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AVERAGE TEMPERATURE AND MULTIPLICITIES OF $e^+e^-$ ANNIHILATION*

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Hadron temperatures of $e^+e^- \rightarrow h\bar{h}$ from PEP experiments at 29 GeV are estimated using the $P_\perp$ distributions; rather small fluctuations are found for temperatures of $\pi,K,...,\Xi$ with respect to the average $\bar{T} = 196 \pm 7$ MeV. A semi-empirical formula including the quark content of $h$ is proposed to account for multiplicities of $\pi,K,...,\Omega$ in terms of a unique temperature $\bar{T}$. The formula is further extended to charmed particles $D, F, J/\psi$ and $\Lambda_c$ without free parameters. The property $\bar{T} \sim E_{cm}^{1/4}$ holds for other experiments at DESY and CESR.

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1. Introduction

An important property of $e^+e^-$ annihilation into hadrons:

$$e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q} \rightarrow hh, \quad (1)$$

due to Chou and Yang,\(^1\) is that the total angular momentum of (1) is equal to that of the virtual photon $\gamma^*$, so that there is only one partition temperature.\(^2\) Clearly, this property, on general grounds of angular momentum conservation, holds also for the conventional temperature determined by the transverse momentum distributions of hadrons, excluding the leading particle.

As a variety of particles has been observed in $e^+e^-$ annihilation, from PEP, DESY and CESR experiments (Tables I and II) the question arises: What is the behavior of the temperature for various hadrons, $\pi, K, \ldots, \Omega$ etc. with masses different by more than one order of magnitude? How to account for their multiplicities covering almost three orders of magnitude, with only one temperature? How to specify this unique temperature?

In this paper, we present results of analyses along these lines, using currently available data\(^3\) (Sec. 2). Temperatures estimated from $P_{\perp}$ distributions are found to fluctuate very little with respect to the average, Fig. 1. To account for the hadron multiplicities of $e^+e^-$ annihilation, we consider a semi-empirical formula, Eq. (5), based on the thermodynamical model\(^4\) including the quark content of the hadron under investigation (Sec. 3). The computed multiplicities agree well with the measurements (Table I).

We will see, Sec. 4, that the average temperature $\bar{T}$ of hadrons thus obtained may be regarded as the only one temperature of $e^+e^-$, since all multiplicities computed with (5) assuming $T = \bar{T}$ turn out to be in agreement with the data as well (Tables I and II). The formula (5) is further extended to include charmed particle D, F, $J/\psi$ and $\Lambda_c$ without other free-parameters, (Sec. 5).

Remarks are made on the energy dependence of the multiplicities (Sec. 4), as well as properties of the temperature related multiplicities in general (Sec. 6).
2. Temperature Estimation

Consider the transverse momentum $P_\perp$ of a hadron of mass $m$ from $e^+e^-$ annihilation (1) with respect to the jet axis. As $P_\perp$ is Lorentz invariant, we may write its distribution in the fireball system as follows:

$$\frac{d\sigma}{dP_\perp^2} \sim \int_0^\infty e^{-E/T} dP_\parallel = m_\perp K_1\left(\frac{m_\perp}{T}\right)$$  \hfill (2)

where $T$ denotes the temperature, $m_\perp = \sqrt{P_\perp^2 + m^2}$ and $K_1$ the modified Bessel function of the second kind, which may be approximated as follows

$$\frac{d\sigma}{dP_\perp^2} \sim \sqrt{m_\perp} e^{-m_\perp/T}.$$  \hfill (3)

The validity of this distribution has been tested extensively for various particles of $e^+e^-$ annihilation, as well as for $pp$, $\pi p$, and other collisions. We therefore use (3) to estimate hadron temperatures $T_h$ of inclusive $e^+e^- \rightarrow hh$ at 29 GeV of PEP experiments for $P_\perp < 2$ GeV/c, corresponding to about three times $<P_\perp>$ of baryons of PEP experiments we are dealing with.

