Why real interest rates, cost of capital and price/earnings ratios vary across countries

Bhagwan Chowdhry a,*, Sheridan Titman b

a The Anderson School at UCLA, 110 Westwood Plaza, Los Angeles, CA 90095-1481, USA
b McCombs School of Business, University of Texas at Austin, Austin, TX 78712-1178, USA

Abstract

This paper examines how productivity changes affect real rates of return and price/earnings ratios in a small open economy. The model provides conditions under which increased productivity in a country’s traded goods sector causes prices of non-traded goods to increase relative to the price of traded goods. Under these conditions, real rates of interest decline and the production of certain non-traded durable goods (such as capital equipment and housing) immediately increase. This ‘overconstruction’ has two effects. First, the increase in capital relative to labor in these sectors increases the marginal product of labor and hence immediately causes an increase in wages and the prices of non-traded goods. Second, an ‘oversupply’ of capital goods and housing depresses their rental rates in the current period, thereby increasing their price to rental ratios or equivalently, their price/earnings ratios. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

A number of studies document significant differences in real rates of interest across countries (e.g. Mishkin, 1984). Attracting particular attention was the lower real rates of interest in Japan, relative to the US, during the 1980s. This observed difference led several government officials, journalists as well as some economists to conclude...
that Japan, as a result, had a lower cost of capital during that period and thus, a competitive advantage over the US.\textsuperscript{1} A related issue that also received considerable attention was the extremely high price/earnings ratios observed in Japan.\textsuperscript{2} While the connection between price/earnings ratios and real interest rates across countries has been informally discussed,\textsuperscript{3} we are not aware of any past attempts to formally consider the underlying causes of differences in real rates and how these causes relate to observed differences in price/earnings ratios across countries. It is clear that, ceteris paribus, a decrease in real interest rates causes price/earnings ratios to increase since future earnings are discounted at lower rates. However, one might expect that the factors which cause real interest rates to differ across countries would cause earnings growth rates to also differ, so the relation between real interest rates and price/earnings ratios may not be so straightforward.

A number of authors have attributed differences in real rates to differences in risk premiums. For example, Darby (1986) argues: “The incomplete linkage of real interest rates internationally appears to reflect risk premiums...”.

Others have attributed differences in real rates to barriers to capital flows. Koraczyk (1985) states:

There are two major reasons why expected real interest may fail to be equal across countries... In a world with risk averse investors, differences in risk will lead to differences in expected returns... A differential in expected real returns across countries may also be due to market segmentation or barriers to capital movements across currencies.

These closed economy explanations for differences in real rates could suggest a positive as well as a negative relation between real interest rates and price/earnings ratios. For example, a country expecting greatly increased productivity in the future might experience an investment boom driving up real interest rates. However, the anticipation of increased earnings could more than offset the increased discount rates, implying increased rather than decreased price/earnings ratios. The fact that cross-country differences in real rates have persisted despite dramatic increases in capital mobility suggests that market segmentation may not be the dominant cause of these differences. The recent experience in Hong Kong provides perhaps the best example of how real rates can differ significantly in the absence of barriers to capital flows. The Hong Kong dollar is directly linked to the US dollar, and hence, because there are no restrictions on the movement of capital in and out of Hong Kong, international arbitrage ensures that the nominal interest rates in the two currencies are essentially equal. However, since the rate of inflation has been consistently higher in Hong


\textsuperscript{3} See, for instance, Frankel (1991).
Kong than in the US, the real interest rate must necessarily be lower in Hong Kong than in the US. Obviously, since the currencies are essentially the same, the differences in real rates cannot be due to currency risk (major devaluation risks, if they existed, would be reflected in the nominal rates).

To understand the difference between the Hong Kong and US real rates, we must understand how two countries with linked monetary policies have such different inflation rates. The current inflation in Hong Kong is mainly due to price increases of goods and services, such as housing and restaurant meals, that cannot be traded internationally (see Wong et al. (1991)). In this sense, the Hong Kong experience appears to be consistent with an observation (in Frankel, 1991) that has been attributed to Balassa (1964) and Samuelson (1964):

...a rapidly growing country tends (i) to experience an increase in the price of its nontraded goods relative to its internationally traded goods (because of higher productivity growth in the traded-goods sector, or else because nontraded goods are superior goods in consumption), and therefore (ii) to exhibit an apparent real appreciation of its currency when the deflation is done using CPIs which include a large share of nontradable goods within them.

It is straightforward to show that an anticipated increase in the real value of a country’s currency, as described by Balassa and Samuelson, implies a decrease in the country’s ex ante real rate of interest if nominal interest rates and exchange rates are determined in frictionless capital markets. This decrease in real rates experienced in a small open economy should be contrasted to the likely response to a productivity increase in a closed economy. In the absence of international capital flows, an anticipated increase in future productivity is likely to lead to an increase in the real rate of interest as individuals attempt to borrow more to shift consumption from the future, when they will be wealthier, to the present, when the marginal utility of consumption is higher.

To explore the relation between differences in real rates and differences in price/earnings ratios across countries we develop a simple model that is consistent with the observations of Balassa (1964) and Samuelson (1964). The two-countries model examines a small open economy, such as Hong Kong, and a large economy that can be thought of as either the rest of the world or the United States. The model considers durable and non-durable goods and services that either are or are not internationally traded. The model also considers non-traded capital goods, such as office buildings, steel mills and car factories as well as traded capital goods such as hammers and tractors. Although frictions can cause the prices of the non-traded goods as well as labor to differ across countries, we assume that there are no frictions

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4 At the end of 1997 Hong Kong nominal rates did increase relative to US rates reflecting concerns about a possible devaluation of the Hong Kong dollar. When these fears subsided the Hong Kong nominal rates fell to a level close to the level in the United States.

5 Also, see Marston (1987).
in the international markets for financial claims. Our analysis indicates that if productivity in an open economy’s traded goods sector increases, wage rates in the economy must also increase. Under certain fairly plausible conditions, this productivity shock leads to an increase in the prices of non-traded goods, such as haircuts, housing and factories. This in turn implies a strengthening of the country’s currency in real terms and a corresponding reduction in its ex post real rate of interest.

