Title
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Journal
Production and Operations Management, 24(8)

ISSN
1059-1478

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Publication Date
2015-08-01

DOI
10.1111/poms.12327

Peer reviewed

Competition and Coordination in a Two-Channel Supply Chain

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November 2014

Abstract

We study competition and coordination in a supply chain in which a single supplier both operates a direct channel and sells its product through multiple differentiated retailers competing in quantities. We study analytically the supply chain with symmetric retailers and find that the supplier generally prefers to have as many retailers as possible in the market, even if the retailers' equilibrium retail price is lower than that of the supplier, and even if the number of retailers and their cost or market advantage prevent sales through the direct channel. We find that the two-channel supply chain may be subject to inefficiencies not present in the single-channel supply chain and study coordination. We show that several contracts known to coordinate a single-channel supply chain do not coordinate the two-channel supply chain; thus we propose a linear quantity discount contract and demonstrate its ability to perfectly coordinate the two-channel supply chain with symmetric retailers. We provide some analytical results for the supply chain with asymmetric retailers and propose an efficient solution approach for finding the equilibrium. In a numerical study of the asymmetric chain we find that the supplier still benefits from having more retailers in the market and that linear quantity discount contracts can mitigate supply chain inefficiency, though they no longer achieve perfect coordination.

Keywords: Supply Chain Management, Game Theory, Dual Channel, Incentives and Contracting

1 Introduction

1.1 Motivation

The increasing consolidation of consumer goods retail into "superstores" has given these large retailers significant power to drive consumer choices. For example, close to 6% of U.S. retailing

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happens through a single retailer, Walmart, in both its online and physical stores (Walmart, 2012), and in addition to Walmart and other general superstores (Target, Meijer, etc.), the last two decades have also seen the growth of specialty superstores, such as Lowe's and Home Depot in the home improvement sector, Best Buy in consumer electronics, and Barnes & Noble in book-selling.

This presents a dilemma for suppliers that control their own retail channels. Some, like Nike and Apple, have exclusive physical stores that sell their products directly to end consumers in addition to a presence in superstores. Others take advantage of e-commerce to reach customers in a wide geographical region with comparatively low investment. In either case, these suppliers that extend into the retail space find themselves competing with the retailers to whom they sell products for the dollars of the end consumer. In some cases, the supplier may even find its price being undercut.

Take, for example, books published by Random House. While books may be purchased directly from Random House's website, the site also directs consumers to the sites of other retailers carrying the same title, many at lower prices. An August 2011 search for the best-selling title Ready Player One shows a price of \$24.00 from Random House, but only \$13.66 from Barnes & Noble, and \$14.67 from Walmart. It is difficult to believe that a consumer would choose to pay an additional \$10.33 to buy the book from Random House, and yet, Random House maintains the availability of this title through its direct retail channel, while providing it to other retailers who sell the same book at a lower price.

In this paper, we examine the motivations of suppliers like Random House that sell to one or more downstream retailers with which they also compete via a direct channel. We consider the supplier to be the exclusive source of a product, and allow for a degree of differentiation at the retail level, representative of differences in both the product itself and the consumer experience in purchasing through one retail firm or the other. Though we focus primarily on suppliers with a retail presence, our model equally applies to earlier stages in the supply chain, such as a component manufacturer and assembler that also sells components to other assemblers.

We first analyze the supply chain with symmetric retailers and find what conditions make it favorable for a supplier to sell to its "competitors" and the pricing and quantity strategies at equilibrium. In addition, we find that common contracts known to coordinate a single-channel supply chain fail to coordinate a supply chain with two channels in operation, and suggest a linear quantity discount contract as an effective alternative. We then consider the supply chain with asymmetric retailers, obtaining some analytical results and providing an efficient solution approach. We then analyze it numerically and find that the qualitative insights developed with symmetric retailers hold under asymmetry. All proofs are found in the Electronic Companion.

1.2 Review of Literature

Since Spengler (1950) introduced the idea of “double marginalization,” many have studied how vertical integration affects a supply chain’s quantities, prices, and profits (Perry, 1989). In the last two decades, the number of suppliers vertically integrating their supply chain by creating their own “direct” channels to reach end customers has increased, and with it, the volume of literature on the topic (Tsay and Agrawal, 2004b), a trend largely due to the ease with which a supplier may compete online (Peterson et al., 1997).

Chiang et al. (2003) and Dumrongsiri et al. (2008) assume that the direct channel is at an inherent disadvantage to the retail channel with respect to customer preference, exclusive of price, and find that the direct channel can be used by the supplier as a mechanism for influencing the retailer’s price. Bernstein et al. (2009) similarly assume that nothing will be sold through the direct channel when its price is higher than that of the indirect channel, but conclude that the direct channel may instead be used for branding efforts. Chen et al. (2008) assume a common heterogenous price for the two channels find that the dual channel structure can be used to segment the market and manage inventory risk when channels compete in service. Cattani et al. (2006) and Arya et al. (2007) assume price homogeneity among the two channels, both finding that the supplier may operate a direct channel that increases profits for both firms, the supplier benefiting from additional revenue and the retailer benefiting from a wholesale price reduction. In contrast to these works, we find that it is sometimes beneficial for the supplier to operate a direct channel even if its price is undercut by that of the retailers, a notable difference from the aforementioned papers, and a result consistent with the motivating example introduced in Section 1.1.

A related stream of literature focuses on strategies for retailers operating more than one channel in competition with other retailers, most often an internet channel alongside a physical store channel (Cattani et al., 2004). Schoenbachler and Gordon (2002) conclude that a consumer-centric view of channel offerings is required to profitably operate multiple channels. Bernstein et al. (2008) show that the addition of an internet channel will typically result in lost profits at equilibrium, while Huang and Swaminathan (2009) find that there is little downside to granting the new channel autonomy in pricing decisions. In contrast to these works that investigate a retailer’s options in choosing its channel strategy, we focus on both the supplier and retailer tier of the supply chain, and the *supplier’s* decision whether or not to enter the retail market.

Also of interest is the large body of recent literature focused on supply chain coordination mechanisms, with the goal of increasing efficiency by providing incentives to align decisions with those of a centralized supply chain (Cachon, 2003). Pasternack (1985) was one of the first to show how coordination mechanisms could improve supply chain efficiency in his work on buy-back mechanisms. Other early work on supply chain coordination proposed quantity discounts as a means of improving the efficiency of a channel consisting of a single supplier and a single retailer (Rosenblatt and Lee 1985, Dolan 1987, Munson and Rosenblatt 1998). Weng (1995) and Bernstein and Federgruen (2005) coordinate supply chains with a single supplier and multiple retailers, the former using a quantity discount contract and the latter using a "price-discount sharing" (PDS) scheme, in which the wholesale price to a retailer is a function of that retailer's retail price. Similarly, Cachon and Lariviere (2005) analyze revenue sharing contracts, finding that the supply chain can be coordinated, and that profits may be arbitrarily allocated in a supply chain consisting of a single supplier and multiple differentiated retailers engaging in Cournot competition. None of these authors, however, consider the two-channel supply chain in their works.

Boyaci (2005) and Tsay and Agrawal (2004a) bring together literature on the two-channel supply chain and coordination mechanisms. Tsay and Agrawal (2004a) model a direct channel in competition with a retail channel in which the allocation of demand between the two channels is constant, total demand is dependent on sales effort in both channels, and retail price is fixed at the same value in both channels. They find that a wholesale price contract must be dependent on sales efforts to coordinate such a supply chain, but note the practical difficulty of implementing any scheme that requires the supplier to monitor the retailer's effort. Boyaci (2005) assumes price and demand to be exogenous, though both may differ by channel, and suggests a "penalty" contract in which the retailer pays the supplier a unit penalty per missed sale to achieve perfect coordination. Like the effort-dependent contract of Tsay and Agrawal (2004a), such a contract may present implementation issues because of its requirement that the supplier know the retailer's lost sales. Our work differs from both of these papers in that we assume price to be endogenous and the proportion of the demand realized in the direct channel changes with the decisions made in both channels. We therefore find that a wholesale price contract dependent on quantity alone (the "linear quantity discount" contract we describe in Section 4) is sufficient to coordinate the supply chain with symmetric retailers, and requires no complex tracking of the retailers' activities.

