danks (1986). CH$^+$ forms through reaction 5.1 with a rate constant:

$$k_f = 1.5 \times 10^{-10} \times \exp(-4640 \text{ K}/T_{\text{eff}}).$$

(5.24)
Figure 5.4: Distribution of log $v_d$ for our model in physical units. The blue circles are the simulation data, and the solid line is a best-fit normal distribution with a mean of 4.04 and a standard deviation of 0.89.

Once it forms, it will be quickly destroyed by reactions with H, H$_2$, and electrons. The rates for these processes are

\[
\begin{align*}
\text{CH}^+ + \text{H} & \rightarrow \text{C}^+ + \text{H}_2 \quad k_{\text{HI}} = 1.5 \times 10^{-10} \\
\text{CH}^+ + \text{H}_2 & \rightarrow \text{CH}_2^+ + \text{H} \quad k_{\text{H}_2} = 1.2 \times 10^{-9} \\
\text{CH}^+ + \text{e} & \rightarrow \text{C} + \text{H} \quad k_e = 1.5 \times 10^{-7},
\end{align*}
\]

where we have adopted the values used by the Meudon PDR code.\(^3\)

Because the electron fraction $x_e \approx x_i \approx 10^{-4}$, removal of CH$^+$ by electrons is not crucial and we ignore it in our calculations. However, destruction by both atomic and molecular hydrogen are both important. Balancing the rate of formation with the rate of destruction

\(^3\)http://pdr.obspm.fr/PDRcode.html
5.3. RESULTS


\[ n_{\text{C}+} n_{\text{H}_2} k_f = n_{\text{CH}^+} n_{\text{HI}} k_{\text{HI}} + n_{\text{C}+} n_{\text{H}_2} k_{\text{H}_2}, \]

we can derive

\[ n_{\text{CH}^+} = x(C^+) \frac{x(\text{H}_2)}{1 - 2x(\text{H}_2)} \frac{k_f}{k_{\text{HI}}} \left( 1 + \frac{k_{\text{H}_2}}{k_{\text{HI}}} \frac{x(\text{H}_2)}{1 - 2x(\text{H}_2)} \right)^{-1} n_{\text{H}} \]

\[ \approx 3.4 \times 10^{-4} \left( \frac{n_{\text{H}}}{30 \text{ cm}^{-3}} \right) \times \exp \left( \frac{-4640 \text{ K}}{T_{\text{eff}}} \right) \quad (5.28) \]

5.3 Results

5.3.1 Temperature and CH\(^+\) abundance

The result of our temperature calculation is summarized in Figures 5.5 and 5.6. Figure 5.5 shows the mass- and volume-weighted differential probability distribution functions of \( \log T \) and \( \log T_{\text{eff}} \). Figure 5.6 shows the cumulative distribution functions of the same quantities. Both \( T \) and \( T_{\text{eff}} \) can take a large range of values spanning over 3 orders of magnitude. The distributions of both quantities appear to be bimodal, showing peaks at several tens and several hundreds of K. The large majority of the mass lies at low \( T \), but a small fraction is found in high temperature “pockets” - precisely the arrangement proposed by Falgarone & Puget (1995). An interesting feature of these distributions is the importance of density variations in setting the temperature. The difference between the mass- and volume weighted distributions shows that high values of \( T \) and \( T_{\text{eff}} \) tend to be found in relatively low-density gas. This is easily understood from equations (5.10) and (5.22). From (5.10), the heating processes are all proportional to \( n_{\text{H}} \). Furthermore, at low \( T \), the dominant cooling processes are C\(^+\) and O lines, for which the cooling power goes like \( n_{\text{H}}^2 \). Accordingly, cooling can balance heating at relatively low temperatures unless \( n_{\text{H}} \) is small. Similarly, the drift velocity \( v_d \) is inversely proportional to \( n_{\text{H}}^2 \) for a fixed ionization fraction (equation (5.22)), meaning that it is likely to be small except in low density regions. In our calculations, we find that the volume-weighted mean temperature \( T_{\text{V}} \approx 280 \text{ K} \) exceeds the density-weighted value \( T_{\text{M}} \sim 65 \text{ K} \) by more than a factor of 4. Similarly, the volume-weighted median \( T_{50,\text{V}} \approx 160 \text{ K} \) exceeds the mass-weighted median \( T_{50,\text{M}} \approx 35 \text{ K} \) by a similar value. This difference is particularly dramatic for the high-temperature tails of the distribution: while approximately 7\% of the volume in our box has a \( T_{\text{eff}} \) greater than 1000 K, only 0.3\% of the mass does.

We thus expect CH\(^+\) to mostly be found in low-density regions. We show in Figure 5.7 the regions of \( T - v_d \), \( T - n_{\text{H}} \), and \( v_d - n_{\text{H}} \) phase space in which most of the CH\(^+\) mass is contained. For instance, in the middle panel, we have created 256\(^2 \) logarithmically-spaced bins spanning the full range of temperatures and densities found in the simulation output. The color scale shows the fraction of the total mass in each of these \( T - n_{\text{H}} \) bins, while the black contours show the regions of phase space that contain the top 99\%, 90\%, 50\%, 10\%, and 1\% of the the CH\(^+\). The left and right panels repeat this procedure for \( T - v_d \) and \( v_d - n_{\text{H}} \), respectively. These plots confirm our expectation that regions with \( T \gtrsim 1000 \text{ K} \) and \( v_d \gtrsim \sigma_{3D} \approx 4 \text{ km s}^{-1} \) tend to be found at low density, with typical values of \( n_{\text{H}} \sim \) a few H
5.3. RESULTS

Figure 5.5: Blue solid line - mass-weighted differential distribution function of log $T$. Green solid line - the volume-weighted distribution of same. Dotted lines - the colors have the same meaning as before, but the distribution of log $T_{\text{eff}}$ has been plotted instead.

nuclei per cm$^3$. They also show that, while these regions are rare, they contain almost all of the CH$^+$ molecules.
Figure 5.6: Same as Figure 5.5, but showing the cumulative rather than the differential distribution functions. Specifically, $P(> \log T)$ shows the fraction of the mass or volume with values of $\log T$ greater than the corresponding value on the x-axis. As before, the solid lines show the distribution of $\log T$ and the dotted lines the distribution of $\log T_{\text{eff}}$. 
5.3. RESULTS

Figure 5.7: Left - the color scale shows the fraction of the total mass in each logarithmically-spaced $T - v_d$ bin. The black contours show the region of phase space in which 99, 90, 50, 10, and 1 percent of the $\text{CH}^+$ is found. Middle - same, but for $T - n_H$. Right - same, but for $v_d - n_H$. 

Mass Fraction $[\text{bin}^{-1}]$ 

$v_d [\text{cm s}^{-1}]$ 

$T [\text{K}]$ 

$n_H [\text{cm}^{-3}]$ 

$T [\text{K}]$ 

$n_H [\text{cm}^{-3}]$
The middle panel of Figure 5.7 illustrates the importance of the intermittency in $\Gamma_{\text{Turb}}$ in setting the gas temperature. The lower curve of this diagram traces out a line that corresponds to what the temperature as a function of density would be if we only considered $\Gamma_{\text{PE}}, \Gamma_{\text{CR}},$ and the various cooling processes described in section 2.3. Most of the gas in the simulation receives little heating from turbulence dissipation and thus lies at or near this line. A small fraction of the gas, however, is heated to significantly higher temperatures by the effects of turbulence, and this gas comprises the bulk of the material that is heated above $\sim 1000$ K. We find that without $\Gamma_{\text{Turb}},$ the fraction of mass in the simulation heated to $T > 1000$ K would drop by approximately a factor of 20.

Sheffer et al. (2008), following Ritchey et al. (2006), used the ratio of $N_{\text{CH}^+}$ to $N_{\text{CH}}$ along with $G_0$ and $x$(H$_2$) as an empirical probe of the gas density along a number of diffuse sight lines. The resulting density estimates were generally quite low, with typical $n_H \sim 3$ cm$^{-3}$, and in some cases much lower than estimates for $n_H$ inferred from C$^+$ excitation for the same sight lines (Sonnentrucker et al. 2002, 2003). Sheffer et al. (2008) interpret this as saying that a significant portion of the extinction and atomic hydrogen are associated with purely atomic regions that contain no CH$^+$, and that the corresponding increase in $G_0$ increases the estimate for $n_H$. Our result suggests an alternative explanation: that the CH$^+$ really is predominately found at low $n_H$, while the C$^+$ observations are probing something closer to the mean density.

### 5.3.2 Sight line analysis

Weselak et al. (2008) presented a sample of 53 CH$^+$-containing sight lines, 50 of which also had measurements of $N_H$ and $N_{\text{H}_2}$. For the purpose of comparing with observations, we will adopt those 50 sight lines as our observational sample. We limit ourselves to this sample so that the CH$^+$ column densities would all be calculated in a consistent way, but the distribution of columns in Weselak et al. (2008) is quite similar to that of Sheffer et al. (2008), which included both original data and results from many previous studies (Federman et al. 1997; Gredel 1997; Knauth et al. 2001; Pan et al. 2004).

Our model, by construction, has the same mean values of $\bar{N}_H$ and $\bar{N}_{\text{H}_2}$ as the Weselak et al. (2008) sample. The mean and median CH$^+$ column density for those sight lines were $1.2 \times 10^{13}$ and $1.1 \times 10^{13}$ cm$^{-2}$, respectively. The corresponding values in our models are $1.2 \times 10^{13}$ and $7.6 \times 10^{12}$ cm$^{-2}$. The means agree almost exactly, but the median in our sample is lower by about 40%. To further compare against the observational sample, we generate 2500 synthetic observations by casting 50 groups of 50 rays through our computational domain. The rays are uniformly distributed over the box with orientations randomly chosen from one of the $x$, $y$, and $z$ directions. For each group, we construct a histogram of $N_{\text{CH}^+}$ using 20 bins spaced evenly of the range $\log N_{\text{CH}^+} = 10$ to $\log N_{\text{CH}^+} = 15$ and compare it against the corresponding histogram for the observational sample. The result is shown in Figure 5.8. The error bars indicate the $1-\sigma$ variation in the number of sight lines per bin. The number of groups was set at 50 because that number was sufficient for the error bars to be converged: the maximum change in the $1-\sigma$ error over all the bins was 15%, and the mean
change was only 1%. We find that, while our model agrees well with the mean and median of the observational sample, it does tend to over-produce both very large and very small values. However, it is not clear that the sight lines in Weselak et al. (2008) constitute a truly random sampling in that they were chosen because CH$^+$ lines were detectable and the H and H$_2$ column densities were measurable. This is likely not the case for the very low column lines in our model.

![Figure 5.8](image)

**Figure 5.8:** Blue - average histogram of CH$^+$ column densities over 2500 lines of sight passing through the simulation volume. The error bars indicate the 1 $-$ $\sigma$ range of variation over all 50 samples of 50 sight lines each. Red - histogram of the CH$^+$ column densities from the 50 sight line sample in Weselak et al. (2008).

From our sample of 2500 rays, we choose 8 “typical” sight lines for more detailed investigation as follows. First, we randomly draw a ray from the sample. Next, we keep it only if both $N_{\text{CH}^+}$ and $N_{\text{H}}$ are within 50% of the overall sample means. Otherwise, we throw it away and draw again. We stop when 8 rays have been selected. The properties of these sight lines are described in Table 5.3.2. Here, the notation $\bar{x}_{99}$ means the average value of $x$ in the cells that are in the top 99% of the CH$^+$ number density distribution. Thus, $\bar{n}_{99}$, $\bar{T}_{M,99}$, and $\bar{v}_{d,99}$ are the mean density, mass-weighted temperature, and drift velocity in the regions.
which contain 99% of the CH\(^{+}\). We find that, consistent with Figure 5.7, almost all of the CH\(^{+}\) is in these sight lines is found in low-density, high-temperature, high-drift pockets of gas, with typical values of \(\bar{n}_{H,99} \approx 1 - 3 \text{ cm}^{-3}\), \(\bar{T}_{M,99} \approx 600 - 800 \text{ K}\), and \(\bar{v}_{d,99} \approx 2.7 - 4.4 \text{ km s}^{-1}\). These temperatures and velocities are quite similar to those obtained in the TDR models of Godard et al. (2009) using very different techniques.