Since a shift in productivity generally evolves only slowly over time, the future productivity path can be somewhat anticipated by investors who immediately incorporate this information into the prices of capital assets. In our model, in anticipation of future productivity improvements, resources are immediately shifted into producing non-traded durable goods, such as housing and factories, to take advantage of anticipated price increases in the future. The increased investment in these durable goods has two effects. First, the increased capital relative to labor in the current period increases the marginal product of labor implying that wages and the prices of non-traded goods increase prior to the productivity increase. Second, an ‘oversupply’ of capital goods and housing depresses their rental rates in the current period thereby increasing their price to rental ratios or equivalently, their price/earnings ratios. We show that under certain fairly plausible conditions this also implies that the ex ante real rate of interest also decreases in the small open economy.

Our analysis of price/earnings ratios is thus consistent with the intuition mentioned at the outset (see also Frankel, 1991), which attributes the higher \( P/E \) ratios in countries such as Japan to the lower rates used to discount their future earnings. However, as we show, this intuition is somewhat oversimplified. Our analysis indicates that \( P/E \) ratios are high when the costs of producing capital goods, and hence the price of capital goods, is expected to increase in the future. If, however, the capital goods sector is expected to experience a strong increase in productivity in the future, the future cost of producing the capital goods would be expected to decrease, and our results would be reversed. Investors will tend to invest less in capital of this type today, preferring to wait until its price is cheaper. As a result, the rental rate for this type of capital will be high, and the price to rental (or \( P/E \)) ratio will be low, even when real interest rates are low.

In the next section, we present a model that formalizes our intuition and derive its implications. In Section 3, we calibrate a multi-period extension of the model to examine if observed high price/rental ratios observed in many countries can arise from empirically reasonable assumptions about parameter values. We discuss some extensions and their implications in Section 4. Concluding remarks are presented in Section 5.

2. The model

We consider a two-period, perfect certainty model with two countries; a small country such as Hong Kong—and the rest of the world (or a large country such as the United States). Our analysis focuses on the small country, which both produces
and consumes what we call traded goods and non-traded goods. The traded goods are characterized as products which can be freely traded across countries, without transaction costs, at prices determined on the world market. From the perspective of the small country, these prices are exogenously determined. In contrast, the non-traded goods cannot be imported or exported so their prices must be determined within the small country. In addition to making a distinction between traded and non-traded goods, we will be separately analyzing durables and non-durables as well as consumption and capital goods (which are used in the production of consumption and other capital goods). In summary, the model assumes that the individuals in these economies consume and produce goods and services that are characterized as follows:

- Non-traded goods and services, denoted $N$. These are further classified as follows:
  - Non-durables, denoted $B$ (barbers’ services). These are goods such as haircuts, household help, restaurants, fresh vegetables, etc. that cannot be stored over time.
  - Durable consumption goods, such as houses, denoted $H$.
  - Durable capital goods, called factories and denoted $F$. Examples are a steel mill, a car factory or a commercial building.
- Traded consumption and capital goods denoted $T$. Examples of these are cars, steel, investment banking services, etc. For simplicity, and without any loss of generality in our results, we assume that the traded goods are non-durable and last only one period.

It should be noted that by ruling out uncertainty, we have ruled out the possibility that a country’s monetary policy affects its real rate of interest and its real exchange rate. Barriers to capital flows created by uncertainty about changing currency values as well as different real rates caused by differences in risk premia are also ruled out by this assumption. Our results are not likely to hold if a country’s monetary policy responds to productivity shocks in a way that affects both the real exchange rate and the real rate of interest. However, if we assume that monetary policy has no effect on real exchange rates and interest rates, then our results should hold in a more general model that allows for uncertain exchange rates.

2.1. Purchasing power parity and real interest rates: a review

Let us denote the home currency as H$ and the foreign currency as FX. Let $S_t$ denote the spot price in period $t$ of the foreign currency FX in terms of the home currency H$. We will use a superscript asterisk to denote all FX prices and no superscript to denote H$ prices. We define the real exchange rate, $E_t = S_t P^*_t / P_t$, where $P$ and $P^*$ denote the general price levels. One can thus write

$$\frac{E_t}{E_0} = \frac{S_t P^*_t / P_t}{S_0 P^*_0 / P_0},$$

where $\Pi$ and $\Pi^*$ denote one plus the overall rates of inflation. If purchasing power
parity (PPP) holds in levels, then, by definition, the real exchange rate equals one. If PPP holds in the rate of change form then, by definition, the real exchange rate does not change from one period to the next.

The FX prices of traded goods are determined in the world markets and are specified exogenously. Let $P_t$ and $P^*_t$ denote the prices of the traded good in period $t$.

**Assumption 1** The law of one price holds for the traded goods.

Therefore, $P_t = S_t P^*_t$ for each period $t$. This implies that

$$\frac{S_t}{S_0} = \frac{\Pi_t}{\Pi^*_t}.$$  

where $\Pi_t$ and $\Pi^*_t$ denote one plus the rates of inflation in the prices of traded goods. Substituting Eq. (2) into Eq. (1), we get

$$\frac{E_1}{E_0} = \frac{\Pi^*_t \Pi^*}{\Pi^*_t \Pi^*}.$$  

(3)

If we let $i$ denote the nominal H$ interest rate, the real domestic interest rate $r$ can be defined as

$$1 + r = \frac{1 + i}{\Pi}.$$  

(4)

Similarly, if we let $i^*$ denote the nominal FX interest rate, the real world interest rate $r^*$ can be defined as

$$1 + r^* = \frac{1 + i^*}{\Pi^*}.$$  

(5)

**Assumption 2** There is perfect capital mobility.

No arbitrage opportunities and perfect certainty implies that the following interest rate parity condition holds:

$$\frac{1 + i}{1 + i^*} = \frac{S_t}{S_0}.$$  

Substituting from Eq. (2) and rearranging, the interest rate parity condition implies

$$1 + i = \frac{1 + i^*}{\Pi_t \Pi^*_t}.$$  

(6)

The above equation implies that the nominal rates deflated by changes in traded goods prices should be equal. However, as we shall see, the real interest rates, calcul-
lated by deflating nominal rates by the overall price index, need not be equated across countries.

Dividing Eq. (4) by Eq. (5), substituting from Eq. (6) and rearranging, we get

\[
\frac{1+r}{1+r^*} = \frac{\Pi_r \Pi^*}{\Pi \Pi^*_T}.
\]  

(7)

From Eqs. (3) and (7), we get the following,

\[
\frac{E_1}{E_0} = \frac{1+r}{1+r^*} \frac{\Pi_r \Pi^*}{\Pi \Pi^*_T}.
\]  

(8)

The above equations provide a good illustration of the relation between PPP and the equivalence of real interest rates across countries: non-traded goods and services that create violations of PPP also generate differences in real rates. We thus obtain the following proposition which can also be found in various forms in Adler and Dumas (1983), Adler and Lehman (1983), Dumas (1992), Dornbusch (1983, 1988), Hsieh (1982), Mussa (1982), Officer (1976), Roll (1979) and Stulz (1987) among others.

