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DUALITY DETERMINATION OF ISOSCALAR MESON-NUCLEON COUPLINGS

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ABSTRACT

It is shown that the suppression of nucleon Born term contributions to Finite Energy Sum Rules for isoscalar meson-nucleon amplitudes is possible if and only if the \( \eta NN \) coupling is weak and the \( fNN, dNN \) couplings conserve helicity. A duality prediction for the diffractive \( pp \to pN^{*}(1400) \) cross-section is shown to be in good agreement with experiment.

It has been realized for a long time [1] that duality requires a large nucleon Born term contribution in Finite Energy Sum Rules (FESR's) to be cancelled by nearby resonances or suppressed by kinematic factors. Thus, for the crossing-odd \( \eta N \to nN \) t-channel helicity amplitudes \( A'(-) \) and \( B(+), \) the \( N(938) \) and \( \Delta(1232) \) contributions to zeroth-moment FESR's cancel quite accurately. For the crossing-even amplitudes, on the other hand, the nucleon contribution is suppressed by a factor \( \nu = \frac{1}{2}(s - u) \) present in first-moment FESR's. The \( N - \Delta \) cancellation mechanism was recently studied [2, 3] for a large number of processes of the type \( R'B' \to R''B'' \) (fig. 1), where \( R', R'' \) are any \( I = 1 \) mesons or reggeons \( (\rho - A_2, \sigma - B, A_1) \) and \( B', B'' \) are \( N(938) \) or \( \Delta(1232). \) It was found that the cancellation can indeed occur in practically all processes and helicity amplitudes, and that this requirement uniquely fixes the helicity structure and relative strengths of the couplings \( R'NN, R'N\Delta \) and \( R'\Delta. \) The couplings that are experimentally known agree very well with these duality predictions.

In this letter we extend the analysis of ref. 3 by allowing either or both of the mesons \( R', R'' \) in fig. 1 to have \( I = 0, \) namely \( R', R'' = f - \omega \) or \( \eta. \) The s-channel isospin can then take only one value, and no \( N - \Delta \) cancellation is possible. Consequently the only allowable large \( R'NN \) couplings are those which result in a suppressed Born term contribution to the FESR's. We find in this way that the \( \eta NN \) coupling must be small, and that the \( fNN, dNN \) couplings have to be dominantly helicity non-flip. Furthermore, by imposing semi-local duality on the \( \Phi N \to \Phi N \) amplitude \( (\Phi = \text{pomeron}) \), we can understand all the characteristic features of the diffractive \( N^{*}(1400) \) enhancement, namely its steep and moderate \( t \)-dependence for small and large \( |t|, \) respectively, as well as the structure in the intermediate region \( t \approx -0.2 \text{ GeV}^2. \)

* The only exception being two amplitudes in the \( R'\Delta \to R''\Delta \) processes [3].
Consider first the elastic process \( \eta N \to \eta N \), with the \( \eta \)'s
\( \pi \)-shell \( (j^P = 0^-) \). We can write a zeroth-moment FESR for the invariant
amplitude \( B \), related as usual to the \( t \)-channel helicity-flip amplitude
by
\[
\mathcal{T}_{+-} = \frac{1}{2m}(\frac{t}{m^2} - t)^{-1/2}\sqrt{\mathcal{G}} B
\]
where \( \mathcal{G} \) is the Kibble function,
\[
\mathcal{G} = -t[(s - m^2)^2 - 2m^2(s + m^2) + st + \mu^2]
\]
and \( m \) is the nucleon \( (\eta) \) mass. Duality then requires the nucleon
correction to the FESR to be of the same order of magnitude as (or
smaller than) the higher-mass resonance contributions. Present evidence
indicates that the latter are quite small; this is in particular
true for the well-established \( S_{11}(1535) \), whose dominant decay mode

\[
* \text{Assuming the nucleon Born term and the } S_{11} \text{ FESR contributions to be equal would correspond to } G_{\eta NN}^2/4\pi = 0.29.
\]

is \( \eta N \). Although the precise magnitudes of the couplings of the other
\( N^* \)'s to \( \eta N \) are rather uncertain, it seems clear that their FESR
contributions in \( \eta N \to \eta N \) are no larger than in \( \pi N \to \pi N \). The

\[
* \text{A graphic illustration of how small the } N^* \text{ resonance contributions are, compared to the } N \text{ and the } A_1 \text{ in the FESR for the } \pi N \to \pi N B^{(*)}
\]
amplitude is given in fig. 3 of ref. 2.

\[
* \text{The absence of a } \Delta(1232) \text{ contribution in } \eta N \to \eta N \text{ then requires the Born term to be an order of magnitude smaller than in } \pi N \to \pi N. \text{ Thus}
\]
a conservative upper limit for the \( \eta NN \) coupling is
\[
G_{\eta NN}^2/4\pi \leq 1 \ldots 2.
\]

Different ways of extracting the \( \eta NN \) coupling from the scarce
data give varying results [5]. Perhaps the most reliable estimate is
provided by analyses of \( \pi N \to \eta N \) near threshold [6], which indicate
a very small value, \( G_{\eta NN}^2/4\pi \leq 0.5 \). \text{SU}(3) with } \text{F/(F + D) = 0.4} [7]
gives for the octet part of the coupling \( G_{\eta NN}^2/4\pi = 1.8 \).