The “temperature” \(\mu_{H}v_{d}^{2}/3k\) associated with a 4 km s\(^{-1}\) drift velocity is \(\approx 900 \text{ K}\), comparable to \(\bar{T}_{M,99}\). Both effects thus appear to be important for building up CH\(^{+}\) columns in excess of \(10^{13} \text{ cm}^{-2}\). To gauge the relative importance of the two effects, we repeat our calculation with \(v_{d}\) computed as above, but with \(T\) fixed at 35 K. The result is that the mean CH\(^{+}\) column drops from \(\approx 1.2 \times 10^{13} \text{ cm}^{-2}\) to \(\approx 8.0 \times 10^{12} \text{ cm}^{-2}\). If we repeat the same experiment with \(T\) computed as above, but ignoring the effects of \(v_{d}\), the CH\(^{+}\) column drops dramatically, down to \(\approx 5.0 \times 10^{11} \text{ cm}^{-2}\). Thus, the CH\(^{+}\) chemistry in our model appears to be mainly driven by ion-neutral drift, with the kinetic temperature making a secondary, but not negligible, contribution to the total column.

Finally, to gauge the spatial extent of these regions, we show in Figure 5.9 the result of integrating \(N_{CH^{+}}\) and \(N_{H}\) along each of the 8 rays in Table 5.3.2. All of the rays show the same general behavior: there are large regions that make basically no contribution to CH\(^{+}\) column, punctuated by a few thin zones where the CH\(^{+}\) abundance is substantial. The typical sight line intersects approximately 2-4 of these regions. This analysis further confirms the view of Falgarone & Puget (1995) - that the cold ISM contains isolated patches of hot, chemically active gas, and that these regions are crucial to understanding diffuse cloud chemistry.

### 5.3.3 H\(_2\) Emission

At \(T \gtrsim 1000 \text{ K}\), significant numbers of H\(_{2}\) molecules can be excited to \(J \geq 2\) rotational levels, producing observable emission in the \(J = 2 \rightarrow 0\), \(J = 3 \rightarrow 1\), and \(J = 4 \rightarrow 2\) lines. This emission is known to be correlated with CH\(^{+}\) column density (Frisch & Jura...
Figure 5.9: CH$^+$ column density versus total column density $N_H$ integrated along the same eight sight lines shown in Table 5.3.2. The quantities $N_{\text{CH}^+}(d)$ and $N_H(d)$ are the CH$^+$ and total column densities integrated up though path length $d$ along each ray. The x-axis has been normalized by the total column density $N_H(\ell_0)$ to fit all the rays on the same plot.

1980; Lambert & Danks 1986; Jensen et al. 2010), and has been interpreted as observational evidence for the intermittent dissipation of MHD turbulence (Falgarone et al. 2005). Because diffuse clouds are typically below the critical densities of the $J = 3$ and higher lines, the level populations are in general non-thermal, and the level populations depend on both $T$ and the gas density. Observations of H$_2$ excitation can thus help to constrain the temperature and density structure in models of CH$^+$ production in diffuse clouds. To compare our results against these observations, we calculate the H$_2$ rotational level populations by balancing collisional excitation with collisional de-excitation and spontaneous emission for the first 20 rotational levels of H$_2$. We solve the resulting eigenvalue problem using SciPy’s scipy.linalg.eig routine. The full code used for this calculation is available at https://bitbucket.org/atmyers/H2Emmision.

Observations of $N_{\text{H}_2}(J = 0)$ and $N_{\text{H}_2}(J = 1)$ indicate that ortho- (odd $J$) and para- (even $J$) hydrogen are not generally in found in the equilibrium 3:1 ratio in the ISM. For example, in the three sight lines presented in Gry et al. (2002), $\gamma = N_{\text{H}_2}(J = 1)/N_{\text{H}_2}(J = 0)$
5.3. RESULTS

Figure 5.10: Excitation diagram for the first 5 rotational levels of \( \text{H}_2 \) from our model compared to observed values. The y-axis shows the mean column densities in each rotational level divided by the degeneracy factors \( g_J \). The x-axis is the energy of each level expressed as a temperature. Red circles - Lacour et al. (2005). Green circles - Gry et al. (2002). Blue circles - Ingalls et al. (2011). Black circles - our model. We have scaled our results up by a factor of 3 to match the mean \( \text{H}_2 \) column density of the observations. The data have been shifted slightly along the x-axis for clarity.

The result of this calculation for \( J = 0 \) to 4 is shown in Figure 5.10, compared against the \( \text{H}_2 \) excitation observations from Gry et al. (2002), Lacour et al. (2005) and Ingalls et al. (2011). We have scaled all the column densities in our model up by a factor of 3 to match the mean \( \text{H}_2 \) column in the above papers. For all of the levels, our model result lies within

is 0.7, 0.6, and 0.4. Likewise, in the four lines from Lacour et al. (2005), \( \gamma = 0.3, 1.6, 0.6, \) and 0.9. Nehmé et al. (2008) found \( \gamma = 0.7 \) towards HD 102065, and Ingalls et al. (2011) found that this assumption fit their data for 6 nearby sight lines as well. In this paper, we will therefore fix the ortho:para ratio at 0.7 and treat ortho-\( \text{H}_2 \) and para-\( \text{H}_2 \) as separate species. For the energy levels \( E_J \), we treat both forms of \( \text{H}_2 \) as quantum rotors with rotational temperature \( T_r = 85.3 \) K. We used the Einstein A coefficients from Wolniewicz et al. (1998) and the collisional excitation rates from Le Bourlot et al. (1999). The degeneracy factors are \( g_J = (2J + 1) \) (para) and \( g_J = 3(2J + 1) \) (ortho).
the spread given by the observations. The mean $J = 1$ to $J = 0$ rotational temperature in our calculation, defined by
\[
\frac{N_{\text{H}_2}(J = 1)}{N_{\text{H}_2}(J = 0)} = \exp \frac{-171 \text{ K}}{T_{10}},
\]
is $\approx 67$ K, very close to the mean value for the sample of 38 sight lines in Rachford et al. (2009) based on FUSE measurements. Because there is a wide range of temperatures present and because the $J = 2$ and higher lines are generally not in thermal equilibrium, however, our results for higher lines are not well-described by a single rotation temperature.

5.4 Conclusions

The intermittent dissipation of turbulence in the ISM has been proposed as an explanation for the high ($> 10^{13}$ cm$^{-2}$) CH$^+$ column densities commonly observed along diffuse molecular sight lines (Falgarone et al. 1995). Turbulence can aid the production of CH$^+$ both by heating a small percentage of the gas directly (Lambert & Danks 1986; Falgarone et al. 1995; Pan & Padoan 2009) and by leading to large drift velocities (Spaans 1995; Federman et al. 1996; Sheffer et al. 2008) within localized regions of intense dissipation. Detailed dynamical and chemical models of these intense dissipation events have had much success in modeling the chemical properties of diffuse sight lines given reasonable assumptions about their properties and frequency in the ISM (Joulain et al. 1998; Godard et al. 2009).

We have re-assessed this question by post-processing a direct numerical simulation of MHD turbulence, scaled to have $n_H = 30$ cm$^{-3}$, $\ell_0 \approx 20$ pc, and $\sigma_{1D} \approx 2.3$ km s$^{-1}$. Although our approach is quite different to the above models, our work corroborates their results in several ways. Specifically:

1. We have solved an energy balance equation cell-by-cell for the temperature. While most of the mass in our cloud is cold ($\lesssim 35$ K), a small fraction ($\lesssim 1\%$) of the mass has been heated to temperatures in excess of 1000 K.

2. Similarly, we have computed the drift velocity in our simulation using an approximate analytic expression. While on average the drift is negligible, in isolated regions it can reach values equal to or exceeding the large scale RMS gas velocity of $\sim 4$ km s$^{-1}$.

3. Both of these effects combine to easily produce CH$^+$ column densities in excess of $10^{13}$ cm$^{-2}$. We find that overall, the drift is most important, in that it alone accounts for about $2/3$ of the CH$^+$ in the box, but the contribution from the gas temperature is not negligible.

4. Our work highlights the importance of including density variations in physical and chemical models of the ISM. 90\% of the CH$^+$ is found in cells with densities of $\approx 4$ cm$^{-3}$ or less - significantly lower than the mean value. These cells make up $\approx 5\%$ of the volume and $\approx 0.2\%$ of the mass in the simulation and have typical temperatures and
drift velocities of $\approx 700 - 800$ K and $\approx 3 - 4$ km $s^{-1}$. These values are quite similar to the TDR models of Godard et al. (2009), despite the difference of our approaches.

5. We have estimated the CH$^+$ column density through our model and compared the resulting distribution to the sample of sight lines presented in Weselak et al. (2008). Our mean CH$^+$ column of $1.2 \times 10^{13}$ cm$^{-2}$ agrees very well with the Weselak et al. sample, although the median is lower by $\approx 40\%$.

6. Finally, we computed the expected H$_2$ rotational line emission from these hot regions, and found that it is consistent with observations of diffuse molecular sight lines.

Acknowledgments

Support for this research was provided by NASA through NASA ATP grants NNX09AK31G and NNX13AB84G (CFM and PSL) and the NSF through grants AST-0908553 and AST-1211729 (ATM and CFM). This research was also supported by grants of high performance computing resources from the National Center of Supercomputing Application through grant TG-MCA00N020. We have used yt$^4$ (Turk et al. 2011) as well as the SciPy$^5$ family of Python scientific libraries for data analysis and plotting.

$^4$http://yt-project.org/
$^5$http://www.scipy.org/
Chapter 6

Star Cluster Formation in Turbulent, Magnetized Dense Clumps with Radiative and Outflow Feedback

Abstract

We present three ORION simulations of star cluster formation in a 1000 $M_\odot$, turbulent molecular cloud clump, including the effects of radiative transfer, protostellar outflows, and magnetic fields. Our simulations all use self-consistent turbulent initial conditions and vary the mean mass-to-flux ratio relative to the critical value over $\mu_\Phi = 2$, $\mu_\Phi = 10$, and $\mu_\Phi = \infty$ to gauge the influence of magnetic fields on star cluster formation. We find, in good agreement with previous studies, that magnetic fields corresponding to $\mu_\Phi = 2$ lower the star formation rate by a factor of $\approx 2.4$ and reduce the amount of fragmentation by a factor of $\approx 2$ relative to the zero-field case. We also find that the field increases the characteristic sink particle mass, again by a factor of $\approx 2.4$. The magnetic field also increases the degree of clustering in our simulations, such that the maximum stellar densities in the $\mu_\Phi = 2$ case are higher than the others by again a factor of $\approx 2$. This clustering tends to encourage the formation of multiple systems, which are more common in the rad-MHD runs than the rad-hydro run. The companion frequency in our simulations is consistent with observations of multiplicity in Class I sources, particularly for the $\mu_\Phi = 2$ case. Finally, we find evidence of primordial mass segregation in our simulations reminiscent of that observed in star clusters like the Orion Nebula Cluster.  

1This chapter will be submitted to the Monthly Notices of the Royal Astronomical Society as part of Star Cluster Formation in Turbulent, Magnetized Dense Clumps with Radiative and Outflow Feedback by Myers, Klein, Krumholz, & McKee, and is reproduced with the permission of all coauthors.
6.1 Introduction

Most stars form in groups (Lada & Lada 2003; Bressert et al. 2010), but theoretical (e.g., Shu 1977; McKee & Tan 2002, 2003) and numerical (Hennebelle & Fromang 2008; Krumholz et al. 2007b, 2010; Myers et al. 2011; Cunningham et al. 2011; Myers et al. 2013) treatments of star formation frequently consider stars forming in isolation. While these models are an important building block, they cannot capture the interaction effects likely to be important in real regions of star formation. For example, in Krumholz et al. (2012b), who considered the collapse of a relatively massive ($1000 M_\odot$) molecular cloud clump, the presence of a few massive stars affected the temperature structure of the entire cluster. A true understanding of star formation requires considering the clustered mode of formation commonly encountered in nature.

