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BEAM-BEAM EFFECTS*

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Beam-Beam Effects

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Abstract. A review of the beam-beam effects in storage rings is given with emphasis on experimental results and new simulation techniques.

INTRODUCTION

The term beam-beam effects is usually used to designate different phenomena associated with interactions of counter-rotating beams in storage rings. Typically, we speak about beam-beam effects when such interactions lead to an increase of the beam core size or to a reduction of the beam lifetime or to a growth of particle's population in the beam halo and a correspondent increase of the background. Although observations of beam-beam effects are very similar in most of storage rings, it is very likely that every particular case is largely unique and machine-dependent. This constitutes one of the problems in studying the beam-beam effects, because the experimental results are often obtained without characterizing a machine at the time of the experiment. Such machine parameters as a dynamic aperture, tune dependencies on amplitude of particle oscillations and energy, betatron phase advance between the interaction points and some others are not well known, thus making later analysis uncertain.

We begin our discussion with demonstrations that beam-beam effects are closely related to non linear resonances. Then, we will show that a non linearity of the space charge field is responsible for the excitation of these resonances. After that, we will consider how beam-beam effects could be intensified by machine imperfections. Then, we will discuss a leading mechanism for the formation of the beam halo and will describe a new technique for beam tails and lifetime simulations. We will finish with a brief discussion of the coherent beam-beam effects.

BEAM-BEAM NON LINEAR RESONANCES

Non linear resonances $\ell Q_x + m Q_y + n Q_5 = k$ (where $Q_x, Q_y$ are the betatron tunes and $Q_5$ is the synchrotron tune and $\ell, m, n, k$ are integer numbers) play a cardinal role in beam-beam effects. Lots of experiments support this statement. One example is given in the Fig.1, where we see two sets of measurements of the
vertical beam size and rates of particles losses [1]. All measurements were done in the same area of a tune plane, but with different beam currents. A tune scan with low currents (Fig.1a) shows only machine imperfection resonances. A tune scan with moderate currents (below the beam-beam limit) shows several additional resonances, which are driven by beam-beam effects (Fig.1b). Notice that the resonances which are visible in a picture of particle losses are different from the resonances which are visible in a picture of the vertical beam size. This means that such effects as beam core blowup and reduction of the lifetime are not obviously correlated. One can also notice that almost all resonances in Fig.1b are odd ones. Recall for ideal head-on collisions, we must expect only even resonances. In fact, a horizontal separation of electron and positron orbits at the interaction point (IP) on a level of $0.1\sigma_x$, where $\sigma_x$ is a horizontal beam size at the IP, was later discovered in these experiments.

Protons and antiprotons are more sensitive to the beam-beam effects. For example, an operation of SPS in collider mode showed that even 16-th order resonances could notably affect the lifetime of protons when the operation point is close to them [2] (see Fig.2).

What is the primary cause of the beam-beam resonances? The experiments show that it is a non linearity of a space charge field. For example, this fact is demonstrated in experiments with beams of unequal emittances performed at the SPS at CERN. When the emittance of the antiproton beam was made smaller than that of the proton beam, the less intense antiproton beam affected strongly the life
time of the proton beam reducing it to 69 sec [3]. With equal emittances the lifetime of protons was 600 sec. A reduction of the lifetime could be explained by the fact that in a case of unequal emittances there are more protons sitting in a region of strong non linearity of the space charge field (see Fig.3).

Similar experiments performed at HERA at DESY showed that the lifetime of protons fell from 100 hours to 0.5 hour when the electron beam size at the IP was reduced to about 1/4 of the proton beam size [4].

Another experiment was done on VEPP-4 in Novosibirsk when a vertical beam size of a “weak” (less intense) positron beam was measured as a function of the separation of electron and positron orbits at the IP [5]. In Fig.4 one can see that maximum blow-up of the vertical beam size took place when core of the positron beam was in one of two regions of strong non linearity of the space charge field of the electron beam.
Figure 4. Positron beam vertical size versus the vertical orbit separation at the IP. Maximum blow-up take place when the core of the positron beam is sitting in one of two regions of strongest non linearity of the space charge field of the electron beam.

A spurious separation of the beam orbits at the IP is one example of an imperfection which violates a symmetry of the beam-beam interaction. Potentially, it leads to excitation of odd resonances as we already saw in the beginning of this paragraph. A symmetry of beam-beam interaction could also be broken by a spurious dispersion at the IP and by any asymmetry in a storage ring leading to the different betatron phase advances between the IP's (if there are more than one IP in the ring). The sensitivity of beam-beam effects to the betatron phase advance errors was studied on VEPP-2M in Novosibirsk [6]. In a special experiment an ultimate beam-beam parameter was measured as a function of a difference of betatron phase advances between two IP's (see Fig.5).