1.3 Contributions

We make the following specific contributions in this paper:

1. We find the supply chain equilibrium quantities when the retailers are symmetric in cost and demand characteristics.
2. We prove that a linear quantity discount contract coordinates the symmetric supply chain.
3. We show numerically that when the retailers are not symmetric, some of our analytical findings for symmetric retailers continue to hold.

2 The Model

We consider a single supplier that exclusively supplies a single product to N retailers. In addition, the supplier also operates a direct retail channel. These $N+1$ firms competing in quantity form an oligopoly over the end market.

The retail price at a given firm (one of the retailers or the supplier direct channel) is affected by the total quantity released to the market, and therefore is a function of both the firm's quantity and its competitors' quantities. We also assume that there is some degree of differentiation in the customer experience at different firms, and that some customers prefer each firm, exclusive of price. This is a reasonable assumption when one considers the many reasons a customer may choose a retail outlet: some specific to bricks-and-mortar stores (location, staff, hours of operation), some specific to online venues (shipping policies, fulfillment times, level of product detail), and some common to both (return policies, credit card acceptance, availability of other products). We therefore use a linear inverse demand model that allows us to capture this differentiation:

$$p = \alpha - Bq, \tag{1}$$

where $p = (p_0, p_1, \cdots, p_N)$ is the vector of retail prices, with p_0 representing the retail price at the supplier direct channel and $p_1, \cdots, p_N$ representing the prices at the N retailers, $q = (q_0, q_1, \cdots, q_N)$ is the vector of quantities, with q_0 representing the quantity at the supplier and $q_1, \cdots, q_N$ representing the quantities at the N retailers, $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_N)$, with α_0 representing the maximum selling price at the supplier and $\alpha_1, \cdots, \alpha_N$ representing the maximum selling price at the retailers, and B in $\mathbb{R}^{(N+1)\times(N+1)}$ is the symmetric price sensitivity matrix given by:

$$B = \begin{bmatrix} \beta_0 & \gamma_{01} & \cdots & \gamma_{0N} \\ \gamma_{10} & \beta_1 & \cdots & \gamma_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N0} & \gamma_{N1} & \cdots & \beta_N \end{bmatrix}.$$

The coefficients $\beta_0, \cdots, \beta_N > 0$ represent the price sensitivity of demand at each firm with respect to its own quantity, and $\gamma_{ij} > 0$ represents the cross-sensitivity of demand at firm i with respect to the quantity at firm j. Because we assume that the change in a competitor's quantity affects a firm's price less than a change in that firm's own quantity, we add the restriction that $\gamma_{ij} < \beta_i \; \forall i, j$ (Vives, 2001). This is broadly applicable when both channels exist as physical locations; customers must physically travel from one to another, at a cost. However, even if both channels are online, a customer may face switching costs in the form of setting up a new account, not being able to bundle shipping with other products, etc. This demand model is common throughout the recent operations management and economic literature (Vives 2001, Carr and Karmakar 2005, Cachon and Kök 2010)[1]. Note that we model competition via quantities to reflect the fact that order quantity decisions often have to be made in advance of the selling season, and prior to pricing decisions, especially when independent retailers place an order from an external supplier. Kreps and Scheinkman (1983) state that the Cournot competition model fits best situations where retailers "simultaneously and independently make quantity decisions" and then bring these quantities to the market, letting the price be determined by the quantities on the market. They show that price competition a la Bertrand would require that quantity decisions follow the realization of demand, which is not always realistic when retailers order from an external supplier. In addition, they show that quantity competition is equivalent to quantity precommitment followed by price competition, thus our model can be viewed as similar to a price competition model as long as retailers and supplier first commit in terms of quantities.

Production incurs a fixed per-unit cost, c_A, and retailing incurs a fixed per-unit cost, c_i, for firm i. We assume that the cost of retailing includes all variable costs, such as inbound shipping, and that the retailers may be asymmetric in such costs. For ease of notation, we let $c'_i = c_i + c_A$ represent the total cost of a unit sold through firm i. The quantity $\nu_i = \alpha_i - c'_i$ represents the maximum product margin of a unit sold through firm i. This maximum product margin is the difference between one unit's maximum selling price and its total cost to the supply chain. Without loss of generality, we assume the maximum product margin to be positive for each channel, as otherwise no item would be produced and sold through that channel.

The supplier and the N retailers engage in a Stackelberg game where the supplier is Stackelberg leader. The supplier chooses the total quantity, Q, to be sold through retailers and the retail quantity q_0 to be sold in the direct channel. The retailers then simultaneously react by choosing their retail quantities q_i, which then determine the retail prices of all firms, while the wholesale price w is set to clear the wholesale market.

For tractability in developing our analytical results, in the next two sections we assume that all N retailers are symmetric. In Section 5, we investigate the effects of retailer asymmetry.

[1] In practice, there is some uncertainty in price. However, as long as the firms are risk-neutral, neither an additive nor a multiplicative uncertainty has an impact on expected utility maximization and our results continue to hold. For ease of exposition, we therefore present the price as a deterministic function of demand.

3 Equilibrium Analysis With Symmetric Retailers

In this section we consider all retailers to be symmetric in terms of cost characteristics and demand parameters, i.e. $\alpha_1 = \cdots = \alpha_N$, and likewise for $\beta_i, \gamma_{ij}, c_i, c_i'$, and ν_i. We denote this by eliminating the subscripts on $\alpha, \beta, \gamma, c, c'$, and ν to indicate they apply to all retailers, while maintaining the use of 0 as the subscript for the supplier. While this is a restrictive assumption that is not satisfied in a strict sense in reality, it allows us to obtain analytically managerial insights that we test numerically in Section 5 when the symmetry assumption is relaxed. Further, the difference in cost and market positions among retailers is likely to be much less significant than the difference in cost and market positions between a supplier and its retailers. For example, Walmart and Target are much more alike than Walmart and Random House.

The ratio $\rho = \frac{\nu_0}{\nu}$ of maximum margins gives an indication of the relative strength of the supplier's direct channel compared to the indirect channel. Also, the notations $\delta = 2\beta + \gamma(N-1)$ and $\Delta = \beta + \gamma(N-1)$ simplify expressions that appear often in our results.

3.1 The Decentralized Case

We first determine the equilibrium solution of the Stackelberg game described above. The supplier first chooses the total wholesale quantity and the direct channel retail quantity, anticipating the market-clearing wholesale price and the retailers' quantities, then all retailers choose their quantities $q_i,\ i = 1, \ldots, N$. The profit to the supplier, Π_0, and the profit to retailer i, Π_i, are therefore given by[2]:

$$\Pi_0 = q_0(\alpha_0 - \beta_0 q_0 - \gamma Q - c_0') + Q(w - c_A) \tag{2}$$

$$\Pi_i = q_i(\alpha - \beta q_i - \gamma \sum_{\substack{j=0 \\ j \neq i}}^{N} q_j - c - w), \quad i = 1, \cdots, N, \tag{3}$$

and the supplier's optimization problem is constrained by the wholesale market clearing condition: $\sum_{i=1}^{N} q_i(w) = Q$, where $q_i(w)$ denotes the quantity selected by retailer i when the wholesale price is w and we recall that Q is the total quantity selected by the supplier to be sold through retailers. We denote $\Pi_T = \Pi_0 + \sum_i \Pi_i$ the total supply chain profits.

Proposition 3.1. *The equilibrium supply chain structure, profits, quantities, and prices are given in Table 1.*

[2] While retail prices cannot, in practice, take negative values, we omit this as an explicit constraint, and instead show that at the equilibrium and at the centralized optimum, prices are positive (see Appendix A, Proof of Prop. 3.1).