Simulations of star cluster formation that include magnetic effects have typically ignored radiative transfer (Li & Nakamura 2006; Wang et al. 2010), while simulations that include radiation have frequently ignored magnetic fields (Offner et al. 2009; Krumholz 2011; Hansen et al. 2012; Krumholz et al. 2012b). An important exception is Price & Bate (2009), which studied the collapse of a 50 $M_\odot$ molecular cloud including both magnetic and radiative effects. Non-zero field strengths can, among other things, reduce the overall rate of star formation (Price & Bate 2009; Padoan & Nordlund 2011; Federrath & Klessen 2012), suppress fragmentation (Hennebelle et al. 2011; Commerçon et al. 2011; Federrath & Klessen 2012; Myers et al. 2013), and influence the core mass spectrum (Padoan et al. 2007), while radiative feedback is likely crucial to picking out a characteristic mass scale for fragmentation (Bate 2009a; Myers et al. 2011; Krumholz 2011; Krumholz et al. 2011, 2012b). In this paper, we extend the work of Krumholz et al. (2012b) by including magnetic fields, and of Price & Bate (2009) by including self-consistently turbulent initial conditions, protostellar outflows, forming a statistically meaningful sample of stars, and following the protocluster evolution until a steady state is reached. The outline of this paper is as follows: we describe our numerical setup in Section 6.2, report the results of our simulations in Section 6.3, discuss our results in Section 6.4, and conclude in Section 6.5.

6.2 Simulations

We have performed six simulations of star formation in turbulent molecular cloud clumps, aimed towards quantifying the effects of varying the magnetic field strength. The first three simulations have a maximum resolution of $\Delta x_f \approx 46$ AU and have either a strong, weak, or zero magnetic field. The next three are identical, except that the resolution is $\Delta x_f \approx 23$ AU instead. As the high-resolution simulations are necessarily more computationally expensive, we integrate them for a shorter period and use them mainly to check for convergence at early times. The parameters of all six runs are summarized in Table 1.

Our simulations consist of two distinct phases: a driving phase, in which we generate turbulent initial conditions using a simplified set of physics, and a collapse phase, in which we
follow the gravitational collapse and subsequent star formation. In this section, we summarize our numerical approach and describe the initial conditions for each of these phases in turn.

6.2.1 Numerical Methods

We use our code ORION to solve the equations of gravito-radiation-magnetohydrodynamics in the two-temperature, mixed-frame, gray flux-limited diffusion (FLD) approximation. ORION uses adaptive mesh refinement (AMR) (Berger & Colella 1989) to focus the computational effort on regions undergoing gravitational collapse, and sink particles (Lee et al. (2013), see also Bate et al. (1995), Krumholz et al. (2004), Federrath et al. (2010a)) to represent matter that has collapsed to densities higher than we can resolve on the finest level of refinement. ORION uses Chombo as its core AMR engine, the HYPRE family of sparse linear solvers, and an extended version of the Constrained Transport scheme from PLUTO (Mignone et al. 2012) to solve the MHD sub-system (Li et al. 2012). The output of our code is the gas density $\rho$, velocity $\mathbf{v}$, magnetic field $\mathbf{B}$, the non-gravitational energy per unit mass $e$, the gravitational potential $\phi$, and the radiation energy density $E_R$, defined on every cell in the AMR hierarchy.

The equations and algorithms that govern our simulations, as well as our choices of dust opacities, flux limiters, and refinement criteria, are with one exception identical to those in Myers et al. (2013). For a complete description of our numerical techniques, see that paper and the references therein. The exception is that, in the present work, we have also included the sub-grid protostellar outflow model of Cunningham et al. (2011). In short, in addition to accreting matter from the grid, the sink particles in these simulations also inject a portion of the accreted matter back to the simulation domain at high velocity in the direction given by the sink particle’s angular momentum vector. Specifically, each sink ejects 21% of the mass it accretes back into the gas at a velocity of $1/3$ the Keplerian speed at the stellar surface, $v_{k,i} = \sqrt{GM_i/r_i}$, where $M_i$ and $r_i$ are the mass and radius of the $i$th sink particle. These parameters were selected so that the momentum flux would be consistent with observed values (Cunningham et al. 2011), without the wind speed dominating the Courant time step. Additionally, the wind-shocked gas in our calculations is capable of reaching temperatures higher than the dust sublimation temperature ($\gtrsim 10^3$ K). Under such conditions, FLD becomes a poor approximation to the gas cooling rate. To remedy this, we make one further change from Myers et al. (2013): when the gas temperature $T_g$ in a cell exceeds $10^3$ K, we remove energy from that cell at a rate given by $(\rho/m_H)^2 \Lambda(T_g)$ and deposit it into the radiation field, where $m_H$ is the hydrogen mass and $\Lambda(T_g)$ is the line cooling function from Cunningham et al. (2006).

We use periodic boundary conditions on all gas variables and on the gravitational potential $\phi$. The lone exception is the radiation energy density $E_R$. Periodic boundary conditions would trap radiation inside the simulation volume, which is not realistic. Instead, we use Marshak boundary conditions equivalent to surrounding the box in a radiation bath with temperature $T_r = 10$ K.
6.2. SIMULATIONS

Table 6.1: Simulation Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>$\mu_\Phi$</th>
<th>$B_0$</th>
<th>$\beta_0$</th>
<th>$M_{A,0}$</th>
<th>$\mu_{\Phi,\text{rms}}$</th>
<th>$B_{\text{rms}}$</th>
<th>$\beta_{\text{rms}}$</th>
<th>$M_{A,\text{rms}}$</th>
<th>$N_0$</th>
<th>$\Delta x_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>$\infty$</td>
<td>0.00</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>0.00</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>128</td>
<td>46</td>
</tr>
<tr>
<td>Weak</td>
<td>10.0</td>
<td>0.16</td>
<td>0.24</td>
<td>3.8</td>
<td>2.8</td>
<td>0.57</td>
<td>0.02</td>
<td>1.1</td>
<td>128</td>
<td>46</td>
</tr>
<tr>
<td>Strong</td>
<td>2.0</td>
<td>0.81</td>
<td>0.01</td>
<td>0.8</td>
<td>1.9</td>
<td>0.84</td>
<td>0.01</td>
<td>0.8</td>
<td>256</td>
<td>23</td>
</tr>
<tr>
<td>Hydro$_{23}$</td>
<td>$\infty$</td>
<td>0.00</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>0.00</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>256</td>
<td>23</td>
</tr>
<tr>
<td>Weak$_{23}$</td>
<td>10.0</td>
<td>0.16</td>
<td>0.24</td>
<td>3.8</td>
<td>2.8</td>
<td>0.57</td>
<td>0.02</td>
<td>1.1</td>
<td>256</td>
<td>23</td>
</tr>
<tr>
<td>Strong$_{23}$</td>
<td>2.0</td>
<td>0.81</td>
<td>0.01</td>
<td>0.8</td>
<td>1.9</td>
<td>0.84</td>
<td>0.01</td>
<td>0.8</td>
<td>256</td>
<td>23</td>
</tr>
</tbody>
</table>

Note. — Col. 3: mean magnetic field. Col. 4: mean plasma $\beta_0 = 8\pi \rho c_s^2 / B_0^2$. Col. 6: resolution of the base grid. Col. 7: maximum resolution at the finest level. All runs have $M_c = 1000 \, M_\odot$, $L = 0.46$ pc, $\Sigma_c = 1$ g cm$^{-2}$, $\sigma_v = 1.2$ km s$^{-1}$, $M = 11.1$, and 4 levels of refinement.

6.2.2 Initial Conditions

We begin with a uniform, isothermal gas inside a box of size $L = 0.46$ pc. The initial gas temperature is $T_g = 10$ K and the initial density $\bar{\rho}$ is $6.96 \times 10^{-19}$ g cm$^{-3}$, or $n_H = 2.97 \times 10^4$ hydrogen nuclei per cm$^{-3}$. The gravitational free-fall time

$$t_{\text{ff}} = \sqrt{\frac{3\pi}{32G\bar{\rho}}}$$

computed using the mean density is $t_{\text{ff}}(\bar{\rho}) \approx 80$ kyr. The corresponding total mass of the clump $M_c = 1000 \, M_\odot$, and the clump surface density $\Sigma_c = 1$ g cm$^{-2}$.

These parameters are identical to those of Krumholz et al. (2012b). In addition, our MHD runs have an initially uniform magnetic field with strength $B_0$ oriented in the $z$ direction. The strength of this field can be expressed using the magnetic critical mass, $M_\Phi$, which is the maximum mass that can be supported against gravitational collapse by the magnetic field. In terms of the magnetic flux threading the box $\Phi = B_0 L^2$:

$$M_\Phi = c_\Phi \frac{\Phi}{G^{1/2}} ,$$

where $c_\Phi = 1/2\pi$ for a sheet-like geometry (Nakano & Nakamura 1978) and $\approx 0.12$ for a uniform spherical cloud (Mouschovias & Spitzer 1976; Tomisaka et al. 1988). In this paper, we take $c_\Phi = 1/2\pi$. The ratio of the mass in the box to the critical mass, $\mu_\Phi = M/M_\Phi$, thus divides the parameter space into magnetically sub-critical ($\mu_\Phi < 1$) cases, for which the field is strong enough to stave off collapse, and magnetically super-critical ($\mu_\Phi > 1$) cases, which will collapse on a timescale of the order of the mean-density gravitational free-fall time. Note
that $\mu_\Phi$ here refers to the box as a whole, and not to the individual cores and clumps that form within.

Observations of the Zeeman effect in both OH lines (Troland & Crutcher 2008) and CN $N = 1 \rightarrow 0$ hyperfine transitions (Falgarone et al. 2008) show that the typical value of $\mu_\Phi$ is $\approx 2$. While these observations do not rule out the existence of sub-critical magnetic fields in some star-forming regions, they do suggest that the typical mode of star formation involves fields that are not quite strong enough to support clouds by themselves over timescales longer than $\sim t_{ff}(\bar{\rho})$. Additionally, Crutcher et al. (2010) suggest, based on a statistical analysis of observed line-of-sight magnetic field components, that values of $\mu_\Phi$ much more supercritical than $\mu_\Phi = 2$ may not be rare. In this paper we thus do two MHD runs, called Weak and Strong. Weak has an initial magnetic field strength of $B_0 = 0.16$ mG, corresponding to $\mu_\Phi = 10$. Strong, which is in fact closer to the mean observed $\mu_\Phi$, has $B_0 = 0.81$ mG and $\mu_\Phi = 2$. The corresponding values for the plasma parameter, $\beta_0 = 8\pi \rho c_s^2/B_0^2$, and the 3D Alfvén Mach number, $M_{A,0} = \sqrt{2\pi \rho c_s^2}/B_0$ are shown in Table 1. We also do a run called Hydro, in which we set $B_0 = 0.0$ mG ($\mu_\Phi = \infty$). Note that, because the Weak run is initially super-Alfvénic, there is some amplification of the initial magnetic field during the driving phase. We thus also show in Table 1 the root-mean-squared magnetic field, $B_{\text{rms}}$, as well as the values of $M_{A, \beta}$, and $\mu_\Phi$ corresponding to $B_{\text{rms}}$ instead of $B_0$.