We begin with the Mark II data of $\Xi^-$ production. Here, we also investigate the charged $\pi$'s in the same event as $\Xi$, in order to check if the temperature is actually the same for both particles. We find by (3) $T_\pi = 193 \pm 4$ MeV. As for $\Xi$, because of low statistics, we have to use instead its $<P_\perp^2> = 0.490 \pm 0.067$ (GeV/c)$^2$ and deduce $T$ by $<P_\perp> = 2MT$ for $T \ll m$ according to (2). We get $T_\Xi = 182 \pm 26$ MeV comparable to $T_\pi$ within large errors. These two temperatures are shown in Fig. 1 by full circles; their values are listed in Table I. We will further discuss this estimate of $T_\Xi$ in Sec. 4.
Next, we consider the two sets of data of TPC Collaboration: one with $\pi^+\pi^-, K^+K^-$ and $p\bar{p}$. The values of $T_h$ fits with (3) are shown in Fig. 1 by open circles and open triangles, respectively.

Comparing the temperatures thus obtained, Table I, we find that they are practically equal within statistical errors, their average being:

$$\bar{T} = 196 \pm 7 \text{ MeV}$$

shown by the shaded band in Fig. 1. That this average $\bar{T}$ may be regarded as the "only one temperature for $e^+e^-$ annihilation" will be discussed in Sec. 4.

3. Hadron Multiplicity

Knowing the temperature $T_h$ of hadron $h$ from $e^+e^-, \pi, K$, etc., we may estimate the cross-section according to (2):

$$\sigma(m,T) \sim T^3 \left( \frac{m}{T} \right)^2 K_2 \left( \frac{m}{T} \right)$$

with appropriate statistical weights of the spin $J$ and the isospin $I$ of $h$. However, such a formula is not adequate to describe the multiplicities $n$ listed in Table I. We therefore propose to modify (4) by including valence-quark content of the hadron under consideration as follows

$$n = C_2 \frac{e^{\Gamma/T}}{(2I+1)(2J+1)} \cdot \sigma(m,T) \cdot u^{\lambda} s^{\mu} c^{\nu}$$

where $T$ stands for $T_h$, for simplicity; $\lambda, \mu$ and $\nu$ are numbers of $u/d, s$ and $c$ quarks constituent of the hadron; $\Gamma$ characterizes the enhancement in case of a resonance, $\Gamma = 0$ for stable particles; the coefficient $C_2$ refers to a pair of particle-antiparticle such as $\pi^+\pi^-, \Lambda\bar{\Lambda}$ etc.,
with the understanding that $C_2 \to C_2/2$ for $K$ mesons $K^+K^-$ or $K^{0*}\bar{K}^{0*}$ etc. to account for the associated production, and this is also the case for self-charge conjugate particles such as $\phi = s\bar{s}$, $J/\psi = c\bar{c}$ in Tables I and II.

As regards the quark parameters in (5), consider first hadrons with light quarks $u/d$ and $s$, so that we set temporarily $v = 0$, leaving the charmed particles to be discussed later (Sec. 5). Now, if we compare the multiplicity ratio $p/\pi^+ = 0.051 \pm 0.007$ and $K^+/\pi^+ = 0.14 \pm 0.06$ of the TPC data in Table I with $p/\pi = 0.094 u$ and $K^+/\pi^+ = 0.33 s/u$, according to (5), we get $u = 0.54 \pm 0.04$, and $s/u = 0.43 \pm 0.02$, consistent with the percentages of light quarks constituting all the hadrons of $e^+e^-$ annihilations taken together, abstraction being made of gluons as we are dealing with final state hadrons. Indeed, according to the group structure of hadrons, we get either $u = 2/3$ and $s/u = 1/2$ in the case of $SU(3)$ or $u = 1/2$ and $s/u = c/u = 1/2$ in the case of $SU(4)$.

Since $s/u = 1/2$ in both cases of $SU(3)$ and $SU(4)$, we therefore assume a priori this value for our formula (5) and proceed to estimate the remaining parameter $u$ and the coefficient $C_2$ using the experimental data $n$ and the temperatures $T_h$ of the hadrons in Table I, except $\Xi$, because its temperature is seemingly underestimated (cf. infra). We find:

$$u = 0.55 \pm 0.05, \quad s/u = 1/2$$

$$C_2 = (7.70 \pm 1.20) \times 10^3 \ (GeV)^{-3}$$

Note that $u$ is halfway between $2/3$ and $1/2$ as is expected from $SU(3)$ and $SU(4)$ mentioned above.

The values of multiplicities thus computed using the temperatures $T_h$ determined by the $P_\perp$ distribution are listed in Table I. They are comparable to the measurements with experimental errors.
As regards the $K_0^*$ (892) resonance, we find by (5), the parameter $\Gamma$ to be $\sim 266$ MeV, seemingly too large compared to its 51 MeV width. Further discussion on the resonance problem will be resumed in Sec. 4.