Domain	$\rho \leq \rho_{min}$	$\rho_{min} < \rho < \rho_{max}$	$\rho \geq \rho_{max}$
Structure	Wholesale Supplier	Two-Channel	Monopoly Retailer
q_0	N/A	$\frac{\delta\nu_0-\gamma N\nu}{2(\beta_0\delta-\gamma^2 N)}$	$\frac{\nu_0}{2\beta_0}$
q_i	$\frac{\nu}{2\delta}$	$\frac{\beta_0\nu-\gamma\nu_0}{2(\beta_0\delta-\gamma^2 N)}$	N/A
p_0	N/A	$\frac{\alpha_0+c_0'}{2}$	$\frac{\alpha_0+c_0'}{2}$
p_i	$\frac{\Delta(\alpha+c')+2\beta\alpha}{2\delta}$	$(\frac{1}{2})(\alpha+c'+\frac{\beta(\beta_0\nu-\gamma\nu_0)}{\beta_0(2\beta-\gamma)+\gamma N(\beta_0-\gamma)})$	N/A
Π_0	$\frac{\nu^2 N}{4\delta}$	$\frac{\delta\nu_0^2+(\beta_0\nu-2\gamma\nu_0)N\nu}{4(\beta_0\delta-\gamma^2 N)}$	$\frac{\nu_0^2}{4\beta_0}$
Π_i	$\frac{\beta\nu^2}{4\delta^2}$	$\frac{\beta(\beta_0\nu-\gamma\nu_0)^2}{4(\beta_0\delta-\gamma^2 N)^2}$	N/A

Table 1: Equilibrium Quantities, Prices, and Profits for the Decentralized Supply Chain

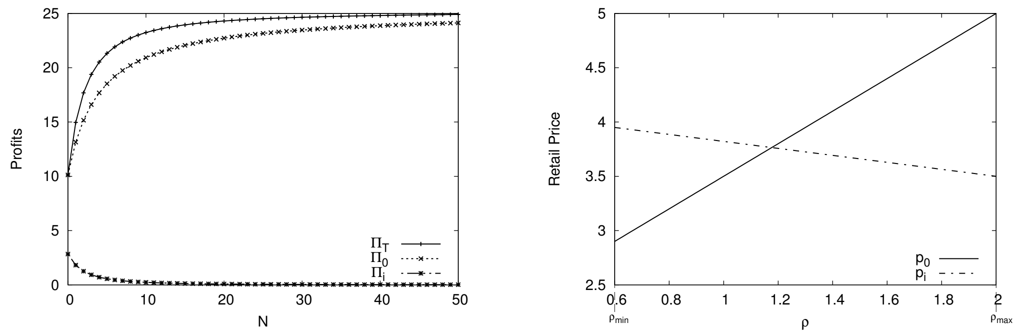
The equilibrium supply chain structure, as shown in Table 1, depends solely on parameter ρ. When $\rho \leq \frac{\gamma N}{\delta} = \rho_{min}$, the supplier's direct channel is weak compared with the indirect channel and thus the supplier exits the retail market and acts only as a wholesale supplier to its retailers. When $\rho \geq \frac{\beta_0}{\gamma} = \rho_{max}$, the supplier's direct channel is strong enough to make retailers exit the market, leaving the supplier as a monopoly retailer. Therefore, the two-channel case occurs if $\rho_{min} < \rho < \rho_{max}$. This indicates that a two-channel equilibrium with both channels in operation exists only when the quantity available at either a retailer or the supplier has a limited influence on the price at the others.

Increasing the value of N increases the retail market size, while intensifying competition between the direct and indirect channels. In addition, the value of N affects the structure of the supply chain by determining the value of ρ_{min}. Changing the number of retailers thus has a non-trivial overall effect on the supplier's total profit. It is straightforward to show from Table 1 that, as retailers enter the market, the supplier earns more in wholesale revenue than is lost in direct retail revenue; thus, to the supplier, the expansion of the indirect retail market is worth the loss of direct retail market share (see Figure 1a). We note that this holds true even when the supplier's price is undercut (see Figure 1b), a finding that helps explain our motivating example of books sold by Random House.

Further, if $\rho \leq 1$, additional entrants eventually force the supplier out of the retail market. However, the supplier's profit continues to increase in N, as enough new wholesale revenue replaces the direct retail revenue no longer earned. Therefore, the supplier benefits from the presence of as many retailers as possible in the market, even if it must eliminate the direct channel at equilibrium.

Corollary 3.2. *The total supply chain profit is monotonically non-decreasing in N. The profit to each retailer is decreasing in N, and the combined retailer profits are unimodal in N.*

It is easy to show that the growth in total demand as N increases due to additional retailers capturing more consumers has a beneficial effect on the total supply chain profit, though the profit



(a) Profits vs. N, where $\alpha_0 = 11$ and $\alpha = 12$ (thus $\nu_0 = 9$, $\nu = 10$, and $\rho = .9$).

(b) Prices vs. ρ, where $N = 3$ and $\alpha = 3$ ($\nu = 1$ and ν_0 varies from .6 to 2, causing ρ to vary from $\rho_{min} = .6$ to $\rho_{max} = 2$).

Figure 1: Profits vs. Number of Retailers and Price vs. ρ in Equilibrium, where $c_0 = c_A = c = 1, \beta_0 = 2$, $\beta = 1.5$, and $\gamma = 1$.

to each retailer is decreasing because of more intense competition. The combined retailer profit is unimodal, thus, the growth in the total profit beyond the maximum retailer profit accrues entirely as a benefit to the supplier.

3.2 The Centralized Case and the Price of Anarchy

As a benchmark for the decentralized equilibrium, we consider a centralized supply chain where a single central decision-maker chooses the prices and quantities for all firms, with the goal of maximizing the total system profit. This profit, $\widetilde{\Pi}_T$, is given by:

$$\widetilde{\Pi}_T = q_0(p_0 - c_0') + \sum_{i=1}^{N} q_i(p_i - c'). \tag{4}$$

Proposition 3.3. *The centralized optimal profits, quantities, and prices are given in Table 2.*

Domain	$\rho \leq \rho_{min}^C$	$\rho_{min}^C < \rho < \rho_{max}$	$\rho \geq \rho_{max}$
Structure	Wholesale Supplier	Two-Channel	Monopoly Retailer
q_0	N/A	$\frac{\Delta\nu_0 - \gamma N \nu}{2(\beta_0\Delta - \gamma^2 N)}$	$\frac{\nu_0}{2\beta_0}$
q_i	$\frac{\nu}{2\Delta}$	$\frac{\beta_0\nu - \gamma\nu_0}{2(\beta_0\Delta - \gamma^2 N)}$	N/A
p_0	N/A	$\frac{\alpha_0 + c_0'}{2}$	$\frac{\alpha_0 + c_0'}{2}$
p_i	$\frac{\alpha + c'}{2}$	$\frac{\alpha + c'}{2}$	N/A
Π_T	$\frac{\nu^2 N}{4\Delta}$	$\frac{\Delta\nu_0^2 + \beta_0 N \nu^2 - 2\gamma N \nu_0 \nu}{4(\beta_0\Delta - \gamma^2 N)}$	$\frac{\nu_0^2}{4\beta_0}$

Table 2: Optimal Quantities, Prices, and Profits for the Centralized Supply Chain

As in the decentralized case, the centralized chain structure depends on ρ and the supplier acts as a monopoly retailer when $\rho \geq \rho_{max}$. However, the supplier exits the direct retail market when $\rho < \frac{\gamma N}{\Delta}$, denoted as ρ^C_{min}. The centralized supply chain thus has a two-channel structure if $\rho^C_{min} < \rho < \rho_{max}$. Note that ρ^C_{min} can be interpreted as a measure of the inverse of a retailer's market power: $1/\rho^C_{min} = 1 + (1/N)(\beta/\gamma - 1)$, and the higher N the more competitive the market is, while the closer γ is to β, the less differentiated retailers are.

We define the efficiency of the system, η, as the ratio of the total profit in the decentralized case and the total profit in the centralized case, a ratio frequently of interest in the supply chain literature (Lariviere and Porteus 2001, Netessine and Zhang 2005, Farahat and Perakis 2009, Martínez-de-Albéniz and Simchi-Levi 2013). The closed form expressions for the efficiency are given in Table 3. The efficiency serves as a measure of supply chain performance. Literature on the "price of anarchy," which measures the efficiency lost to selfish behavior (typically, 1 - η: see Roughgarden and Tardos, 2002, Johari and Tsitsiklis, 2002, or Perakis and Roels, 2007), seeks to quantify this inefficiency and uses the price of anarchy as a motivation for coordination mechanisms. In what follows, we study characteristics of the supply chain efficiency and we show that the dual channel efficiency behaves differently than the single-channel efficiency.