Molecular clouds and the clumps they contain are also observed to have significant non-thermal velocity dispersions (e.g. Hennebelle & Falgarone 2012; McKee & Ostriker 2007), which are generally explained by invoking the presence of supersonic turbulence. Turbulence is frequently modeled in simulations of star formation by generating a velocity field with the desired power spectrum (say, $P(k) \propto k^{-2}$ for supersonic Burgers turbulence) in Fourier space and then superimposing this field on a pre-determined smooth density distribution (e.g. Krumholz et al. 2007a; Bate 2009a; Wang et al. 2010; Girichidis et al. 2011; Myers et al. 2013). While this approach captures some of the effects of turbulence on cloud collapse, such as providing “seeds” for fragmentation, it has the downside that density and velocity fields are not self-consistent at time $t = 0$. This lack of initial sub-structure in the density field permits collapse on the order of a free-fall time (Krumholz et al. 2012b). While this may be appropriate for simulations at the scale of individual cores, it is not appropriate for simulations at the scale of dense clumps or GMCs, as these structures convert only a small percentage of their mass to stars per free-fall time (Zuckerman & Evans 1974; Krumholz & Tan 2007; Krumholz et al. 2012a). Here, we instead follow the approach used in, e.g. Offner et al. (2009), Federrath & Klessen (2012), Hansen et al. (2012), and Krumholz et al. (2012b): we generate initial conditions using a driven turbulence simulation, and then switch on gravity and allow the gas to collapse. This ensures that the density and velocity fields are self-consistent at time zero.

During the driving phase, we turn off self-gravity, particles, and radiative transfer, leaving just the ideal MHD equations. We set $\gamma = 1.0001$, so that the gas is very close to isothermal during this phase. For the driving pattern, we use a 512³ perturbation cube generated in Fourier space according to method in Dubinski et al. (1995). This pattern has a flat power spectrum in the range $1 \leq kL/2\pi \leq 2$, where $k$ is the wavenumber. We also perform a
6.2. SIMULATIONS

Helmholtz decomposition and keep only the divergence-free portion of the driving velocity, as in e.g. Padoan & Nordlund (1999), Ostriker et al. (1999, 2001), Kowal et al. (2007), Lemaster & Stone (2009), and Collins et al. (2012). We then drive the turbulence using the method of Mac Low (1999) for two crossing times. The resulting initial states for the collapse phase are illustrated in Figure 6.1. Note that the initial conditions for Weak, for which $\mathcal{M}_A = 6.4$, contains much more structure in the magnetic field than those for Strong ($\mathcal{M}_A = 1.3$), in which the turbulence is not strong enough to drag around field lines significantly.

![Figure 6.1](image)

*Figure 6.1:* Turbulent initial conditions for our three main runs. The colors indicate column density, while the mass-weighted, plane-of-sky magnetic field orientations are over-plotted as black arrows.

Our choice of a solenoidal (divergence-free) driving pattern requires some discussion. The purpose of the driving is to mimic the effects of turbulence cascading down to our dense clump from larger scales. Since this is necessarily somewhat artificial, one would hope that the choice of driving pattern had little effect on the nature of the fully-developed turbulence. However, the presence of large-scale compressive motions in the driving has a significant effect on the density probability distribution function (Federrath et al. 2008), the fractal density structure (Federrath et al. 2009), and the star formation rate (Federrath & Klessen 2012). The latter is of particular importance here. The turbulent runs in Krumholz et al. (2012b), which used initial conditions quite similar to our Hydro run, had star formation rates that were too high by an order of magnitude. If the IMF peak is determined by the temperature structure imposed by protostellar accretion luminosities (Krumholz 2011), then overestimating the star formation rate likely means overestimating the characteristic stellar mass as well. Our choice of solenoidal driving helps bring the star formation rate closer to the observed values (Section 3.2), so the level of radiative feedback is probably more realistic in these calculations. Furthermore, even turbulence that is driven purely compressively will have approximately half the power in solenoidal modes in the inertial range for hydrodynamic, supersonic turbulence (Federrath et al. 2010b), and magnetic fields further decrease the
compressive fraction (Kritsuk et al. 2010; Collins et al. 2012). We thus expect that, whatever the driving mechanisms responsible for maintaining GMC turbulence on large scales, it would be mostly (but not purely) solenoidal by the time it cascades down to the $\approx 0.46$ pc scales of our box. At the end of the driving phase, our simulations have 29%, 22%, and 14% of the total power in compressive motions in the Hydro, Weak, and Strong runs, respectively.

After generating the initial conditions, we move on to modeling the collapse phase. We coarsen the turbulence simulations above from $N_0 = 512$ to either $N_0 = 256$ for the high-resolution runs or $N_0 = 128$ for our main runs. We switch on gravity, sink particles, and radiation, and also set $\gamma = 5/3$ instead of $\gamma = 1.0001$, appropriate for a gas of $\text{H}_2$ that is too cold for the rotational and vibrational degrees of freedom to be accessible. This also allows the temperature to vary according to the outcome of our radiative transfer calculation. We summarize the results of the collapse phase in the next section.

6.3 Results

We begin by describing the evolution of the large-scale morphology of our clumps in section (Sec. 6.3.1). We then discuss the overall rate of star formation (Sec. 6.3.2), compare our sink particle mass distributions to the stellar IMF (Sec. 6.3.3) and to the protostellar mass functions of McKee & Offner (2010) (Sec. 6.3.4), examine the magnetic field geometry on the scale of individual stellar cores (Sec. 6.3.5) and the accretion history of individual protostars (Sec. 6.3.6), describe the primordial mass segregation observed in our simulations (Sec. 6.3.7), and finally discuss the multiplicity of our simulated star systems (Sec. 6.3.8). Unless otherwise stated, the results in this section are from our main Hydro, Weak, and Strong calculations at $\Delta x \approx 46$ AU. We discuss numerical convergence in section (Sec. 6.3.2).

6.3.1 Global Evolution

In Figures 6.2 through 6.4, we show the evolution of the column density $\Sigma$ and density-weighted mean gas temperature $T$, defined as $\Sigma = \int_{-L/2}^{L/2} \rho dx$ and $T = \int_{-L/2}^{L/2} \rho T_g dx / \Sigma$. Because star formation proceeds at different rates in the three runs (see Sec. 6.3.2) we compare the simulations based on the total mass that has been converted into stars, rather than the elapsed time. Figures 6.2 through 6.4 show snapshots of the runs when the total mass in stars is 5, 10, 15, and 20 $M_\odot$. The global morphology of all three calculations is quite similar to the non-magnetic, turbulent simulations presented in Krumholz et al. (2012b). In all three runs, the turbulence creates a network of over-dense, filamentary regions. As time passes, these dense regions collapse gravitationally and begin to fragment into isolated cores of gas. The cores collapse to form stars, leading to the appearance that stars tend to be strung along the gas filaments. Comparing the late-time distribution of stars in run Strong to those of run Weak and Hydro, two effects jump out. First, there are many more stars in Hydro than in the either Weak or Strong. Second, the magnetic field appears to confine star formation to take place within a smaller surface area in the $\mu_\Phi = 2$ case than in the others, so that the star
Figure 6.2: Density-weighted mean temperature (left) and column density (right) for the Hydro run. Projected sink particle positions have been over-plotted as white dots.
6.3. RESULTS

Figure 6.3: Same as Figure 6.2, but for the Weak MHD run instead.

particles tend to be found at higher surface density, and there are large regions with no stars at all. The reason for this behavior is simple: when the box as a whole is only magnetically
6.3. RESULTS

Figure 6.4: Same as Figure 6.2, but for the Strong MHD run instead.

supercritical by a factor of 2, then there are relatively large sub-regions within the domain that are magnetically sub-critical. These regions are not able to collapse to form stars on
timescales comparable to $t_{\text{ff}}$. We return to this point in section 6.3.7.

![Figure 6.5: T-\(\rho\) phase plots for all three runs. The columns, from left to right, correspond to the Hydro, Weak, and Strong runs, while the rows, from top to bottom, show the state of the simulations at the points at which 5, 10, 15, and 20 $M_\odot$ of stars have formed. The colors show the amount of mass in each $T-\rho$ bin.](image)

The evolution of the gas temperature is interesting as well. At $t = 0$, the gas in the simulations is uniformly at 10 K. As stars form, they also heat up their surrounding
6.3. RESULTS

Figure 6.6: $T$-$\rho$ phase plots for all three runs. The columns, from left to right, correspond to the Hydro, Weak, and Strong runs, while the rows, from top to bottom, show the state of the simulations at 0.1, 0.2, 0.3, and 0.4 $t_{\text{ff}}$. The colors show the amount of mass in each $T$-$\rho$ bin.

environments. When the mass in stars is $5 \, M_\odot$, the high-temperature regions are confined to the cores of gas around the individual protostars. As the simulations evolve and the protostars grow in mass, the heated regions grow and begin to overlap. By the time 20 $M_\odot$ of stars have formed, even regions far from any protostars have begun to be heated above the
background temperature of 10 K, although the median gas temperatures are still a quite cool 11-12 K.

We examine the temperature structure in our simulations more quantitatively in Figures 6.5 and 6.6. These plots are constructed as follows. First, we create a set of 2-dimensional bins in $\rho - T_g$ space. We have chosen the bins to be logarithmically spaced in both $\rho$ and $T_g$, covering a range from $10^{-20}$ to $10^{-12}$ g cm$^{-3}$ in density and $10^{0.5}$ to $10^{2.5}$ K in temperature. Each bin is 0.025 dex wide in both $\rho$ and $T_g$, so that there are 320 density bins and 80 temperature bins. Then, we loop over every cell in the simulation. If a cell is not covered by a finer level of refinement (i.e. it is at maximum available resolution), we examine its density and temperature and add its mass to the appropriate bin. Otherwise, we skip it and move on. Figures 6.5 and 6.6 thus show the distribution of gas mass with both density and temperature, in units of $M_\odot$ dex$^{-2}$.

We have performed this calculation for all three of our runs, comparing each at equal stellar masses (Figure 6.5) and at equal times (Figure 6.6). The differences between the three runs are particularly dramatic when the runs are compared at equal evolution times, because one of the effects of the magnetic field is to delay the rate of star formation (Section 6.3.2). However, even when compared at equal stellar mass (Figure 6.5), there is still less hot gas in the Strong run than the others. This is likely due to the overall lower accretion rate in the Strong run, since accretion luminosity is the dominant source of heating. The excess hot gas in the Weak run at early times is a small-sample size effect: there are only a few stars present at early times, and the Weak field run happens to form a few stars particularly early in its evolution (see Section 6.3.2). At later times, when there are dozens of stars in each run, the temperature structures of Weak and Hydro look quite similar.

### 6.3.2 Star Formation

We now consider the properties of the sink particles formed in our simulations. In this section, we consider sink particles to be "stars" when their masses exceed 0.05 $M_\odot$. This threshold corresponds to the approximate mass at which second collapse occurs (Masunaga et al. 1998; Masunaga & Inutsuka 2000). Below this mass, our code will merge sink particles if one enters the accretion zone of another, so only sinks with masses greater than 0.05 $M_\odot$ are ensured to be permanent over the course of the simulations. With that caveat, we display the total number of stars $N_*$ and the star formation efficiency (SFE) versus time in our simulations in Figures 6.7 and 6.8. We have taken the definition of the SFE to be the total mass in stars divided by the total mass of the cluster, including both gas and stars:

$$\text{SFE} = \frac{M_*}{M_{\text{gas}} + M_*} = \frac{M_*}{M_c}. \quad (6.3)$$

There is a monotonic decrease in both the SFE and $N_*$ at a given time with magnetic field strength. The reduction in $N_*$ between the $\mu_\Phi = \infty$ and $\mu_\Phi = 2$ cases is approximately a factor of 2. This agrees well with the simulations of Hennebelle et al. (2011), which found the same reduction in the number of fragments (a factor of $\approx 1.5 - 2$ between $\mu_\Phi = 2$ and $\mu_\Phi = 120$).
using quite different numerical schemes and initial conditions. For example, Hennebelle et al. (2011) used an isothermal equation of state with a barotropic switch at high density, compared to our FLD radiative transfer, and took as initial conditions a spherical cloud with velocity perturbations, compared to our turbulent box initial conditions. This factor of $\approx 2$ also agrees with the isothermal calculations of Federrath & Klessen (2012), whose initial conditions were similar to our own. There is evidence from numerical simulations (Commerçon et al. 2011; Myers et al. 2013) that a combination of magnetic fields and radiative heating from accretion luminosity onto massive protostars can much more dramatically suppress fragmentation in the context of massive ($\gtrsim 100 \, M_\odot$) core collapse, but as we do not form stars anywhere near as massive as those in Myers et al. (2013) in these runs, this effect is not dramatic here.