Next we consider the nucleon contribution to FESR's for the
reggeon amplitudes \( \eta N \to R''N \) (fig. 1), where \( R'' = \omega - f, \pi - B \) or
\( A_1 \) (all that follows works out in an identical way for the amplitudes
\( \omega N \to R''N \) as expected from exchange degeneracy between the \( \omega NN \) and
\( \eta NN \) couplings). Since we know phenomenologically that the \( \omega NN \) and
\( \eta NN \) couplings are not small, we are led to expect the existence of a
kinematic suppression factor in front of the nucleon contribution. As
we shall see, the contribution from the helicity non-flip \( \eta NN \) coupling
is indeed suppressed in this way, while there is no corresponding
suppression of the helicity flip coupling. From this follows that the
(\( \eta NN \) and \( \omega NN \) couplings must be dominantly helicity non-flip, in
agreement with phenomenological results [8].

The general formulas for a resonance contribution derived in
[3] can be directly applied here. The angles \( \chi, \psi' \) and \( \psi'' \) are
proportional to the mass difference between the resonance and the
external baryons and can thus be set equal to zero, just as in the
application to the \( N - A \) cancellation. Since we are interested in
zeroth moment FESR's (which do not have extra suppression
factors of \( \psi \)), we need only consider \( s - u \) crossing odd
amplitudes. Forming the combination of helicity amplitudes that also
In the t-channel, we find using eqs. (2.43), (2.59), (3.1) and (3.9) of ref. 3 that the residue of the nucleon Born term in the \( fN \rightarrow R^*N \) amplitude is

\[ T_f^{\prime}(t', \lambda') = (2t' + 1) \left( \begin{array}{c} \lambda_3 \\ \lambda_6 \end{array} \right) e^{i\lambda'/2} T_{3s}^{(t')} \left( \begin{array}{c} \lambda_3 \\ \lambda_6 \end{array} \right) e^{-i\lambda'/2} \]

where the quantity in parenthesis is a \( 3j \)-symbol, and repeated helicity indices are summed over. The overall t-channel multipole amplitude \( T_f^{\prime}(t, \lambda; I) \) is defined in an analogous way [3] in terms of the t-channel cm. helicity amplitude for the process. Finally, the curly brackets in eq. (3) denote \( 6j \)-symbols [9].

In order to find the nucleon contribution to an \( fN \rightarrow R^*N \) FESR we must still remove the kinematic singularity at \( \rho = 0 \) from the expression (3). This is actually where the crucial difference lies between a reggeon amplitude and a spin zero amplitude like \( \eta N \rightarrow \eta N \), and which allows the \( fNN \) coupling to be large. For a spin zero amplitude the kinematic singularity is of the form \( 1/\rho^2 \), while for a reggeon amplitude it is in general \( 1/\rho^{4-2n} \), \( n \) being the t-channel helicity flip. The reasons for this are discussed at length in appendix B of ref. 3; we shall not comment further on them here. It implies, however, that the singularity-free amplitude which satisfies an FESR is obtained by multiplying the helicity amplitude by a factor \( \rho^{4-2n}/2 \). Since \( \rho \) is proportional to \( (s - m^2)^2 \) for small values of the momentum transfers (cf. eq. (2) with \( t' = t'' = \mu^2 \)),
This factor suppresses the nucleon contribution. Thus it is sufficient to investigate whether the nucleon Born term as given by eq. (3) is small; this already provides an upper limit on the FESR contribution.

The converse is also true: If the Born term (5) is finite, then the nucleon FESR contribution is not suppressed. To see this one has to consider the exact expression for \( \mathcal{T}_o^{(-)}(\ell, \lambda; I) \) with \( \psi, \psi' \neq 0 \), as these angles contain negative powers of \( \sqrt{\mathcal{J}} \) which cancel the \( \frac{\mathcal{J}}{|\lambda|/2} \) factor. By looking at a few examples one may convince oneself that the conclusions based on the present, somewhat oversimplified, treatment are correct.