**Proposition 1** The following statements are equivalent:

1. *Purchasing power parity holds in the rate of change form (i.e. the real exchange rate stays constant from one period to the next).*
2. *The domestic real interest rate is equal to the world real interest rate (r=r*).*
3. *The ratio of the rate of inflation in the prices of traded goods to the overall rate of inflation is the same domestically as it is in the rest of the world (\(\Pi_T/\Pi=\Pi_T^*/\Pi^*\)).*

As the above proposition indicates, when price changes are certain, differences in real rates of interest across countries must be driven by differences in the rates of appreciation of traded and non-traded goods in the two economies. Since our primary interest is in studying the smaller economy, we can simplify our analysis, without loss of generality, by making the following assumption.

**Assumption 3** The world rate of inflation in the prices of traded goods is equal to the overall world rate of inflation, i.e. \(\Pi_T^*=\Pi^*\).

This assumption implies that

\[
\frac{1+i}{\Pi_T} = 1+r^*.
\]  

(9)
2.2. Production technology

The production of all consumption and capital goods requires two factors of production, labor \((L)\) and capital \((K)\). Although labor is assumed to be homogeneous, capital comes in two forms, traded and non-traded. Our definition of non-traded capital is capital that requires local labor for its production. A factory would be an example of this. Although financial claims for a factory can be traded internationally, the actual physical assets are not mobile. One could also think of a corporation’s organization (or the training costs of its employees) as non-traded capital. The traded capital can be thought of as airplanes, tractors, hammers, screwdrivers and other tools.

To keep the model tractable we assume that producers are competitive and their production functions require only one form of capital and exhibit constant returns to scale. We assume that the production of traded goods requires a factory (non-traded capital) but does not require traded capital. This particular assumption does not affect our main results. In addition, we assume that the production of non-traded goods requires various tools (i.e. traded capital goods) but does not require a factory (i.e. non-traded capital goods).

In summary, the following assumption describes the production functions considered in our model.

Assumption 4 The period 0 production function for all of the capital and consumption goods are described by the following Cobb–Douglas specification:

\[
K^{\alpha_G}L^{1-\alpha_G}, \quad G \in \{N,T\},
\]

where \(K_{G_0}^{G}\) is the capital good, either a non-traded factory \((K_T=F)\) or a traded good \((K_N=T)\), \(L_{G_0}\) is the period 0 input of labor and the parameter \(\alpha_G \in (0,1)\) measures the capital intensity of the production of good \(G\). The labor in the two countries should be thought of as a non-traded good, i.e. the wage rates need not be the same in the two countries.

Our model examines the effect of a productivity shock that affects the small country’s production functions in period 1, leaving productivity in the large country unchanged. Perhaps the smaller country initially has access to less sophisticated technology than the larger country, but through information flows, the technological differences narrow over time raising productivity in the smaller country. These productivity shocks are assumed to be perfectly anticipated at date 0, generating an immediate effect on the domestic economy. To simplify our analysis we will assume that the productivity shock lasts only one period. After period 1, productivity growth rates in the two countries are assumed to be identical (and equal to zero). Specifically, we assume the following.\(^6\)

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\(^6\) One might alternatively consider the possibility of the small country benefiting from an increase in the demand for a traded good that it produces. For example, Hong Kong investment bankers have recently experienced an increase in the demand for their services in China. Directly modeling an increased demand of this type would add to the complexity of our model. However, we think that a productivity increase...
Assumption 5 The production function in period 1 onwards is of the following form:
\[ \Psi_G K_{G_1}^{\alpha_G} L_{G_1}^{1-\alpha_G}, \quad G \in \{N,T\}, \]
where \( K_{G_1} \) and \( L_{G_1} \) denote the inputs in period 1 of capital and labor respectively, and \( \Psi_G \) measures the increase in productivity of good \( G \)'s production.

The technology flows affect the productivity of different goods differently. For example, the non-traded non-durable goods and services (\( B \)), such as hair cuts, may not benefit from the technology improvements and hence, show no gain in productivity (\( \Psi_B = 1 \)). In contrast, productivity in the non-traded durable goods (housing), the non-traded capital goods (factories) and the traded goods sectors are likely to benefit from the technology shock (\( \Psi_H, \Psi_F, \Psi_T > 1 \)), but possibly by different amounts.

Finally, we must make the following assumption to guarantee the existence of an equilibrium:

Assumption 6 Labor is not supplied with perfect elasticity.

What we are ruling out with this assumption is the possibility that the small country imports an infinite amount of labor at a fixed price in response to a productivity increase. We assume that the price of labor increases as the demand for labor increases, however, apart from that, we are making no assumptions about the elasticity of the supply of labor.

2.3. Rents, wage rates and prices of non-traded goods

Most of our results follow directly from the first order conditions for profit maximization. These first order conditions allow us to solve for the competitive wage rates in periods 0 and 1, as functions of the marginal product of labor in the various sectors times the price of the goods. Since wages are equal in all sectors, we have the following equalities:

\[ W_0 = P_{T_0} (1-\alpha_T) \left( \frac{F_0}{L_{T_0}} \right)^{\alpha_T} = P_{N_0} (1-\alpha_N) \left( \frac{K_{N_0}}{L_{N_0}} \right)^{\alpha_N} \quad \forall N \in \{B,H\}, \quad (10) \]

\[ W_1 = P_{T_1} \Psi_T (1-\alpha_T) \left( \frac{F_1}{L_{T_1}} \right)^{\alpha_T} = P_{N_1} \Psi_N (1-\alpha_N) \left( \frac{K_{N_1}}{L_{N_1}} \right)^{\alpha_N} \quad \forall N \in \{B,H\}, \quad (11) \]

where \( W_0 \) and \( W_1 \) denote the wage rates in period 0 and 1, respectively.