Figure 6.7: Star formation efficiency (SFE) versus free-fall time for the Hydro (blue), Weak (green) and Strong (red) runs. The solid lines are from the high-resolution simulations, the dashed from the low. The black dotted lines demonstrate the slope of the low-resolution curves computed at SFE $= 0.02$, which we use to determine the star formation rate below. The free-fall time has been computed using the mean density.

We also show in Figures 6.7 and 6.8 the SFE and $N_*$ versus free-fall time for the three high-resolution runs used in our convergence study. We find that as far as we have been able
to run our high-resolution models, there is excellent convergence in the mass in stars as a function of time, and good convergence in \( N^* \) as well. The largest discrepancy in \( N^* \) occurs in the Hydro run at \( t = 0.25 \, t_{\text{ff}} \), when the low-resolution run contains \( \approx 50\% \) more stars than the high-resolution run.

Next, we examine the star formation rate (SFR) in our runs. The dimensional SFR, \( \dot{M}_* \), is simply the rate at which gas is converted into stars, in e.g. \( M_\odot \, \text{yr}^{-1} \). There are various definitions of the dimensionless SFR, \( \epsilon_{\text{ff}} \), in the literature; the most straightforward approach (e.g. Krumholz & McKee 2005; Padoan & Nordlund 2011; Federrath & Klessen 2012) is to normalize \( \dot{M}_* \) by what the star formation rate would be if all the mass in the box was converted to stars in a mean-density gravitational free-fall time:

\[
\epsilon_{\text{ff}, \bar{\rho}} = \frac{\dot{M}_*}{\dot{M}_c / t_{\text{ff}, \bar{\rho}}},
\]

where \( t_{\text{ff}, \bar{\rho}} \) is Equation (6.1) evaluated at \( \bar{\rho} \). However, because of the compressive effects of supersonic turbulence, most of the mass is actually at higher densities than \( \bar{\rho} \). One could alternatively define some density threshold, \( \rho_{\text{thresh}} \), evaluate \( t_{\text{ff}} \) at that density, and define the
relevant mass to be all the mass at $\rho_{\text{thresh}}$ or higher. Krumholz et al. (2012b) take $\rho_{\text{thresh}}$ to be the mass-weighted mean density, $\langle \rho \rangle$, and therefore define:

$$
\epsilon_{\text{ff},(\rho)} = \frac{\dot{M}_*}{\langle \rho \rangle (1/2) M_c/t_{\text{ff},(\rho)}},
$$

(6.5)

where $t_{\text{ff},(\rho)}$ is the free-fall time (Equation 6.1) computed using $\langle \rho \rangle$, and the factor of $1/2$ accounts for the fact that, for a log-normal mass distribution, half the cloud mass is above $\langle \rho \rangle$.

The first of these definitions is more analogous to extragalactic CO observations, in which the mass is taken to be all the mass in the beam, while the second is more analogous to observations of the SFR based on high-critical density tracers like HCN. We report both forms of $\epsilon_{\text{ff}}$ in Table 6.2 below, where we have evaluated $\langle \rho \rangle$ at the instant gravity is switched on. Note that, while $\bar{\rho}$ is the same in all of our runs, $\langle \rho \rangle$ is not: the magnetic field keeps material from getting swept up along field lines, such that the value of $\langle \rho \rangle$ generally decreases with increasing magnetic field strength (Padoan & Nordlund 2011).

The SFEs in Figure 6.7 all follow the same pattern: a period of relatively slow star formation in which the SFR increases with time, followed by a period with a roughly constant rate of star formation. The initial “ramp up” period lasts for 0.4 to 0.5 $t_{\text{ff},(\rho)}$ after gravity is switched on, with the magnetic field strength slightly delaying the onset of steady-state star formation relative to the field-free case. To compute $\dot{M}_*$, we have evaluated the slope for each run at the time at which the mass in stars is 20 $M_\odot$, since all of the runs are in the linear phase at that SFE. These slopes are indicated by the dotted lines in Figure 6.7. We show the resulting values of $\dot{M}_*$, $\epsilon_{\text{ff},\bar{\rho}}$, and $\epsilon_{\text{ff},(\rho)}$ in Table 6.2. We find that the magnetic field decreases the SFR by a factor of $\approx 2.4$ over $\mu_{\Phi} = \infty$ to $\mu_{\Phi} = 2$, for both definitions of $\epsilon_{\text{ff}}$. The $\approx 2.4$ reduction agrees well with previous studies of the SFR in turbulent, self-gravitating clouds (Price & Bate 2009; Padoan & Nordlund 2011; Federrath & Klessen 2012). Likewise, our value of $\epsilon_{\text{ff},\bar{\rho}} = 0.17$ in the Hydro case is comparable to the value of 0.14 reported in the solenoidally driven, pure HD run in Federrath & Klessen (2012). This suggests that the radiative and outflow feedback processes included in this work have not dramatically altered the SFR over the time we have run, although a direct numerical experiment confirming this would be desirable.

Our Hydro run is almost identical to the “TuW” run from Krumholz et al. (2012b). The exception is the turbulent driving pattern, which is solenoidal here and was a “natural” 1:2 mixture of compressive and solenoidal modes (i.e., 1/3 of the total power was in compressive motions) in Krumholz et al. (2012b). Federrath & Klessen (2012) found that mixed forcing increased the star formation rate by a factor of $\sim 3$ - 4 over the pure solenoidal case, depending on the random seed used to generate the driving pattern. If we compare our $\epsilon_{\text{ff},(\rho)}$ to that of run TuW in Krumholz et al. (2012b), we see that our driving pattern itself resulted in an $\approx 2.3$ reduction in the star formation rate, similar to the Federrath & Klessen (2012) result. However, even with this reduction, the lowest SFR reported in this work of 0.05 in the Strong run is still slightly higher than the typically observed value of 0.01 (Krumholz & Tan 2007; Krumholz et al. 2012a). Because of the sensitivity of the SFR to the details of
6.3. RESULTS

Table 6.2: Summary of the star formation in each run.

<table>
<thead>
<tr>
<th>Name</th>
<th>$t_f/t_{ff,\bar{\rho}}$</th>
<th>$t_{ff,\bar{\rho}}$</th>
<th>$M_{*,t_f}$</th>
<th>$N_{*,t_f}$</th>
<th>$\dot{M}_s$</th>
<th>$\epsilon_{ff,\bar{\rho}}$</th>
<th>$\epsilon_{ff,\langle \rho \rangle}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>0.45</td>
<td>80.0</td>
<td>28.5</td>
<td>23.9</td>
<td>89</td>
<td>2.2</td>
<td>0.17</td>
</tr>
<tr>
<td>Weak</td>
<td>0.68</td>
<td>80.0</td>
<td>29.8</td>
<td>33.8</td>
<td>81</td>
<td>1.2</td>
<td>0.07</td>
</tr>
<tr>
<td>Strong</td>
<td>0.78</td>
<td>80.0</td>
<td>33.4</td>
<td>32.2</td>
<td>92</td>
<td>0.9</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note. — Col 2. - the final simulation time. Col. 3 - in kyr. Col 4. - the total mass in stars at $t_f$. Col 5. - the number of stars at $t_f$. Col. 6 - in $10^{-3} M_\odot$ yr$^{-1}$.

the driving, which is in any event is only a rough approximation to the true drivers of GMC turbulence, we believe that the raw numbers in Table 6.2 are to be taken less seriously than the trend with magnetic field strength, which appears to be robust for both solenoidal (this work, Padoan & Nordlund (2011)) and naturally mixed (Federrath & Klessen 2012) driving.

6.3.3 The Initial Mass Function

The stellar initial mass function (IMF) (e.g. Chabrier 2005) is one of the most basic observable properties of stellar populations, and serves as an important constraint on numerical simulations of star formation. In this section, we examine the distribution of sink particle masses in our simulations, and compare the result against the observed IMF. To begin, we show in Figure 6.9 the evolution of the median, 25th percentile, and 75th percentile sink particle masses in each of our runs.

There are two points to make about this plot. First, in each of our runs, the 25th, 50th, and 75th percentile sink masses have all leveled off to well-defined values after about 10 to 20 $M_\odot$ of gas has been converted into stars. Even though most of the sink particles in our calculations are still accreting at the time we stop running, this growth is counterbalanced by the fact the new stars are continuously forming, so that the population as a whole has approached a steady-state distribution. Second, the median particle mass appears to monotonically increase with magnetic field strength, from $m_c = 0.05 M_\odot$ in the Hydro run to $m_c = 0.09 M_\odot$ in Weak and $m_c = 0.12 M_\odot$ in Strong. Thus, the magnetic field increases the median mass by a factor of $\approx 2.4$ over the range $\mu_\Phi = \infty$ to $\mu_\Phi = 2$.

Next, we examine the full distribution of sink particle masses. In Figure 6.10, we show the differential mass distributions for our simulated clusters at the point at which the mass in stars is 20 $M_\odot$. We find that the distributions are well-fit by a log-normal:

$$
\Psi(m) \propto \exp\left[-\frac{(\log m - \log m_c)^2}{2\sigma^2}\right],
$$

(6.6)

where $m_c$ is the median mass for each run given above and $\sigma = 0.55$ is the log-normal width.
We normalize the above expression so that $\int_{\log m_1}^{\log m_2} \Psi(m) d\log(m)$ gives the fraction of stars with $\log m$ between $\log m_1$ and $\log m_2$. If we take $m_c = 0.2 M_\odot$, this is equivalent to the low-mass limit of the Chabrier (2005) IMF. However, even our Strong run, which has the largest $m_c$, is lower than the Chabrier (2005) characteristic mass by a factor of 1.7. The Weak and Hydro medians are smaller by factors of 2.2 and 4.0, respectively.

McKee (1989) considered an approximate expression for the maximum stable mass for a finite temperature cloud in the presence of magnetic fields:

$$M_{cr} \approx M_{BE} + M_\Phi,$$

where $M_{BE} = 1.18 c_s^2 / \sqrt{G^3 \rho}$ is the Bonnor–Ebert mass and $M_\Phi$, as defined above, is the maximum stable mass for a pressureless cloud supported by magnetic fields. It is instructive
to compute the typical value of $M_{cr}$ in our simulations. If we write $M_{cr} = \mu_{\Phi,\text{core}} M_\Phi$, where $\mu_{\Phi,\text{core}}$ is the mass-to-flux ratio at the core scale, rather than the entire box, then:

$$
M_{cr} = \frac{\mu_{\Phi,\text{core}}}{\mu_{\Phi,\text{core}} - 1} M_{\text{BE}}.
$$

(6.8)

The Bonnor-Ebert mass for each of our runs, evaluated at the mass-weighted mean density, is 0.098 $M_\odot$, 0.102 $M_\odot$, and 0.114 $M_\odot$ for Hydro, Weak, and Strong, respectively. To estimate the value of $\mu_{\Phi,\text{core}}$, we use $\mu_{\Phi,\text{rms}}$. The resulting estimates for $M_{cr}$ are 0.10 $M_\odot$, 0.16 $M_\odot$, and 0.23 $M_\odot$ - approximately a factor of 2 higher than our simulation results for the median stellar mass. The factor of $\sim 2.3$ increase from $\mu_\Phi = \infty$ to $\mu_\Phi = 2$ is quite close to the increase in $m_c$ we observe in our simulations.