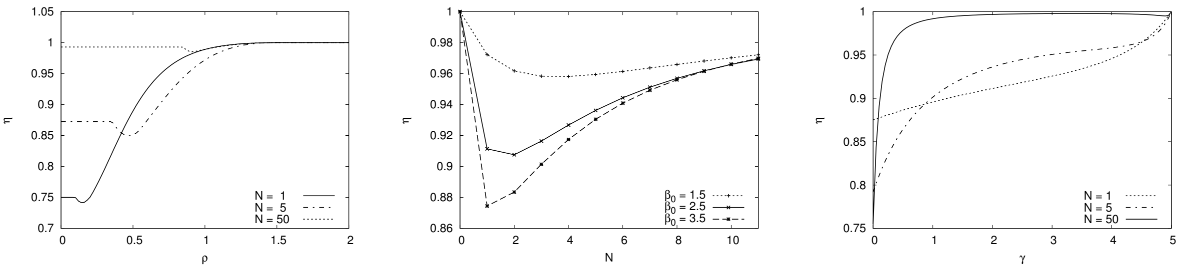
	$0 \leq \rho \leq \rho_{min}$	$\rho_{min} < \rho \leq \rho^C_{min}$	$\rho^C_{min} < \rho < \rho_{max}$	$\rho \geq \rho_{max}$
η	$\frac{(\beta+\delta)(\beta_0\delta-\gamma^2N)N}{\delta^2(\beta_0N-2\gamma N\rho+\Delta\rho^2)}$	$\frac{\Delta R}{N(\beta_0\delta-\gamma^2N)^2}$	$\frac{(\beta_0\Delta-\gamma^2N)R}{2(\beta_0\delta-\gamma^2N)^2(2\beta_0N-(4\gamma N+\delta\rho-\gamma(N-1)\rho)\rho)}$	1
In ρ	Constant	Unimodal	Monotone Increasing	Constant
In N	Monotone Increasing	Not Monotone	Not Monotone	Constant
In γ	Monotone Increasing	Not Monotone	Not Monotone	Constant

Table 3: Monotonicity Properties of Efficiency, where $R = \beta_0N(\beta_0(\beta+\delta) - \gamma^2N) - 2\gamma N(\beta_0(\beta+\delta) - \gamma^2N)\rho + (\beta_0\delta^2 - \gamma^2N\Delta)\rho^2$

Proposition 3.4. *The efficiency is not monotone in ρ, N, or γ.*

We summarize the monotonicity properties of efficiency in Table 3. We first consider the effect of ρ on the efficiency, illustrated in Figure 2a. In the range $0 \leq \rho \leq \rho_{min}$, η is constant in ρ, and takes a value between $\frac{3}{4}$, at $N = 1$, and 1, at $N = \infty$. This echoes results developed by previous authors for a single-channel supply chain that $\frac{3}{4}$ is the minimum efficiency of a system in which a single supplier interacts with N non-differentiated symmetric retailers in competition (Roughgarden and Tardos, 2002, Johari and Tsitsiklis, 2002, Martínez-de-Albéniz and Simchi-Levi, 2008, Adida and DeMiguel, 2011).

When ρ increases beyond ρ_{min}, the decentralized system moves from the wholesale supplier structure to the two-channel structure, and the efficiency is unimodal in this range, reaching a minimum between ρ_{min} and ρ^C_{min}. In particular, we observe that the efficiency of the two-channel supply chain falls below that of a single-channel supply chain when $\rho_{min} < \rho < \rho^C_{min}$. Therefore, it is evident that the supplier's sub-optimal presence in the retail market lowers efficiency: products sold through the direct channel yield a lower maximum product margin than the indirect channel.



(a) Efficiency vs. ρ, where $\alpha = 12$, $c_0 = c_A = c = 1$, $\beta_0 = 1.5$, $\beta = 5$, and $\gamma = 1$ (α_0 varies from 2 to 22, causing ρ to vary from 0 to 2).

(b) Efficiency vs. N, where $\beta = 2.5$, and $\gamma = 1$, where $\alpha_0 = \alpha = 9$, $c_0 = c_A = c = 1$ (thus $\nu_0 = 7$, $\nu = 7$, and $\rho = 1$).

(c) Efficiency vs. γ, where $\beta_0 = \beta = 5$, $\alpha_0 = \alpha = 9$, $c_0 = c_A = c = 1$ (thus $\nu_0 = 7$, $\nu = 7$, and $\rho = 1$).

Figure 2: Efficiency vs. ρ, N, and γ.

Figures 2b and 2c illustrate the non-monotonicity of efficiency with respect to both N and γ in the range[3] $\rho_{min}^C < \rho < \rho_{max}$. These results are unique to a dual channel supply chain, as for a single-channel supply chain with a single supplier and multiple retailers, Adida and DeMiguel (2011) show that efficiency is increasing in the number of retailers and decreasing in retailer differentiation (i.e. an increase in γ). This difference can be understood as an effect of the direct channel: only in the two-channel supply chain may the intensity of retail competition also bring about a decrease in the number of units sold via the direct channel and avoiding double marginalization.

The two-channel supply chain therefore contains several sources of inefficiency not found in the single-channel supply chain, but that can be remedied:

1. When $\rho_{min} < \rho < \rho_{min}^C$, the inefficiency can be reduced by removing the supplier from the retail market.
2. When β_0 is large enough to cause a dip in efficiency for low values of N, if the number of retailers is rather small, the efficiency may be improved by *reducing* further the number of retailers.
3. When N is large enough that efficiency is not monotonically increasing in γ, efficiency can be improved by *decreasing* the intensity of competition, through means such as the removal of links to third-party retailers from Random House's website in our introductory example.

We caution, however, that these tactics are applicable in a limited number of situations, thus we turn our attention to coordinating contracts as a potentially more robust method of improving efficiency.

[3] In this numerical example, $\rho = 1$ so it is higher than both $\rho_{min} = \frac{N}{N+9}$ and $\rho_{min}^C = \frac{N}{N+4}$, and lower than $\rho_{max} = 1.5$, so the system has a two-channel structure in equilibrium independently of the number of retailers for both the decentralized and centralized cases.

4 Coordination Mechanisms With Symmetric Retailers

Based on the above discussion, we consider ways to improve the efficiency of a two-channel supply chain when $\rho_{min} < \rho < \rho_{max}$, i.e. when the decentralized supply chain has a two-channel structure in equilibrium, and the centralized supply chain's optimal structure is either two-channel (for $\rho_{min}^C \leq \rho < \rho_{max}$) or single-channel with the supplier acting only as a wholesale supplier (for $\rho_{min} < \rho < \rho_{min}^C$).

For the contracts we discuss herein, we assume that the contract parameters governing compensation from the retailer to the supplier are negotiated before the start of the game (similarly to Cachon and Lariviere, 2005), then the supplier chooses its retail quantity, and finally the retailers choose their retail quantities. Further, due to symmetry among retailers, we assume that the same contract is offered to all retailers.

4.1 Applying Common Contracts to the Two-Channel Supply Chain

We find that no fixed per-unit wholesale price contract coordinates the two-channel supply chain, but unlike the one-channel supply chain (Gallego et al., 2011), this holds true even when the margin is eliminated for one of the channels. Similarly, an all-units quantity discount, in which the supplier offers a wholesale price discount on all units purchased if a retailer orders more than a specified breakpoint (Dolan, 1987) also fails to coordinate the two-channel supply chain, though it can achieve perfect coordination in a single-channel supply chain.

Revenue sharing contracts (Cachon and Lariviere, 2005) and linear price discount sharing (PDS) contracts (Bernstein and Federgruen, 2005) fare only slightly better. In a revenue sharing contract, the retailers get a discount on the wholesale price, in exchange for returning a percentage of their revenue to the supplier. In a linear PDS contract, the wholesale price is discounted by an amount linearly proportional to the discount the retailers offer on their retail prices. These two classes of contracts are equivalent, in that they generate the same prices, quantities, and profits for all firms. When $\rho_{min} < \rho < \rho_{min}^C$, there exists a revenue sharing contract or linear PDS contract that perfectly coordinates the supply chain. Its parameters and resulting profits are given in Table 4. However, when $\rho_{min}^C \leq \rho < \rho_{max}$, the supply chain cannot be perfectly coordinated with a revenue sharing contract or linear PDS contract. Therefore, these contracts only coordinate the two-channel supply chain when the supplier is forced out of the retail market, and fail to coordinate the supply chain when both channels are in operation.