We shall now consider the processes \( fN \rightarrow R'N \), with

\[ R' = f, n, \rho \text{ or } A_1 \]. Recalling [3] that the two \( fNN \) couplings allowed by parity are \( T_{NN}^f(0, 0) \) (helicity non-flip) and \( T_{NN}^f(1, 1) = - T_{NN}^f(1, -1) \) (helicity flip). Looking first at \( fN \rightarrow fN \), we see from eq. (3) that the nucleon Born term for the \( c = +, \ell = 1 \) amplitude \( \mathcal{T}_+^{(-)}(1, 1; 0) \) (i.e., natural parity helicity flip in the t-channel) is proportional to \( |T_{NN}^f(1, 1)|^2 \). Since this contribution will appear unsuppressed in the FESR, the helicity flip coupling \( T_{NN}^f(1, 1) \) must be small, and can be ignored in the following.

Choosing the nonflip \( fNN \) coupling \( \lambda' = \lambda = 0 \), and using the \( \pi NN \), \( \rho NN \) and \( A_1 NN \) couplings determined in [2], we may evaluate the last two factors in eq. (3) for each process,

\[
\begin{align*}
\mathcal{T}_N \rightarrow fN & : [1 - \sigma(-1)^\ell] [1 + \sigma] & \text{(4a)} \\
\mathcal{T}_N \rightarrow nN & : [1 - \sigma(-1)^\ell] [1 - \sigma] & \text{(4b)} \\
\mathcal{T}_N \rightarrow \rho N & : [1 - \sigma(-1)^\ell] [1 - \sigma] & \text{(4c)} \\
\mathcal{T}_N \rightarrow A_1 N & : [1 + \sigma(-1)^\ell] [1 + \sigma]. & \text{(4d)}
\end{align*}
\]

But since \( \ell' = 0 \), the \( 6J \) symbol in eq. (5) is zero unless \( \ell = \ell'' \), i.e. \( \ell = 0 \) for \( fN \rightarrow fN \) and \( \ell = 1 \) for \( fN \rightarrow R', \rho N \text{ or } A_1 N \). Each of the expressions (4) thus vanishes and the nucleon Born term is suppressed in all FESR's (determined in detail by small factors that were neglected in our approximation).

We have thus found that if, and only if, the helicity non-flip \( fNN \) coupling is dominant, the nucleon Born term will be suppressed in all \( fN \rightarrow R'N \) FESR's. Furthermore, to obtain this result one must use the same \( \pi NN \), \( \rho NN \) and \( A_1 NN \) helicity couplings that were required previously [3] for the \( N - \Delta \) cancellation to take place in the amplitudes \( R'N \rightarrow R'N \) with \( I' = I'' = 1 \). This is even more gratifying because for those amplitudes it was observed that in the first moment FESR's the \( N \) and \( \Delta \) contributions built up a non-zero t-channel exchange only when \((-1)^\ell + I = +1\). Taking such a semilocal duality seriously implies that the \( f \) and \( \omega \), having \( I = 0 \), must couple to nucleons with \( \ell = 0 \), i.e. the same coupling that we get in this investigation. Similarly, the \( \pi NN \) coupling must be small because only \( \ell = 1 \) is kinematically allowed.

It is straightforward to extend the above calculation to the processes \( fN \rightarrow R^'\Delta \) and \( f\Delta \rightarrow R^'\Delta \), using the formulas of ref. 3. One finds again that the \( N \) and \( \Delta \) Born terms are suppressed in the FESR's for all the (numerous:) helicity amplitudes, provided that one uses the previously established couplings and that the dominant \( f\Delta \) coupling has \( \ell = \chi = 0 \) and satisfies

\[ T_{\Delta\Delta}^f(0, 0) = - 2 T_{NN}^f(0, 0) \].
According to the above analysis, the nucleon Born term will always be suppressed in isoscalar meson-baryon FESR's. In practice, this means that the contributions from the Born term and from the higher-mass resonances are comparable in magnitude. It is then interesting to ask how "local" the duality relation is between the s-channel contributions and the t-channel Regge exchange. We shall illustrate this point using the forward elastic pomeron (pP) amplitude \( \tau N \rightarrow \pi N \), which is "measured" by the data on the inclusive pp \( \rightarrow px \) process at high energy (we assume that the pomeron can be treated as a normal reggeon).

The data [10, 11, 12, 13] on the diffractive production of low-mass \( (M^2 < 2.5 \text{ GeV}^2) \) inelastic states in pp \( \rightarrow px \) show three characteristic features: (i) A very sharp t-dependence* for

* Note that \( t \) now denotes the momentum transfer in the inclusive process, i.e. the \( (\text{mass})^2 \) of \( \pi \) in \( \tau \pi N \rightarrow \pi \pi N \).