![Figure 6.10](image)

*Figure 6.10:* Differential mass distributions for the Hydro, Weak, and Strong runs. The blue histograms are the simulation data, while the black solid and dotted lines are log-normal distributions (Equation (6.6)) with either the simulated value of $m_c$ (solid), or the Chabrier (2005) value (dotted).

It is not surprising that our sink particles undershoot the Chabrier (2005) IMF somewhat - many of the sinks in Figure 6.10 formed only recently, and practically all of them are still accreting mass. The more relevant comparison is thus to the protostellar mass function (PMF), $\Psi_P$, in McKee & Offner (2010), which gives the mass distribution of a population of still-embedded Class 0 and I protostars. We compare our simulations to these theoretical PMFs in the next section.

### 6.3.4 The Protostellar Mass Function

The PMF depends on three factors: the functional form of $\dot{N}_s(t)$, the distribution of final stellar masses (i.e., the IMF), and the accretion history of the individual protostars, which can be calculated from various theoretical models of star formation. For example, competitive accretion (CA) (Zinnecker 1982; Bonnell et al. 1997), makes a different prediction about a star’s accretion time as a function of its final mass than the turbulent core (TC) model of
McKee & Tan (2002, 2003), so a population of still-accreting protostars with the same IMF and functional form of $\dot{N}_*(t)$ will have a different mass distribution under the two theories.

McKee & Offner (2010) provide PMFs for two functional forms of $\dot{N}_*$, one where it is constant and one where it exponentially accelerates with time. In our simulations, we have a $\dot{N}_*$ that is approximately linear with time (Figure 6.8), at least after an initial transient phase of $\approx 0.2 t_{ff}$, so we will not include any adjustments for accelerating star formation in our comparisons. We also do not include the “tapered accretion” models considered in McKee & Offner (2010), as we find that the accretion rates in our simulations are well-described by non-tapered accretion (see Section 6.3.6). We have also followed McKee & Offner (2010) in assuming that the distribution of final stellar masses $\Psi_C$ follows the Chabrier (2005) stellar IMF. We consider PMFs associated with three basic accretion models - the TC model, the CA model, and the isothermal sphere (IS) model of Shu (1977) - and two more complex models - two-component turbulent core (2CTC) model of McKee & Tan (2003) and the two-component competitive accretion (2CCA) model of Offner & McKee (2011). 2CTC is a generalization of the TC model that limits to IS for low masses and TC for high masses, while 2CCA similarly interpolates for the IS and CA models. Having fixed $\Psi_C$ and the form of $\dot{N}_*(t)$, the only other parameter that enters into the “basic” PMFs is the upper mass limit of stars that will form in the cluster, $m_u$. In our comparison, we set $m_u = 6 \, M_\odot$, which is larger than the most massive protostar we form in these simulations and about the mass of the most massive core identified in section 6.3.5. The two component models contain an additional parameter: the ratio $R_{\dot{m}}$ of accretion rates for the either the TC or CA models to the IS model for a $1 \, M_\odot$ star. We have taken $R_{\dot{m}} = 3.6$ for 2CTC and $R_{\dot{m}} = 3.2$ for 2CCA, which correspond to the fiducial parameter choices in McKee & Offner (2010) and Offner & McKee (2011).

We show the distribution of protostellar masses for the Hydro, Weak, and Strong models in Figure 6.11. To make a clean comparison across the three runs, we have plotted the results at the times for which the total mass in stars is $20 \, M_\odot$, or SFE = 0.02. The earliest time this occurs is $t \approx 0.4 t_{ff}$ in the Hydro run, so we are well-outside the initial “turn-on” phase during which the assumption of constant $\dot{N}_*$ is inappropriate. We also show in Figure 6.11 the five theoretical PMFs from above. The TC and CA models seem to predict more low-mass protostars than we find in our simulations, and the IS model predicts a median mass that is too large by about 0.5 dex. The two-component models, however, agree well with the median mass found in our Strong simulation. The Hydro run does not compare well with any of the theoretical models, mainly because its median mass is too low - lower than the Weak run by a factor of $\approx 2$ and the Strong run by a factor of $\approx 3$ for this snapshot. This increase in the typical protostellar mass due to the magnetic field appears to be necessary to get good agreement with the two-component PMFs.

To examine the degree of agreement with the two-component PMFs over the entire evolution of the cluster, we perform a Kolmogorov-Smirnov (K-S) test comparing our simulated protostar populations to the 2CTC and 2CCA PMFs for all our data outputs. The results are shown in Figure 6.12. The Strong MHD run, after the initial transient phase, attains statistical consistency with both PMFs. This agreement appears to be steady with time,
6.3. RESULTS

Figure 6.11: Protostellar mass distributions in our simulations at \( M_* \approx 20 \, M_\odot \) compared to the theoretical PMFs in McKee & Offner (2010). The blue histograms are the simulation data. The green solid curve is the PMF associated with the TC model, the red solid curve the CA model, and the blue solid curve is the IS model. The green and red dotted curves are the 2CTC and 2CCA PMFs, respectively.

hovering around a K-S \( p \)-value of 0.1. The \( p \)-value for the Weak run is also relatively stable, although the agreement with the predicted PMFs is not as good. The Hydro distribution never reaches a steady \( p \)-value > \( 10^{-4} \) for any of the models we consider. Note that, as the PMFs predicted by the 2CTC and 2CCA models are quite similar, our simulated PMFs cannot be said to favor one accretion history model over the other.

6.3.5 Core Magnetic Field Structure

It is also useful to examine the geometry of the magnetic field in the cores formed in our simulations. From the Weak and Strong field MHD runs, we select the four most massive protostars at the time \( t = 0.4t_{\text{ff}} \). These range in mass from 0.3 to 1.8 \( M_\odot \) at this point in the calculation. In Figure 6.13, we show column density maps overlaid with density-weighted, projected magnetic field vectors showing the central 3000 AU surrounding each protostar. Figure 6.14 shows the same cores convolved with a 1000 AU Gaussian beam to ease comparison with observations. As in Krumholz et al. (2012b), we find that all the protostars are found near the centers of dense structures similar to the cores identified in dust thermal emission maps. The typical size of the cores, by inspection, is about 0.005 pc. We calculate the core mass by adding up all the mass (in gas and in the central sink particle) within a sphere of 0.005 pc radius around the protostar. The resulting core masses range from about 2 \( M_\odot \) to 6 \( M_\odot \).

In the strong run, we find that the magnetic field geometry always follows the “hourglass” structure commonly observed in regions of low-mass (Girart et al. 2006; Rao et al. 2009) and high-mass (Girart et al. 2009; Tang et al. 2009) star formation. We see examples of this in the Weak case (the left two panels of Figure 6.13) but we also see examples of highly
6.3. RESULTS

Figure 6.12: K-S test results comparing our simulated protostar populations to the 2CTC (solid) and 2CCA (dotted models. The y-axis shows the p-value returned by the test, and the x-axis shows time.

...disordered field geometry (the right two panels). This is in part due to the greater ability of the protostellar outflows to disrupt magnetic field lines in the Weak field case. Note that, because our wind model caps the wind velocity at 1/3 the Keplerian value, this tendency for the winds to disrupt the field lines is if anything underestimated in our simulations.

In general, dust polarization maps of star-forming cores tend to reflect magnetic fields that are quite well-ordered. If Crutcher et al. (2010) is correct, and cores with $\mu_\Phi \gtrsim 10$ are not rare, then chaotic magnetic field geometries like those shown in the bottom panels of 6.13 should not be rare, either. Crutcher et al. (2010) argues for a flat distribution of field strengths from approximately twice the median value down to very near 0 $\mu$G. If this is true, and the median field corresponds to $\mu_\Phi = 2$, then a flat distribution implies that $\approx 10\%$ of cores should have $\mu_\Phi \geq 10$. 

...
6.3. RESULTS

Figure 6.13: Top - zoomed in views of the four most massive protostars in the Strong field calculation at $t = 0.4 \, t_{ff}$. The window size has been set to 3000 AU. The color scale shows the logarithm of the column density, and the black arrows show the mass-weighted, plane-of-sky magnetic field vectors. The masses of the protostars have been indicated in each panel. Bottom - same, but for the Weak MHD run.

6.3.6 Turbulent Core Accretion

The turbulent core (TC) model of McKee & Tan (2002; 2003) is a generalization of the singular isothermal sphere (Shu 1977) that was developed in the context of massive stars. In this model, both the gravitationally bound clump of gas where a cluster of stars is forming and the cores that form individual stars and star systems are assumed to be supersonically turbulent. The predicted accretion rate in the TC model is:

$$\dot{m}_* = 1.2 \times 10^{-3} \left(\frac{m_{*,f}}{30 \, M_\odot}\right)^{3/4} \Sigma_{cl}^{3/4} \left(\frac{m_*}{m_{*,f}}\right)^{1/2} M_\odot \, \text{yr}^{-1};$$

(6.9)

where we have increased the normalization constant by a factor of 2.6 from the McKee & Tan (2003) value to account for subsonic contraction, as per Tan & McKee (2004). In the above equation, $\dot{m}_*$ is the instantaneous mass accretion rate onto the protostar, $m_*$ is the protostar’s current mass, $m_{*,f}$ is the final mass of the star once it is done accreting, and $\Sigma_{cl}$ is the surface density of the surrounding molecular clump, which we identify with the mean surface density in our simulations, $\Sigma_c = 1 \, \text{g cm}^{-2}$. 

Figure 6.14: Same as Figure 6.13, except convolved with a 1000 AU Gaussian beam. The size of the beam is indicated by the white circle.

Figure 6.15: Plots of $\dot{m}_*$ versus $m_*$ for the four most massive protostars in each run. The solid colored lines correspond to the individual protostars, while the black solid line is the average $\dot{m}_*$ of all four. The black dashed line is the theoretical prediction of the TC model (see text), while the black dotted line is the best $\dot{m}_* \propto m_*^{1/2}$ power-law fit to the data. These lines overlap almost exactly for the Hydro case. The accretion rates have been smoothed over a 500-year timescale for clarity.

The TC model includes the effects of magnetic fields in an approximate way. Its prediction
6.3. RESULTS

is that the effect of the field strength on the accretion rate should be quite modest. The value quoted above takes the magnetic field into account for a “typical” field strength, for which $\mathcal{M}_A$ is $\approx 1$. According to McKee & Tan (2003), the accretion rate in the field-free case would be only $\approx 6\%$ higher, assuming that $\alpha_{\text{vir}}$ is kept constant as the magnetic field strength is varied.

To test this, we select the four most massive stars (as these are the stars for which the TC model should be most applicable) at the end of our Hydro, Weak, and Strong runs, and plot $\dot{m}_* \times m_*$ over the accretion history of the protostars. We compare the simulation results to Equation 6.9. As we also hold $\alpha_{\text{vir}}$ constant across our runs, the TC model predicts that Equation 6.9 with the stated normalization should be quite accurate for all the runs, whatever the field strength. To estimate $m_{*,f}$, we take the sink particle mass at the most evolved time and add in all the gas remaining in the surrounding 1000 AU core, although this neglects material entrained by outflows and potential competition with nearby partners. The resulting average $m_{*,f}$ over the four most massive protostars is $2.0\, M_\odot$ for Hydro, $4.2\, M_\odot$ for Weak, and $2.6\, M_\odot$ for Strong.

The result is shown in Figure 6.15. Overall, our simulation results agree quite well with the TC model, both in terms of the predicted power-law slope, $\dot{m}_* \propto m_*^{1/2}$, and the predicted normalization. There is a noticeable reduction in the accretion rate relative to Equation 6.9 with magnetic field strength, but overall this effect is small compared to the size of the fluctuations in the simulation data. To characterize the error in the TC prediction, we fit a $\log \dot{m}_* = C + (1/2) \log m_*$ power-law to the mean accretion rates for the four-star sample in each run (the solid black curves in Figure 6.15). The resulting fits are compared against Equation (6.9) in Figure 6.15. We find that the normalization of the best-fit power-law, $C$, is lower than the prediction of Equation (6.9) by 12% in the Hydro run, 35% in the Weak MHD run, and 44% in the Strong MHD run. It is not surprising that the measured accretion rates in the magnetic cases differ somewhat from the prediction in Equation (6.9), since the latter is based on the assumptions that (1) the Alfvén Mach number is unity in the star-forming cores, whereas we set only the initial $\mathcal{M}_A$ in the entire turbulent box; and (2) the mass-to-flux ratio in the star-forming cores is similar to that estimated by Li & Shu (1997).