Similarly, the marginal product of the traded capital, which is used in the production of non-traded goods, must be identical in each of the non-traded goods sec-
tors and equal to the price of the traded capital (which lasts only one period). Therefore,

\[ \begin{align*}
P_{T_0} &= P_{N_0} \alpha_N \left( \frac{L_{N_0}}{K_{N_0}} \right)^{1-\alpha_N} \forall N \in \{B, H\} , \\
P_{T_1} &= P_{N_1} \Psi_N \alpha_N \left( \frac{L_{N_1}}{K_{N_1}} \right)^{1-\alpha_N} \forall N \in \{B, H\} .
\end{align*} \] (12) (13)

In addition to examining real exchange rates and real rates of interest, we will be examining how productivity shocks affect price/earnings ratios, or equivalently, the ratio of the rental rates on factories to their selling prices. Since factories are used to produce the traded good, the competitive rent on factory units equals the marginal product of factory units times the price of the traded good. Thus,

\[ \begin{align*}
R_{F_0} &= P_{T_0} \alpha_T \left( \frac{L_{T_0}}{F_0} \right)^{1-\alpha_T} , \\
R_{F_1} &= P_{T_1} \Psi_T \alpha_T \left( \frac{L_{T_1}}{F_1} \right)^{1-\alpha_T} ,
\end{align*} \] (14) (15)

where \( R_{F_0} \) and \( R_{F_1} \) denote the rents on factories in periods 0 and 1, respectively.

We now express the date 0 value of the factories (as well as houses) as the discounted value of their future rents:

\[ P_{N_0} = R_{N_0} + \frac{1-\delta_N}{1+i} P_{N_1} \forall N \in \{H, F\} , \] (16)

where \( \delta_N \in [0,1] \) denotes the rate at which the factories and houses depreciate per period and \( R_{N_0} \) is the per period rental income. Notice that to avoid confusion about real rates of interest and real prices we are discounting nominal prices in the above equations at the nominal rate of interest. Also notice that since, in the model, there is no uncertainty, the nominal rate of return on all assets equals the risk-free nominal rate \( i \) (and therefore the real rate of return on all assets is also identical).

Finally, we assume that, after period 1, productivity growth rates in the two countries are identical and equal to zero. Therefore, it must be the case that

\[ \frac{R_{F_{t+1}}}{R_{F_t}} = \Pi_T \forall t \geq 1 . \]

The price of factories, in period 1, is the discounted sum of all rental income. Thus,

\[ P_{F_1} = R_{F_1} + (1-\delta_F) \frac{\Pi_T}{(1+i)} R_{F_1} + (1-\delta_F)^2 \frac{\Pi_T^2}{(1+i)^2} R_{F_1} + \ldots , \]

which simplifies to
The equilibrium is characterized by Eqs. (1)–(17). To solve for the equilibrium we must first determine the prices in period 1, and then work backwards to solve for the prices in period 0. The period 1 equilibrium requires a solution for the wage rate and the prices of the three non-traded goods (factory units, houses and barbers). These prices are determined as functions of the parameters in the production functions and the price of the traded good which is determined exogenously. The solutions for these prices assume that the firms select the cost minimizing production functions and are perfectly competitive and thus earn zero profits.

It should be noted that this framework allows us to solve for prices without having to explicitly solve the consumer’s maximization problem. The fact that the traded goods’ price as well as interest rates are exogenous implies that the small country’s consumers’ consumption/savings decisions do not affect prices. Our model is thus consistent with the small country experiencing either trade surpluses or deficits, which would be affected by their savings rate. To model two large countries, such as the US and Japan, one would need to solve the consumer’s problem explicitly, since traded good prices and interest rates would be affected by productivity shocks in these countries. Modeling the consumer’s problem considerably complicates the model and is not likely to affect the central results in the paper.

An additional notable feature of this partial equilibrium model is that the equilibrium wage rate is determined without any assumptions about the supply of labor (except that the supply curve cannot be perfectly elastic). Because of our constant returns to scale production functions, the zero profit condition and the inelastic demand for the traded good, the demand for labor from the traded goods sector is also perfectly elastic. If labor costs are slightly less, then profits can be earned in this sector and an infinite amount of labor will be demanded. Conversely, if wage rates are only slightly higher, profits in this sector will be negative so that there will be no labor demanded. Hence, the price of traded goods determines the wage rate, regardless of the supply of labor.  

2.4. Implications

The following two propositions provide the paper’s central results. In particular, the propositions specify how productivity shocks affect real rates of interest, the price/earnings ratios of manufacturing firms and the price to rental ratio of housing.

Proposition 2 If the production of factories is at least as labor intensive as the

\[ P_{F_1} = \frac{(1+r^*)}{(r^*+\delta)} \]  

(17)
production of traded goods \((\alpha_T \leq \alpha_r)\) and if the productivity increase next period in the traded goods sector is larger than the productivity increase next period in the production of factories \((\Psi_T > \Psi_r)\) then the following propositions hold.

1. The shock to period 1 productivity causes the price of factories in period 0, \(P_{F0}\), and the wages in period 0, to rise immediately in period 0 and the rental rates on factories, \(R_{F0}\), to fall in period 0. The magnitude of the rise (or the fall) is larger:
   1.1. the larger is the increase in productivity in the traded goods sector, \(\Psi_T\), and
   1.2. the smaller is the increase in productivity in the production of factories, \(\Psi_r\).

2. The price to rental ratio for factories is larger in period 0 than in period 1. \(((P_{F0}/R_{F0}) > (P_{F1}/R_{F1}))\).

3. The rate of increase in prices of factories between period 0 and 1 exceeds the rate of inflation in traded goods’ prices \(((P_{F1}/P_{F0}) > \Pi_T)\).

4. The price to rental ratio for factories in period 0, \((P_{F0}/R_{F0})\), and the rate of increase in the prices of factories between period 0 and 1, \((P_{F1}/P_{F0})\), are larger:
   4.1. the larger is the increase in productivity in the traded goods sector, \(\Psi_T\),
   4.2. the smaller is the increase in productivity in the production of factories, \(\Psi_r\), and
   4.3. the larger is the labor intensity of producing factories, \((1-\alpha_F)\).
   4.4. the larger is the labor intensity of producing traded goods, \((1-\alpha_T)\).

**Proof:** See Appendix A.

The preceding proposition states that when factory costs are expected to increase in period 1, the production of factory units will increase in period 0, which has the immediate effect of increasing capital to labor ratios driving up wages, driving down rents, and increasing price/rent ratios. Changes that tend to increase period 1 factory prices will tend to magnify this effect on wages and rents. As we show in the next proposition, similar conditions on the production functions in the non-traded consumption goods sector implies that real rates of interest will also be negatively affected by an anticipated increase in productivity.