\( |t| \leq 0.15 \text{ GeV}^2 \), corresponding to exponential slopes

\( 3 \times 15 \ldots 20 \text{ GeV}^2 \), (ii) Structure in the form of a dip or break

around \( |t| \approx 0.2 \text{ GeV}^2 \), and (iii) A slope \( B = 4 \ldots 7 \text{ GeV}^2 \) in \( t \) for larger \( |t| \). At larger masses, \( M^2 > 2.5 \text{ GeV}^2 \), the slopes in \( t \) are \( B = 4 \ldots 7 \text{ GeV}^2 \) for all \( t \) and depend at most weakly on \( M \). The \( N \)-dependence of the cross-section suggests a dominance of the triple-pomeron term (ppp) [10, 14]. In fact, the \( 1/v \)-behavior required by the \( CPP \) term \( (v = M^2 - m^2 - t, m = \text{nucleon mass}) \) interpolates the data quite well down to \( M^2 = 2.5 \text{ GeV}^2 \).

These features of the data suggest that the cut-off \( \Lambda \) in the Finite Mass Sum Rule integral can be taken at \( M^2 = 2.5 \text{ GeV}^2 \);

\[ -t \frac{d \sigma}{d t} (pp \rightarrow pp) + \int dN \frac{d^2 \sigma}{dtdN} (pp \rightarrow px) = N \alpha_{p\pi p} \]  \( \text{(5)} \)

Here we separated the nucleon Born term contribution, corresponding to elastic pp scattering. It is suppressed by a factor of \( v_N = -t \) in this first-moment sum rule. The second term in (5) is then essentially the contribution from the familiar "\( N^*(1400) \)" enhancement.

The sum rule suggests that the unusual features of the \( N^*(1400) \) differential cross-section, listed above, can be understood from the requirement that the \( N^* \), together with the Born term, should build up the \( CPP \) term. Indeed, the large slope at small \( |t| \) is necessary because of the rapidly increasing Born term contribution (which is big and proportional to \(-t\)). At larger \( |t| \), on the other hand, the elastic cross-section is small and the \( N^*(1400) \) builds up most of the rhs. in (5), hence its slope in \( t \) is similar to that of the \( CPP \) term.

In order to test eq. (5) more quantitatively, we use a parametrisation of the pp \( \rightarrow pp \) cross-section obtained from a fit to data at \( P_{lab} = 175 \text{ GeV/c} \) [16];

\[ \frac{d \sigma}{d t} (pp \rightarrow pp) = 72.1 \exp (10.95 t + 2.31 t^2) \text{ mb/GeV}^2 \]

For the \( CPP \) coupling we use the value obtained at small \( |t| \) in the pd \( \rightarrow dx \) experiment [10], modified by a \( t^2 \) term in the exponent as determined by the larger \( |t| \) ISR data [14];
The $N^*(1400)$ differential cross section predicted by eq. (5)

\[ G_{N^*p} = 3.50 \exp (6.5 t + 1.5 t^2) \, \text{mb/GeV}^2 \].

We write the second term in (5) as \( v_{N^*} \frac{d \sigma_{N^*}}{dt} \), with

\[ v_{N^*} = 1 \, \text{GeV}^2 - t. \]

is shown in fig. 2 (smooth curve). The slope and normalization for
\( t \leq 0.1 \, \text{GeV}^2 \) agree within 10% with the low-mass data on $pd \to dx$
[10] **. At the position of the dip, the Born term builds up more than
\( \gg \) of the $t_{FR}$ term. Even a slight change in the normalization of
the $t_{FR}$ term (6) (by \( \pm 20\% \)) could thus transform the dip into
a break, in agreement with the t-distribution suggested by the data [11]
on the exclusive channel $pn \to p(n-p)$ (histogram).

Finally, it is interesting to note that a dip at \( t \approx -0.2 \) is
often associated with a Bessel function zero, in a picture where the
\( pp \to N^*(1400) \) reaction is assumed to be peripheral [18]. Such an
interpretation is not in contradiction with our mechanism, provided
one assumes that the sum rules, which are derived assuming the exchange
of a factorizable Regge pole, remain valid in the presence of absorption.
Also, the duality considerations above imply that at least part of
the t-channel pomeron exchange is built up by the s-channel nucleon contribu-
tion. This indicates that the standard two-component duality assumption
may have to be modified for amplitudes with external pomerons.

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FIGURE CAPTIONS

Fig. 1. An s-channel resonance contribution to the reggeon amplitude $R' B' \to R''$. The external particles 1, 2, 5 and 6 may be taken to be spinless.

Fig. 2. The differential cross-section for the low-mass ($M^2 < 2.5$) inelastic enhancement in $pp \to pX$, determined by eq. (5) as the difference between the triple-pomeron and nucleon contributions (smooth curve). The two data points are from the $pd \to dX$ experiment [10]. The histogram shows the t-distribution for the exclusive process $pn \to p(x-p)$ with $M^2(x-p) < 2.4 \text{ GeV}^2$ [11], normalized to agree with the smooth curve for small $|t|$. 
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