6.3.7 Mass Segregation

Much of our knowledge of the detailed inner structure of star clusters comes from studies of the Orion Nebula Cluster (ONC), which at $\sim 400$ pc is close enough to Earth to be easily observable. One interesting property of the ONC is that, with the exception of relatively massive stars like those that comprise the Trapezium, stars appear to be distributed throughout the cluster independently of mass. However, stars more massive than $\approx 3\, M_\odot$ appear to only be found in the center of the cluster, where the stellar surface density is highest (Huff & Stahler 2006). Allison et al. (2009b) also studied mass segregation in the ONC, finding a similar pattern, but with a threshold of $\approx 5\, M_\odot$ below which there was no significant segregation instead. Monceros R2 (Carpenter et al. 1997) and NGC 1983 (Sharma et al. 2007) show similar behavior. In this section, we investigate whether our simulations
Figure 6.16: Stellar density $n_\ast$ versus stellar mass $m_\ast$ for all the stars in our simulations. The blue circles correspond to the Strong run, the green plus symbols to Weak, and the red crosses to Hydro. The colored lines show the median stellar density in each run.

display this pattern of mass segregation as well.

Following Bressert et al. (2010), we define the stellar density around a sink particle out to the $N$th neighbor as:

$$ n_\ast(N) = \frac{N - 1}{(4/3)\pi r_N^3}, $$

where $r_N$ is the distance to the $N$th closest sink. The choice of $N$ is somewhat arbitrary; in what follows, we take $N = 9$ for all numerical results, and verify that our qualitative conclusions are not sensitive to this choice over the range $N = 4$ to $N = 20$. Likewise, we define the stellar surface density as:

$$ N_\ast(N) = \frac{N - 1}{\pi r_N^2}, $$
as this quantity is closer to what observers measure. For every star in our simulations, we compute \( n_* \) and \( N_* \) and plot these quantities versus the star mass \( m_* \) in Figures 6.16 and 6.17. There are two interesting features revealed in these plots. First, although the Strong and Weak runs have advanced to approximately the same time and have approximately the same number of stars, the stars in the Strong run tend to be found at higher stellar densities. The Strong run has 11 stars with \( n_* = 1 \times 10^5 \) stars pc\(^{-3} \) or greater, while neither of the other runs do. The mean and median \( n_* \) are higher in the Strong run as well (see Table 6.3). This trend is also visible in Figures 6.3 and 6.4, where the star formation appears more clustered in Strong than in Weak, in that roughly the same number of stars are confined to a smaller surface area.

The second is that, with a few exceptions, all the stars with \( m_* > 1 \, M_\odot \) are found in regions of relatively high stellar density. To make this more quantitative, we compute the mean and median values of \( n_* \) twice, once for stars with \( m_* < 1 \, M_\odot \), and again for stars with \( m_* > 1 \, M_\odot \). We do the same for the stellar mass density, \( \rho_* \), defined as the total stellar

\[ \text{stars / pc}^2 \]

\[ 10^3 \rightarrow 10^4 \]

\[ m_* [M_\odot] \]

\[ 10^{-1} \rightarrow 10^1 \]

\textit{Figure 6.17:} Same as Figure 6.16, but for the stellar surface density \( N_* \) instead.
### Table 6.3: Stellar surface density around >1 $M_\odot$ versus <1 $M_\odot$ stars

<table>
<thead>
<tr>
<th>Name</th>
<th>$n_{*&lt;50}$</th>
<th>$n_{*&lt;}$</th>
<th>$\bar{n}_*$</th>
<th>$n_{*&gt;}$</th>
<th>$\rho_{*&lt;50}$</th>
<th>$\rho_*$</th>
<th>$\rho_{*&gt;}$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>2.0</td>
<td>–</td>
<td>8.6</td>
<td>–</td>
<td>0.5</td>
<td>–</td>
<td>2.5</td>
<td>–</td>
</tr>
<tr>
<td>Weak</td>
<td>2.1</td>
<td>74.1</td>
<td>9.0</td>
<td>64.6</td>
<td>0.5</td>
<td>117.1</td>
<td>8.5</td>
<td>96.7</td>
</tr>
<tr>
<td>Strong</td>
<td>8.2</td>
<td>226.0</td>
<td>31.4</td>
<td>160.5</td>
<td>3.1</td>
<td>250.9</td>
<td>25.6</td>
<td>171.8</td>
</tr>
<tr>
<td>All</td>
<td>2.7</td>
<td>74.1</td>
<td>16.8</td>
<td>91.6</td>
<td>0.7</td>
<td>113.7</td>
<td>12.5</td>
<td>110.0</td>
</tr>
</tbody>
</table>

Note. — Col. 1: median number density for stars with $m_* < 1 M_\odot$ in units of 1000 stars per pc$^3$. Col. 2: same, but for stars with $m_* > 1 M_\odot$. Col. 3 and 4: same, but the mean instead of the median. Col. 5 through 8: same as Col. 1 through 4, but for the stellar mass density in units of 1000 $M_\odot$ per pc$^{-3}$ instead of the number density. Col. 9: $p$-value returned by a K-S test comparing the $m_* < 1 M_\odot$ and $m_* > 1 M_\odot$ distributions.

With the exception of the Hydro run, which only has 2 stars with $m_* > 1 M_\odot$, the mean $n_*$ for super-solar stars is larger than those of sub-solar stars by about a factor of $\approx 6$, while the median is larger by a factor of $\approx 30$. If we compare the stellar mass densities instead, the effect is even more pronounced. We have also indicated in Figures 6.16 and 6.17 the median value of $n_*$ for sub-solar stars in each run. In the Strong run, only one >1 $M_\odot$ star lies in a region where the stellar surface density is below the median for all the <1 $M_\odot$ stars in the run. In the Weak run, none do.

We also show in Table 6.3 the $p$-value associated with a two-sided Kolmogorov-Smirnov (K-S) test comparing the distributions of $n_*$ for <1 $M_\odot$ and >1 $M_\odot$ stars. For the Weak and Strong MHD cases, we can reject the null hypothesis that the two populations are drawn from the same underlying distribution at the 0.05% and 5% confidence levels, respectively. For the Hydro case, this number is not particularly meaningful, since there are only a couple of stars larger than 1 $M_\odot$. Finally, for the combined sample of all the stars formed in all three runs, the $p$-value that <1 $M_\odot$ stars and >1 $M_\odot$ stars have the same distribution is only 2.6 $\times 10^{-6}$.

Note that the same is not true if we repeat this procedure with a different mass threshold. For example, if we compare the distribution of $n_*$ around stars with 0.05 $M_\odot$ < $m_*$ < 0.4 $M_\odot$ to that of stars with 0.4 $M_\odot$ < $m_*$ < 1.0 $M_\odot$ (the threshold of 0.4$M_\odot$ was picked because it divides the stars in the mass range 0.05 to 1.0 $M_\odot$ into two groups of approximately equal mass), we get K-S $p$-values of 0.30, 0.59, and 0.56, for the Hydro, Weak, and Strong runs, respectively. In other words, the data for stars with 0.05 $M_\odot$ < $m_*$ < 0.4 $M_\odot$ are consistent with being drawn from the same underlying distribution as those with 0.4 $M_\odot$ < $m_*$ < 1.0 $M_\odot$. There appears to be a real difference in our simulation between <1 $M_\odot$ stars, which
are found at both low and high stellar density independent of mass, and > 1 $M_\odot$ stars, which are much more likely to be found at high $n_*$.

Our threshold value of 1 $M_\odot$ is lower than the threshold for the ONC by about a factor of 3. It is perhaps not surprising that we do not agree with the ONC value quantitively, since the most massive star in our simulations is $\approx 5.2$ $M_\odot$, while $\theta^1$ Orionis C, the most massive member of the Trapezium, is $\approx 37$ $M_\odot$ (Kraus et al. 2009). Nonetheless, we do reproduce the fact that beyond some threshold mass, stars are much more likely to form in the center of clusters. Interestingly, this same basic pattern has been observed for protostellar cores as well. In a study of dense cores in the $\rho$ Ophiuchi cloud complex, Stanke et al. (2006) found no mass segregation for starless cores with masses $\lesssim 1$ $M_\odot$, but the most massive cores were only found in the dense, inner region. Finally, although N-body processing can produce the mass segregation observed in the ONC on timescales comparable to the cluster age of a few Myr (e.g. Allison et al. 2009a), insufficient time (only 56,000 kyr) has elapsed for this effect to be important here. The mass segregation in our simulations is primordial, rather than dynamical.

6.3.8 Multiplicity

Finally, we consider the multiplicity of the stars formed in our simulations. To divide our star particles into gravitationally bound systems, we use the algorithm of Bate (2009a) (see also Bate 2012; Krumholz et al. 2012b). We start with a list of all the stars in each simulation. For each pair, we compute the total center-of-mass frame orbital energy. We then replace the most bound pair with a single object that has the total mass, net momentum, and center-of-mass position of its constituent stars. We repeat this procedure until there are no more bound pairs. The only exception is that we do not create systems with more than four stars - if combining the most bound pair of objects would create a system with 5 or more stars, we combine the next most bound pair instead. At the end of this process, there are no more pairs that can be combined, either because they are not gravitationally bound or because combining them would result in more than 4 stars in a system.

We then compute both the multiplicity fraction MF (Hubber & Whitworth 2005; Bate 2009a; Krumholz et al. 2012b):

$$\text{MF} = \frac{B + T + Q}{S + B + T + Q},$$

(6.12)

where $S$, $B$, $T$, and $Q$ are the numbers of single, binary, triple, and quadruple star systems, and the companion fraction (e.g. Haisch et al. 2004):

$$\text{CF} = \frac{B + 2T + 3Q}{S + B + T + Q},$$

(6.13)

which is the number of companions per system. The CF is often reported in observations, but the MF has the desirable property that it does not change if a high-order system is re-classified as a binary or vice-versa.

\footnote{Our qualitative results are essentially the same if we choose a slightly different limit.}
6.3. RESULTS

The results of this calculation for our three runs are shown in Table 6.4. We find that there is a clear trend towards more multiplicity with stronger magnetic fields. This likely related to the phenomenon discussed in 6.3.1 and 6.3.7. Stellar clustering is more dense in the Strong run than the others, with roughly the same number of stars packed into a smaller volume, so the availability of potential partners tends to be greater in run Strong.

At this point, we mention a few caveats of this analysis. First, in our simulations, we are only marginally able to resolve binaries closer than our sink accretion radius of $r_{\text{sink}} = 4\Delta x_f \approx 184$ AU. Due to the way our sink particle algorithm works, binaries are unable to form within this distance. Likewise, binaries where one of the partners forms outside of this distance but falls in before exceeding the minimum merger mass of $0.05M_\odot$ would be counted as a single star in these simulations. Stars that form outside $r_{\text{sink}}$ and exceed this threshold before falling in are able to form binary systems closer than $r_{\text{sink}}$. However, there is an additional problem, which is that sink-sink gravitational forces are softened on a scale of $0.25\Delta x_f \approx 11.5$ AU and gas-sink gravitational forces on a scale of $\Delta x_f \approx 46$ AU. We thus have only a limited ability to resolve binaries with separations $\lesssim 200$ AU.