**Proposition 3** If the production of a non-traded consumption good (houses and other non-durable goods such as haircuts) is at least as labor intensive as the production of factories, which in turn is at least as labor intensive as the production of traded goods \((\alpha_N \leq \alpha_r \leq \alpha_T)\), and if the productivity increase next period in the non-traded consumption goods sector is no larger than the productivity increase next period in the production of factories, which in turn is smaller than the productivity increase next period in the production of traded goods \((\Psi_N \leq \Psi_r < \Psi_T)\), then the following propositions hold:

1. The price to rental ratio for houses is larger in period 0 than in period 1 \(((P_{H0}/R_{H0}) > (P_{H1}/R_{H1}))\).
2. The rate of increase in prices of the non-traded good between period 0 and 1 is at least as large as the rate of increase in the prices of factories which exceeds the inflation in traded goods’ prices \((\frac{P_{N1}}{P_{N0}}) \geq (\frac{P_F}{P_{F0}}) > \Pi_f\).

3. The price to rental ratio for houses in period 0 \((\frac{P_H}{R_H})\) and the rate of increase in the prices of the non-traded good between period 0 and 1 \((\frac{P_{N1}}{P_{N0}})\) are larger:
   3.1. the larger is the increase in productivity in the traded goods sector, \(\Psi_f\),
   3.2. the smaller is the increase in productivity in the production of the non-traded good, \(\Psi_N\), and
   3.3. the larger is the labor intensity of producing the non-traded good \((1 - \alpha_N)\).

4. The fourth proposition consists of three parts:
   4.1. the domestic overall rate of inflation exceeds the domestic rate of inflation in the prices of traded goods \((\Pi > \Pi_f)\);
   4.2. the domestic real rate of interest is smaller than the world real rate of interest \((r < r^*)\);
   4.3. the domestic currency appreciates in real terms between period 0 and 1.

**Proof:** See Appendix A.

The results in the preceding two propositions are consistent with the casual intuition discussed in the introduction about the relation between the low real rates and high price/earnings ratios in Japan. The intuition is that with lower real interest rates, the discounted present value of future earnings is greater, raising the price/earnings ratio. However, it should be stressed that it is the inflation rate on capital goods, rather than the rate on consumer goods, that is relevant for determining price/earnings ratios.

The inflation rate in the non-traded capital goods and consumer goods sectors need not be the same. Conditions can exist where consumer prices do not increase (e.g., when productivity increases sufficiently in the non-traded consumer goods sector), but capital goods prices do increase as a result of the productivity shock. In such a case, real rates, as traditionally measured, will not decrease but price/earnings ratios will increase. Alternatively, conditions exist where real rates do fall but price/earnings ratios decrease. Hence, the results provided in the previous propositions are parameter specific. To illustrate this point, the following proposition is presented which provides conditions under which a positive productivity shock causes price/earnings ratios to decrease.

**Proposition 4:** If the production of factories is as labor intensive as the production of traded goods \((\alpha_f = \alpha_T)\) and if the productivity increase in the next period in the production of factories is larger than the productivity increase in the traded goods sector next period \((\Psi_f > \Psi_T)\) then the following propositions hold:

1. The shock to period 1 productivity causes the price of factories in period 0, \(P_{F0}\), and the wages in period 0, to fall in period 0 and the rental rates on factories, \(R_{F0}\), to rise in period 0.
2. The price to rental ratio for factories is smaller in period 0 than in period 1 \((P_{F0}/R_{F0}) < (P_{F1}/R_{F1})\).

3. The rate of increase in prices of factories between period 0 and 1 is smaller than the rate of inflation in traded goods’ prices \((P_{F1}/P_{F0}) < \Pi_T\).

**Proof:** See Appendix A.

### 3. Calibrating the model: can the high Japanese price/earnings ratios be justified?

In the previous section we established that the price/rental ratios in an economy would increase if productivity was expected to grow faster in the traded goods sector than in the non-traded goods sector. In addition to differences in productivity growth rates, price/rental ratios were shown to depend, among other things, on the labor intensities of the production of traded and non-traded goods as well as on the labor intensities of the production of the non-traded factories. In this section we present a calibration exercise that illustrates the importance of these labor intensities in the determination of price/rental ratios. This exercise also demonstrates that the fairly large differences between the price/rental ratios in Japan and the United States can arise from plausible assumptions about the level and the duration of productivity shocks in the traded goods sector.

For this calibration exercise, we consider a multi-period extension of our model. Suppose the productivity shock lasts for \(N\) periods. We calculate the price/rental ratios by solving the dynamic programming problem recursively backwards, i.e. we first solve the price/rental ratio in period \(N−1\). The equilibrium is characterized by Eqs. (10)–(17) which represent a system of eight simultaneous equations with eight unknown variables, \(P_{F0}, P_{F1}, R_{F0}, R_{F1}, F_0/L_{T0}, F_1/L_{T1}, K_{F0}/L_{F0}, K_{F1}/L_{F1}\). We then solve the period \(N−2\) problem using \(P_{F0}\) as an input for \(P_{F1}\) in the current period. We now have Eqs. (10), (12), (14) and (16) and four unknown variables \(P_{F0}, R_{F0}, F_0/L_{T0}, K_{F0}/L_{F0}\). We proceed analogously for periods \(N−3, N−4, \text{etc.}\) until we obtain the price/rental ratio for the current period.

Without loss of generality, we assume zero inflation for traded goods:

\[\Pi_T = 1\]

so that

\[1 + i = 1 + r^*.\]

We normalize the traded goods prices

\[P_{T0} = P_{T1} = 1,\]

and for simplicity, we further assume zero depreciation and no productivity improvements in the production of factories and non-traded goods.
\[ \delta_T = 0, \quad \Psi_T = \Psi_N = 1. \]

Our calibration exercise uses as its basis empirical estimates documented in Marston (1987) and calculates price/rental ratios as a function of the number of years the productivity growth differentials are expected to persist. Marston (1987) documents that the productivity growth rate in the traded sector exceeded the productivity growth rate in non-traded sector by much more in Japan than in the United States which at least partially explains the lower real rate of interest in Japan. Specifically, Marston, using the OECD International Sectoral Data Base, documented that over the 1973–83 period, Japanese “productivity growth in the traded sector was 73.2 percent greater than in the nontraded sector”, whereas “[p]roductivity in the US traded goods sector grew by 13.2 percent faster than in the US nontraded sector”\(^8\). This implies that in Japan the productivity growth differential between the traded and non-traded sectors exceeded this differential in the US by roughly 5 percent per year over this 10 year period. Our calculations, made from the perspective of investors looking forward after 1983, assume that these productivity growth rate differences will last for another 1 to 15 years. The calculations also assume that the nominal (and real) interest rate in the United States is 5 percent, which corresponds to a benchmark price/rental ratio of 21 in the United States\(^9\).