4. The Average Temperature

We have found that the hadron temperatures $T_h$ of $e^+e^-$ annihilation at $E_{cm} = 29$ GeV are consistent with the average temperature $\bar{T} = 196 \pm 7$ MeV, Fig. 1. Thus, we may regard $\bar{T}$ as the temperature of $e^+e^-$ annihilation and use it to compute the multiplicities of hadrons using (5) and $T = \bar{T}$, including also $\phi$, $f_2$, $\Lambda$, and $\Omega$, for which $P_\perp$ distributions are not available. The results are listed in Table I. Note especially the computed $n$ for $f_2$, in good agreement with the datum, assuming $\Gamma = 176$ MeV.

A comparison of the multiplicities thus obtained with the experimental data indicates that it is indeed appropriate to use the average temperature $\bar{T}$ to describe $e^+e^- \rightarrow hh$. In this regard, it is worth noting that the multiplicity of $\Xi$ here computed using $\bar{T}$ instead of $T_h$ in Table I agrees rather well with the data, suggesting an underestimation of $T_\Xi$ by $<P_\perp^2>$ in Sec. 2, due to paucity of statistics as mentioned above, Sec. 2.9.

As is well known, the temperature determined by $P_\perp$ in Eq. (2) satisfies the Stefan's law. Thus the dependence of $\bar{T}$ on the cm energy $E_{cm}$ (GeV) is as follows

$$\bar{T} = 196 (E_{cm}/29)^{1/4} \text{ MeV}$$

(7)

This property enables us to analyze ratios of multiplicities at other energies without extra information, the coefficient $C_2$ being cancelled out.

Consider first the TASSO data at 34 GeV, their ratio

$$\Xi/\Lambda = 0.084 \pm 0.026$$
is comparable to $0.085 \pm 0.031$ of PEP data. As the temperature is now $T = 204$ MeV, we find by (5) $\Xi/\Lambda = 0.12$ in agreement with the experimental data.

Next, consider their $\rho^0/\pi^+$ ratio:

$$\frac{\rho^0}{\pi^+} = \frac{0.73 \pm 0.6}{5.15 \pm 0.20} = 0.14 \pm 0.01 .$$

We deduce by (5) $\Gamma = 165$ MeV comparable to the $\rho^0$ width $153 \pm 5$ MeV. This checks our parametrization of $\Gamma$. Another case is $f_2 (1270)$, as has been mentioned above.

Turn now to $e^+e^-$ annihilation at a lower energy corresponding to $E_{cm} = 10$ and $10.5$ GeV of experiments by ARGUS collaboration\(^1\) and CLEO Collaboration,\(^2\) corresponding to $T = 150$ MeV. The results are summarized in Table II. Here again, we find excellent agreement between the computed multiplicities by (5) and the experimental data. Especially, the case of $\Omega$ is worth noting, its multiplicity being about three orders of magnitude less than $\pi$. Note that $\Omega/\Lambda$ depends on $T$ approximately as $\exp\left[-(m_\Omega - m_\Lambda)/T\right]$, thus it increases $\geq 3.4$ times at $T = 196$ MeV of the PEP energy.

Finally, we note that the temperature dependence of the cross-section (4) is $\sigma \sim T^{3/2}$, leading to $\sigma \sim E_{cm}^{3/8}$ in view of (7), and that $n_\pi \sim E_{cm}^{1/2}$ as is well known. Consequently, the energy dependence of the coefficient $C_2$ of our empirical formula (5) is expected to be $C_2 \sim E_{cm}^{-1/8}$.