	Revenue Sharing	Linear PDS	Linear Quantity Discount
	$\phi = \frac{\delta(\rho^C_{min}-\rho)}{\beta\rho^C_{min}}$	$\zeta = \frac{\delta(\rho-\rho_{min})}{\beta\rho^C_{min}}$	$s = \frac{\delta(\rho-\rho_{min})}{2\rho-\rho^C_{min}}$
	$w^r = \frac{A}{2\beta\gamma N}$	$z = \frac{A}{2\beta\gamma N}$	$w^o = \alpha - c - \frac{\gamma\rho^C_{min}(N-1)\nu}{2\Delta(2\rho-\rho^C_{min})}$
$\widehat{\Pi}_0$	$\frac{\nu^2(\delta\rho-\beta\rho^C_{min})}{4\gamma\Delta}$	$\frac{\nu^2(\delta\rho-\beta\rho^C_{min})}{4\gamma\Delta}$	$\frac{\nu^2 N((2\beta+3\gamma(N-1))\rho-(\beta+2\gamma(N-1))\rho^C_{min})}{4\Delta^2(2\rho-\rho^C_{min})}$
$\widehat{\Pi}_i$	$\frac{\gamma\nu^2(N-1)(\rho-\rho^C_{min})}{4\Delta^2(2\rho-\rho^C_{min})}$	$\frac{\nu^2\delta(\rho^C_{min}-\rho)}{4\gamma\Delta N}$	$\frac{\nu^2\delta(\rho^C_{min}-\rho)}{4\gamma\Delta^2 N}$

Table 4: Contract Parameters and Resulting Profits for $\rho_{min} < \rho < \rho^C_{min}$, where $A = (2\beta(\Delta(c'+c_A)+\alpha\gamma(N-1)) + \gamma^2(\alpha+c')(N-1)^2)\rho^C_{min} - \delta\rho(\delta c' + \alpha\gamma(N-1))$

4.2 Linear Quantity Discount Contract

In the linear quantity discount contract, the per-unit discount is a linear function of the number of units purchased by a retailer (Rosenblatt and Lee, 1985). This contract involves two parameters: w^o, the maximum wholesale price, and s, the discount per unit, resulting in a wholesale price per unit, $w = w^o - sq$. Parameters w^o and s are fixed, and the supplier chooses its retail quantity before the retailers choose theirs. Under this scheme, the profits are given by

$$\widehat{\Pi}_0 = q_0(p_0 - c_0 - c_A) + \sum_{i=1}^{N} q_i((w^o - sq_i) - c_A) \tag{5}$$

$$\widehat{\Pi}_i = q_i(p - c - (w^o - sq_i)). \tag{6}$$

Theorem 4.1. *When $\rho_{min} < \rho < \rho^C_{min}$, there exists a linear quantity discount contract that perfectly coordinates the supply chain. Its parameters and resulting profits are given in Table 4.*

When $\rho^C_{min} \leq \rho < \rho_{max}$, the linear quantity discount contract with $s = \beta - \epsilon$ and $w^o = \frac{\beta_0\delta(\alpha-c+c_A)-\gamma(\beta\nu_0+\gamma N(\alpha-c+c_A))}{2(\beta_0\delta-\gamma^2 N)}$ perfectly coordinates the supply chain when ϵ approaches zero, and this results in limiting profits of $\widehat{\Pi}_0 = \widetilde{\Pi}_T$ and $\widehat{\Pi}_i = 0$.

When $\rho^C_{min} < \rho < \rho^C_{max}$, the wholesale price is uniformly equal to $\frac{\alpha-c+c_A}{2}$, while the optimal price for the retailers is $\frac{\alpha+c+c_A}{2}$. Thus, each retailer is selling at a price exactly equal to its marginal cost. However, unlike in the revenue sharing and linear PDS contracts, this zero margin is not presupposed by the contract itself, but is rather a result of the quantity decision, and a deviation from the optimal quantity would have a negative effect on their profits. Thus, this contract achieves perfect coordination in this range, though retailers earn less than in the decentralized case with no contract. This is a significant limitation of the contract, as the retailers have no incentive to participate in a supply chain that offers them no profit. However, this may be remedied with a transfer payment from the supplier, a mechanism previously proposed by a variety of authors (Dada and Srikanth 1987, Bernstein and Federgruen 2003, Ha and Tong 2008). Because the total supply chain profit is larger with the contract than without, a linear quantity discount contract in combination with a fixed transfer payment can perfectly coordinate the two-channel supply chain at a Pareto improvement to all firms.

5 Asymmetry of Retailers

In Sections 3 and 4, we assumed that all retailers have similar cost and market positions, i.e. $\alpha_i, \beta_i, \gamma_{ij}, c_i, c_i'$, and ν_i are symmetric among retailers. In this section, we relax this assumption and we examine the effect of asymmetry in these parameters. We first reformulate the retailers' equilibrium problem as an LCP (linear complementarity problem) and the supplier's problem as an MPEC (mathematical program with equilibrium constraints). Under mild conditions, we provide a Linear Program equivalent to the retailers' LCP, which we use to obtain the set of non-idle retailers, i.e. retailers that choose a non-zero quantity at equilibrium, in a very efficient manner computationally. Given this set, we obtain analytically some structural results on the retailers' equilibrium solution for given decisions made by the supplier in the first stage. We then run a series of numerical experiments to gain insight into the supply chain's behavior when the retailers are asymmetric and differentiated.

5.1 Analysis

First, and similarly to Adida and DeMiguel (2011), we formulate the retailers' equilibrium problem as an LCP and the supplier's problem as an MPEC (Luo et al., 1996). Then, we analyze the retailers' equilibrium that is the response to a given wholesale price decision made by the supplier, by using an equivalent Linear Programming formulation (as shown in Mangasarian, 1976 and Cottle and Pang, 1978, and as suggested in Federgruen and Hu, 2014). Finally, we summarize an efficient solution method for solving the supplier's problem.

5.1.1 Formulation

We denote

$$\bar{B} = \begin{bmatrix} \beta_1 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & \beta_2 & & \gamma_{2N} \\ \vdots & & \ddots & \\ \gamma_{N1} & \gamma_{N2} & & \beta_N \end{bmatrix}, \quad D = \begin{bmatrix} \beta_1 & & 0 \\ & \ddots & \\ 0 & & \beta_N \end{bmatrix},$$

$\alpha = (\alpha_1, \cdots, \alpha_n)^T, \quad c = (c_1, \cdots, c_N)^T, \quad \gamma_0 = (\gamma_{01}, \cdots, \gamma_{0N})^T, \quad \bar{\gamma}_0 = (\gamma_{10}, \cdots, \gamma_{N0})^T \quad q = (q_1, \cdots, q_N)^T,$

and e is the column vector of size N with all components equal to 1.

Given subsets I and I' of $\{1, \cdots, N\}$, for a vector x and a matrix M, we define as $x_{(I)}$ and $M_{(II')}$ respectively the subvector indexed by I and submatrix indexed by the subset of rows I and the subset of columns I'. We make the following assumption.

Assumption 5.1. *The matrix $\bar{B} + D$ is positive definite.*

Retailer i selects the non-negative quantity that maximizes its profit:

$$\max_{q_i \geq 0} \quad q_i(\alpha_i - \beta_i q_i - \sum_{j \neq i} \gamma_{ij} q_j - \gamma_{i0} q_0 - c_i - w).$$

The retailers' equilibrium problem may be reformulated as an LCP (Adida and DeMiguel, 2011):

$$0 \leq [(\bar{B} + D)q - \alpha + q_0 \bar{\gamma}_0 + c + we] \ \perp q \geq 0. \tag{7}$$

Under Assumption 5.1, this LCP has a unique solution (Cottle et al., 2009).