As with resonance excitation, the non linearity of the space charge field is responsible for a detuning (dependence of betatron tunes from amplitudes of particle oscillations). Detuning, if it is not too large, is usually a positive factor, since it limits the resonance growth of amplitudes (a particle that is in resonance at one amplitude moves out of resonance when its amplitude changes). However, machine non linearity also creates a detuning which could potentially cancel the detuning due to the space charge field. If this happens on resonance, then this resonance could easily drive particles to large amplitudes.

The experimental study of the effect of a machine cubic non linearity on a strength of beam-beam non linear resonances was done on the VEPP-4 in Novosibirsk [7]. In one example the loss rates of positrons were measured in the two tune scans across the resonance $7Q_x = 60$, which appeared due to the small orbit separation at the IP. A tune scan was made with positive and then with
negative cubic machine non linearity. The result is shown in Figure 6, where we see a big difference between these two cases.

![Figure 6 Positron loss rate and specific luminosity, Lsp, in a tune scan across the resonance Qx = 60. The magnitude of the machine related detuning is a) δQx = 0.00016 at A_x / σ_x = 1 and b) δQx = -0.00016 at A_x / σ_x = 1.]

**RESONANCE STREAMING AND PHASE CONVECTION**

A plausible condition for reaching large amplitudes is particle transport along the resonance [8]. It is illustrated in the Figure 7. Resonances present themselves in the phase space as long "tubes". Figure 7 shows a drawing of a projection of one such "tube" into the amplitude plane. A particle, trapped inside an isolated resonance "tube", performs small amplitude libration. The direction of this libration in the amplitude plane is given by the resonance vector, which, in general, makes some angle ϕ with a tangent line to the resonance curve. In the presence of external noise and/or damping the resonance "tubes" of sum resonances (where m and ℓ are positive) provide easy routes from the beam core to large amplitudes. For example, if due to damping the particle moves from a to b, then a center of libration moves from A to B, i.e. the average particle motion over long time consists in a motion up along the resonance line.

![Figure 7. A projection of the resonance "tube" into the amplitude plane. Particle displacement from a to b results in displacement of the libration center from A to B.]

![Figure 8. Two trajectories demonstrating the effect of resonance streaming.]

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A computer simulation of the resonance streaming was performed in a model including a non linear map and damping. Figure 8 shows trajectories of two particles. The first particle was launched on resonance and it moved up against damping. The second particle was launched near the resonance and it moved to the beam center in accord with damping.

Thus, particle density distribution far from the beam core is strongly affected by the resonance transport. When damping and noise are weaker than the resonance, then particles stream to large amplitudes. They fall off the resonance when noise and damping become dominant. After leaving the resonance particles can damp or can be attracted by another resonance and stream to even higher amplitudes. Eventually, due to diffusion the damped particles will again reach the resonance region and stream to large amplitudes. Thus, moving back and forth in phase space particles form closed fluxes. This phenomenon is called phase convection [9].

BEAM-BEAM SIMULATIONS

A substantial progress in a simulation of the beam halo and the calculation of lifetime in electron-positron colliders has been made during last few years. Currently, there are two beam-beam programs [10,11], which are able to determine the beam lifetime limit on a level of ten hours with a statistical confidence of a few percent by tracking only about 10 millions particle-turns. Along with these calculations, the programs also determine the particle density distribution.

Both programs emerged from the same ideas [12] and base their algorithms on the presence of a random component (such as quantum fluctuations) in the particle motion. Here we reproduce a description of the algorithm realized in [10]. In a first step the program defines the particle density distribution in the vicinity of a beam center by means of a conventional tracking of one particle with zero initial coordinates. The number of particle-turns used for this first step is equivalent to several hundred damping times. Then, contour lines of equal density of the density distribution function in the amplitude plane are defined and one of these lines is chosen as a special boundary. This line divides the amplitude plane into internal and external regions (see Figure 9a). Then tracking continues, but now when the particle crosses the boundary, all of its six phase space coordinates are stored. These crossing points, called outflights in [10], form a halo above the border line shown in the Figure 9a. After accumulation about 50 thousand outflights the program proceeds to the next step. In this step a new border line at larger amplitudes is chosen and tracking begins from one of the outflight points. Now the particle motion below the first border line (region I in Figure 9b) is not tracked. If the particle moves into that region, then tracking of this particle is interrupted and tracking of a new particle is started from a randomly chosen outflight point. During this step the program defines more precisely the particle density distribution in the area above first boundary and accumulates statistics of the outflight through the second border line. This step also lasts several hundred damping times. At the end of this step regions I and II merge into one enlarged closed region and a new border line at larger amplitudes is chosen. Then the process is repeated. Thus, with each new step the region where particle motion is not tracked is increased and most of the CPU time is spent for tracking in new
regions where statistics is poor and not in the regions where it is already sufficient.