Fig. 1 plots the price/rental ratio of Japanese non-traded capital (factories) as a function of the number of future years that Japan will experience extraordinary (relative to the United States) growth in its traded good sector (relative to the non-traded goods sector). The figure provides separate plots that assume different capital intensities of the production of the factories (the \( \alpha_T \) parameter in the Cobb–Douglas production function) and the production of the traded goods (the \( \alpha_F \) parameter in the Cobb–Douglas production function). These capital intensities are assumed to range from 0.10 to 0.50.

The plots shown in Fig. 1 illustrate the degree to which price/rental rates are sensitive to the number of years that the productivity growth differential is expected to increase. As one would expect, the price/rental ratio is higher when the differential is expected to persist longer. The plots also illustrate how labor intensities affect price/rental ratios. When the production of non-traded capital is more labor intensive, the capital more efficiently stores labor that will be more valuable in the future. Hence, investors will overbuild the more labor intensive capital in these situations, driving up their prices and driving down their rental rates. The labor intensity of the

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\(^8\) Marston used OECD data on total value added and corresponding employment figures to generate productivity series for each of ten sectors of the Japanese and US economies. Of the ten sectors, two were classified as traded and six as non traded (one energy-intensive sector was deleted).

\(^9\) A 5% real rate of interest may appear to be high compared with historical estimates. However, because there is no uncertainty in our model, choosing a more reasonable real rate, say 2 to 3%, would imply a price/rental ratio of 34–51 which is high compared with empirical estimates for the United States. Assuming a higher real rate of 5% implies a benchmark price/rental ratio of 21 which is empirically more plausible. Intuitively, choosing a higher real rate amounts to accounting for uncertainty in earnings and rental rates.
traded good affects price/rental ratios through its effect on future wage rates. If the production of the traded good is more labor intensive, then wages increase more for any given increase in productivity. Hence, the incentive to store labor is higher which, in turn, causes price/rental ratios to increase.

The price/rental ratios plotted in Fig. 1 demonstrate that the high price/earnings and price/rental ratios observed in Japan during the mid to late 1980s could have been justified by extrapolating the growth rates from the previous 10 year period. Because of various accounting differences it is difficult to directly compare US and Japanese price/earnings ratios (French and Poterba, 1991 discuss these problems and provide estimates of adjusted price/earnings ratios that can be used for comparison purposes); however, price/rental ratios observed for commercial real estate do not have these accounting problems. Jones, Lang and Wootten, a commercial real estate brokerage firm, reported that Japanese price/rental ratios ranged from about 35 to 60 in the 1985 to 1987 time period (Jones, Lang and Wootten, 1997), which is consistent with the numbers given in Fig. 1.

Of course, with the benefit of hindsight we now know that the assumed productivity growth rates were overly optimistic and that Japanese stock and real estate prices were too high (given what we now know). As we know, Japanese stock and real estate prices fell considerably when these assumed productivity growth rates were not realized.
4. Extensions and implications

4.1. Home bias and dynamic portfolio choices

Although an unexpected boost in productivity will make most individuals in an economy better off, our analysis suggests that there can also be some unfortunate distributional implications. For example, a retired individual living off past savings would be made worse off from a productivity increase if it reduced the real rate of return on his savings.

One could analyze the portfolio implications of uncertain future productivity shocks in this framework by adding an initial date where individuals form portfolios taking into account how uncertain productivity shocks affect both asset prices and discount rates. The insights provided by Merton (1971) and others about the need to hedge against shifts in the investment opportunity set would be especially relevant in this setting. Although we have not solved such a model, we would conjecture that older individuals saving for their retirement would like to tilt their portfolios towards assets whose values increase in the event of an unanticipated productivity shock since such a shock would lower the real rate of return that they would receive on their savings.

This line of reasoning has implications for what has come to be known in the international investments literature as the home-country bias. One observation that has been made by a number of authors is that even when capital markets are open, investors prefer to invest in their home markets. While this may not seem rational when viewed from the perspective of a static model, a home-country bias will arise, as we have described above, if investors are concerned about hedging against changes in the real rate of return in their countries. In contrast to earlier work on this home bias, our analysis suggests that the home-country market portfolio may not provide the most effective hedge against changes in the opportunity set, and depending on their point in the life cycle, different investors will have different tendencies to invest in the local market. Older investors, who are likely to be concerned about hedging against changes in the real rate of interest, should invest in stocks that represent claims on non-traded durable goods that contains a large amount of embedded labor. Intuitively, investors can hedge against increased prices resulting from increased local labor costs by buying assets which embed local labor. However, younger investors, who are expected to be net sellers of local labor in the future, may be expected

10. For example, Cooper and Kaplanis (1991) document that domestic equities accounted for 98% of US investors’ portfolios in 1990. Similarly, Japanese (87%), UK (79%) and West German (75%) investors strongly favor domestic assets in their portfolios. In Hong Kong, which is very small, very open, and is not a very diversified economy, investors still invest close to half of their stock portfolio in Hong Kong stocks.

to hold very different portfolios. We would expect that these investors would be more internationally diversified to reduce their exposure to risks that are highly correlated with their own human capital.

4.2. Implications relating to Japan

As we mentioned in the introduction, barriers to capital movements, resulting perhaps from currency risks, have been the most popular explanations for the low real rates and high price/earnings multiples observed in Japan during the 1980s. However, we believe that the arguments presented in this paper provide a more convincing explanation for these phenomena. One popular argument was that the Japanese interest rates were kept artificially low because of a combination of high savings rates in Japan, large deficits and low savings rates in the US, and currency risks that acted as a barrier to capital flows from Japan to the US. Although this argument sounds plausible in theory, ex post evidence indicates that Japanese interest rates were in fact artificially high during this period (or at least higher than they would have been if people had perfect foresight). The ex post rate of return on default-free Yen denominated bonds turned out to be substantially higher than the rate of return on default-free dollar denominated bonds. If investors had perfect foresight and had anticipated the productivity improvements realized in Japan during the 1980s, international arbitrage would have forced Japanese nominal rates down to a level where Japanese real rates of interest would have been much lower than they actually were and quite possibly negative. It is likely that barriers to capital flows increased rather than decreased Japan’s real interest rate.

It should be stressed that it is differences in the changes in productivity, both across countries and across sectors within the countries, that affect real rates of interest, \( \frac{P}{E} \) ratios and investment expenditures. Real rates of interest will be low in a country when its productivity is growing much faster in its traded goods sector (relative to its trading partners) than in its non-traded goods sector. Uniform increases in the productivity of an economy may have no effect on these variables.