Anticipating the retailers equilibrium $q(w, q_0) = (q_1(w, q_0) \cdots q_N(w, q_0))^T$ that is the best response to a market-clearing wholesale price w and direct channel quantity q_0, the supplier selects the non negative total quantity on the indirect channel, Q, and direct channel quantity, q_0, that maximize its profit. The supplier's optimization problem may thus be formulated as an MPEC:

$$\begin{aligned}
\max_{Q, q_0} \quad & q_0(\alpha_0 - \beta_0 q_0 - \gamma_0^T q(w) - c_0') + Q(w - c_A) \\
\text{s.t.} \quad & Q = e^T q(w, q_0) \\
& 0 \leq [(\bar{B} + D)q(w, q_0) - \alpha + q_0 \bar{\gamma}_0 + c + we] \perp q \geq 0 \\
& q_0 \geq 0.
\end{aligned}$$

5.1.2 Retailers' Equilibrium

We now focus on the retailers' equilibrium problem (7) for given decisions w and q_0 made by the supplier. The unconstrained first order conditions may be written as follows:

$$-(\bar{B} + D)q + \alpha - q_0 \bar{\gamma}_0 - c - we = 0.$$

It is clear that under Assumption 5.1, the unique solution of the unconstrained first order conditions is given by

$$q^*(w, q_0) = (\bar{B} + D)^{-1}(\alpha - q_0 \bar{\gamma}_0 - c - we). \tag{8}$$

The quantity vector given in (8) includes in general some negative components and hence may not be feasible. Specifically, all components of the vector $q^*(w, q_0)$ are non negative iff:

$$w(\bar{B} + D)^{-1} e \leq (\bar{B} + D)^{-1}(\alpha - q_0 \bar{\gamma}_0 - c).$$

We define[4]

$$v_i \equiv \frac{[(\bar{B} + D)^{-1}(\alpha - q_0 \bar{\gamma}_0 - c)]_i}{[\bar{B} + D)^{-1} e]_i},$$

where $[x]_i$ denotes the ith component of vector x. Therefore, we have

$$q^*(w, q_0) \geq 0 \text{ iff } w \leq \min_i v_i.$$

Lemma 5.2. *If $q^*(w, q_0) \geq 0$, then $p^* \equiv \alpha - \bar{B}q \geq 0$.*

It follows that if $w \leq \min_i v_i$, the retailers equilibrium is given by Equation (8).

Lemma 5.3. *If $q^*(w), q_0 \leq 0$, then $\alpha - q_0 \bar{\gamma}_0 - c - we \leq 0$.*

If $q^*(w, q_0) \leq 0$, i.e. if $w \geq \max_i v_i$, then $\alpha - q_0 \bar{\gamma}_0 - c - we \leq 0$ and it is clear that $q = 0$ solves the LCP given in (7).

The results above can be interpreted as follows: if the market clearing wholesale price is low enough (i.e., if the total retail quantity selected by the supplier is high enough) and/or the direct channel quantity selected by the supplier is low enough, all retailers participate in the market by selecting non zero quantities. Conversely, if the wholesale price is high enough and/or the direct channel quantity selected by the supplier is high enough, no retailer participates in the market.

When $q^*(w, q_0)$ has at least one negative and one positive component, i.e. in the case when $\min_i v_i < w < \max_i v_i$, some retailer(s), but not all, choose a zero quantity at equilibrium. We call "idle" the retailers that select a zero quantity at equilibrium and we denote K the set of non-idle retailers[5]. It is clear that, once K is known, we can exclude from the market the retailers that are not in set K and the solution of the LCP can be found by setting

$$q_{(K^C)} = 0; \qquad q_{(K)} = (\bar{B}_{(KK)} + D_{(KK)})^{-1}(\alpha_{(K)} - q_0 \bar{\gamma}_{0_{(K)}} - c_{(K)} - we_{(K)}). \tag{9}$$

To identify set K, we formulate a linear program (LP) that is equivalent to the LCP under mild assumptions and use the solution of the LP to obtain the set of idle retailers.

[4]Note that v_i depends on the decision q_0 made by the supplier. For notational simplicity, we omit to make this dependency explicit.

[5]Note that set K depends on the decisions w and q_0 made by the supplier. For notational simplicity, we omit to make this dependency explicit.

Assumption 5.4. *Matrix $(\bar{B}+D)^{-1}$ is a Z-matrix, i.e. has non-positive off-diagonal components.*

The assumption above is not very restrictive. In light of equation (8), this assumption makes intuitive sense: it means that $[\bar{B}+D)^{-1}]_{ij} \leq 0$ for any $i \neq j$, i.e., should the wholesale price be high enough so that all retailers order a non zero quantity at equilibrium, the quantity selected at equilibrium by any given retailer (i) decreases when the fixed part α_j of the price at a competing retailer ($j \neq i$) goes up or the cost c_j at another retailer (j) goes down.

As shown in Mangasarian (1976), under Assumption 5.4, the retailers' equilibrium problem (7) is equivalent to the following LP:

$$\begin{aligned} \min_{q} \quad & e^T(\bar{B}+D)q && (10)\\ \text{s.t.} \quad & (\bar{B}+D)q + q_0\bar{\gamma}_0 - \alpha + c + we \geq 0 \\ & q \geq 0. \end{aligned}$$

Let $\bar{q}$ be the solution of the LP. Since an LP may be easily solved computationally, we can obtain efficiently set K as $K = \{i : \bar{q}_i > 0\}$, and find the quantity solution of the LCP by applying (9).

5.1.3 Supplier's Problem

The supplier selects the total quantity sold on the retailer indirect channel and its own direct channel quantity, anticipating the reaction of the retailers to maximize its profits:

$$\begin{aligned} \max_{Q,q_0} \quad & q_0(\alpha_0 - \beta_0 q_0 - \gamma_0^T q(w) - c_0') + Q(w - c_A) && (11)\\ \text{s.t.} \quad & q_{(K^C)}(w, q_0) = 0 \\ & q_{(K)}(w, q_0) = (\bar{B}_{(KK)} + D_{(KK)})^{-1}(\alpha_{(K)} - q_0\bar{\gamma}_{0_{(K)}} - c_{(K)} - we_{(K)}), \quad K = \{i : \bar{q}_i > 0\}, \quad \bar{q} \text{ solves } (10) \\ & Q = e^T q(w, q_0) && (12)\\ & q_0 \geq 0. \end{aligned}$$

Condition (12) can be re-written to find the wholesale price as

$$w = \frac{e_{(K)}^T(\bar{B}_{(KK)} + D_{(KK)})^{-1}(\alpha_{(K)} - q_0\bar{\gamma}_{0_{(K)}} - c_{(K)}) - Q}{e_{(K)}^T(\bar{B}_{(KK)} + D_{(KK)})^{-1}e_{(K)}}, \tag{13}$$

which can be plugged into the supplier's objective (11) to express the objective as a function of Q and q_0. This problem can then be solved numerically.

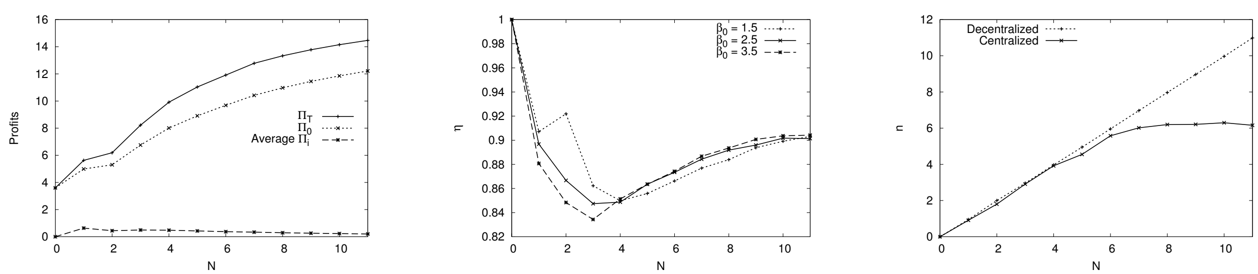
To summarize, we propose a computationally efficient solution approach for solving the MPEC.

The retailers' lower level best response subroutine takes as inputs the supplier's decisions and is solved via a linear program which may be solved efficiently.

5.2 Effect of the Number of Retailers on Profits and Efficiency

In this section, we fix the supplier's parameters (α_0, β_0, γ_{0i}, c_0, and c_A), and an upper and lower bound for each of the retailers' parameters. We then assign to each retailer random parameters drawn from a uniform distribution between these bounds, maintaining the assumptions that $\beta_i > \gamma_{ij}$ and $\gamma_{ij} = \gamma_{ji}\ \forall i, j$ and B is symmetric. The trial is repeated 250 times and we average the relevant output values (e.g. profits, efficiency, etc.). We repeat the process in this manner, varying the supplier's parameters and the bounds on the retailer's parameters, and conclusions are drawn from the complete set of numerical results.

Asymmetric retailers do not necessarily all participate in the retail market. Throughout this section we use N to refer to the total number of retailers considered and n to refer to the number of retailers who have a retail quantity greater than zero. We call the latter "active retailers".



(a) Average Profits vs. N, where $\beta_0 = 2.5$.

(b) Average Efficiency vs. N.

(c) Average Number of Active Retailers vs. N.