The only source of error in the described algorithm is insufficient statistics of the outflight points which must correctly reflect the particle phase space density near the border lines. However, the algorithm has the capability to control the accuracy in parallel with the calculation. This is illustrated in Figure 10. Here we see a contour plot of a particle density distribution function obtained in a typical example of beam-beam calculations. Each contour line is shown three times with solid, dashed and dotted lines. (Not all three are seen everywhere because sometimes the lines overlap). These three lines are the contour lines obtained in three sequential steps of the algorithm and they correspond to increased statistical accuracy in the line's definition. Bold solid lines 5, 10, 15 are border lines. Remarkably, contour lines obtained on the basis of the accumulated statistics of the outflight points are in a good agreement with the same contour lines obtained in previous steps. Thus, we can conclude that statistics of the outflight points is sufficient.

![Figure 9](image9.png)

**Figure 9.** Drawings showing the algorithm of fast tracking technique. a) The outflight points form a halo above the border line in the amplitude plane. In the next step these points will be used as the initial coordinates for tracking. b) Two border lines and several segments of particle trajectories. Region I is the closed region, region II is the internal region, region II is the external region.

![Figure 10](image10.png)

**Figure 10.** A contour plot of a particle density distribution function obtained in one example of beam-beam calculations [10]. Between two lines the particle density drops on a factor $e$. Bold lines 5, 10, 15 are border lines. Solid, dashed and dotted lines show the same contour lines during three sequential steps. (Not all three are seen everywhere because sometimes the lines overlap)
The program [10] was used for a numerical analysis of the experimental results shown in the Figure 6a. Simulations were done for conditions closely resembling the real experiment. Horizontal and vertical beam-beam parameters were chosen as $\xi_x=0.015$ and $\xi_y=0.045$, beam orbits were separated at the IP in the horizontal plane by $0.3 \sigma_x$ and non linear tune shifts due to a cubic machine non linearity were chosen as $\Delta Q_x=0.015$ at $10 \sigma_x$ and $\Delta Q_y=7.5 \times 10^{-5}$ at $10 \sigma_y$. Four calculations were made for four horizontal betatron tunes chosen along the horizontal line in the tune plane across the resonance $7Q_x=60$ at a vertical betatron tune $Q_y=9.613$. Four contour plots of particle density distribution function were obtained (see Figure 11). One can notice a correlation between the simulation and the experiment: larger horizontal tails correspond to the tunes with larger loss rates.

Figure 11. Contour plots of particle density distribution function obtained in a simulation of beam-beam effects with experimental conditions described in the text. Pictures from a) to d) are made for different horizontal betatron tunes specified in the pictures and for vertical betatron tune $9.613$. Arrows show the locations where the non linear resonances disturb the distribution function. The numbers above the arrows stand for $\ell, m, n$. The contour plot in picture e) was obtained for the same conditions as the contour plot in picture b) except for the orbit separation, which was zero in a case of the picture e).
These programs [10,11] perform lifetime simulations for a "weak-strong" regime where it is assumed that particle density distributions in one beam remains unperturbed. At the expense of the increased time of calculations both algorithms could be upgraded to a "strong-strong" regime when both beams are affected by the beam-beam interaction. Although this is a quite desirable option, it is, nevertheless, not really important for the lifetime calculations. In fact, the particle's behavior in a halo does not depend very much from details of particle density distribution in the beam core. For example, beam tail and lifetime simulations were made when horizontal beam size of a "strong" beam was modulated by ±20% at a frequency of twice of the horizontal betatron tune and no dramatic changes in the tail distribution and lifetime were found [14]. This result was obtained by using the program [11].

However, the "strong-strong" regime can include new phenomena which are totally absent in a "weak-strong" regime: this new phenomena are, of course, coherent beam-beam effects.

COHERENT BEAM-BEAM EFFECTS

Coherent beam-beam effects are collective beam instabilities emphasized via the positive feedback loop of beam-beam interactions. The strong non linearity of the space charge field provides favorable conditions for a stable excitation of high order modes of a collective motion. These modes are especially pronounced when the coherent betatron tunes are close to one of the resonances \( \ell Q_x + m Q_y = k \) [14] thus limiting the space for the choice of the operating point in the tune plane. Hopefully, the same non linearity works also as a stabilizing factor forcing fast decoherence of a collective particle motion [15]. However, in the special case of collisions of four space-charge compensated beams [16], this decoherence becomes weak and coherent beam-beam effects are especially strong [17,18].

Modeling of coherent beam-beam effects is difficult, since the beams do not preserve their shapes in coherent oscillations. It means that on every turn the evaluation of a beam space charge field on the basis of the actual spatial particle distribution is required. Such attempts were recently made for the case of nominally round beams (equal beta-functions at the IP and equal emittances in the two transverse dimensions), although the beams were not constrained to remain round [19]. Figure 12 shows one example of simulations performed at betatron tunes just below the six-order resonance.

![Figure 12. Beam sizes as function of turn number. There are period-3 anticorrelated coherent oscillations. On a given turn, one beam has a dense core while the other is hollow. On the next turn the beams have comparable sizes. On the third turn the first beam is hollow and the second is dense, and so on.](image-url)
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Dedication. During his life Jeffrey Tennyson did a lot to clarify the mystery of the beam-beam effects. Sadly, he passed away at a very early age at the prime of his career. I would like to dedicate this review to him.

REFERENCES