These differences in productivity growth rates, which drive our results, are likely to occur in developing economies since technology transfers from developed to less developed countries tend to be focused on the traded goods sector. It should also be noted that since it is the difference between the productivity growth rates in the traded and non-traded goods sector that drive our results, that very sluggish productivity improvements in the non-traded sectors can have the same effect on real exchange rates as rapid productivity improvements in the traded goods sector. For example, the large productivity gains in retailing generated by firms such as Wal-Mart in the US would have the effect of increasing real rates of interest in the US relative to Japan. Perhaps, the inferior productivity gains in Japan’s non-traded sector had as large an effect on its appreciating currency as its superior productivity gains in its traded goods sector.

Our analysis may also provide insights relating to the alleged willingness of Japanese firms to sell (dump) their products on world markets at very low, and in some cases, negative profit margins. We show that firms will indeed sell their output
at lower profit margins if an anticipated increase in the price of capital goods leads them to temporarily ‘overinvest’ in capital equipment. An anticipated increase in the cost of producing capital goods will occur when the traded goods sectors are expected to experience greater productivity gains than the sector that produces capital goods. For example, an anticipated productivity increase in the Japanese automobile sector that is not matched by a comparable productivity gain in the sector that produces capital goods will result in an increase in the price of capital equipment (due to increased labor costs). Anticipating this increase in capital goods prices, the Japanese auto firms will have a tendency to ‘overinvest’ today, in order to avoid the higher capital goods prices in the future. In equilibrium, they are willing to achieve lower initial profits from their capital investments because they anticipate that their capital equipment will appreciate in price over time.

Do these incentives to ‘overinvest’ put US firms at a competitive disadvantage?

Our model assumes that the country experiencing the productivity shock is very small, so that firms in the large country are unaffected by the increase in the small country’s investment in capital goods. However, we would expect that in a general equilibrium setting with two large countries such as Japan and the US, an increase in capital goods investment in Japan could result in an offsetting decrease in capital investment in the US. In some sense we might say that the US firms are at a competitive disadvantage since in such a general equilibrium economic forces give the US firms an incentive to at least temporarily give up market share to their Japanese counterparts with excess capacity.

5. Conclusion

The model presented in this paper analyzes how productivity shocks can affect real interest rates, real exchange rates and price/earnings multiples. Our analysis illustrates how intuition developed from studying large domestic economies may not be applicable to small open economies in an international setting. In the absence of international capital flows, an anticipated increase in future productivity is likely to lead to an increase in the real rate of interest as individuals attempt to borrow more to shift consumption from the future, when they will be wealthier, to the present, when the marginal utility of consumption is higher.

In a small open economy, individuals will also have an incentive to shift income. However, since they are small relative to the world capital markets, they have no effect on world interest rates and their productivity increase will not affect the price of goods that are freely tradeable internationally. In contrast, a productivity shock will affect the prices of the non-traded goods in the economy. Under fairly plausible conditions, a productivity shock will increase the price of non-traded goods, implying an increased rate of inflation and a decreased real rate of interest.

Our analysis indicates that this decline in real rates is generally associated with an increase in price/earnings ratios. Intuitively, investors anticipate the increased cost of producing capital goods in the future, and thus overproduce capital goods in the present period. This leads to both higher costs of producing the capital good as well
as lower equilibrium rental rates on the capital. Our numerical results suggest that these effects can explain the very high price/earnings ratios observed in Japan during the mid-1980s.

These numerical results also illustrate how productivity shocks can differentially affect price/earnings ratios in different industries. For example, industries whose capital goods are produced with more labor-intensive production processes will have higher price/earnings ratios than industries with capital goods that are produced mainly from existing capital.

Our model is directly applicable to the analysis of the situation in Hong Kong during the early to mid 1990s, where real interest rates were negative and also provides insights, discussed above, about the differences in real rates in the US and Japan during the 1980s. The popular ‘segmented markets’ explanation for these differences suggests that real rates were ‘too low’ in Japan (because of their higher savings rate) and ‘too high’ in the US (because of high government deficits). It has been argued that this, in turn, leads to a competitive advantage for Japanese firms over their US counterparts, thereby causing less than the efficient level of investment in the US compared with Japan. In contrast, the different levels of investments in the two countries arise in our model because of an efficient reallocation of resources caused by a productivity shock. The allocation of investment capital in this model follows as a result of unimpeded capital flows across countries rather than as a result of segmented markets. Our analysis, thus, suggests very different policy implications than those suggested by the segmented markets view.

Before closing, we must note that although our analysis is directly applicable to the Hong Kong and Japanese situations described above, one should be cautious about directly applying the analysis to countries like China and India which have experienced substantial productivity improvements in their agricultural sectors. In our model, the productivity improvements in the traded goods sector leads to an increase in the production of the traded goods which in turn generates increased demand for labor and higher wages. In contrast, productivity improvements in agriculture often result in a decrease in the demand for labor. These labor-saving productivity increases could increase the supply of non-farm labor, which would in turn lower wages and reduce the price of non-traded goods. The negative effect on wages coming from the productivity improvements in the agricultural sector could offset the positive effect on wages coming from the productivity increases in the traded goods sector which could offset the reduction in real rates and the increase in price/earnings ratios described in this paper. Perhaps this explains why we do not tend to observe very low real rates and very high price/earnings ratios in many developing countries.

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12 One could show this within the context of our model by assuming that the capital component of the production of the traded (agricultural good), in this case land, is in fixed supply.
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Appendix A

Before proving Propositions 2–4, we must first prove the following four lemmas.

**Lemma 1:** If \( \Psi F = \Psi T = 1 \) or if \( \alpha_T = \alpha_F \) and \( \Psi T = \Psi F \) then \( \frac{P_{F_0}}{R_{F_0}} \) and \( \frac{P_{F_1}}{R_{F_1}} \) are equal to \( \Pi_T \).