5. Charmed Particles

There remains the case where the virtual photon $\gamma^*$ in $e^+e^-$ annihilation (1) gives rise to charm quarks $c\bar{c}$, instead of light quarks $u\bar{u}$ or $s\bar{s}$ as has been considered thus far. In this respect, we have to specify in the formula (5) the mass spectrum $\rho(m_q)$ of the quark-antiquark $q\bar{q}$ involved in $e^+e^-$ annihilation (1) and to retain the factor $c^\nu$ together with $u^\Lambda$ and $s^\mu$ as mentioned before, Sec. 3. As regards $\rho(m_q)$, we may use Hagedorn's model,\(^3\) namely
This amounts to replacing $C_2$ in (5) by

$$C_2 \rightarrow A_2 \equiv C_2 \left( \frac{m_u}{m_c} \right)^{3/2} \cdot e^{-(m_c-m_u)/\bar{T}}$$

where $m_u = (m_u + m_d + m_s)/3 = 0.37$ GeV is the average mass of light quarks, $m_c = 1.61$ GeV the mass of charm quark, and $\bar{T}$ the average temperature of $e^+e^-$ annihilation. For PEP experiments of $E_{cm} = 29$ GeV, $\bar{T} = 196$ MeV, we get

$$A_2/C_2 = 61.5 .$$

With this modification (9), we may analyze the charm particles using (5), with $c/u = 1/2$ as discussed in Sec. 3, and that without other free parameter.

We now apply these considerations to the ratio of $D$ meson to $K$ meson, using (4) to simplify the writing:

$$\frac{n_D}{n_K} = \frac{A_2}{C_2} \cdot \frac{1/2}{1/2 \cdot 1/2} \cdot \frac{\sigma(m_D,\bar{T})}{\sigma(m_K,\bar{T})} \cdot \frac{c}{s}$$

At PEP energy, $\bar{T} = 196$ MeV, we expect $n_D/n_K = 0.502$ in agreement with the measurements of HRS collaboration in terms of the value $R$, i.e., $e^+e^- \rightarrow \mu^+\mu^-$ cross-section $^{14a,15b}$

$$\frac{R(D^0+\bar{D}^0)}{R(K^0+\bar{K}^0)} = \frac{3.35^{+1.2}_{-1.9}}{6.15^{+0.28}_{-0.32}} = 0.50 \pm 0.29$$
with large errors due to D mesons: That this experimental value agrees with 0.50 according to (11) justifies a posteriori (10).

Turn next to the ratio of $F^+ \equiv D_s^+ = \bar{s}\bar{c}$ to $D^0 = \bar{c}\bar{u}$:

\[
\frac{D_s}{D} = \frac{1}{1/2} \cdot \frac{\sigma(m_F, T)}{\sigma(m_D, T)} \cdot \frac{s}{u},
\]

is similar to the $K/\pi$ ratio discussed above, Sec. 3, except that here, the mass difference between $D_s$ (1970) and $D$ (1865) is less important than the case of $K/\pi$. For $T = 196$ MeV, we get $D_s/D = 0.65$ compared to $0.14\pm0.05$ of the HRS Collaboration. Further investigation is needed to understand this discrepancy. Note that our value of $D_s/D$ is between 0.27 and 0.87 of the Lund Model and the Webber Model, see Table II, Ref. 3 and that $D_s/D$ increases with energy and approaches $(m_F/m_D)^{3/2}$ as $T \to \infty$ according to (5).

In this regard, consider $F^+ \to \phi\pi^+$ and $D^0 \to K^+\pi^-$ of the CLEO Collaboration at 10.5 GeV. The $F/D$ ratio is listed in Table II, together with the computed value, using $B(D_s \to K^+\pi^-) = 0.042\pm0.006$ of the recent Mark III experiment. In passing, we have also analyzed $\Lambda_{c}$ of the same CLEO experiment. The result is listed in Table II; the discrepancy between the experimental and the computed value is about twice standard errors.

Finally, we note that $J/\psi$ has been observed by both CLEO and ARGUS experiments, and that the multiplicity $n(J/\psi) < 6\times10^{-4}$ agrees with $3.4\times10^{-4}$ using (5) and (10).

### 6. Conclusion

We have analyzed the temperatures of hadrons of $e^+e^-$ annihilation (1) of PEP experiments at 29 GeV, using the $P_\perp$ distribution (2), Table I, and found the following properties:
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(1) The temperatures of $\pi, K, ..., \Xi$ differ very little from the average $T = 196 \pm 7$ MeV, Fig. 1.

(2) The multiplicities of various hadrons, from $\pi$ to $\Omega$, as well as some charmed particles can be described by the semi-empirical formulae (5) and (9) taking account of the quark content of the hadron, in terms of the average temperature $\overline{T}$ (Table I).

(3) The energy dependence of the average temperature $\overline{T}$ follows the Stefan's law (7); so that to some extent, the energy dependence of hadron multiplicity may be predicted for ARGUS and CLEO experiments at $\sim 10$ GeV (Table II).