Most main-sequence, solar-type stars are members of binaries Duquennoy & Mayor (1991), and young stellar objects such as T Tauri stars (Duquennoy & Mayor 1991; Patience et al. 2002) and Class I sources (Haisch et al. 2004; Duchêne et al. 2007; Connelley et al. 2008; Duchêne & Kraus 2013) have an even greater tendency to be found in bound multiple systems. The sink particles in our simulations are all still embedded, but have begun to heat up their immediate surroundings to temperatures high enough to radiate in the infrared, and thus are most analogous to Class I sources. Observations of multiplicity among Class I objects have difficulty detecting both very tight and very wide binaries, and thus generally report a restricted companion fraction - that is, the CF counting only companions within some range of projected separations. For instance, Connelley et al. (2008) found a CF of 0.43 for

<table>
<thead>
<tr>
<th>Name</th>
<th>$S$</th>
<th>$B$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$MF$</th>
<th>$CF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydro</td>
<td>139</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>Weak</td>
<td>65</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>0.18</td>
<td>0.37</td>
</tr>
<tr>
<td>Strong</td>
<td>66</td>
<td>9</td>
<td>2</td>
<td>8</td>
<td>0.22</td>
<td>0.44</td>
</tr>
<tr>
<td>Hydro$_r$</td>
<td>138</td>
<td>12</td>
<td>3</td>
<td>1</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Weak$_r$</td>
<td>67</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td>Strong$_r$</td>
<td>67</td>
<td>11</td>
<td>5</td>
<td>2</td>
<td>0.21</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note. — Rows 1-3 - all companions, regardless of separation. Rows 4-6 - not counting companions with separations $< 200$ or $> 4500$ AU.
Class I sources in nearby star-forming regions within the range of 100 to 4500 AU, while the high-resolution observations of Duchêne et al. (2007) found CF = 0.47 for 14 to 1400 AU. To compare against these observations, we therefore compute the restricted companion fractions in our simulations in the range 200 - 4500 AU. The lower limit of 200 AU restricts the analysis to binaries that are resolved at our grid resolution. The results of this calculation are also shown in Table 6.4.

The main effect of restricting our analysis to companions in the range of 200 - 4500 AU is to re-classify triple and quadruple star systems as binaries. This is because the higher-order multiples in our simulations are generally hierarchical, with, say, a wide companion orbiting a tighter binary system. Looking for companions only between 200 - 4500 AU misses many of these partners. This effect makes no difference for the MF, but the can change the CF significantly, particularly in the Weak and Strong runs, which without restriction had many triple and quadruple systems. Discounting the companions in the range 100 - 200 AU, Connelley et al. (2008) found a CF of 0.33 in the range 200 - 4500 AU. This is quite close to our Strong run, in which the CF restricted to the same range is 0.32.

Additionally, like Bate (2012) and Krumholz et al. (2012b), the multiplicity fraction in our calculations is a strong function of primary mass, \( m_p \). If we consider only systems in which the primary star exceeds 1 \( M_\odot \), we find that there are 3 such systems in run Hydro, 4 in Weak, and 5 in Strong. Only one of these systems, however, is single. This suggests that the trend for higher multiplicity at higher primary masses, which is well-observed for main-sequence stars, may already be in place during the Class I phase. This is likely related to the phenomenon discussed in section 6.3.7: that stars more massive than \( \approx 1 \ M_\odot \) are much more likely than average to form in regions of high stellar density. Thus, more massive stars tend to form in regions where there are more potential partners for forming binary and other higher multiple systems. Another potential mechanism behind the strong mass dependence of the multiplicity fraction - that more massive stars have higher accretion rates and thus are more likely to be subject to disk fragmentation (Kratter et al. 2010) - is unlikely to be responsible for the trend in our simulations, simply because at a resolution \( \Delta x_f \approx 46 \) AU, we are not able to resolve any disk physics.

### 6.4 Discussion

We find that the magnetic field influences most aspects of cluster formation and early evolution, including the star formation rate, the degree of fragmentation, the median fragment mass, the multiplicity fraction, and the typical stellar density in the cluster. However, the magnitude of these effects are rather modest at \( \mu_F = 2 \), differing from the pure Hydro case at roughly the factor of 2 level. While, in general, our magnetized runs, particularly the \( \mu_F = 2 \) run, compare favorably with observations compared to our non-magnetized run, the differences are not dramatic.

In 6.3.3, we compared the properties of our simulated protostars to the Chabrier (2005) stellar IMF, finding that while our simulations agreed well with the log-normal functional
form, the characteristic masses were lower than the Chabrier (2005) value of $m_c = 0.2 \, M_\odot$ by a factor of 2-4, depending on the magnetic strength. This is to be expected, since even at late times our population of sink particles includes newly-formed objects that are not close to their final masses, and most of the more evolved objects are still accreting significantly. Krumholz et al. (2012b), which used initial conditions similar to our Hydro run but with mixed solenoidal/compressive forcing, found good agreement with the Chabrier (2005) system IMF ($m_c = 0.25$) and the Da Rio et al. (2012) IMF for the Orion Nebula Cluster ($m_c = 0.35 \, M_\odot$). The typical protostar in Krumholz et al. (2012b) was thus significantly larger than the typical protostar in this work.

This difference is likely to due to the varying degree of effectiveness of radiative feedback in our two simulations. Because the star formation rate was higher in Krumholz et al. (2012b), there was considerably more protostellar heating, which pushed the characteristic fragmentation scale to higher masses. If we compare our Figures 6.5 and 6.6 against Figure 10 of Krumholz et al. (2012b), we see that there is significantly less gas that has been heated above the background temperature in our Hydro run than in run TuW of Krumholz et al. (2012b). Quantitatively, Krumholz et al. (2012b) report that 7% of the gas in run TuW is hotter than 50 K at the latest time available. The corresponding values for the most evolved stage of our simulations shown in Figure 6.5 are 0.3% 0.3%, and 0.1% for Hydro, Weak, and Strong, respectively. This difference in protostellar heating made the particle masses in Krumholz et al. (2012b) agree well with the IMF, even though the majority of the sinks were still accreting. With the lower star formation rates in this work, our median sink particle mass drops to something more characteristic of a protostellar mass function, rather than an IMF (see Sec. 6.3.4).

However, our simulations confirm the result of Krumholz et al. (2012b) that when turbulent initial conditions are treated self-consistently, the population of sink particles can approach a steady-state mass distribution (Figure 6.12). The “overheating problem" identified by Krumholz (2011) for simulations in which star formation is too rapid does not occur here, and the characteristic stellar mass is relatively stable with time.

The first simulations of star cluster formation in turbulent molecular clouds to include both magnetic fields and radiative feedback are the smoothed-particle hydrodynamics (SPH) simulations of Price & Bate (2009). Price & Bate (2009) found that the median protostellar mass tended to decrease with increasing magnetic field strength in their radiative calculations, the opposite of the trend reported here, although they cautioned that larger simulations that form more sink particles were necessary before drawing firm conclusions. One potential reason for the difference between our result and Price & Bate (2009) is that the star formation in Price & Bate (2009) occurs in the context of a globally collapsing structure, in which stars form in the center and accrete in-falling gas before getting ejected by N-body interactions. Because this rate of infall is lower with stronger $B$ fields, the typical particle accreted less material before being ejected. In contrast, in our simulations there is no global infall. The typical star forms from a core that results from turbulent filament fragmentation, and the magnetic field increases the typical fragment mass.
6.5 Conclusions

We have presented a set of simulations of star-forming clouds designed to investigate the effects of varying the magnetic field strength on the formation of star clusters. We find that the magnetic field strength influences cluster formation in several ways. First, the magnetic field lowers the star formation rate by a factor of \( \approx 2.4 \) over the range \( \mu_\Phi = \infty \) to \( \mu_\Phi = 2 \), in good agreement with previous studies (Price & Bate 2009; Padoan & Nordlund 2011; Federrath & Klessen 2012). Second, it also suppresses fragmentation, reducing the number of sink particles formed in our simulations by about a factor of \( \approx 2 \) over the same range of \( \mu_\Phi \). This too is in good agreement with previous work (Hennebelle et al. 2011; Federrath & Klessen 2012).

The magnetic field also tends to increase the median sink particle mass, again by a factor of about 2.4 over the range of \( \mu_\Phi = \infty \) to \( \mu_\Phi = 2 \). Even at \( \mu_\Phi = 2 \), however, the median sink mass is still lower than value for the Chabrier (2005) IMF by about 40%, likely because our sinks are still accreting at the time we stop our calculations. On the other hand, our \( \mu_\Phi = 2 \) calculation is statistically consistent with both the two-component turbulent core and the two-component competitive accretion protostellar mass functions from McKee & Offner (2010) and Offner & McKee (2011). In contrast, the pure Hydro simulation does not agree well with either the Chabrier (2005) IMF or any of the PMFs in McKee & Offner (2010).

We also find that the accretion rates onto the most massive stars in our simulations (about \( \sim 2 - 5 \, M_\odot \)) are well-described by the TC model. We have confirmed that these accretion rates depend only weakly on the magnetic field strength, as predicted by McKee & Tan (2003).

We examined the magnetic field geometry in our simulations at the \( \sim 0.005 \, \text{pc} \) scale. In the Strong field case, the field geometry agrees well with observations of low-mass (Girart et al. 2006; Rao et al. 2009) and high-mass (Girart et al. 2009; Tang et al. 2009) star-forming cores, but the magnetic field lines are often quite disordered in the Weak run. If, as suggested by Crutcher et al. (2010), \( \approx 10\% \) of cores have \( \mu_\Phi \gtrsim 10 \), then we would expect a similar fraction of observed cores to reveal disordered fields at the \( \sim 3000 \, \text{AU} \) scale.

Many of the stars in our simulations are members of bound multiple systems, and our Strong field run in particular agrees well with observations of multiplicity in Class I sources over the range of 200 - 4500 AU (Connelley et al. 2008). We find a trend towards increased multiplicity with magnetic field strength that is likely explained by the fact that star formation is more clustered in the Strong run than others, since at \( \mu_\Phi = 2 \) much of the volume is prevented from collapsing gravitationally. Our simulations also reproduce the fact, observed in main-sequence stars, that more massive stars are more likely to be found in multiple systems than their lower-mass counterparts.

Finally, all our simulations exhibit a form of primordial mass segregation like that observed in the ONC, in which only the most massive stars are more likely than average to be found in regions of high stellar density.
Acknowledgments

Support for this work was provided by NASA through ATP grant NNX13AB84G (R.I.K., M.R.K. and C.F.M.) and a Chandra Space Telescope grant (M.R.K.); the NSF through grants AST-0908553 and NSF12-11729 (A.T.M., R.I.K. and C.F.M.) and grant CAREER-0955300 (M.R.K.); an Alfred P. Sloan Fellowship (M.R.K); and the US Department of Energy at the Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344 (A.J.C. and R.I.K.) and grant LLNL-B602360 (A.T.M.). Supercomputing support was provided by NASA through a grant from the ATFP. We have used the YT toolkit (Turk et al. 2011) for data analysis and plotting.
Bibliography

Berger, M. J., & Colella, P. 1989, Journal of Computational Physics, 82, 64
Cesaroni, R., Galli, D., Lodato, G., Walmsley, C. M., & Zhang, Q. 2007, Protostars and Planets V, 197
Dalgarno, A. 2006, Proceedings of the National Academy of Science, 103, 12269
BIBLIOGRAPHY 132

Federrath, C., Roman-Duval, J., Klessen, R. S., Schmidt, W., & Mac Low, M.-M. 2010b, A&A, 512, A81
Jones, E., Oliphant, T., Peterson, P., et al. 2001–, SciPy: Open source scientific tools for Python
Klein, R. I. 1999, Journal of Computational and Applied Mathematics, 109, 123
—. 2008, Nature, 451, 1082
 Landau, L. D., & Lifshitz, E. M. 1959, Fluid mechanics
 Lee, A., Cunningham, A., McKee, C., & Klein, R. 2013, in prep
Mihalas, D., & Klein, R. I. 1982, Journal of Computational Physics, 46, 97
Miyoshi, T., & Kusano, K. 2005, Journal of Computational Physics, 208, 315