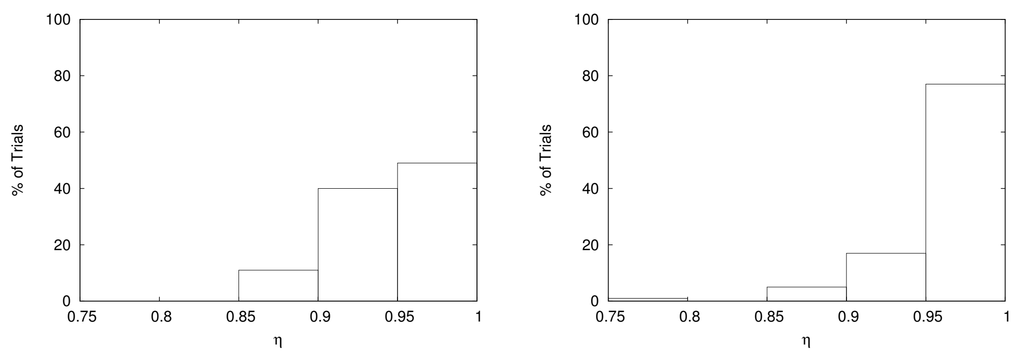
Figure 3: Average Profits, Efficiency and Number of Active Retailers vs. Number of Retailers, where $\alpha_0 = 8, c_0 = c_A = c_i = 1$, and $\alpha_i, \beta_i, i = 1, \cdots, N$, and $\gamma_{ij}, i = 0, \cdots, N, j = 0, \cdots, N$ are drawn from uniform distributions on $[7.5, 10.5], [1, 4]$, and $[.5, 1.5]$, respectively (thus $\nu_0 = 6$ and $\nu_i, i = 1, \cdots, N$, is drawn from a uniform distribution on $[5.5, 8.5]$).

We find that the results obtained numerically in the asymmetric case are consistent with those obtained analytically in the symmetric case: as shown in Figure 3a, the supplier's average profit and the average total profit are increasing in N, while each retailer's average profit is decreasing in N. There may be one or more retailers who leave the retail market, i.e., in general $n < N$, and we find that the larger N, the greater the difference in fraction of active retailers between the centralized and decentralized settings, indicating that a larger portion of the retailers make a suboptimal decision to remain in the market at equilibrium; see Figure 3c. The supplier may also choose to leave the retail market (as occurs when the supply chain contains one or more retailers with large maximum margins), but on average, the supplier still earns more profit when more retailers are present (as shown in Figure 3a).

We also find that, as in the symmetric case, the efficiency is not monotonic in N, but rather drops off upon the initial entry of retailers into the market, and then recovers as additional retailers push the supply chain closer to perfect competition, as shown in Figure 3b.

5.3 Effect of Asymmetry on the Existence of a Coordinating Contract

In this section, we investigate the effect of a linear quantity discount contract on the supply chain with asymmetric retailers. We assume that retailer i is offered a linear quantity discount contract characterized by parameters $s_i, w_i^o,\ i = 1, \cdots, N$ and we search numerically for the set of contract parameters that maximizes the total supply chain profit. We can find contract parameters for each retailer that induce retailer quantities equal to the centralized solution; however, the asymmetry of cross-sensitivity (the γ_{ij}'s) makes it impossible to induce the supplier to choose a quantity equal to its centralized optimal quantity, and thus no contract perfectly coordinates the asymmetric chain.



(a) With Fixed Per-unit Wholesale Price. (b) With Linear Quantity Discount Contracts.

Figure 4: Distribution of Efficiency of the Asymmetric Supply Chain in 100 Randomized Trials, where N is drawn from a discrete uniform distribution from 2 to 6, $c_0 = c_A = 1$, $\beta_0 = 2.5$, and $\alpha_0,\ \alpha_i, i = 0, \cdots, N,\ c_i,\ \beta_i, i = 0, \cdots, N$, and $\gamma_{ij}, i = 0, \cdots, N, j = 0, \cdots, N$ drawn from uniform distributions on $[12, 14]$, $[8, 10]$, $[1.5, 2.5]$, $[1.5, 3.5]$, and $[.5, 1.5]$, respectively.

However, even without achieving perfect coordination, linear quantity discount contracts can still significantly improve the efficiency, and therefore the total profit, of the supply chain. As shown in Figure 4, for 77% of the trials in our experiments, a contract was found that resulted in efficiency of 95% or better. By contrast, the efficiency with a fixed per-unit wholesale price was above 95% for only 50% of the trials. In 62% of the trials, the efficiency improved by at least 1% when a contract was applied. Nevertheless, the fact that the contract proposed here does not perfectly coordinate the decisions with the centralized setting is a clear limitation. Further research on contracts that may fully coordinate asymmetric dual supply chains is necessary to improve upon this result.

6 Conclusions

We have shown that, in a two-channel supply chain, the supplier benefits from additional competition in the retail market provided it is the sole supplier to those competitors and new retailers capture new demand. We therefore conclude that the supplier should sell its product to as many retailers as possible, even if doing so causes nothing to be sold through the direct channel or its own retail price to be undercut. Further, the efficiency of such a supply chain may increase with differentiation when the number of retailers is high, or increase in the number of retailers when differentiation is low, two phenomena not seen in the single-channel supply chain, thus creating counterintuitive strategies for mitigating the loss of efficiency, and motivating the search for a coordinating contract.

While several contracts known to coordinate a traditional supply chain fail when applied to a symmetric two-channel supply chain, we prove that a linear quantity discount contract perfectly coordinates this supply chain. Even though a transfer payment may be required for all firms to adopt such a contract, the increase in total profits makes such an arrangement feasible. When the retailers are asymmetric, the linear quantity discount contract is capable of improving the supply chain efficiency, even though it falls short of achieving perfect coordination.

These insights have practical implications for manufacturers that operate direct channels through which they sell products that are either exclusive or clear market leaders; specifically, we show that their exclusivity may better serve them at the wholesale level, selling to many retailers, rather than at the retail level, acting as a monopoly retailer. Additionally, forging relationships with retailers through coordinating contracts further maximizes the leverage of the manufacturer's exclusivity advantage. As these manufacturers adapt to the increasing prominence of superstores in the U.S. marketplace, they are wise to recognize the value of a strategy inclusive of multiple retailers.

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A Proofs of Propositions and Theorems

Proof of Proposition 3.1. We first consider the case that $q_0 > 0$ and $q_i > 0$. Since Π_i is concave in q_i, maximizing Π_i over q_i to find the retailers' reaction function, the first order condition is

$$\frac{d\Pi_i(q_i)}{dq_i} = \alpha - 2\beta q_i - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j - w - c = 0.$$

Using the symmetry of the retailers, this becomes

$$\frac{d\Pi_i(q_i)}{dq_i} = \alpha - 2\beta q_i - \gamma((N-1)q_i + q_0) - w - c = 0,$$

giving a reaction function of

$$q_i(w) = \frac{\alpha - c - w - \gamma q_0}{2\beta + \gamma(N-1)}.$$

The wholesale market clearing condition implies $Q = Nq_i(w)$, or

$$w = \alpha - c - \gamma q_0 - \frac{2\beta + \gamma(N-1)}{N} Q.$$

We next maximize Π_0 over Q and q_0. It is straightforward to obtain that the Hessian is negative definite and hence, after simplifying the first order conditions, we obtain $q_0^* = \frac{\delta\nu_0 - \gamma N\nu}{2(\beta_0\delta - \gamma^2 N)}$, $q_i^* = Q^*/N = \frac{\beta_0\nu - \gamma\nu_0}{2(\beta_0\delta - \gamma^2 N)}$ and $w^* = \frac{\alpha - c + c_A}{2}$.

If the resulting q_i is such that $q_i \leq 0$, (i.e. $\frac{\beta_0\nu - \gamma\nu_0}{2(\beta_0\delta - \gamma^2 N)} \leq 0$), we then take $q_i = 0$: the supplier acts as a monopoly retailer. In this case, Π_0 is concave in q_0, and therefore, we use the first-order condition to maximize the profit over q_0, and find that $q_0^* = \frac{\nu_0}{2\beta_0}$.