In view of these properties, the average temperature $\overline{T}$ may be regarded as the quasi-equilibrium temperature of $e^+e^-$ annihilation (1), which is the "only-one-temperature" of $e^+e^-$ annihilation predicted by Chou and Yang. Finally, we recall that $\overline{T}$ here found describes the equipartition of energies among produced hadrons in their rest frame, as has been discussed elsewhere. 5, 6
Acknowledgements

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References and Footnotes


Table I - Hadron multiplicities $n$ and temperatures $T_h$ of $e^+e^-$ annihilation at 29 GeV. Data from Mark II, TPC and HRS Collaborations, Refs. 7, 8 and 14. Computed $n$ according to Eq. (5).

<table>
<thead>
<tr>
<th>$h^-h^-$</th>
<th>$n$ Experim.</th>
<th>$T_h$ (MeV)</th>
<th>$n$ computed</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>193±14</td>
<td></td>
<td></td>
<td>associated with $\Xi$, Ref. 7b</td>
</tr>
<tr>
<td>$\Xi\Xi$</td>
<td>0.017±0.006</td>
<td>182±26</td>
<td>0.010</td>
<td>0.020</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>10.7±0.06</td>
<td>195±12</td>
<td>10.5</td>
<td>10.3</td>
</tr>
<tr>
<td>$K^0\bar{K}^0$</td>
<td>1.35±0.13</td>
<td>201±14</td>
<td>1.83</td>
<td>1.62</td>
</tr>
<tr>
<td>$K^0\bar{K}^0$</td>
<td>0.49±0.08</td>
<td>193±33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>10.5±0.2</td>
<td>198±13</td>
<td>10.7</td>
<td>10.3</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>1.43±0.6</td>
<td>193±17</td>
<td>1.54</td>
<td>1.62</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>0.53±0.07</td>
<td>208±18</td>
<td>0.56</td>
<td>0.38</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.084±0.022</td>
<td></td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.11±0.04</td>
<td></td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>$\Lambda\bar{\Lambda}$</td>
<td>0.20±0.02</td>
<td></td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>$\Omega\bar{\Omega}$</td>
<td>0.016±0.008</td>
<td></td>
<td>0.005</td>
<td></td>
</tr>
</tbody>
</table>

(a) $\Xi$: HRS 0.016±0.006 Ref. 14a; TPC 0.020±0.009 Ref. 3.
(b) $K^0$: HRS 1.58±0.08 Ref. 14b.
(c) $\Omega$: TPC 0.027±0.017 Ref. 3.
Table II - Ratios of multiplicities of hadrons of $e^+e^-$ at 10 GeV, continuum of $\Upsilon$, data from ARGUS, Ref. 10 and CLEO, Ref. 11. Computated ratios by (5), (9).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Computed Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ARGUS</strong></td>
<td></td>
</tr>
<tr>
<td>$\Lambda/\pi = 0.32\pm0.06$</td>
<td>0.39</td>
</tr>
<tr>
<td>$[(\Sigma^++\Sigma^-)/\Lambda = 0.10\pm0.01$</td>
<td>0.12 neglect $\Gamma = 51$ MeV</td>
</tr>
<tr>
<td>$\Xi^-/\Lambda = 0.10\pm0.02$</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Omega^-/\Lambda = (5.4\pm1.8)\times10^{-3}$</td>
<td>$5.33\times10^{-3}$</td>
</tr>
<tr>
<td><strong>CLEO</strong></td>
<td></td>
</tr>
<tr>
<td>$F^+/D^0 = 0.34\pm0.10$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\Lambda_c/D^0 = 1.06\pm0.37$</td>
<td>0.44</td>
</tr>
<tr>
<td>$J/\psi &lt; 6\times10^{-4}$</td>
<td>$3.4\times10^{-4}$</td>
</tr>
</tbody>
</table>
**Figure Caption**

1. Temperature of hadrons of $e^+e^-$ annihilation at 29 GeV determined by the $P_\perp$ distribution, Eq. (3). Data of Mark II Collaboration, Ref. 7, and TPC Collaboration, Ref. 8. The shaded band shows the average $\bar{T} = 196\pm7$ MeV.
$e^+e^- \rightarrow h \bar{h} 29 \text{ GeV}$

$T = 196 \pm 7 \text{ MeV}$