**Proof:** From Eqs. (11) and (13), we get

\[
\Psi_T (1 - \alpha_T) \left( \frac{F_1}{L_{T_1}} \right)^{\alpha_T} = \frac{(1 - \alpha_N) K_{N_1}}{\alpha_N} \forall N \in \{B, H\}. \tag{18}
\]

Substituting in Eq. (17) for \( P_{F_1} \) from Eqs. (13) and (18) and for \( R_{F_1} \) from Eq. (15) and simplifying, we get,

\[
\left( \frac{F_1}{L_{T_1}} \right)^{1 - \alpha_T \alpha_F} = \Psi_T^{\alpha_T} \Psi_F A \frac{1 + r^*}{r^* + \delta_F}, \tag{19}
\]

where

\[ A = \alpha_T \left( \frac{1 - \alpha_F}{1 - \alpha_T} \right)^{1 - \alpha_F}. \]

From Eqs. (10) and (12), we get,

\[
(1 - \alpha_T) \left( \frac{F_0}{L_{T_0}} \right)^{\alpha_T} = \frac{(1 - \alpha_N) K_{N_0}}{\alpha_N} \forall N \in \{B, H\}. \tag{20}
\]

Now, from Eqs. (16) and (17), we have

\[
P_{F_0} = R_{F_0} \left( 1 - \delta_F \right) \frac{1 + r^*}{1 + i} \frac{R_{F_1}}{r^* + \delta_F}. \tag{21}
\]

Substituting for \( P_{F_0} \) from Eqs. (12) and (20) and for \( R_{F_0} \) from Eq. (14) and simplifying, we get,
\[
\left( \frac{F_0 - \delta_F}{L_{T_0}} \right)^{1-\alpha_F} = A \left[ 1 + \frac{1 - \delta_F}{r^* + \delta_F} \pi_{RF} \right],
\]

where

\[
\pi_{RF} = \frac{R_{F_1}/R_{F_0}}{\Pi_T}
\]

Substituting in Eq. (22) from Eqs. (14), (15), (19) and (23) and simplifying, we get,

\[
\left( \frac{\pi_{RF}^{\alpha_F}}{\Psi_T} \right)^{1-\alpha_F} = \Psi_F = \left[ \frac{1 - \delta_F}{\pi_{RF}} + \frac{1 - \delta_F}{r^* + \delta_F} \right] r^* + \delta_F.
\]

For \(N=F\), dividing Eq. (16) by \(R_{F_0}\) and simplifying, we get

\[
\frac{P_{F_0}}{R_{F_0}} = 1 + \frac{1 - \delta_F}{r^* + \delta_F} \pi_{RF}.
\]

If \(\Psi_T = \Psi_F = 1\) or if \(\alpha_F = \alpha_T\) and \(\Psi_F = \Psi_T\), then from Eq. (24), \(\pi_{RF} = 1\). Substituting \(\pi_{RF} = 1\) in the above equation implies that

\[
\frac{P_{F_0}}{R_{F_0}} = 1 + \frac{1}{r^* + \delta_F} \frac{P_{F_1}}{R_{F_1}} = \frac{P_{F_1}}{R_{F_1}}.
\]

For \(N=F\), dividing Eq. (16) by \(P_{F_0}\), we get,

\[
1 = \frac{R_{F_0}}{P_{F_0}} + \frac{1 - \delta_F}{P_{F_0}} \frac{P_{F_1}}{1 + \delta_F}.
\]

Substituting from Eq. (26) and simplifying we get \(P_{F_1}/P_{F_0} = \Pi_T\).

\textbf{Lemma 2} \(\pi_{RF}\) is increasing in \(\Psi_T\) and decreasing in \(\Psi_F\), \(\alpha_F\) and \(\alpha_T\).

\textbf{Proof:} Follows from implicitly differentiating Eq. (24) and simplifying.

\textbf{Lemma 3:} The price to rental ratio in period 1 for factories, \(P_{F_1}/R_{F_0}\) and the rate of increase in the price of factories from period 0 to 1, \(P_{F_1}/P_{F_0}\) are increasing in \(\Psi_T\) and decreasing in \(\Psi_F\) and \(\alpha_F\).
Lemma 4  The price of factories in period 0, $P_{F0}$, and wages in period 0, $W_0$, are increasing in $\Psi_T$ and decreasing in $\Psi_F$. The rental rate on factories in period 0, $R_{F0}$ is decreasing in $\Psi_T$ and increasing in $\Psi_F$.

Proof: Results for $W_0$ and $R_{F0}$ follow directly from Eqs. (10), (14) and (22) and Lemma 2. To see the result for $P_{F0}$, we substitute for $R_{F0}$ from Eq. (14) in Eq. (25) and simplify to obtain:

$$P_{F0} = P_{T0} \alpha_T a T A^{-\delta_F(1-\delta_F)/(1-\delta_T)} \pi_{R_F} + \frac{1}{\pi_{R_F}} r^* + \delta_F.$$ (28)

the result then follows from Lemma 2.

Proof of Proposition 2 Follows from Lemmas 1–4.

Proof of Proposition 3 From Eqs. (18) and (20), we have

$$\frac{K_{N1}/L_{N1}}{K_{N0}/L_{N0}} = \Psi_T \left( \frac{F_1/L_{T1}}{F_0/L_{T0}} \right)^{\alpha_T} \forall N \in \{B,H,F\}. \tag{29}$$

Substituting from Eqs. (14), (15) and (23) and rearranging, we get

$$\Psi_T \left( \frac{F_1/L_{T1}}{F_0/L_{T0}} \right)^{\alpha_T} = \left( \frac{\pi_{R_F}^{\delta_F}}{\Psi_T} \right)^{-1/(1-\alpha_T)}. \tag{30}$$

From Eqs. (10) and (11), we get

$$\frac{P_{N1}}{P_{N0}} = \frac{P_{F1} \Psi_F (K_{N1}/L_{N1})^{\alpha_F-\alpha_N}}{P_{F0} \Psi_N (K_{N0}/L_{N0})^{\alpha_F-\alpha_N}} \forall N \in \{B,H\}. \tag{31}$$

Substituting from Eqs. (29) and (30) and rearranging, we get

$$\frac{P_{N1}}{P_{N0}} = \frac{P_{F1} \Psi_F (1-\delta_F/(1-\delta_T)) r^* + \delta_F}{P_{F0} \Psi_N \pi_{R_F}^{\delta_F} r^* + \delta_F} \forall N \in \{B,H\}. \tag{31}$$

Since the term in the square brackets in Eq. (31) is strictly greater than one (because
\( \pi_{RF} > 1 \) and \( \Psi_F \geq 1 \), the results about \( P_{N1}/P_{N0} \) follow immediately by inspection. Also, since
\[
1 = \frac{R_{N0}}{P_{N0}} + \frac{1 - \delta_N P_{N1}}{1 + i P_{N0}},
\]
the results for \( P_{N0}/R_{N0} \) follow.

**Proof of Proposition 4** Follows from Lemmas 1–4.

**References**