If the resulting q_0 is such that $q_0 \leq 0$, (i.e. $\frac{\delta\nu_0 - \gamma N\nu}{2(\beta_0\delta - \gamma^2 N)} \leq 0$), we then take $q_0 = 0$: the supplier acts as a wholesale supplier, and Π_0 is concave in Q and Π_i is concave in q_i. We proceed as we did in the two-channel case to get a reaction function of $q_i = \frac{\nu - w}{2\beta + \gamma(N-1)}$. Maximizing Π_0 over the wholesale quantity, we get, from the first order conditions, $Q^* = \nu N/(2\delta)$, $w^* = \frac{\nu}{2} + c_A$ and $q_i^* = \frac{\nu}{2\delta}$. The prices and profits follow from the quantities and wholesale prices in all three cases, and it is easily verified that the assumption of $\nu_0 > 0$ and $\nu > 0$ is sufficient to ensure the non-negativity of all prices and profits. □

Proof of Proposition 3.3. The total profit is given in Equation (4). When $q_0 > 0$ and $q_i > 0$, the total profit function is concave in q_0 and q_i, and the first-order conditions are

$$\frac{\partial\Pi_T(q)}{\partial q_0} = \alpha_0 - 2\beta_0 q_0 - 2\gamma \sum_{i=1}^{N} q_i - c_0' = 0$$

$$\frac{\partial\Pi_T(q)}{\partial q_i} = \alpha - 2\beta q_i - 2\gamma \sum_{\substack{j=1 \\ j\neq i}}^{N} q_j - c' = 0.$$

Using the symmetry of retailers, the first-order conditions become

$$\frac{\partial\Pi_T(q)}{\partial q_0} = \alpha_0 - 2\beta_0 q_0 - 2\gamma N q_i - c_0' = 0$$

$$\frac{\partial\Pi_T(q)}{\partial q_i} = \alpha - 2\beta q_i - 2\gamma((N-1)q_i + q_0) - c' = 0.$$

Solving for quantities yields $q_0^* = \frac{\Delta\nu_0 - \gamma N\nu}{2(\beta_0\Delta - \gamma^2 N)}$ and $q_i^* = \frac{\beta_0\nu - \gamma\nu_0}{2(\beta_0\Delta - \gamma^2 N)}$.

As in the decentralized case, if the expression obtained above for q_0 is negative, we let $q_0 = 0$ ($\rho \le \rho^C_{min}$); the total profit function is then concave in q_i and maximizing over q_i, the first order condition and the symmetry of retailers give $q_i^* = \frac{\nu}{2\Delta}$. If the expression obtained above for q_i0 is negative, we let $q_i = 0$ ($\rho \ge \rho_{max}$); the centralized case is then identical to the monopoly case.

The optimal retail prices and maximum profits can be found from the optimal quantities for all three ranges of ρ. □

Proof of Proposition 3.4. In the range $0 \le \rho \le \rho_{min}$, the efficiency is given by

$$\eta = \frac{\Delta(3\beta + \gamma(N-1))}{\delta^2},$$

which is constant in ρ and increasing in N and γ ($\frac{\partial\eta}{\partial\gamma} = \frac{2\beta^2}{\delta^3} > 0$).

In the range $\rho_{min} < \rho \le \rho^C_{min}$, the efficiency is

$$\eta = \frac{\Delta\left(\beta_0 N(\beta_0(3\beta + \gamma(N-1)) - \gamma^2 N) + 2\gamma N(\beta_0(\gamma - 3\beta) + \gamma N(\gamma - \beta_0))\rho + \kappa\rho^2\right)}{N(\beta_0\delta - \gamma^2 N)^2},$$

where $\kappa = \beta_0(2\beta-\gamma)^2 + (2\beta_0(\beta-\gamma) + \beta(2\beta_0 - \gamma + \gamma^2)\gamma N + (\beta_0 - \gamma)\gamma^2 N^2$. Clearly, η is a quadratic function of ρ, and because κ is positive, η is convex in ρ. At $\rho = \frac{\gamma N(\beta_0(\gamma-3\beta)+\gamma N(\gamma-\beta_0))}{\kappa}$, the partial derivative of η with respect to ρ is zero. We next show that this value is between ρ_{min} and ρ^C_{min}:

$$\frac{\gamma N(\beta_0(\gamma-3\beta) + \gamma N(\gamma-\beta_0))}{\kappa} - \rho_{min} = \frac{(\beta_0(2\beta-\gamma) + \gamma N(\beta_0-\gamma))\beta\gamma N}{\delta\kappa} \ge 0,$$
$$\rho^C_{min} - \frac{\gamma N(\beta_0(\gamma-3\beta) + \gamma N(\gamma-\beta_0))}{\kappa} = \frac{\beta_0\beta^2\gamma N}{\Delta\kappa} \ge 0.$$

We therefore conclude that efficiency is unimodal over ρ in this range, as illustrated in Figure 2a.

In the range $\rho^C_{min} < \rho \le \rho_{max}$, the efficiency and its partial derivative with respect to ρ are

$$\eta = \frac{(\beta_0\Delta - \gamma^2 N)(\beta_0 N(\beta_0(3\beta+\gamma(N-1)) - \gamma^2 N) + 2\gamma N(\beta_0(\gamma-3\beta) + \gamma N(\gamma-\beta_0))\rho + (\beta_0\delta^2 - \gamma^2 N\Delta)\rho^2)}{(\beta_0(\gamma-2\beta) + \gamma N(\gamma-\beta_0))^2(\beta_0 N + \rho(-2\gamma N + \beta\rho + \gamma(N-1)\rho))}$$
$$\frac{\partial\eta}{\partial\rho} = \frac{2\beta_0\beta^2 N(\beta_0(\beta-\gamma) + \gamma N(\beta_0-\gamma))\rho(\beta_0 - \gamma\rho)}{(\beta_0\delta - \gamma^2 N)^2(\beta_0 N + \rho(\gamma(\rho(N-1) - 2N) + \beta\rho))^2} \ge 0,$$

so efficiency is monotonically increasing in ρ.

In the range $\rho > \rho_{max}$, the centralized system acts exactly as the decentralized system, and the efficiency is therefore 1. □

Proof of Theorem 4.1. If we assume a linear quantity discount contract where $w = p^o - s.q_i$, where $p^o = \frac{\beta_0(\delta(\alpha-c+2c_A)+\gamma(N-1)c_A))-\gamma(\beta\nu_0+\gamma N(\alpha-c+3c_A))}{2(\beta_0\Delta-\gamma^2 N)}$, and $s = \beta_i - \epsilon$, retailer i's profit is

$$\Pi_i = q_i(\alpha - \gamma(\sum_{j=0}^{N} q_j) - p^o + \epsilon q_i).$$

Finding the reaction function, plugging it into the supplier's profit function, and solving for both quantities gives us quantities for q_0 and q_i as functions of ϵ. As ϵ goes to zero, the quantities become

$$q_0 = \frac{\Delta\nu_0 + \gamma N\nu}{2(\beta_0\Delta - \gamma^2 N)}$$
$$q_i = \frac{\beta\nu - \gamma\nu_0}{2(\beta_0\Delta - \gamma^2 N)},$$

which are equal to the centralized quantities. □

Proof of Lemma 5.2. We have

$$\begin{aligned}
p^*(w) &= \alpha - \bar{B}q^*(w) \\
&= \alpha - \bar{B}(\bar{B}+D)^{-1}(\alpha - q_0\bar{\gamma}_0 - c - we) \\
&= [I - \bar{B}(\bar{B}+D)^{-1}]\alpha + \bar{B}(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) \\
&= [(\bar{B}+D) - \bar{B}](\bar{B}+D)^{-1}\alpha + \bar{B}(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) \\
&= D(\bar{B}+D)^{-1}\alpha + \bar{B}(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we)
\end{aligned}$$

If $q^*(w) \geq 0$, then

$$\begin{aligned}
&(\bar{B}+D)^{-1}\alpha \geq (\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) \\
\Rightarrow &D(\bar{B}+D)^{-1}\alpha \geq D(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) \\
\Rightarrow &p^*(w) \geq D(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) + \bar{B}(\bar{B}+D)^{-1}(q_0\bar{\gamma}_0 + c + we) = q_0\bar{\gamma}_0 + c + we \geq 0.
\end{aligned}$$

□

Proof of Lemma 5.3. If $q^*(w) \leq 0$, then $(\bar{B}+D)^{-1}(\alpha - q_0\bar{\gamma}_0 - c - we) \leq 0$. Because the coefficients of $\bar{B}+D$ are non negative, left multiplying the inequality by $\bar{B}+D$ leads to the result. □